EE 382V: Social Computing	Fall 2018

Homework 2: November 9

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1.1 Question 1

Say whether the claim is true or false with a brief justification.

(a) If player A in a two-person game has a dominant strategy S_A, then there is a pure strategy Nash equilibrium in which player A plays S_A and player B plays a best response to S_A.
(b) In a Nash equilibrium of a two-player game each player is playing an optimal strategy, so the two players strategies are social-welfare maximizing.

1.1.1 Answer 1a

True. If player A is playing a dominant strategy S_A then player A will not increase his payout by playing a different strategy. Then if player B is playing the best response to S_A, player's B payout will not increase by playing a different strategy. Since neither player's payout increases by playing a different strategy, their strategies for a Nash equilibrium.player A's and player B's payout does not increase by playing a different strategy.

1.1.2 Answer 1b

False. Consider the Prisoner's Dilemna game

```
                    Player B
                    C         NC
            C      -1,-1       -10,0
Player A    NC      0,-10       -4,-4
```

There is a Nash Equilibrium at (C,C) = (-4,4) but the socially optimal solution is (N,C) =(-1,-1).

1.2 Question 2

Find all pure strategy Nash equilibria in the game below. In the payoff matrix below the rows correspond to player A's strategies and columns correspond to player B's strategies. The first entry in each box is player A's payoff and the second entry is player B's payoff.

```
                    Player B
                    L         R
            U      1,2       3,2
```

```
Player A     D       2,4       0,2
```

1.2.1 Answer 2

Pure strategy Nash Equilibrias:
`DL = (2,4)`
`UR = (3,2)`

1.3 Question 3

In this question we will consider several two-player games.

(a) Find all pure (non-randomized) strategy Nash equilibria for the game described by the payoff matrix below.

```
                     Player B
                     L         R
             U      2,15      4,20
Player A     D      6,6       10,8
```

(b) Find all pure (non-randomized) strategy Nash equilibria for the game described by the payoff matrix below.

```
                     Player B
                     L         R
             U       3,5       4,3
Player A     D       2,1       1,6
```

(c) Find all Nash equilibria for the game described by the payoff matrix below.

```
                     Player B
                     L         R
             U       1,1       4,2
Player A     D       3,3       2,2
```

1.3.1 Answer 3a

Pure strategy Nash Equilibria:
`DR = (10,8)`

1.3.2 Answer 3b

Pure strategy Nash Equilibria:
`UL = (3,5)`

1.3.3 Answer 3c

Pure strategy Nash Equilibria:
`UR = (4,2) DL = (3,3)`

Let p represent the probability that Player A chooses strategy S_U, and $(1-p)$ represent the probability that Player A chooses strategy S_D. Likewise, let q represent the probability that Player B chooses strategy T_L, and $(1-q)$ represent the probability that Player B chooses strategy T_R.

Player A expected payoff for Strategy S_U: $1*q+4*(1-q)=4-3q$
Player A expected payoff for Strategy S_D: $3*q+2*(1-q)=2+q$
Indifference: $4-3q=2+q$
$2=4q$
$q=\frac{1}{2}$

Player B expected payoff for Strategy S_L: $1*p+3*(1-p)=3-2p$
Player B expected payoff for Strategy S_R: $2*p+2*(1-p)=2$
Indifference: $3-2p=2$
$1=2p$
$p=\frac{1}{2}$

Mixed strategy Nash equilibria probabilistic vector:
$((\frac{1}{2},\frac{1}{2}),(\frac{1}{2},\frac{1}{2}))$