# mtool estimators floating precision

### Simulation Settings

The cause-specific hazards of the outcome of interest and the competing risk follow proportional hazards models, specifically:

$$\alpha_{01} = 0.5t \exp(\beta_{01} Z)$$

$$\alpha_{02} = t \, \exp(\beta_{02} Z)$$

where both cause-specific hazards have the form of a Weibull distribution and a common set of covariates. Cause 1 is the one of interest with an incidence rate of 54 % while cause 2 has an incidence rate of 28 % with a common uniform censoring rate of  $\sim 15\%$ .

```
knitr::opts_chunk$set(cache = T)

# independent normal variables
p <- 20
n <- 400
rho <- 0.5

# Very sparse case
# Common betas for both competing risks
beta <- c(0.5, rep(0, 18), 0.5)
zero_ind1 <- which(beta == 0)
nonzero_ind1 <- which(beta != 0)

# Generate iid X's
X <- matrix(rnorm(p*n), nrow = n, ncol = p)
# XB matrix
suma <- X %*% beta</pre>
```

```
# Function to generate survival times
create.times <- function(n, ch, sup.int = 100) {</pre>
  times <- numeric(n)</pre>
  i <- 1
  while (i <= n)
  { u <- runif(1)
  if (ch(0, -log(u)) * ch(sup.int, -log(u)) < 0)
  { times[i] <- uniroot(ch, c(0, sup.int), tol = 0.0001, y= -log(u))$root
  i <- i + 1
  else {
    cat("pos")
  }}
  times
}
# binomial probability of cause 1
binom.status <- function(ftime, n, a01, a02, size = 1)
{ prob <- a01(ftime) / (a01(ftime) + a02(ftime))
out <- rbinom(n, size, prob)</pre>
out }
# Cause-specific proportional hazards
times <- vector()</pre>
f.status <- vector()</pre>
for (i in seq len(n)) {
alpha.1 \leftarrow function(t) \{ ((0.5*t)*exp(suma[i])) \}
alpha.2 <- function(t) { t*exp(suma[i]) }</pre>
cum.haz <- function(t, y) { stats::integrate(alpha.1, lower=0.001, upper=t,</pre>
                                                subdivisions=1000)$value +
 stats::integrate(alpha.2, lower=0.001, upper=t,
                  subdivisions=1000)$value - y }
times[i] <- create.times(1, cum.haz)</pre>
f.status[i] <- binom.status(times, 1, alpha.1, alpha.2) + 1
}
# Censoring
cens.times <- runif(n, 0, 6)</pre>
```

```
# Censoring in status variable
  f.status <- as.numeric(times <= cens.times) * f.status</pre>
  prop.table(table(f.status))
f.status
     0
            1
0.1775 0.5450 0.2775
  # times with censoring
  times <- pmin(times, cens.times)</pre>
  # Dataset
  sim.dat <- data.frame(time = times, status = f.status)</pre>
  sim.dat <- cbind(sim.dat, X)</pre>
  colnames(sim.dat)[3:22] <- paste0("X", seq_len(p))</pre>
  head(sim.dat)
                           Х1
                                        Х2
                                                    ХЗ
                                                                 Х4
       time status
1 0.9573900
                 0 -0.2833227  0.49315714 -0.39741951  2.03625495 -0.7032940
2 1.1799018
                 2 0.3232338 1.71616792 -0.46033546 0.02252274 0.3078185
                 1 1.7249138 -0.81680140 -0.56385883 0.94053695 -0.2924841
3 0.8318170
4 0.7128943
                 1 0.2462367 -0.43702486 -1.07616818 -0.11868349 -0.6220214
5 0.3465138
                 0 -1.2528001 -0.07444809 0.02754773 1.39543413 1.3102259
                 1 -0.3435119 2.05859431 -0.81325204 0.95979719 0.8135085
6 1.8083344
          Х6
                     Х7
                                Х8
                                            Х9
                                                      X10
                                                                  X11
                                                                             X12
1 1.3812699 -1.4583283 1.5471274 -0.4759714 -1.1536913 0.9986666 -0.3782998
2 1.0390017 0.6989621 -1.1535188 1.1049859 -0.2081242 1.4297879 -0.9685742
3 -0.5955198  0.5454273 -1.0567628 -0.9781166 -2.9286698  0.8496133  0.7936185
4 0.8488174 -1.3804222 0.6297198 -1.8816258 0.3714670 0.6091019 0.3082478
5 -2.9057474 -0.2645360 -0.1934075 -0.2205454 -1.4219697 1.3002518 -0.5440021
6 -1.1784667 0.9474378 1.9692580 0.1082714 -1.4709294 -0.3398231 1.6439283
         X13
                    X14
                                  X15
                                            X16
                                                       X17
                                                                    X18
1 - 0.5719765 \quad 0.7106656 \quad -0.805159653 \quad 0.1649621 \quad -0.7901370 \quad -0.69944603
2 -0.2328593  0.2757856 -1.059329592  1.6586922  0.1182002 -1.81396615
3 0.4405046 -0.7697308 -0.007615668 0.2647511 0.2657026 -1.42275830
```

## 2. Floating precision of mtool estimators

```
# Split into training and test sets
train.index <- caret::createDataPartition(sim.dat$status, p = 0.70, list = FALSE)
train <- sim.dat[train.index.]</pre>
validation <- sim.dat[-train.index,]</pre>
surv_obj_train <- with(train, Surv(time, as.numeric(status), type = "mstate"))</pre>
cov_train <- cbind(train[3:22])</pre>
# Create case-base dataset
cb_data_train <- create_cbDataset(surv_obj_train, as.matrix(cov_train))</pre>
# Apply to validation set
# First fit
lambdagrid \leftarrow rep(0.01, 150)
cvs res <- mclapply(lambdagrid, function(lambda val) {</pre>
fit_val1 <- fit_cbmodel(cb_data_train, regularization = 'elastic-net',</pre>
                                              lambda = lambda val , alpha = 1)
 \}, mc.cores = 4)
lambdagrid \leftarrow rep(0.009, 150)
cvs_res1 <- mclapply(lambdagrid, function(lambda_val) {</pre>
fit_val1 <- fit_cbmodel(cb_data_train, regularization = 'elastic-net',</pre>
                                              lambda = lambda_val , alpha = 1)
 \}, mc.cores = 4)
```

```
non_zero_coefs <- unlist(mclapply(cvs_res,</pre>
  function(x) {return(x$no_non_zero)}, mc.cores = 4))
  non_zero_coefs1 <- unlist(mclapply(cvs_res1,</pre>
  function(x) {return(x$no_non_zero)}, mc.cores = 4))
  # Display different outputs
  cvs_res[[1]]
$coefficients
22 x 2 sparse Matrix of class "dgCMatrix"
 [1,] 0.1188583 0.03100094
 [2,] .
 [3,] .
 [4,] .
 [5,] .
 [6,] .
 [7,] .
 [8,] .
[9,] .
[10,] .
[11,] .
[12,] .
[13,] .
[14,] .
[15,].
[16,] .
[17,].
[18,] .
[19,].
[20,] 0.1460524 0.10126200
[21,] 0.3274119 0.39797117
[22,] -0.1295394 -0.41744877
$no_non_zero
[1] 8
  cvs_res[[which(non_zero_coefs != 8)[1]]]
```

```
$coefficients
22 x 2 sparse Matrix of class "dgCMatrix"
 [1,] 1.188379e-01 0.03094678
 [2,] .
 [3,] .
 [4,].
 [5,] .
 [6,] .
 [7,] .
 [8,] .
 [9,] .
[10,] .
[11,] .
[12,] .
[13,] .
[14,]
[15,] 4.438434e-08 .
[16,].
[17,]
[18,] 1.443084e-08 .
[19,] .
[20,] 1.461072e-01 0.10135319
[21,] 3.275139e-01 0.39809244
[22,] -1.295302e-01 -0.41744088
$no_non_zero
[1] 10
  prop.table(table(non_zero_coefs))
non_zero_coefs
         8
                                10
0.94666667 0.046666667 0.006666667
  prop.table(table(non_zero_coefs1))
non_zero_coefs1
                                10
                                            11
0.860000000 0.106666667 0.026666667 0.006666667
```

## Fixing the issue

Increased the tolerance from 1e-4 to 1e-5 and the niter\_inner\_mtplyr to 15 (from 5) (number of stochastic updates used to estimate the full gradient)

#### Running the same test again with new function

```
lambdagrid \leftarrow rep(0.02, 150)
  cvs_res <- mclapply(lambdagrid, function(lambda_val) {</pre>
  fit_val1 <- fit_cbmodel(cb_data_train, regularization = 'elastic-net',</pre>
                                               lambda = lambda_val , alpha = 1)
   \}, mc.cores = 4)
  non_zero_coefs <- unlist(mclapply(cvs_res,</pre>
  function(x) {return(x$no_non_zero)}, mc.cores = 4))
  prop.table(table(non_zero_coefs))
non_zero_coefs
1
  lambdagrid <- rep(0.009, 150)
  cvs_res1 <- mclapply(lambdagrid, function(lambda_val) {</pre>
  fit_val1 <- fit_cbmodel(cb_data_train, regularization = 'elastic-net',
                                               lambda = lambda_val , alpha = 1)
   \}, mc.cores = 4)
  non_zero_coefs1 <- unlist(mclapply(cvs_res1,</pre>
  function(x) {return(x$no_non_zero)}, mc.cores = 4))
  prop.table(table(non_zero_coefs1))
```

```
non_zero_coefs1
7
1
  #' Multinomial deviance
  # '
  #' @param cb_data Output of \code{create_cbDataset}
  #' @param fit_object Output of \code{fit_cbmodel}
  #' @return Multinomial deviance
  multi_deviance <- function(cb_data, fit_object) {</pre>
    X <- as.matrix(cbind(cb_data$time, cb_data$covariates))</pre>
     fitted_vals <- as.matrix(X %*% fit_object$coefficients)</pre>
    pred_mat <- VGAM::multilogitlink(fitted_vals,</pre>
                                         inverse = TRUE)
     # Turn event_ind into Y_mat
    Y_fct <- factor(cb_data$event_ind)</pre>
    Y_levels <- levels(Y_fct)</pre>
    Y_mat <- matrix(NA_integer_, ncol = length(Y_levels),</pre>
                      nrow = nrow(X)
     for (i in seq_len(length(Y_levels))) {
       Y_mat[,i] <- (Y_fct == Y_levels[i])</pre>
     }
     dev <- VGAM::multinomial()@deviance(pred_mat, Y_mat,</pre>
                                            w = rep(1, nrow(X)))
    return(dev)
```