STANFORD UNIVERSITY
DEPARTMENT of ELECTRICAL ENGINEERING

EE 102B   Spring 2013
Problem Set #3

Assigned: April 17, 2013
Due Date: April 24, 2013

Reading: In *DSP First*, Chapter 6 on the frequency response and notes posted on the website as "Chapter 66".

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due by 5pm in Packard 263. It can be handed in late up to 5pm on Friday, April 26.

PROBLEM 3.1*:

We now have three ways of describing an LTI system: the difference equation; the impulse response, $h[n]$; and the frequency response, $H(e^{j\hat{\omega}})$. In the following, you are given one of these representations and you must find the other two.

(a) $y[n] = (x[n] + 2x[n-2] + x[n-4])$.

(b) $h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]$.

(c) $H(e^{j\hat{\omega}}) = [1 + \cos(2\hat{\omega})]e^{-j\hat{\omega}3}$. *Hint: Expand the cosine using Euler's formula.*

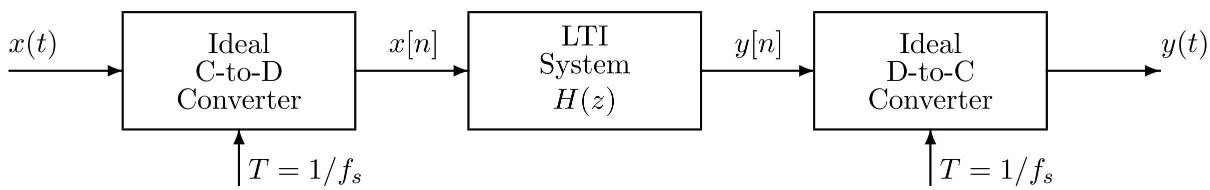
PROBLEM 3.2*:

The input to the C-to-D converter in the figure below is

$$x(t) = 10 + 6\cos(2000\pi t + \pi/8) + 4\cos(6000\pi t - \pi/3)$$

The frequency response of the LTI system is

$$H(e^{j\hat{\omega}}) = (1 - e^{-j\hat{\omega}2})$$



(a) If $f_s = 10000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.

(b) If $f_s = 5000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter. *Note that even when aliasing distortion occurs, we can still determine the effect of the system on the input $x[n]$ and therefore we can determine $y(t)$ from $y[n]$.*

PROBLEM 3.3*:

The following MATLAB program is an implementation of an LTI system for a specific input signal. Your job is to convert this program back to the mathematical equations that describe the signals and systems that it represents.

```
nn = 0:16000;
xx = 3 + 2*cos(0.75*pi*nn-pi/4) + 11*cos(1.5*pi*nn-pi/3);
yy = conv([1,0,0,0,-1]/4,xx);
soundsc(yy,8000)
```

(a) What is the difference equation that relates the input and output of the system?

(b) What is the impulse response $h[n]$ of the system?

(c) What is the system function $H(e^{j\hat{\omega}})$ of the system that is implemented by the `conv( )` statement?

(d) Plot or sketch the spectrum of the signal $x[n]$ corresponding to `xx`. Your plot should cover the range $-\pi \leq \hat{\omega} \leq \pi$.

(e) Neglecting the end effects in the convolution, determine $y(t)$ that describes the signal produced by the `soundsc( )` statement.

PROBLEM 3.4*:

Consider the linear time-invariant system defined by the following difference equation:

$$y[n] = \sum_{k=0}^{4} x[n-k]$$

(a) By inspection, find a simple expression for the frequency response $H(e^{j\hat{\omega}})$ of the system.

(b) Show that your answer in (a) can be expressed in the form

$$H(e^{j\hat{\omega}}) = \frac{\sin(5\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j\hat{\omega}2}.$$

(c) Sketch the frequency response (magnitude and phase) as a function of frequency from the formula above (or plot it using `freqz( )`).

(d) Suppose that the input is

$$x[n] = 10 + 10\cos(\hat{\omega}_0 n) \text{ for } -\infty < n < \infty$$

Find a non-zero frequency $0 < \hat{\omega}_0 < \pi$ for which the ouput $y[n]$ is a constant for all n, i.e.,

$$y[n] = c \qquad \text{for } -\infty < n < \infty$$

and find the value for c. (In other words, the sinusoid is removed by the filter.)

(e) Is your answer to part (d) unique; i.e., can you find another value of $0 < \hat{\omega}_0 < \pi$ for which the output $y[n] = c$?

PROBLEM 3.5*:

When the DTFT $X(e^{j\hat{\omega}})$ is given, it is possible to determine the corresponding discrete-time sequence $x[n]$ by inverse DTFT. Likewise, when the sequence $x[n]$ is given, it is possible to determine the corresponding DTFT $X(e^{j\hat{\omega}})$. In the parts (a) to (c) below, determine $x[n]$, and in parts (d) to (f) below, determine $X(e^{j\hat{\omega}})$.

(a) $X(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} - 2e^{-j\hat{\omega}3} + e^{-j\hat{\omega}5}$

(b) Determine $x[n]$ when $X(e^{j\hat{\omega}}) = \begin{cases} e^{-j\hat{\omega}7} & \text{for } 0 \leq |\hat{\omega}| \leq 0.4\pi \\ 0 & \text{for } 0.4\pi < |\hat{\omega}| \leq \pi \end{cases}$

(c) $X(e^{j\hat{\omega}}) = 12je^{-j\hat{\omega}6}\sin(4\hat{\omega})$

(d) $x[n] = (0.8)^n u[n-4]$

(e) $x[n] = p[n] - p[n-8]$, where $p[n] = u[n] - u[n-8]$. Recall that $u[n]$ is the unit-step signal.

(f) $x[n] = 2\dfrac{\sin(0.1\pi n)}{\pi n}\cos(0.3\pi n)$, for $-\infty < n < \infty$. Hint: Use the frequency-shift property.