EE 102B
Lecture 2
Solving Signals and Systems Problems
(Thinking About Thinking)

Ronald W. Schafer
Department of Electrical Engingeering
Stanford University

A Relevant Quote

"It is by the solution of problems that the investigator tests the temper of his steel; he finds new methods and new outlooks, and gains a wider and freer horizon."

David Hilbert, 1862-1943

This quote from the great German mathematician Hilbert reminds us that the primary way to learn a mathematics-based subject like DSP is to WORK PROBLEMS. Work as many as possible, and work them CAREFULLY and THOUGHTFULLY.

Thinking in the Language of DSP

Many analysis-type problems will be assigned to give you opportunities to learn how to apply the DSP theory and the style of thinking that is common in the field.

Also, with the widespread availability of MATLAB, it is reasonable to assign design-oriented problems that may require a combination of thought and a computer to solve. A typical assignment will have one or more such problems.

Problem Solving

Signal processing relies on mathematics for the representation of engineering problems, and often solutions to signal processing analysis and design problems are indistinguishable from the solutions to "math" problems. G. Polya** presents a process for solving math problems that can easily be adapted for solution of DSP problems:

1. Understand the problem
2. Devise a plan for solution
3. Carry out the plan
4. Look back and learn

** G. Polya, *How to Solve It,* Princeton University Press, 1985

1. Understand the problem

- What is the unknown? What are the givens? What is the condition?
- Are the givens sufficient? Or are they redundant?
- Draw a figure.
- Introduce suitable notation.
- Express the condition in mathematical form.

2. Devise a plan

- You must find a connection between what is given and what is to be found.
- Have you seen the same problem in a slightly different form?
- Can you adapt the solution to a related problem?
- Can you solve a special case first?

3. Carry out your plan

- Check each step.
  - Can you see clearly that each step is correct?
  - Can you PROVE that each step is correct?

4. Look back

- Can you check the result?
- Can you check your argument?
- Can you now see the answer at a glance?
- Can you now see another way to solve the problem (probably more simply)?
- Can you use the method or the result in another problem?
- Can you generalize the result to cover a broader range of problems?

Example

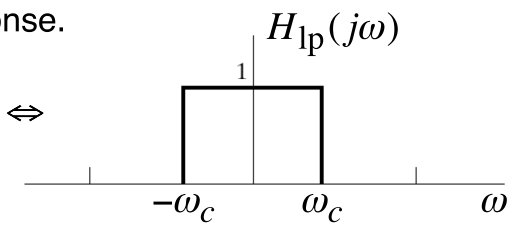
Find the output of an ideal lowpass filter with cutoff frequency $\omega_c = 0.5\pi$, when the input is

$$x(t) = 2\frac{\sin(0.4\pi t)}{\pi t} + 5\delta(t-20) + 10\cos(0.6\pi t)$$

Understand the problem.

- The ideal lowpass filter is an LTI system that can be characterized by its impulse response and/or frequency response.

$$h_{\text{lp}}(t) = \frac{\sin\omega_c t}{\pi t} \quad \Leftrightarrow$$



- Our problem is to determine the output, call it $y(t)$, when $\omega_c = 0.5\pi$, and the input is

$$x(t) = 2\frac{\sin(0.4\pi t)}{\pi t} + 5\delta(t-20) + 10\cos(0.6\pi t)$$

Make a plan.

- Some possible methods of solution are:
  A. Convolve $h_{\text{lp}}(t)$ with $x(t)$ to get $y(t)$.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h_{\text{lp}}(t-\tau)\,d\tau$$

  B. Determine the FT of $x(t)$, multiply by the frequency response and then determine the IFT of the product to give $y(t)$.

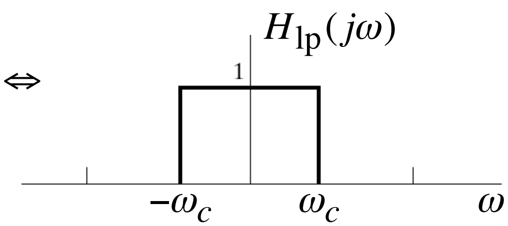
$$Y(j\omega) = H(j\omega)X(j\omega)$$

  C. Use the principle of superposition. Find the output due to each of the three components of the input by the easiest method and superimpose (add) them to get $y(t)$.

Execute Plan C

$$h_{\text{lp}}(t) = \frac{\sin\omega_c t}{\pi t} \quad \Leftrightarrow$$

$$\omega_c = 0.5\pi$$



- Find the output when the input is

$$x(t) = 2\frac{\sin(0.4\pi t)}{\pi t} + 5\delta(t-20) + 10\cos(0.6\pi t)$$

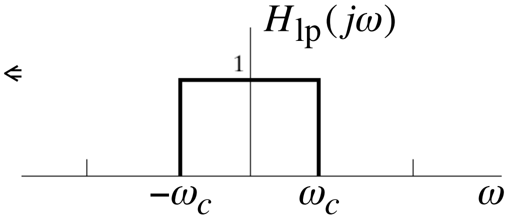
$$x(t) = x_1(t) + x_2(t) + x_3(t) \Rightarrow$$

$$y(t) = y_1(t) + y_2(t) + y_3(t)$$

Execute Plan C – I

$$h_{lp}(t) = \frac{\sin \omega_c t}{\pi t} \Leftarrow$$

$$\omega_c = 0.5\pi$$



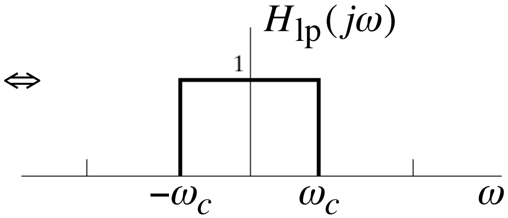
- Find the output when the input is

$$x_1(t) = 2\frac{\sin(0.4\pi t)}{\pi t}$$

Execute Plan C - II

$$h_{lp}(t) = \frac{\sin \omega_c t}{\pi t} \Leftrightarrow$$

$$\omega_c = 0.5\pi$$

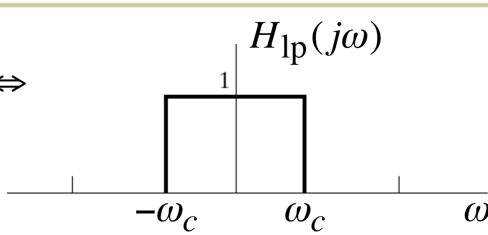


- Find the output when the input is
  $x_2(t) = 5\delta(t-20)$

Execute Plan C - III

$$h_{lp}(t) = \frac{\sin \omega_c t}{\pi t} \Leftrightarrow$$

$$\omega_c = 0.5\pi$$

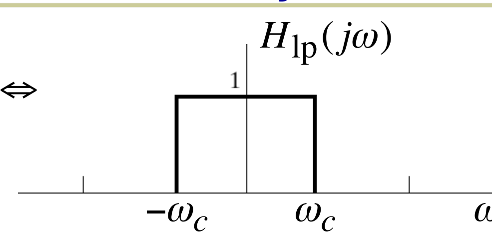


- Find the output when the input is
  $x_3(t) = 10\cos(0.6\pi t)$

Solution Summary

$$h_{lp}(t) = \frac{\sin \omega_c t}{\pi t} \Leftrightarrow$$

$$\omega_c = 0.5\pi$$



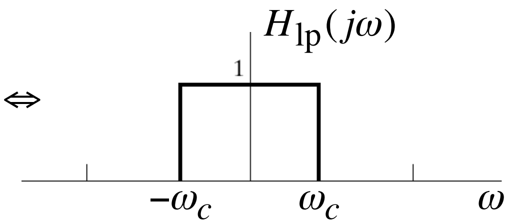
- Find the output when the input is

$$x(t) = 2\frac{\sin(0.4\pi t)}{\pi t} + 5\delta(t-20) + 10\cos(0.6\pi t)$$

$$\boxed{\begin{gathered} y(t) = 2\frac{\sin(0.4\pi t)}{\pi t} + 5\frac{\sin[0.5\pi(t-20)]}{\pi(t-20)} \\ \text{since } 0.4\pi < \omega_c < 0.6\pi \end{gathered}}$$

Generalize the Result - I

$$h_{\mathrm{lp}}(t) = \frac{\sin \omega_c t}{\pi t} \quad \Leftrightarrow$$

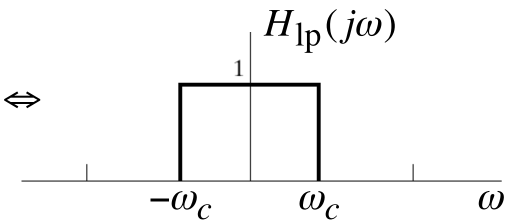


- Find the output when the input is

$$x(t) = A\frac{\sin(\omega_1 t)}{\pi t} + B\delta(t-\tau_d) + C\cos(\omega_2 t)$$

Generalize the Result - I

$$h_{\mathrm{lp}}(t) = \frac{\sin \omega_c t}{\pi t} \quad \Leftrightarrow$$



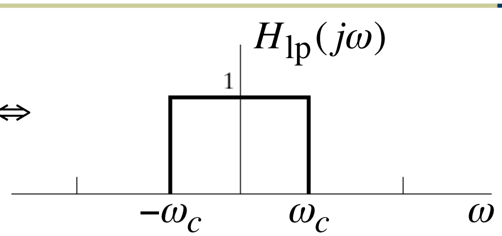
- Find the output when the input is

$$x(t) = A\frac{\sin(\omega_1 t)}{\pi t} + B\delta(t-\tau_d) + C\cos(\omega_2 t)$$

$$\boxed{y(t) = A\frac{\sin(\omega_1 t)}{\pi t} + B\frac{\sin[\omega_c(t-\tau_d)]}{\pi(t-\tau_d)} + C\cos(\omega_2 t) \quad \text{if } \omega_c > \omega_1 \text{ and } \omega_2}$$

Generalize the Result - II

$$h_{\mathrm{lp}}(t) = \frac{\sin \omega_c t}{\pi t} \quad \Leftrightarrow$$



- Find the output when the input is

$$x(t) = A\frac{\sin(\omega_1 t)}{\pi t} + B\delta(t-\tau_d) + C\cos(\omega_2 t)$$

What if $\omega_c < \omega_1$ and ω_2?

How to Solve DSP Problems

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