1

$$\mathbf{x} = \left(\frac{3k+1}{2k}, 2, 0, \frac{1}{k} \right)^t$$

$\ell_\infty \rightarrow$

$|2| = 2$

$|0| = 0$

$$\left| \frac{3k+1}{2k} \right| = \frac{3k}{2k} + \frac{1}{2k} = \frac{3}{2} + \frac{1}{2k} \;;\; k \neq 0$$

$-1 \rightarrow 1$

$1 \rightarrow \frac{3}{2} + \frac{1}{2} = 2$

$2 \rightarrow \frac{3}{2} + \frac{1}{2 \cdot 2} \longrightarrow \frac{3}{2} + \frac{1}{4} = \frac{6}{4} + \frac{1}{4} = \frac{7}{4}$

$\rightarrow$ máx : 2

$\left| \frac{1}{k} \right| \Rightarrow \quad k \rightarrow \infty \Rightarrow \frac{1}{k} \rightarrow 0$

$\max(1/k) = 1$

$$\|\mathbf{x}\|_\infty = 2$$

②

x : solución real.

$\tilde{x}$: solución aproximada.

$\|x - \tilde{x}\|_\infty$

$$\left\| \begin{pmatrix} 0 \\ \pi \\ \pi \end{pmatrix} - \begin{pmatrix} 0'1 \\ 3'18 \\ 3'10 \end{pmatrix} \right\|_\infty = \left\| \begin{matrix} -0'1 \\ \pi - 3'18 \\ \pi - 3'10 \end{matrix} \right\|_\infty = \begin{matrix} \longrightarrow 0'1 \\ \longrightarrow |\pi - 3'18| \sim |3'14 - 3'18| = |-0'04| = 0'04 \\ |\pi - 3'10| \sim |3'14 - 3'10| = 0'04 \end{matrix}$$

$\|x - \tilde{x}\|_\infty = 0'1$

$\|A\tilde{x} - \mathbf{b}\|_\infty$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2\pi \\ 0 \\ \pi \end{pmatrix}$$

$$\left\| \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0'1 \\ 3'18 \\ 3'10 \end{pmatrix} - \begin{pmatrix} 2\pi \\ 0 \\ \pi \end{pmatrix} \right\|_\infty$$

$$\left\| \begin{pmatrix} 6'38 \\ -0'02 \\ 3'20 \end{pmatrix} - \begin{pmatrix} 2\pi \\ 0 \\ \pi \end{pmatrix} \right\|_\infty \longrightarrow \left\| \begin{pmatrix} 6'38 & - & 2\pi \\ -0'02 & & \\ 3'20 & - & \pi \end{pmatrix} \right\|_\infty \sim \left\| \begin{pmatrix} 0'1 \\ -0'02 \\ 0'06 \end{pmatrix} \right\|_\infty \begin{matrix} \longrightarrow 0'1 \\ \longrightarrow 0'02 \\ \longrightarrow 0'06 \end{matrix}$$

$\|A\tilde{x} - \mathbf{b}\|_\infty = 6'38 - 2\pi$

3

$\|A\|_2 = \sqrt{\rho(A^t A)}$, calcola $\|A\|_2$

$$A^t = \begin{vmatrix} -1 & 0 & 0 \\ 2 & 3 & 0 \\ 0 & 4 & 7 \end{vmatrix}$$

$$\sqrt{\rho\left(\begin{vmatrix} -1 & 0 & 0 \\ 2 & 3 & 0 \\ 0 & 4 & 7 \end{vmatrix} \cdot \begin{vmatrix} -1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{vmatrix}\right)} =$$

$$\sqrt{\rho\left(\begin{vmatrix} 1 & -2 & 0 \\ -2 & 13 & 12 \\ 0 & 12 & 65 \end{vmatrix}\right)} =$$

$$\det\left(\begin{vmatrix} 1 & -2 & 0 \\ -2 & 13 & 12 \\ 0 & 12 & 65 \end{vmatrix} - \lambda I\right) = 0$$

$$\det\left(\begin{vmatrix} 1-\lambda & -2 & 0 \\ -2 & 13-\lambda & 12 \\ 0 & 12 & 65-\lambda \end{vmatrix}\right) = 0$$

$$\begin{matrix} 1-\lambda & -2 & 0 \\ -2 & 13-\lambda & 12 \end{matrix}$$

$$(1-\lambda)(13-\lambda)(65-\lambda) - (1-\lambda)12^2 - (-2)^2(65-\lambda) = 0$$

$$((1-\lambda)(13-\lambda) - 4)(65-\lambda) - 144(1-\lambda)$$

$$((13 - \lambda - 13\lambda + \lambda^2) - 4)(65-\lambda) - 144(1-\lambda)$$

$$(9 - 14\lambda + \lambda^2)(65-\lambda) - 144(1-\lambda)$$

$$585 - 9\lambda - 910\lambda + 14\lambda^2 + 65\lambda^2 - \lambda^3 - 144 + 144\lambda$$

$$441 - 775\lambda + 79\lambda^2 - \lambda^3 = 0$$

$\lambda_1 \cong 67'6 \longrightarrow$ max: $67'6 \longrightarrow \|A\|_2 \cong \sqrt{67'6} \sim 8'22$

$\lambda_2 \cong 10'8$

$\lambda_3 \cong 0'61$

④

$$|C| \text{ y } C^{-1} \text{ si } C = A^t \cdot A$$

$$A = \begin{vmatrix} -1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{vmatrix}$$

$$C = A^t \cdot A = \begin{vmatrix} -1 & 0 & 0 \\ 2 & 3 & 0 \\ 0 & 4 & 7 \end{vmatrix} \cdot \begin{vmatrix} -1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 0 \\ -2 & 13 & 12 \\ 0 & 12 & 65 \end{vmatrix}$$

$$C^{-1} := \begin{vmatrix} 1 & -2 & 0 \\ -2 & 13 & 12 \\ 0 & 12 & 65 \end{vmatrix} \; f_2 \to f_2 + 2f_1 \; \begin{vmatrix} 1 & -2 & 0 \\ 0 & 9 & 12 \\ 0 & 12 & 65 \end{vmatrix} \; f_2 \to \frac{1}{3} f_2 \; \begin{vmatrix} 1 & -2 & 0 \\ 0 & 3 & 4 \\ 0 & 12 & 65 \end{vmatrix} \; f_3 \to f_3 - 4f_2$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \qquad \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \qquad \begin{vmatrix} 1 & 0 & 0 \\ 2/3 & 1/3 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & -2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 49 \end{vmatrix} \; f_3 \to \frac{1}{49} f_3 \; \begin{vmatrix} 1 & -2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 1 \end{vmatrix} \; f_2 \to f_2 - 4f_3 \; \begin{vmatrix} 1 & -2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{vmatrix} \; f_2 \to \frac{1}{3} f_2$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 2/3 & 1/3 & 0 \\ -8/3 & -4/3 & 1 \end{vmatrix} \qquad \begin{vmatrix} 1 & 0 & 0 \\ 2/3 & 1/3 & 0 \\ -8/147 & -4/147 & 1/49 \end{vmatrix} \qquad \begin{vmatrix} 1 & 0 & 0 \\ 130/147 & 65/147 & -4/49 \\ -8/147 & -4/147 & 1/49 \end{vmatrix}$$

$$\begin{vmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \; f_1 \to f_1 + 2f_2 \; \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 0 \\ \frac{130}{441} & \frac{65}{441} & \frac{-4}{147} \\ \frac{-8}{147} & \frac{-4}{147} & \frac{1}{49} \end{vmatrix} \qquad \begin{vmatrix} \frac{701}{441} & \frac{130}{441} & \frac{-8}{147} \\ \frac{130}{441} & \frac{65}{441} & \frac{-4}{147} \\ \frac{-8}{147} & \frac{-4}{147} & \frac{1}{49} \end{vmatrix} = \boxed{C^{-1}}$$

$$|C| \to \begin{vmatrix} 1 & -2 & 0 \\ -2 & 13 & 12 \\ 0 & 12 & 65 \end{vmatrix} \to \left.\begin{array}{l} + \; 13 \cdot 65 \\ \\ - \; 12^2 + 65 \cdot 4 \end{array}\right\} \boxed{441 = |C|}$$

$$\begin{matrix} 1 & -2 & 0 \\ -2 & 13 & 12 \end{matrix}$$

(5)

Número 88008 : A

Número 58303 : B

Número 31938 : C

$$\left.\begin{array}{l} A+B+C = 5 \\ 2A + 5\cdot B + 2C = 13 \\ A = B \end{array}\right\} \quad \left.\begin{array}{l} 2A + C = 5 \\ 7A + 2C = 13 \end{array}\right\} \longrightarrow$$

$$\left(\begin{array}{cc|c} 2 & 1 & 5 \\ 7 & 2 & 13 \end{array}\right) f_1 \to 2f_1$$

$$\left(\begin{array}{cc|c} 4 & 2 & 10 \\ 7 & 2 & 13 \end{array}\right) f_2 \to f_2 - f_1$$

$$\left(\begin{array}{cc|c} 4 & 2 & 10 \\ 3 & 0 & 3 \end{array}\right)$$

$$\boxed{A = 1} \boxed{= B}$$

$$4A + 2C = 10$$

$$2A + C = 5$$

$$2 + C = 5$$

$$\boxed{C = 3}$$

6

4 Promociones : P_1 , P_2 , P_3 , P_4

$$\left.\begin{array}{l} P_1+P_2+P_3+P_4=220 \\ P_3=40+P_4 \\ P_1+P_2=50+P_3 \\ P_1+2P_2=150+P_4 \end{array}\right\} \quad \left.\begin{array}{l} P_1+P_2+P_3+P_4=220 \\ P_3-P_4=40 \\ P_1+P_2-P_3=50 \\ P_1+2P_2-P_4=180 \end{array}\right\} \quad \left\{\begin{array}{l} P_1+P_2+P_3+P_4=220 \\ P_1+P_2-P_3=50 \\ P_3-P_4=40 \\ P_1+2P_2-P_4=180 \end{array}\right.$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 1 & 1 & -1 & 0 & 50 \\ 0 & 0 & 1 & -1 & 40 \\ 1 & 2 & 0 & -1 & 180 \end{array}\right) \begin{array}{l} f_2 \to f_2 - f_1 \\ \\ f_4 \to f_4 - f_1 \end{array} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 0 & 0 & -2 & -1 & -170 \\ 0 & 0 & 1 & -1 & 40 \\ 0 & 1 & -1 & -2 & -70 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 0 & 1 & -1 & -2 & -70 \\ 0 & 0 & 1 & -1 & 40 \\ 0 & 0 & -2 & -1 & -170 \end{array}\right) f_4 \to f_4 + 2f_3 \quad \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 0 & 1 & -1 & -2 & -70 \\ 0 & 0 & 1 & -1 & 40 \\ 0 & 0 & 0 & -3 & -90 \end{array}\right) f_4 \to -\frac{1}{3} f_4$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 0 & 1 & -1 & -2 & -70 \\ 0 & 0 & 1 & -1 & 40 \\ 0 & 0 & 0 & 1 & 30 \end{array}\right) \begin{array}{l} f_1 \to f_1 - f_4 \\ f_2 \to f_2 + 2f_4 \\ f_3 \to f_3 + f_4 \end{array} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 190 \\ 0 & 1 & -1 & 0 & -10 \\ 0 & 0 & 1 & 0 & 70 \\ 0 & 0 & 0 & 1 & 30 \end{array}\right) \begin{array}{l} f_1 \to f_1 - f_3 \\ f_2 \to f_2 + f_3 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 120 \\ 0 & 1 & 0 & 0 & 60 \\ 0 & 0 & 1 & 0 & 70 \\ 0 & 0 & 0 & 1 & 30 \end{array}\right) f_1 \to f_1 - f_2 \quad \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 60 \\ 0 & 1 & 0 & 0 & 60 \\ 0 & 0 & 1 & 0 & 70 \\ 0 & 0 & 0 & 1 & 30 \end{array}\right) \begin{array}{l} P_1 = 60 \\ P_2 = 60 \\ P_3 = 70 \\ P_4 = 30 \end{array}$$

$P_1 + P_2 + P_3 + P_4 \stackrel{?}{=} 220 \longrightarrow 60+60+70+30 \stackrel{?}{=} 220$ ✓

$P_3 \stackrel{?}{=} 40 + P_4 \longrightarrow 70 \stackrel{?}{=} 40 + 30$ ✓

$P_1 + P_2 \stackrel{?}{=} 50 + P_3 \longrightarrow 60 + 60 \stackrel{?}{=} 50 + 70$ ✓

$P_1 + 2P_2 \stackrel{?}{=} 150 + P_4 \longrightarrow 60 + 2 \cdot 60 \stackrel{?}{=} 150 + 30$ ✓

⑦

$$\begin{cases} 2x - y + 3z = 4 \\ 3x + 2y - z = 3 \\ x + 3y - 4z = -1 \end{cases}$$

$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 4 \\ 3 & 2 & -1 & 3 \\ 1 & 3 & -4 & -1 \end{array}\right)$$

a)

$$A^* = \left(\begin{array}{ccc|c} 2 & -1 & 3 & 4 \\ 3 & 2 & -1 & 3 \\ 1 & 3 & -4 & -1 \end{array}\right) \xrightarrow[f_3 \to f_3 - \frac{1}{2}f_1]{f_2 \to f_2 - \frac{3}{2}f_1} \left(\begin{array}{ccc|c} 2 & -1 & 3 & 4 \\ 0 & 7/2 & -11/2 & -3 \\ 0 & 7/2 & -11/2 & -3 \end{array}\right) f_3 \to f_3 - f_2$$

$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 4 \\ 0 & 7/2 & -11/2 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right) \longrightarrow \text{Rg}\,A^* = \text{Rg}\,A \neq \text{nº incognitas.}$$

b)

$$\left.\begin{array}{r} 2x - y + 3z = 4 \\ 7/2\,y - 11/2\,z = -3 \\ z = \lambda \end{array}\right\} \quad \left.\begin{array}{r} 2x - y + 3z = 4 \\ 7y - 11z = -6 \\ z = \lambda \end{array}\right\} \quad \begin{array}{l} 2x - y + 3\lambda = 4 \\ 7y - 11\lambda = -6 \end{array}$$

$$y = -\frac{6}{7} + \frac{11}{7}\lambda$$

$$2x + \frac{6}{7} - \frac{11}{7}\lambda + 3\lambda = 4$$

$$2x = \frac{22}{7} + \frac{10}{7}\lambda$$

$$x = \frac{11}{7} + \frac{5}{7}\lambda$$

Solución:

$$\begin{cases} x = \frac{11}{7} + \frac{5}{7}\lambda \\ y = -\frac{6}{7} + \frac{11}{7}\lambda \\ z = \lambda \end{cases}$$

8

$$\begin{aligned} x + y + z + t &= 220 \\ x - y \quad\quad + 2t &= 60 \\ z - 2t &= 10 \\ 3x \quad + z \quad\quad &= 250 \end{aligned}$$

a)

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 1 & -1 & 0 & 2 & 60 \\ 0 & 0 & 1 & -2 & 10 \\ 3 & 0 & 1 & 0 & 250 \end{array}\right) \quad f_2 \to f_2 - f_1$$

$1 - (1) = 0$ $\qquad 60 - (220) = -160$

$-1 - (1) = -2$

$0 - (1) = -1$

$2 - (1) = 1$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 0 & -2 & -1 & 1 & -160 \\ 0 & 0 & 1 & -2 & 10 \\ 3 & 0 & 1 & 0 & 250 \end{array}\right) \quad f_4 \to f_4 - 3f_1$$

$3 - 3(1) = 0$

$0 - 3(1) = -3$

$1 - 3(1) = -2$

$0 - 3(1) = -3$

$250 - 3(220) = -410$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 0 & -2 & -1 & 1 & -160 \\ 0 & 0 & 1 & -2 & 10 \\ 0 & -3 & -2 & -3 & -410 \end{array}\right) \quad f_2 \to -\frac{1}{2} f_2$$

$-(0)/2 = 0$ $\qquad -(-160)/2 = 80$

$-(-2)/2 = 1$

$-(-1)/2 = 1/2$

$-(1)/2 = -1/2$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 0 & 1 & 1/2 & -1/2 & 80 \\ 0 & 0 & 1 & -2 & 10 \\ 0 & -3 & -2 & -3 & -410 \end{array}\right) \quad f_4 \to f_4 + 3f_2$$

$-3 + 3(1) = 0$

$-2 + 3(1/2) = -2 + 3/2 = \frac{-4+3}{2} = -1/2$

$-3 + 3(-1/2) = -3 - 3/2 = \frac{-6-3}{2} = -9/2$

$-410 + 3(80) = -410 + 240 = -170$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 0 & 1 & 1/2 & -1/2 & 80 \\ 0 & 0 & 1 & -2 & 10 \\ 0 & 0 & -1/2 & -9/2 & -170 \end{array}\right) \quad f_4 \to -2f_4$$

$-2\left(-\frac{1}{2}\right) = \boxed{1}$

$-2(-9/2) = \boxed{9}$

$-2(-170) = \boxed{340}$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 0 & 1 & 1/2 & -1/2 & 80 \\ 0 & 0 & 1 & -2 & 10 \\ 0 & 0 & 1 & 9 & 340 \end{array}\right) \quad f_4 \to f_4 - f_3$$

$1 - 1 = \boxed{0}$

$9 - (-2) = \boxed{11}$

$340 - 10 = \boxed{330}$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 0 & 1 & 1/2 & -1/2 & 80 \\ 0 & 0 & 1 & -2 & 10 \\ 0 & 0 & 0 & 11 & 330 \end{array}\right) f_4 \to \frac{1}{11} f_4$$

$\frac{1}{11}(11) = 1$

$\frac{1}{11} \cdot 330 = 30$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 0 & 1 & 1/2 & -1/2 & 80 \\ 0 & 0 & 1 & -2 & 10 \\ 0 & 0 & 0 & 1 & 30 \end{array}\right) f_2 \to 2f_2$$

$2(1) = 2$

$2(1/2) = 1$

$2(-1/2) = -1$

$2(80) = 160$

Es compatible determinado ya que

Rango A = Rango A^* = nº incognitas

b)

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 0 & 2 & 1 & -1 & 160 \\ 0 & 0 & 1 & -2 & 10 \\ 0 & 0 & 0 & 1 & 30 \end{array}\right) f_3 \to f_3 + 2f_4$$

$1 + 2(0) = 1$

$-2 + 2(1) = 0$

$10 + 2(30) = 70$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 220 \\ 0 & 2 & 1 & -1 & 160 \\ 0 & 0 & 1 & -2 & 10 \\ 0 & 0 & 0 & 1 & 30 \end{array}\right)$$

$f_1 \to f_1 - f_4$ $\quad 220 - 30 = 190$

$f_2 \to f_2 + f_4$ $\quad 160 + 30 = 190$

$f_3 \to f_3 + 2f_4$ $\quad 10 + 60 = 70$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 190 \\ 0 & 2 & 1 & 0 & 190 \\ 0 & 0 & 1 & 0 & 70 \\ 0 & 0 & 0 & 1 & 30 \end{array}\right)$$

$f_1 \to f_1 - f_3$ $\quad 190 - 70 = 120$

$f_2 \to (f_2 - f_3)\frac{1}{2}$ $\quad (190-70)\,1/2 = 60$

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 120 \\ 0 & 1 & 0 & 0 & 60 \\ 0 & 0 & 1 & 0 & 70 \\ 0 & 0 & 0 & 1 & 30 \end{array}\right)$$

$f_1 \to f_1 - f_2$ $\quad 120 - 60 = 60$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 60 \\ 0 & 1 & 0 & 0 & 60 \\ 0 & 0 & 1 & 0 & 70 \\ 0 & 0 & 0 & 1 & 30 \end{array}\right) \begin{array}{l} x = 60 \\ y = 60 \\ z = 70 \\ t = 30 \end{array}$$

c) $x + y + z + t \stackrel{?}{=} 220 \rightarrow 60 + 60 + 70 + 30 \stackrel{?}{=} 220$ ✓

$x - y + 2t \stackrel{?}{=} 60 \longrightarrow 60 - 60 + 2 \cdot 30 \stackrel{?}{=} 60$ ✓

$z - 2t \stackrel{?}{=} 10 \longrightarrow 70 - 2 \cdot 30 \stackrel{?}{=} 10$ ✓

$3x + z \stackrel{?}{=} 250 \longrightarrow 3 \cdot 60 + 70 \stackrel{?}{=} 250$ ✓