Discovering Dark Energy With Type Ia Supernovæ

Description

A type Ia Supernova (SNe Ia) is the result of the explosion of a White Dwarf that reaches the Chandrasekhar mass ($\sim 1.4 M_\odot$). It is believed that the physics underlying SNe Ia are always the same, producing the same luminosity. They are considered standard candles and therefore powerful probes that can be used to determine the distance to high redshift objects. When comparing different supernovæthey only differ in observed magnitude due to the fact that they are at different distances from us. One quantifies the distance using the distance modulus (μ) which is the difference between the observed magnitude (m) and the absolute magnitude (M), i.e.

$$\mu \equiv m - M = 5\log_{10}\left(\frac{D_L}{\mathrm{Mpc}}\right) + 25\,, \tag{1}$$

where D_L is the Luminosity distance. In a ΛCDM Universe, with flat spatial curvature ($\Omega_K = 0$), the luminosity distance can be determined from the expression

$$D_L(z) = \frac{c}{H_0}(1+z)\int_0^z \mathrm{d}\bar{z}\frac{1}{\sqrt{\Omega_m(1+\bar{z})^3 + (1-\Omega_m)}}\,, \tag{2}$$

where c is the speed of light, in km/s, H_0 is the value of the Hubble constant today, in km/s/Mpc^{-1}, and Ω_m is the matter content of the Universe today. Note that since radiation contribution is negligible in the late universe we have omitted it from Eq. 2. Hence, by comparing the observed redshift of a supernova to its inferred distance we can map the expansion rate of the Universe.

In this project we will go through the introductory steps to Statistical Cosmology. Our goal is to determine which value of Ω_m is the best one describing some observational data. We do so by computing the distance between the observational dataset and a model (i.e. a value of Ω_m). One commonly uses the χ^2 to find such distance, which is given by

$$\chi^2 = \sum_{i=1}^{N}\frac{\left(\mu_i^{\mathrm{obs}} - \mu_i^{\mathrm{model}}\right)^2}{\sigma_{\mu,i}^2}\,, \tag{3}$$

where μ^{obs} is the observed data, μ^{model} is the predicted distance modulus for a given model at the observed redshift and σ_μ is the uncertainty on the measured data. N is the number of objects in the observed dataset and the index i corresponds to the ith object in the data.

For this project you will be provided with a dataset of SNe Ia in a file called "sndata.txt". Each line contains data for a single SNe Ia where the first column is the observed redshift (z), the second the distance modulus (μ) and the third column the uncertainty in the distance modulus observation (σ_μ).

Exercises

1. Read the dataset and produce a scatter plot of distance modulus against redshift for these SNe Ia with the corresponding errors bars. Do not forget to label properly your axes and ensure that each datapoint is visible as well as its error bars.

2. Define two functions which allow you to compute the luminosity distance (Eq. 2) and the distance modulus (Eq. 1) for difference values of redshift z, Hubble parameter today H_0 and amount of matter Ω_m.

3. Plot D_L *vs* z for $H_0 = 70$ km s^{-1} Mpc^{-1} and $\Omega_m = 0.3$. Use $z \in [0, 2.0]$. Label the axes properly and use the correct units.

4. Fix $H_0 = 70$ km s^{-1} Mpc^{-1} and plot on the scatter plot from exercise 1 the predicted μ for different amounts of matter $\Omega_m = 0.1, 0.3$ and 1.0. Include a legend to distinguish the 3 lines.

5. Define a function that allows you to compute χ^2 as a function of the cosmological parameter Ω_m.

6. To find the most appropriate value of Ω_m that best fits the data, we need to try out a range of trial values of Ω_m and evaluate the χ^2 for each one. Try this for a range of Ω_m between [0,1]. The lowest χ^2 value corresponds to the best fitting model. What value to you get? Plot χ^2 *vs* Ω_m.