STANFORD UNIVERSITY
EE 102B Spring-2013

Lecture 19

Block Processing and Spectrum Analysis

May 15, 2013

ASSIGNMENTS

- Reading for this Lecture:
  - SPF: Section 12-3, Chapter 66-6 thru 66-9
  - S&S:
  - HW#06 is due by 5pm today, May 15, in Packard 263. No late penalty if handed in by 5pm Friday, May 17.
- Lab #05 is due by 5pm, Friday, May 17, in Packard 263. Disregard the “verfied/signed” line on the report page. New Lab 05 is posted.

Office Hours for Course Staff – Come see us.

- Ron Schafer: Mon. and Weds. 2:00-4:00 pm, Packard 211.
- Dookun Park: Tues. 1:30 ~ 3:30 pm in Packard 107
- Ruo Yu Gu (Roy): Mon. 4:00 ~ 6:00 pm in Packard 106

Lecture Objective

- Review sampling theorem for the DTFT
- Review discrete-time convolution and the DFT
- Block processing with the DFT
  - Segmenting a signal
  - Convolution with a segmented signal
- Spectrum analysis

A SAMPLING THEOREM FOR THE DTFT

Summary of DTFT Sampling Theorem

- Sample DTFT to get DFT

$$X[k] = \sum_{n=-\infty}^{\infty} x[n]e^{-j(2\pi/N)kn} = X(e^{j\omega_k}) \quad k = 0,1,\ldots,N-1$$

sum on all non-zero samples

- Reconstruction by IDFT

$$\tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j(2\pi/N)kn} = \sum_{r=-\infty}^{\infty} x[n-rN]$$

- Exact reconstruction if no overlap; i.e.,

$$\tilde{x}[n] = x[n] \quad 0 \le n \le N-1, \text{ if } x[n] \ne 0 \text{ only for } 0 \le n \le N-1$$

USING THE DTFT SAMPLING THEOREM FOR TIME-DOMAIN CONVOLUTION

Convolution of Finite-Length Sequences

- Let h[n] be of length P and x[n] of length L

$$y[n] = x[n] * h[n] = \sum_{m=0}^{L-1} x[m]h[n-m]$$

- y[n] also has finite length, L+P-1 samples

$$y[n] = 0 \quad \text{for } n < 0 \text{ and for } n > L+P-2$$

Sampling the DTFT of a Convolution

$$y[n]=h[n]*x[n] \Leftrightarrow Y(e^{j\hat{\omega}})=H(e^{j\hat{\omega}})X(e^{j\hat{\omega}})$$

$$Y(e^{j(2\pi/N)k})=H(e^{j(2\pi/N)k})X(e^{j(2\pi/N)k})$$

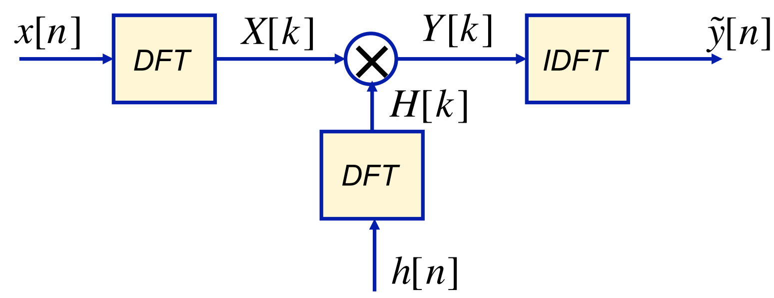
Compute the sampled DTFT by computing the DFTs

$$X[k]=\sum_{n=0}^{L-1} x[n]e^{-j(2\pi/N)kn} \qquad H[k]=\sum_{n=0}^{P-1} h[n]e^{-j(2\pi/N)kn}$$

$$\tilde{y}[n]=\frac{1}{N}\sum_{k=0}^{N-1}\underbrace{H[k]X[k]}_{Y[k]}e^{j(2\pi/N)kn}=\sum_{r=-\infty}^{\infty} y[n-rN]$$

$$\boxed{\tilde{y}[n]=y[n] \quad 0\le n\le N-1 \quad \text{if } N\ge L+P-1}$$

Convolution Using the DFT



BLOCK PROCESSING OF LONG SIGNALS

Long (Continuing) Signals

- Sometimes we have signals that go on for a long time; e.g., symphony performance, news broadcast, lecture in EE102B, seismograph signal, …
- Usually it is not the entire signal but rather its local variations that interest us.
- This leads to trying to track variations in a signal; e.g., speech encodes information in a changing acoustic pattern.

Time-Dependent (Short Time) DTFT and DFT

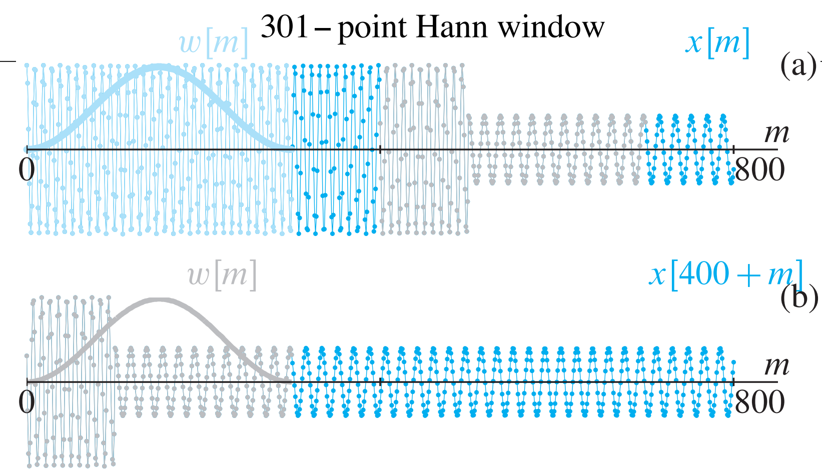
- Definition: short-time DTFT

$$X(e^{j\hat{\omega}},n) = \sum_{m=0}^{L-1} w[m]x[m+n]e^{-j\hat{\omega}n} \qquad 0 \le \hat{\omega} < 2\pi$$

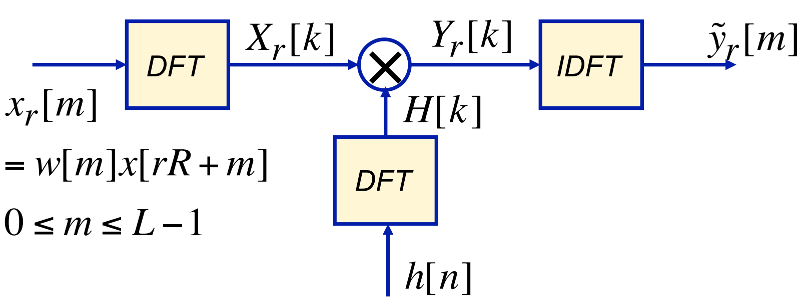
- Definition: short-time DFT

$$X[k,n] = \sum_{m=0}^{L-1} w[m]x[m+n]e^{-j(2\pi/N)kn} \qquad k = 0,1,\ldots,N-1$$

- w[m]x[m+n] focuses attention on the signal around time n.



Convolution Using the DFT

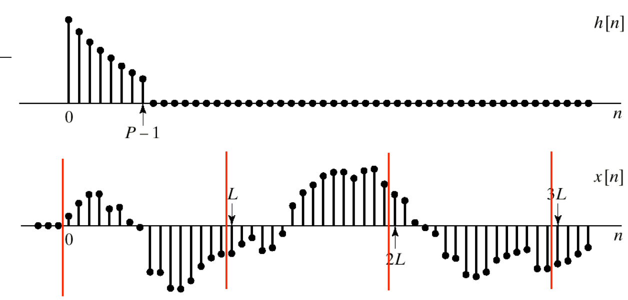


$x_r[m]$

$= w[m]x[rR+m]$

$0 \le m \le L-1$

Block Convolution



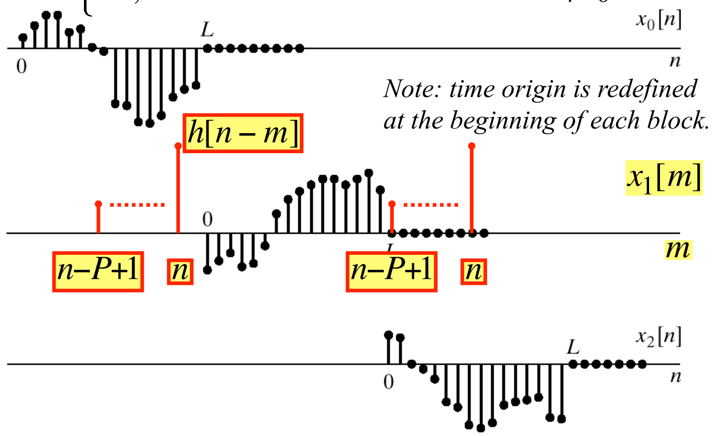
$$x[n] = \sum_{r=0}^{\infty} x_r[n-rL] \rightarrow y[n] = x[n]*h[n] = \sum_{r=0}^{\infty} y_r[n-rL]$$

$$x_r[m] = \begin{cases} x[m+rL], & 0 \le m \le L\text{-}1 \\ 0, & \text{otherwise} \end{cases} \qquad \rightarrow \; y_r[n] = x_r[n]*h[n]$$

Rectangular window

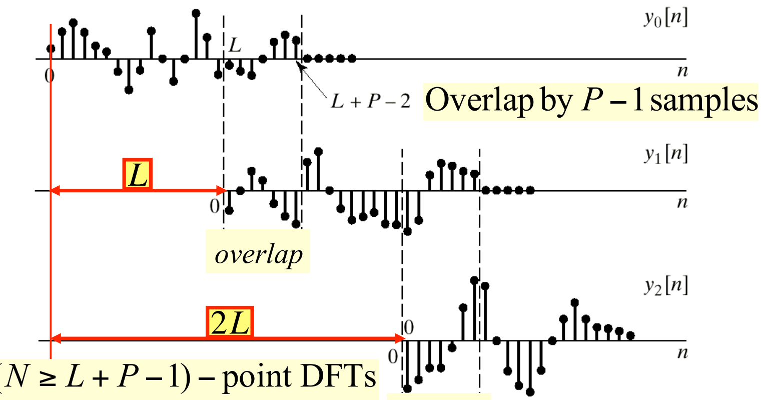
Segmenting the Input

$$x_r[n] = \begin{cases} x[n+rL], & 0 \le n \le L-1 \\ 0, & \text{otherwise} \end{cases} \qquad x[n] = \sum_{r=0}^{\infty} x_r[n-rL]$$



Putting the Output Pieces Together

$$y[n] = \sum_{r=0}^{\infty} y_r[n-rL] \qquad y_r[n] = x_r[n] * h[n]$$



Use $(N \ge L+P-1)$ – point DFTs to do the convolutions.

WINDOWING

Multiplication in the Time-Domain

- Multiplication in the time domain

$$y[n] = w[n]x[n]$$

- Convolution in the frequency domain

$$Y(e^{j\hat{\omega}}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})W(e^{j(\hat{\omega}-\theta)})\,d\theta$$

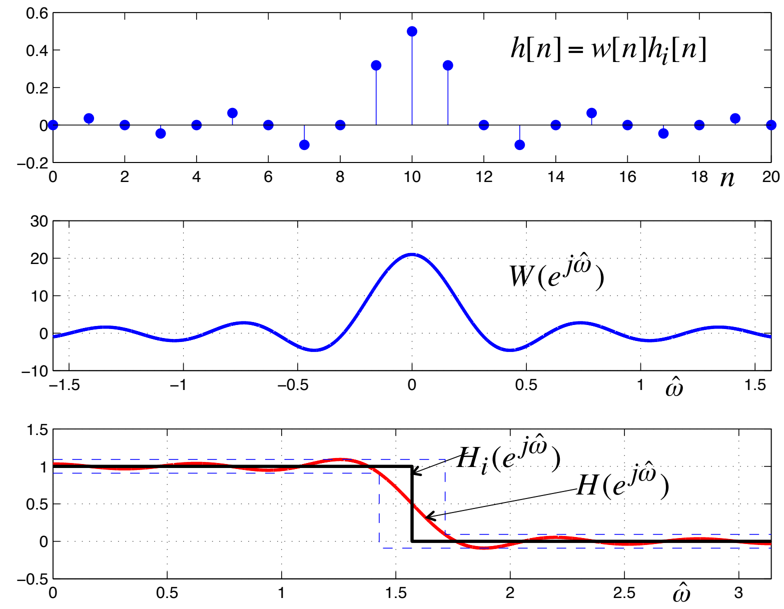
Rectangular Window

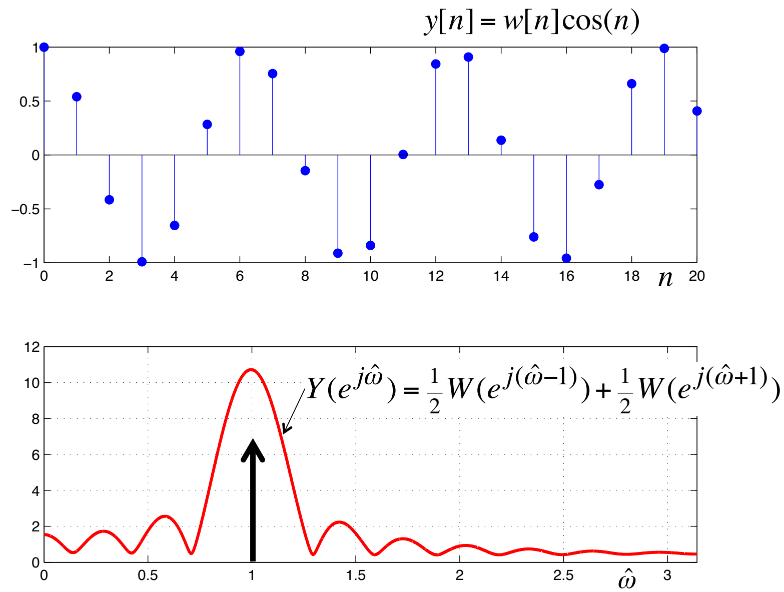
- Time window

$$w[n]=\begin{cases}1 & 0\le n\le L-1\\ 0 & \text{otherwise}\end{cases}$$

- DTFT of Window

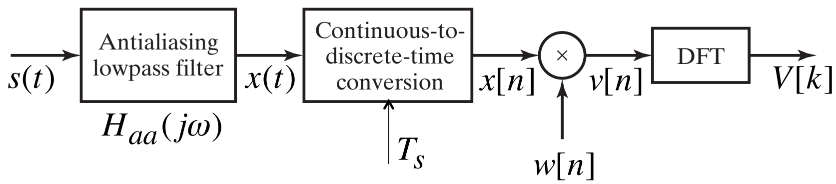
$$W(e^{j\hat{\omega}})=\frac{\sin(\hat{\omega}L/2)}{\sin(\hat{\omega}/2)}e^{-j\hat{\omega}(L-1)/2}$$





SPECTRUM ANALYSIS OF CONTINUOUS-TIME SIGNALS

Spectrum Analysis of Continuous-Time Signals



Work out the details of the relationship between V[k] and S(jω) to verify your understanding of all parts of this important DSP system as well as how the parts fit together to give an estimate of the spectrum.

Examples to try with spectrum_analysis_demo

- W=[2*pi/16], A=[1], TH=[0], B=0, N=32, L=32, swin=r
- W=[2*pi/16], A=[1], TH=[0], B=0, N=32, L=32, swin=h
- W=[2*pi/16], A=[1], TH=[0], B=0, N=32, L=16, swin=r
- W=[2*pi/14, 4*pi/15], A=[1,.75], TH=[0,0], B=0, N=64, L=64, swin=r
- W=[2*pi/16, 4*pi/16], A=[1,.75], TH=[0,0], B=0, N=64, L=64, swin=r
- W=[2*pi/14, 4*pi/15], A=[1,.75], TH=[0,0], B=0, N=64, L=32, swin=k
- W=[2*pi/14, 4*pi/15], A=[1,.75], TH=[0,0], B=0, N=64, L=64, swin=k

Time-Dependent (Short Time) DTFT and DFT

- Definition: short-time DTFT

$$X(e^{j\hat{\omega}},n) = \sum_{m=0}^{L-1} w[m]x[m+n]e^{-j\hat{\omega}n} \qquad 0 \le \hat{\omega} < 2\pi$$

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$$X[k,n] = \sum_{m=0}^{L-1} w[m]x[m+n]e^{-j(2\pi/N)kn} \qquad k = 0,1,\ldots,N-1$$

- w[m]x[m+n] focuses attention on the signal around time n.