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EE 301
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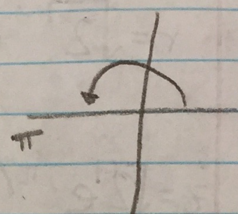
Homework #1

1)

a) $\frac{1}{2} e^{j\pi}$, $e^{iy} = \cos y + i\sin y$

$z = re^{i\theta} = \frac{1}{2} e^{j\pi} = \frac{1}{2}\left(\underset{-1}{\cos\pi} + \underset{0}{\cancel{i\sin\pi}}\right)$



$= -\frac{1}{2}$

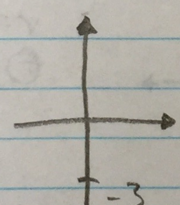
b) $z = \sqrt{2}\, e^{-j\,9\pi/4} = \sqrt{2}\left(\cos(-9\pi/4) + i\sin(-9\pi/4)\right)$

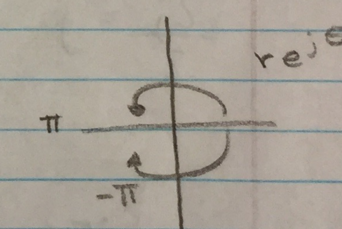
$= \sqrt{2}\left(\cos(9\pi/4) - i\sin(9\pi/4)\right)$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}}{\sqrt{2}} - i\frac{\sqrt{2}}{\sqrt{2}} = 1 - i$

2)

a) $-3j$

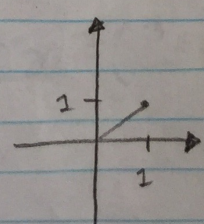




$r = 3$ $\quad \cancel{\cos\left(\frac{-\pi}{2}\right)} + i\sin\left(\frac{-\pi}{2}\right) = -i$

$z = 3e^{-j\pi/2}$

b) $1 + j$



$r = \sqrt{2}$, $\theta = \frac{\pi}{4}$

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

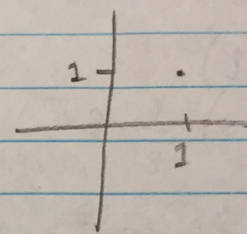
$z = \sqrt{2}\, e^{j\pi/4}$

$z = \sqrt{2}\, e^{j\pi/4}$

1-1

c) $j(1-j)$, where $j^2=-1$ $\quad r=\sqrt{1^2+1^2}=\sqrt{2}$

$j-j^2=j-(-1)=j+1=1+j$



$r=\sqrt{2}$ $\quad \Theta=\frac{\pi}{4}$, $z=\sqrt{2}\,e^{j\pi/4}$

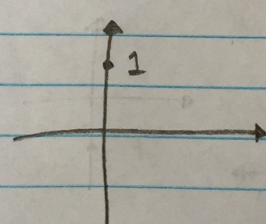
$z=\sqrt{2}\,e^{j\pi/4}$

d) $\frac{1+j}{1-j}\cdot\frac{1+j}{1+j}=\frac{(1+j)(1+j)}{(1-j)(1+j)}$

$(1+j)(1+j)=1+j+j+j^2=1+2j+(-1)=2j$

$(1-j)(1+j)=1+\cancel{j}-\cancel{j}-j^2=1-(-1)=1+1=2$

$\frac{2j}{2}=j$



$r=1$
$\Theta=\frac{\pi}{2}$

$z=e^{j\pi/2}$

3)
a) $zz^*=r^2$, $\quad z=re^{j\theta}$, $\quad z^*=re^{-j\theta}$

$zz^*=(re^{j\theta})(re^{-j\theta})=\frac{(r\cancel{e^{j\theta}})(r)}{\cancel{e^{j\theta}}}=r^2$

b) $\frac{z}{z^*} = \frac{re^{j\theta}}{re^{-j\theta}} = \frac{\cancel{r}}{\cancel{r}}(e^{j\theta})(e^{j\theta}) = e^{2j\theta}$

c) $Re\{z\} = \frac{1}{2}(z + z^*)$

$\frac{1}{2}(z + z^*) = \frac{1}{2}(x + \cancel{jy} + x - \cancel{jy}) = \frac{2x}{2} = x$

and $Re\{z\}$ is x. Therefore, $Re\{z\} = \frac{1}{2}(z + z^*)$

d) $Im\{z\} = \frac{1}{2j}(z - z^*)$

$\frac{1}{2j}(z - z^*) = \frac{1}{2j}(\cancel{x} + jy - \cancel{x} + jy) = \frac{\cancel{2j}y}{\cancel{2j}} = y$

and $Im\{z\}$ is y. Therefore, $Im\{z\} = \frac{1}{2j}(z - z^*)$

4) $e^{j\theta} = \cos\theta + j\sin\theta$

a) $\frac{1}{2}(e^{j\theta} + e^{-j\theta}) = \frac{1}{2}(\cos\theta + \cancel{j\sin\theta} + \cos\theta - \cancel{j\sin\theta})$

$= \frac{\cancel{2}\cos\theta}{\cancel{2}} = \cos\theta$

Therefore, $\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$

2-1

b) $\frac{1}{2j}(e^{j\theta} - e^{-j\theta}) = \frac{1}{2j}(\cancel{\cos}\theta + j\sin\theta - \cancel{\cos}\theta + j\sin\theta)$

$= \frac{1}{\cancel{2}j}(\cancel{2}j\sin\theta) = \sin\theta$

Therefore, $\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$

c) $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$, where
$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \frac{1}{2}(1 + \cos^2\theta - \sin^2\theta)$, where
$1 = \cos^2\theta + \sin^2\theta$

$= \frac{1}{2}(\cos^2\theta + \cos^2\theta + \cancel{\sin^2\theta} - \cancel{\sin^2\theta})$

$= \frac{\cancel{2}\cos^2\theta}{\cancel{2}} = \cos^2\theta$

Therefore, $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

d) $\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$\sin(\theta + \phi) = \mathrm{Im}\{e^{i(\theta+\phi)}\} = \mathrm{Im}\{e^{i\theta}e^{i\phi}\}$

$= \mathrm{Im}\{(\cos\theta + i\sin\theta)(\cos\phi + i\sin\phi)\}$

$= \mathrm{Im}\{\cos\theta\cos\phi + i\cos\theta\sin\phi + i\sin\theta\cos\phi$
$- \sin\theta\sin\phi\}$

$= \sin\theta\cos\phi + \cos\theta\sin\phi$

because $\sin\theta = \mathrm{Im}\{e^{i\theta}\} = \mathrm{Im}\{\cos\theta + i\sin\theta)$

therefore, $\sin(\theta + \phi) = \mathrm{Im}\{e^{i(\theta+\phi)}\}$

As a result,

$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$, as desired