STANFORD UNIVERSITY
EE 102B Spring-2013

Lecture 20

Spectrum Analysis and the Spectrogram

May 17, 2013

ASSIGNMENTS

- Reading for this Lecture:
  - SPF: Section 12-3, Chapter 66-6 thru 66-9 and Chapter 13
  - S&S:
  - HW#07 is due by 5pm Wednesday, May 22, in Packard 263.
  - Lab #05 is due by 5pm, today, May 17, in Packard 263.

Office Hours for Course Staff – Come see us.

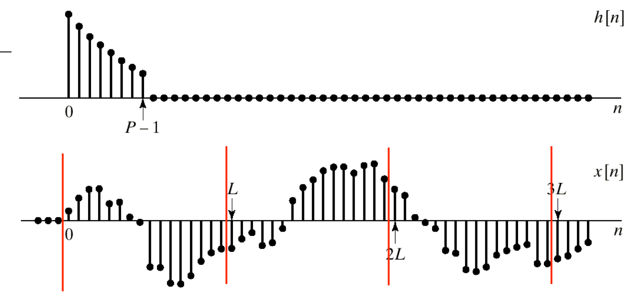
- Ron Schafer: Mon. and Weds. 2:00-4:00 pm, Packard 211.
- Dookun Park: Tues. 1:30 ~ 3:30 pm in Packard 107
- Ruo Yu Gu (Roy): Mon. 4:00 ~ 6:00 pm in Packard 106

Lecture Objective

- Review of block processing
- Windowing
- Examples using spectrum_analysis_demo_GUI
- Short-time Fourier analysis
- The spectrogram

REVIEW OF BLOCK PROCESSING OF LONG SIGNALS

Block Convolution



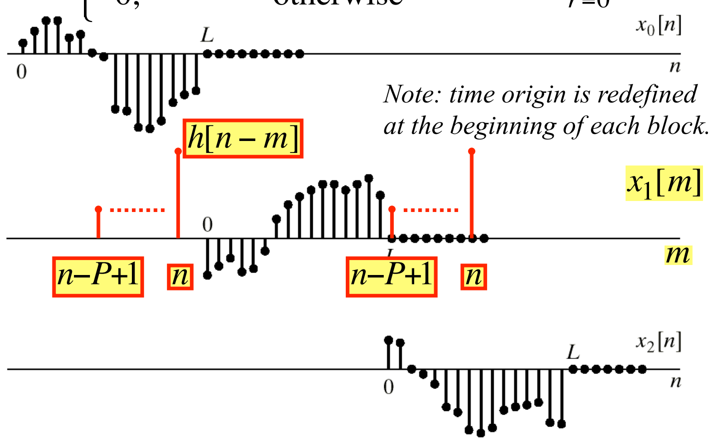
$$x[n] = \sum_{r=0}^{\infty} x_r[n-rL] \rightarrow y[n] = x[n]*h[n] = \sum_{r=0}^{\infty} y_r[n-rL]$$

$$x_r[m] = \begin{cases} x[m+rL], & 0 \le m \le L-1 \\ 0, & \text{otherwise} \end{cases} \quad \rightarrow \quad y_r[n] = x_r[n]*h[n]$$

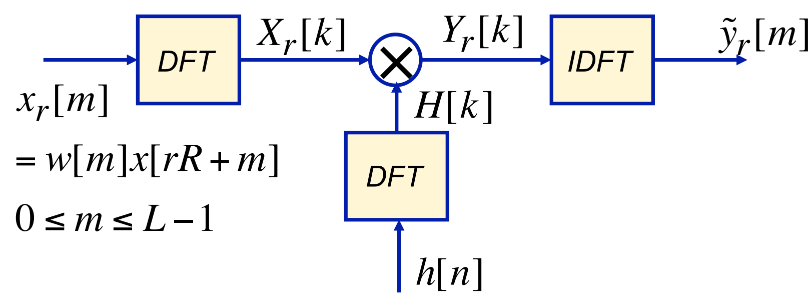
Rectangular window

Segmenting the Input

$$x_r[n] = \begin{cases} x[n+rL], & 0 \le n \le L-1 \\ 0, & \text{otherwise} \end{cases} \qquad x[n] = \sum_{r=0}^{\infty} x_r[n-rL]$$



Convolution Using the DFT

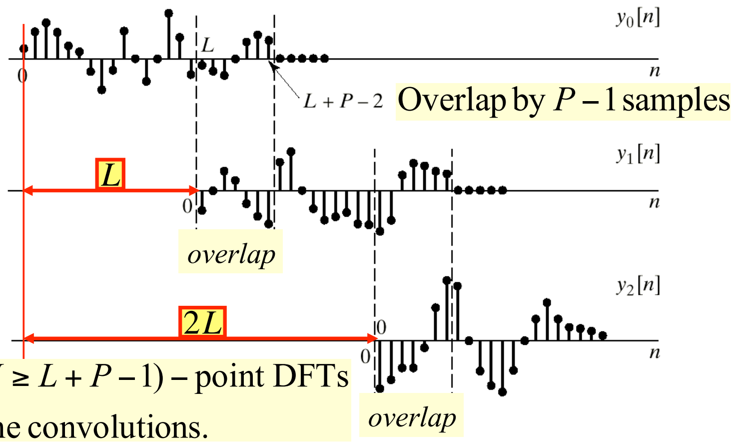


$$x_r[m] = w[m]x[rR+m]$$

$$0 \le m \le L-1$$

Putting the Output Pieces Together

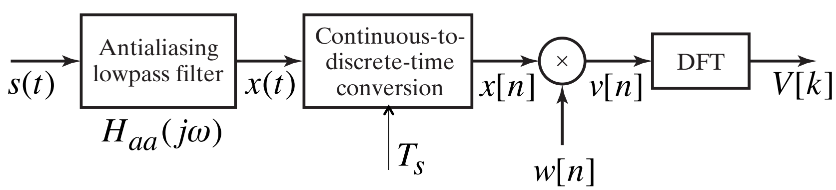
$$y[n] = \sum_{r=0}^{\infty} y_r[n - rL] \qquad y_r[n] = x_r[n] * h[n]$$



Use $(N \geq L + P - 1)$ – point DFTs to do the convolutions.

SPECTRUM ANALYSIS OF CONTINUOUS-TIME SIGNALS

Spectrum Analysis of Continuous-Time Signals



Work out the details of the relationship between $V[k]$ and $S(j\omega)$ to verify your understanding of all parts of this important DSP system as well as how the parts fit together to give an estimate of the spectrum.

WINDOWING

Multiplication in the Time-Domain

- Multiplication in the time domain

$$y[n] = w[n]x[n]$$

- Convolution in the frequency domain

$$Y(e^{j\hat{\omega}}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})W(e^{j(\hat{\omega}-\theta)})d\theta$$

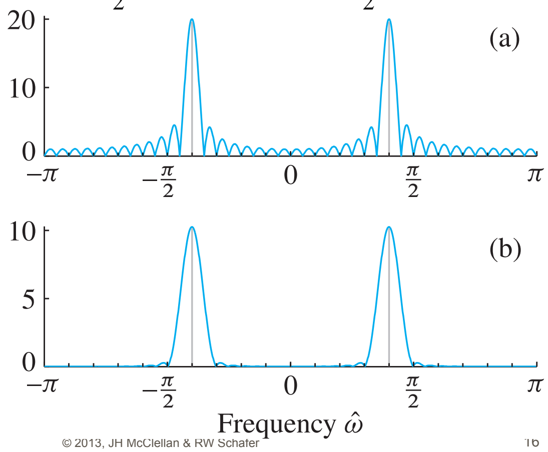
Windowing a Sinusoid

$$x_w[n] = w[n]\cos(0.4\pi n) \Leftrightarrow$$

$$X_w(e^{j\hat{\omega}}) = \tfrac{1}{2}W(e^{j(\hat{\omega}+0.4\pi)}) + \tfrac{1}{2}W(e^{j(\hat{\omega}-0.4\pi)})$$

40-point Rectangular Window

40-point Hann Window



Examples to try with spectrum_analysis_demo

- W=[2*pi/16], A=[1], TH=[0], B=0, N=32, L=32, swin=r
- W=[2*pi/16], A=[1], TH=[0], B=0, N=32, L=32, swin=h
- W=[2*pi/16], A=[1], TH=[0], B=0, N=32, L=16, swin=r
- W=[2*pi/14, 4*pi/15], A=[1,.75], TH=[0,0], B=0, N=64, L=64, swin=r
- W=[2*pi/16, 4*pi/16], A=[1,.75], TH=[0,0], B=0, N=64, L=64, swin=r
- W=[2*pi/14, 4*pi/15], A=[1,.75], TH=[0,0], B=0, N=64, L=32, swin=k
- W=[2*pi/14, 4*pi/15], A=[1,.75], TH=[0,0], B=0, N=64, L=64, swin=k

PLOTTING THE SHORT-TIME SPECTRUM
THE SPECTROGRAM

Example 1

- Consider the continuous-time time-varying sinusoidal signal

$$x(t) = \begin{cases} 5\cos(2\pi f_0 t) & 0 \le t < T_1 \\ 2\cos(2\pi f_1 t) & T_1 \le t < T_2 \\ 2\cos(2\pi f_2 t) & T_2 \le t < T_3 \\ 0.5\cos(2\pi f_3 t) & T_3 \le t < T_4 \end{cases}$$

$$f_0 = 211,\ f_1 = 111,\ f_2 = 800,\ f_3 = 400 \text{ Hz}$$

Example 1

- Sample x(t) with sampling rate f_s = 2000.

$$x[n] = x(nT_s) = \begin{cases} 5\cos(\hat{\omega}_0 n) & 0 \le n < 499 \\ 2\cos(\hat{\omega}_1 n) & 500 \le n < 2999 \\ 2\cos(\hat{\omega}_2 n) & 3000 \le n < 4999 \\ 0.5\cos(\hat{\omega}_3 n) & 5000 \le n \le 9999 \end{cases}$$

$$\hat{\omega}_0 = 0.211\pi, \hat{\omega}_1 = 0.111\pi, \hat{\omega}_2 = 0.8\pi, \hat{\omega}_3 = 0.4\pi$$

$$T_1 f_s = 500, T_2 f_s = 3000, T_3 f_s = 5000, T_4 f_s = 1000,$$

Time-Dependent (Short Time) DTFT and DFT

- Definition: short-time DTFT

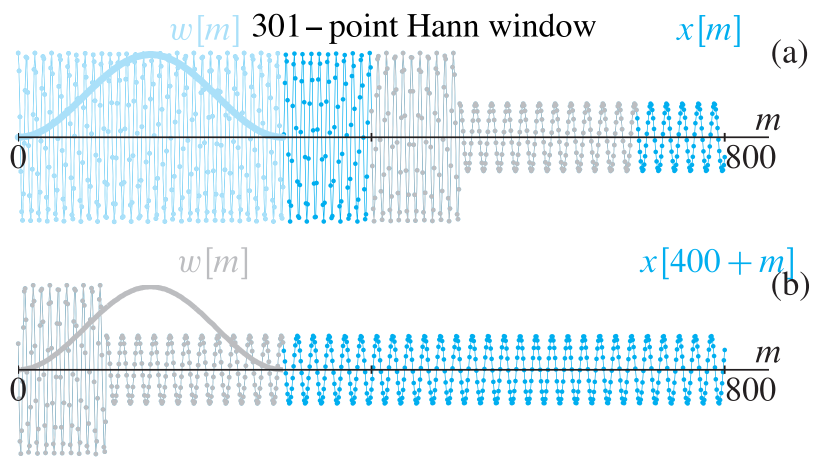
$$X(e^{j\hat{\omega}}, n] = \sum_{m=0}^{L-1} w[m]x[m+n]e^{-j\hat{\omega}n} \qquad 0 \le \hat{\omega} < 2\pi$$

- Definition: short-time DFT

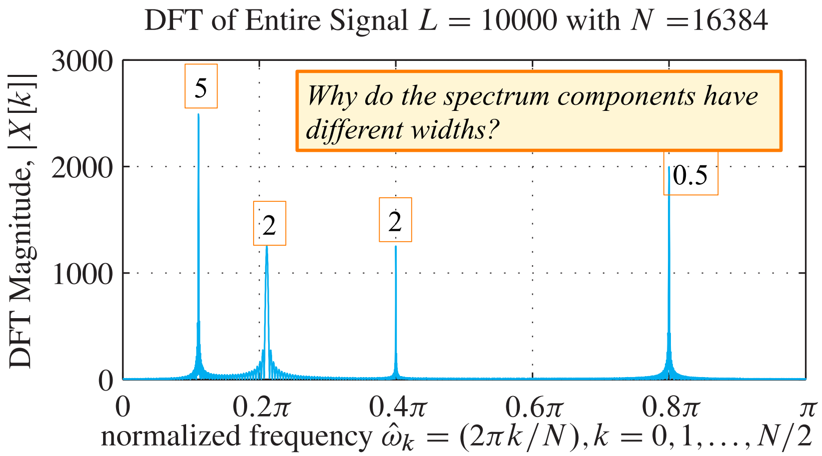
$$X[k,n] = \sum_{m=0}^{L-1} w[m]x[m+n]e^{-j(2\pi/N)kn} \qquad k = 0,1,\ldots,N-1$$

- w[m]x[m+n] focuses attention on the signal around time n.

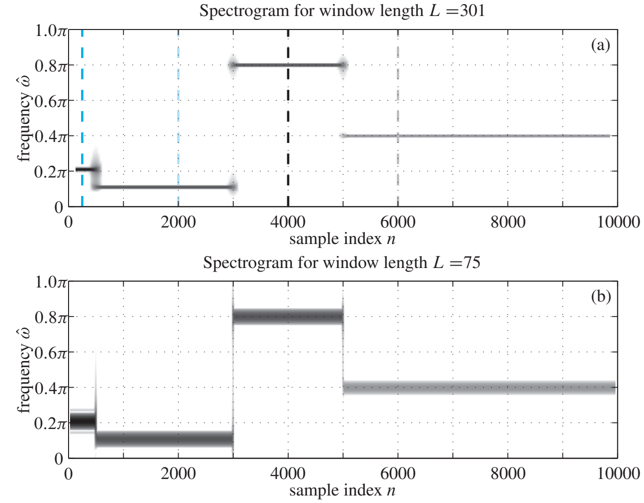
Illustration of Short-Time Analysis



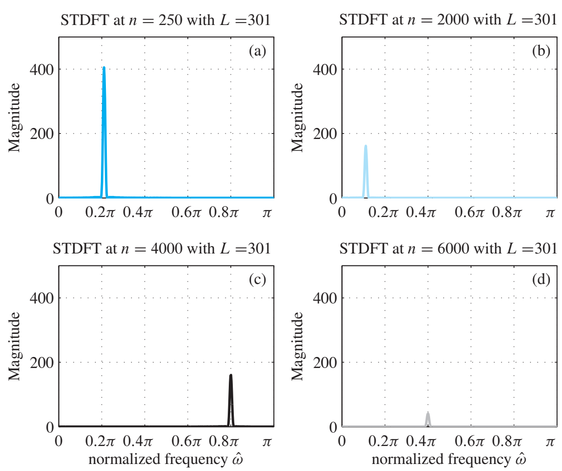
Long-Term Spectrum of Example 1



Spectrogram for Example 1



Spectrogram Slices for Example 1

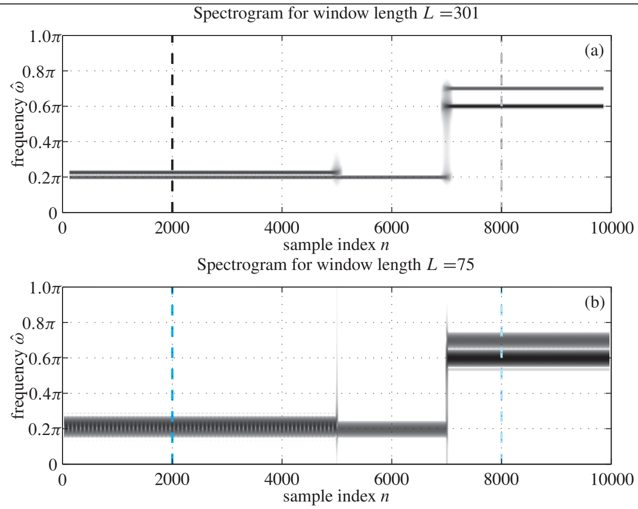


Example 2

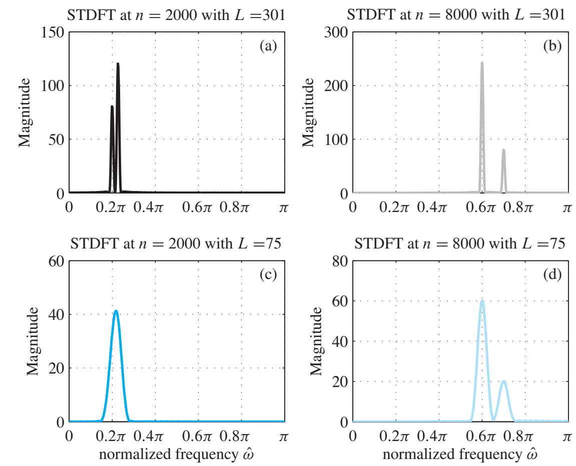
- The sampled signal is

$$x[n] = \begin{cases} \cos(0.2\pi n) + 1.5\cos(0.227\pi n) & 0 \le n < 5000 \\ \cos(0.2\pi n) & 5000 \le n < 7000 \\ 3\cos(0.6\pi n) + \cos(0.7\pi n) & 7000 \le n < 10000 \end{cases}$$

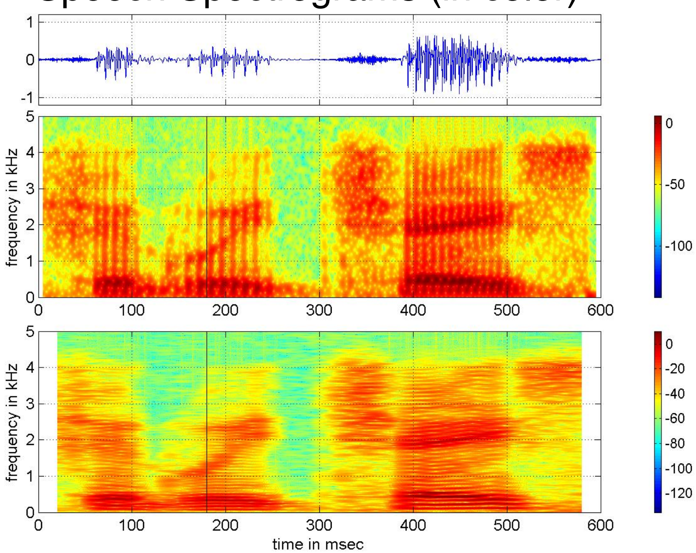
Spectrogram for Example 2



Spectrogram Slices for Example 2



Speech Spectrograms (in color)



Wideband (short window)

Narrow-band (long window)

