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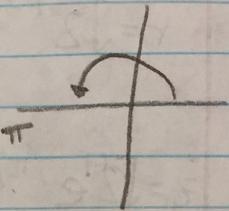
EE 301

Professor Fariba Ariaei

## Homework #1

1)

a)  $\frac{1}{2} e^{j\pi}$ ,  $e^{jy} = \cos y + j \sin y$

$$z = r e^{j\theta} = \frac{1}{2} e^{j\pi} = \frac{1}{2} \left( \cos \pi + j \sin \pi \right)$$


The diagram shows a Cartesian coordinate system with a horizontal real axis and a vertical imaginary axis. A vector labeled  $z$  originates from the origin. It has a length of  $\frac{1}{2}$  and is directed along the negative real axis at an angle of  $\pi$  radians from the positive real axis.

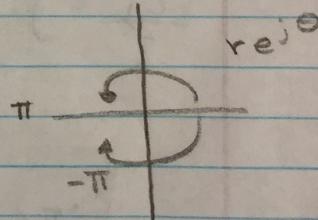
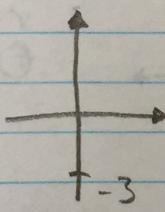
$$= -\frac{1}{2}$$

b)  $z = \sqrt{2} e^{-j9\pi/4} = \sqrt{2} (\cos(-9\pi/4) + j \sin(-9\pi/4))$   
 $= \sqrt{2} (\cos(9\pi/4) - j \sin(9\pi/4))$

$$= \sqrt{2} \left( \frac{1}{\sqrt{2}} - j \frac{1}{\sqrt{2}} \right) = \frac{\sqrt{2}}{\sqrt{2}} - j \frac{\sqrt{2}}{\sqrt{2}} = 1 - j$$

2)

a)  $-3j$



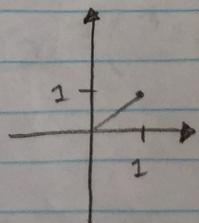
$$r = 3 \quad \cos\left(\frac{-\pi}{2}\right) + j \sin\left(\frac{-\pi}{2}\right) = -i$$

$$z = 3 e^{-j\pi/2}$$

b)  $1+j$

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$z = \sqrt{2} e^{j\pi/4}$$



$$r = \sqrt{2}, \theta = \frac{\pi}{4}$$

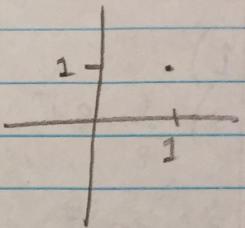
$$z = \sqrt{2} e^{j\pi/4}$$

1-1

c)  $j(1-j)$ , where  $j^2 = -1$   $r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$$j - j^2 = j - (-1) = j + 1 = 1 + j$$

$$r = \sqrt{2}, \theta = \frac{\pi}{4}, z = \sqrt{2} e^{j\pi/4}$$



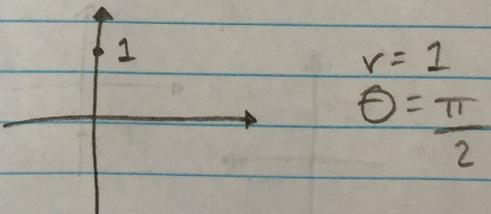
$$z = \sqrt{2} e^{j\pi/4}$$

d)  $\frac{1+j}{1-j} \cdot \frac{1+j}{1+j} = \frac{(1+j)(1+j)}{(1-j)(1+j)}$

$$(1+j)(1+j) = 1+j+j+j^2 = 1+2j+(-1) = 2j$$

$$(1-j)(1+j) = 1+j-j-j^2 = 1-(-1) = 1+1=2$$

$$\frac{2j}{2} = j$$



$$r = 1, \theta = \frac{\pi}{2}$$

$$z = e^{j\pi/2}$$

3)

2)  $zz^* = r^2, z = re^{j\theta}, z^* = re^{-j\theta}$

$$zz^* = (re^{j\theta})(re^{-j\theta}) = \cancel{e^{j\theta}}(r)(\cancel{e^{-j\theta}}) = r^2$$

(2)

$$b) \frac{z}{z^*} = \frac{re^{j\theta}}{re^{-j\theta}} = \cancel{r}(e^{j\theta})(e^{-j\theta}) = e^{2j\theta}$$

$$c) \operatorname{Re}\{z\} = \frac{1}{2}(z + z^*)$$

$$\frac{1}{2}(z + z^*) = \frac{1}{2}(x + jy + x - jy) = \frac{2x}{2} = x$$

and  $\operatorname{Re}\{z\}$  is  $x$ . Therefore,  $\operatorname{Re}\{z\} = \frac{1}{2}(z + z^*)$

$$d) \operatorname{Im}\{z\} = \frac{1}{2j}(z - z^*)$$

$$\frac{1}{2j}(z - z^*) = \frac{1}{2j}(x + jy - x + jy) = \frac{2jy}{2j} = y$$

and  $\operatorname{Im}\{z\}$  is  $y$ . Therefore,  $\operatorname{Im}\{z\} = \frac{1}{2j}(z - z^*)$

$$4) e^{j\theta} = \cos\theta + j\sin\theta$$

$$2) \frac{1}{2}(e^{j\theta} + e^{-j\theta}) = \frac{1}{2}(\cos\theta + j\sin\theta + \cos\theta - j\sin\theta)$$

$$= \frac{2\cos\theta}{2} = \cos\theta$$

$$\text{Therefore, } \cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$

2-1

$$\text{b) } \frac{1}{2j} (e^{j\theta} - e^{-j\theta}) = \frac{1}{2j} (\cos\theta + j\sin\theta - \cos\theta + j\sin\theta)$$
$$= \frac{1}{2j} (2j\sin\theta) = \sin\theta$$

Therefore,  $\sin\theta = \frac{1}{2j} (e^{j\theta} - e^{-j\theta})$

$$\text{c) } \cos^2\theta = \frac{1}{2} (1 + \cos 2\theta), \text{ where}$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \frac{1}{2} (1 + \cos^2\theta - \sin^2\theta), \text{ where}$$
$$1 = \cos^2\theta + \sin^2\theta$$

$$= \frac{1}{2} (\cos^2\theta + \cos^2\theta + \sin^2\theta - \sin^2\theta)$$

$$= \frac{2\cos^2\theta}{2} = \cos^2\theta$$

Therefore,  $\cos^2\theta = \frac{1}{2} (1 + \cos 2\theta)$

$$\text{d) } \sin(\theta + \phi) = \sin\theta \cos\phi + \cos\theta \sin\phi$$

$$\sin(\theta + \phi) = \text{Im}\{e^{i(\theta+\phi)}\} = \text{Im}\{e^{i\theta} e^{i\phi}\}$$

$$= \text{Im}\{(\cos\theta + i\sin\theta)(\cos\phi + i\sin\phi)\}$$

$$= \text{Im}\{\cos\theta \cos\phi + i\cos\theta \sin\phi + i\sin\theta \cos\phi - \sin\theta \sin\phi\}$$

$$= \sin\theta \cos\phi + \cos\theta \sin\phi$$

(3)

$$\text{because } \sin \theta = \operatorname{Im} \{ e^{i\theta} \} = \operatorname{Im} \{ \cos \theta + i \sin \theta \}$$

$$\text{therefore, } \sin(\theta + \phi) = \operatorname{Im} \{ e^{i(\theta+\phi)} \}$$

As a result,

$$\sin(\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi, \text{ as desired}$$