Large dust particles around comets

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Abstract: A expression to estimate the maximum diameter, d_{max}, of the boulders ejected from comets is shown.. Special attention is devoted to discuss the influence of the nucleus rotation in the determination of the size of the largest particles. An increase of the d_{max} about 40% is found for comets spinning with a six hours period. Particularly, these expressions are applied to the Comet 67P/Churyumov-Gerasimenko, the current target for the Rosetta mission. Also, the equation of motion is numerically integrated. Gravitational, gas drag and radiation forces, together with the inertial forces, are included. An application to Comet 1P/Halley and 46P/Wirtanen is made. Flying times in pseudo-stable orbits are obtained and the calculated values for the total mass injected into the orbits are shown..

Keywords: Comets – Dust particles

1. INTRODUCTION

The study of the comets has always been considered very interesting by the prevailing belief that these are among the most primitive bodies in the Solar System. Comets eject dust particles when they are close enough to the sun because of the sublimation of ices on the surface of the nuclei. These particles range from microns to meters in size. Large particles can cause serious hazard to spacecraft that are close to the vicinity of cometary nuclei. When Giotto visited Comet Halley in 1986, it was hit by a large dust particle. The spacecraft angular momentum vector was shifted by 0.9 degree. The mass of the particle was estimated to be between 0.1 to 1 g, i.e., a size of 0.3-0.6 cm assuming a density of 1 g cm^{-3}. We show here (section 2) an expression to obtain the maximum diameter, d_m, that can be lifted from the nucleus surface. Particularly, we study in section 3 the influence of the nucleus rotation on the determination of d_m, which can be important if the comet is fast spinning. We make an application to the comet 64P/Churyumov-Gerasimenko, the target of the Rosetta mission (section 4).

As is known the dust particles lifted from the cometary surface fly away constituting the coma and the tail. Nevertheless, a small fraction of those particles may stay orbiting the nucleus for some time. In order to study this possibility, we numerically integrate the equation of motion and we show some results of trajectories as examples for two comets: 1P/Halley and 46P/Wirtanen (section 5). We obtain the flying times of the ejected particles for different conditions and we also discuss the orbital stability.

2. MAXIMUM DIAMETER OF LIFTED GRAINS.

We consider the gravitational cometary nucleus and Sun force, the solar radiation pressure force, the drag force and the inertial forces. As the ejection occurs only if $\mathbf{F}_R.\mathbf{n} \geq 0$ (where $\mathbf{F}_R$ is the total force acting on the particle and **n** is an unitary normal vector to the surface) we obtain (Molina et al., 2008):

$$d_{\max} = \frac{1}{\rho_d g_{eff}} \frac{3C_D \dot{m}_g v_g}{16\pi \ R^2} \qquad (1)$$

where C_D is the drag coefficient, $\dot{m}_g$ is the gas mass loss rate, v_g is gas velocity, ρ_d is the grain density, and R is the radius of the nucleus. The effective gravitational acceleration is $g_{eff} = g - \Omega^2 R \cos^2 \phi$, where g is the gravitational acceleration of the comet, Ω is the angular velocity of the rotation of the comet, and ϕ is the latitude on the surface.

In early works (Whipple, 1951, for example) C_D was taken to be constant. Henderson (1976) gave drag coefficient correlating equations for spherical particles over a wide range of flow conditions. C_D is function of the relative Mach number defined as a difference between the gas and the particle Mach number. A common used value is 2, although other values less than five could also be considered (Wallis, 1982).

The dust grain density will depend on each comet. Values between 400 to 1200 kg m^{-3} are usually adopted.

The thermal velocity, $v_{th} = (8kT/\pi m)^{1/2}$ where k is the Boltzmann constant, T is the temperature and m is the molecular weigh, is used to obtain v_g. Here we take $v_{th} = \frac{1}{2} v_g$ as adopted by Huebner and Weigert (1966) and others later on.

A good knowledge of the radius of the comet nucleus is necessary because of the R^2 dependence in the equation. However, the crucial quantity in equation 1 is $\dot{m}_g$. It is equal to $\frac{\pi F R^2}{n r^2 H}$ (Whipple, 1951), where F is the solar constant (F=0.032 cal cm^{-2} s^{-1} at 1 AU) and H=450 cal/g. Considering numerical values for a 1.8 km radius of the cometary nucleus at 1 AU from the Sun and assuming n=1, it is 7.2 10^3 kg s^{-1}. There are very active comets and comets with a low gas emission from the nucleus. Besides, the gas mass loss rate is mainly due to some places of the surface called active areas.

As $g = G4\pi\rho_n R/3$, where G is the universal gravitational constant, d_m also depends on the density of the nucleus ρ_n. As in the case of ρ_d, this quantity is often unknown, although a value near 1000 kg m^{-3} is usually assumed.

3. ROTATION

As g_{eff} must be positive, only rotation periods larger than a critical period $\tau_{crit} = (3\pi/\rho_n G)^{\frac{1}{2}}$ are possible, otherwise the comet would fly apart. Writing the nucleus density in kg m^{-3}, the cometary nucleus must be spinning with a period $\tau > 3.76.10^5 \rho_n^{-\frac{1}{2}}$. Thus, assuming a density of 1000 kg m^{-3}, the nucleus must be spinning with a period larger than 3.3 hours.

Whipple (1982) found that the average rotation period of 46 comets is 15 hr, ranging from 4.1 and 120 h. Comet rotation periods increase with increasing radius of the nucleus; small comets rotate very fast and the large ones rotate slowly. It is need to say here that we are assuming that the cometary nucleus are strengthless bodies. Nevertheless, the tensile strengths of the cometary nuclei have been estimated to be very small. The tensile strength of a human hair is 3 orders of magnitude greater than that of a cometary nucleus (Jewitt, 1992).

Recently, Molina (2010) has shown the importance of the nucleus rotation in the determining of the maximum size of the particle that can be lifted from a cometary nucleus surface. The percentual relative increase in d_m as a function of the nucleus density is shown in figure 1. For a comet rotating with a period of about 6 hours (e.g. Comet 46/P Wirtanen) the biggest particle ejected from the surface of the nucleus have a diameter 43 per cent higher than if the comet did not rotate.

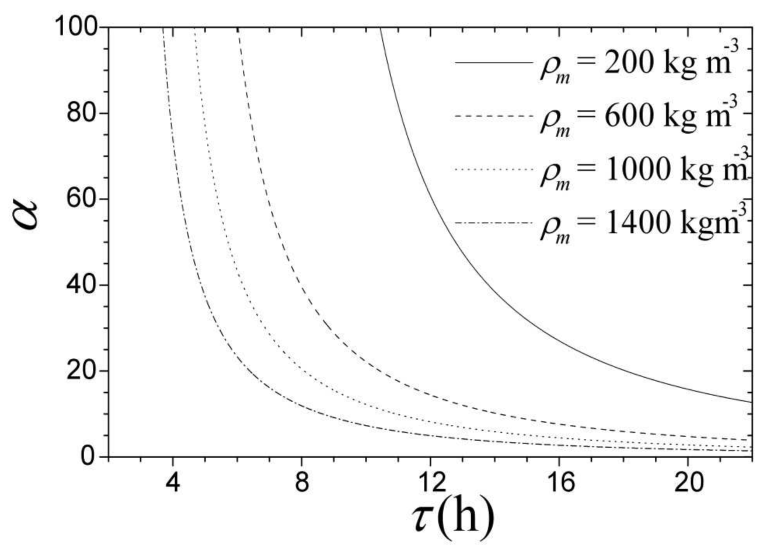

Fig.1. Percentual relative increase in d_{max} due to cometary rotation for different values of the nucleus density.

4. COMET 67P/CHURYUMOV-GERASIMENKO

Comet 67P/Churyumov-Gerasimenko (hereafter comet C-G) appeared in the list of ten selected comets accessible by spacecrafts made by Yeomans (1985). However, it was in 2003 when comet C-G was selected as the target of the Rosetta mission., after the cancellation of the launch to the comet 46P/Wirtanen. The Rosetta spacecraft will rendezvous and land upon the surface of comet C-G in late 2014. That is why it is important to illustrate d_m obtained with the present model. Although first estimations favoured a near 3 km nucleus radius, more precise studies indicate a radius less than 2 km. Lamy et al. (2007) determined the size and shape of the nucleus of this comet from several light curves. They concluded that the comet is an irregular body with an effective radius of 1.72 km and they found that the comet is rotating around a principal axis with a period of 12.4-12.7 hours. They estimated a nuclear density of 370 kg m^{-3} although densities between 100 and 500 kg m^{-3} were reported by Davidsson and Gutierrez (2005). Tubiana et al. (2008) determined a much higher rotation period of 12.7407 ± 0.0011 h and an effective radius of 2.38 ± 0.04 km. Lamy et al. (2007) reported production rate measurements by several works relative to 1982 and to 1996 apparition and also a least-squared fitted analytical curve. For the gas production rate, we consider the maximum value of the above mentioned curve near perihelium of $7 \cdot 10^{27}$ molecules per second and, according with equation 4 of that paper, it corresponds to a heliocentric distance of 1.34 AU. In the absence of published information for the grain density, ρ_d, we assume it is equal to the nucleus density, ρ_n. We assume the mean value of the interval reported by Davidson and Gutierrez (2005): 300 kg m^{-3}. Considering a temperature of 250 K, the corresponding value for the gas velocity is 270 m s^{-1}.

Following Fulle(1997) we assume an anisotropic ejection as the third power of the cosine of the zenith angle. The centrifugal force enhances the value of d_m by a factor of 28.8 %, giving, finally a value of d_m = 0.3 m.

5. ORBITS

As mentioned in the introduction, some grains lifted from the surface of the nucleus can orbit the nucleus for a long time. We will consider that the grain is in an pseudo-stable orbit when it is flying at least a time comparable to the revolution period of the comet. We study here the flying times and the orbital trajectories for two comets: 1P/Halley and 46P/Wirtanen.

5.1. Flying times

The equation of motion can be written as

$$\frac{d^2\mathbf{r_d}}{dt^2} = \left(\frac{3C_D\dot{m}_g v_g}{16\pi C_p Q_p}\beta - GM_c\right)\frac{\mathbf{r_d}}{r_d^3} + \beta M_S G\frac{\mathbf{r_c}}{r_c^3} + \frac{\mu M_S G}{r_c^3}\left(\frac{3(\mathbf{r_d}\cdot\mathbf{r_c})}{r_c^2}\mathbf{r_c} - \mathbf{r_d}\right) + $$
$$+ \mathbf{\Omega}^2 \cdot \mathbf{r_d} - (\mathbf{\Omega}\cdot\mathbf{r_d})\cdot\mathbf{\Omega} + 2(\mathbf{v_d} \times \mathbf{\Omega}) \qquad (2)$$

Here, $\mathbf{r_d}$ is the nucleus to the grain vector, C_D is the drag coefficient, $\dot{m}_g$ is the gas mass loss rate, v_g is the gas velocity, C_p= 1.19 10^{-3} kg m^{-2}, Q_p is the scattering efficiency of the grain, β is the radiation pressure and gravitational force with the Sun ratio, G is the gravitational constant, M_c is the mass of the comet, M_S is the mass of the Sun, $\mathbf{r_c}$ is the Sun to comet vector, μ = $1-\beta$, $\mathbf{\Omega}$ is the angular velocity of the rotation of the comet and $\mathbf{v_g} = \frac{d\mathbf{r_d}}{dt}$ being t the time. In the above expression we have used the same nomenclature as that employed by Fulle (1997) except for the three last terms containing $\mathbf{\Omega}$, which are due to the rotation of the cometary nucleus, and they are not present in the work by Fulle. However, they can be important as we show in this paper.

We consider initially the comet at aphelion. The comet then starts its journey toward the Sun. The particle is characterized by a given β and will be located at a certain latitude ϕ. We have integrated the equation of motion using a fourth-order-Runge-Kutta method with a time step of a hundredth of the rotational period of the nucleus. The parameter to vary are β, ϕ and the obliquity I , for which we select three values: +0º, +20º, and +40º. We define a ϕ-β grid having a resolution of 1º in latitude and 0.1 in log β . For each ϕ-β combination we compute the time at which the grain is lifted and the time from ejection till the particle either comes back to the surface or travels a distance long enough from the nucleus ($r_d \geq 10^6$ m). We use here two model: a rotational model and a co-rotational mode (as that used by Fulle (1997)).

The comet 46P was discovered by C.A. Wirtanen in January 1948 studying its proper motion in plates obtained at the Lick Observatory. Although this comet is not now the target of any space mission it is among the best observed short period comets and has been studied from the ultraviolet to radio wavelengths from ground and from space. It rotates very fast with a spinning period of 6 hour. In Figure 2 we show two maps of flying times of dust particles as a function of β and ϕ. The upper diagram shows the results for the rotational model for a value of the obliquity of 0º, and the lower diagram those ones corresponding to the co-rotational

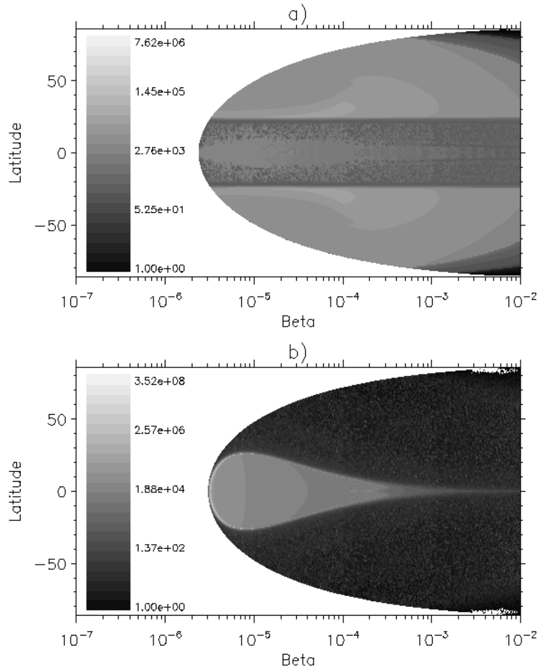

Fig.2. Flying times in seconds of particles ejected from Comet 46P/Wirtanen as a function of β and ϕ. The upper panel (a) corresponds to the rotational model for $I = 0$°, and the lower panel (b) corresponds to the co-rotational model.

model. It is easy to note the existence of large differences between the results given by the two mentioned models. For some areas, the flying times are very much longer in the case of the co-rotational model (even longer than the orbital period) than in the case of the rotational model. The existence of these areas is agree with the findings by Fulle (1997) as well as that the long orbital periods are found for latitudes equal to ±25º.

For a non-zero obliquity the ϕ-β diagram shows important changes respect to the case I =0º. The flying times found for I = 40º reach the double value that if the obliquity is 0.

We apply our models to comet 1P/Halley that, as is known, is very much larger and very much slower in rotation than the comet 46P/Wirtanen.

Figure 3 shows the flying times of particles ejected from Comet 1P/Halley. For the case of co-rotational motion (low part) a tadpole-shaped feature again appears, although now is broader in latitude that in the case of the Comet 46P/Wirtanen. Also appear here regions with flying times values longer than twice the orbital period, but now these regions are located a higher latitudes near 60º. Also the ϕ-β diagrams show a behaviour with the obliquity dependence similar to that found for the comet 46P/Wirtanen.

5.2. Trajectories

We show here examples of trajectories of particles in pseudo-stable orbits found for both mentioned comets.

Figure 4 shows the trajectory of a particle ejected from the Comet 46P/Wirtanen that came down in the surface after a full orbital period.

Figure 5 show the trajectory of a particle ejected from the Comet 1P/Halley. The behaviour is now very different. The orbit is quasi-keplerian. The reason is that Comet 1P/Halley reach very large distances from the Sun and then, the predominant force is due to the gravitation of the cometary nucleus. There are many differences between the properties of the trajectories of particles ejected from one and other of the mentioned comets. In the case of Comet 46P/Wirtanen the distance from the particle to the surface of the comet is continuously varying as well as the angular momentum of the particle. However, these changes only appear near perihelion in the case of particle ejected from the Comet 1P/Halley, because then all forces play an important role, perturbing the orbit and producing instabilities that could make the particle to escape or to be injected in other quasi-keplerian orbit with a different orbital plane.

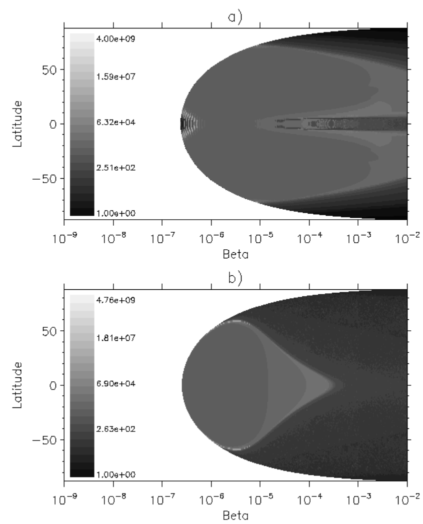

Fig.3. Flying times in seconds of particles ejected from Comet 1P/Halley as a function of β and ϕ. The upper panel (a) corresponds to the rotational model for $I = 0$º, and the lower panel (b) corresponds to the co-rotational model.

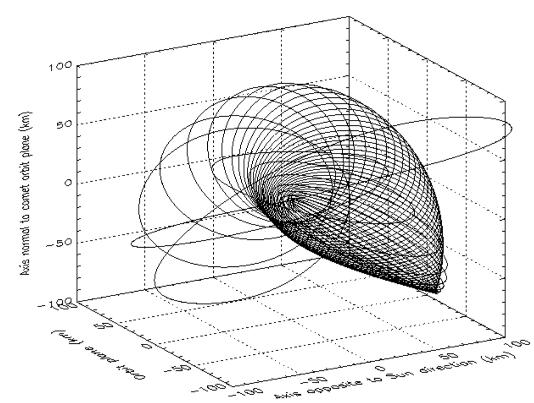

Fig.4. Orbital simulation for a particle from comet 46P/Wirtanen with $\beta=5.75\times10^{-6}$, I=20º, and ϕ=12º.

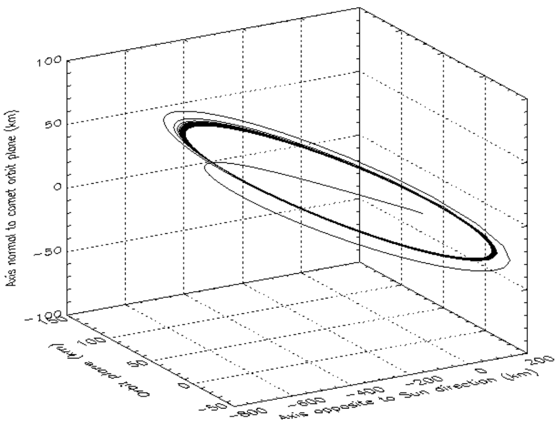

Fig.5. Orbital simulation for a particle from comet 1P/Halley with β=3.00x10^{-7}, I=0º, and ϕ =-8º.

Acknowledgments

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