STANFORD UNIVERSITY
DEPARTMENT of ELECTRICAL ENGINEERING

EE 102B Spring 2013
Problem Set #2

Assigned: April 10, 2013
Due Date: April 17, 2013

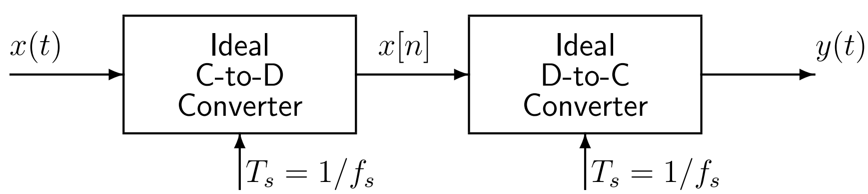
Reading: In *SP First*, Chapter 5 on *FIR Filters* and Chapter 6 on *Frequency Response.*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due by 5pm on April 17, 2013. It can be handed in up to 5pm on Friday, April 19 with a 10% penalty per day late. No credit will be given after that time.

PROBLEM 2.1*:



We can do some interesting things with sampling. One of them is that we can change the period of a periodic waveform. This problem illustrates how this can be done for the specific periodic input signal

$$x(t) = 2\cos(2\pi(33)t) + \cos(2\pi(99)t).$$

In all the following parts, assume that the sampling frequency is $f_s = 30$ Hz. Note that this sampling rate *does not* satisfy the conditions of the Shannon sampling theorem, so aliasing will occur.

(a) Plot the spectrum of the periodic continuous-time signal $x(t)$. What is the fundamental frequency of $x(t)$?

(b) Determine an expression for the discrete-time signal $x[n]$ as a sum of discrete-time cosine signals. Be sure that all of the normalized frequencies are positive and less than π radians. Plot the spectrum of $x[n]$ over the range of normalized frequencies $-\pi \le \hat{\omega} \le \pi$.

(c) Now the continuous-time output signal $y(t)$ that is created by the ideal D-to-C converter operating with sampling rate $f_s = 30$ Hz will also be a sum of cosine signals. Write an expression for $y(t)$ and plot its spectrum. What is the fundamental frequency of $y(t)$?

(d) How are the fundamental frequencies of $x(t)$ and $y(t)$ related? Do you think that it would be possible to change the fundamental frequency by a different factor by using a different sampling rate?

PROBLEM 2.2:

Consider a continuous-time signal

$$x(t) = A\cos(\omega_0 t + \phi)$$

We know that this signal is periodic with period $T_0 = 2\pi/\omega_0$; i.e. $x(t + T_0) = x(t)$ for all t. Now suppose that $x(t)$ is sampled to obtain the sequence

$$x[n] = x(nT_s) = A\cos(\omega_0 nT_s + \phi) = A\cos(\hat{\omega}_0 n + \phi)$$

where $\hat{\omega_0} = \omega_0 T_s$.

Now a discrete-time signal is periodic with period N if $x[n + N] = x[n]$ for all n, where N is necessarily an integer.

(a) Will $x[n]$ be periodic for all possible sampling rates? If not, what condition on T_s is sufficient to ensure that $x[n]$ is periodic with period N?

(b) If $\omega_0 = 2000\pi$, what value of T_s will result in a periodic sequence with period $N = 100$?

PROBLEM 2.3*:

This problem is concerned with finding the output of an FIR filter for a given input signal. A linear time-invariant system is described by the difference equation

$$y[n] = \sum_{k=0}^{3} (0.5)^k x[n-k] \tag{1}$$

(a) Determine the filter coefficients $\{b_k\}$ of this FIR filter.

(b) Find the impulse response, $h[n]$, for this FIR filter. The impulse response is a discrete-time signal, so make a (stem) plot of $h[n]$ versus n.

(c) Use the difference equation in (1) to compute the output $y[6]$ (i.e., the output at time $n = 6$) when the input is

$$x[n] = \begin{cases} 0 & n < 0 \\ (n+1) & 0 \le n \le 4 \\ -4 & 5 \le n \le 10 \\ 0 & n \ge 11. \end{cases}$$

Show how you computed $y[6]$ from the given information. A sketch of $x[n]$ may be useful for this.

(d) Use `conv( )` and `subplot( )` write a MATLAB program (and turn a printout of the program with a copy of the plot) to plot the input and output as a function of n.

PROBLEM 2.4:

Consider a system defined by $\quad y[n] = \sum_{k=N_1}^{N_2} b_k x[n-k]$

In other words, only the coefficients $b_{N_1}, b_{N_1+1}, \ldots, b_{N_2}$ are non-zero.

Suppose that the input $x[n]$ is non-zero only for $N_3 \le n \le N_4$. Use sketches of $h[k]$ and $x[n-k]$ for different values of n to show that $y[n]$ is non-zero at most over a finite interval of the form $N_1 + N_3 \le n \le N_2 + N_4$. How many non-zero samples can there be in the output sequence $y[n]$?

Hint: consult Figs. 5.4, 5.5 and 5.6 in the book for the sliding window interpretation of the FIR filter.

PROBLEM 2.5*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

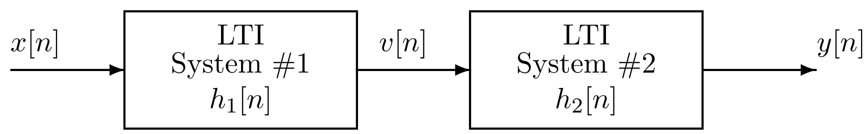


Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 has impulse response,

$$h_1[n] = \begin{cases} 0 & n < 0 \\ 1 & n = 0 \\ -1 & n = 1 \\ 0 & n > 1 \end{cases}$$

and System #2 is described by the difference equation

$$y[n] = 0.25v[n] + 0.25v[n-1] + 0.25v[n-2] + 0.25v[n-3] \tag{2}$$

(a) Determine the difference equation of System #1; i.e., the equation that relates $v[n]$ to $x[n]$.

(b) When the input signal $x[n]$ is an impulse, $\delta[n]$, determine the signal $v[n]$ and make a plot. Show that the resulting output is the given impulse response $h_1[n]$.

(c) From the difference equation in (2), determine $h_2[n]$, the impulse response of System #2.

(d) Determine the impulse response of the overall cascade system, i.e., find $y[n]$ when $x[n] = \delta[n]$.

(e) From the impulse response of the overall cascade system as obtained in part (d), obtain a single difference equation that relates $y[n]$ directly to $x[n]$ in Fig. 1.

PROBLEM 2.6*:
For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

(a) $y[n] = x[n]\cos(0.2\pi n)$

(b) $y[n] = -x[n+1] + x[n] - x[n-1]$

(c) $y[n] = |x[-n]|$

PROBLEM 2.7*:
Consider a system implemented by the following MATLAB program:

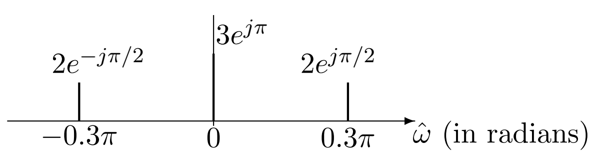
```
% xx.mat is a binary file containing the
% vector of input samples called ‘‘xx’’
load xx
yy1 = conv(ones(1,4),xx);
yy2 = conv([1,-1,-1,1],xx);
ww = yy1 + yy2;
yy = conv(ones(1,3),ww);
```

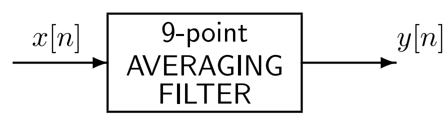
The overall system from input `xx` to output `yy` is an LTI system composed of three LTI systems.

(a) Draw a block diagram of the system that is implemented by the program above. Be sure to indicate the impulse responses of each of the three component systems.

(b) The overall system is an LTI system. What is its impulse response, and what is the difference equation that is satisfied by the input $x[n]$ and output $y[n]$?

PROBLEM 2.8:
A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.





(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

(b) Determine a formula for the output signal $y[n]$.