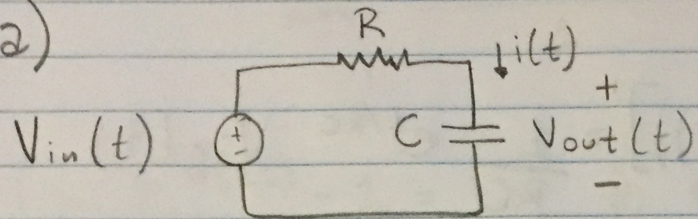
Lab 2 Q3 and Q7

Q3) 1^{st} order differential equation:

a)



$$i(t) = \frac{V_{in}(t) - V_{out}(t)}{R}$$

Current through capacitor → $i(t) = C \cdot \frac{d\,V_{out}(t)}{dt}$

$$\frac{V_{in}(t) - V_{out}(t)}{R} = C \cdot \frac{d\,V_{out}(t)}{dt}$$

$$V_{in}(t) - V_{out}(t) = R \cdot C \cdot \frac{d\,V_{out}(t)}{dt}$$

$$V_{in}(t) = RC\,\frac{d\,V_{out}(t)}{dt} + V_{out}(t)$$

b) $\frac{d\,V_{out}(t)}{dt} + \frac{1}{RC} V_{out}(t) = \frac{1}{RC} V_{in}(t)$

Now, I will use the method of the integrating factor by creating a function called $\mu(t)$.

$$\mu(t) = e^{\int \frac{1}{RC} dt} = e^{t/RC}$$

$$\underbrace{e^{t/RC}\frac{dV_{out}(t)}{dt} + \frac{1}{RC}e^{t/RC}V_{out}(t)}_{\text{product rule}} = \frac{1}{RC}e^{t/RC}V_{in}(t)$$

$$\left[(e^{t/RC})(V_{out}(t))\right]' = \frac{1}{RC}e^{t/RC}V_{in}(t)$$

$$e^{t/RC}\cdot V_{out}(t) = V_0 + \int_0^t \frac{1}{RC}e^{\tau/RC}V_{in}(\tau)\,d\tau$$

$$V_{out}(t) = V_0e^{-t/RC} + \int_0^t \frac{1}{RC}e^{(\tau-t)/RC}V_{in}(\tau)\,d\tau$$

$$V_{out}(t) = V_0e^{-t/RC} + \int_0^t \frac{1}{RC}e^{-(t-\tau)/RC}V_{in}(\tau)\,d\tau,$$

$t \geq 0$

Q7) Obtain pulse response part 2: Compute $\lim_{\Delta \to 0} y_\Delta(t)$. You can use l'Hopital's rule to evaluate $\lim_{\Delta \to 0} \frac{e^{\Delta/RC} - 1}{\Delta}$.

$$\lim_{\Delta \to 0} \frac{e^{\Delta/(RC)} - 1}{\Delta} = \lim_{\Delta \to 0} \frac{\frac{1}{RC} e^{\Delta/RC}}{1}$$

Then, plug in zero in delta:

$$\frac{1}{RC} e^{0/RC} = \frac{1}{RC}$$