Tarea No. 1

3) $t^5 y^{(4)} - t^3 y'' + 6y = 0$

Cuarto Orden, No lineal

6) $\frac{d^2R}{dt^2} = -\frac{K}{R^2}$

Segundo Orden, No lineal

9) $(y^2-1)dx + xdy = 0$; en y; en x

$\frac{dy}{dx} = \frac{-(y^2-1)}{x}$ $\qquad$ $\frac{dx}{dy} = \frac{-x}{y^2-1}$

En y es No lineal $\qquad$ En x es lineal

12) $\frac{dy}{dt} + 20y = 24$; $\quad y = \frac{6}{5} - \frac{6}{5}e^{-20t}$

$y' = \frac{120}{5}e^{-20t} \rightarrow 24e^{-20t} \rightarrow 24e^{-20t} + 20\left(\frac{6}{5} - \frac{6}{5}e^{-20t}\right) = 24$

$\cancel{24e^{-20t}} + 24 - \cancel{24e^{-20t}} = 24 \rightarrow 24 = 24$

$y = \frac{6}{5} - \frac{6}{5}e^{-20t}$ Si es solucion de $\frac{dy}{dt} + 20y = 24$

15) $(y-x)y' = y-x+8; \quad y = x+4\sqrt{x+2}$

$$y' = 1 + \frac{2}{\sqrt{x+2}} \rightarrow x+4\sqrt{x+2}\left(1+\frac{2}{\sqrt{x+2}}\right) \rightarrow 4\sqrt{x+2}+8$$

$$4\sqrt{x+2}+8 = y-x+8$$

18) $2y' = y^3 \cos x; \quad y = (1-\operatorname{Sen} x)^{-1/2}$

$$y' = \frac{\cos x}{2(1-\operatorname{Sen} x)^{3/2}} \rightarrow \cancel{2}\left(\frac{\cos x}{\cancel{2}(1-\operatorname{Sen} x)^{3/2}}\right) = (1-\operatorname{Sen} x)^{-1/2}$$

21) $\frac{dP}{dt} = P(1-P); \quad P = \frac{C_1 e^t}{1+C_1 e^t}$

$$P' = \frac{C_1 e^t}{(1+C_1 e^t)^2} \rightarrow 1-P = \frac{1}{1+C_1 e^t} \rightarrow * P \rightarrow \frac{C_1 e^t}{(1+C_1 e^t)^2}$$

$$\frac{C_1 e^t}{(1+C_1 e^t)^2} = \frac{C_1 e^t}{(1+C_1 e^t)^2} \rightarrow$$ Si es solucion

24) $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 12x^2$

$y = c_1x^{-1} + c_2x + c_3 x\ln x + 4x^2$

$y' = -c_1x^{-2} + c_2 + c_3(\ln x + 1) + 8x$

$y'' = 2c_1x^{-3} + c_3x^{-1} + 8$

$y''' = -6c_1x^{-4} - c_3x^{-2}$

$x^3(-6c_1x^{-4} - c_3x^{-2}) + 2x^2(2c_1x^{-3} + c_3x^{-1} + 8) - x(-c_1x^{-2} + c_2 + c_3(\ln x + 1) + 8x)$
$+ (c_1x^{-1} + c_2x + c_3x\ln x + 4x^2) \to 12x^2$

$12x^2 = 12x^2$ Es Solución

27) $y' + 2y = 0$ $\quad y = e^{mx}$

$y' = me^{mx} \to me^{mx} + 2e^{mx} = 0 \to e^{mx}(m+2) = 0$

$m = -2$

30) $2y'' + 7y' - 4y = 0$ $\quad y = e^{mx}$

$y' = me^{mx}, \quad y'' = m^2e^{mx}$

$2m^2e^{mx} + 7me^{mx} - 4e^{mx} = 0 \to e^{mx}(2m^2 + 7m - 4) = 0 \to 2m^2 + 7m - 4 = 0$

$m = \frac{-7 \pm \sqrt{49 + 32}}{4} \to \quad m = \frac{1}{2} \quad m = -4$

33) $3xy' + 5y = 10 \qquad y = C \qquad y' = 0$

$3x(0) + 5C = 10 \rightarrow C = 10/5 \rightarrow \underline{C = 2}$

36) $y'' + 4y' + 6y = 10 \qquad y = C \qquad y' = 0 \qquad y'' = 0$

$0 + 4(0) + 6C = 10 \rightarrow C = 10/6 \rightarrow \underline{C = 5/3}$

~~40) Construya una ecuación diferencial que usted asegure~~
~~tenga una sola solución y=0~~

39) Construya una ecuacion diferencial que no tenga ninguna solucion real

$\underline{(y')^2 + y^2 + 1 = 0}$

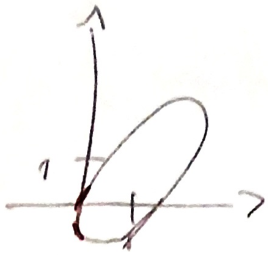
42) $\dfrac{d^2y}{dx^2} = y \rightarrow$ Las funciones que satisfacen son Combinaciones de exponenciales

$\underline{y(x) = C_1 e^{x} + C_2 e^{-x}}$

$\dfrac{d^2y}{dx^2} = -y \rightarrow$ Combinaciones senos y cosenos

$\underline{y(x) = C_1 \cos x + C_2 \sin x}$

45)



48) $\frac{dy}{dx} = \frac{y-x^2}{y^2-x}$

?

51) $\frac{dy}{dx} = f(x)$

$y = \int f(x)\,dx + C \rightarrow \frac{dy}{dx} = 2x \rightarrow y = x^2 + C$

$\frac{dy^2}{dx^2} = f(x)$

$\frac{dy}{dx} = \int f(x)\,dx + C_1; \quad y = \iint f(x)\,dx\,dx + C_1 x + C_2$

$\frac{d^2y}{dx^2} = \cos x \rightarrow y = -\cos x + C_1 x + C_2$

54) $y = C_1 x + C_2 x^2$

$$x^2 y'' - 2xy' + 2y = 0$$

57) $\frac{dy}{dx} = y(a - by) \quad (a, b > 0)$

a) $\frac{dy}{dx} = 0 \to y(a - by) = 0 \to \quad y = 0 \; ; \; y = \frac{a}{b}$

b) $y' > 0 \to y(a - by) > 0$ si $0 < y < \frac{a}{b}$, $y' > 0$ (creciente)

Si $y < 0$ o $y > \frac{a}{b}$, $y' < 0$ (decreciente)

c) $y = \frac{a}{2b}$ $\quad y'' = (a - 2by) y' = (a - 2by) y (a - by)$

$y'' = 0$ en $y = \frac{a}{2b}$ (Si $y' \neq 0$)

60) $x^3y''' + 2x^2y'' + 20xy' - 78y = 0$

$$y = 20\frac{\cos(5\ln x)}{x} - 3\frac{\operatorname{sen}(5\ln x)}{x}$$

$$y' = \frac{u'x - u}{x^2} \rightarrow u' = -20 \cdot 5 \frac{\sin(5\ln x)}{x} - 3 \cdot 5 \cdot \frac{\cos(5\ln x)}{x} =$$

$$\frac{-100\sin(5\ln x) - 15\cos(5\ln x)}{x}$$

$$y' = \frac{\left(\frac{-100\sin(5\ln x) - 15\cos(5\ln x)}{x} \cdot x\right) - (20\cos(5\ln x) - 3\sin(5\ln x))}{x^2}$$

$$y' = \frac{-97\sin(5\ln x) - 35\cos(5\ln x)}{x^2} \leftarrow \text{Derivada "}$$

$$y'' = \frac{v'x^2 - 2xv}{x^4} \rightarrow v' = -97 \cdot 5 \cdot \frac{\cos(5\ln x)}{x} + 35 \cdot 5 \cdot \frac{\sin(5\ln x)}{x} = \frac{-485\cos(5\ln x) + 175\sin(5\ln x)}{x}$$

$$y'' = \frac{\left(\frac{-485\cos(5\ln x) + 175\sin(5\ln x)}{x} \cdot x^2\right) - 2x(-97\sin(5\ln x) - 35\cos(5\ln x))}{x^4}$$

$$y'' = \frac{-415\cos(5\ln x) + 369\sin(5\ln x)}{x^3} \leftarrow y'''$$

$$y''' \rightarrow w(x) = -415\cos(\ln x) + 369\sin(5\ln x) \Rightarrow y'' = \frac{w(x)}{x^3}$$

$$y''' = \frac{w'x^3 - 3x^2w}{x^6} \rightarrow w' = 415 \cdot 5 \cdot \frac{\sin(5\ln x)}{x} + 369 \cdot 5 \cdot \frac{\cos(5\ln x)}{x} =$$

$$\frac{2075\sin(5\ln x) + 1845\cos(5\ln x)}{x}$$

$$y''' = \frac{(2075\sin(5\ln x) + 1845\cos(5\ln x))x^2 - 3x^2(-415\cos(5\ln x) + 369\sin(5\ln x))}{x^6}$$

$$y''' = \frac{968\sin(5\ln x) + 3090\cos(5\ln x)}{x^4}$$

Sustituimos: $x^3y''' + 2x^2y'' + 20xy' - 78y = 0$

$$x^3\left(\frac{968\sin(5\ln x) + 3090\cos(5\ln x)}{x^4}\right) + 2x^2\left(\frac{-415\cos(5\ln x) + 369\sin(5\ln x)}{x^3}\right) +$$

$$20x\left(\frac{-97\sin(5\ln x) - 35\cos(5\ln x)}{x^2}\right) - 78\left(\frac{20\cos(5\ln x) - 3\sin(5\ln x)}{x}\right) =$$

$$\frac{(968 + 738 - 1940 + 234)\sin(5\ln x) + (3090 - 830 - 700 - 1560)\cos(5\ln x)}{x} =$$

$$\frac{0\cdot\sin(5\ln x) + 0\cdot\cos(5\ln x)}{x} = 0$$

Por lo tanto la funcion $y = \frac{20\cos(5\ln x)}{x} - \frac{3\sin(5\ln x)}{x}$

es solucion de la ecuación $x^3y''' + 2x^2y'' + 20xy' - 78y = 0$