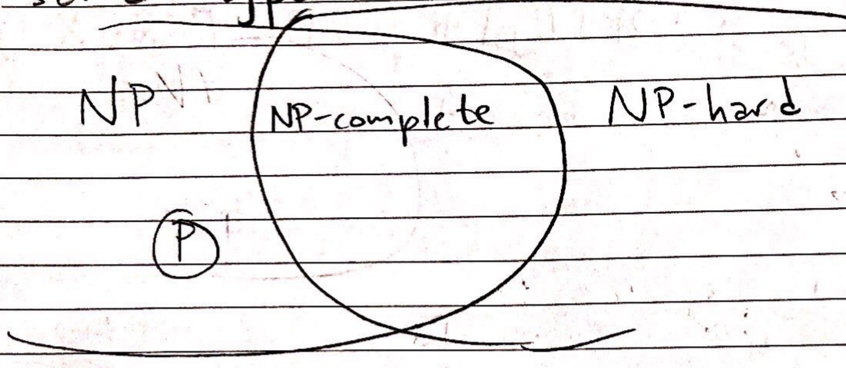
CSCI 570 - HW 11

1) True. The reason for this to be true is because NP-complete problems are also NP-hard. In addition, only problems of the same type can be reduced to each other.



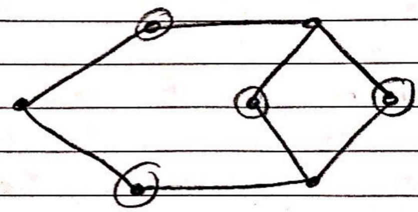
2) True. This is true because NP-hard cannot be reduced to NP-complete. Also, NP cannot be reduced to NP-complete. This is due to the fact that they will not satisfy the requirements of the other classification. In example, NP-hard problems, which are not NP-complete, do not satisfy the NP requirements. Also, NP problems, which are not NP-complete, do not satisfy the NP-hard problems. Therefore, NP-complete problems can only be reduced from another NP-complete problem. For example, B can only be reduced from A, where both B and A are NP-complete, in $A \leq_p B$.

3) False. This is false because there are problems in NP-hard that do not belong to NP-complete. Therefore, there are problems in NP-hard problems that do not belong to NP. As a result, not every NP-hard problem can be solved in polynomial time.

4) Clique reduction from the Independent set.

$G \rightarrow \boxed{C} \rightarrow 4$

$G \rightarrow \boxed{IS} \rightarrow 4$



From the original graph G, get its complement graph $\bar{G}$. The complement graph will fill all the missing edges and remove all the edges that were previously there. Then, I will use the blackbox of Clique on the complement graph $\bar{G}$ to compute $\bar{G}$'s largest Clique, which the magnitude of the Clique will be equal to the magnitude of the largest Independent Set in the original graph G.

Half-Clique reduction from Clique.

From the original graph G, add $2m - |V|$ nodes to the original graph to create $\bar{G}$. I will not add edges from the recently added nodes to the original graph G. With this methodology of modifying the graph, the Half-Clique blackbox will yield the same result as the Clique blackbox.

5) 1.- Prove that $X \in NP$.
2.- Choose a problem Y that is known to be NP-complete.
3.- Prove that $Y \leq_p X$.

Big-Ham-Cycle

Certificate: get an ordered list of vertices of the Hamiltonian Cycle which should contain a weight of the edges that is at least half of the total sum of the weights of all the edges in the graph.

Certifier: - check that all vertices are visited only once.
- there has to be an edge between every pair of adjacent vertices in the ordered list.
- there has to be an edge between the last vertex and the first vertex.
- check that the edges from the Hamiltonian Cycle add up to at least half of the total sum of all the edges in the graph.

Reduction: Grab an edge an assume it belongs to the Hamiltonian Cycle. Add a weight of $|E|$ to the selected edge and add a value of 1 to the rest of the edges. That way if that edge belongs to the

Hamitonian Cycle, the value of the Hamiltonian Cycle is going to be larger than $\frac{E}{E+(E-1)}$, which is approximately larger than one half.