Eidgenössische Technische Hochschule Zürich
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Complex Social Systems:
Modeling Agents, Learning, and Games

Opinion Formation
Comparing Different Models

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Abstract

As humans opinions and beliefs permeate our social encounters and actions, the things we say, and the decisions we take. For this reason, their understanding is of fundamental importance in the field of social science, but due to their complexity, our knowledge about how social interaction shapes personal opinions is limited. For this reason, the field of opinion dynamics has attracted the interest of both social and exact sciences. The main objective is to model the opinion of a group of agents via a simple description of how they interact with one another. Studying the condition with which it is possible to reach consensus and polarization. Several models have been proposed through the years that mainly build on one another. The goal of this work is to provide an overview and comparison of their basic assumptions, motivations as well as limitations of these different methodologies, and potential for expanding on them by future work.

Contents

1 Individual contributions

Lukas Keller: Implemented the classical model & wrote paragraphs 3.1, 4.1 and 5.1. Collected all the python code for the different models, adjusted the implementations such that they work in a similar way and are easier to use & put it all in one file. Prepared the part of the presentation on the classical model.
Duy Lai: Introduced the context of voting behavior and modelled with the modified co-evolution network. Implemented the model with Matlab. Wrote paragraphs 4.3, 5.3 and 6.3. Prepared the part of the presentation on the co-evolution network model.
Javier Naya: Implementation of the bifurcation diagrams for the bounded confidence model, as well as different probability distributions as initial conditions. Wrote the abstract, introduction, bifurcation part of bounded confidence alongside Agnese. Conclusion and bifurcation diagrams in the presentation.
Valentyna Pavliv: Implemented the susceptibility model (classical model with weights on opinions). Translated Matlab code into Python code in order to standardize all code. Prepared the slides of the presentation and the introduction part.
Agnese Sacchi: implemented the bounded confidence model, in particular the opinion dynamics and change in opinion for the uniform, symmetric interval, uniform, asymmetric interval and opinion dependent interval. Wrote paragraphs 3.2, 4.2 and 5.2 (excluding the

part on the bifurcation diagram) together with Javier. Prepared the part of the presentation on the bounded confidence model.

2 Introduction and Motivations

Opinions have always been one of the individualizing attributes an individual possesses, shaped by past experiences and interactions with other individuals, the dynamic evolution of such ideas is of great interest in the realm of social sciences. With the advent of the information age and an increasing number of ways to interact and share opinions, the quantification of these interactions can lead to insights that can enhance decision making at the political and societal level. The range of relevant issues includes topics such as minority opinion spreading, collective decision making, the emergence of political parties, etc. [1].

Given the complexity of human interaction, the creation of opinion dynamic models that can accurately describe it is of great interest to the scientific community. Starting from psychological theories of human interaction such as imitation of peers, social pressure to conform with group norms, confirmation bias, etc [2]. All these well known behaviors form the motivation behind the opinion dynamic models. From here the expectation of these models is to reproduce experimental behaviors from surveys and studies [3], and being able to answer the question of 'Why, despite tendencies to convergence, differences between individuals and groups continue to exist'[4]. In particular, these models study the formation of a consensus from the initial distribution of opinions, clustering of groups with different opinions, and polarization with extreme views [5].

One of the first attempts to create such models comes from Degroot [6], who in 1974 proposed what we now call the classical model. Introducing a vector of opinions for the agents, and modeling their interaction via realizations of a stochastic matrix acting on the opinions of agents. Further development of this model was made by Friedkin and Johnsen [7], adding additional weight to an individual's initial opinion. The common assumption being qualitative evidence that highlights the role of imitation among peers and social pressure to conform with group norms.

The bounded confidence model, on the other hand, takes into account the confirmation (or similarity) bias, where individuals tend to ignore the opinions of others completely if their views differ by a significant amount. Initially proposed by Deffuant and Weisbuch [8] in the now known as DW model, only two agents could interact at a time, where they would either ignore or average their opinions depending on the distance between these. The model we will study today, proposed by Hegselman and Krause [9] and known as the HW model, considers that all agents interact with each other at each time step. There is also a generalization by Urbig [10] where there is an arbitrary number of interactions possible and bridges the gap between the HW and DW models.

The bounded confidence model is one of the most popular models of opinion dynamics due to its relatively simple description that can nevertheless reproduce consensus, clustering, and polarization behaviors via the modification of the model parameters. Modified versions of it have been used in studying the understanding of social networks [11] and e-commerce [12]. The influence of opinion leaders [13] and radicalization [8], the use of representatives from groups in decision making [14], the influence of 'open mindedness' of agents in the reaching of a consensus in a community [15], and also the influence of noise and individualization in pluralization and clustering behavior [16]

A critical difference between the classical and the bounded confidence model is the way the clustering is formed. For the former, interactions between the agents are fixed, thus there are only opinion changes possible. Meanwhile, in the latter, the shape of the community is formed based on the opinion of the agents, with the caveat of interacting with every other agent at the same time. To form a bridge between the models we also implemented a co-evolution network [17], in which both the structure and opinions are dynamic. Furthermore, we modified the model to work with a discrete 'distance' between opinions, to have a point of comparison with the previous two models.

As we can see, there is a wide variety of models that have been developed to study opinion dynamics. But, as Flache et al. mention in their review: 'A central problem of the literature on social influence dynamics is that there are many models but little is known about their relation to each other' [2]. So the objective of this work is to shed light on the basic assumptions, motivations, and limitations for the previously described models. The third section will provide a theoretical description of each of the three models, followed by an explanation of the parameters and methodology used for the simulation. Then a discussion about the results obtained and finally a summary where the findings are discussed.

3 Description of the Model

An important distinction to make about the following models is the types of opinions their agents can hold. Both the classical and the bounded confidence model have a continuous distribution of possible opinions, usually set between [0,1]. This means the models are useful for representing topics such as political bias (left or right), level of agreement/disagreement with a legislation, or even the estimated price of an item or stock. While useful this can also be a limitation when trying to describe issues such as party representation or even personal taste such as favorite bands or movies. As will be described bellow, the co-evolution network approach works with a discrete opinion space, providing a complementary approach as the previous models. In our case, we will utilize a discretized spaced with equally distanced points (such as 1 to 10), where we can implement a 'distance' measure and as such have a comparison between the previous models.

3.1 The classical model

Probably the most basic of continuous opinion dynamics models is the classical model which calculates opinions of a population by multiplying an opinion vector by a constant stochastic matrix [6]:

$$x(t+1) = Ax(t) \text{ for } t \in \{0, 1, 2, \cdots N\}$$

Here $x(t)$ is a vector containing the opinion of the population (of size N) at time t. The opinion is represented by a real number in the interval $[0, 1]$, thus the name "continuous model". The matrix A is a stochastic matrix (i.e. each row sums up to 1) that represents the connections between different agents and how they value each others opinion. This means that for each step the agent will average out the opinion of their connections, thus reaching a consensus with them, but not forming any new connections or modifying their existing network.

This classical model is very basic and easy to analyze also analytically (it reduces to calculating matrix powers). This of course limits the strength of the model in accurately predicting the opinion of certain populations. However, it is useful in answering basic questions and creating models that require simple boundary conditions. Its lack of different parameters benefits this by making it less susceptible to small changes in the input.

One use case would be modeling different communities inside a given population by subdividing the matrix A into smaller sub-matrices arranged on the diagonal, called opinion fragmentation by Hegselmann [9]. Here an example with a population of 4:

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

As an example of how one would use this model we tried answering the question of how a celebrity, that acts as a link between multiple communities and influences the change in opinion of a given population.

3.2 Bounded Confidence Model

Considering an opinion profile $x(t)$ at time t , we can update the vector via the following rule

$$x(t+1) = \frac{1}{|I(i, x(t))|} \sum_{j \in I(i,x(t))} x_j(t)$$

where $I(i, x) = \{1 \leq j \leq n||x_i - x_j| \leq \epsilon_i\}$ is the set containing agents whose opinions fall within an interval ϵ of agent i's opinion. In the case where the confidence interval is asymmetric we have $I(i, x) = \{1 \leq j \leq n| - \epsilon_l \leq x_j - x_i \leq \epsilon_r\}$. In this project we consider three different types of confidence interval:

1. Uniform, symmetric: the confidence interval $[-\epsilon, \epsilon]$ is symmetric and equal for every agent, i.e. the opinion of agent i will only be influenced by opinions x_j with $|x_i - x_j| \leq \epsilon$.

2. Uniform, asymmetric: the confidence interval $[-\epsilon_l, \epsilon_r]$ is again equal for every agent but asymmetric with $\epsilon_l \neq \epsilon_r$ and $0 \leq \epsilon_l, \epsilon_r \leq 1$. The opinion of agent i will only be influenced by opinions x_j with $-\epsilon_l \leq x_j - x_i \leq \epsilon_r$.

3. Opinion dependent, asymmetric: in 2. we only considered asymmetries which are independent of the opinion an agent holds. However, it is a common phenomenon that people whose opinion is to the right/left (which in our model corresponds to a value x_i closer to 1/0) listen more to people whose views are further right/left, while being sceptical of opinions which are more left/right. This translates into a model where opinions to the left have a confidence interval with $\epsilon_r < \epsilon_l$ and vice versa. For the special case of center opinions ($x = 0.5$) the confidence interval will be symmetric.

3.3 Monte Carlo approach – Co-evolution network

A particular inadequacy of the classical models for opinions is that individuals may not "switch sides" at some random time. Since opinion dynamics, as do other social fields, usually exhibits stochastic nature, a probabilistic (so-called Monte Carlo) approach is therefore more realistic. A more adequate model, which could be that proposed in [17] is thus implemented. This model is also modified here in order to simulate the voting behaviors introduced in Section 4.3.

Concretely, let us assume a network of N vertices, representing N individuals, joined by M edges representing the connections between the agents. There are G discrete opinions where the opinion of individual i is denoted as g_i. A side remark is that opinions may now go extinct since they are presented discretely. We also assume here that the number of possible opinions and edges scale in proportion to the number of individuals, i.e. we define the parameters $\gamma := \frac{N}{G}$ and $\bar{k} := \frac{2M}{N}$, where $\bar{k}$ is the mean degree, or number of neighbors, of the vertices. In other words, it is $\sum_{i=1}^{N} k_i = 2M$, since each edge is counted twice, once for each of its ends. The M edges of the network are initially placed uniformly at random between vertex pairs, and opinions are assigned to vertices uniformly at random. At each step, we either

1. Pick a vertex i at random. If the degree k_i, i.e the number of neighbors of i, is zero, do nothing. Otherwise, with probability ϕ, select at random one of the edges attached to i and move the other end of that edge to a vertex chosen randomly from the set of all vertices having opinion g_i.

2. Otherwise (i.e., with probability $1-\phi$) pick a random neighbor j of i and set g_i equal to g_j with the *joint probability* $P(x \leq \phi_1) \times P(x \leq \phi_2)$, where

   - $\phi_1 = \frac{5/3}{|g_j - g_i(t=0)|}$ proportional to the inverse of distance between g_j and the *initial* $g_i(t=0)$,
   - $\phi_2 = \mathcal{N}\left(\frac{M}{2}, M \times 0.3; k_j\right)$, the normal distribution of mean $M/2$ and variance $M \times 0.3$ evaluated at k_j,
   - x is a random number taken from the continuous uniform distribution $\mathcal{U}(0,1)$.

The algorithm repeats step 2 until both criteria are satisfied while at each loop, a new j is picked.

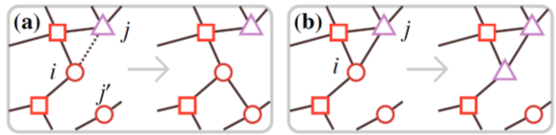


Figure 1: Illustration of the co-evolution network model. [17]

4 Implementation

4.1 The classical model

To answer the question of how celebrities influence opinion dynamics between different communities without other connections between them, we chose a population size of 80 and divided those up into 5 communities. (This size seemed like a good compromise between big population size for accurate results and small size for more readable results.) The values of the matrix A are chosen at random inside each community, while laying a greater weight on their own opinion. We simulated the change in opinion in this population as a control and then added different numbers of celebrities to the existing population. A celebrity influences a given agent with a certain probability (0.3) and with a connection strength in a given interval ($[0, 0.2]$).

We also redid the same simulation after adding some random connections between agents of different communities to represent the real world more accurately (e.g. social networks or contacts outside your community).

4.2 Bounded Confidence

We looked at the bounded confidence model with uniform, symmetric and asymmetric intervals for different values of ϵ resp. ϵ_l, ϵ_r to gain an insight on different phase transitions. Additionally, in the case of symmetric intervals we plotted the change in opinion from one period to another, i.e. $\Delta_i(t-1) = x_i(t) - x_i(t-1)$.

To implement the opinion dependent interval we introduce the opinion dependent biases $\beta_l, \beta_r > 0$ with $\beta_l + \beta_r = 1$. These divide the total confidence interval $\bar{\epsilon}$ into two asymmetric parts and are determined by the opinion dependent function $f(x) = mx + \frac{1-m}{2}$, with $0 \leq m \leq 1$. The total confidence interval $\bar{\epsilon}$ and the slope m are the two parameters we can vary in this model, the latter determines the strength of the bias, for $m = 0$ we don't have any bias. Hence, we define $\beta_r(x) = f(x)$, $\beta_l(x) = 1 - \beta_r(x)$ and $\epsilon_r(x) = \beta_r(x)\bar{\epsilon}$ and $\epsilon_l(x) = \beta_l(x)\bar{\epsilon}$.

In addition to studying individual trajectories of opinions, it is insightful to see how the final opinion profiles for the agents vary depending on their opinion range ϵ. The resulting figure is called a bifurcation diagram, where the y-axis represents the final opinion clusters for each value of ϵ. This is possible because we know that after a finite amount the solution will always converge to an answer. To implement the diagram different initial conditions were considered; first a uniform opinion distribution, which although not realistic is a consistent way to study the model; a gaussian distribution, which would be expected if we were sampling the opinion of a large population, and finally a bimodal gaussian with the peaks on two different extremes in order to simulate polarization in a particular topic.

4.3 Co-evolution network & voting behaviors

Following the 2020 United States elections, an example of how one might use this model is comparing the American electoral system with the European one. One may wonder how American political opinions are divided mainly into two blocks, namely democrats and republicans. It turns out that, in the United States, as in most of the English-speaking countries, it is the first-past-the post (FPTP) system which is used as voting method. And the FPTP is a particular method of the so-called plurality voting.

In these systems, individuals tend to vote for the candidate whom they believe, has the higher chance to win. In fact, plurality means that the candidate must receive at least 50% of the votes in order to win. In order for plurality to be achieved, individuals will align their political opinions with one another for the winning cause. Given that opinions are distributed uniformly over the scope, this mechanism results in a consensus of two political blocks. While on the contrary, in most of the European countries, it is the proportional representation system which is used.

Back to the model, if $\phi \to 0$, it is step 2 where the individuals' opinions change, which is the most probable. Consequently it represents the plurality voting case, namely in the United States or the United Kingdom. While if $\phi \to 1$, the individuals do not change their opinions but only exchange connections. This case thus corresponds to the proportional representation, which is present in most European countries.

In terms of statistical physics, the cases $\phi \to 0$ and $\phi \to 1$ respectively correspond to the ordered and disordered states while ϕ can be regarded as the order parameter. In [17], by using the method of finite-size scaling, the critical value ϕ_c is determined to be 0.458 ± 0.008.

5 Simulation Results and Discussion

5.1 The classical model

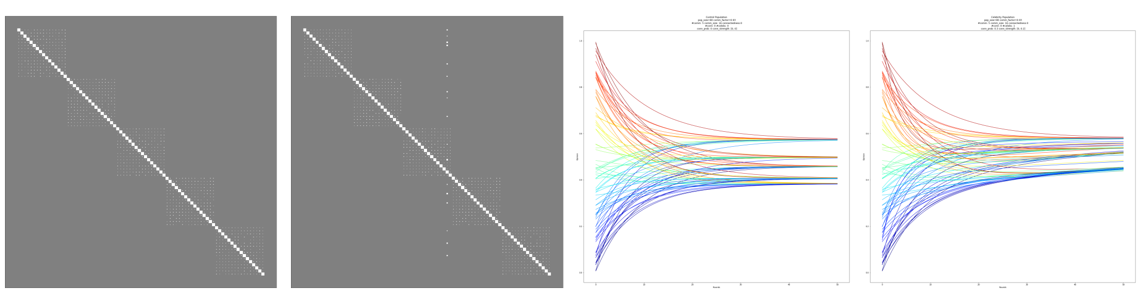
We use existing code [18] to visualize the matrix A in a so called hinton diagram, which indicates the connection strength between the agents by the size of the white squares.

As we can see in Fig 2 the opinions in the control population converge to five distinct values. But while the graph with one celebrity still shows some clustering around certain values, the clear difference between the communities begins to disappear. If we add four celebrities to the population we can see in Fig 3 that this effect gets even stronger, here almost all opinions start converging to the same value. Clearly indicating that celebrities (i.e. agents that strongly influence a lot of other agents) have a strong effect on the change in opinion of a population.

Adding connections between different communities, the different opinions already converge to each other a lot more in the control population. However adding one celebrity as in Fig 4 the observed effect from above clearly still occurs, but it now takes less celebrities until the opinions quickly converge to one single value. This happens in Fig 5 pretty fast with just three celebrities.

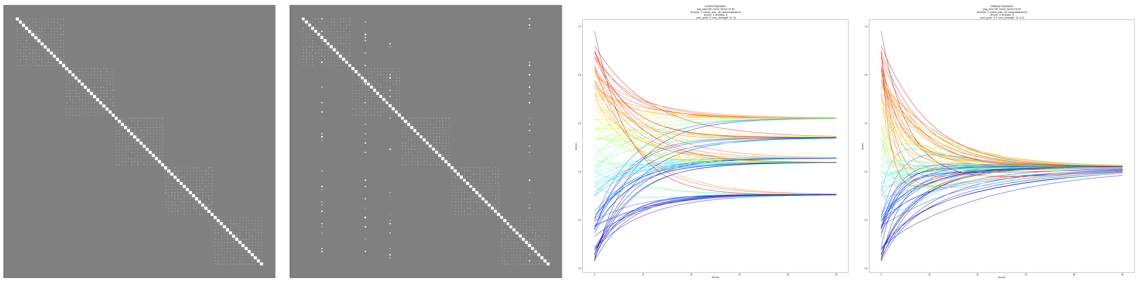
The findings here corroborate results from Degroot's orignal paper, that if all states are connected in some way or another, then the consensus will always be reached in finite time. Which means the only way to obtain clustering behaviour is to separate the communities such that they don't have any contact with each other at all, and thus creating two independent classical models.

This experiment shows a good use for this basic model. However, there clearly are limitations: with larger population sizes the problem of creating a fitting matrix A becomes increasingly difficult. Especially modeling different communities in a population is possible (as done above), however in the real world communities change over time and in relation to the opinions of the population at a given time. This is where the bounded confidence model comes to play.



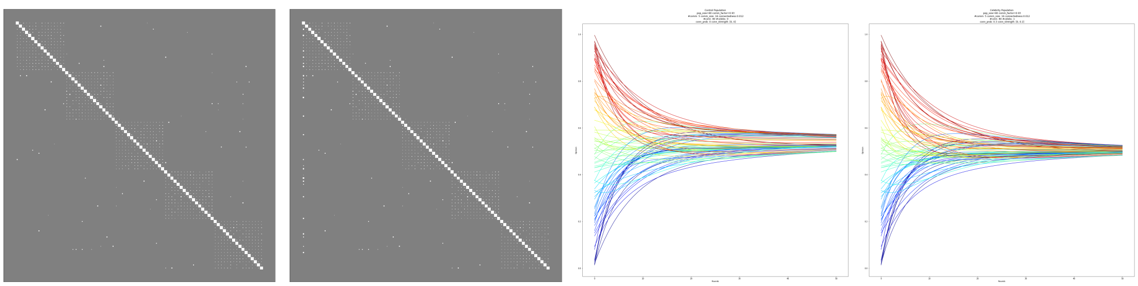
(a) control (b) 1 celebrity (c) control (d) 1 celebrity

Figure 2: Basic population, 1 celebrity



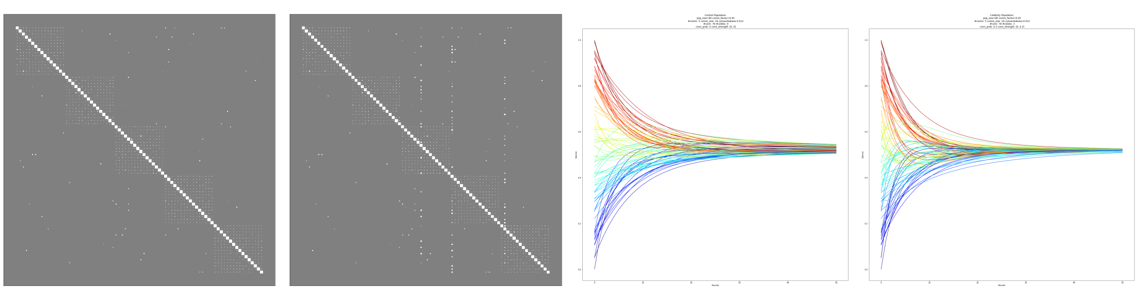
(a) control (b) 4 celebrities (c) control (d) 4 celebrities

Figure 3: Basic population, 4 celebrities



(a) control (b) 1 celebrity (c) control (d) 1 celebrity

Figure 4: Population with more connections, 1 celebrity



(a) control (b) 3 celebrities (c) control (d) 3 celebrities

Figure 5: Population with more connections, 3 celebrities

5.2 Bounded confidence

For the uniform, symmetric interval we took a population of 625 and values of $\epsilon = 0.01$, 0.15 and 0.25. As we can see from Fig 7 while for a value $\epsilon = 0.01$ we can count up to 30 different surviving opinions, for a higher value of $\epsilon = 0.15$ the agents end up converging to four different opinions. Finally, for $\epsilon = 0.25$ consensus is reached. These results show that by increasing the value of ϵ we go through three different phases: we start with fragmentation (when we have a large number of surviving opinions) to then step into polarisation and finally reach consensus.

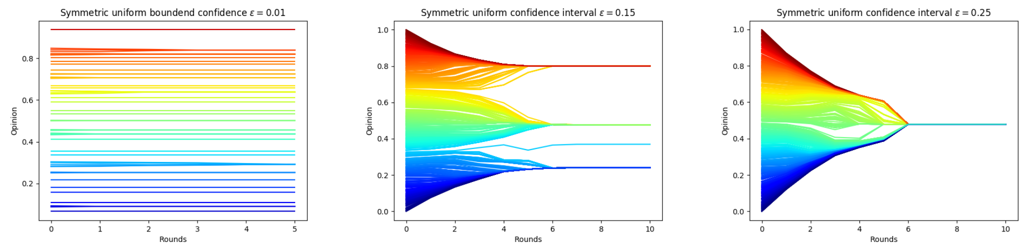
This can be more clearly interpreted via the bifurcation diagram, where for low values of epsilon there are several clusters of final opinions that are formed, while once over a certain value of epsilon (around $\epsilon \sim 0.25$) there is a transition where consensus is always reached. Thus consensus and clustering can be both seen in these diagrams for several initial conditions. For the truncated normal (we use $\mu = 0.5$ and $\sigma = 0.23$) we can observe that a lower ϵ value is needed before reaching a consensus, expected given the higher population density near central opinions. And for the bimodal normal ($\mu_1 = 0.15, \mu_2 = 0.8$ and $\sigma_1 = \sigma_2 = 0.15$). Bifurcation diagrams for asymmetric and asymmetric dependent models can be found in the github repository.

Then, we looked at the change in opinion from one period to another, i.e. $\Delta_i(t-1) = x_i(t) - x_i(t-1)$. As you can see in Fig 8 for $\epsilon = 0.01$ the plot appears to be quite messy, but for higher values of ϵ we can recognize a pattern. First, the more extreme opinions (shades of blue and red) undergo considerable changes while the change in central opinions (shades of green) is barely visible. As time goes on however, the trend is inverted and opinions at the center are the ones which make bigger jumps.

For the uniform, asymmetric interval we chose an asymmetry that favours opinions to the right i.e. $\epsilon_l < \epsilon_r$. We looked at the dynamics for values a) $\epsilon_l = 0.02, \epsilon_r = 0.04$, b) $\epsilon_l = 0.03, \epsilon_r = 0.15$ and c) $\epsilon_l = 0.1, \epsilon_r = 0.25$. From Fig 9 one can easily see that by increasing the confidence interval we step again over three different phases: going from plurality, over to polarity and to conformity. Move over, as one might expect, the asymmetry favours opinions to the right i.e. opinions with values closer to 1.

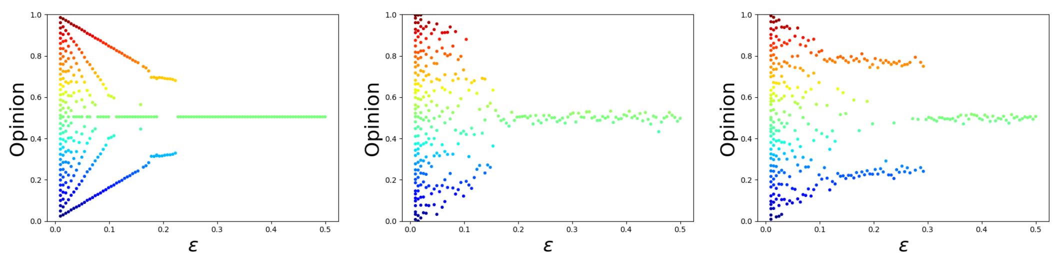
Finally, for the opinion dependent asymmetry we first looked at how the opinion dynamics change for different values of m and $\bar{\epsilon}$. In Fig 10 we can see the effect that the strength of the bias has on the opinion dynamics: as the bias increases the surviving opinions will be more and more polarised towards the extremes and the central opinions might even die out completely. While in the absence of bias ($m = 0$) consensus is reached, already for values of $m = 0.2$ there is a polarisation in opinions, which keeps increasing for higher biases. On the other hand, in Fig 11 we see how the opinion dynamics changes for increasing confidence intervals. As in the case of the uniform, symmetric interval, we witness the phase transition from fragmentation to polarisation and finally to consensus. However, compared to the uniform, symmetric bounded confidence model, the phase transitions now take place for bigger confidence intervals, the bias in fact, "fights against" consensus and towards polarisation, as we have just seen.

The models introduced above build one on top of the other up to the opinion dependent interval, which is slightly more complex, yet more realistic. There are however, several limitations to these models. First of all, unlike in the classical model, there is no way to constrain the connections physically and to simulate the behaviour of different communities. Moreover, while from experience we know that a personal interaction is more likely to influence someone's opinion than when interacting with them online, in the model there is no differentiation in the types of interactions. One last limitation we identified, is that in this model equal weight is put on the opinions of the interacting agents.



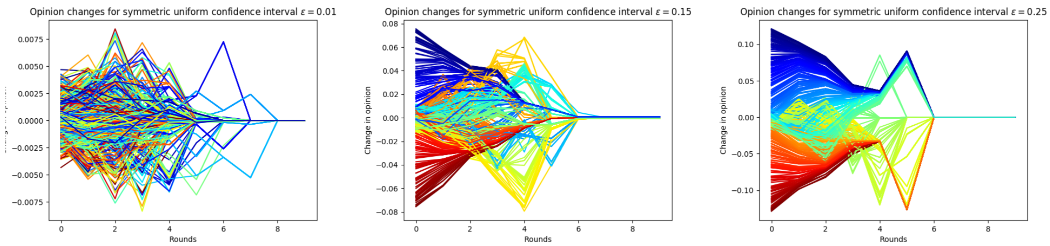
(a) $\epsilon = 0.01$ (b) $\epsilon = 0.15$ (c) $\epsilon = 0.25$

Figure 6: Opinion dynamics for uniform, symmetric confidence interval.



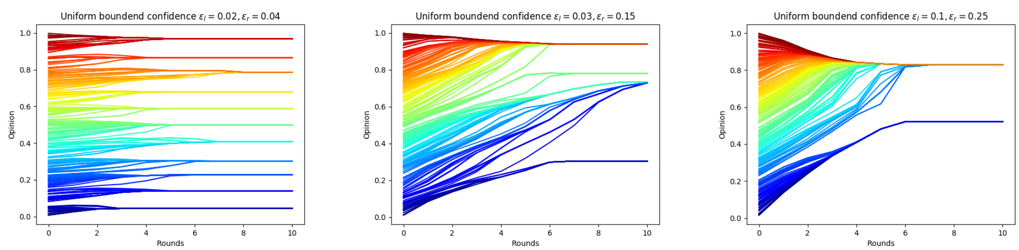
(a) Uniform (b) Truncated Normal (c) Bi-modal Normal

Figure 7: Bifurcation diagram for symmetric bounded confidence model. Population size of 200 and 100 different values of equally spaced ϵ used. Several initial distributions shown



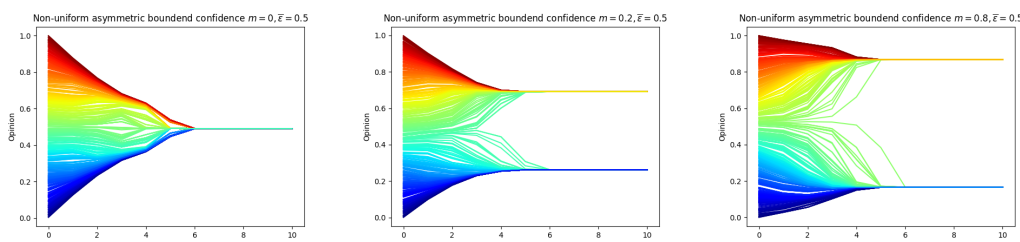
(a) $\epsilon = 0.01$ (b) $\epsilon = 0.15$ (c) $\epsilon = 0.25$

Figure 8: Change in opinion over time for the opinion dynamics of Fig 7.



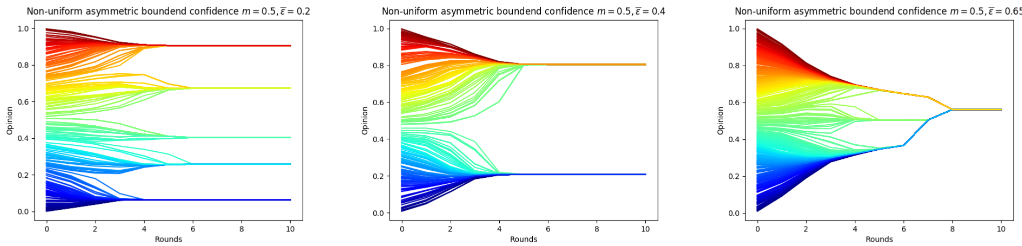
(a) $\epsilon_l = 0.02, \epsilon_r = 0.04$ (b) $\epsilon_l = 0.03, \epsilon_r = 0.15$ (c) $\epsilon_l = 0.1, \epsilon_r = 0.25$

Figure 9: Opinion dynamics for uniform, asymmetric confidence interval.



(a) $m = 0, \bar{\epsilon} = 0.5$ (b) $m = 0.2, \bar{\epsilon} = 0.5$ (c) $m = 0.8, \bar{\epsilon} = 0.5$

Figure 10: Opinion dependent asymmetry with fix $\bar{\epsilon} = 0.4$

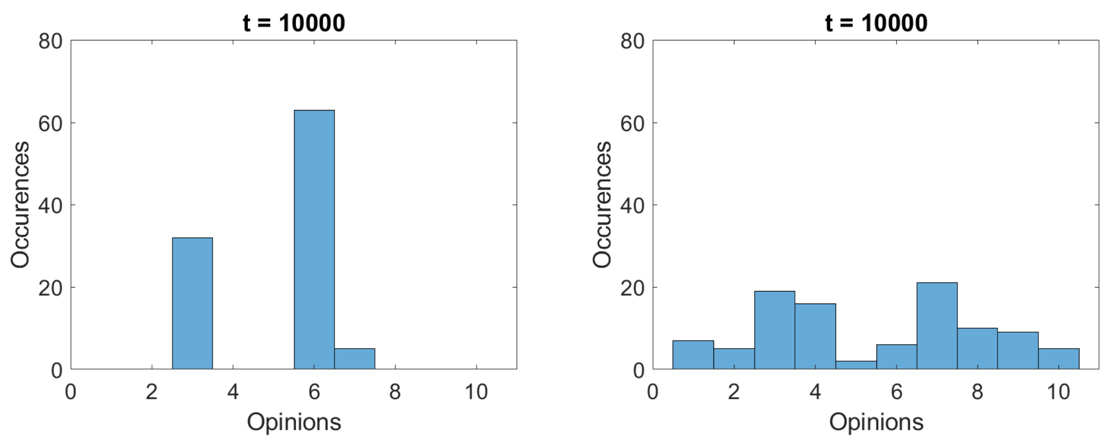


(a) $m = 0.5, \bar{\epsilon} = 0.2$ (b) $m = 0.5, \bar{\epsilon} = 0.4$ (c) $m = 0.5, \bar{\epsilon} = 0.65$

Figure 11: Opinion dependent asymmetry with fix $m = 0.5$

5.3 Voting behaviors

For the following simulations, ϕ is chosen as one small value and one large in order to illustrate the two states while $N = 100$, $\gamma = 10$ and the mean degree $\bar{k} = 4$ are kept fixed. Initially, the opinions are distributed uniformly in both cases.



(a) Plurality voting, $\phi = 0.25$. (b) Proportional representation voting, $\phi = 0.75$.

Figure 12: Political opinions. [*Click on subcaptions for animated evolutions*[1]]

Let us take a look at Figure 12a. With a small value of ϕ below ϕ_c, the consensus wraps up with two blocks in which one has the majority of votes, i.e. above 50%. It is observed that, despite being divided, opinions do not get extremist since their probabilities to change are inversely proportional to their mutual distances. Meanwhile thanks to the Gaussian implemented, a certain block has the most chance to acquire votes when the number of its "members" is half of the population. By the same way, due to the initial uniform distribution, there is a similar block on the other side of the opinion scope while the rest of available opinions die out. This is the mechanism of the plurality voting system.

[1] Thanks to Robert Bemis for the GIF MATLAB code.

Combining with the inverse proportionality in opinion distances, the mechanism prevents a favorable block from getting extremist, thus a potential "dictatorship". The result is then there are two big blocks of moderate opinions.

On the other hand, with a large value of ϕ in Figure 12b, one can see that impositions on the individuals' opinions are largely removed. Instead, they rather exchange connections and probably move to an opinion being neighbor to their own one. The opinions stay more or less randomly distributed as at the beginning.

Speaking of simulating with the same parameters, one does not, however, expect to obtain the same exact result in details since there is stochasticity during the process[2]. Nevertheless, it is important to look for the quantitative (global) behavior, which is produced by the mechanism: the same pattern of opinions evolution is being created and can be recognized when the time of simulation is large enough. It can be said furthermore that the equilibrium state of the system is reached when opinions do not change anymore. This fact may be verified by comparing the two Figures 12, in which opinions in the proportional representation system do not change as much as in the plurality voting one. Hence, in the case of proportional voting, the equilibrium state is reached faster.

6 Summary and Outlook

After looking in depth into three different models for opinion dynamics we gained a good understanding of what are the strengths and weaknesses of each model.

While the classical model is the simplest, it fails to explain how diversity can exist or even increase over time and how opinion clusters can exist, even in highly connected networks. As discussed previously, the limitation with the classical model is of 'physical' constrain, meaning that community members are restricted to only participate in their specific communities (or more broadly, by the chosen design of the matrix A) which is not influenced by time or by the current opinions.

On the other hand, the bounded-confidence model gives a more **dynamic** approach to the process, allowing each agent to choose whom he might interact with. In other words, everyone knows everyone else's opinion but only interacts with those who have similar opinions, more accurately fitting the role of modern social networks or interactions seen in the real world. An advantage of this model is that it can replicate in a simple manner the consensus, clustering and bi-polarization. A limitation however, is that it puts equal weight into the opinions of the interacting agents, so your own opinion is taken into account as much as the others, more akin to interactions with social pressure. Moreover, only one type of interaction is considered amongst agents, even though 'individuals are more open to vicing their opinions online, but social influence is stronger in person' [11].

In conclusion, the bounded confidence model improves the classical model but looses one of it's useful characteristics: the possibility to limit the connections by physical constraints. In the bounded confidence model, in fact, it is assumed that every agent knows all other

[2]The reader shall be aware of this aspect when reproducing the simulation.

agents and simply chooses whether to interact with them or not, this assumptions is, as far as personal relations are concerned, not always realistic. In both of the above mentioned models opinions have to live in a continuous and bounded space, meaning questions like which political party do you support, or, are you in favor of marijuana legalization can't be properly modeled. A stronger limitation of this model is the lack of research quantifying the thresholds and distributions in real populations.

For these scopes is the co-evolution network better suited. In addition, the latter provides deeper insights on how the context influences opinions and vice versa while taking the probabilistic aspect into account, which corresponds more to reality.

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