STANFORD UNIVERSITY
EE 102B Spring-2013

Lecture 07
Frequency Response of
FIR Systems
April 15, 2013

ASSIGNMENTS

- Reading for this Lecture:
  - SPF: Sections 6-1, 6-2, 6-3, 6-4, & 6-5
  - S&S: Sections 2.1 and 3.2
- HW#02 is posted. It is due by 5pm on Wednesday, April 17 in Packard 263.
- Lab #02 is posted. It is due by 5pm, Friday, April 19, in Packard 263

Office Hours for Course Staff – Come see us.

- Ron Schafer: Mon. 2:00-3:00 pm and Weds. 4:00-5:00 pm in Packard 211
- Dookun Park: Tues. 1:30 ~ 3:30 pm in Packard 107
- Ruo Yu Gu (Roy): Mon. 4:00 ~ 6:00 pm in Packard 106
- https://class2go.stanford.edu/EE102B/Spring2013/pages/staff

Lab 01 syn_fourier - 1

$$x(t) = \sum_{k=-N}^{N} a_k e^{j2\pi f_k t}$$

$$x(t) = \sum_{k=1}^{2N+1} a_k e^{j2\pi f_k t}$$

a_k = ak(k)

f_k = fk(k)

- function xx = syn_fourier(tt, ak, fk)

```
xx = exp( tt(:)*(2i*pi*fk(:)') )* ak(:);
```

- tt(:) is a column vector (matrix) regardless of whether tt is a row or column vector.
- X' is the complex conjugate transpose of X. X.' is the non-conjugate transpose.

Lab 01 syn_fourier - 2

$$x(t) = \sum_{k=1}^{2N+1} a_k e^{j2\pi f_k t}$$

- function xx = syn_fourier(tt, ak, fk)

  xx = exp(tt(:)*(2i*pi*fk(:)'))* ak(:);

tt(:) is a column vector (matrix) regardless of whether tt is a row or column vector.

$$\begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_M \end{bmatrix} \begin{bmatrix} 2\pi f_1 & 2\pi f_2 & \cdots & 2\pi f_{2N+1} \end{bmatrix}$$

Lab 01 syn_fourier - 3

$$x(t) = \sum_{k=1}^{2N+1} a_k e^{j2\pi f_k t}$$

- function xx = syn_fourier(tt, ak, fk)

  xx = exp(tt(:)*(2i*pi*fk(:)'))* ak(:);

$$\begin{bmatrix} j2\pi f_1 t_1 & j2\pi f_2 t_1 & \cdots & j2\pi f_{2N+1} t_1 \\ j2\pi f_1 t_2 & j2\pi f_2 t_2 & \cdots & j2\pi f_{2N+1} t_2 \\ \vdots & \vdots & \vdots & \vdots \\ j2\pi f_1 t_M & j2\pi f_2 t_M & \cdots & j2\pi f_{2N+1} t_M \end{bmatrix}$$

Lab 01 syn_fourier - 4

$$x(t) = \sum_{k=1}^{2N+1} a_k e^{j2\pi f_k t}$$

- function xx = syn_fourier(tt, ak, fk)

  xx = exp(tt(:)*(2i*pi*fk(:)'))* ak(:);

$$\begin{bmatrix} e^{j2\pi f_1 t_1} & e^{j2\pi f_2 t_1} & \cdots & e^{j2\pi f_{2N+1} t_1} \\ e^{j2\pi f_1 t_2} & e^{j2\pi f_2 t_2} & \cdots & e^{j2\pi f_{2N+1} t_2} \\ \vdots & \vdots & \vdots & \vdots \\ e^{j2\pi f_1 t_M} & e^{j2\pi f_2 t_M} & \cdots & e^{j2\pi f_{2N+1} t_M} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{2N+1} \end{bmatrix}$$

Lab 01 syn_fourier -5

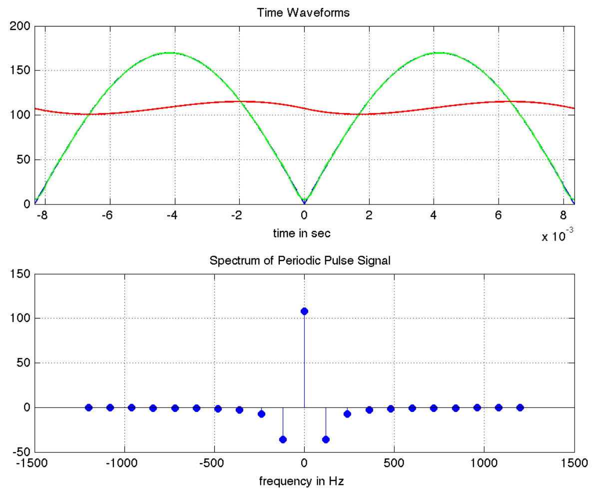
$$x(t) = \sum_{k=1}^{2N+1} a_k e^{j2\pi f_k t}$$

- function xx = syn_fourier(tt, ak, fk)

  xx = exp(tt(:)*(2i*pi*fk(:)'))* ak(:);

$$\begin{bmatrix} \sum_{k=1}^{2N+1} a_k e^{j2\pi f_k t_1} \\ \sum_{k=1}^{2N+1} a_k e^{j2\pi f_k t_2} \\ \vdots \\ \sum_{k=1}^{2N+1} a_k e^{j2\pi f_k t_M} \end{bmatrix} = \begin{bmatrix} x(t_1) \\ x(t_2) \\ \vdots \\ x(t_M) \end{bmatrix}$$

Full-Wave Rectifier Simulation



TWO QUICK QUESTIONS

- FIR Filter is “FIRST DIFFERENCE” filter:
  - $y[n] = x[n] - x[n-1]$
  - Find output when input = unit-step signal?

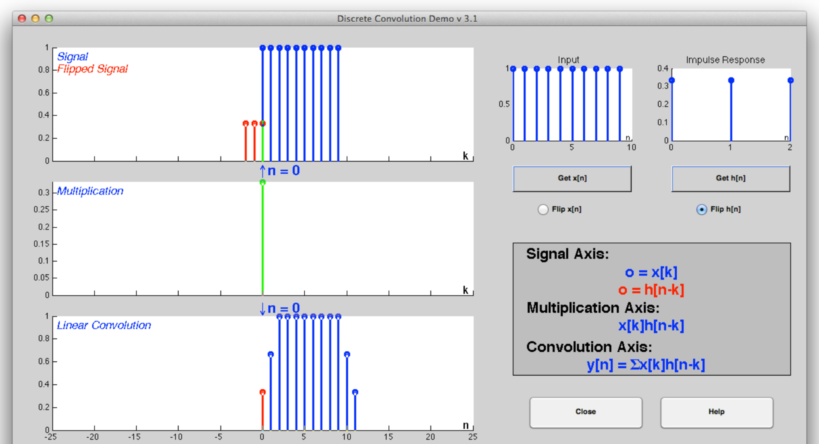
$x[n] = u[n]$

$$y[n] = u[n] - u[n-1] = ?$$

- Convolve shifted unit impulse signals

$$s[n] = \delta[n-2] * \delta[n-5] = ?$$

DCONVDEMO: MATLAB GUI



LECTURE OBJECTIVES

- SINUSOIDAL INPUT SIGNAL
  - DETERMINE the FIR FILTER OUTPUT
- FREQUENCY RESPONSE of FIR
  - PLOTTING vs. Frequency
  - MAGNITUDE vs. Freq
  - PHASE vs. Freq

MAG

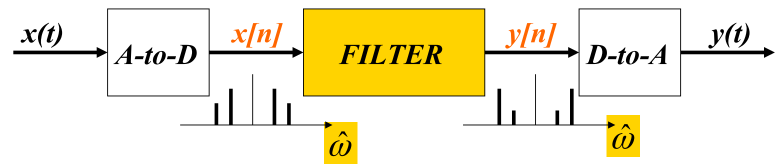
PHASE

$$H(e^{j\hat{\omega}}) = \left|H(e^{j\hat{\omega}})\right| e^{j\angle H(e^{j\hat{\omega}})}$$

DOMAINS: Time & Frequency

- **Time-Domain: "n" = time**
  - x[n] discrete-time signal
  - x(t) continuous-time signal
- **Frequency Domain (sum of sinusoids)**
  - Spectrum vs. f (Hz)
    - ANALOG vs. DIGITAL
  - Spectrum vs. omega-hat
- Move back and forth QUICKLY

DIGITAL "FILTERING"



- CONCENTRATE on the **SPECTRUM**
- SINUSOIDAL INPUT
  - INPUT x[n] = SUM of SINUSOIDS
  - Then, OUTPUT y[n] = SUM of SINUSOIDS

FILTERING EXAMPLE

- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

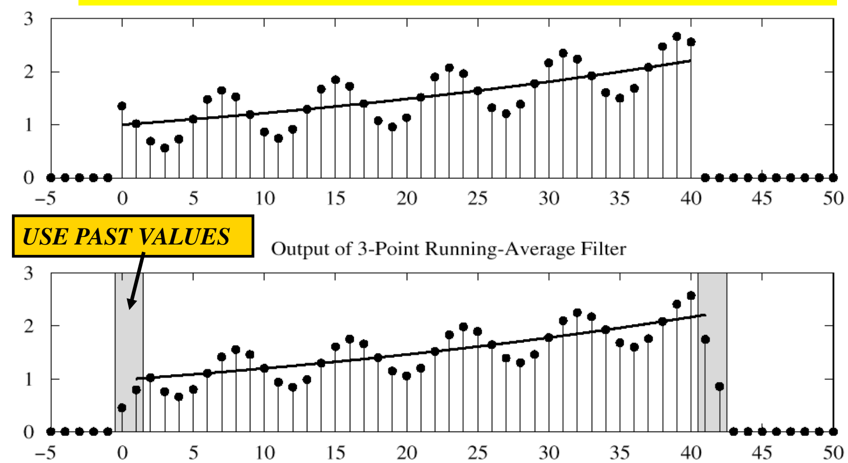
$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$

- *3-point AVERAGER*
  - *Changes A slightly*

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$

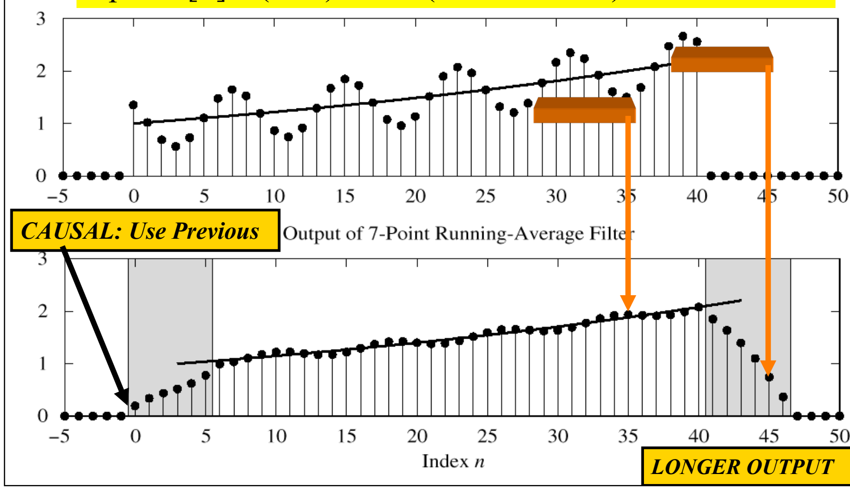
3-pt AVG EXAMPLE

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$



7-pt FIR EXAMPLE (AVG)

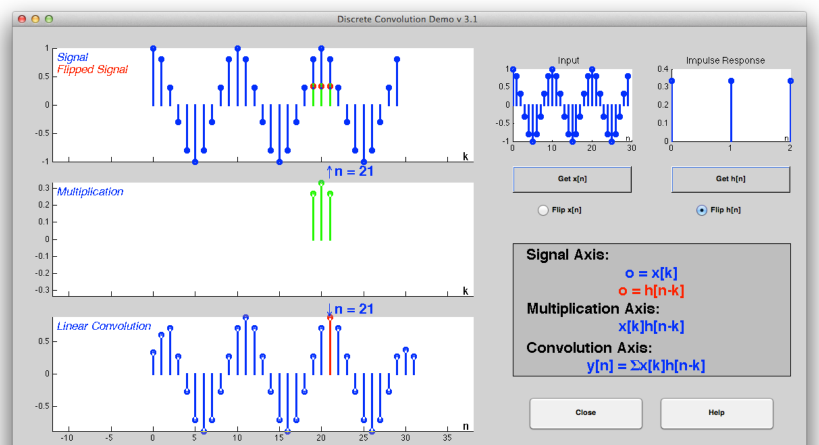
$$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$



SINUSOIDAL RESPONSE

- INPUT: x[n] = SINUSOID
- OUTPUT: y[n] will also be a SINUSOID
  - Different Amplitude and Phase
  - SAME Frequency
- AMPLITUDE & PHASE CHANGE
  - Called the FREQUENCY RESPONSE

DCONVDEMO: MATLAB GUI



COMPLEX EXPONENTIAL

$$x[n] = Ae^{j\varphi}e^{j\hat{\omega}n} \qquad -\infty < n < \infty$$

x[n] is the input signal—a complex exponential

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

FIR DIFFERENCE EQUATION

COMPLEX EXP OUTPUT

- Use the FIR "Difference Equation"

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} b_k A e^{j\varphi} e^{j\hat{\omega}(n-k)}$$

$$= \left(\sum_{k=0}^{M} b_k e^{j\hat{\omega}(-k)} \right) A e^{j\varphi} e^{j\hat{\omega} n}$$

$$= H(e^{j\hat{\omega}}) A e^{j\phi} e^{j\hat{\omega} n}$$

FREQUENCY RESPONSE

- *At each frequency, we can DEFINE*

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega} k}$$

FREQUENCY RESPONSE

- Complex-valued formula
  - Has MAGNITUDE vs. frequency
  - And PHASE vs. frequency
  - Alternatively, REAL and IMAGINARY parts

EXAMPLE 6.1

$$\{b_k\} = \{1,\ 2,\ 1\}$$

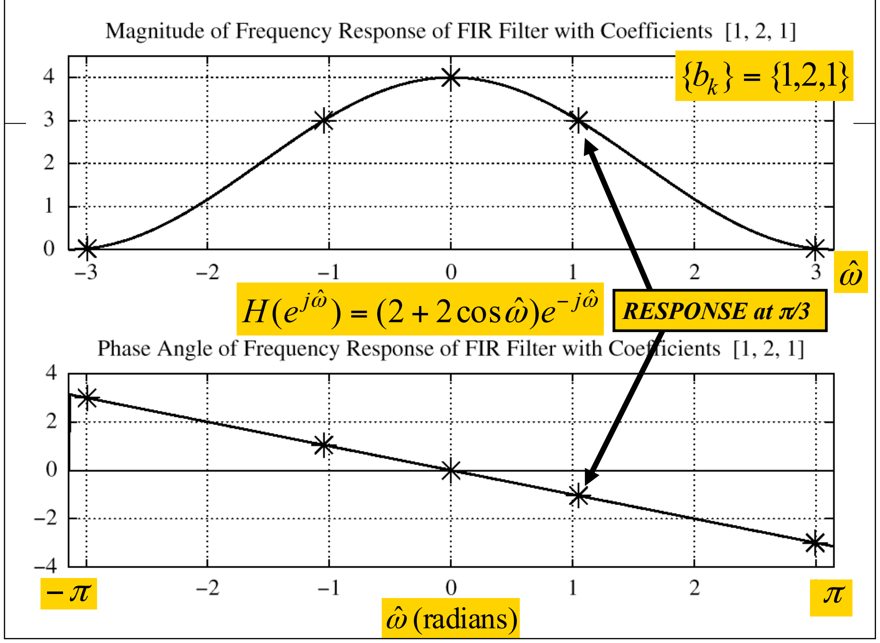
$$H(e^{j\hat{\omega}}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$
$$= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}})$$
$$= e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega})$$

EXPLOIT SYMMETRY

Since $(2 + 2\cos\hat{\omega}) \geq 0$

Magnitude is $\left|H(e^{j\hat{\omega}})\right| = (2 + 2\cos\hat{\omega})$

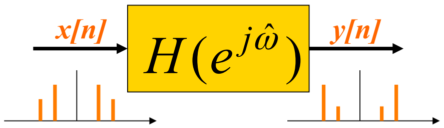
and Phase is $\angle H(e^{j\hat{\omega}}) = -\hat{\omega}$



EXAMPLE 6.2

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known

and $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$



$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$

Evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/3$

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

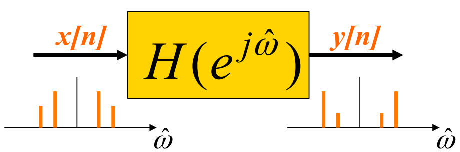
$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @\,\hat{\omega} = \pi/3$$

$$y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4}e^{j(\pi/3)n} = 6e^{-j\pi/12}e^{j(\pi/3)n}$$

EXAMPLE: COSINE INPUT

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known

and $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$



$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

EX: COSINE INPUT

Find $y[n]$ when $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4}) = e^{j(\pi n/3 + \pi/4)} + e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

Use Linearity

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow y[n] = y_1[n] + y_2[n]$$

EX: COSINE INPUT (ans-2)

Find $y[n]$ when $x[n]=2\cos(\frac{\pi}{3}n+\frac{\pi}{4})$

$$H(e^{j\hat{\omega}})=(2+2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$y_1[n]=H(e^{j\pi/3})e^{j(\pi n/3+\pi/4)}=3e^{-j(\pi/3)}e^{j(\pi n/3+\pi/4)}$$

$$y_2[n]=H(e^{-j\pi/3})e^{-j(\pi n/3+\pi/4)}=3e^{j(\pi/3)}e^{-j(\pi n/3+\pi/4)}$$

$$y[n]=3e^{j(\pi n/3-\pi/12)}+3e^{-j(\pi n/3-\pi/12)}$$

$$\Rightarrow y[n]=6\cos(\tfrac{\pi}{3}n-\tfrac{\pi}{12})$$

General Result for Sinusoidal Input - I

$$x[n]=A\cos(\hat{\omega}_0 n+\phi)$$

$$=\frac{A}{2}e^{j\phi}e^{j\hat{\omega}_0 n}+\frac{A}{2}e^{-j\phi}e^{-j\hat{\omega}_0 n}$$

$$y[n]=\frac{A}{2}e^{j\phi}\,|H(e^{j\omega_0})|\,e^{j\angle H(e^{j\omega_0})}e^{j\hat{\omega}_0 n}$$

$$+\frac{A}{2}e^{-j\phi}\,|H(e^{-j\omega_0})|\,e^{j\angle H(e^{-j\omega_0})}e^{-j\hat{\omega}_0 n}$$

General Result for Sinusoidal Input - II

$$y[n]=\frac{A}{2}e^{j\phi}\,|H(e^{j\omega_0})|\,e^{j\angle H(e^{j\omega_0})}e^{j\hat{\omega}_0 n}$$

$$+\frac{A}{2}e^{-j\phi}\,|H(e^{-j\omega_0})|\,e^{j\angle H(e^{-j\omega_0})}e^{-j\hat{\omega}_0 n}$$

$$y[n]=$$

$$A\,|H(e^{j\omega_0})|\cos\left[\hat{\omega}_0 n+\phi+\angle H(e^{j\omega_0})\right]$$

MATLAB: FREQUENCY RESPONSE

- `HH = freqz(bb,1,ww)`
  - VECTOR **bb** contains Filter Coefficients
  - SP-First: `HH = freekz(bb,1,ww)`
- FILTER COEFFICIENTS $\{b_k\}$

$$\boxed{H(e^{j\hat{\omega}})=\sum_{k=0}^{M}b_k e^{-j\hat{\omega}k}}$$

Time & Frequency Relation

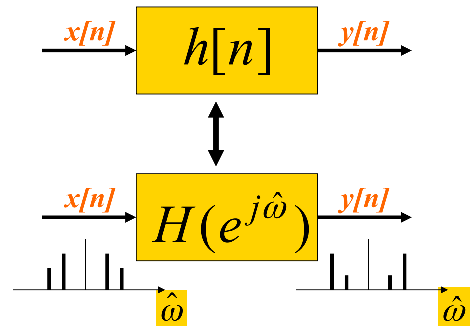
- Get Frequency Response from h[n]
  - Here is the FIR case:

$$\boxed{H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k}}$$

IMPULSE RESPONSE

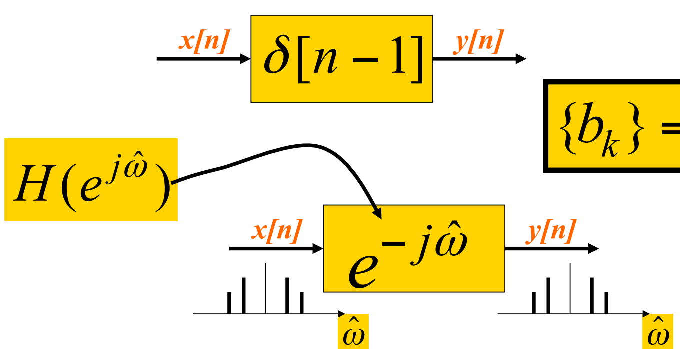
BLOCK DIAGRAMS

- Equivalent Representations



UNIT-DELAY SYSTEM

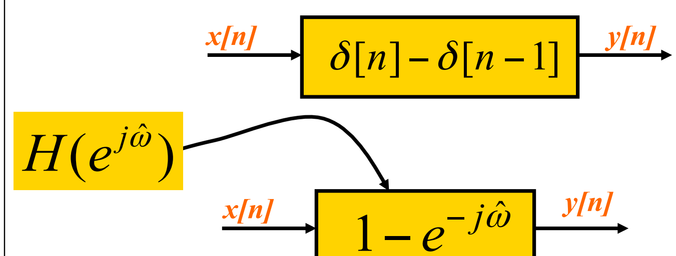
Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-1]$



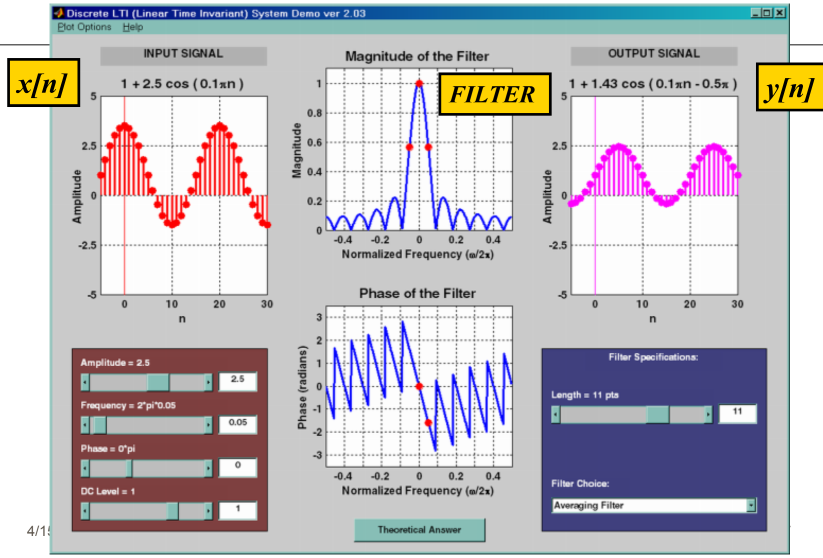
$$\boxed{\{b_k\} = \{\,0,\ 1\,\}}$$

FIRST DIFFERENCE SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for the Difference Equation: $y[n] = x[n] - x[n-1]$

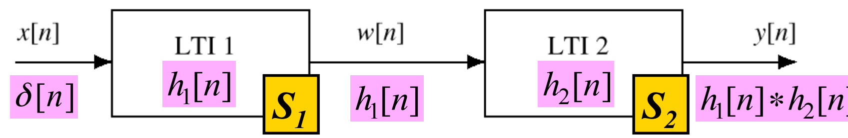


DLTI Demo with Sinusoids



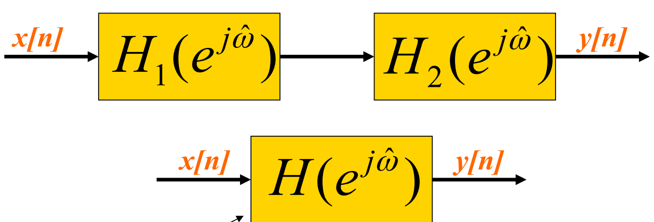
CASCADE SYSTEMS

- Does the order of S_1 & S_2 matter?
  - WHAT ARE THE FILTER COEFFS? $\{b_k\}$
  - WHAT is the overall FREQUENCY RESPONSE ?



CASCADE EQUIVALENT

- MULTIPLY the Frequency Responses



EQUIVALENT SYSTEM

$$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}})$$