STANFORD UNIVERSITY
DEPARTMENT of ELECTRICAL ENGINEERING

EE 102B   Spring 2013
Problem Set #4

Assigned: April 24, 2013
Due Date: May 1, 2013

Reading: In *DSP First*, Chapter 6 on the frequency response and notes posted on the website as "Chapter 66".

$\Longrightarrow$ **Please check the "Class2Go Forum" often. All official course announcements are posted there with email reminders.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due by 5pm, Wednesday, May 1, in Packard 263. It can be handed in with late penalties up to 5pm on Friday, May 3.

PROBLEM 4.1*:
We have seen that a right-sided exponential signal is described by the DTFT pair

$$x_R[n] = a^n u[n] \Longleftrightarrow X_R(e^{j\hat{\omega}}) = \frac{1}{1 - ae^{-j\hat{\omega}}} \qquad |a| < 1. \tag{1}$$

Now consider a left-sided signal defined by

$$x_L[n] = -b^n u[-n-1] \tag{2}$$

(a) Make a sketch (or MATLAB plot) of $x_L[n]$ for the case $b = 2$.

(b) Determine a general (closed form) expression for $X_L(e^{j\hat{\omega}})$ in terms of $\hat{\omega}$ and b. Be sure to state any conditions that must be met for the existence of the DTFT, $X_L(e^{j\hat{\omega}})$. You can obtain the desired expression for $X_L(e^{j\hat{\omega}})$ either by summing the defining DTFT power series or by starting with Eq. (1) and employing various properties of the DTFT.

(c) Compare your result in (b) to $X_R(e^{j\hat{\omega}})$ in Eq. (1). How are they the same? How are they different. If you were given a couplex function in the form

$$X(e^{j\hat{\omega}}) = \frac{1}{1 - ce^{-j\hat{\omega}}},$$

and told that it is the DTFT of a signal $x[n]$, how could you tell whether it corresponds to a left- or a right-sided exponential signal?

(d) Use the results of Eq.(1), (b), (c) and properties of the DTFT to determine the time-domain signal corresponding to each of the following DTFTs.

(i) $X(e^{j\hat{\omega}}) = \dfrac{-2e^{-j\hat{\omega}}}{1 - 2e^{-j\hat{\omega}}}$

(ii) $X(e^{j\hat{\omega}}) = \dfrac{1 - 2e^{-j\hat{\omega}3}}{1 - 0.5e^{-j\hat{\omega}}}$

(iii) $X(e^{j\hat{\omega}}) = \dfrac{2}{1 - 2e^{j\hat{\omega}}}$

PROBLEM 4.2*:

Consider the cascade connection of two LTI discrete-time systems shown in Fig. 1.

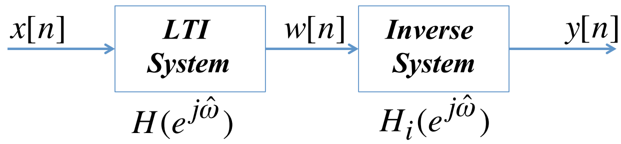


Figure 1: Inverse filtering.

The frequency response of the first system is

$$H(e^{j\hat{\omega}}) = \frac{1 - be^{-j\hat{\omega}}}{1 - ae^{-j\hat{\omega}}} \qquad \text{where } |a| < 1. \tag{3}$$

(a) Determine an equation for the impulse response $h[n]$ of the first LTI system as the sum of two right-sided sequences.

(b) We wish to find the frequency response, $H_i(e^{j\hat{\omega}})$, of the second system so that $y[n] = x[n]$; i.e., we want the second system to undo the effects of the first system. Determine an expression for the frequency response of the second system such that the condition is met. When the condition is met, the second system is termed the *inverse system* for the first system.

(c) Under what condition(s) will the inverse system found in (b) be a stable and causal system? For this case, determine the impulse response $h_i[n]$ for the inverse system.

(d) For the condition(s) of (c), write the difference equation that relates $y[n]$ to $w[n]$.

PROBLEM 4.3:

Now suppose that the first system in Fig. 1 has frequency response $H(e^{j\hat{\omega}}) = 1 + \alpha e^{-j\hat{\omega}n_0}$.

(a) Determine an equation for the impulse response $h[n]$ of the first LTI system.

(b) Determine the frequency response, $H_i(e^{j\hat{\omega}})$, of the second system so that $y[n] = x[n]$; i.e., we want the second system to be the inverse system to the first system.

(c) For this case, determine the impulse response $h_i[n]$ for the inverse system. *Hint: In part (c), expand $H_i(e^{j\hat{\omega}})$ in a power series, and from the power series coefficients, obtain an expression for the sequence $h_i[n]$.* Under what condition(s) will the inverse system be a stable and causal system?

(d) For the condition(s) of (c), write the difference equation that relates $y[n]$ to $w[n]$.

PROBLEM 4.4*:

An LTI system has frequency response

$$H(e^{j\hat{\omega}}) = \frac{e^{-j\hat{\omega}} - 0.5}{1 - 0.5e^{-j\hat{\omega}}}$$

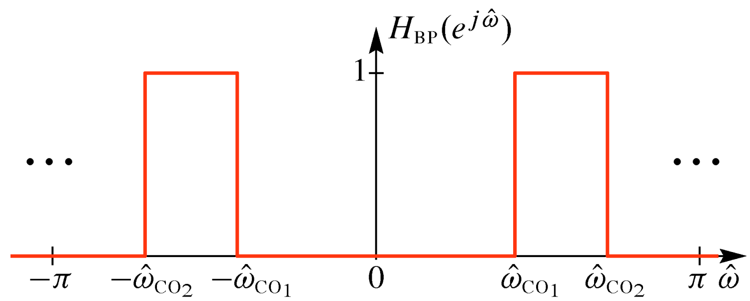
(a) Determine the output $y[n]$ when the input is

$$x[n] = \cos(0.4\pi n) \qquad -\infty < n < \infty$$

(b) Show that $|H(e^{j\hat{\omega}})| = 1$ for all $\hat{\omega}$. That is, the system is an *allpass* filter.

(c) Determine the recurrence formula that can be used to compute the output.

PROBLEM 4.5*:
Consider the ideal bandpass filter shown in the following figure:



There are several ways that you can represent this frequency response in terms of ideal lowpass filters. When you do so, you will obtain different (equivalent) expressions for the impulse response.

(a) Represent $H_{\text{BP}}(e^{j\hat{\omega}})$ as the difference of two ideal lowpass filters. Determine an expression for the impulse response $h_{\text{BP}}[n]$.

(b) Represent $H_{\text{BP}}(e^{j\hat{\omega}})$ as the sum of two frequency-shifted ideal lowpass filters. Determine another expression for the impulse response $h_{\text{BP}}[n]$.

(c) Use a `stem( )` plot in MATLAB to verify that the two expressions give the same impulse response for the case $\hat{\omega}_{\text{co}_1} = .2\pi$ and $\hat{\omega}_{\text{co}_2} = .4\pi$. Plot both on the same stem plot over the range $-20 \leq n \leq 20$. You should only see one plot if they are the same.

(d) Define the rectangular-windowed impulse response of a causal bandpass filter as

$$h_{wbp}[n] = \begin{cases} h_{\text{BP}}[n - n_d] & 0 \leq n \leq 40 \\ 0 & \text{otherwise} \end{cases}$$

where $h_{\text{BP}}[n]$ is the ideal bandpass impulse response found in (a) or (b). How should n_d be selected so that the resulting frequency response $H_{wbp}(e^{j\hat{\omega}})$ has linear phase? Give an equation for $\angle H_{wbp}(e^{j\hat{\omega}})$.

(e) Sketch or plot $|H_{wbp}(e^{j\hat{\omega}})|$ for the cutoff frequencies of (c).

PROBLEM 4.6:

Determine the impulse response, $h_{\mathrm{MB}}[n]$, of the ideal multi-band filter whose frequency response is defined over the base band $-\pi \leq \hat{\omega} < \pi$ as

$$H_{\mathrm{MB}}(e^{j\hat{\omega}}) = \begin{cases} 2 & 0 \leq |\hat{\omega}| \leq \hat{\omega}_{\mathrm{co}_1} \\ 0 & \hat{\omega}_{\mathrm{co}_1} \leq |\hat{\omega}| \leq \hat{\omega}_{\mathrm{co}_2} \\ 1 & \hat{\omega}_{\mathrm{co}_2} \leq |\hat{\omega}| \leq \pi \end{cases}$$

PROBLEM 4.7*:

The autocorrelation function for a signal $x[n]$ is defined as

$$c_{xx}[n] = x[-n] * x[n] = \sum_{k=-\infty}^{\infty} x[-k]x[n-k] = \sum_{k=-\infty}^{\infty} x[k]x[n+k]$$

(a) Determine an expression for the autocorrelation function for the sequence $x[n] = u[n] - u[n-5]$. *Do not leave your result as a sum.* Draw a sketch showing $x[k]$ and $x[n+k]$ for n fixed. Use this sketch to aid you in evaluating the autocorrelation function for all n.

(b) Plot $c_{xx}[n]$ obtained in (a) and verify that $c_{xx}[-n] = c_{xx}[n]$ and $\max_n\{c_{xx}[n]\} = c_{xx}[0]$.

(c) Determine the energy density spectrum $C_{xx}(e^{j\hat{\omega}})$ for the signal in (a). Verify that it is a real and even function of $\hat{\omega}$, and sketch it for $-\pi \leq \hat{\omega} < \pi$.

(d) Repeat (a)-(d) for the signal $y[n] = a^n u[n]$.