Doctoral Programme in Economics

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Problem set 3

Latest version

Macroeconomics II

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Question 1

I will assume that agents know their type before making their decisions. The problem of a tpye θ agent born at period t is:

$$\begin{aligned} \max_{c_{t+1}, n_t} \quad & u(c_{+1}) - g(n_t) \\ \text{s.t.} \quad & m_t = p_t \theta n_t \\ & p_{t+1} c_{t+1} = m_t(1+\sigma) \end{aligned} \tag{1}$$

The competitive equilibrium are allocation $\left(\{c_t^i, m_t^i, n_t^i\}_{t=1}^{\infty}\right)_{i \sim f(\theta)}$ and prices $\{p_t\}_{t=1}^{\infty}$ such that:

1. Given prices, allocation solves the problem of the agent (1) for every period and every agent type.
2. markets clear in every period:
   (a) Market for money:
   $$M_t = \int m_t^i f(\theta) d\theta \tag{2}$$
   (b) Market for goods:
   $$\int c_{t^i f(\theta) d\theta = \int \theta n_t^i f(\theta) d\theta}(3)$$

The first order condition for the agent is:

$$\frac{p_t}{p_{t+1}} \theta^i (1+\sigma) u' \left(\frac{p_t}{p_{t+1}} \theta^i (1+\sigma) n_t^i \right) = g'(n_t^i) \tag{4}$$

From the market clearing in the money market:

$$M_t = \int m_t^i f(\theta) d\theta = \int p_t \theta^i n_t^i f(\theta) d\theta = p_t \underbrace{\int \theta^i n_t^i f(\theta) d\theta}_{E[n_t]} = p_t E[n_t] \tag{5}$$

$$p_t = \frac{M_t}{E[n_t]} \tag{6}$$

$$p_{t+1} = \frac{M_{t+1}}{E[n_{t+1}]} = \frac{M_t(1+\sigma)}{E[n_{t+1}]} \tag{7}$$

$$\frac{p_t}{p_{t+1}} = \frac{E[n_{t+1}]}{E[n_t](1+\sigma)} \tag{8}$$

Plugging in equation 8 in 4:

$$E[n_{t+1}] \theta^i u' \left(E[n_{t+1}] \frac{\theta^i n_t^i}{E[n_t]} \right) = g'(n_t^i) E[n_t] \tag{9}$$

From equation (9) one can clearly see that agents' decisions are independent of the money growth σ and hence, that money is neutral.

Question 2

The planner maximizes total welfare subject to resource constraint, this is:

$$\max_{\{c_t,n_t\}_{t=1}^{\infty}} \quad \sum_{t=1}^{\infty} U_t = \sum_{t=1}^{\infty} u(c_{t+1}) - g(n_t)$$
$$\text{s.t.} \quad c_t = A_t n_t \quad \forall t \tag{10}$$

The first order conditions of this problem are:

$$[c_t]: \qquad u'(c_t) - \lambda_t = 0$$
$$[n_t]: \qquad -g'(n_t + \lambda_t A_t = 0 \tag{11}$$

Therefore we have that the optimal allocation is given by:

$$A_t u'(A_t n_t) = g'(n_t) \equiv \xi(A_t) \tag{12}$$

The planner optimal allocation is $(n_t = \xi(A_t), c_t = A_t \xi(A_t))_{t=1}^{\infty}$.

I will assume that there is no money growth. A SREE is a pair of functions $n(A)$, $p(A)$ such that, given $p(\cdot)$, $n(A)$ solves:

$$\max_n \quad E\left[u\left(\frac{p(A)}{p(A')}An\right) \mid A\right] - g(n) \quad \forall A \tag{13}$$

and market clears:

$$An(A) = \frac{M}{p(A)} \qquad \forall A \tag{14}$$

The first order condition is:

$$E\left[\frac{p(A)}{p(A')}Au'\left(\frac{p(A)}{p(A')}An(A)\right) \mid A\right] = g'(n(A)) \tag{15}$$

From the market clearing condition, we know that

$$p(A) = \frac{M}{An(A)} \tag{16}$$

Therefore,

$$\frac{p(A)}{p(A')} = \frac{A'n(A')}{An(A)} \tag{17}$$

Combining equations 15 and 17:

$$E\left[A'n(A')u'(A'n(A') \mid A\right] = g'(n(A))n(A) \quad \forall A \tag{18}$$

Now, assume that A is iid. Then, equation 18 becomes

$$E\left[A'n(A')u'(A'n(A'))\right] = g'(n(A))n(A) \quad \forall A \tag{19}$$

There is no A in the LHS, which means that it is constant. Denote

$$E\left[A'n(A')u'(A'n(A'))\right] = k \tag{20}$$

we have that

$$g'(n(A))n(A) = k \quad \forall A \implies n(A) = \bar{n} \tag{21}$$

So, at equilibrium, we have that:

$$n(A) = \bar{n} \qquad p(A) = \frac{M}{A\bar{n}} \qquad y(A) = A\bar{n} \tag{22}$$

This is, employment is constant and independent of A, output is increasing in A (since people always work the same amount of hours, the more productive they are the more output there is) and prices are decreasing in A (since supply of money is constant, an increase in A leads to an increase in production and therefore a decrease of prices in order to make demand equal supply).

This is not equal to the planners allocation. In the planner allocation employment depends (positively) on the technology shock, as I showed at the beginning of the exercise. Recall the optimal equation of the planners problem:

$$Au'(An(A)) = g'(n(A)) \tag{23}$$

Because of the gross substitutes assumption, we know that the LHS is increasing in A. Therefore, it has to be that the RHS is also increasing in A. Since $g(\cdot)$ is strictly convex, we have that $n(A)$ is increasing in A.

Question 3

Let z denote the state at period t: $z = (M, x)$. A SREE are functions, $n(z)$, $p(z)$ and $c(z)$ such that:

- Inidivual optimization: Given $p(z)$, $n(z)$ and $s(z)$ solve the agent maximization problem:

$$\begin{aligned} \max_{n,m,c^y,c^o} \quad & u(c^y, c^o) - g(n) \\ \text{s.t.} \quad & p(z)c^y + m = p(z)n(z) \\ & p(z')c^o = mx' \end{aligned} \tag{24}$$

We can rewrite this problem as:

$$\max_{n,m} \quad u\left(n(z) - \frac{m}{p(z)}, \frac{mx'}{p(z')}\right) - g(n) \tag{25}$$

- Market clears:
  - Market for money:

$$Mx = m \tag{26}$$

  - Market for goods:

$$n(z) = c(z) + \frac{Mx}{p(z)} \quad \forall z \tag{27}$$

The first order conditions of the problem are:

$$E\left[u_1\left(c(z), \frac{mx'}{p(z')}\right)\right] = g'(n(z)) \quad \forall z \tag{28}$$

$$E\left[u_1\left(c(z), \frac{mx'}{p(z')}\right)\right] = E\left[u_2\left(c(z), \frac{mx'}{p(z')}\right)\frac{p(z)x'}{p(z')}\right] \quad \forall z \tag{29}$$

From 27:

$$p(z) = \frac{Mx}{n(z) - c(z)} \implies \frac{p(z)}{p(z')} = \frac{n(z') - c(z')}{n(z) - c(z)}\frac{1}{x'} \tag{30}$$

Plugging in 26 and 30 in 29:

$$E\left[u_1\left(c(z), n(z') - c(z')\right)\right] = E\left[u_2\left(c(z), n(z') - c(z')\right)\frac{n(z') - c(z')}{n(z) - c(z)}\right] \tag{31}$$

Let me assume for simplicity that the utility function is of the form:

$$u(c^y, c^o) = u(c^y) + u(c^o) \tag{32}$$

Then, 31 becomes:

$$u'(c(z))[n(z) - c(z)] = E\left[u'(n(z') - c(z'))[n(z') - c(z')]\right] \quad \forall z \tag{33}$$

Now, I have to find two equilibria:

1. **Money is neutral:**

   Consider the following functions:

$$n(z) = \bar{n} \qquad c(z) = \bar{c} \qquad p(z) = \frac{Mx}{\bar{n} - \bar{c}} \tag{34}$$

   and $\bar{n}$ and $\bar{c}$ are such that:

$$u'(\bar{c}) = g'(\bar{n}) \tag{35}$$

   With this functions, money would be neutral. Consumption and employment does not depend on the monetary shock. Clearly, markets would clear and the first first order condition would hold (this is so by construction). If we look at the optimal equation (33), we would have that:

$$u'(\bar{c})[\bar{n} - \bar{c}] = u'(\bar{n} - \bar{c})[\bar{n} - \bar{c}] \implies 2\bar{c} = \bar{n} \tag{36}$$

   So we have that $2\bar{c} = \bar{n}$ and $u'(\bar{c}) = g'(\bar{n})$. Two equations and two unknowns, and we would have solved for an equilibrium where money is neutral.

2. **Prices are independent of current money shock.**

   Consider:

$$n(z) = n(M)x \quad c(z) = c(M)x \tag{37}$$

   This implies, following equation 30, that:

$$p(z) = \frac{Mx}{n(z) - c(z)} = \frac{Mx}{(n(M) - c(M))x} = \frac{M}{n(M) - c(M)} \implies p(M) \tag{38}$$

   So we have that prices do not react to current monetary shocks. However, this recall that at equilibrium it has to be that:

$$u'(c(z)) = g'(n(z))\forall z \tag{39}$$

   With this functions, we would have that,

$$u'(c(M)x) = g'(n(M)x)\forall M, x \tag{40}$$

   However, recall that u is concave and g is convex. This implies that, $\forall \tilde{x} > x$:

$$u'(c(M)x) = g'(n(M)x) \implies u'(c(M)\tilde{x}) < g'(n(M)\tilde{x}) \tag{41}$$

   So an equilibrium in which prices are independent of current money shock cannot exists.

Question 4

Recall from the notes, in the Lucas Model we have the following IC and MC:

$$E\left[\frac{p_t}{p_{t+1}} x_{t+1} u'(c_{t+1})\right] = g'(n_t) \tag{42}$$

$$M_{t-1} x_t = p_t \theta_t n_t \tag{43}$$

Denote $z = x/\theta$. Gross complements implies that $cu'(c)$ is decreasing in c. Make the following guess:

$$p(M, z) = M\phi(z) \qquad n(M, z) = \varphi(z) \tag{44}$$

With these function, we arrive to the following equilibrium equation:

$$E_{\theta', z', \theta | z}\left[\frac{\theta' \varphi(z')}{\theta} u'\left(\frac{\theta' \varphi(z')}{\theta}\right)\right] = g'(\varphi(z))\varphi(z) \tag{45}$$

If shocks are iid, we will have that money is neutral in two situation:

1. z reveals x and θ completely. Then, a change in x does not affect at all the expectation of the LHS and hence $\varphi(z) = \varphi(\theta)$.
2. z does not reveal any information of x and θ. Then, again a change in x does not change nothing in the expectation, which is indeed constant. Then, $\varphi(z) = k$.

As soon as a change in x affects that expectation, by revealing information of θ or because shocks are correlated, then the labour supply will be affected by the monetary shock. The fact of gross complementarity does not change anything, only the direction in which employment reacs, but it will react as well.