Stanford University
Winter 2012-2013

Signal Processing and Linear Systems I

Lecture 2: Signal Characteristics and Models

January 7, 2013

Signal Characteristics and Models

- Types of signals and systems
- Operations on the time dependence of a signal
  - Time scaling
  - Time reversal
  - Time shift
  - Combinations
- Signal characteristics
- Periodic signals
- Complex signals
- Signals sizes
- Signal Energy and Power

Signals

- Typical think of signals in terms of communication and information
  - radio signal
  - broadcast or cable TV
  - audio
  - electric voltage or current in a circuit
- More generally, *any* physical or abstract quantity that can be measured, or influences one that can be measured, can be thought of as a signal.
  - tension on bike brake cable
  - roll rate of a spacecraft
  - concentration of an enzyme in a cell
  - the price of dollars in euros
  - the federal deficit

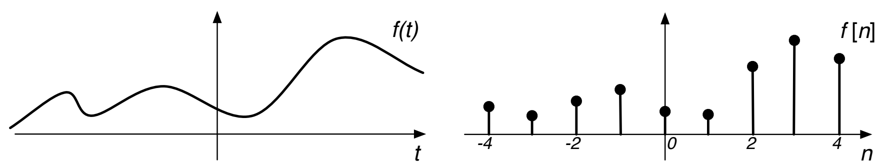
  Very general concept.

Systems

- Typical systems take a signal and convert it into another signal,
  - radio receiver
  - audio amplifier
  - modem
  - microphone
  - cell telephone
  - cellular metabolism
  - national and global economies
- Internally, a system may contain many different types of signals.
- The systems perspective allows you to consider all of these together.
- In general, a *system* transforms *input signals* into *output signals*.

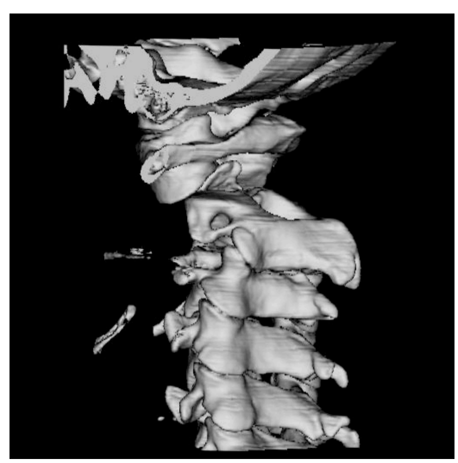
Continuous and Discrete Time Signals

- Most of the signals we will talk about are functions of time.
- There are many ways to classify signals. This class is organized according to whether the signals are continuous in time, or discrete.
  - A *continuous-time* signal has values for all points in time in some (possibly infinite) interval. This is the topic of ee102a.
  - A *discrete time* signal has values for only discrete points in time. This is the main topic of ee102b.



- Signals can also be a function of space (images, volumes) or of space and time (video), and may be continuous or discrete in each dimension.


CT Data


3D Surface Render

Images from *Computed Tomography* by Willi Kalender, Wiley, 2006

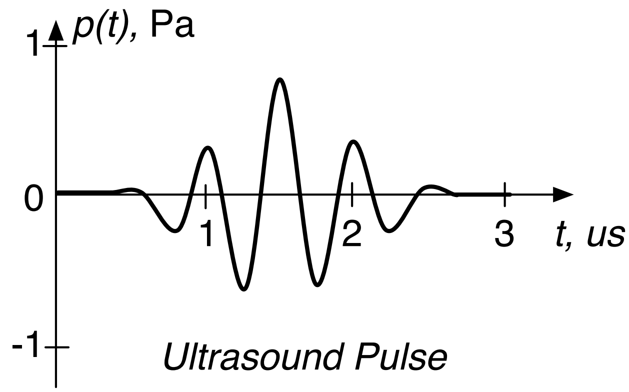
Types of Systems

Systems are classified according to the types of input and output signals

- *Continuous-time system* has continuous-time inputs and outputs.
  - AM or FM radio
  - Conventional (all mechanical) car
- *Discrete-time system* has discrete-time inputs and outputs.
  - PC computer game
  - Matlab
  - Your mortgage
- Hybrid systems are also very important (A/D, D/A converters).
  - You playing a game on a PC
  - Modern cars with ECU (electronic control units)
  - Most commercial and military aircraft

Continuous Time Signals

- Function of a time variable, something like t, τ, t_1.
- The entire signal is denoted as v, $v(.)$, or $v(t)$, where t is a dummy variable.



Ultrasound Pulse

Discrete Time Signals

- Fundamentally, a discrete-time signal is sequence of samples, written

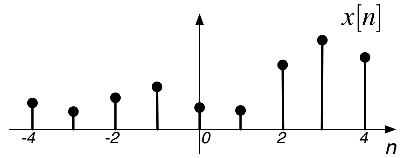
$$x[n]$$

where n is an integer over some (possibly infinite) interval.

- Often, at least conceptually, samples of a continuous time signal

$$x[n] = x(nT)$$

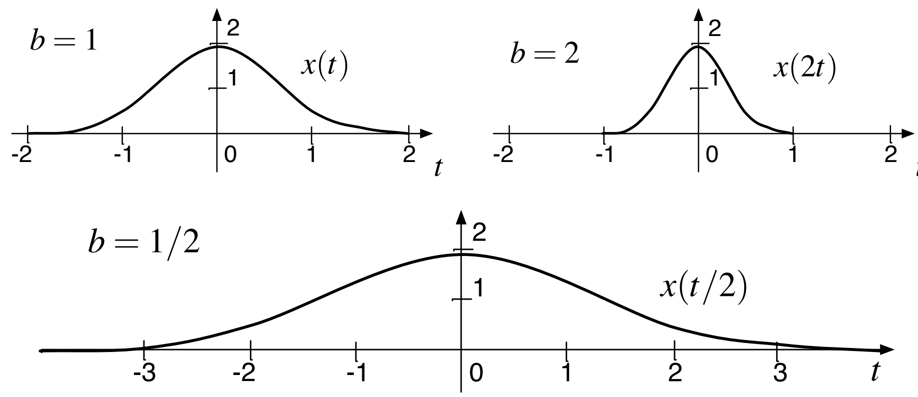
where n is an integer, and T is the *sampling period*.



- Discrete time signals might not represent uniform time samples (NYSE closes, for example)
- There might not be an underlying continuous time signal (NYSE closes, again)
- There might not be any underlying physical reality (PC computer game)

Operations on Signals: Time Scaling, Continuous Time

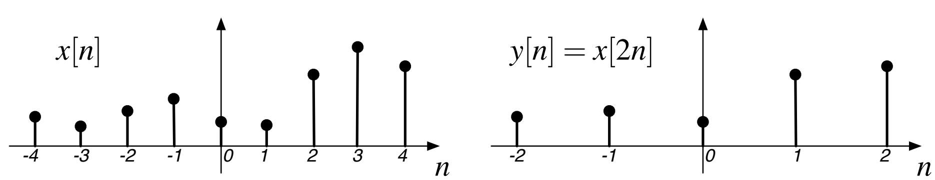
A signal $x(t)$ is scaled in time by multiplying the time variable by a positive constant b, to produce $x(bt)$. A positive factor of b either expands $(0 < b < 1)$ or compresses $(b > 1)$ the signal in time.



Time Scaling, Discrete Time

The discrete-time sequence $x[n]$ is *compressed* in time by multiplying the index n by an integer k, to produce the time-scaled sequence $x[nk]$.

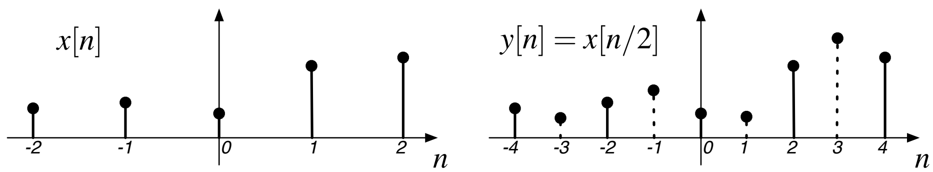
- This extracts every k^{th} sample of $x[n]$.
- Intermediate samples are lost.
- The sequence is compressed.



Called *downsampling*, or *decimation*.

The discrete-time sequence $x[n]$ is *expanded* in time by dividing the index n by an integer m, to produce the time-scaled sequence $x[n/m]$.

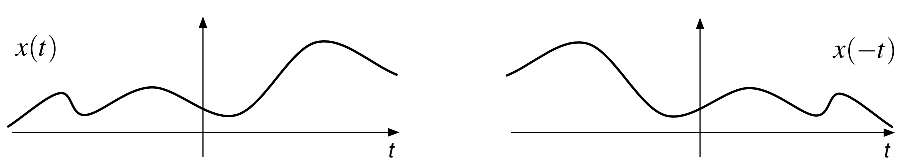
- This specifies every m^{th} sample.
- The intermediate samples must be synthesized (set to zero, or interpolated).
- The sequence is expanded.



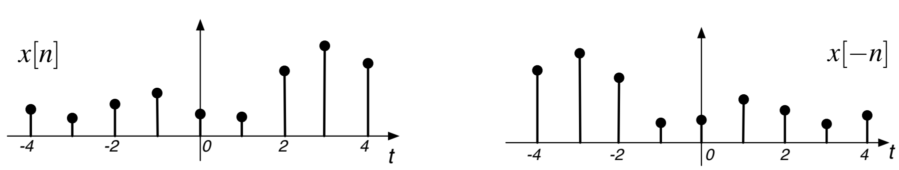
Called *upsampling*, or *interpolation*.

Time Reversal

- Continuous time: replace t with $-t$, time reversed signal is $x(-t)$



- Discrete time: replace n with $-n$, time reversed signal is $x[-n]$.

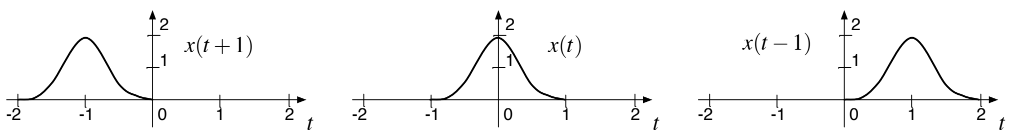


- Special case of time scaling with $b = -1$.

Time Shift

For a continuous-time signal $x(t)$, and a time $t_1 > 0$,

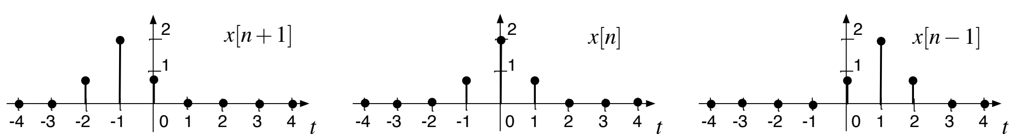
- Replacing t with $t - t_1$ gives a *delayed* signal $x(t - t_1)$
- Replacing t with $t + t_1$ gives an *advanced* signal $x(t + t_1)$



- May seem counterintuitive. Think about where $t - t_1$ is zero.

For a discrete time signal $x[n]$, and an integer $n_1 > 0$

- $x[n - n_1]$ is a delayed signal.
- $x[n + n_1]$ is an advanced signal.
- The delay or advance is an integer number of sample times.

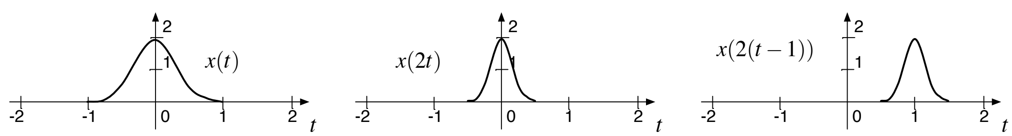


- Again, where is $n - n_1$ zero?

Combinations of Operations

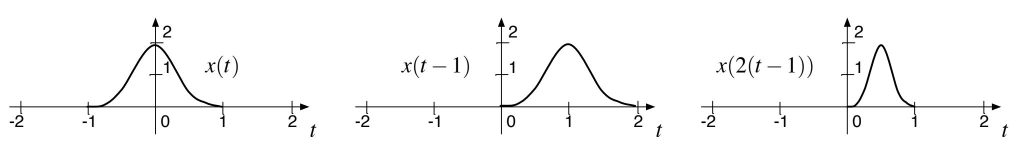
- Time scaling, shifting, and reversal can all be combined.
- Operation can be performed in any order, but care is required.
- This *will* cause confusion.
- Example: $x(2(t-1))$

Scale first, then shift
Compress by 2, shift by 1

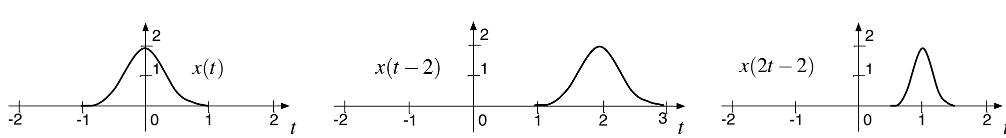


Example $x(2(t-1))$, continued

Shift first, then scale
Shift by 1, compress by 2 *Incorrect*

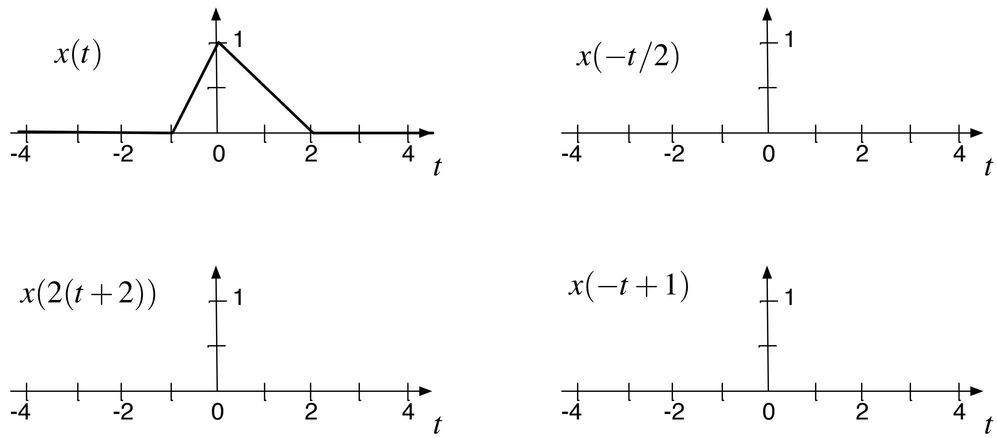


Shift first, then scale
Rewrite $x(2(t-1)) = x(2t-2)$
Shift by 2, scale by 2 *Correct*



Where is $2(t-1)$ equal to zero?

Try these yourselves ....



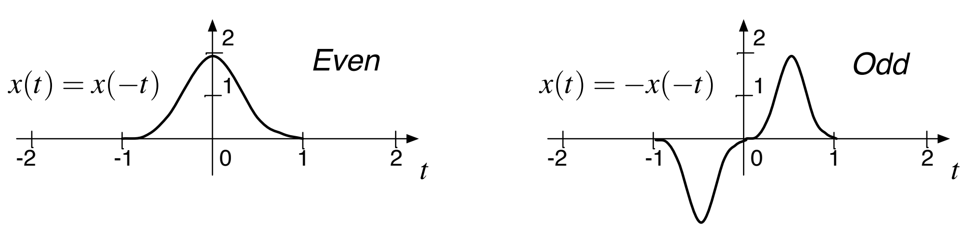
Even and Odd Symmetry

- An *even* signal is symmetric about the origin

$$x(t) = x(-t)$$

- An *odd* signal is antisymmetric about the origin

$$x(t) = -x(-t)$$



- Any signal can be decomposed into even and odd components

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$
$$x_o(t) = \frac{1}{2}[x(t) - x(-t)].$$

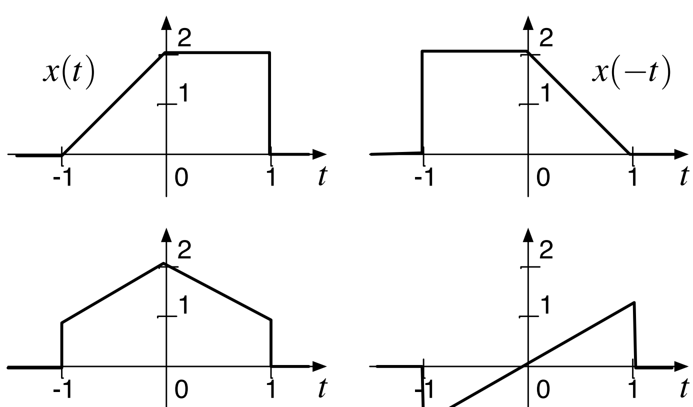
Check that

$$x_e(t) = x_e(-t),$$
$$x_o(t) = -x_o(-t),$$

and that

$$x_e(t) + x_o(t) = x(t).$$

- Example

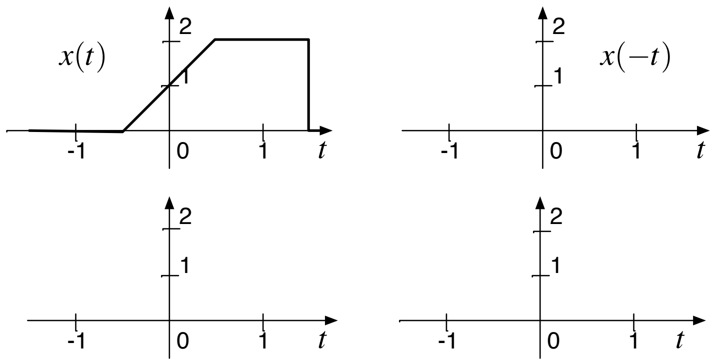


$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

- Same type of decomposition applies for discrete-time signals.

The decomposition into even and odd components depends on the location of the origin. Shifting the signal changes the decomposition.

Plot the even and odd components of the previous example, after shifting $x(t)$ by $1/2$ to the right.

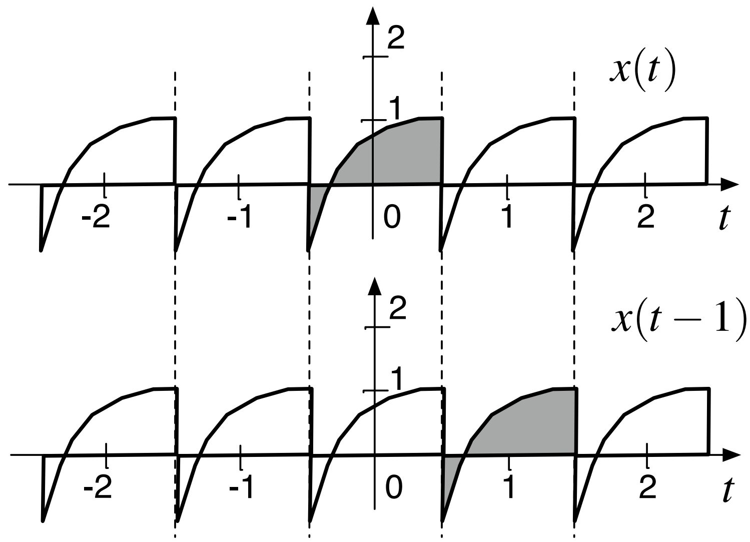


$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] \qquad x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Periodic Signals

- Very important in this class.
- Continuous time signal is periodic if and only if there exists a $T_0 > 0$ such that
$$x(t + T_0) = x(t) \qquad \text{for all } t$$
T_0 is a period of $x(t)$ in time.
- A discrete-time signal is periodic if and only if there exists an integer $N_0 > 0$ such that
$$x[n + N_0] = x[n] \qquad \text{for all } n$$
N_0 is the period of $x[n]$ in sample spacings.
- The smallest T_0 or N_0 is the *fundamental period* of the periodic signal.

Example:



Shifting $x(t)$ by 1 time unit results in the same signal.

- Common periodic signals are sines and cosines

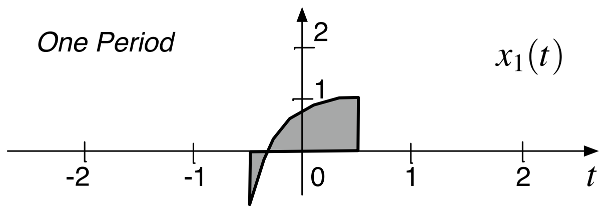
$$\begin{aligned} x(t) &= A\cos(2\pi t/T_0 - \theta) \\ x[n] &= A\cos(2\pi n/N_0 - \theta) \end{aligned}$$

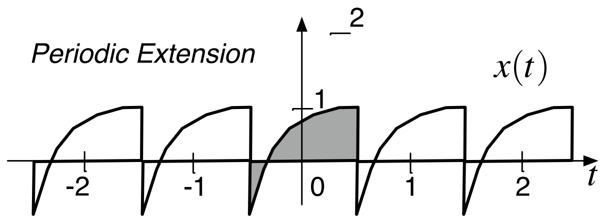
- An *aperiodic* signal is a signal that is not periodic.

- Seems like a simple concept, but there are some interesting cases
  - Is
  $$x[n] = A\cos(2\pi na - \theta)$$
  periodic for any a?
  - Is the sum of periodic discrete-time signals periodic?
  - Is the sum of periodic continuous-time signals periodic?

Periodic Extension

- Periodic signals can be generated by *periodic extension* by any segment of length one period T_0 (or a multiple of the period).

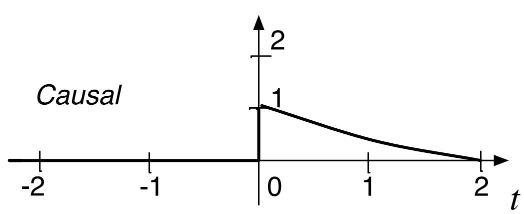




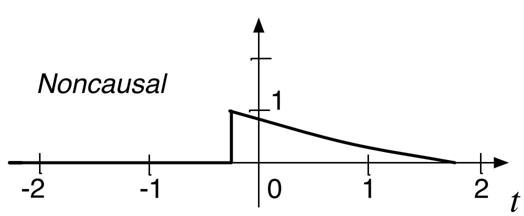
- We will often take a signal that is defined only over an interval T_0 and use periodic extension to make a periodic signal.

Causal Signals

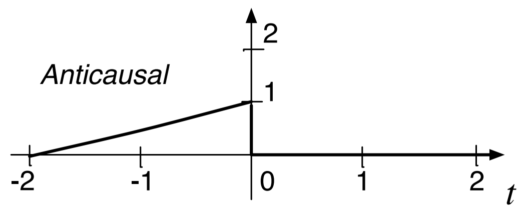
- *Causal signals* are non-zero only for $t \geq 0$ (starts at $t = 0$, or later)



- *Noncausal signals* are non-zero for some $t < 0$ (starts before $t = 0$)



- *Anticausal signals* are non-zero only for $t \leq 0$ (goes backward in time from $t = 0$)



Complex Signals

- So far, we have only considered real (or integer) valued signals.
- Signals can also be complex

$$z(t) = x(t) + jy(t)$$

where $x(t)$ and $y(t)$ are each real valued signals, and $j = \sqrt{-1}$.

- Arises naturally in many problems
  - Convenient representation for sinusoids
  - Communications
  - Radar, sonar, ultrasound

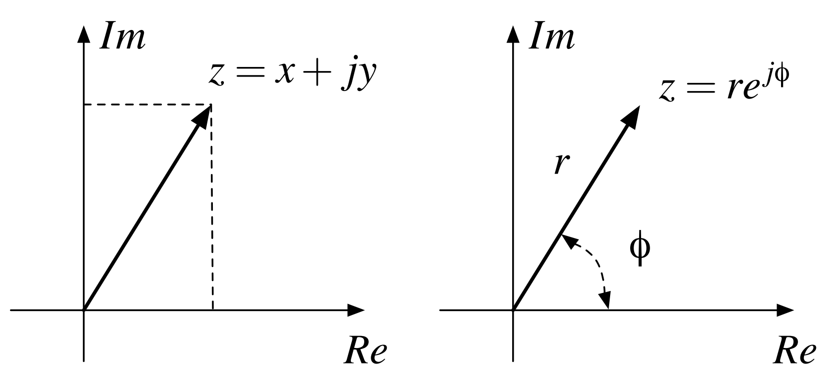
Review of Complex Numbers

Complex number in Cartesian form: $z = x + jy$

- $x = \Re z$, the *real part* of z
- $y = \Im z$, the *imaginary part* of z
- x and y are also often called the *in-phase* and *quadrature* components of z.
- $j = \sqrt{-1}$ (engineering notation)
- $i = \sqrt{-1}$ (physics, chemistry, mathematics)

Complex number in polar form: $z = re^{j\phi}$

- r is the *modulus* or *magnitude* of z
- ϕ is the *angle* or *phase* of z
- $\exp(j\phi) = \cos\phi + j\sin\phi$



- complex exponential of $z = x + jy$:

$$e^z = e^{x+jy} = e^x e^{jy} = e^x(\cos y + j \sin y)$$

Know how to add, multiply, and divide complex numbers, and be able to go between representations easily.

Signal Energy and Power

If $i(t)$ is the current through a resistor, then the energy dissipated in the resistor is

$$E_R = \lim_{T\to\infty} \int_{-T}^{T} i^2(t)\ R\ dt$$

This is energy in *Joules*.

The signal energy for $i(t)$ is defined as the energy dissipated in a 1 Ω resistor

$$E_i = \lim_{T\to\infty} \int_{-T}^{T} i^2(t) dt$$

The *signal energy* for a (possibly complex) signal $x(t)$ is

$$E_x = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt.$$

In most applications, this is not an actual energy (most signals aren't actually applied to 1Ω resistor).

The average of the signal energy over time is the *signal power*

$$P_x = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt.$$

Again, in most applications this is not an actual power.

Signals are classified by whether they have finite energy or power,

- An *energy signal* $x(t)$ has energy $0 < E_x < \infty$
- A *power signal* $x(t)$ has power $0 < P_x < \infty$

These two types of signals will require much different treatment later.

Properties of Energy and Power Signals

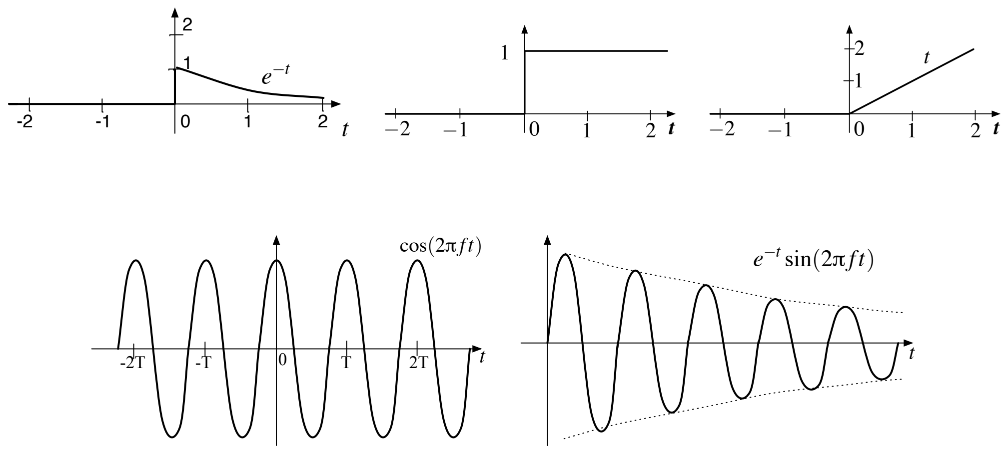
An energy signal $x(t)$ has zero power

$$\begin{aligned} P_x &= \lim_{T\to\infty} \frac{1}{2T} \underbrace{\int_{-T}^{T} |x(t)|^2\, dt}_{\to E_x < \infty} \\ &= 0 \end{aligned}$$

A power signal has infinite energy

$$\begin{aligned} E_x &= \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2\, dt \\ &= \lim_{T\to\infty} 2T \underbrace{\frac{1}{2T}\int_{-T}^{T} |x(t)|^2\, dt}_{\to P_x > 0} = \infty. \end{aligned}$$

Classify these signals as power or energy signals



A bounded periodic signal.
A bounded finite duration signal.