Zumobot case study

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29.6.2017

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Introduction

One of the most used study cases in Control Theory is the Inverted Pendulum analysis, as it present an unstable open-loop characteristic but is also possible to stabilize on a closed-loop configuration.

Here we present the anyalis of the Zumobot 32U4, a small robot available on the market. It is equiped with two DC motors, an Arduino-based board with several periferials.

TODO: Complete

1 System analysis

An inverted pendulum can be represented as a cart moving on an horizontal axis, connected to a rigid body pendulum.

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2 Mathematical development

Through the literature is possible to find several approaches to obtain a model for the IP.

To fully model the dynamic behavior of the zumo robot, we need to consider the equations that govern the movement as rigid body.

2.1 First approach - Sum of forces

The first approach considered here is described in [1], as follows:

The equations of motion are obtained from the sum of forces in the cart for the horizontal direction.

$$F - b \cdot \dot{x} - N = M \cdot \ddot{x} \tag{1}$$

Now considering the pendulum itself, the force applied in the horizontal direction due to the momentum of the pendulum is determined as:

$$\tau = r \cdot F = I \cdot \ddot{\theta} \tag{2}$$

Given the fact that the moment of inertia of a pendulum of mass m is defined as $I = m \cdot L^2$, the previous equation can be rewritten as:

$$F = \frac{I \cdot \ddot{\theta}}{r} = \frac{m \cdot l^2 \cdot \ddot{\theta}}{l} = m \cdot l \cdot \ddot{\theta} \tag{3}$$

Obtaining the component of the force defined in 3 in the horizontal direction:

[1] This is a footnote

$$F = m \cdot l \cdot \ddot{\theta} \cdot \cos\theta \tag{4}$$

Now, the component of the centripetal force acting on the pendulum is similar to the one in 3, but the horizontal component of this force is:

$$F = m \cdot l \cdot \dot{\theta}^2 \cdot \sin\theta \tag{5}$$

Summing the defined forces present in the horizontal direction of the pendulum in 4 and 5, we obtain the following expression:

$$N = m \cdot \ddot{x} + m \cdot l \cdot \ddot{\theta} \cdot \cos\theta - m \cdot l \cdot \dot{\theta}^2 \cdot \sin\theta \tag{6}$$

Now we can substitute 6 into 1, we obtain the first equation of motion:

$$F = (M + m)\ddot{x} + b \cdot \dot{x} + m \cdot l \cdot \ddot{\theta} \cdot \cos\theta - m \cdot l \cdot \dot{\theta}^2 \cdot \sin\theta \tag{7}$$

3 System model

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4 Regulator

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5 Implementation

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References

[1] K. Sultan, *The Inverted Pendulum, Analysis, Design and Implementation.* IEEE, 2003.