```
ClearAll["Global`*"]
```

Lambda System three state ladder

Bandwith, its inverse, initial and final times as well as the time difference between the lasers are as follows

```
τ = 6;
1/τ
```

$\frac{1}{6}$

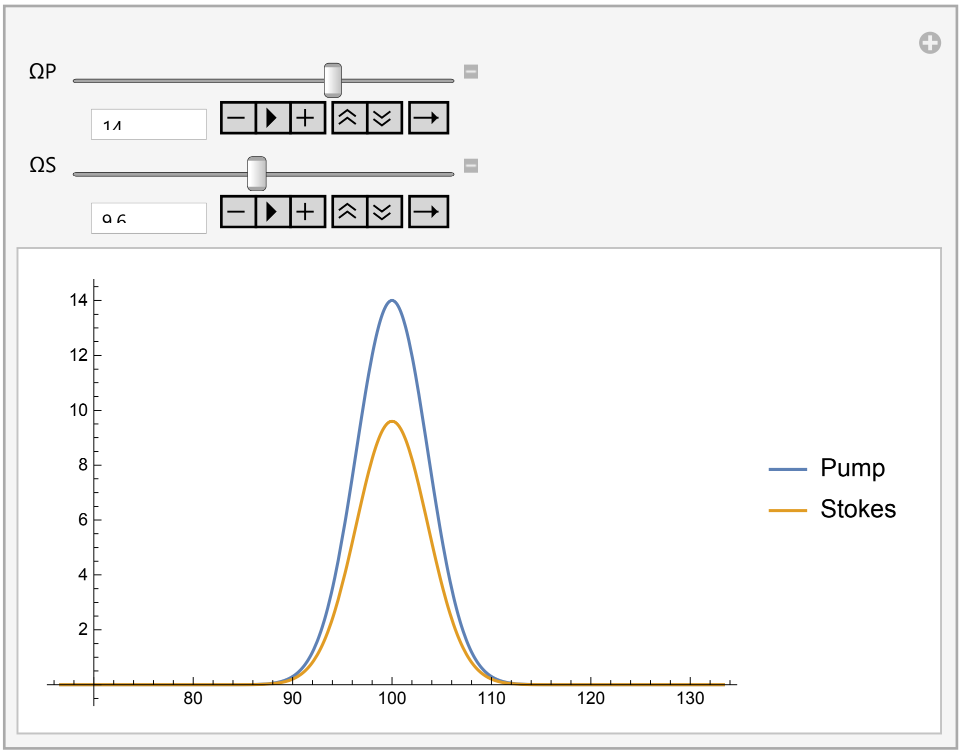
```
ti = 0;
tf = 200;
Δt = 0;
```

The gaussian equations for both Rabi frequencies are

```
Ω1[t_] := ΩP * Exp[-2 * Log[2] * (t - (tf - ti) / 2)^2 / τ^2];
Ω2[t_] := ΩS * Exp[-2 * Log[2] * ((t + Δt) - (tf - ti) / 2)^2 / τ^2];
```

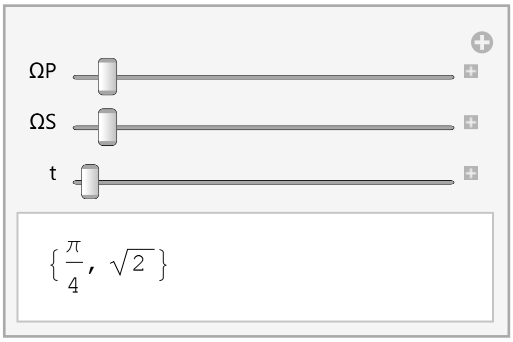
Both pulses are separated by Δt.

```
Manipulate[Block[{Ω1, Ω2, t},
  Ω1[t_] := ΩP * Exp[-2 * Log[2] * (t - (tf - ti) / 2)^2 / τ^2];
  Ω2[t_] := ΩS * Exp[-2 * Log[2] * ((t + Δt) - (tf - ti) / 2)^2 / τ^2];
  Plot[{Ω1[t], Ω2[t]}, {t, ti + 2 * tf / τ, tf - 2 * tf / τ}, PlotRange → All,
   PlotLegends → {"Pump", "Stokes"}]], {{ΩP, 1}, 0, 20, 0.4}, {{ΩS, 1}, 0, 20, 0.4}]
```



The following representation gives the value of the angle of mix Φ =Arctan $\frac{\Omega_P}{\Omega_S}$and the geometrical average of the Rabi frequencies.

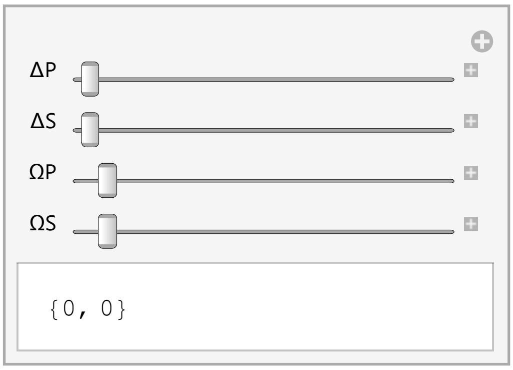
```
Manipulate[{ArcTan[ΩP * Exp[-2 * Log[2] * (t - (tf - ti) / 2)^2 / τ^2] /
    (ΩS * Exp[-2 * Log[2] * ((t + Δt) - (tf - ti) / 2)^2 / τ^2])], Sqrt[ΩP^2 + ΩS^2]},
  {{ΩP, 1}, 0, 20}, {{ΩS, 1}, 0, 20}, {t, ti + 2 * tf / τ, tf - 2 * tf / τ}]
```



First we send a pulse from the Stokes laser so Φ=0. Then a pump laser pulse which is still incident on the sample when the Stokes pulse has already passed so Φ=$\frac{\pi}{2}$ which is a gradual change given the shape of the pulses.

The frequencies that are associated with the gain of population and the loss of population for any state are, respectively

```
Manipulate[{(ΔP * ΩP^2 + ΔS * ΩS^2) / (ΩP^2 + ΩS^2), (ΔP - ΔS) * ΩP * ΩS / (ΩP^2 + ΩS^2)},
  {{ΔP, 0}, 0, 2, 0.1}, {{ΔS, 0}, 0, 2, 0.1}, {{ΩP, 1}, 0, 20, 1}, {{ΩS, 1}, 0, 20, 1}]
```



The Hamiltonian is

```
HA[t_] := {{2 ΔP, Ω1[t], 0}, {Ω1[t], 0, Ω2[t]}, {0, Ω2[t], 2 ΔS}}
HA[t] // MatrixForm
```

$$\begin{pmatrix} 2\,\Delta P & 2^{-\frac{1}{18}(-100+t)^2}\,\Omega P & 0 \\ 2^{-\frac{1}{18}(-100+t)^2}\,\Omega P & 0 & 2^{-\frac{1}{18}(-100+t)^2}\,\Omega S \\ 0 & 2^{-\frac{1}{18}(-100+t)^2}\,\Omega S & 2\,\Delta S \end{pmatrix}$$

Where ΔP,ΔS are the detuning between states 1 and 2, and 2 and 3 respectively. Also Ω1 and Ω2 are gaussian functions and we have applied RWA to both interactions undergone by level 2 with levels 1 and 3. Probability loss is neglected.

Let us solve now the Schrödinger eq for 3 states $C_i' = HA * C_i$ so as to find the population in each level.

```
Manipulate[Block[{c1Solved, c2Solved, c3Solved , c1, c2, c3, t, s, Ω1, Ω2, HA},
  Ω1[t_] := ΩP * Exp[-2 * Log[2] * (t - (tf - ti) / 2)^2 / τ^2];
  Ω2[t_] := ΩS * Exp[-2 * Log[2] * ((t + Δt) - (tf - ti) / 2)^2 / τ^2];
  HA[t_] := {{2 ΔP, Ω1[t], 0}, {Ω1[t], 0, Ω2[t]}, {0, Ω2[t], 2 ΔS}};
  s = NDSolve[{{c1'[t], c2'[t], c3'[t]} == -ⅈ HA[t].{c1[t], c2[t], c3[t]},
     c1[ti] == 1, c2[ti] == 0, c3[ti] == 0}, {c1[t], c2[t], c3[t]}, {t, ti, tf}];
  c1Solved = c1[t] /. s[[1, 1]];
  c2Solved = c2[t] /. s[[1, 2]];
  c3Solved = c3[t] /. s[[1, 3]];
   Plot[{c1Solved * Conjugate[c1Solved], c2Solved * Conjugate[c2Solved],
     c3Solved * Conjugate[c3Solved]}, {t, ti + 2 * tf / τ, tf - 2 * tf / τ},
    PlotRange → All, PlotLegends → {"P1", "P2", "P3"}]]
 , {{ΔP, 2}, -20, 20, 2}, {{ΔS, 4}, -20, 20, 2},
 {{ΩP, 20}, 0, 100, 5}, {{ΩS, 20}, 0, 100, 5}]
```