

A TRULY MULTIVARIATE APPROACH TO MANOVA

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ABSTRACT

All too often researchers perform a Multivariate Analysis of Variance (MANOVA) on their data and then fail to fully recognize the true multivariate nature of their effects. The most common error is to follow the MANOVA with univariate analyses of the dependent variables. One reason for the occurrence of such errors is the lack of clear pedagogical materials for identifying and testing the multivariate effects from the analysis. The current paper consequently reviews the fundamental differences between MANOVA and univariate Analysis of Variance and then presents a coherent set of methods for plumbing the multivariate nature of a given data set. A completely worked example using genuine data is given along with estimates of effect sizes and confidence intervals, and an example results section following the technical writing style of the American Psychological Association is presented. A number of issues regarding the current methods are also discussed.

INTRODUCTION

Multivariate statistical methods have grown increasingly popular over the past twenty-five years. Most graduate programs in education and the social sciences now offer courses in multivariate methods. Statistical software such as SAS and SPSS that provide canned routines for conducting even the most complex multivariate analyses can also be found on the computers of modern educational researchers, psychologists, and sociologists, among other scientists. A wide array of multivariate textbooks written for both novices and experts can likewise be found on the bookshelves of these scientists. The continued proliferation of multivariate statistical procedures can no doubt be attributable to the belief that models of nature and human behavior must often account for multiple, inter-related variables that are conceptualized simultaneously or over time. Multivariate Analysis of Variance (or MANOVA) is one particular technique for analyzing such multivariable models.

In MANOVA the goal is to maximally discriminate between two or more distinct groups on a *linear combination* of quantitative variables. For instance, a psychologist may wish to investigate how children educated in Catholic schools differ from children educated in public schools on a number of tests that measure:

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(1) reading, (2) mathematics, and (3) moral reasoning skills. Using MANOVA the psychologist could examine how the two groups differ on a linear combination of the three measures. Perhaps the Catholic school children score higher on moral reasoning skills relative to reading and math when compared to the public school children? Perhaps the Catholic school children score higher on both moral reasoning skills and math relative to reading when compared to the public school children? These potential outcomes, or questions, are multivariate in nature because they treat the quantitative measures simultaneously and recognize their potential inter-relatedness. The goal of conducting a MANOVA is thus to determine how quantitative variables can be combined to maximally discriminate between distinct groups of people, places, or things. As will be discussed below this goal also includes determining the theoretical or practical meaning of the derived linear combination—or combinations—of variables.

Many excellent journal articles and book chapters have been devoted to MANOVA in the past twenty-five years. Chapters by Huberty and Petoskey (2000) and Weinfurt (1995), for instance, provide lucid introductions to this complex and conceptually powerful statistical procedure. Multivariate textbooks by Stevens (2001), Tabachnick and Fidell (2006), and Hair, et al. (2006), to name a few, also provide outstanding treatments of MANOVA. Despite these resources, however, a central premise of the current paper is that many published applications of MANOVA fail to exploit the conceptual advantage of conducting a multivariate, rather than univariate, analysis. Recent reviews (Huberty & Morris, 1989; Keselman, et al., 1998; Kieffer, Reese, & Thompson, 2001), for instance, have shown that studies employing MANOVA to explore group differences on multiple quantitative variables often fail to realize the multivariate nature of the reported effects. Instead, authors tend to resort to “follow-up” univariate statistical analyses to make sense of their findings (viz., following a significant MANOVA with multiple ANOVAs). One potential reason for these bad habits of data analysis is a paucity of clear examples that demonstrate appropriate procedures. Consequently, we draw upon the work of Richard J. Harris (1993, 2001; Harris, M., Harris, R., & Bochner, 1982) and Carl J. Huberty (Huberty, 1984; Huberty & Smith, 1982; Huberty & Petoskey, 2000; also, see Enders, 2003) in the current paper to demonstrate a general strategy for conducting MANOVA. This strategy focuses on the *linear combinations* of variables, or *multivariate composites*, that are the numerical and conceptual basis of any multivariate analysis; subsequently, specific techniques for identifying and testing these composites for statistical significance will be shown. An approach for interpreting and labeling the multivariate composites will also be presented, and an example write-up of MANOVA results that follows APA style will be provided.

MANOVA vs. ANOVA

Simply defined, MANOVA is the multivariate generalization of univariate ANOVA. In the latter analysis mean differences between two or more groups are examined on a single measure. For instance, a psychologist may wish to study the mean differences of ethnic groups on a continuous measure of implicit racism, or an educator may wish to examine the differences between boys and girls regarding their mean performance on a test of mathematical reasoning ability. In comparison, and as stated above, the goal in MANOVA is to examine mean differ-

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ences on *linear combinations* of multiple quantitative variables. Ethnic groups, for instance, could be compared on a combination of explicit and implicit measures of racism, or boys and girls could be compared with regard to their mean performances on a combination of mathematical, spatial, and verbal reasoning tasks. In both instances the variables would be analyzed simultaneously (i.e., multivariately) rather than individually (i.e., univariately).

Because the differences between these univariate and multivariate procedures can best be explicated by way of example, we will proceed with a complete analysis of genuine data. Specifically, we will draw data from a Master's thesis by Iwasaki (1998) in which the personality traits of different cultural groups were examined.¹ In addition to other measures, college students in Iwasaki's study completed the NEO PI-r (Costa & McCrae, 1992), a popular questionnaire that measures the Big Five personality traits: Neuroticism, Extraversion, Openness-to-Experience, Agreeableness, and Conscientiousness. The students were also classified into three groups:

- EA: European Americans (Caucasians living in the United States their entire lives)
- AA: Asian Americans (Asians living in the United States since before the age of 6 years)
- AI: Asian Internationals (Asians who moved to the United States after their 6th birthday)

The three groups form mutually exclusive categories, and the personality questionnaire yields quasi-continuous trait scores (viz., they may range in value from 0 to 192) that are assumed to represent an interval scale. The categorical grouping variable will herein be referred to as the *independent variable*, and the quasi-continuous trait measures will be referred to as the *dependent variables*. Note this terminology will be used throughout solely for the sake of convenience and is not intended to imply a causal ordering of the variables. As is well known, attributing cause is a logical and theoretical task that extends beyond the bounds of statistical analysis.

The goal in univariate ANOVA is to examine differences in group means on a single, continuous variable. Therefore each dependent variable (Big Five trait score) is analyzed and interpreted separately. The results for the univariate tests of overall differences among the EA, AA, and AI groups from the SPSS MANOVA procedure (which can also be used to conduct ANOVAs) are as follows:

| EFFECT .. GRP | | | | | | | |
|---------------------------------------|----------|----------|----------|----------|-------|--------|---------|
| Univariate F-tests with (2,200) D. F. | | | | | | | |
| Var | Hypo. SS | Error SS | Hypo. MS | Error MS | F | Sig. F | eta-sqr |
| Neu | 456.85 | 90650.37 | 228.43 | 453.25 | .50 | .605 | .01 |
| Ext | 12180.69 | 68087.41 | 6090.35 | 340.44 | 17.89 | .000 | .15 |
| Ope | 8773.97 | 58283.59 | 4386.99 | 291.42 | 15.05 | .000 | .13 |
| Agr | 6550.10 | 61033.79 | 3275.05 | 305.17 | 10.73 | .000 | .10 |
| Con | 1297.48 | 68134.10 | 648.74 | 340.67 | 1.90 | .152 | .02 |

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Using a Bonferroni adjusted *a priori* *p*-value of .01 (.05/5), the population means for the three groups are judged to be unequal on the extraversion, openness-to-experience, and agreeableness traits. The largest univariate effect is noted for extraversion, for which 15% ($\eta^2 = .15$) of the variability in the extraversion trait scores can be explained by group membership. This effect seems small, although it could be judged as large using Cohen's (1988) conventions (.01 = small, .06 = medium, .14 = large). It should also be noted the homogeneity of population variances assumption was tested for each analysis and no violations were noted.

Each of the statistically significant univariate, omnibus effects could be followed by complex or paired comparisons to clarify the nature of the mean differences between the EA, AA, and AI groups. The results from such analyses would be interpreted separately for each of the Big Five trait scores because the univariate approach essentially treats any potential correlations among the dependent variables as meaningless.

By comparison a multivariate approach takes into account the inter-correlations among the Big Five personality traits. As can be seen in the following SPSS CORRELATION output, a number of the trait scores are modestly correlated:

Correlations

| | Neuroticism | Extraversion | Openness | Agreeableness | Conscientiousness |
|-------------------|-------------|--------------|----------|---------------|-------------------|
| Neuroticism | 1.000 | -.255** | -.019 | -.097 | -.397** |
| Extraversion | -.255** | 1.000 | .363** | .011 | .371** |
| Openness | -.019 | .353** | 1.000 | .232** | .125 |
| Agreeableness | -.097 | .011 | .232** | 1.000 | .097 |
| Conscientiousness | -.397** | .371** | .125 | .097 | 1.000 |

** Correlation is significant at the 0.01 level (2-tailed)

Reasoning multivariately with the same data, the question becomes: In what way or ways can the Big Five traits be combined to discriminate among the three groups? Perhaps a combination of high extraversion and low neuroticism separates the groups, or perhaps a combination of high extraversion, high agreeableness, and low conscientiousness discriminates among the three groups? These questions demonstrate how a multivariate frame of mind entails considering the dependent variables simultaneously rather than separately. Whether or not such questions are justified or meaningful is an issue that must be addressed by any researcher confronted with the prospect of conducting a MANOVA. In the current example this issue manifests itself as follows: Are we truly interested in examining the multivariate, linear combinations of Big Five traits, or are we content with considering each trait separately? Another way of considering the issue regards the intent to interpret the multivariate effect that might underlie the data. For the current example, if we have no intention of *interpreting* the multivariate composites (that is, the linear combinations of traits—the dependent variables), then the univariate analyses above are perfectly sufficient. There is certainly no shame in

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conducting multiple ANOVAs and separately interpreting the results for each dependent variable. It is more than a methodological *faux pas*, however, to conduct a MANOVA with no intent of interpreting the multivariate combination of variables.

In fact, two common errors seem to be associated with the failure to accurately discriminate between univariate and multivariate approaches toward data analysis. First, many researchers believe that conducting a MANOVA will provide protection from Type I error inflation when conducting multiple univariate ANOVAs. Following this erroneous reasoning, for instance, we would first conduct a MANOVA for the personality data above and, if significant, judge the statistical significance of the univariate ANOVAs based on their unadjusted observed p -values rather than their Bonferroni-adjusted p -values. Although such an analysis strategy is common in the literature, it is not to be recommended because the Type I error rate will only be properly controlled when the null hypothesis is true (Bray & Maxwell, 1982), which is an unlikely occurrence in practice and therefore an unrealistic assumption. Type I error inflation can be controlled through the use of a Bonferroni adjustment or a fully post hoc critical value derived from the results of a MANOVA, but the researcher must make the extra effort to compute the critical values against which to judge each univariate F-test (see Harris, 2001, and below). To reiterate, simply running a MANOVA prior to multiple ANOVAs will not generally provide appropriate protection against Type I error inflation. The extra step of computing the Bonferroni-adjusted critical values or the special MANOVA-based post hoc critical value must also be taken.

Second, many researchers believe that ANOVA should be used as a follow-up procedure to MANOVA for interpreting and understanding the multivariate effect. While common, this analysis strategy is based on the misconception that results from multivariate analyses are simply additive functions of the results from univariate analyses. As will be described below, however, the multivariate information from a MANOVA is contained in the linear combinations of dependent variables that are generated from the analysis. Conducting an ANOVA on each of the dependent variables following a MANOVA completely ignores these linear combinations. Furthermore, the conceptual meaning of the results from a series of ANOVAs will not necessarily match the conceptual meaning of the results from a MANOVA. In other words, the multivariate nature of the results will not necessarily emerge from a series of univariate analyses. Techniques for identifying and exploring the linear combinations of variables that result from a MANOVA are therefore of critical importance.

CONDUCTING THE MANOVA

Returning to the Big Five trait example, let us decide to pursue a truly multivariate approach. In other words, let us commit to examining the linear combinations of personality traits that might differentiate between the European American (EA), Asian American (AA), and Asian International (AI) students. Assuming that no *a priori* model for combining the Big Five traits is available, these six steps will consequently be followed:

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1. Conduct an omnibus test of differences among the three groups on linear combinations of the five personality traits.
2. Examine the linear combinations of personality traits embodied in the discriminant functions.
3. Simplify and interpret the strongest linear combination.
4. Test the simplified linear combination (multivariate composite) for statistical significance.
5. Conduct follow-up tests of group differences on the simplified multivariate composite.
6. Summarize results in APA style

If an existing model for combining the traits were available, a priori, then an abbreviated approach, which will be discussed near the end of this paper, would be undertaken.

Step 1: Conducting the Omnibus MANOVA.

The omnibus null hypothesis for this example posits the EA, AA, and AI groups are equal with regard to their population means on any and all linear combinations of the Big Five personality traits. This hypothesis can be tested using any one of the major computer software packages. In SPSS for Windows the General Linear Model (GLM) procedure can be used or the dated MANOVA routine can be run through the syntax editor. There are several slight advantages to using the MANOVA syntax; consequently these procedures are employed herein, and the complete annotated syntax statements can be found in the Appendix. The multivariate results from SPSS MANOVA are:

| EFFECT .. GRP | | | | | |
|---|--------|-----------|------------|----------|--------|
| Multivariate Tests of Significance (S = 2, M = 1 , N = 97) | | | | | |
| Test Name | Value | Approx. F | Hypoth. DF | Error DF | Sig. F |
| Pillais | .41862 | 10.42982 | 10.00 | 394.00 | .000 |
| Hotellings | .53723 | 10.47592 | 10.00 | 390.00 | .000 |
| Wilks | .62327 | 10.45320 | 10.00 | 392.00 | .000 |
| Roys | .25313 | | | | |
| Note.. F statistic for WILKS' Lambda is exact. | | | | | |

As can be seen, four test statistics are reported for the group effect: "Pillais", "Hotellings", "Wilks", and "Roys." Huberty (1994, pp. 183-189) offers a detailed discussion of these four statistics, and the first three tests indicate the multivariate effect is statistically significant for the current data (all 'Sig. F' values, that is, p 's < .001). Wilks' Lambda is arguably the most popular multivariate statistic, and Tabachnik and Fidell (2006) generally support reporting it instead of the other values.

The analysis strategy recommended in this paper, however, employs Roy's g.c.r. A number of details regarding this statistic must therefore be clarified. First, it is often reported in two different metrics. In the SPSS output shown above,

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which was generated with the MANOVA routine, it is reported as a measure of association strength, $\theta = .25$, that indicates the proportion of overlapping variance between the independent variable and the first linear combination of dependent variables. In other words, θ is equivalent to the well-known η^2 measure of association strength. SPSS output generated from the GLM option in the pull-down menus appears in a different format:

Multivariate Tests^c

| Effect | Value | F | Hypotheses df | Error df | Sig. |
|--------------------------|---------|-----------------------|---------------|----------|------|
| Intercept Pillai's Trace | .995 | 7270.287 ^a | 5.000 | 196.000 | .000 |
| Wilk's Lambda | .005 | 7270.287 ^a | 5.000 | 196.000 | .000 |
| Hotelling's Trace | 185.466 | 7270.287 ^a | 5.000 | 196.000 | .000 |
| Roy's Largest Root | 185.466 | 7270.287 ^a | 5.000 | 196.000 | .000 |
| GRP Pillai's Trave | .419 | 10.430 | 10.000 | 394.000 | .000 |
| Wilk's Lambda | .623 | 10.453 ^a | 10.000 | 392.000 | .000 |
| Hotelling's Trace | .537 | 10.476 | 10.000 | 390.000 | .000 |
| Roy's Largest Root | .339 | 13.354 ^b | 5.000 | 197.000 | .000 |

a. Exact statistic

b. The statistic is an upper bound on F that yields a lower bound on the significance level.

c. Design: Intercept + GRP

As can be seen, except for "Roy's Largest Root", the test values are equal to those generated by the MANOVA routine. The value for Roy's g.c.r. from GLM is reported as an eigenvalue ($\lambda = .34$) which can be easily computed from the θ value:

$$\lambda = \frac{\theta}{1 - \theta}, \quad \theta = \frac{\lambda}{1 + \lambda}$$

When using a program other than SPSS, the researcher should be certain to explore the software manuals to determine which metric is being reported. Alternatively, as will be shown in the next step in the procedures, the value of θ can be computed "manually" with compute statements.

The second issue regarding Roy's g.c.r. is the F-value and hypothesis test generated by the GLM procedure. This test is an upper bound that may unfortunately lead to dramatically high Type I error rates (Harris, personal communication, October 26th, 2005). It should consequently be avoided, and the tabled values for θ reported by Harris (1985, 2001) should instead be used. A program reported by Harris (1985, p. 475) can also be used to compute the observed p -value for Roy's g.c.r. (in the θ metric) with s , m , and n degrees of freedom.² These values are computed as:

$$\begin{aligned} s &= \min(df_{\text{effect}}, p) \\ m &= (|df_{\text{effect}} - p| - 1) / 2 \\ n &= (df_{\text{error}} - p - 1) / 2 \end{aligned}$$

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where,

p = number of dependent variables

$df_{\text{effect}} = k - 1$

$df_{\text{error}} = N - k$

N = number of observations

k = number of groups

For the current data $p = 5$, $k = 3$, and $N = 203$. Consequently, $df_{\text{effect}} = 2$, $df_{\text{error}} = 200$, $s = \min(2, 5) = 2$, $m = (|2 - 5| - 1) / 2 = 1$, and $n = (200 - 5 - 1) / 2 = 97$ as reported in the MANOVA output above. Harris' program yields a critical θ value (θ_{crit}) of .07056 for a critical p -value (p_{crit}) of .05. The observed θ of .25313 exceeds θ_{crit} and is therefore statistically significant. Entering various p -values through trial-and-error in Harris' program reveals the observed p -value to be less than .0005.

Finally, now that Roy's statistic has been explained, the other omnibus tests of the multivariate effect can be succinctly described for pedagogical purposes as follows:

Pillai's Trace is the sum of the effect sizes for the discriminant functions; that is, $\Sigma \theta_i$. A value approaching s indicates a large omnibus effect, and when $s = 1$ Pillai's Trace will equal Roy's θ .

Hotelling's Trace is similar to Pillai's Trace, but is based on eigenvalues; namely, $\Sigma \lambda_i$. The magnitude of Hotelling's Trace is difficult to interpret since it has no set range, although when $s = 1$ the result will equal Roy's λ .

Wilks' Lambda (Λ) is based on overlapping variances, or effect sizes, namely, $\Pi(1 - \theta_i)$. Opposite of the other test statistics, values near 0 indicate large omnibus effects.

These three tests differ from Roy's test by combining, in some manner, the information for all the discriminant functions produced from the analysis for a given effect.

Step 2: Examining the Linear Combinations

As was stated at various points above the multivariate effect is conveyed through the linear combinations of Big Five traits. These linear combinations are defined by the discriminant function coefficients that can be requested from most computer programs in both raw and standardized form. The coefficients from SPSS MANOVA for the current example are reported as follows:

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| EFFECT .. GRP (Cont.) | | |
|---|----------|---------|
| Raw discriminant function coefficients | | |
| Function No. | | |
| Variable | 1 | 2 |
| N | -.00580 | -.00700 |
| E | -.06027 | -.00017 |
| O | .04239 | -.04230 |
| A | -.02758 | -.03102 |
| C | .00748 | -.00956 |
| Standardized discriminant function coefficients | | |
| Function No. | | |
| Variable | 1 | 2 |
| N | -.12356 | -.14893 |
| E | -1.11200 | -.00309 |
| O | .72369 | -.72218 |
| A | -.48180 | -.54193 |
| C | .13811 | -.17640 |

The discriminant function coefficients are regression weights that are multiplied by the Big Five scale scores (N, E, O, A, C) in original or z-score units to create the multivariate composites referred to throughout this manuscript. Consequently, these regression weights are the heart and soul of MANOVA because they represent exactly how the dependent variables are combined to maximally discriminate between the EA, AA, and AI groups. Depending on the number of groups and the number of dependent variables, one or more linear combinations, or multivariate composites, will be generated. The value of s degrees of freedom will in fact indicate the number of multivariate composites produced. In the current example, two composites are produced based on the three groups and five personality traits. These two composites are furthermore uncorrelated (orthogonal) and ordered in terms of their “strength”; that is, the extent to which they overlap with the independent variable.

Using the unstandardized coefficients above, the first multivariate composite can be written and computed as follows:

Composite #1 = (N)(-.0058) + (E)(-.06027) + (O)(.04239) + (A)(-.02758) + (C)(.00748).

This new variable can be entered as a single dependent variable in an ANOVA, yielding the following results:

| Source of Variation | SS | DF | MS | F | Sig of F |
|---------------------|--------|-----|-------|-------|----------|
| WITHIN CELLS | 200.01 | 200 | 1.00 | | |
| GRP | 67.79 | 2 | 33.89 | 33.89 | .000 |
| (Total) | 267.80 | 202 | 1.33 | | |

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A measure of association strength between the independent variable and multivariate composite, η^2 , can then be computed:

$$\eta^2 = \frac{(F_{observed})(df_{between})}{(F_{observed})(df_{between}) + df_{within}} = \frac{(33.89)(2)}{(33.89)(2) + 200} = .25$$

The result, .25, is equal to Roy's g.c.r. reported above as θ . Computing the multivariate composite and conducting an ANOVA thus demonstrates that Roy's g.c.r. is a measure of effect size for the first linear composite. The second multivariate composite is orthogonal to (i.e., uncorrelated with) the first and can be written and computed as follows:

Composite #2 = (N)(-.007) + (E)(-.00017) + (O)(-.0423) + (A)(-.03102) + (C)(-.00956). Conducting an ANOVA and computing η^2 yields:

| Source of Variation | SS | DF | MS | F | Sig of F |
|---------------------|--------|-----|-------|-------|----------|
| WITHIN CELLS | 199.88 | 200 | 1.00 | | |
| GRP | 39.64 | 2 | 19.82 | 19.83 | .000 |
| (Total) | 239.52 | 202 | 1.19 | | |

$$\eta^2 = \frac{(F_{observed})(df_{between})}{(F_{observed})(df_{between}) + df_{within}} = \frac{(19.83)(2)}{(19.83)(2) + 200} = .17$$

As mentioned above, the strength of association for this second composite is lower than the first. Nonetheless, it too can be tested for statistical significance using the same m and n degrees of freedom for testing the first composite, but $s = \min(k - j, p - j + 1)$, where j is equal to the composite's ordinal value. In this case the second composite ($j = 2$) is being tested for statistical significance, and $s = \min(3 - 2, 5 - 2 + 1) = 1$, and $\theta_{crit} = .04702$ for $p_{crit} = .05$. The second composite is therefore also statistically significant since $.17 > .04702$. It is also noteworthy that the sum of the η^2 values for the first (.25) and second (.17) composites equals .42, which is the value for the Pillai's Trace multivariate statistic above. Pillai's Trace thus differs from Roy's g.c.r. by testing group differences on the *complete set* of linear combinations generated from the analysis. Wilks' Lambda and Hotelling's Trace similarly provide tests of the complete set of multivariate composites. The omnibus nature of these three tests is a distinct disadvantage in the current approach, however, which focuses on testing and interpreting the *individual* discriminant functions.

Step 3: Simplifying and Interpreting the First Linear Combination

The next step in the analysis involves interpreting the multivariate composites defined by the discriminant functions. As was noted above, the first composite will always yield the highest θ (i.e., η^2) value, and in many genuine data sets the remaining composites can be ignored because of their small effect sizes. The second composite in the current example, however, shares 17% of its variance with the independent variable, which is nearly as high as the percentage for the first composite (25%). Nonetheless, solely for the sake of convenience, we will simplify and interpret only the first composite for the personality data.

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It is often useful to determine the “multivariate gain” of the composite under consideration over the univariate approach toward the same data. The gain is determined by comparing the θ value for the composite to the corresponding values from the univariate F-tests. For the current data the largest univariate η^2 value was .15 for Extraversion, which is modestly lower than .25 for the first multivariate composite. The multivariate gain over simple univariate analyses was thus .10; in other words, the multivariate effect was 10 percentage points higher than the strongest univariate effect in terms of shared variance.

As with any estimate of effect size the researcher must draw upon his or her experience and theoretical framework as well as existing literature to judge the importance of the multivariate gain. This judgement will also go hand-in-hand with the conceptual interpretation or labeling of the multivariate composite. The reader is most likely familiar with the process of interpreting and labeling multivariate composites in the realm of Exploratory Factor Analysis (EFA). In EFA one begins with a pool of items and attempts to identify a set of common factors believed to represent theoretically meaningful constructs (e.g., personality traits, clinical syndromes, dimensions of intelligence, etc.) that underlie the original items. Through a process of examining pattern, structure, or factor score coefficients the factors are “interpreted”, which is to say they are labeled or named. The factors themselves are mathematically determined as multivariate functions of the original items in the analysis and are thus similar to the discriminant functions in MANOVA. Consequently, the methods commonly employed to interpret factors can be used to interpret multivariate composites. In factor analysis, for example, an arbitrary criterion is often used (e.g., $|\cdot.30|$, $|\cdot.40|$) to judge pattern or structure coefficients so that “salient” items may be identified for a given factor. Once the salient items are identified, their content is examined for a common theme which is then named and used as the factor label.

In MANOVA this process of labeling should begin with an examination of simplified versions of the discriminant function coefficients. If the dependent variables are on different scales the standardized function coefficients and standardized variables (z-scores) should be used when interpreting and computing the simplified composite variable. If the dependent variables are all on the same scale, as in the current data, the raw (i.e., unstandardized) coefficients and raw scores should be preferred. The first composite is thus simplified by focusing only on the relatively large raw discriminant function coefficients. The full function is repeated here:

Composite #1 = (N)(-.0058) + (E)(-.06027) + (O)(.04239) + (A)(-.02758) + (C)(.00748). Clearly, the coefficients for Neuroticism and Conscientiousness are relatively small and near zero. Converting these small coefficients to zero yields:

Simplified Composite #1 = (N)(0) + (E)(-.06027) + (O)(.04239) + (A)(-.02758) + (C)(0). As is done in interpreting factors differences between the relatively large function coefficients are ignored. In other words, the coefficients are changed to unity while their signs are retained:

Simplified Composite #1 = (E)(-1) + (O)(1) + (A)(-1) = O - (E + A). The rationale behind this simplifying process is to round to zero those coefficients that are relatively small because they are assumed to be deviating from zero well within the bounds of sampling variability (Einhorn & Hogarth, 1975; Grice, 2001; Wainer, 1976), although no statistical test of this assumption exists. Furthermore,

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the differences among the large coefficients are assumed to be within the bounds of sampling variability, and thus important information is not lost by converting these values to 1s and -1s consistent with their original signs (see Rozeboom, 1979, for further discussion of this topic). Again, this is the same process used when interpreting factors in factor analysis, when creating sum scores from a factor analysis or multiple regression analysis, and generating contrast coefficients in analysis of variance from an examination of means.

In words then, the multivariate composite that discriminates between the European American, Asian American, and Asian International students is higher Openness-to-Experience relative to lower Extraversion and Agreeableness. A sensible label to apply to this novel multivariate composite is “Reserved-Openness.” Individuals who score high on this composite are quietly or reservedly open to new experiences, whereas individuals who score low on this composite can be described as gregariously traditional (i.e., extraverted, agreeable, and low on openness). The composite can thus be interpreted as Reserved-Openness vs. Gregarious-Traditionalism.

The nature of this multivariate composite can further be understood by examining the means in Figure 1. As can be seen in the highlighted (i.e., the “boxed”) portions of the graph European Americans rate themselves higher on Extraversion and Agreeableness relative to Openness-to-Experience, whereas Asian Americans and Asian Internationals rate themselves higher on Openness-to-Experience relative to Extraversion and Agreeableness. It is thus the pattern of means, or more specifically the differences in patterns of means, that is captured by the simplified, multivariate composite. When reporting the results of the analyses for this particular study, the multivariate effect could possibly be discussed with respect to cultural differences between Asian and Caucasian Americans in terms of their personality types. Types, in the realm of personality psychology, are considered to be multivariate clusters of traits or other stable personal characteristics.

The simplification and interpretation process is perhaps the most important stage of the MANOVA since it provides the bridge from a purely statistical effect to a theoretically meaningful effect. If at this point in the analysis the multivariate composite (i.e., the discriminant function) can not be labeled or theoretically interpreted, a switch to separate univariate analyses would be prudent. Otherwise, the researcher will be faced with a situation in which the multivariate effect is potentially large and statistically significant, but conceptually meaningless. Because of the importance of interpretation in the current approach toward MANOVA, a number of pointers for interpreting the multivariate function will be presented below.

Step 4: Testing the Simplified Multivariate Composite for Statistical Significance.

Do the EA, AA, and AI groups differ significantly in their means on the simplified composite, Reserved-Openness? Recall the three groups differed significantly on the full composite, as indicated by the Roy's *g.c.r.* test ($\theta = .25, p < .0005$). The mathematics underlying MANOVA will insure the θ values are maximized for each of the linear combinations of dependent variables, subject to the condition that each is uncorrelated with preceding discriminant functions. The multivariate composite produced from the simplification process is essentially a

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crude approximation of the first exact discriminant function, and it will always yield a lower θ value that must be tested for statistical significance. The formula can be found in Harris (2001, p. 222):

$$F_{crit} = \frac{df_{error} \theta_{crit}}{(1 - \theta_{crit}) df_{effect}}$$

Using Harris' g.c.r. program, $\theta_{crit} = .0706$ for $s = 2$, $m = 1$, $n = 97$, and $p_{crit} = .05$.

$$F_{crit} = \frac{df_{error} \theta_{crit}}{(1 - \theta_{crit}) df_{effect}} = \frac{(200)(.0706)}{(1-.0706)(2)} = 7.60$$

The simplified composite, Reserved-Openness, can be computed in SPSS and entered as the dependent variable in an ANOVA:

| Source of Variation | SS | DF | MS | F | Sig of F |
|---------------------|-----------|---------------------------|----------|-------|----------|
| WITHIN CELLS | 104562.38 | 200 | 522.81 | | |
| GRP | 28955.74 | 2 | 14477.87 | 27.69 | .000 |
| (Total) | 133518.12 | 202 | 660.98 | | |
| R-Squared = .217 | | Adjusted R-Squared = .209 | | | |

The observed F-statistic ($F_{obs} = 27.69$) exceeds $F_{crit} = 7.60$ and is therefore statistically significant. The results consequently indicate the three EA, AA, and AI groups differ in terms of their population means on the multivariate composite Reserved-Openness. Moreover, the η^2 value is .22, which compares favorably to .25 for the full composite. The multivariate gain of the simplified composite (.15 compared to .22) is still substantial and similar to the multivariate gain of the full composite (.15 compared to .25). In other words, very little overlapping variance with the independent variable was lost in the simplification process.

The direction of this effect can be understood by first computing the range of values that are possible on the simplified composite. The original scores on the Big Five scales could range in value from 0 to 192. The lowest possible score for Reserved-Openness is therefore -384 [viz., $0 - (192 + 192)$], and the highest possible score is equal to 192 [viz., $192 - (0 + 0)$]. The European Americans ($M = -129.67$, $SD = 21.85$) scored approximately 24 scale points lower, on average, than the Asian American ($M = -107.68$, $SD = 26.03$) and Asian International ($M = -103.25$, $SD = 21.21$) students on the Reserved-Openness composite. On a 576-point scale, this average difference seems to reflect a modest, or small, effect size. On the other hand, the observed scores on the Reserved-Openness composite ranged from -179 to -28 for all 203 participants. Compared to this observed range, the approximate 24-point mean difference might be interpreted as more theoretically or practically significant.

Step 5: Conducting Follow-up Tests on the Simplified Multivariate Composite.

As in univariate ANOVA, an omnibus multivariate effect for three or more groups should be followed by pairwise comparisons or tests of complex contrasts.

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Moreover, in univariate ANOVA the choice of an adjustment procedure (e.g., Tukey's HSD, or Scheffe') for controlling the Type I error rate will depend on two factors: (1) the type of contrasts, pairwise or complex; and (2) whether the contrasts are planned or constructed after examining the results. The same factors must be considered in MANOVA when contrasting the groups on the simplified multivariate composite. Additionally, one must consider whether the simplified multivariate composite was planned or constructed after examining the discriminant function coefficients.

In the current example of Iwasaki's personality data, the multivariate composite Reserved-Openness was constructed in a purely post hoc fashion. Let us further investigate, post hoc, a complex contrast and pairwise comparisons between the EA, AA, and AI groups. The complex contrast entails a comparison of the average AA and AI means with the EA multivariate composite mean [viz., $(1)(AA) + (1)(AI) - (2)(EA)$].³ Given the fully post hoc nature of the multivariate composite and the follow-up tests, Harris (2001) recommends a Scheffe'-style adjustment equal to the product of the g.c.r.-based F_{crit} value above and df_{effect} : $(7.60)(2) = 15.20$. This adjustment is admittedly conservative, but it is the price that must be paid when a priori theory is not available for constructing the multivariate composite or the group contrasts. More will be said about adjustments for Type I error rates below.

For the current data the results for the complex comparison and all possible pairwise comparisons on the simplified composite ("Comp 1") generated from SPSS GLM appear as follows:

Contrast Coefficients

| Contrast | Group | | |
|----------|--------------------|----------------------|-----------------|
| | European Americans | Asian Internationals | Asian Americans |
| 1 | -1 | .5 | .5 |
| 2 | -1 | 1 | 0 |
| 3 | -1 | 0 | 1 |
| 4 | 0 | -1 | 1 |

Contrast Tests

| Contrast | | | Value of Contrast | Std. Error | t | df | Sig. (2-tailed) |
|-------------------|---------------------------------|---|-------------------|------------|--------|---------|-----------------|
| Reserved-Openness | Assume equal variances | 1 | 24.2024 | 3.3347 | 7.258 | 200 | .000 |
| | | 2 | 26.4167 | 3.7725 | 7.002 | 200 | .000 |
| | | 3 | 21.9881 | 4.0382 | 5.445 | 200 | .000 |
| | | 4 | -4.4286 | 4.0740 | -1.087 | 200 | .278 |
| | Does not assume equal variances | 1 | 24.2024 | 3.3100 | 7.312 | 160.290 | .000 |
| | | 2 | 26.4167 | 3.5520 | 7.437 | 144.982 | .000 |
| | | 3 | 21.9881 | 4.2978 | 5.116 | 106.810 | .000 |
| | | 4 | -4.4286 | 4.2838 | -1.034 | 104.810 | .304 |

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The tests for the contrasts are reported as t-values and must therefore be compared to the square root of 15.20, which equals 3.90 ($t^2 = F$ for single degree of freedom contrasts). The results clearly show the European Americans are distinct from the Asian Americans and Asian Internationals on the multivariate composite. Specifically, the European Americans scored lower, on average, than the Asian International and Asian American students on the Reserved-Openness composite. The population means for the Asian groups were concluded to be equal.

Given the American Psychological Association's recent efforts (Wilkinson, et al., 1999) to encourage researchers to compute and report estimates of effect size as well as confidence intervals, these statistics should be derived and interpreted for the follow-up contrasts as well. As demonstrated above θ is a measure of association strength that can be reported as an indicator of the magnitude of effect. Similarly, η^2 values can easily be computed for the t-values obtained from the contrasts using the well-known formulae:

$$\eta^2_{contrast} = \frac{t^2_{contrast}}{t^2_{contrast} + df_{error}} = \frac{F_{contrast}}{F_{contrast} + df_{error}}$$

The η^2 result for the complex comparison, for instance, is .21 and indicates a large effect using Cohen's conventions.

Computing confidence intervals for contrasts of means on the simplified multivariate composite is more difficult and may require a modicum of matrix algebra (see Harris, 2001, p. 221). The confidence interval must take into account the contrast coefficients, the weights used to create the multivariate composite, and the matrix of residuals from the MANOVA. Specifically, the formula for a multivariate contrast is as follows:

$$\mathbf{c}\bar{\mathbf{X}}\mathbf{a}' \pm \sqrt{\sum \frac{c_j^2}{n_j} (\mathbf{aEa}')\lambda_{crit}}$$

where \mathbf{c} is a row vector of k contrast coefficients, \mathbf{a} is a row vector of p weights used to define the multivariate composite, $\bar{\mathbf{X}}$ is a $k \times p$ matrix of group means on the dependent variables, \mathbf{E} is a $p \times p$ matrix of residuals (i.e., the error matrix from MANOVA), and λ_{crit} is transformed from the θ_{crit} value used for the omnibus test. The c_j 's and n_j 's are the contrast coefficients and sample sizes for the groups, respectively.

Fortunately, as pointed out by a reviewer of this manuscript, the equation above can be simplified so that computing confidence intervals is a manageable task:

$$\text{Value of Contrast} \pm \text{std. error} \sqrt{(df_{error})\lambda_{crit}},$$

where 'Value of Contrast', 'std. error', and df_{error} are taken from the SPSS 'Contrast Tests' table above (24.2024, 3.3347, and 200, respectively) for the first contrast. Using θ_{crit} (.0706) from above,

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$$\lambda_{crit} = \frac{\theta_{crit}}{1 - \theta_{crit}} = \frac{(.0706)}{(1-.0706)} = .0760$$

Thus,

$$24.2042 \pm 3.3347 \sqrt{(200)(.0760)},$$

and the 95% confidence interval for contrasting the EAs with the Aas and Ais on the Reserved-Openness multivariate composite can be written as:

$$11.20 < \mu_{\text{Comp(AA,AI)}} - \mu_{\text{Comp(EA)}} < 37.20.$$

The center of the confidence interval is located at 24.20, the mean difference between the EA and averaged AA and AI groups on the Reserved-Openness composite which can range in value from -384 to 192 (576 units). The width of this confidence interval is only 26 units, or 4.5% (26 / 576) of the scale range, and is therefore a highly precise confidence interval.

Step 6: Summarizing Results using APA style.

When reporting the results of a MANOVA in APA style it is important to provide the overall tests of statistical significance, the full discriminant function coefficients, and the simplified multivariate composite. The theoretical or conceptual interpretation of the composite must also be presented along with the statistical tests of the composite and follow-up comparisons. For the overall tests of significance, Roy's g.c.r. must be reported when following the approach outlined in this paper. Wilks' Lambda or other tests can also be reported, although they are superfluous in this context. A complete example write-up of the analysis of Iwasaki's data above can be found in the Appendix. The reader will find the example includes brief assessments of several of the assumptions underlying MANOVA. Such assessments are also important, although they were not described above.

Three assumptions underlie significance testing in MANOVA: (1) independence of observations, (2) multivariate normality of the group population dependent variables, and (3) homogeneity of group population variance-covariance matrices. Each of these assumptions should be assessed as part of the analysis. Stevens (2002, Chapter 6) offers an excellent discussion of these assumptions as does Tabachnick and Fidell (2006, Section 9.3). The participants' observations in Iwasaki's study were determined to be independent across and within groups (e.g., the participants completed the questionnaires separately and were not related). Although multivariate normality cannot be assessed in SPSS, univariate normality was evaluated for each of the Big Five variables within each of the three groups. All but two of the Kolmogorov-Smirnov tests were not statistically significant (p 's > .05), indicating that most of the variables were normally distributed. Although these results for univariate normality do not guarantee multivariate normality, they at least make the latter assumption more reasonable. Moreover, the simplified multivariate composite was itself tested and found to follow a normal distribution within the bounds of typical sampling variability, thus buttressing the statistical conclusions made for the composite. Lastly, Box's M test for equality of covariance matrices was not statistically significant at the .05

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level, indicating that the group population covariance matrices could be assumed equal. Stevens discusses potential adjustments for violations of each of the three assumptions, and Harris (2001, pp. 237-238) discusses the relevance of these assumptions for the g.c.r. test, specifically. It is worth noting briefly that Harris addresses the common criticism against the g.c.r. test regarding its sensitivity to violations of multivariate normality and/or homogeneity of covariance matrices. He points out that creating, interpreting, and testing simplified composites offsets the problems of the g.c.r. test.

ADDITIONAL ISSUES

Subsequent Discriminant Functions

Although the second function was tested for statistical significance, only the first discriminant function was examined in detail above. Certainly, the second function could have been examined using all of the procedures that were applied to the first function. The functions are independent and could yield distinct and interesting multivariate information regarding group differences, keeping in mind that the functions will be rank-ordered with respect to their proportion of overlap with the independent variable. In other words, the first function will always possess the highest θ (or η^2) value, followed by the second, and so on. A quick comparison of Roy's g.c.r. value, reported as θ , and Pillai's trace from the omnibus MANOVA will give an indication of the strength of the first discriminant function compared to the remaining functions. The researcher must then decide if pursuing the subsequent functions is worthwhile both conceptually and statistically. Are the subsequent functions interpretable? Are they statistically significant? Recall from above that subsequent functions can be tested using the same m and n degrees of freedom for the first composite, but $s = \min(k - j, p - j + 1)$, where j is equal to the composite's ordinal value. Can the other functions be simplified easily? Such questions must be answered by the researcher in the context of his or her study when deciding to pursue the subsequent functions.

Strategies for interpreting the multivariate composites and results

Perhaps the most challenging aspect of the current approach is interpreting the discriminant functions; that is, making conceptual or theoretical sense of the multivariate composites generated by the analysis. Following the advice of Harris (2001) the interpretation process must begin with the discriminant function coefficients, and with a measure of good fortune the process will end with these coefficients. If an investigator is looking for additional information to help solidify or shore up a composite label the *structure coefficients*, which are the correlations between the multivariate composites and original measures, may also be computed and examined. The structure coefficients can be requested using the DISCRIM option in SPSS, which is accessible from the pull-down menus in Windows. A simpler and arguably more appropriate strategy, however, is to compute the correlations between the simplified composite (which represents the interpreted multivariate effect) and the dependent variables. For Iwasaki's data, for instance, the simplified composite was computed as a new variable in SPSS and then simply

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correlated with the original Big Five scale scores. In this example the signs and relative magnitudes of these correlations (i.e., structure coefficients) were similar to the discriminant function coefficients for Neuroticism ($r = -.15$), Extraversion ($r = -.64$), Openness-to-Experience ($r = .50$), and Agreeableness ($r = -.67$). The correlation between Reserved-Openness and Conscientiousness ($r = -.50$) was negative and relatively large in absolute magnitude even though the discriminant function coefficient for Conscientiousness was near zero ($b < .01$). Individuals who scored relatively high on the multivariate composite scored, on average, relatively low on the Conscientiousness scale. In other words, the Gregarious-Traditional individuals tended to report relatively higher levels of Conscientiousness than Reserved-Open individuals. This correlation is certainly consistent with the traditionalism aspect of the multivariate construct and thus supports the interpretation of the discriminant function. Such interpretive congruence between the discriminant function coefficients and the structure coefficients will not always occur, and there is a body of literature discussing this intriguing fact of multivariate statistics. While some authors argue vehemently in support of using primarily structure coefficients to interpret multivariate composites, our strategy relies almost exclusively on the discriminant function coefficients as the basis for the interpretation process (see Harris, 2001). If the structure coefficients are examined at all, they are used only in a secondary role in an attempt to clarify or enhance the theoretical understanding of the multivariate composite.

Another aid to the interpretation process is the reflective nature of the signs of the raw and standardized discriminant function coefficients. In other words, the signs of the coefficients are arbitrary and can be reflected without loss of meaning. For example, the above Reserved-Openness composite could have originally been computed as $(E)(1) + (O)(-1) + (A)(1)$ rather than $(E)(-1) + (O)(1) + (A)(-1)$. The three groups would therefore differ in terms of higher Extraversion and Agreeableness relative to lower Openness-to-Experience (Gregarious-Traditionalism). It is important to note that *the signs for all of the variables in the composite must be reversed* if this strategy is employed. In our experience, simply reversing the signs can at times provide the necessary insight for deriving an interpretation when it is not readily evident with the original discriminant function coefficients. Consider the second multivariate composite for the current data, which could be simplified to $(1)(O) + (1)(A)$. What personality type might we apply to a person who is high in openness-to-experience and agreeableness? Reversing the signs of the coefficients $[(-1)(O) + (-1)(A)]$ changes the task to inquiring what type of person is closed to new experiences and disagreeable? It seems the label "Rigid" applies to this composite, and the opposite label "Flexible" would thus apply to the opposite pole. The new composite can therefore be scored as either an index of Rigidity ($-O + -A$) or Flexibility ($O + A$) without loss of meaning.

When working with the unstandardized discriminant function coefficients and original dependent variables, manipulating the scaling of the simplified multivariate composite can also greatly aid the interpretation process. MANOVA maximizes the differences between group means on linear combinations of the dependent variables. Consequently, it can be very useful to center the original variables before computing the multivariate composite. The centered scaling will generate the same η^2 values for the multivariate composite and the same results for post hoc comparisons of groups. Each dependent variable is centered by

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subtracting its mean from the individual scores. For instance, the mean for Extraversion for all 203 participants in the example above is equal to 116.73. A compute statement in SPSS can be written to center the Extraversion (E) scale scores:

COMPUTE E_center = E - 116.73.

Reserved-Openness would then be computed from the centered variables, $O_center - (E_center + A_center)$. The primary benefit of centering the variables is interpreting group differences on the multivariate composite. The means for the EA ($M = -15.44$), AA ($M = 6.55$), and AI ($M = 10.98$) groups more clearly indicate that the AIs and AAs score relatively high on Reserved-Openness whereas the EAs score relatively low.

When the dependent variables are measured on different scales, the standardized discriminant function coefficients are often easier to simplify and interpret. As standardized coefficients they are derived from a MANOVA conducted on the z-scores of the dependent variables, thus removing differences in their scaling and variance. The multivariate composite should consequently be computed from the z-scores rather than the original variables; for example, $(1)(Oz) + (-1)(Ez) + (-1)(Az)$, keeping in mind the η^2 value for the standardized composite will likely differ from the η^2 value for the original composite. Similar to centered scores, however, working with the standardized scores may also facilitate thinking about the groups in terms of their relative, rather than their absolute, performances on the dependent variables and on the multivariate composite. For instance, the means for the EA ($M = -.73$), AA ($M = .31$), and AI ($M = .50$) groups on the standardized composite again clearly indicate that the AIs and AAs score relatively high on Reserved-Openness whereas the EAs score relatively low. The conceptual or theoretical nature of the multivariate composite may therefore be easier to understand when switching from a scale-based to a standardized perspective.

Lastly, a topic related to the scaling of the simplified weights used in the multivariate composite is that of complex weighting schemes. In most instances the discriminant function coefficients can be simplified to -1s, 0s, and 1s because it is easier to think of whole and equivalent units of Extraversion, Agreeableness, etc. than of fractional or unequal units of these variables. Some analyses, however, may call for more complex weighting schemes in which one or more of the variables is given greater weight in the simplified composite. For instance, given the relatively large discriminant function coefficient for Extraversion in the first function above, the simplified composite could have been computed as $(-2)(E) + (1)(O) + (-1)(A)$. It may be that the label Reserved-Openness is better captured by greater weight given to extraversion relative to agreeableness. Such judgments would of course be driven primarily by logic and theory, although the η^2 values for the composites derived from different weighting schemes could be computed and compared. Furthermore, if a fully post hoc critical value is employed, as above, an infinite number of such composites can be computed and compared. A conceptually meaningful multivariate composite derived from a complex weighting scheme that also yields a high η^2 value may finally be preferred over a competing composite derived from equal weights. Given the long history of evidence showing that complex weighting schemes are generally no more effec-

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tive, practically speaking, than complex weighting schemes, it is nonetheless reasonable to expect simple and equal weighting schemes to perform adequately.

More complex designs

The essence of conducting a truly multivariate analysis of variance entails an examination of the multivariate composite or composites of the dependent variables that are generated from the analysis. Depending on the number of groups and the number of dependent variables, the number of composites generated will vary. Furthermore, with factorial designs (i.e., designs with two or more independent variables) distinct sets of multivariate composites will be generated for each interaction and main effect. The task of the researcher then becomes interpreting, simplifying, and analyzing the composite or composites for each effect in the analysis. Suppose differences between men and women were examined in the example above. The inclusion of this additional independent variable would yield a 2×3 (gender by group) factorial MANOVA with five dependent variables. The s degree of freedom parameter is computed as $\min(df_{\text{effect}}, \# \text{ dependent variables})$ and indicates the number of independent discriminant functions computed for each effect. The gender main effect would yield one discriminant function, the group main effect would yield two functions, and the interaction would yield two functions. All of these discriminant functions would be distinct, and would possibly yield different simplified multivariate composites with different interpretations. Obviously, the burden can become great, and the researcher must then return to the critical point made above: Is a truly multivariate question being asked? In other words, does the researcher have reason to expect significant multivariate gain (both statistically and conceptually) from the MANOVA compared to conducting a series of univariate factorial ANOVAs? If the answer is “yes”, then the considerable work involved with simplifying and interpreting the discriminant functions produced by the factorial MANOVA must be undertaken with patience. Alternatively, Harris (2001) suggests the factorial MANOVA can initially be ignored to create a single simplified composite for all effects. For instance, the suggested 2×3 MANOVA for Iwasaki’s data could be “reduced” to a oneway MANOVA with 6 groups (EA males, EA females, AI males, AI females, AA males, AA females), which would produce 5 discriminant functions that could be simplified and interpreted. Of course the first function would yield the highest θ value and may be the only multivariate composite worth pursuing. Regardless, the simplified composite (or composites) can then be examined using the procedures above in which univariate ANOVA procedures, with the appropriate critical values, are employed to test the two main effects and the interaction for the simplified composite(s). This approach saves a substantial amount of effort compared to simplifying and labeling different composites for each effect in the factorial MANOVA.

Post hoc vs. a priori tests

The example above was fully post hoc, meaning that interpreting and constructing the simplified, multivariate composite was done after an examination of the results from the MANOVA and the EA, AI, and AA group contrasts were

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conducted after an examination of the means on the simplified composite. When considering the issue of post hoc versus a priori analyses in the current approach, then, two factors must be considered: (1) the construction of the multivariate composite, and (2) the construction of group contrasts. Harris (2001, p. 222) crosses these two factors to produce a table reporting the critical values for different scenarios:

| | | Dependent Variable Composite <i>a priori</i>? | |
|---|------------|--|--|
| | | Yes | No |
| Independent Variable Contrast <i>a priori</i>? | Yes | $F_{\alpha(1, df_{error})}$ Bonferroni adjustment can be applied to α for multiple comparisons | $\frac{p \cdot df_{error}}{df_{error} - p + 1} F_{\alpha}(p, df_{error} - p + 1)$ Bonferroni adjustment can be applied to α for multiple comparisons |
| | No | $df_{effect} \cdot F_{\alpha(df_{effect}, df_{error})}$ | $df_{effect} \cdot F_{crit}$ |

The value for F_{crit} is computed as:

$$F_{crit} = \frac{df_{error} \theta_{crit}}{(1 - \theta_{crit}) df_{effect}}$$

The values for df_{effect} , df_{error} , etc. are defined above under Step 1. F_{α} refers to the critical value from the F-distribution with the indicated degrees of freedom and an a priori p-value equal to α . As can be seen in the table, the multivariate composite can be generated prior to or after the omnibus MANOVA. It is conceivable to generate a multivariate composite on a priori grounds, particularly when replicating previous results. In the example above, it is also conceivable the Reserved-Openness composite could have been theoretically anticipated based an understanding of differences between American and Asian cultures. If the composite were created prior to the analysis and the group contrasts were also planned, then the critical value in the upper lefthand corner of the table [viz., $F_{\alpha}(1, df_{error})$] would have been used to test the statistical significance of the various results. As with univariate ANOVA the choice of critical values is to maximize power, and the most powerful tests will usually be those in which the composites and group contrasts are constructed on an a priori basis.

When the multivariate composite is constructed prior to the analyses, we would further recommend a number of omnibus post hoc tests in which the dependent variables are removed, individually, from the multivariate composite and the resulting η^2 values recorded. In this way the importance of each variable to the a priori composite can be assessed. A large drop in η^2 , for instance, would indicate that the variable is an important component of the multivariate composite. Unfortunately, a test of statistical significance for the individual variables in the composite is not available in the context of the methods employed herein.

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Multicollinearity and Singularity among Predictors

Some textbook authors (e.g., Stevens, 2002, and Tabachnick & Fidell, 2006) argue that high correlations among the dependent variables are problematic. When two or more of the dependent variables are completely redundant (either considered pairwise or in linear combination), they are said to be *singular*, and this problem should be resolved by removing the problematic dependent variables. When the variables are highly—but not perfectly—related they are said to suffer from multicollinearity. This condition is problematic because it tends to result in high variability in the discriminant function coefficients, much like multicollinearity “increases the variances of the regression coefficients” (Stevens, 2002, p. 92) in multiple regression. But how much multicollinearity among the dependent variables should be tolerated? Common rules of thumb are 80% or 90% overlap, while some might consider even moderate overlap (25% - 50%) to be too high. It is our opinion that other than singularity or near singularity (>90% overlap), the multicollinearity issue should not weigh heavily in the decision process behind the analysis. The beauty of multivariate statistical models is that they incorporate the interrelations among variables, and to argue that the dependent variables in a MANOVA, for example, must be nearly orthogonal is to argue that one should always opt for conducting independent univariate analyses. In other words, when the variables are independent (i.e., uncorrelated) the results from multivariate analyses are completely predictable from univariate or bivariate analyses of those same variables. Consider a case in which the outcome variable in a multiple regression is standardized, and the predictor variables are also standardized and uncorrelated with one another. In this instance the regression weights will equal the bivariate correlations between each predictor and the outcome variable. Insisting that multicollinearity be low is thus tantamount to insisting that one’s multivariate results match a series of univariate analyses performed on the same dependent variables. Such reasoning leaves us to wonder why we should bother with multivariate statistics at all.

Stepdown Analysis

A much recommended method for following up a significant omnibus MANOVA is to conduct what is referred to as a *stepdown analysis*. This procedure is equivalent to a series of analyses of covariance (ANCOVAs) in which the dependent variables are evaluated in terms of their unique overlap with the independent variable. Stevens (2002) and Tabachnick & Fidell (2006) both offer discussions of this type of analysis. An important feature of stepdown analysis is that the order of variables in the ANCOVAs is of utmost importance and must be driven by a clear rationale. The methods above, by comparison, permit the researcher to examine the importance of the dependent variables simultaneously and is a relatively straightforward alternative to stepdown analysis. The similarities between the above procedures and Discriminant Function Analysis and multivariate profile analysis are also readily apparent and reveal the intimate connections between these methods. Lastly, conducting a stepdown analysis requires the additional rigid assumptions (e.g., homogeneity of regression slopes) associated with ANCOVA. For these reasons, we prefer the methods above to stepdown analysis.

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NOTES

1. The data set, which includes the independent and dependent variables, can be downloaded from the first author's website: <http://psychology.okstate.edu/faculty/jgrice/personalitylab/>
2. The program "gcrcomp" can be downloaded from the first authors' website: <http://psychology.okstate.edu/faculty/jgrice/personalitylab/>
3. An argument could certainly be made that contrasting the EA with the combined AA and AI groups is obvious enough to be considered as a priori rather than post hoc. The approach taken herein and the analyses reported above, however, are distinct from the analysis strategy reported in Iwasaki's (1998) thesis. We therefore chose a conservative route for the multivariate analyses.

Author Note

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MULTIVARIATE ANALYSIS OF VARIANCE

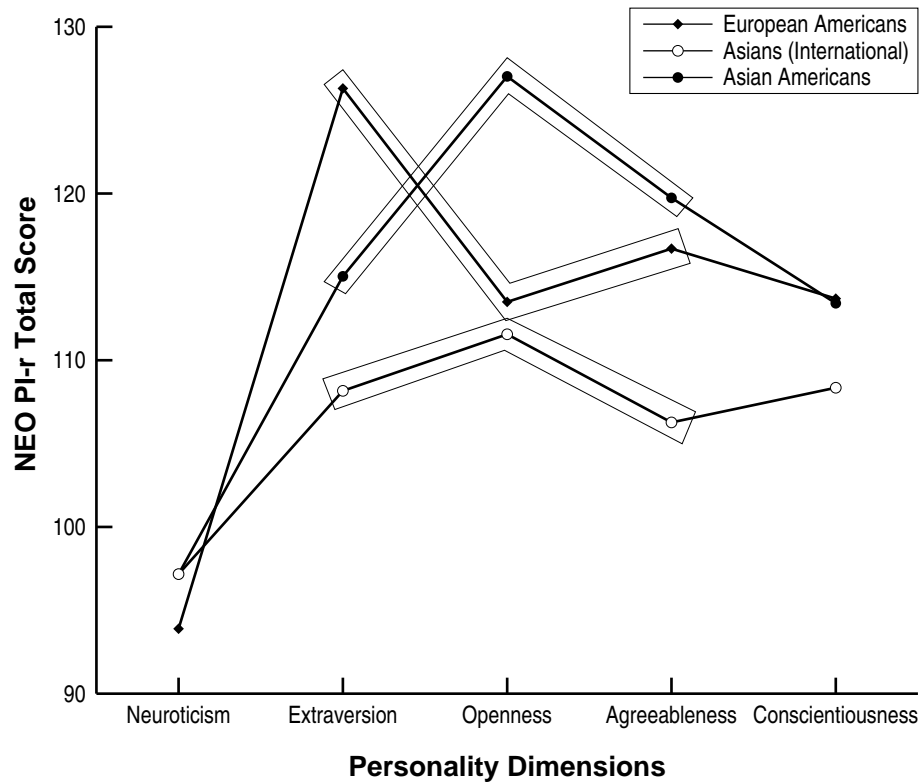


Figure 1. Means for European American, Asian International, and Asian American students on Big Five personality traits as measured by the NEO PI-r.

APPENDIX

Annotated Syntax Statements for SPSS

```

Title 'Grice & Iwasaki MANOVA results'.
Subtitle 'Step 1 Analyses'.
* Both univariate and multivariate results will be printed from the MANOVA
* command below.
MANOVA neoa c by grp(0,2)
/print cellinfo(means) homogeneity
/discrim(raw stan) alpha(1.0) /* 'alpha(1.0)' insures all discriminant
                                functions will be printed */
/design.

```

APPLIED MULTIVARIATE RESEARCH

* The following GLM procedure will yeild the same results but will not print
* the discriminant function coefficients. It reports Roy's g.c.r. as an
* eigenvalue (lambda) rather than a proportion of overlap (theta).

GLM

```
n e o a c BY grp
/METHOD = SSTYPE(3)
/INTERCEPT = INCLUDE
/PRINT = DESCRIPTIVE ETASQ HOMOGENEITY
/CRITERIA = ALPHA(.05)
/DESIGN = grp .
```

Subtitle 'Step 2 Analyses'.

* Compute the 1st full multivariate composite.

```
COMPUTE Compl=(N * -0.0058) + (E * -0.06027) + (O * 0.04239) + (A * -0.02758)
+ (C * 0.00748).
```

```
MANOVA Compl BY grp(0,2)
```

```
/design. /* using MANOVA to conduct univariate ANOVA on 'Compl'. */
```

```
COMPUTE Comp2=(N * -.007) + (E * -.00017) + (O * -.0423) + (A * -.03102)
+ (C * -.00956).
```

```
MANOVA Comp2 BY grp(0,2)
```

```
/design. /* Using MANOVA to conduct univariate ANOVA on 'Comp2'. */
```

Subtitle 'Step 3 Analyses'.

```
COMPUTE Simpl=(E * -1) + (O * 1) + (A * -1).
```

```
VARIABLE LABEL Simpl 'Reserved-Openness'.
```

Subtitle 'Step 4 Analyses'.

* Test Simpl, the simplified multivariate composite (Reserved-Openness) for

* statistical significance.

* F-critical (p = .05) for the test is 7.60.

```
MANOVA Simpl BY grp(0,2)
```

```
/print cellinfo(means)
```

```
/design.
```

Subtitle 'Step 5 Analyses'.

* Follow-up Tests comparing three groups on simplified multivariate composite

* (Reserved-Openness).

```
ONEWAY Simpl BY grp
```

```
/contrast = -1 .5 .5
```

```
/contrast = -1 1 0
```

```
/contrast = -1 0 1
```

```
/contrast = 0 -1 1
```

```
/statistics descriptives
```

```
/plot means
```

```
/missing analysis.
```

Subtitle 'Additional Analyses'.

* Computing structure coefficients; i.e., correlations between multivariate

* composite and DVs.

```
CORR Simpl with N E O A C.
```

MULTIVARIATE ANALYSIS OF VARIANCE

APA STYLE EXAMPLE

A one-factor, between-subjects multivariate analysis of variance (MANOVA) was conducted. The Big Five personality trait scores from the NEO PI-r served as the dependent variables in the analysis, and the ethnic groups (European Americans, Asian Americans, Asian Internationals) comprised the independent variable. Evaluation of the homogeneity of variance-covariance matrices and normality assumptions underlying MANOVA did not reveal any substantial anomalies, and the *a priori* level of significance was set at .05. The bivariate correlations for the dependent variables across all 203 participants are presented in Table 1.

Results from the MANOVA were statistically significant according to Wilks' Λ (.62), $F(10, 392) = 10.45, p < .001$. Furthermore, Roy's greatest characteristic root (g.c.r.) was statistically significant ($s = 2, m = 1, n = 97, p < .001$) and indicated that the independent variable and first multivariate combination of dependent variables shared 25 percent of their variance. Univariate means and standard deviations and the unstandardized discriminant function coefficients for the first multivariate combination are reported in Table 2. As can be seen, the coefficients (w_s) indicate the EA, AA, and AI groups differed as a function of relatively high Openness-to-Experience ($w_s = .04$) compared to lower reported levels of Extraversion ($w_s = -.06$) and Agreeableness ($w_s = -.03$). The coefficients for Neuroticism ($w_s = -.01$) and Conscientiousness ($w_s = .01$) were relatively small in absolute value. Following the MANOVA analysis strategy recommended by Harris (2001), a simplified multivariate composite was created from the centered dependent variables with extreme discriminant function coefficients. For the current data the simplified composite was equal to: $(-1)(\text{Extraversion}) + (-1)(\text{Agreeableness}) + (1)(\text{Openness-to-Experience})$, or Openness - (Extraversion + Agreeableness). Conceptually, this combination of traits represents something akin to a personality type that we labeled "Reserved-Openness". We labeled the opposite of this type "Gregarious-Traditionalism".

As can be seen in Figure 1, the three groups differed in the patterns of the Extraversion, Openness-to-Experience, and Agreeableness traits. Specifically, the European Americans reported higher levels of Extraversion and Agreeableness compared to Openness-to-Experience; in other words, they exhibited Gregarious-Traditionalism. The opposite pattern of means was observed for the Asian American and Asian International students who exhibited Reserved-Openness. Indeed, the three groups differed on the simplified multivariate composite (based on the centered variables) representing Reserved-Openness, $F(2, 200) = 27.69, p < .001, \eta^2 = .22$, according to a fully post hoc criterion for statistical significance (Harris, 2001). Furthermore, using a Scheffe'-adjusted critical value to control for Type I error inflation, we conducted several follow-up contrasts. The European Americans ($n = 75, M = -15.44, SD = 21.86$) were found, on average, to score lower on Reserved-Openness than the Asian International ($n = 72, M = 10.98, SD = 26.03$) and Asian American ($n = 56, M = 6.55, SD = 21.21$) students combined (Mean difference for contrast, $M_{\text{contrast}} = 24.20, p < .05, \eta^2 = .21, CI_{.95}: 11.19, 37.21$). Moreover, the EAs were found to score lower than the AIs ($M_{\text{contrast}} = 26.42, p < .05, \eta^2 = .20, CI_{.95}: 11.71, 41.12$) and AAs ($M_{\text{contrast}} = 21.99, p < .05, \eta^2 = .13, CI_{.95}: 6.25, 37.73$), considered separately. The mean difference between the AIs and AAs was not statistically significant ($M_{\text{contrast}} = -4.43, p > .05, \eta^2 = .01, CI_{.95}: -20.31, 11.45$). While the estimated effect sizes were small, the 95% confidence intervals were precise when compared to the possible range of values on the simplified multivariate composite.

APPLIED MULTIVARIATE RESEARCH

Table 1. Intercorrelations among Big Five Personality Traits

| Trait | N | E | O | A | C |
|-------------------|------|-------|------|------|-------|
| Neuroticism | 1.00 | -.26* | -.02 | -.10 | -.40* |
| Extraversion | | 1.00 | .35* | .01 | .37* |
| Openness | | | 1.00 | .23* | .13 |
| Agreeableness | | | | 1.00 | .10 |
| Conscientiousness | | | | | 1.00 |

* $p < .001$

Table 2. Means, Standard Deviations, and Discriminant Function Coefficients for EA, AA, and AI Groups on the Big Five Personality Traits

| Personality Trait | Group | <i>M</i> | <i>SD</i> | w_s |
|-------------------|-------|----------|-----------|--------|
| Neuroticism | All | 95.91 | 23.17 | -.0058 |
| | EA | 93.95 | 22.83 | |
| | AA | 97.04 | 22.07 | |
| | AI | 97.07 | 18.88 | |
| Extraversion | All | 116.53 | 22.09 | -.0603 |
| | EA | 126.23 | 18.46 | |
| | AA | 114.98 | 20.45 | |
| | AI | 108.19 | 16.73 | |
| Openness | All | 116.22 | 20.60 | .0424 |
| | EA | 113.40 | 17.12 | |
| | AA | 126.98 | 19.11 | |
| | AI | 111.36 | 15.24 | |
| Agreeableness | All | 113.75 | 20.61 | -.0276 |
| | EA | 116.84 | 18.69 | |
| | AA | 119.68 | 19.78 | |
| | AI | 106.42 | 13.88 | |
| Conscientiousness | All | 111.62 | 20.81 | .0075 |
| | EA | 113.87 | 15.53 | |
| | AA | 113.34 | 22.56 | |
| | AI | 108.38 | 17.72 | |

Note. EA = European American (N = 75); AA = Asian American (N = 56); AI = Asian International (N = 72). w_s = coefficients from first unstandardized discriminant function.