University of Southern California Ming Hsieh Department of Electrical Engineering EE 348L - Electronic Circuits Spring 2016

Homework 2 Solutions

Done

2.2 Refer to Fig. P2.2.

$$v_{+} = v_{I} \times \frac{1 \text{ k}\Omega}{1 \text{ k}\Omega + 1 \text{ M}\Omega} = v_{I} \frac{1}{1001}$$

$$v_{O} = Av_{+} = Av_{I} \frac{1}{1001}$$

$$A = 1001 \frac{v_{O}}{v_{I}}$$

$$= 1001 \times \frac{4}{1}$$

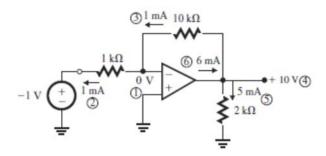
$$A = 4004 \text{ V/V}$$

2.8

Circuit	$v_o/v_i(V/V)$	$R_{\rm in}$ (k Ω)
a	$\frac{-100}{20} = -5$	20
b	-5	20
c	-5	20
d	-5	20

Note that in circuit (b) the $20\text{-}k\Omega$ load resistance has no effect on the closed-loop gain because of the zero output resistance of the ideal op amp. In circuit (c), no current flows in the $20\text{-}k\Omega$ resistor connected between the negative input terminal and ground (because of the virtual ground at the inverting input terminal). In circuit (d), zero current flows in the $20\text{-}k\Omega$ resistor connected in series with the positive input terminal.

2.16 The circled numbers indicate the order of the analysis steps. The additional current supplied by the op amp comes from the power supplies (not shown).



2.25 Refer to Fig. P2.25.

$$\frac{V_o}{V_i} = \frac{-R_2/R_1'}{1 + \frac{1 + R_2/R_1'}{A}} \tag{1}$$

where

$$R'_1 = R_1 || R_c$$

Thus,

$$\frac{1}{R_1'} = \frac{1}{R_1} + \frac{1}{R_c}$$

Substituting in Eq. (1),

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1} \frac{1 + \frac{R_1}{R_c}}{1 + \frac{1 + \frac{R_2}{R_1} + \frac{R_2}{R_c}}{A}}$$

To make $\frac{V_o}{V_i} = -\frac{R_2}{R_1}$, we have to make

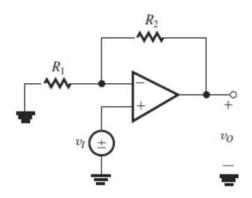
$$\frac{R_1}{R_c} = \frac{1 + \frac{R_2}{R_1} + \frac{R_2}{R_c}}{A}$$

That is,

$$A\frac{R_1}{R_c} = 1 + G + G\frac{R_1}{R_c}$$

which yields

$$\frac{R_c}{R_1} = \frac{A - G}{1 + G}$$
 Q.E.D



$$\frac{v_O}{v_I} = 1 + \frac{R_2}{R_1}$$

(a)
$$\frac{v_0}{v_1} = 1 = 1 + \frac{R_2}{R_1}$$

Set $R_2 = 0 \Omega$ and eliminate R_1

(b)
$$\frac{v_0}{v_1} = 2 = 1 + \frac{R_2}{R_1}$$

$$\frac{R_2}{R_1} = 1$$
; set $R_1 = R_2 = 10 \text{ k}\Omega$

(c)
$$\frac{v_0}{v_1} = 21 = 1 + \frac{R_2}{R_1}$$

$$\frac{R_2}{R_1} = 20$$
; set $R_1 = 10 \text{ k}\Omega$, $R_2 = 200 \text{ k}\Omega$

(d)
$$\frac{v_0}{v_1} = 100 = 1 + \frac{R_2}{R_1}$$

$$\frac{R_2}{R_1}$$
 = 99; set R_1 = 10 k Ω , R_2 = 990 k Ω

2.47 Refer to the circuit in Fig. P2.47:

(a) Using superposition, we first set $v_{P_1} = v_{P_2}, \ldots, = 0$. The output voltage that results in response to $v_{N_1}, v_{N_2}, \ldots, v_{N_R}$ is

$$v_{ON} = -\left[\frac{R_f}{R_{N1}}v_{N1} + \frac{R_f}{R_{N2}}v_{N2} + \dots + \frac{R_f}{R_{Nn}}v_{Nn}\right]$$

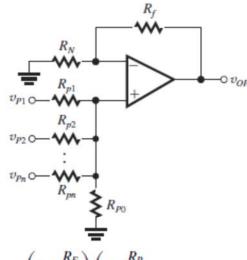
Then we set $v_{N1} = v_{N2} = \cdots = 0$, then:

$$R_N = R_{N1} \parallel R_{N2} \parallel R_{N3} \parallel \cdots \parallel R_{Nn}$$

The circuit simplifies to that shown below.

$$v_{OP} = \left(1 + \frac{R_f}{R_N}\right) \times \left(v_{P1} - \frac{1/R_{P1}}{\frac{1}{R_{P0}} + \frac{1}{R_{P1}} + \dots + \frac{1}{R_{Pn}}}\right)$$

$$+v_{P2}\frac{1/R_{P2}}{\frac{1}{R_{P0}}+\cdots+\frac{1}{R_{Pn}}}\cdots+v_{Pn}\frac{1/R_{Pn}}{\frac{1}{R_{P0}}+\cdots+\frac{1}{R_{Pn}}}$$



$$v_{OP} = \left(1 + \frac{R_F}{R_N}\right) \left(v_{P1} \frac{R_P}{R_{P1}} + v_{P2} \frac{R_P}{R_{P2}} + \dots + \frac{R_P}{R_{P_R}} v_{P_R}\right)$$

where

$$R_p = R_{p_0} \| R_{p_1} \| \cdots \| R_{p_n}$$

when all inputs are present:

$$v_O = v_{ON} + v_{OP}$$

$$= -\left(\frac{R_f}{R_{N1}}v_{N1} + \frac{R_f}{R_{N2}}v_{N2} + \cdots\right) + \left(1 + \frac{R_f}{R_N}\right)\left(\frac{R_P}{R_{P1}}v_{N1} + \frac{R_P}{R_{P2}}v_{N2} + \cdots\right)$$

(b)
$$v_0 = -4v_{N1} + v_{P1} + 3v_{P2}$$

$$\frac{R_f}{R_{N1}} = 4, R_{N1} = 10 \text{ k}\Omega \Rightarrow R_f = 40 \text{ k}\Omega$$

$$\left(1 + \frac{R_f}{R_N}\right) \left(\frac{R_P}{R_{P1}}\right) = 1 \Rightarrow 5\frac{R_P}{R_{P1}} = 1 \tag{1}$$

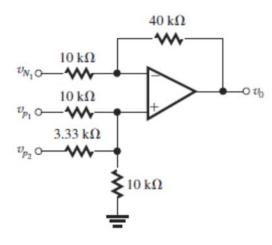
$$\left(1 + \frac{R_f}{R_N}\right) \left(\frac{R_P}{R_{P2}}\right) = 3 \Rightarrow 5 \frac{R_P}{R_{P2}} = 3 \tag{2}$$

Substituting for $\frac{1}{R_p}$, $\frac{1}{R_p}=\frac{1}{R_{p_0}}+\frac{1}{R_{p_1}}+\frac{1}{R_{p_2}}$ in

Eqs. (1) and (2) and selecting (arbitrarily)

 $R_{p_0} = 10 \text{ k}\Omega$ results in $R_{p_1} = 10 \text{ k}\Omega$ and

 $R_{P2} = 3.33 \text{ k}\Omega$. The result is the following circuit:



Done

$$2.49 \ v_{+} = v_{1} \frac{R_{4}}{R_{3} + R_{4}}$$

$$\frac{v_O}{v_I} = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{R_4}{R_3 + R_4}\right) = \frac{1 + R_2/R_1}{1 + R_3/R_4}$$

Done

2.60
$$R_1 = R_3 = 5 \text{ k}\Omega, R_2 = R_4 = 100 \text{ k}\Omega$$

Equation (2.15):
$$\frac{R_4}{R_3} = \frac{R_2}{R_1} = 20$$

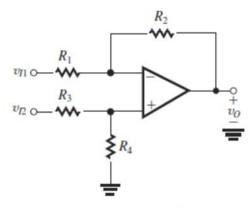
From Eq. (2.16),

$$v_O = \frac{R_2}{R_1} v_{id}$$

$$A_d = \frac{v_O}{v_{id}} = \frac{R_2}{R_1} = 20 \text{ V/V}$$

From Eq. (2.20)

$$R_{id} = 2R_1 = 2 \times 5 \text{ k}\Omega = 10 \text{ k}\Omega$$



The two resistance ratios $\frac{R_2}{R_1}$ and $\frac{R_4}{R_3}$ differ by 1%.

$$\therefore \frac{R_2}{R_1} = 0.99 \frac{R_4}{R_3}$$

Now for this case, A_{cm} can be found from Eq. (2.19)

$$\begin{split} A_{cm} &= \left(\frac{R_4}{R_4 + R_3}\right) \left(1 - \frac{R_2}{R_1} \times \frac{R_3}{R_4}\right) \\ &\simeq \frac{100}{100 + 5} \times \left(1 - 0.99 \frac{R_4}{R_3} \times \frac{R_3}{R_4}\right) \end{split}$$

$$A_{cm} = 0.0095$$

Neglecting the effect of resistance variation on A_d ,

$$A_d = \frac{R_2}{R_1} = \frac{100}{5} = 20 \text{ V V}$$

$$CMRR = 20 \log \left| \frac{A_d}{A_{cm}} \right|$$

$$=20 \log \left| \frac{20}{0.0095} \right|$$

$$= 66.4 \, dB$$

Done

2.68 (a) Refer to Fig. P2.68 and Eq. (2.19):

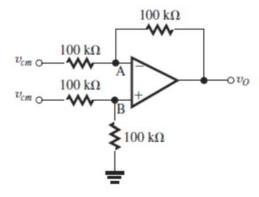
$$A_{cm} = \frac{R_4}{R_3 + R_4} \left(1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \right)$$
$$= \frac{100}{100 + 100} \left(1 - \frac{100.100}{100.100} \right)$$

$$A_{cm}=0$$

Refer to 2.17:
$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

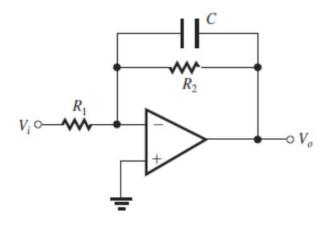
$$\Rightarrow A_d = \frac{R_2}{R_1} = 1$$

(b)
$$v_A = v_B$$



$$v_A = v_{cm} \frac{100}{100 + 100}$$

$$v_A = \frac{v_{CM}}{2}$$
 and $v_B = \frac{v_{CM}}{2}$
 $\Rightarrow -5 \text{ V} \le v_{CM} \le 5 \text{ V}$



Let
$$Z_2 = R_2 \parallel \frac{1}{sC}$$
 and $Z_1 = R_1$

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{Y_1}{Y_2} = -\frac{1/R_1}{\frac{1}{R_2} + sC}$$

$$= -\frac{(R_2/R_1)}{1 + sCR_2}$$

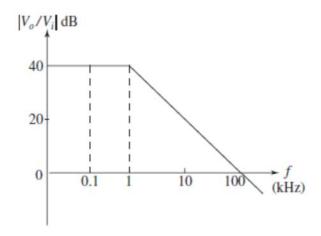
This function is of the STC low-pass type, having a dc gain of $-\frac{R_2}{R_1}$ and a 3-dB frequency

$$\omega_0 = \frac{1}{CR_2}$$

$$R_{\rm in} = R_1 = 10 \,\mathrm{k}\Omega$$

$$dc gain = 40 dB = 100$$

$$\therefore 100 = \frac{R_2}{R_1} \Rightarrow R_2 = 100R_1 = 1 \text{ M}\Omega$$

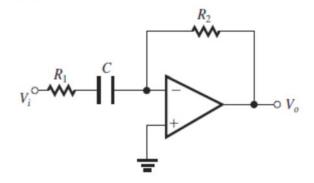


3-dB frequency at 1 kHz

$$\therefore \omega_0 = 2\pi \times 1 \times 10^3 = \frac{1}{CR_2}$$

$$C = \frac{1}{2\pi \times 1 \times 10^3 \times 10^6} = 0.16 \text{ nF}$$

From the Bode plot shown above, the unity-gain frequency is 100 kHz.



$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1 + \frac{1}{sC}}$$

Thus,

$$\frac{V_o}{V_i} = -\frac{(R_2/R_1)s}{s + \frac{1}{CR_1}}$$

which is that of an STC high-pass type.

High-frequency gain
$$(s \to \infty) = -\frac{R_2}{R_1}$$

3-dB frequency
$$(\omega_{3dB}) = \frac{1}{CR_1}$$

For a high-frequency input resistance of 1 k Ω , we select $R_1 = 1$ k Ω . For a high-frequency gain of 40 dB,

$$\frac{R_2}{R_1} = 100 \Rightarrow R_2 = 100 \text{ k}\Omega$$

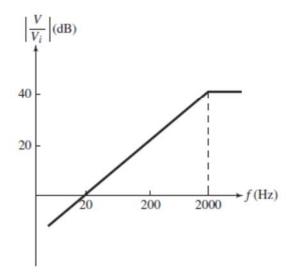
For $f_{3dB} = 2$ kHz,

$$\frac{1}{2\pi \, CR_1} = 2 \times 10^3$$

$$\Rightarrow C = 79 \text{ nF}$$

The magnitude of the transfer function reduces from 40 dB to unity (0 dB) in two decades. Thus

$$f$$
 (unity gain) = $\frac{f_{3dB}}{100} = \frac{2000}{100} = 20 \text{ Hz}$



Done

P.H2:

