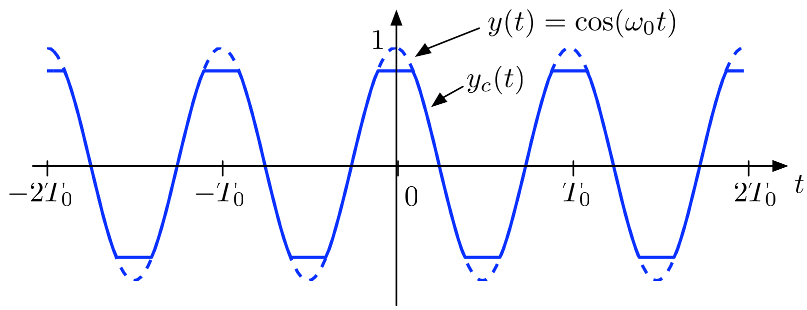
EE102A Winter 2012-13 — Pauly

Signal Processing and Linear Systems I

Problem Set #4

Due: Wednesday, February 6 at 5 PM.

1. An amplifier has an input of $x(t) = \cos(\omega_0 t)$. Unfortunately, the output is limited by the rail voltage of the amplifier, and the result is the clipped waveform $y_c(t)$ shown below:



We will use the Fourier series to characterize the effect of this clipping.

a) Is the amplifier a linear system? Is it time-invariant?

b) We can think of the clipped signal as the ideal signal $y(t) = \cos(\omega_0 t)$ plus a distortion term $y_d(t)$, so that

$$y_c(t) = y(t) + y_d(t).$$

Sketch the error signal $y_d(t)$.

c) What is the symmetry of the Fourier series coefficients of the error term $y_d(t)$? Are they real, imaginary, or complex?

d) Which of the Fourier series coefficients of $y_d(t)$ are non-zero? For example, is D_0 non-zero, or D_1 non-zero? You don't need to calculate the coefficients.
Hint: When considering the Fourier series coefficients integral over the whole period $-T_0/2$ to $T_0/2$, look for symmetries over *half* the period, such as $-T_0/2$ to 0, and 0 to $T_0/2$.

2. $f(t)$ is a periodic signal with a period T_0, and has a Fourier series with coefficients $\{D_n\}$.

(a) What are the Fourier series coefficients for the delayed signal $f(t - \tau)$?

(b) What are the Fourier series coefficients for the time scaled signal $f(at)$. Note that the period of $f(at)$ is T_0/a.

(c) You find that all of the odd Fourier series coefficients are zero (i.e. $D_1, D_{-1}, D_3, D_{-3}, \cdots$ are all zero). What can you conclude about $f(t)$?

(d) Two functions $f(t)$ and $\tilde{f}(t)$ have Fourier series coefficients D_n and $\tilde{D}_n$, respectively, and

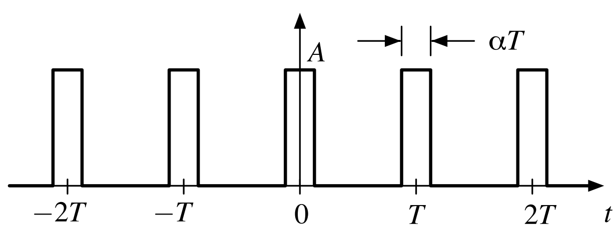
$$\begin{aligned} \left|\tilde{D}_n\right| &= |D_n| \\ \angle\tilde{D}_n &= \angle D_n + n\frac{\pi}{2} \end{aligned}$$

Find a simple expression for $\tilde{f}(t)$ in terms of $f(t)$.

3. Switching amplifiers are a very efficient way to generate a time-varying output voltage from a fixed supply voltage. They are particularly useful in high-power applications.

The basic idea is that an output voltage a is generated by rapidly switching between zero and the supply voltage A. The output is then lowpass filtered to remove the harmonics generated by the switching operation. For our purposes we can consider the lowpass filter as an integrator over many switching cycles, so the output voltage is the average value of the switching waveform.. Varying the switching rate varies the output voltage. In this problem we will only consider the case where the desired output voltage is constant.

We can analyze this system with the Fourier series. If the output pulses are spaced by T, the waveform the amplifier generates immediately before the lowpass filter is



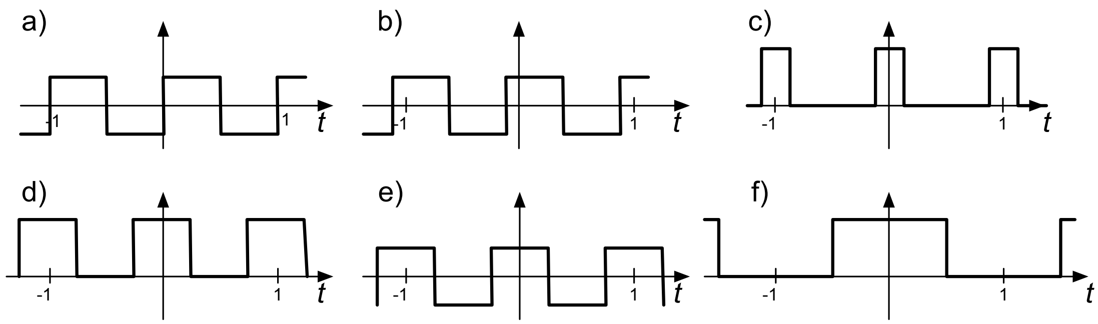
The duty cycle of the switching amplifier is α, and the width of the pulses is αT. When $\alpha = 1$, the amplifier is constantly on and produces its maximum output A.

(a) Reducing the duty cycle reduces the output voltage. After the lowpass filter, only the zero-frequency spectral component D_0 remains, and this will be the output voltage. Find the value of D_0 as a function of the duty cycle α. If the desired output voltage is a and the supply voltage is A, what should α be?

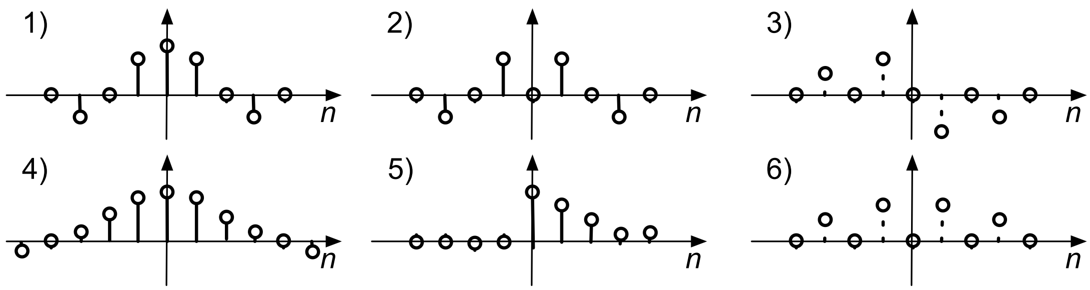
(b) The lowpass filter must suppress (average out) the harmonics generated by the switching waveform. The first harmonic is the most difficult to suppress since it tends to be large, and is closest in frequency. Find an expression for the amplitude of the first harmonic D_1 as a function of the duty cycle α.

(c) What duty cycle α results in the largest magnitude first harmonic?

4. Six different periodic signals (a-f) are plotted below. Assume they are all real.



Six different Fourier series coefficient spectra (1-6) are plotted below. Solid lines are real coefficients, and dashed lines are imaginary.

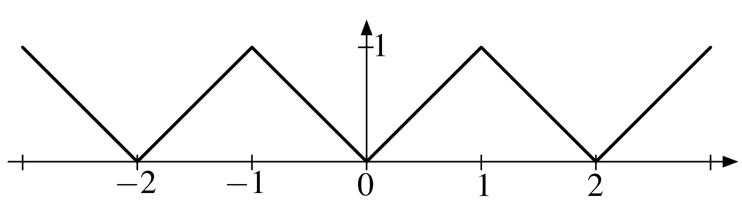


Each spectrum may correspond to one or more of the signals above, or maybe none at all. For each spectrum, decide which signals (if any) it corresponds to. In each case, provide a very brief description of your reasoning.

5. For the lab, we are going to need the Fourier series of a triangle waveform, in addition to the sawtooth waveform described in class. When we found the Fourier series for the sawtooth waveform we used the integral

$$\int_0^1 te^{-j\omega t}dt = \begin{cases} \frac{1}{2} & \text{if } \omega = 0 \\ \frac{je^{-j\omega}}{\omega} + \frac{e^{-j\omega}-1}{\omega^2} & \text{if } \omega \neq 0 \end{cases}.$$

Use this result to show that the Fourier series coefficients for the triangle waveform



are

$$D_n = \begin{cases} \frac{1}{2} & \text{if } n = 0 \\ \frac{(-1)^n - 1}{(n\pi)^2} & \text{if } n \neq 0 \end{cases}$$

Simplify this further if you can. Which terms are non-zero?

Laboratory 4

In this lab we will use matlab to compute the Fourier series for several signals, and compare the errors that results from the approximation of the signals by truncated Fourier series. The first signal will be the sawtooth waveform we discussed in class. This has has a discontinuity that causes Gibbs oscillations on either side of the discontinuity. The second is the triangle signal that you found the Fourier series for in Problem 1b. This has the same values as the sawtooth waveform for $t = [0, 1)$. However, since it is continuous, it is much better behaved.

Task 1: Write an m-file that evalutates a Fourier series Write an m-file that takes a set of Fourier series coefficients, a fundamental frequency, and a vector of output times, and computes the truncated Fourier series evaluated at these times. The declaration and help for the m-file might be

```
function fn = myfs(Dn,omega0,t)
%
%  fn = myfs(Dn,omega0,t)
%
%  Evaluates the truncated Fourier Series at times t
%
%   Dn     -- vector of Fourier series coefficients
%               assumed to run from -N:N, where length(Dn) is 2N+1
%   omega0 -- fundamental frequency
%   t      --  vector of times for evaluation
%
%   fn     --  truncated Fourier series evaluated at t
%
```

The output of the m-file should be

$$f_N(t) = \sum_{n=-N}^{N} D_n e^{j\omega_0 n t}$$

The length of the vector `Dn` should be $2N + 1$. You will need to calculate N from the length of `Dn`.

Task 2: Evaluate the Fourier series of the sawtooth waveform Verify the output of your routine by checking the Fourier series coefficients for the sawtooth waveform described in the class notes. Use the matlab `subplot` command to put multiple plots on a page. In particular the command

```
>> subplot(311)
```

indicates that you are generating a 3x1 array of plots (the first two coefficients) and that the current output should go to the first of these plots (the third coefficient). The other three plots can be addressed by changing the last index. For example, to plot the third of the three plots,

```
>> subplot(313)
```

Calculate the Fourier series coefficients of the sawtooth waveform for $N = 8$ using the expression from the lecture notes, and store them in a vector `Dn`. Plot the coefficients using a `stem` plot

```
>> subplot(311)
>> n = [-8:8]
>> stem(n,real(Dn),'-')
>> hold
>> stem(n,imag(Dn),'--')
```

In this case `Dn` is complex, so you need to plot each. The `hold` command simply puts the next plot on the same axes. The third argument to the stem plot specifies the line type, just as in `plot`. Then define a vector of the times you want to evaluate, and the fundamental frequency

```
>> t = [-2:0.01:2]
>> omega0 = 2*pi;
```

Then plot the ideal function, and the truncated Fourier series approximation

```
>> ft = mod(t,1);
>> fn = myfs(Dn,omega0,t);
>> subplot(312)
>> plot(t,ft,t,fn);
```

In the third subplot, plot the approximation error,

```
>> subplot(313)
>> plot(t,ft-fn);
```

Note the size of the error, for comparison with the next task. Make sure you label the axes of the plots as you go along.

Task 3: Evaluate the Fourier series of the triangle waveform Repeat the steps of Task 2 for the case of the triangular signal from this homework. Compute the integral square error for Task 2 and this task by squaring the error, and summing. For example, for the Task 2 signals,

```
>> e2 = sum((ft-fn).^2)*0.01;
```

where the 0.01 is the time sample width. What is the ratio of the integral square error for the two signals?