EE102A Winter 2012-13 Pauly
Signal Processing and Linear Systems I

Problem Set #5

Problem Set Due: Wednesday Feb. 13

1. Determine whether the assertions are true or false, and provide a supporting argument.

   (a) The convolution of two real and odd signals is real and odd.

   (b) The convolution of an even signal and an odd signal is an odd signal.

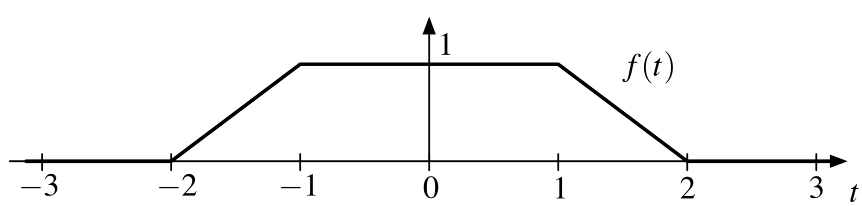
   (c) $f(t)$ and $g(t)$ are real signals, and

   $$h(t) = (f * g)(t).$$

   If we define scaled versions of these signals $f_a(t) = f(at)$, $g_a(t) = g(at)$, and $h_a(t) = h(at)$, then

   $$h_a(t) = (f_a * g_a)(t).$$

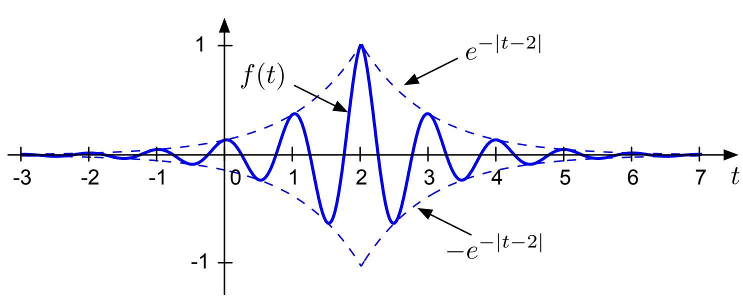
2. The signal $f(t)$ is plotted below:



   (a) Find an expression for $f(t)$ in terms of sums or convolutions of scaled and shifted $\mathrm{rect(t)}$ and $\Delta(t)$ signals.

   (b) Find the Fourier transform of $f(t)$. Simplify the result as much as possible.

3. The signal $f(t)$ is plotted below:

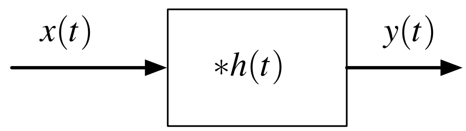


This can be written as

$$f(t) = e^{-|t-2|}\cos(2\pi t).$$

Find $F(j\omega)$, the Fourier transform of $f(t)$. You don't need to combine terms for your answer.

4. A linear system has the block diagram



The input to the system consists of two signals

$$x(t) = f(t) + g(t)$$

which are

$$\begin{aligned} f(t) &= \mathrm{sinc(t/2)} \\ g(t) &= \cos(3\pi t)\mathrm{sinc}^2(t/2) \end{aligned}$$

Your job is to design the system $h(t)$ so that only $g(t)$ appears in the output, and f(t) is completely suppressed

$$y(t) = g(t)$$

a) Find the spectrum of the input $X(j\omega)$.

b) Sketch the spectrum of the input $X(j\omega)$, and the spectrum of your new system $H(j\omega)$. Label the axis clearly.

c) Write an expression for $H(j\omega)$.

d) Find the impulse response of your system $h(t)$.

5. Generalized Parseval's Theorem

(a) Given two possibly complex signals $f_1(t)$ and $f_2(t)$ with Fourier transforms $F_1(j\omega)$ and $F_2(j\omega)$, show that

$$\int_{-\infty}^{\infty} f_1(t)f_2^*(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(j\omega)F_2^*(j\omega)d\omega$$

This is another form of Parseval's theorem, which reduces to the form we discussed in class if $f_1(t) = f_2(t)$.

(b) If $f_1(t)$ and $f_2(t)$ are real, show

$$\int_{-\infty}^{\infty} f_1(t)f_2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(j\omega)F_2(-j\omega)d\omega$$

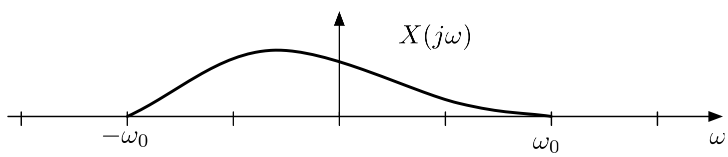
(c) Use this result to show that

$$\int_{-\infty}^{\infty} \text{sinc}(t-n)\text{sinc}(t-k)\, dt = \begin{cases} 1 & k = n \\ 0 & k \neq n \end{cases}$$

where n and k are integers. This shows that shifted sincs are orthogonal functions! This will be very important when we discuss reconstructing a continuous signal from its samples.

Hint: Recall that the integral of a complex exponential over an integer number of periods is zero.

6. *Spectral Widths* Let $x(t)$ be a signal whose spectrum is identically zero outside the range $-\omega_0 \leq \omega \leq \omega_0$. An example of such a spectrum is shown below.



For the following signals, determine the range over which their spectrum is non-zero.

a) $y(t) = x(t) + x(t-1)$.

b) $y(t) = \dfrac{dx(t)}{dt}$.

c) $y(t) = x(t)\cos(\omega_0 t)$.

d) $y(t) = x(t)e^{jb_0 t}$, where b_0 is a positive real constant.

e) $y(t) = x^3(t) * x^2(t)$.