TDHCF MVC Time Mapping Protocol

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1 Time Coordinate Mapping and Active Phase Dynamics

To resolve the coordinate ambiguity between the bulk advanced/retarded time and the holographic global time during active MVC phases (where $\dot{\mu} \neq 0$), we establish the following explicit mapping protocol. This ensures that the non-conservation current Q_μ derived in the bulk (Vaidya gauge) is correctly projected onto the brane observables.

1.1 Coordinate Definitions

- **Bulk Null Coordinate (v):** We adopt the ingoing Eddington-Finkelstein coordinate v for the bulk metric definition (ds^2_{bulk}), such that the mass parameter is naturally $\mu = \mu(v)$. This captures the causal inflow of the bulk fluid (null dust) thickening the horizon.

- **Global Holographic Time (T):** The time coordinate of the asymptotic boundary ($r \to \infty$) and the reference frame for the holographic dual field theory.

- **Brane Proper Time (τ):** The intrinsic time measured by an observer comoving with the brane at position $y = 0$ (or $r = r_b$).

1.2 The Chain Rule Protocol

Strictly distinguishing v from T is required to preserve causality during rapid horizon updates. We impose the chain rule for the evolution of the mass parameter:

$$\left.\frac{d\mu}{dT}\right|_{\text{obs}} = \frac{d\mu}{dv} \cdot \left(\frac{dv}{dT}\right)_\Sigma, \tag{1}$$

where:

- $\frac{d\mu}{dv}$ is the geometric mass loss rate appearing directly in the Vaidya curvature tensor ($R_{\mu\nu} \propto \dot{\mu}(v)$).

- $\left(\frac{dv}{dT}\right)_\Sigma$ is the gravitational redshift/Doppler factor relating the null coordinate to global time at the brane location Σ.

1.3 Operational Approximation for Quasi-Static Regimes

For the specific case of the TDHCF quasi-static regime, where the brane velocity is non-relativistic relative to the bulk frame ($\dot{r}_b \approx 0$) and the horizon evolution is slow compared to the curvature scale ($L\dot{\mu} \ll 1$), we approximate the factor as:

$$\left(\frac{dv}{dT}\right)_\Sigma \approx \sqrt{f(r_b)} \approx 1 \quad (\text{for } r_b \gg r_h), \tag{2}$$

allowing the identification $\mu(v) \simeq \mu(T)$ only in the asymptotic limit. However, for precise Q_0 calculation near the MVC threshold, the full form (1) is retained in the derivations to satisfy local covariance.

1.4 Relation to Proper Time

Finally, physical rates measured on the brane relate to global time via the standard Lorentz/redshift factor $\gamma = dT/d\tau$:

$$\frac{d\mu}{d\tau} = \frac{d\mu}{dT}\frac{dT}{d\tau} = \frac{d\mu}{dv}\left(\frac{dv}{dT}\right)_{\Sigma}\frac{dT}{d\tau}. \tag{3}$$

This explicitly closes the variable identification gap in the bulk-to-brane projection.