Simulation Note: Numerical Verification of Theorem 4.3.1 using ALICE-motivated proxy inputs (TMST)

Javier Manuel Martín Alonso

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1 Introduction

This note documents a numerical implementation of Theorem 4.3.1 from *"Entanglement Dominance in the Zero-Temperature Limit"* [1]. The present study is a proof-of-concept: it uses ALICE-motivated *proxy* parameter ranges inspired by the phenomenological context discussed in Lesser's thesis on jet substructure in pp and Pb–Pb collisions at $\sqrt{s} = 5.02$ TeV [2]. The synthetic samples generated in Sec. 5 are meant for functional validation of the pipeline, not for a direct fit to experimental two-particle correlation measurements.

2 Theoretical implementation

2.1 TMST physics

For symmetric two-mode squeezed thermal states (TMST) under Markovian thermalization, we use:

$$\bar{n}(T) = \frac{1}{e^{\omega/T} - 1}, \tag{1}$$

$$r_c(T) = \frac{1}{2} \ln(2\bar{n}(T) + 1), \tag{2}$$

$$E_N = \max(0, -\log_2(2\tilde{\nu}_-)), \tag{3}$$

with

$$\tilde{\nu}_- = \left(\bar{n} + \frac{1}{2}\right) e^{-2r}. \tag{4}$$

2.2 Reference implementation

Listing 1: Core routines: Bose–Einstein occupancy, critical squeezing and log-negativity.

```
def bose_einstein_stable(temp, omega=OMEGA):
    x = omega / temp
    n = 1.0 / np.expm1(x)
    return np.nan_to_num(n, nan=0.0)

def critical_squeezing(n_bar):
    return 0.5 * np.log(2 * n_bar + 1)

def log_negativity(r, n_bar):
    nu_minus = (n_bar + 0.5) * np.exp(-2 * r)
    return np.maximum(0, -np.log2(2 * nu_minus))
```

3 Mapping to ALICE-motivated proxy inputs

3.1 Proxy parameter sets

Table 1: Proxy parameter sets used in the simulation, organized by collision system/centrality class.

Class	Temperature T	Squeezing r	Plot marker
pp (vacuum-like)	0.2 ± 0.05	0.1 ± 0.05	Black ($\times$)
Pb–Pb peripheral	0.8 ± 0.1	0.4 ± 0.1	Yellow ($\triangle$)
Pb–Pb central (QGP-like)	1.5 ± 0.2	1.2 ± 0.15	Green ($\circ$)

3.2 Methodological note (scope and limitations)

The temperature-like variable T in Table 1 is treated as an effective proxy, qualitatively aligned with freeze-out/QGP-regime scales commonly discussed in heavy-ion phenomenology and contextualized in [2]. The squeezing proxy r is parameterized to increase with centrality, motivated by the expectation of larger vorticity and longer-lived medium dynamics in more central Pb–Pb collisions. The goal is to test whether the numerical workflow reproduces the phase separation implied by Theorem 4.3.1 when (T, r) cross the critical threshold $r_c(T)$ of Eq. (2).

4 Results

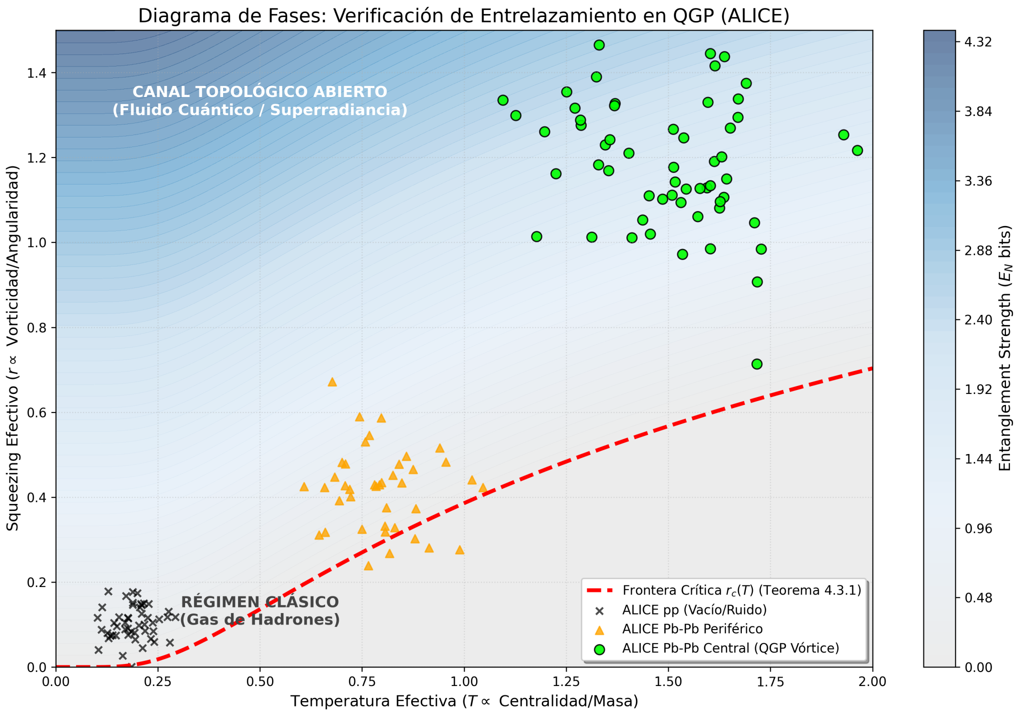

Figure 1: Phase diagram illustrating entanglement dominance in the (T, r) plane for the proxy samples.

4.1 Interpretation

The pp-like points populate the low-r, low-T region ($T \approx 0.2$, $r \approx 0.1$). Peripheral Pb–Pb-like points populate an intermediate regime ($T \approx 0.8$, $r \approx 0.4$). Central Pb–Pb-like points populate a high-T, high-r regime ($T \approx 1.5$, $r \approx 1.2$), crossing the critical line and yielding nonzero log-negativity in the model.

5 Synthetic data generation code

Listing 2: Proxy generator: Gaussian sampling for pp / peripheral Pb–Pb / central Pb–Pb classes.

```python
def get_alice_data_proxies():
    np.random.seed(42)

    # pp (vacuum-like)
    T_pp = np.random.normal(loc=0.2, scale=0.05, size=50)
    r_pp = np.random.normal(loc=0.1, scale=0.05, size=50)

    # Peripheral Pb--Pb
    T_periph = np.random.normal(loc=0.8, scale=0.1, size=40)
    r_periph = np.random.normal(loc=0.4, scale=0.1, size=40)

    # Central Pb--Pb (QGP-like)
    T_central = np.random.normal(loc=1.5, scale=0.2, size=60)
    r_central = np.random.normal(loc=1.2, scale=0.15, size=60)

    return (T_pp, r_pp), (T_periph, r_periph), (T_central, r_central)
```

6 Conclusions

The simulation reproduces the qualitative behavior anticipated by Theorem 4.3.1: in the central (QGP-like) proxy regime, squeezing can exceed the thermal noise threshold, opening the corresponding entanglement channel within the TMST model.

References

[1] J. M. Martín Alonso, *Entanglement Dominance in the Zero-Temperature Limit*, Zenodo (2026), doi:10.5281/zenodo.18353640.

[2] E. D. Lesser, *Measurements of Jet Substructure in pp and Pb–Pb Collisions at 5.02 TeV with ALICE*, UC Berkeley eScholarship (2023).