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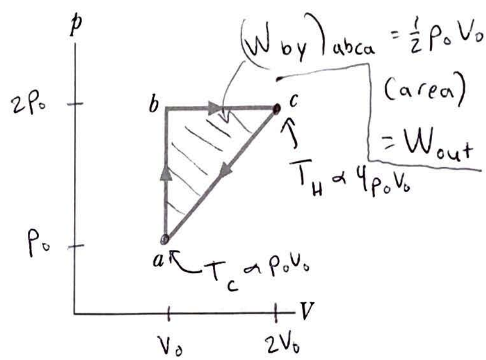
1. (15 points, 5 points each): The following graph shows a new kind of car engine. Suppose that $(p, V) = (p_0, V_0)$ at point a, and $(p, V) = (2p_0, 2V_0)$ at point c. A diatomic gas with rotational but not vibrational modes excited (throughout the entire cycle) moves from a to b to c and straight back to a as shown:

$C_V = \frac{5}{2}R$

$C_p = C_V + R = \frac{7}{2}R$



Your answers to the following should be in terms of p_0 and V_0 alone.

(a) Find the heat added to the gas for all three steps of the cycle (positive if heat is added to the gas, negative if heat leaves the gas).

(b) Find the efficiency of this engine.

(c) Find the maximum efficiency possible for any engine running between these two temperatures.

(a) $\underline{a \to b}$ $\quad Q_{ab} = nC_V\,\Delta T = \frac{5}{2}(nR\Delta T) = \frac{5}{2}\Delta(PV) = \frac{5}{2}\left[(2P_0)(V_0) - P_0V_0\right]$

$\boxed{Q_{ab} = \frac{5}{2}P_0V_0}$

$\underline{b \to c}$ $\quad Q_{bc} = nC_p\,\Delta T = \frac{7}{2}(nR\Delta T) = \frac{7}{2}\Delta(pV) = \frac{7}{2}\left[(2p_0)(2V_0) - (2p_0)V_0\right]$

$\boxed{Q_{bc} = 7P_0V_0}$

(have to use 1st law of thermo)

$\underline{c \to a}$ $\quad (\Delta E_{th})_{abca} = -(W_{by})_{abca} + Q_{ab} + Q_{bc} + Q_{ca}$

$0 = -\frac{1}{2}P_0V_0 + \frac{5}{2}P_0V_0 + 7P_0V_0 + Q_{ca}$

$\Rightarrow \boxed{Q_{ca} = -9P_0V_0}$

(b) $\eta = \frac{W_{out}}{Q_H} = \frac{(W_{by})_{abca}}{Q_{ab} + Q_{bc}} = \frac{\frac{1}{2}P_0V_0}{\frac{5}{2}P_0V_0 + 7P_0V_0}$

$\boxed{\eta = \frac{1}{19} \approx 0.05}$

(c) $\eta_{Carnot} = \frac{T_H - T_C}{T_H} = \frac{4p_0V_0 - P_0V_0}{4p_0V_0}$

$\boxed{\eta_{Carnot} = \frac{3}{4} = 0.75}$

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2. (20 points, 5 points each): A refrigerator operates via the following 3-step cycle:

- An isothermal expansion to four times its initial volume. ($A \to B$)
- An adiabatic compression that cuts the volume in half. ($B \to C$)
- An isobaric process that cuts the volume in half again, returning to the initial point. ($C \to A$)

(a) Draw this cycle on a (labelled) pV diagram. Call the initial volume and pressure V_0 and p_0. By "labelled," we mean axes, points, processes, and direction of the cycle (arrow on each process). For each point, it should be clear what the volume and pressure are in terms of p_0 and V_0.

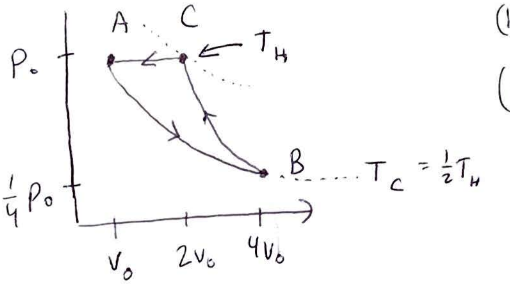
(b) Solve for the adiabatic constant for this gas. This value is actually impossible (in three dimensions). Explain why.

(c) Find the Carnot coefficient of performance for any refrigerator operating between these two temperature extremes.

(d) You don't have to solve for the actual coefficient of performance of this refrigerator[1], but explain which heat(s) from which step(s) you would use to plug into Q_C for the formula $K = Q_C/W$.

[1] Actually, you'd get a value bigger than K_{Carnot} because of the issue raised in part (b)

(a)



(b) Adiabatic $\Rightarrow$ $PV^\gamma = \text{constant}$

$$\left(\tfrac{1}{4}P_0\right)(4V_0)^\gamma = (P_0)(2V_0)^\gamma$$

$$\tfrac{1}{4}(4^\gamma) = 2^\gamma$$

$$2^\gamma = 4 \Rightarrow \boxed{\gamma = 2}$$

$\gamma = \frac{f+2}{f} \Rightarrow f = 2$ degrees of freedom

impossible because in 3D, minimum of 3 degrees of freedom so $\gamma \leq 5/3$

(c) $K_{\text{Carnot}} = \dfrac{T_C}{T_H - T_C} = \dfrac{\frac{1}{2}T_H}{T_H - \frac{1}{2}T_H} = \boxed{1}$

(d) Heat is delivered to the gas during $A \to B$ process : $\boxed{Q_{C\,(\text{cold})} = Q_{A\to B} > 0}$

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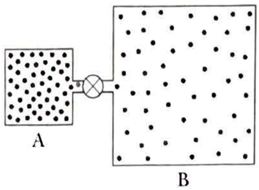
3. (15 points, 5 points each) Rigid container A is in thermal equilibrium with rigid container B. Energy in the form of heat is allowed to transfer between the two containers, but no particles are allowed to pass through. Container A is filled with Argon (Ar, 39.95 g/mol); container B is filled with Oxygen molecules that can rotate but not vibrate (O_2, and the molar mass of an Oxygen atom is 16.00 g/mol). The volume of container B is five times larger than that of container A. Both containers have the same number of molecules.



(a) If the pressure in container A is p_A, what is the pressure in container B? Your answer should be in terms of p_A alone.

(b) If the rms speed of gas molecules in container A is v_A, what is the rms speed of molecules in container B? Your answer should be in terms of v_A alone.

(c) If the total thermal energy in container A is $E_{th,A}$, what is the total thermal energy in container B? Your answer should be in terms of $E_{th,A}$ alone.

(a) $p_A V_A = nRT = p_B V_B \Rightarrow \boxed{p_B = \left(\frac{V_A}{V_B}\right) p_A = \frac{1}{5} p_A}$

(b) $(KE)_{trans} = \frac{3}{2} k_B T$ same for both $\Rightarrow \frac{3}{2} k_B T = \frac{1}{2} m_A v_A^2 = \frac{1}{2} m_B v_B^2$

$$v_B = \left(\sqrt{\frac{m_A}{m_B}}\right) v_A = \left(\sqrt{\frac{m_{Ar}}{m_{O_2}}}\right) v_A = \left(\sqrt{\frac{39.95}{32.00}}\right) v_A \simeq \boxed{1.117\, v_A}$$

(c) $E_{th} = \frac{f}{2} nRT$ $\quad E_{th,A} = \frac{3}{2} nRT$ (monoatomic) and $E_{th,B} = \frac{5}{2} nRT$ (diatomic medium temp)

$$\boxed{E_{th,B} = \frac{5}{3} E_{th,A}}$$

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4. (15 points, 5 points each) 1.00 mol of diatomic gas adiabatically expands such that it cools from 300.0 K to 270.0 K. Assume that, throughout the expansion, rotational modes are excited in the gas, but not vibrational modes. The initial volume of the gas is V_0.

$f = 5$

$C_V = \frac{5}{2}R$

$C_P = \frac{7}{2}R$

$\gamma = \frac{7}{5}$

(a) Find the amount of work that the gas does on its surroundings. You should be able to get a number (your answer should not depend on V_0).

(b) Find the final volume of the gas in terms of V_0.

(c) Suppose, instead, it was 1.00 mol of monoatomic gas that adiabatically expanded such that $T_i = 300.0$ K to $T_f = 270.0$ K (also with initial volume V_0). Does this gas do more work on its surroundings, or less? Explain using a pV diagram.

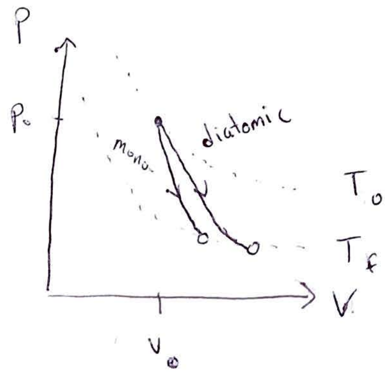
(a) $$W_{\text{by gas}} = -W_{\text{on gas}} = -\Delta E_{th} = -n C_V \Delta T = -(1.00\,\text{mol})\left(\tfrac{5}{2}R\right)(-30\,\text{K})$$

$$= \boxed{+623\text{ J}}$$

(b) $$T_0 V_0^{\gamma-1} = T_f V_f^{\gamma-1} \Rightarrow V_f = V_0\left(\frac{T_0}{T_f}\right)^{\frac{1}{\gamma-1}} = V_0\left(\frac{300.0\text{ K}}{270.0\text{ K}}\right)^{2.5}$$

$$\boxed{V_f \simeq 1.301\,V_0}$$

(c)



This monoatomic gas does $\boxed{\text{less}}$ work on its surroundings (area under the monoatomic graph is less).

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(12 points, 3 points each): 4 Multiple-choice questions / fill-in-the-blanks on various topics.

Directions for multiple-choice questions: COMPLETELY FILL IN THE SQUARE for the answer.
Directions for fill-in-the-blank questions: Your answer should be entirely in the boxed region. Include the number of significant figures ("sig. figs.") requested in the problem.

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5. Which of the following statements regarding engines and refrigerators is FALSE?

- ☐ It is impossible to construct a refrigerator that takes energy in the form of heat from a low temperature reservoir and delivers the same amount of energy to a high temperature reservoir.
  $Q_C = Q_H \Rightarrow W_{in} = 0 \Rightarrow K = \infty$
- ■ You can cool the kitchen by leaving the refrigerator door open.
- ☐ You can heat the kitchen by leaving the oven open.
- ☐ In a refrigerator, you can take out more energy in the form of heat than work that you put in.
  $K = \frac{Q_C}{W_{in}} > 1$

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6. The engine in your car takes in heat to do work. For every 100.0 kJ of energy in the form of heat that your car takes in, it does 20 kJ of work, and spits out 80.0 kJ in the form of heat. The lower heat reservoir of the engine is 300.0 K. What is the temperature of the hot reservoir, assuming your car is a perfect Carnot engine? If your car is not a perfect Carnot engine, is the hot reservoir actually warmer or colder than this?

$\eta = \frac{W_{out}}{Q_H} = \frac{20\text{ kJ}}{100\text{ kJ}} = \frac{1}{5} = \underbrace{\frac{T_H - T_C}{T_H}}_{\text{Carnot}} \Rightarrow T_H = 5T_H - 5T_C$

$T_H = \frac{5}{4} T_C = \frac{5}{4}(300\text{ K}) \simeq 380\text{ K}$

- ☐ 1500 K, warmer.
- ☐ 1500 K, colder.
- ☐ 600 K, colder.
- ☐ 380 K, colder.
- ■ 380 K, warmer.

If your engine is not ideal (Carnot), you need a bigger temp. difference ⇒ hotter reservoir

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7. The surface temperature of a star is 8530 K, and the power output of the star is 5.34×10^{26} W. Assuming the star is a perfect blackbody, what is the radius of the (spherical) star? (express your answer in meters to 3 sig figs).

$P = \sigma A T^4$

$5.34 \times 10^{26}\text{ W} = \left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2\text{K}^4}\right) \times (4\pi) R^2 (8530\text{ K})^4$

$\boxed{3.76 \times 10^{8}\text{ m}}$

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8. Two containers (A and B) initially hold 10 balls. Once a second, one of the balls is chosen at random and switched to the other container. After a long time has passed, you record the number of balls in each container every second. In 100,000 s, approximately how many times do you expect to find that all the balls were in container A?

$$\text{prob}(\text{all balls in A}) = \left(\frac{1}{2}\right)^{10} \approx \frac{1}{1024}$$

- ☐ 1,000
- ☒ 100
- ☐ 10
- ☐ 1
- ☐ 0

$$N\left(\begin{array}{c}\text{all balls in A}\\ \text{after 100k trials}\end{array}\right) = \frac{1}{1024} \times 100{,}000 \approx 100$$