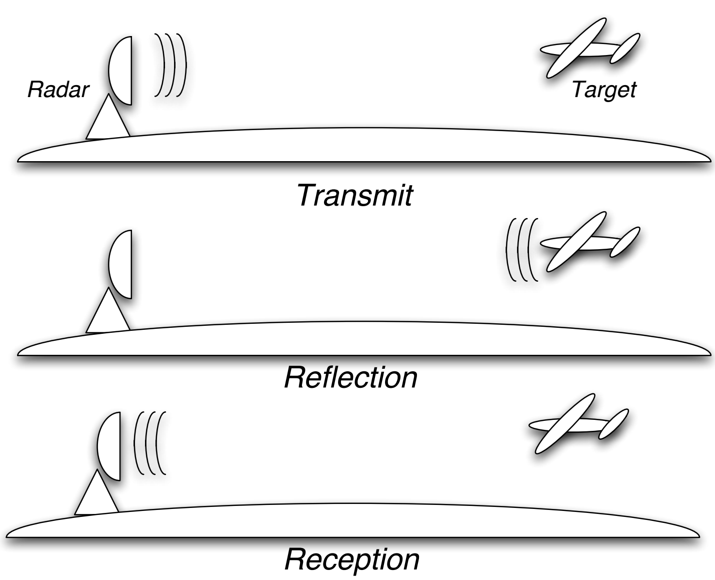
Stanford University
Winter 2012-2013

Signal Processing and Linear Systems I

Applications 1: Introduction to Radar

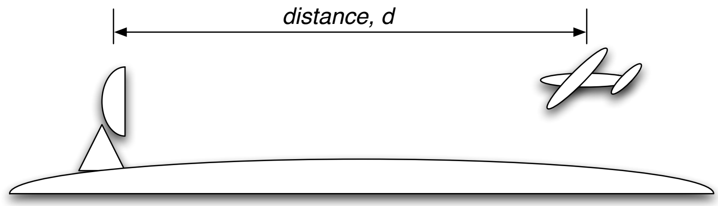
January 23, 2013

Basic Radar Idea



Radar Parameters

The target is a distance d from the antenna



The pulse travels a total distance of $2d$ at the speed of light c, which is 3×10^8 m/s, or 300 m/μs.

The echo returns after a delay of

$$t_d = \frac{2d}{c}$$

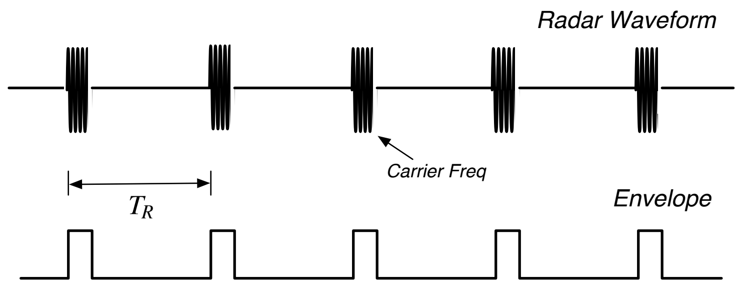
A delay of 1 μs corresponds to a distance of

$$d = ct_d/2 = (300 \text{ m}/\mu\text{s})(1\mu\text{s})/2 = 150 \text{ m}$$

There is also a frequency shift (Doppler shift) that we'll ignore for now, but will come back to later in the course.

Simple Radar Waveform

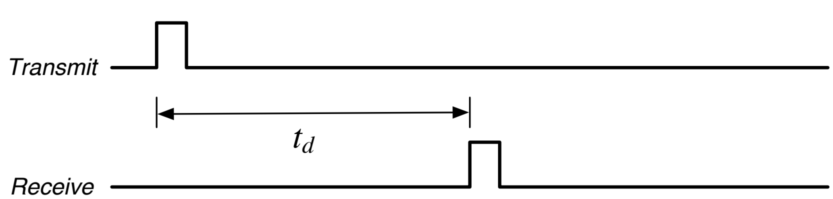
In Lecture 3 we showed how to represent a radar waveform as an envelope mulitplied by a carrier.



We'll show later (Lecture 12, Modulation and Demodulation) we only need to consider the envelope of the signal to understand basic radar signal processing.

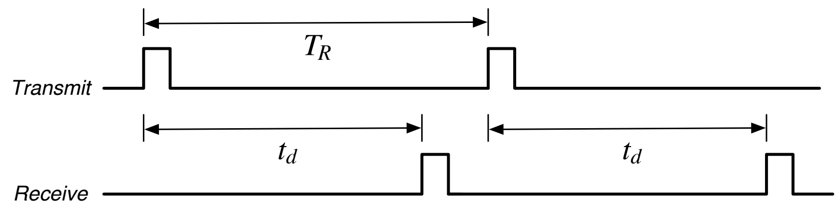
Range Ambiguity

In a simple experiment, we transmit a pulse, and wait for it to come back



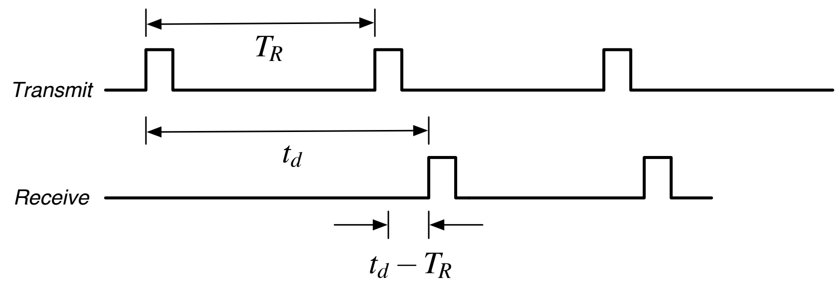
This tells us that there is a target at a distance $d = c\ t_d/2$.

Practically we want to repeat the measurement as rapidly as possible, to track the target



We transmit a new pulse after we receive the previous return.

If we don't wait long enough ...



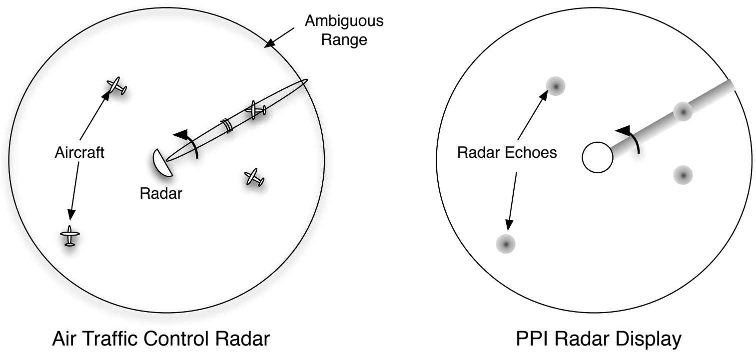
We think there is a target at $c\ (t_d - T_R)/2$, which is much closer!

The targets at $c\ (t_d - T_R)/2$ and $c\ t_d/2$ are *ambiguous*, we can't tell them apart.

To be unambiguous, we need the roundtrip time $2d/c$ to be less than T_R. A given T_R has a maximum usable range.

Radar Display

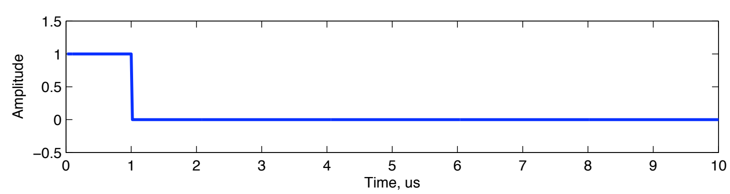
Air traffic control radar gives a top-down 2D view of the airspace



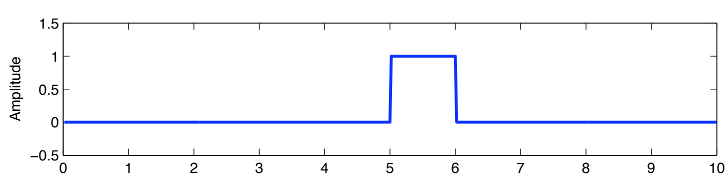
- Radar sends a pulse out, and listens for echoes
- The received signal is written on the display at the same angle
- The radar rotates, and sends out another pulse

Pulse Detection

How do we detect when the pulse has returned? It seems simple if the SNR is high. If we transmit a pulse

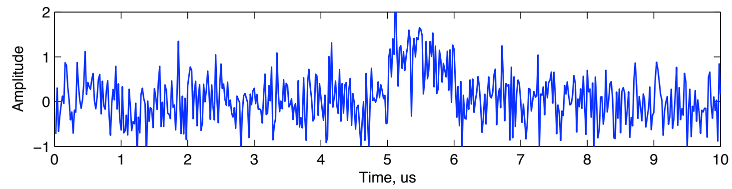


We get a delayed version of the pulses at the receiver



We could measure the time of arrival of the leading edge of the pulse, for example.

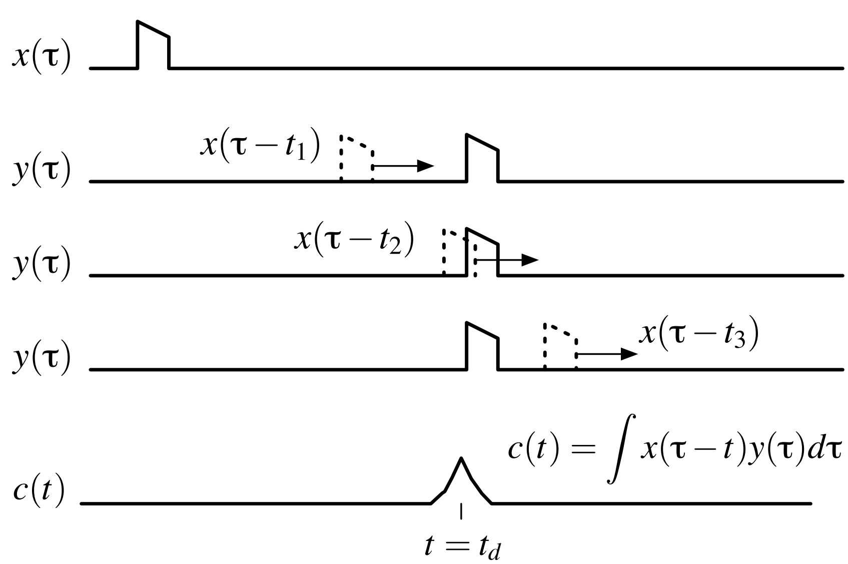
In practice, the returned signal is much smaller than the transmitted signal, and has been corrupted by noise,



Here it is much harder to say where the reflected signal is!

Matched Filters, and Correlation Receivers

Solution is to search for signals that look like the transmitted signal.



If $x(t)$ is the transmit signal, and $y(t)$ is the returned signal, then

$$c(t) = \int_{-\infty}^{\infty} x(\tau - t) y(\tau) d\tau$$

This is the cross-correlation of x and y, which is written as

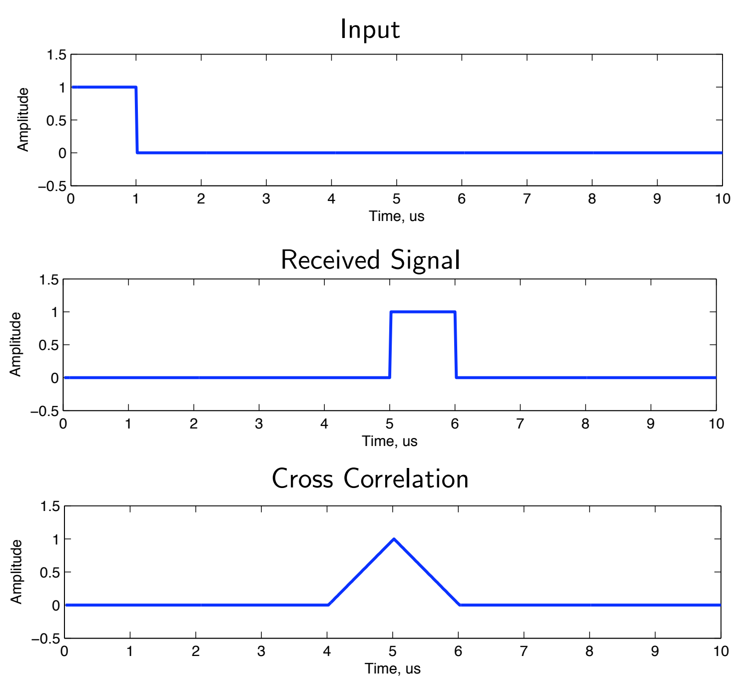
$$(x \star y)(t) = \int_{-\infty}^{\infty} x(\tau - t) y(\tau) d\tau = \int_{-\infty}^{\infty} x(\tau) y(\tau + t) d\tau$$

To look for a reflected signal at time t,

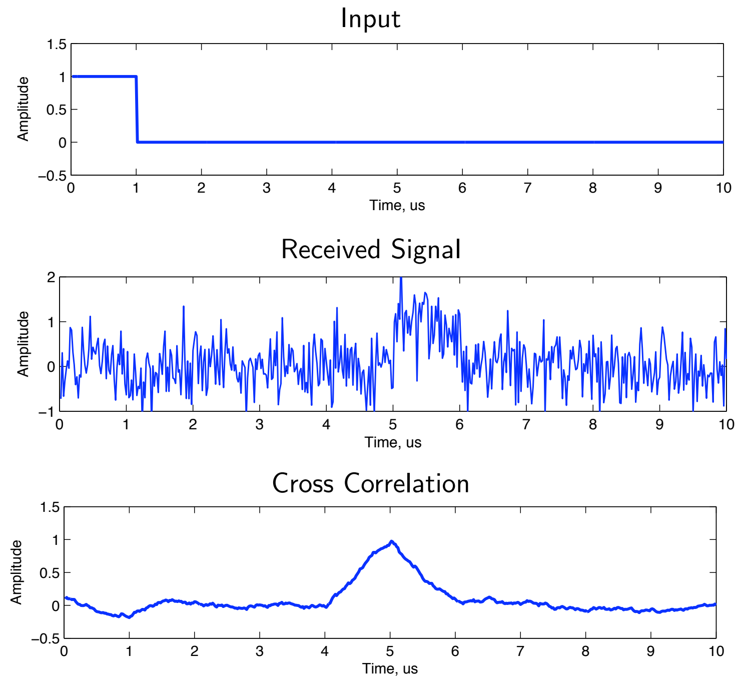
- Shift the transmitted signal to time t,
- Multiply point by point with the received signal,
- Then integrate.

This is just like convolution, except you don't reverse one of the signals.

For the noiseless case before, we get



For the high-noise case,



Cross correlation properties

- Better defined peak, easier to identify time
- Broader peak, twice the duration of the transmit pulse
- Good suppression of noise, optimal under reasonable assumptions

If the received signal at time t_d is really $y(\tau) = x(\tau - t_d)$, then the cross correlation at t_d is

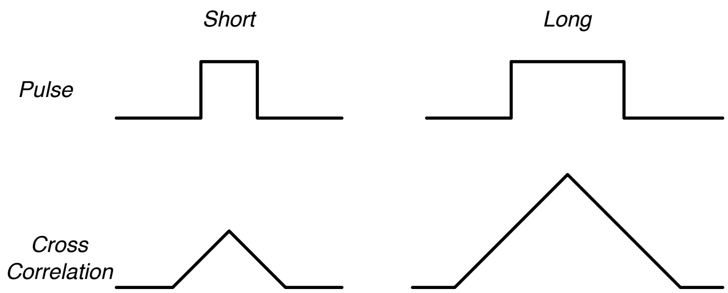
$$c(t_d) = \int x(\tau - t_d)y(\tau)d\tau = \int x(\tau - t_d)x(\tau - t_d)d\tau = \int x^2(\tau)d\tau = E_x$$

which is the energy in the transmitted pulse.

Transmit Pulses

We would like the transmitted pulse to have a large E_x, so that we can detect signals far away.

Peak RF amplitude is limited, so we have to make the pulses longer to increase E_x.



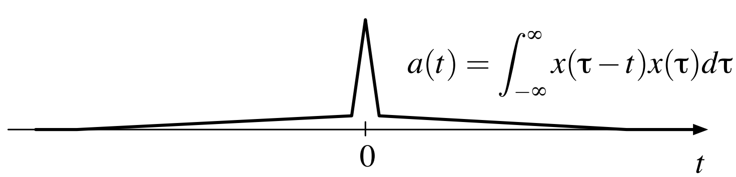
Doubling the pulse length doubles E_x, but makes it harder to accurately identify the return time.

Pulse Compression

Can we have the large E_x of a long pulse, with the temporal resolution of a short pulses?

Remarkably, yes!

What we want are pulses whose cross-correlation with themselves are small, except right at the origin.



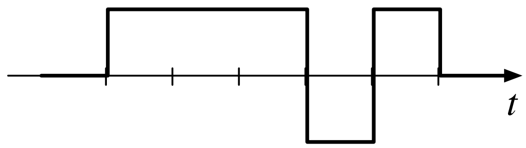
$$a(t) = \int_{-\infty}^{\infty} x(\tau - t)x(\tau)d\tau$$

This is the *autocorrelation* of $x(t)$.

Finding pulses with these properties has been a major research effort in radar.

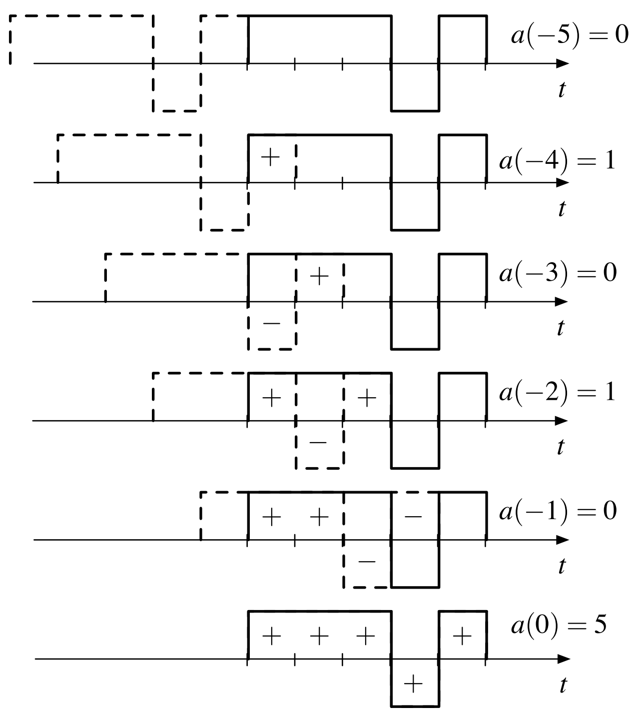
One example are pulses based on *Barker codes*.

A length 5 Barker code is



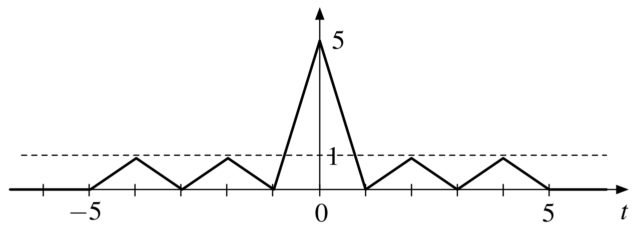
It is described by the amplitudes of the subpulses, in this case $(+1, +1, +1, -1, +1)$.

The remarkable thing about Barker codes, is that the autocorrelation on an N sample code is either ± 1 or 0 everywhere, except at $t = 0$, when it is N!

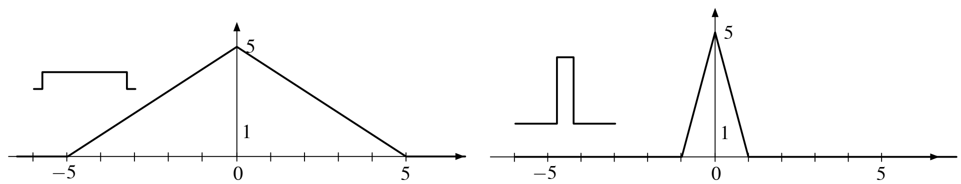


The positive time samples can be found similarly.

The result is



This has the energy of a long rectangular pulse, with the temporal resolution of a high-amplitude short rectangular pulse.



This is called pulse compression, because the autocorrellation is effectively compressed compared to the transmit pulse.

Surprisingly, Barker codes are known for only a few N!

N	Code
2	$(+1, -1)$
3	$(+1, +1, -1)$
4	$(+1, -1, +1, +1)$
5	$(+1, +1, +1, -1, +1)$
7	$(+1, +1, +1, -1, -1, +1, -1)$
11	$(+1, +1, +1, -1, -1, -1, +1, -1, -1, +1, -1)$
13	$(+1, +1, +1, +1, +1, -1, -1, +1, +1, -1, +1, -1, +1)$

Besides radar and pulse compression, Barker codes are also used in communication for spread spectrum modulation.

Summary

Systems that require detecting small signals often use correlation receivers.

Correlation is closely related to convolution.

For radar and communications, we often want signals that have:

- Low peak amplitude
- Large energy (long duration)
- Sharp autocorrelation (good temporal resolution)

These are pulse compression waveforms. Barker codes are one example, but there are many others as we'll see soon.