STANFORD UNIVERSITY
DEPARTMENT of ELECTRICAL ENGINEERING

EE 102B Spring 2013
Problem Set #1

Assigned: April 3, 2013
Due Date: April 10, 2013

Reading: In *SP First*, Review Chapters 9-12 .

$\Longrightarrow$ **Please check the Class2Go website often. All official course announcements are posted there.**

ALL of the **STARRED** problems should be turned in for grading. A solution will be posted to the web on Friday, April 12.

Your homework is due by 5pm on April 10. No late homework will be accepted after 5pm on Friday, April 12.

PROBLEM 1.1*:
The impulse response of a continuous-time linear time-invariant system is

$$h(t) = \delta(t) - 400e^{-400t}u(t).$$

(a) Find the frequency response $H(j\omega)$ of the system. Express your answer as a single rational function with powers of $(j\omega)$ in the numerator and denominator.

(b) Plot the magnitude squared, $|H(j\omega)|^2 = H(j\omega)H^*(j\omega)$, versus ω. Also plot the phase $\angle H(j\omega)$ as a function of ω.

(c) At what frequency ω does the magnitude squared of the frequency response have its largest value? At what frequency is the magnitude squared of the frequency response equal to one half of its peak value? (This is referred to as the 3dB point of the filter since the frequency response magnitude measured in decibels, $10 \log |H(j\omega)|^2$, is 3.01dB smaller at this frequency compared to its peak value when measured in decibels.

(d) Suppose that the input to this system is

$$x(t) = 5 + 10\cos(200\pi t) + \delta(t - 0.05).$$

Use superposition to find the output $y(t)$. *Hint: To find the response of each term, use the easiest method, i.e., impulse response or frequency response.*

PROBLEM 1.2:

A continuous-time LTI system is defined by the following input/output relation:

$$y(t) = -x(t) + 2x(t-T) - x(t-2T). \qquad (1)$$

(a) Find the impulse response $h(t)$ of the system; i.e., determine the output when the input is an impulse.

(b) Substitute your answer for $h(t)$ into the the integral formula

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$$

to find the frequency response. Express your answer in the form $H(j\omega) = R(j\omega)e^{-j\omega t_d}$ where $R(j\omega)$ is a real function of ω and t_d is a real number.

(c) Apply the system definition given in Eq. (1) directly to the input $x(t) = e^{j\omega t}$ for $-\infty < t < \infty$ and show that $y(t) = H(j\omega)e^{j\omega t}$, where $H(j\omega)$ is as determined in part (b); i.e., just substitute $x(t) = e^{j\omega t}$ into Eq. (1).

(d) Sketch the magnitude $|H(j\omega)|$ and phase $\angle H(j\omega)$ as functions of ω.

PROBLEM 1.3:

In each of the following cases, determine the Fourier transform or inverse Fourier transform. Give your answer as a simple formula or a plot. Explain each answer by stating which property and transform pair you used.

(a) Find $X(j\omega)$ when $x(t) = 20e^{-7(t-1)}u(t-1)$.

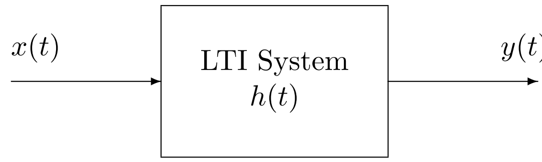
(b) Find $s(t)$ when $S(j\omega) = e^{-j\omega/2}[j\pi\delta(\omega+10\pi) - j\pi\delta(\omega-10\pi)]$.

(c) Find $H(j\omega)$ when $h(t) = \dfrac{d}{dt}\left\{\dfrac{\sin(6\pi t)}{\pi t}\right\}$.

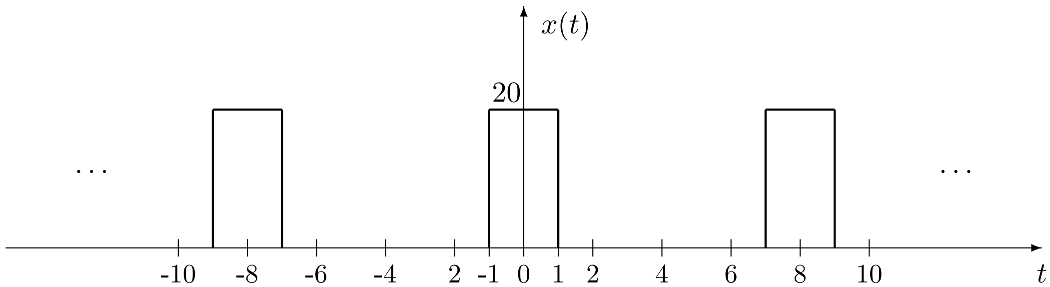
(d) Plot $|H(j\omega)|$ found in part (c) as a function of ω.

PROBLEM 1.4*:

Consider the LTI system below:



The input to this system is the periodic pulse wave $x(t)$ depicted below:



The input can be represented by the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{where} \quad a_k = \begin{cases} \dfrac{20\sin(\pi k/4)}{\pi k} & k \neq 0 \\ 5 & k = 0. \end{cases}$$

(a) Determine ω_0 in the Fourier series representation of $x(t)$. Also, write down the integral that must be evaluated to obtain the Fourier coefficients a_k.

(b) Plot the spectrum of the signal $x(t)$; i.e., make a plot showing the a_k's plotted at the frequencies $k\omega_0$ for $-4\omega_0 \leq \omega \leq 4\omega_0$.

(c) If the frequency response of the system is an ideal lowpass filter

$$H(j\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

where ω_c is the *cutoff frequency*, for what values of ω_c will the output of the system have the form

$$y(t) = A + B\cos(\omega_0 t + \phi)$$

where A and B are nonzero?

(d) If the frequency response of the system is the ideal *highpass* filter

$$H(j\omega) = \begin{cases} 0 & |\omega| < \pi/8 \\ 1 & |\omega| > \pi/8 \end{cases}$$

plot the output of the system, $y(t)$, when the input is $x(t)$ as plotted above. *Hint: First determine what frequency is removed by the filter, and then determine what effect this will have on the waveform.*

(e) If the frequency response of the LTI system is $H(j\omega) = 1 - e^{-j4\omega}$, plot the output of the system, $y(t)$, when the input is $x(t)$ as plotted above. *Hint: In this case it will be easiest to determine the impulse response $h(t)$ corresponding to $H(j\omega)$ and from $h(t)$ you can easily find an equation that relates $y(t)$ to $x(t)$. This will allow you to plot $y(t)$.*

PROBLEM 1.5*:

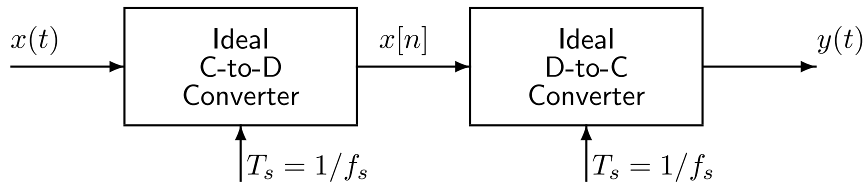


Figure 1: Ideal sampling and reconstruction system.

Shown in the figure above is an ideal C-to-D converter that samples $x(t)$ with a sampling period $T_s = 1/f_s$ to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$. Suppose that $x(t)$ is given by

$$x(t) = [10 + 10\cos(500\pi t - \pi/2)]\cos(2000\pi t)$$

(a) Use Euler's formulas for the cosine functions to expand $x(t)$ in terms of complex exponential signals so that you can sketch the two-sided spectrum of this signal. Be sure to label important features of the plot. Is this waveform periodic? If so, what is the period?

(b) What is the *minimum* sampling rate f_s that can be used in the above system so that $y(t) = x(t)$?

(c) Plot the spectrum of the sampled signal $x[n]$ for the case when $f_s = 5000$. Your plot should include labels on the frequency (on the $\hat{\omega}$ scale), amplitude and phase of each spectrum component.

PROBLEM 1.6*:

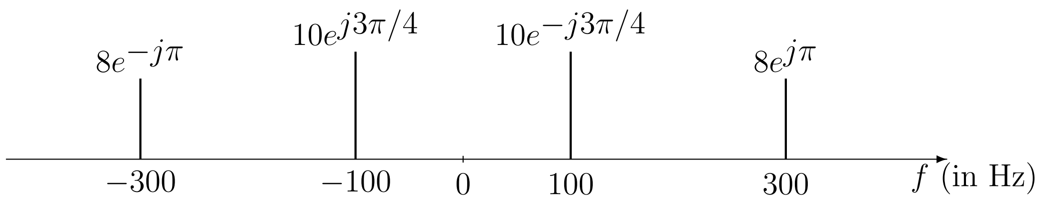
Again consider the ideal sampling and reconstruction system shown in Figure 1 of Problem 1.5.

(a) Suppose that the discrete-time signal $x[n]$ in Figure 1 is given by the formula

$$x[n] = 4\cos(0.3\pi n - \pi/3)$$

If the sampling rate of the C-to-D converter is $f_s = 10000$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs with frequency less than 10000 Hz; i.e., find $x_1(t) = A_1\cos(\omega_1 t + \phi_1)$ and $x_2(t) = A_2\cos(\omega_2 t + \phi_2)$ such that $x[n] = x_1(nT_\mathrm{s}) = x_2(nT_\mathrm{s})$ if $T_\mathrm{s} = 1/10000$ secs.

(b) Now if the input $x(t)$ to the system in Figure 1 of Problem 1.5 has the two-sided spectrum representation shown below,what is the *minimum* sampling rate f_s such that the output $y(t)$ is equal to the input $x(t)$?



(c) Determine the spectrum for $x[n]$ when $f_\mathrm{s} = 300$ samples/sec. Make a plot for your answer. Be sure to label the frequency, amplitude and phase of each spectral component.