EE 382V: Parallel Algorithms	Summer 2017

Lecture 21: July 21

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21.1 Introduction

During this lecture we covered the following topics:

- The Euler Tour Technique

This set of lecture notes will briefly re-examine the topics covered in this lecture, in the order in which they appeared during class.

21.2 Euler Tour Technique

Given some tree, let's say that we want to find the height of the tree. If it was a balanced tree, this would be easy to do. But what if the tree was skewed? The height may not be $log(n)$ in this case. The Euler Tour Technique is a way to find the height of a tree in $log(n)$ time independent of the height.

Let's think of a linked list as a lopsided tree. With pointer jumping, we were able to find the length of the linked list in $log(n)$ time. We can use a similar technique for a tree if we represent it as a linked list.

Given a directed graph, a Euler Tour is a walk along the edges of the graph, starting from any node v, in which every edge is traversed exactly once and ends at the vertex v. A graph has an Euler Circuit if and only if every node has an even degree (an even number of edges).

We can view a tree as a linked list of edges. Let's construct a directed graph from a tree and then define a Euler Tour for the graph. It is sufficient to specify the next edge (successor) for every edge e in the directed graph. Let v_0, v_{m-1} be the neighbors of u. Then $succ(v_i, u)$ is given by $(u, v_{(i+1 \ mod \ m)})$. (This is depth first search).

21.2.1 Implementation of Euler Tour on a Graph from a Tree

Given the tree T shown in Figure 21.1 below, we will construct a Euler Tour. Figure 21.2 shows the adjacency list representation of the T. Now let's make two modifications to Figure 21.2. First, lets make the lists circular instead of null terminating. Second, lets put a bidirectional pointer between edges that represents the same undirected edge, represented by a blue dotted line. This results in the circular adjacency list in Figure 21.3.

In this example $succ(1, 4) = (4, 2)$. In Figure 21.3, $succ$ is the element that is shown by the blue dotted edges followed by black edges. So we can can construct the following Euler Tour directly from Figure 21.3:

$(1, 4) \rightarrow (4, 2) \rightarrow (2, 4) \rightarrow (4, 3) \rightarrow (3, 4) \rightarrow (4, 1) \rightarrow (1, 5) \rightarrow (5, 1) \rightarrow (1, 4)$

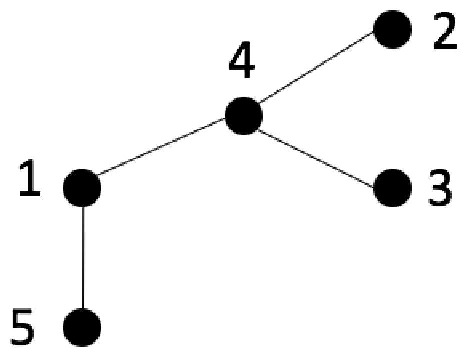

Figure 21.1: A tree T

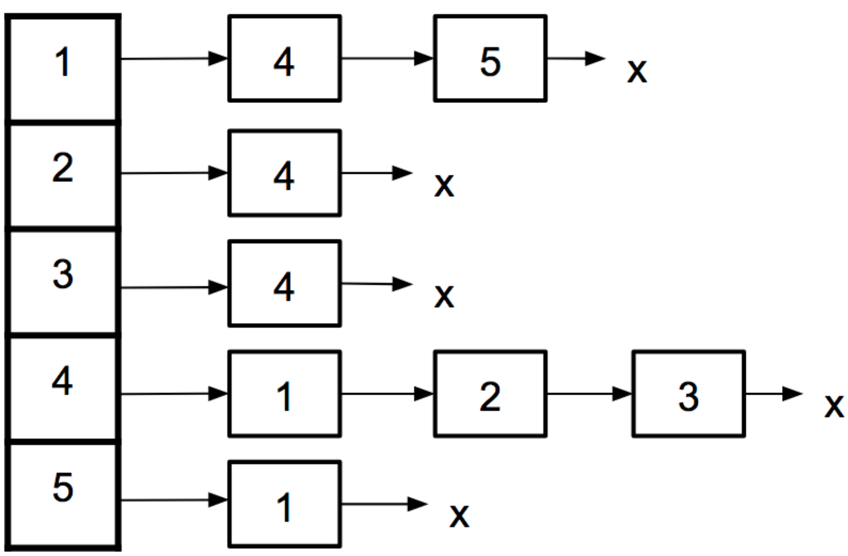

Figure 21.2: The adjacency lists of the tree T

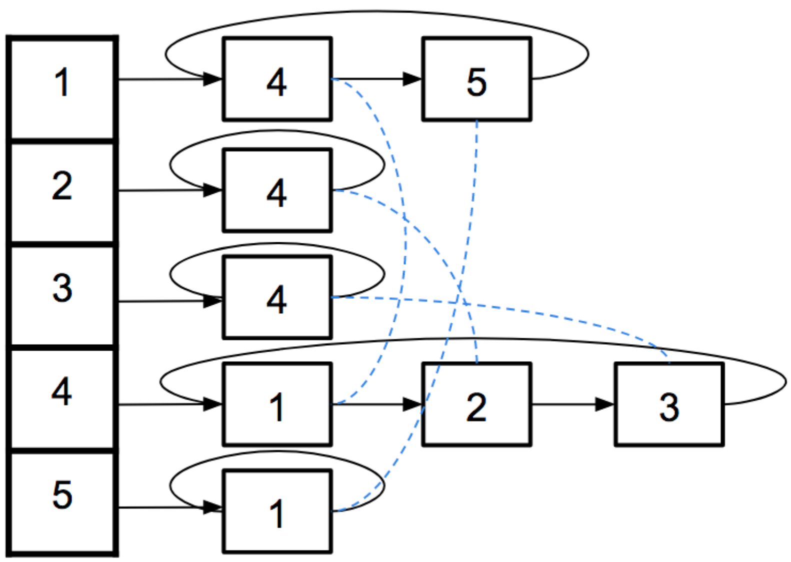

Figure 21.3: The circular adjacency lists of tree T with additional pointers