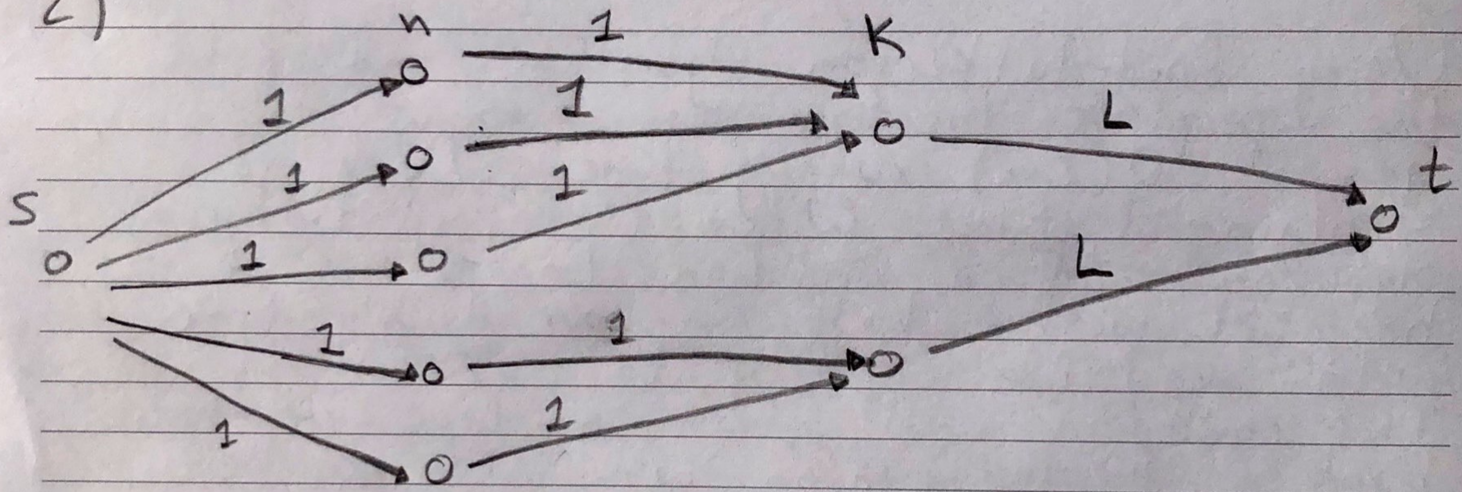
Spring 2019 - HW8

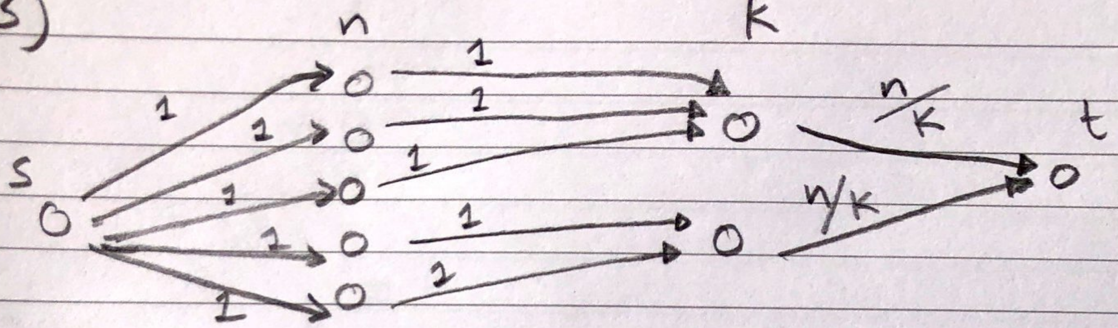
1) I will use Ford-Fulkerson to compute the maximum flow of the graph. This will take $O(mn)$. Then, I will look for the min cut of the graph, which will take me $O(m)$ because I will have to traverse all the edges. Therefore, the algorithm runs in $O(m^2 n)$.

2)



Run Ford-Fulkerson in the graph from above. At the end, if the max flow equals (the flow getting to the sink t) the number of clients then every client will be connected simultaneously to a base station. Ford-Fulkerson in this problem will run in $O(nm)$, which is polynomial.

3)



Run Ford-Fulkerson algorithm in the graph from above. This will take O(mn), which is polynomial. If the maximum flow equals the number of clients, then it is possible to send every injured person to a hospital in such a way that the load on the hospitals is balanced $\lceil n/k \rceil$ as desired.

Practice Problems

Critical edge = reducing capacity → reduces maximum flow.

It is false that with respect to a maximum flow of G, any edge whose flow is equal to its capacity its a critical edge! The reason for this is that the maximum flow can find a different path.

Algorithm: Reduce the capacity of the edge "e" by 1 unit. Then, find a s-t path containing "e" and subtract 1 unit of the s-t flow on that path. For finding the s-t flow, consider "e"'s adjacent nodes to be (u, v) and run BFS from source to find a s-u path and BFS from v-t path, which gives you a s-u-v-t path crossing "e" in O(|m|+|n|).
If there is an augmenting s-t path in Gf then "e" is not a critical edge.