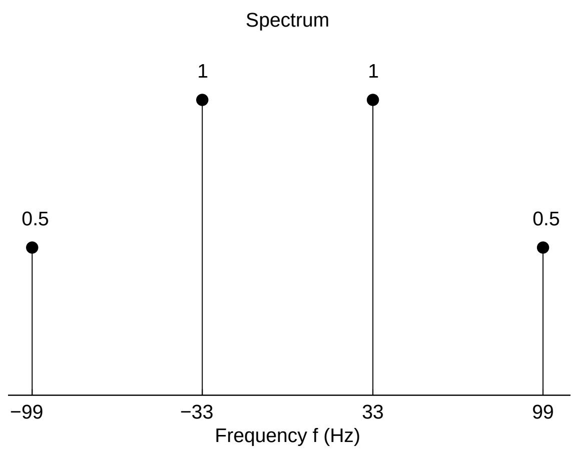
EE102B Signal Processing and Linear Systems II

Solutions to Problem Set Two
2012-2013 Spring Quarter

Problem 2.1 (20 points)

(a)



$$x(t) = \frac{1}{2}(2e^{j2\pi 33t} + e^{j2\pi 99t} + 2e^{-j2\pi 33t} + e^{-j2\pi 99t})$$

$$f_0 = 33 \text{ Hz}$$

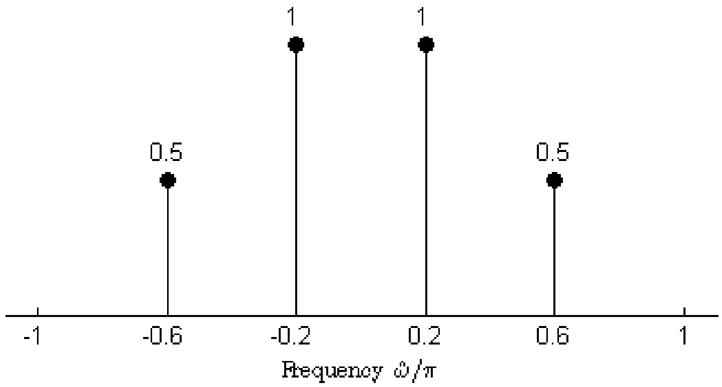
(b) To obtain the spectrum of the discrete signal, we first sample the signal according to $n = f_s t$:

$$x[n] = 2\cos(2\pi 33 \times \frac{n}{30}) + \cos(2\pi 99 \times \frac{n}{30})$$

$$x[n] = 2\cos(2\pi \times 1.1n) + \cos(2\pi \times 3.3n)$$

This can be simplified:

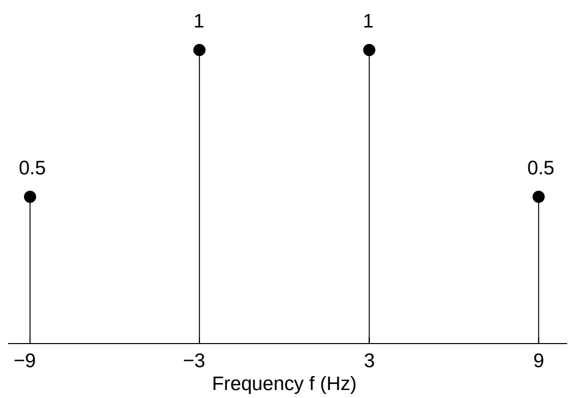
$$x[n] = 2\cos(2\pi \times 0.1n + 2\pi n) + \cos(2\pi \times 0.3n + 3 \times 2\pi n) = 2\cos(0.2\pi n) + \cos(0.6\pi n)$$



(c)

$$n = f_s t$$

$$y(t) = 2\cos(6\pi t) + \cos(18\pi t)$$



$$f_0 = 3 \text{ Hz}$$

(d) Note that for our signal $y(t)$, the fundamental frequency $(f_0)_y$ is indistinguishable from $-(f_0)_y$. From parts b) and c), we can see that $(f_0)_y = (f_0)_x - kf_s$, where k is an integer such that $-\frac{1}{2}f_s \leq (f_0)_y \leq \frac{1}{2}f_s$.

This suggests that to achieve an arbitrary $|(f_0)_y| \leq (f_0)_x$, we can choose f_s as $f_s = (f_0)_x + |(f_0)_y|$

If $|(f_0)_y| \leq \frac{1}{3}(f_0)_x$, we can also choose f_s as $f_s = (f_0)_x - |(f_0)_y|$.

Problem 2.2 (0 points)

(a) For the discrete signal to be periodic with period N, the following must hold for all n:

$$A\cos(\omega_0 n T_s + \phi) = A\cos(\omega_0 (n+N) T_s + \phi)$$

This is true if for some integer k:

$$\omega_0 n T_s = \omega_0 (n+N) T_s - 2\pi k$$

Thus, for the signal to be periodic it is sufficient that (for any integer k and N):

$$T_s = \frac{2\pi k}{\omega_0 N}$$

Equivalently, $\frac{T_s \omega_0}{2\pi} = \frac{k}{N}$ so in other words it is sufficient that $\frac{T_s \omega_0}{2\pi}$ is rational.

(b)

$$T_s = \frac{2\pi k}{\omega_0 N}$$

$$T_s = \frac{2\pi k}{2000\pi \times 100}$$

$$T_s = 10^{-5},\ 2 \times 10^{-5},\ 3 \times 10^{-5} ...$$

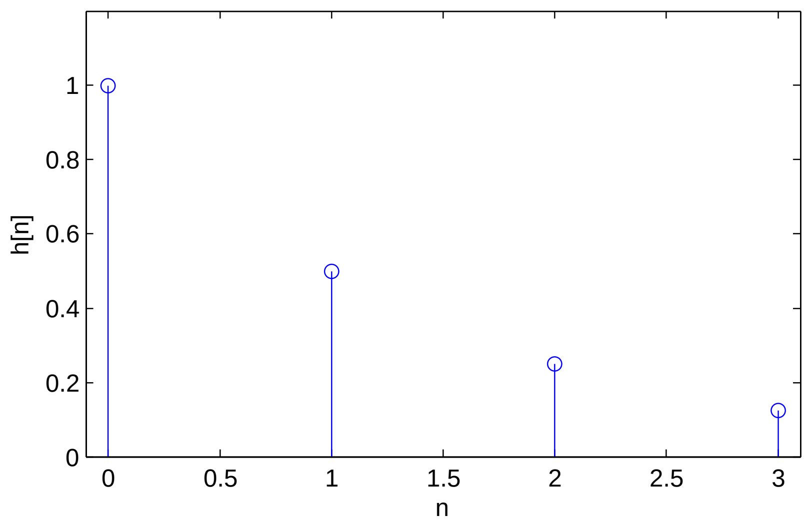
For the fundamental period to be N=100, we choose $k = 1$ and thus $T_s = 10^{-5}$.

Problem 2.3 (20 points)

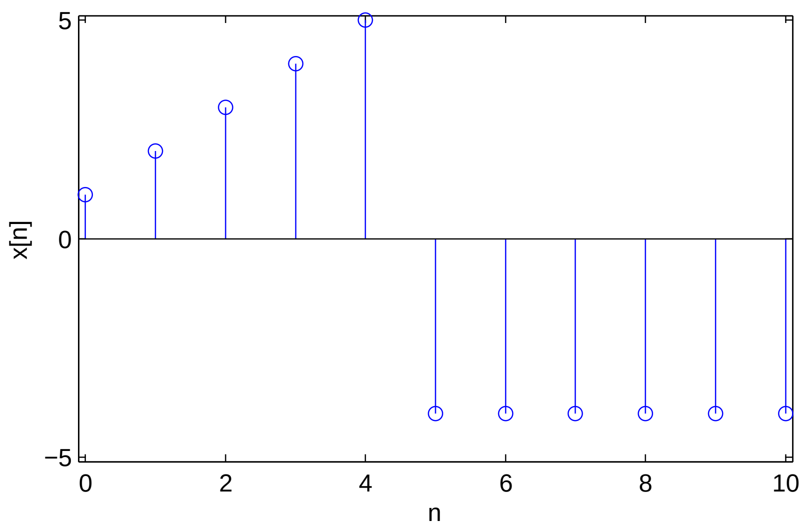
(a) $b_k = (\frac{1}{2})^k$ for $0 \leq k \leq 3$, $b_k = 0$ otherwise.

(b)

$$h[n] = \sum_{k=0}^{3} (\frac{1}{2})^k \delta[n-k]$$



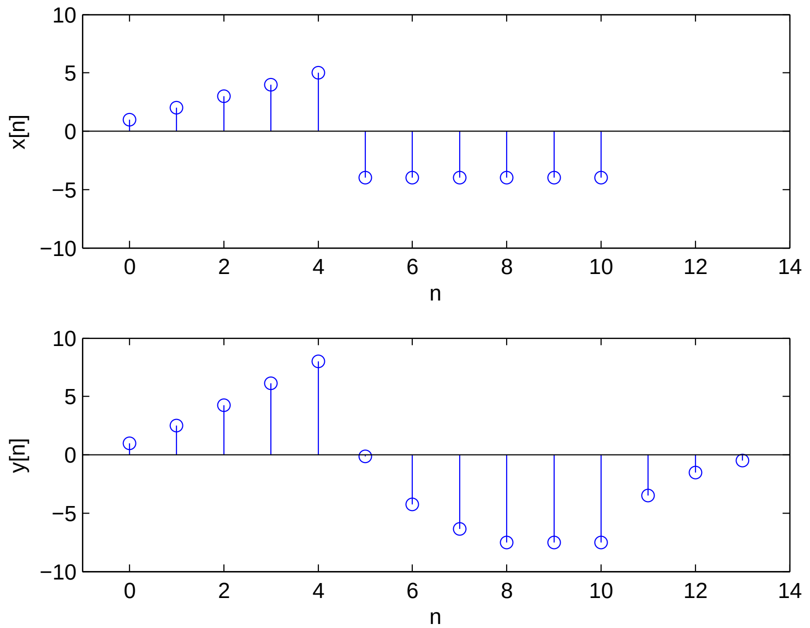
(c)



$$y[6] = (\frac{1}{2})^0 x[6] + (\frac{1}{2})^1 x[5] + (\frac{1}{2})^2 x[4] + (\frac{1}{2})^3 x[3]$$

$$y[6] = -4.25$$

(d)




```
function FIRFilter3d(x,n)
h = (1/2).^(0:3);
y = conv(h,x);
subplot(2,1,1);
stem(n,x); axis([-1 14 -10 10]); xlabel('n'); ylabel('x[n]')
subplot(2,1,2);
stem(n(1):(n(end)+3),y); axis([-1 14 -10 10]); xlabel('n'); ylabel('y[n]')
```

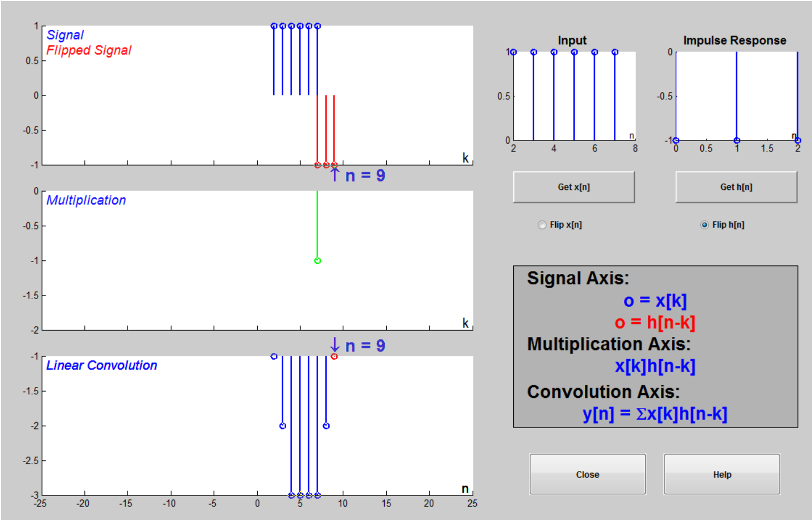
Problem 2.4 (0 points)

Maximize n subject to the constraint that $N_1 \leq k \leq N_2$, $N_3 \leq n-k \leq N_4$ (these constaints ensure at least one element of $y[n] = \sum_{k=N_1}^{N_2} b_k x[n-k]$ is non-zero):

$$N_3 + k \leq n \leq N_4 + k$$

$$n_{max} = N_4 + N_2$$

We come to the same conclusion by using dconvdemo:



Here $N_1 = 0$, $N_2 = 2$, $N_3 = 2$, $N_4 = 7$, and $n_{max} = 9$ as expected.

Similarly,

$$n_{min} = N_3 + N_1$$

Therefore, in the output sequence $y[n]$ the maximum number of non-zero samples is:

$$L = n_{max} - n_{min} + 1 = (N_4 + N_2) - (N_3 + N_1) + 1$$

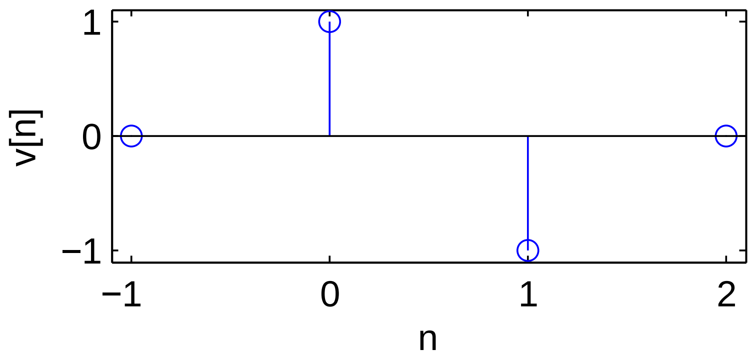
Problem 2.5 (20 points)

(a)

$$v[n] = x[n] - x[n-1]$$

(b)

$$v[n] = \delta[n] - \delta[n-1]$$



(c)

$$h_2[n] = \sum_{k=0}^{3} \frac{1}{4}\delta[n-k]$$

(d)

$$h_1 * h_2[n] = \sum_{k=0}^{3} \frac{1}{4}\delta[n-k] - \sum_{k=0}^{3} \frac{1}{4}\delta[n-k-1]$$

$$h_1 * h_2[n] = \frac{1}{4}\delta[n] - \frac{1}{4}\delta[n-4]$$

(e)

$$y[n] = \frac{1}{4}x[n] - \frac{1}{4}x[n-4]$$

Problem 2.6 (20 points)

(a)

(i) Linear: $(\alpha x_1[n] + \beta x_2[n])\cos(0.2\pi n) = \alpha(x_1[n]\cos(0.2\pi n)) + \beta(x_2[n]\cos(0.2\pi n))$

(ii) Not time-invariant: $x[n-n_0]\cos(0.2\pi n) \neq x[\hat{n}]\cos(0.2\pi\hat{n})\big|_{\hat{n}=n-n_0}$

(LHS: Shift by n_0, then pass through the filter. RHS: Pass through the filter, then shift by n_0)

(iii) Causal: $y[n]$ only depends on $x[n]$

(b)

(i) Linear: $\sum_{k=-1}^{1}(-1)^k(\alpha x_1[n-k]+\beta x_2[n-k]) = \alpha\sum_{k=-1}^{1}(-1)^k x_1[n-k]+\beta\sum_{k=-1}^{1}(-1)^k x_2[n-k]$

(ii) Time-invariant: $\sum_{k=-1}^{1}(-1)^k x[n-k-n_0] = y[n-n_0]$

(iii) Non-causal: $y[n]$ depends on $x[n+1]$, $n < n+1$

(c)

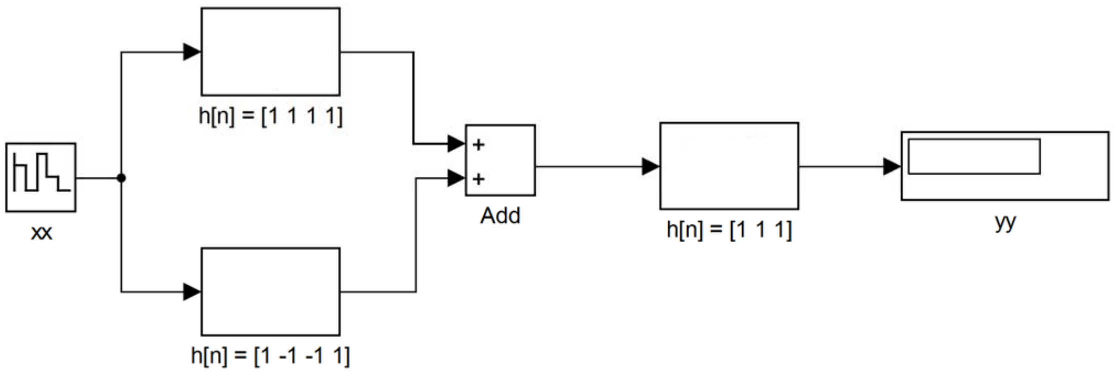
(i) Non-linear: $|\alpha x_1[n] + \beta x_2[n]| \neq \alpha|x_1[-n]| + \beta|x_2[-n]|$

(ii) Not time-invariant: $|x[-(n-n_0)]| \neq y[n-n_0] = |x[-n-n_0]|$

(iii) Non-causal: $y[n]$ depends on $x[-n]$, $n < -n$ for $n < 0$

Problem 2.7 (20 points)

(a) Let us use [1 1 1] as a shortform for $\sum_{k=0}^{2} \delta[n-k]$.



(b)

yy = [1, 1, 1]*ww
yy = [1, 1, 1]*(yy1 + yy2)
yy = [1, 1, 1]*([1, 1, 1, 1]*xx + [1, -1, -1, 1]*xx)
yy = [1, 1, 1]*([2, 0, 0, 2]*xx)
yy = ([1, 1, 1]*[2, 0, 0, 2])*xx
yy = [2, 2, 2, 2, 2, 2]*xx

$$h_T[n] = 2\delta[n] + 2\delta[n-1] + 2\delta[n-2] + 2\delta[n-3] + 2\delta[n-4] + 2\delta[n-5]$$

$$y[n] = 2x[n] + 2x[n-1] + 2x[n-2] + 2x[n-3] + 2x[n-4] + 2x[n-5]$$

Problem 2.8 (0 points)

(a)

$$x[n] = -3 + 2je^{j0.3\pi n} - 2je^{-j0.3\pi n}$$

$$x[n] = -3 - 4\sin(0.3\pi n)$$

(b) We know the following property of LTI filters when the input is a sinusoid (plus a DC):

$$y[n] = -3H(e^{j0}) - 4|H(e^{j0.3\pi})|\sin(0.3\pi n + \angle H(e^{j0.3\pi}))$$

Now, the frequency response of the filter is:

$$H(e^{j\hat{\omega}}) = \frac{\sin((9/2)\hat{\omega})}{9\sin((1/2)\hat{\omega})}e^{-j\hat{\omega}(8/2)}$$

$$H(e^{j0}) = \frac{\sin((9/2) \times 0)}{9\sin((1/2) \times 0)} = 1$$

$$|H(e^{j0.3\pi})| = |\frac{\sin((9/2) \times 0.3\pi)}{9\sin((1/2) \times 0.3\pi)}e^{-j(8/2)\times 0.3\pi}| = 0.2181$$

$$\angle H(e^{j0.3\pi}) = \angle\frac{\sin((9/2) \times 0.3\pi)}{9\sin((1/2) \times 0.3\pi)}e^{-j(8/2)\times 0.3\pi} = \pi - (8/2) \times 0.3\pi = -0.2\pi$$

$$y[n] = -3 - 0.8724\sin(0.3\pi n - 0.2\pi)$$