STANFORD UNIVERSITY
EE 102B Spring-2013

Lecture 15

Review D-T Filtering of C-T Signals and Decimation/ Interpolation

May 3, 2013

ASSIGNMENTS

- Reading for this Lecture:
  - SPF: Section 12-3
  - S&S: Chapter 5
- HW#05 is due by 5pm Wednesday, May 8, in Packard 263.
- Lab #04 is due by 5pm, today, May 3, in Packard 263. Lab #05 is due Friday May 17.
- Mid-term exam on Friday, May 10, in class. Room and exam conditions TBA.

Office Hours for Course Staff – Come see us.

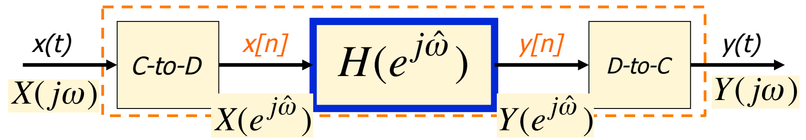
- Ron Schafer: Mon. and Weds. 2:00-4:00 pm, Packard 211.
- Dookun Park: Tues. 1:30 ~ 3:30 pm in Packard 107
- Ruo Yu Gu (Roy): Mon. 4:00 ~ 6:00 pm in Packard 106
- https://class2go.stanford.edu/EE102B/Spring2013/pages/staff

Lecture Objective

- Review discrete-time filtering of continuous-time signals
- Sampling rate changing by discrete-time filtering
  - Decimation
  - Interpolation

REVIEW OF DISCRETE-TIME FILTERING OF CONTINUOUS-TIME SIGNALS

Putting it All Together



$$Y(j\omega) = H_r(j\omega)Y(e^{j\omega T_s}) = H_r(j\omega)H(e^{j\omega T_s})\underline{X(e^{j\omega T_s})}$$

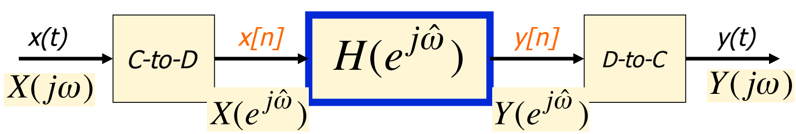
1 for $|\omega| \le \pi/T_s$

$$Y(j\omega) = \cancel{H_r(j\omega)}H(e^{j\omega T_s})\underline{\frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))}$$

If no aliasing occurs in sampling x(t), then it follows that

$$\boxed{Y(j\omega) = H(e^{j\omega T_s})X(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)}$$

DT Filtering of CT Signals

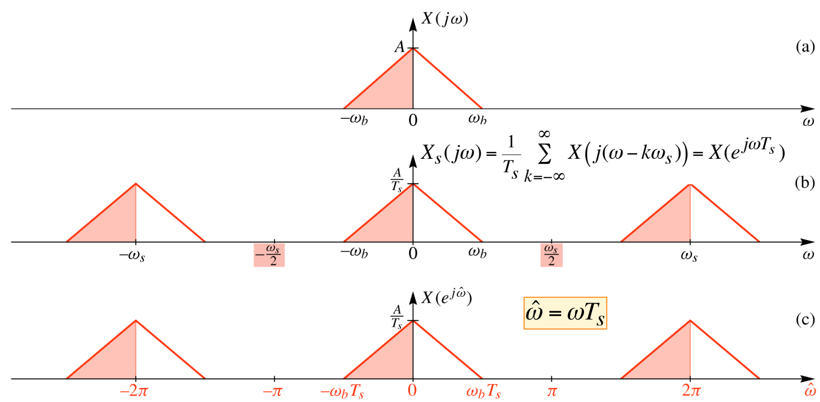


If no aliasing occurs in sampling x(t), then it follows that

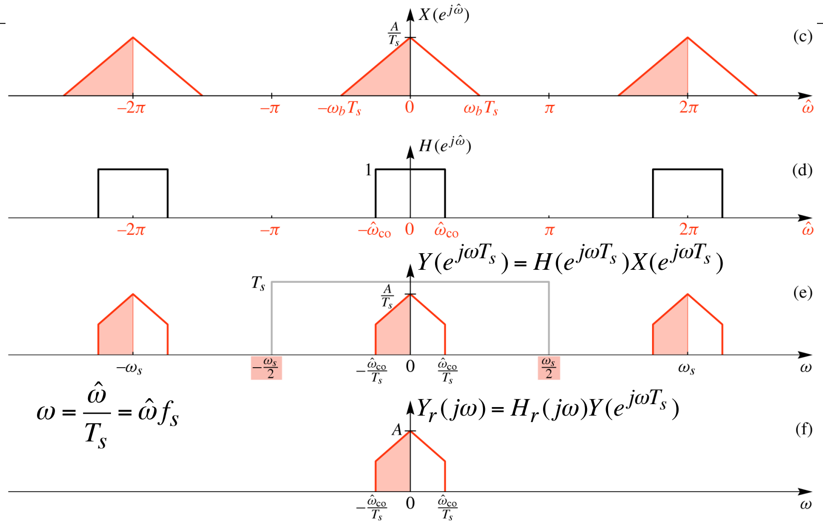
$$\boxed{Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)}$$

$$H_{\text{eff}}(j\omega) = \begin{cases} H(e^{j\omega T_s}) & |\omega| < \frac{1}{2}\omega_s \\ 0 & |\omega| > \frac{1}{2}\omega_s \end{cases}$$

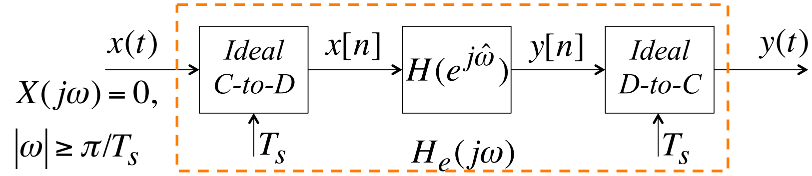
Sampling



Discrete-Time Filtering of Continuous-Time Signals

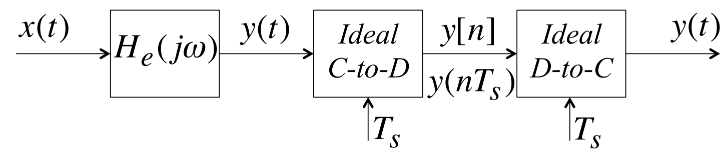


Effective Filter Equivalent System



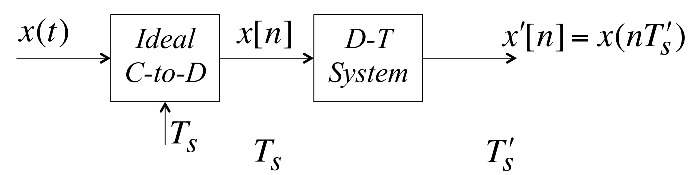
$$H_e(j\omega) = \begin{cases} H(e^{j\omega T_s}) & |\omega| < \pi/T_s \\ 0 & |\omega| > \pi/T_s \end{cases}$$

$$y(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin\left[\frac{\pi}{T_s}(t-nT_s)\right]}{\frac{\pi}{T_s}(t-nT_s)}$$



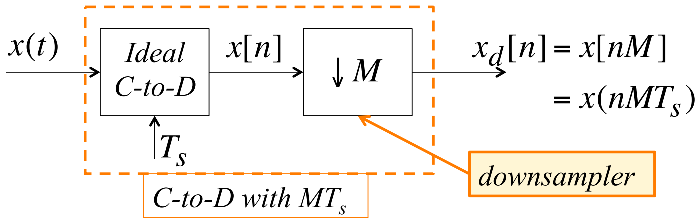
CHANGING THE SAMPLING RATE USING DISCRETE-TIME FILTERING

Sampling Rate Changing by Discrete-Time Processing



$$X'(e^{j\omega T_s'}) = \frac{1}{T_s'} \sum_{k=-\infty}^{\infty} X\left(j(\omega - k\frac{2\pi}{T_s'})\right)$$

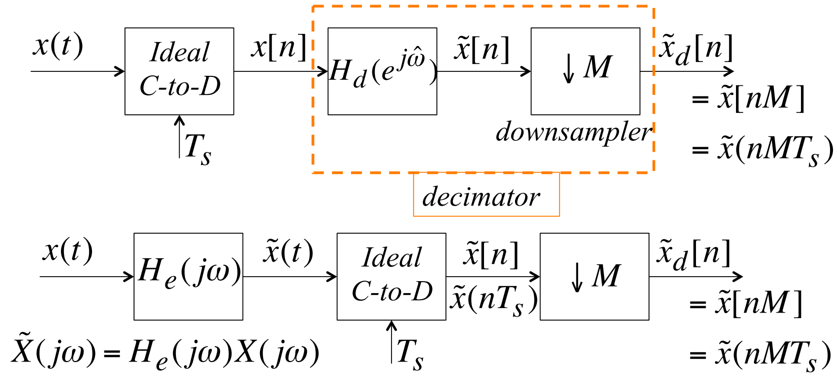
Sampling Rate Reduction by *downsampling*



$$X_d(e^{j\omega MT_s}) = \frac{1}{MT_s}\sum_{k=-\infty}^{\infty} X\left(j(\omega - k\frac{2\pi}{MT_s})\right)$$

We will have aliasing distortion unless the input is M-times over-sampled; i.e, $X(j\omega) = 0, \quad |\omega| \geq \pi/(MT_s)$

Sampling Rate Reduction by Decimation



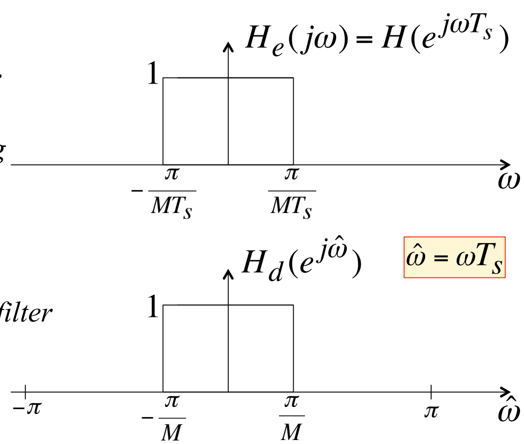
$$\tilde{X}(j\omega) = H_e(j\omega)X(j\omega)$$

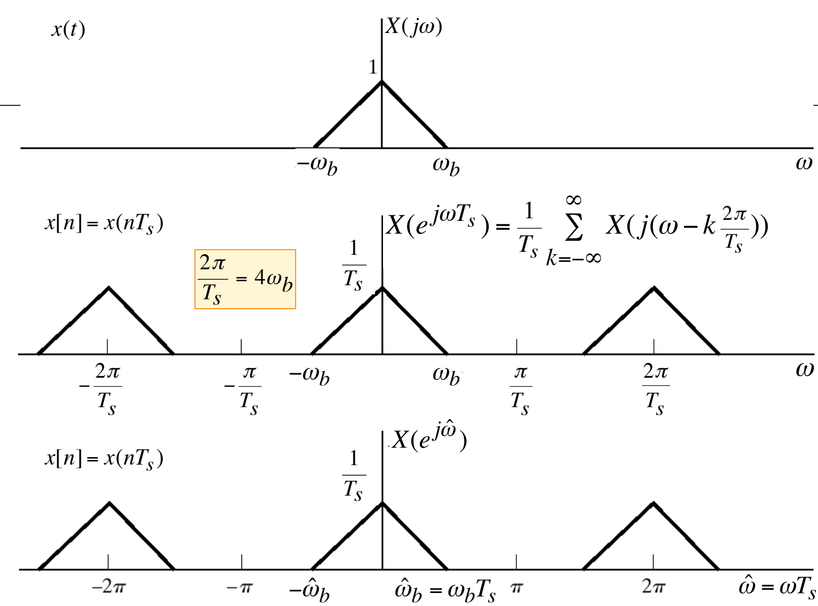
$$\tilde{X}_d(e^{j\omega MT_s}) = \frac{1}{MT_s}\sum_{k=-\infty}^{\infty} \tilde{X}\left(j(\omega - k\frac{2\pi}{MT_s})\right)$$

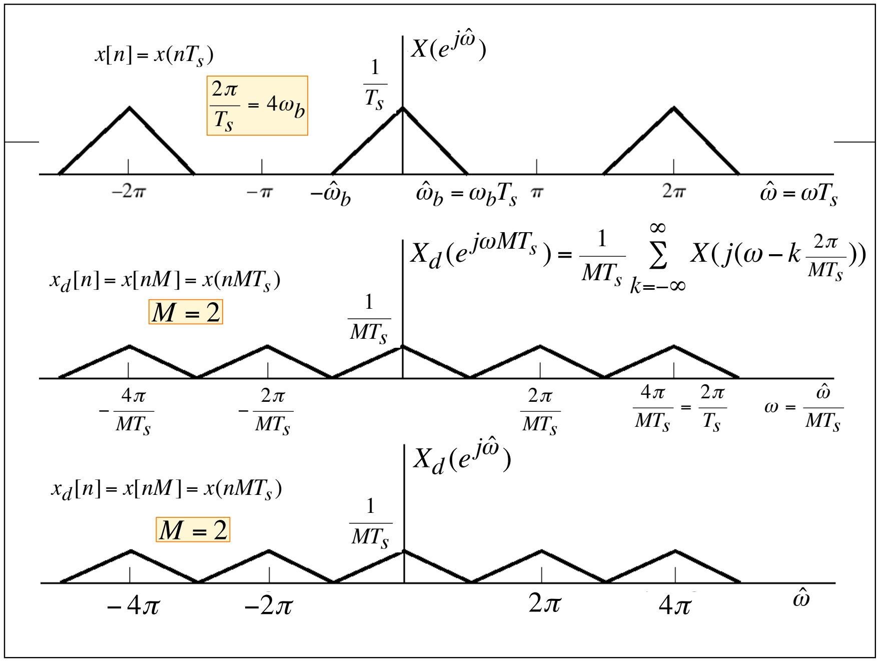
Ideal Filters for Decimation

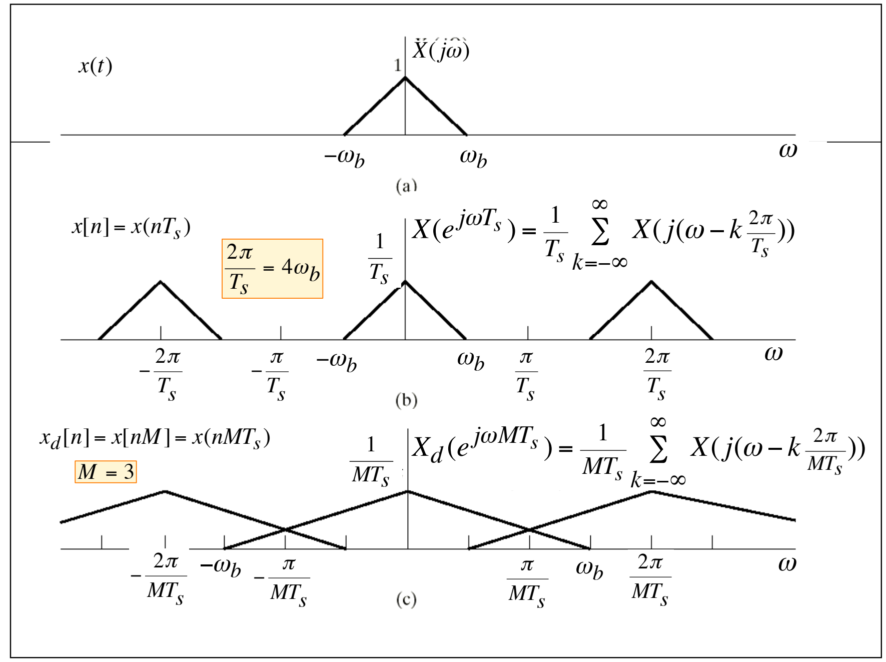
Continuous-time pre-filter required to avoid aliasing with MT_s

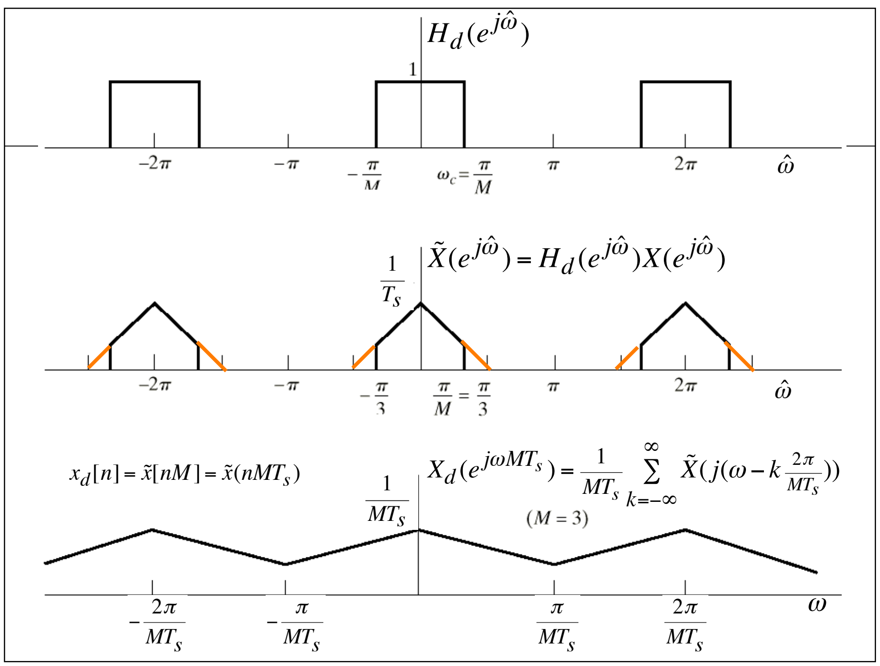
Bandlimiting discrete-time filter



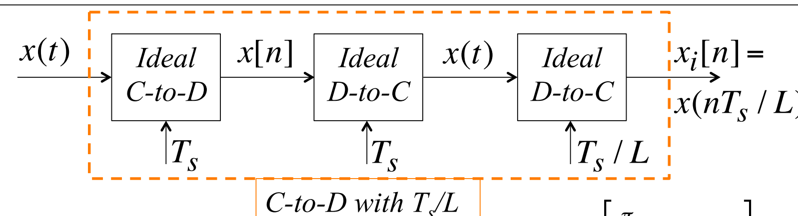








Increasing Sampling Rate by Interpolation - I



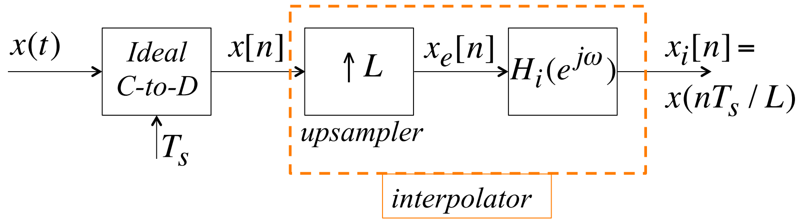
$$x[n] = x(nT_s)$$

$$X(j\omega) = 0, \quad |\omega| \geq \pi / T_s$$

$$x(t) = \sum_{k=-\infty}^{\infty} x[k] \frac{\sin\left[\frac{\pi}{T_s}(t - kT_s)\right]}{\frac{\pi}{T_s}(t - kT_s)}$$

$$x_i[n] = \sum_{k=-\infty}^{\infty} x[k] \frac{\sin\left[\frac{\pi}{T_s}(nT_s/L - kT_s)\right]}{\frac{\pi}{T_s}(nT_s/L - kT_s)} = \sum_{k=-\infty}^{\infty} x[k] \frac{\sin\left[\frac{\pi}{L}(n - kL)\right]}{\frac{\pi}{L}(n - kL)}$$

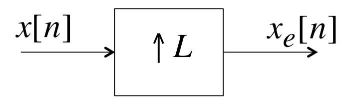
Increasing Sampling Rate by Interpolation - II



$$x_e[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-kL] = \begin{cases} x[n/L] & n = 0, \pm 1, \pm 2, \ldots \\ 0 & \text{otherwise} \end{cases}$$

$$\boxed{x_i[n] = x_e[n] * h_i[n] = \sum_{k=-\infty}^{\infty} x[k] \frac{\sin\left[\frac{\pi}{L}(n-kL)\right]}{\frac{\pi}{L}(n-kL)}}$$

Up-Sampling in the Frequency Domain



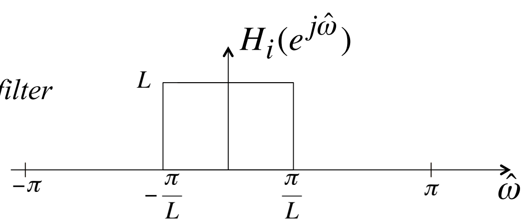
$$x_e[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-kL] = \begin{cases} x[n/L] & n = 0, \pm 1, \pm 2, \ldots \\ 0 & \text{otherwise} \end{cases}$$

$$X_e(e^{j\hat{\omega}}) = \sum_{k=-\infty}^{\infty} x[k]\delta[n-kL]e^{-j\hat{\omega}k}$$

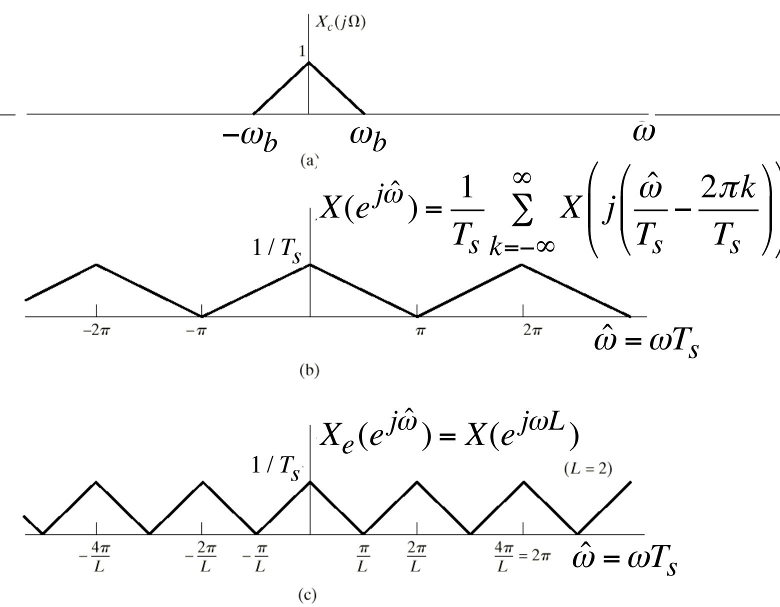
$$= \sum_{k=-\infty}^{\infty} x[k]e^{-j\hat{\omega}kL} = X(e^{j\hat{\omega}L})$$

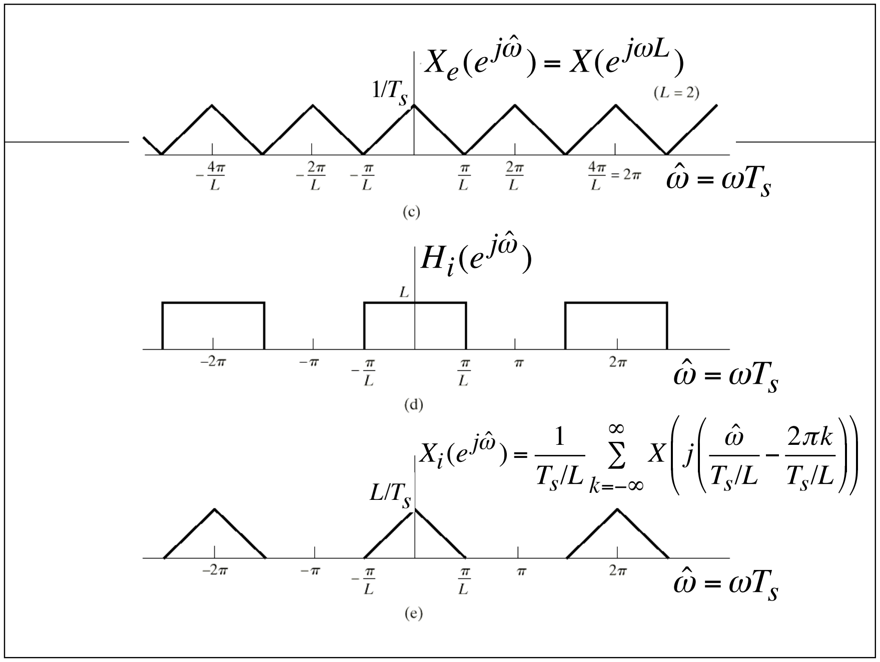
Ideal Filter for Interpolation

Bandlimiting discrete-time filter



$$\boxed{h_i[n] = L\frac{\sin\left[\frac{\pi}{L}n\right]}{\pi n} = \frac{\sin\left[\frac{\pi}{L}n\right]}{\frac{\pi}{L}n}}$$





$X_e(e^{j\hat{\omega}}) = X(e^{j\omega L})$
$1/T_s$
$(L = 2)$
$-\frac{4\pi}{L}$ $-\frac{2\pi}{L}$ $-\frac{\pi}{L}$ $\frac{\pi}{L}$ $\frac{2\pi}{L}$ $\frac{4\pi}{L} = 2\pi$
$\hat{\omega} = \omega T_s$
(c)
$H_i(e^{j\hat{\omega}})$
L
-2π $-\pi$ $-\frac{\pi}{L}$ $\frac{\pi}{L}$ π 2π
$\hat{\omega} = \omega T_s$
(d)
$X_i(e^{j\hat{\omega}}) = \frac{1}{T_s/L}\sum_{k=-\infty}^{\infty} X\left(j\left(\frac{\hat{\omega}}{T_s/L} - \frac{2\pi k}{T_s/L}\right)\right)$
L/T_s
-2π $-\pi$ $-\frac{\pi}{L}$ $\frac{\pi}{L}$ π 2π
$\hat{\omega} = \omega T_s$
(e)