

ELECTROMAGNETIC FIELDS AND WAVES

Conceptual Questions

31.1. (a) Yes, up. Andre sees the same magnetic field as the laboratory field, which points up.

(b) Andre sees an electric field $\vec{E}_A = \vec{V}_{AL} \times \vec{B}_L$, which by the right-hand rule points into the page.

31.2. (a) A magnetic force acting down and an electric force acting up. Sharon sees the same magnetic field as Bill but also sees an electric field $\vec{E}_B = \vec{V}_{BS} \times \vec{B}_S$. (Note that the magnetic and electric forces on the charge are equal and opposite so the total force on the charge is zero.)

(b) In Bill's frame of reference the charge is at rest and therefore has no magnetic force acting on it. Since there is not an electric field present in Bill's reference frame, no forces act on the charge.

31.3. (a) Negative. Your thumb will point opposite to the magnetic field.

(b) Positive. Your thumb will point in the same direction as the magnetic field.

31.4. Negative. The net current through the surface is to the right, and if you curl your fingers along the arrow around the surface, your thumb points to the left.

31.5. Decreasing. Pointing the thumb of your right hand makes your fingers curl ccw, so $\frac{d\Phi_e}{dt}$ must be negative to get a cw \vec{B} .

31.6. (a) No. The direction of \vec{v}_{em} is $\vec{E} \times \vec{B}$ which would be along the $-x$ -axis.

(b) Yes. The Pointing vector requirement is satisfied.

31.7. (a) The right-hand rule has $\vec{E} \times \vec{B}$ point out of the page.

(b) $\vec{E} \times \vec{B}$ points up.

31.8. (a) Since $I \propto E_0^2$, the new intensity is 40 W/m^2 .

(b) For an electromagnetic wave $E_0 \propto B_0$, so 40 W/m^2 .

(c) For electromagnetic waves, doubling one amplitude requires that the other is doubled, so 40 W/m^2 .

(d) Changing the frequency does not change the amplitudes and hence the intensity is unchanged at 10 W/m^2 .

31.9. A loop antenna works by taking advantage of the current produced by magnetic induction in Faraday's law as an electromagnetic wave passes through.

31.10. $I_d > I_c = I_e > I_b > I_a$. The intensity passing through a polarizer is $I_0 \cos^2 \theta$. Polarizer d is aligned with the incident wave ($\theta = 0$) while $\theta = 90^\circ$ for polarizer a . Polarizers c and e are at the same angle θ from the vertical.

Exercises and Problems

Exercises

Section 31.1 E or B? It Depends on Your Perspective

31.1. Model: Apply the Galilean transformation of fields.

Solve: (a) Equation 31.11 gives the Galilean field transformation equation for magnetic fields:

$$\vec{B}_B = \vec{B}_A - \frac{1}{c^2} \vec{v}_{BA} \times \vec{E}_A$$

\vec{B}_A is in the positive \hat{k} direction, $\vec{B}_A = B\hat{k}$. For $B_B > B_A$, $\vec{v}_{BA} \times \vec{E}_A$ must be in the negative \hat{k} direction. Since $\vec{E}_A = E\hat{j}$, \vec{v}_{BA} must be in the negative \hat{i} direction, so that $\vec{v}_{BA} \times \vec{E}_A = -(v_{BA}\hat{i}) \times (E_A\hat{j}) = -v_{BA}E_A\hat{k}$. The rocket scientist will measure $B_B > B_A$ if the rocket moves along the $-x$ -axis.

(b) For $B_B = B_A$, $\vec{v}_{BA} \times \vec{E}_A$ must be zero. The rocket scientist will measure $B_B = B_A$ if the rocket moves along either the $+y$ -axis or the $-y$ -axis.

(c) For $B_B < B_A$, $\vec{v}_{BA} \times \vec{E}_A$ must be in the positive \hat{k} direction. The rocket scientist will measure $\vec{v}_{BA} \times \vec{E}_A$, if the rocket moves along the $+x$ -axis.

31.2. Model: Apply the Galilean transformation of velocity.

Solve: (a) In the laboratory frame, the speed of the proton is

$$v = \sqrt{(1.41 \times 10^6 \text{ m/s})^2 + (1.41 \times 10^6 \text{ m/s})^2} = 2.0 \times 10^6 \text{ m/s}$$

The angle the velocity vector makes with the positive y -axis is

$$\theta = \tan^{-1} \left(\frac{1.41 \times 10^6 \text{ m/s}}{1.41 \times 10^6 \text{ m/s}} \right) = 45^\circ$$

(b) In the rocket frame, we need to first determine the vector \vec{v}' . Equation 31.1 yields:

$$\vec{v}_{PR} = \vec{v}_{PL} + \vec{v}_{LR} = (1.41 \times 10^6 \hat{i} + 1.41 \times 10^6 \hat{j}) \text{ m/s} + (-1.00 \times 10^6 \hat{i}) \text{ m/s} = (0.41 \times 10^6 \hat{i} + 1.41 \times 10^6 \hat{j}) \text{ m/s}$$

The speed of the proton is

$$v_{PL} = \sqrt{(0.41 \times 10^6 \text{ m/s})^2 + (1.41 \times 10^6 \text{ m/s})^2} = 1.47 \times 10^6 \text{ m/s}$$

The angle the velocity vector makes with the positive y -axis is

$$\theta' = \tan^{-1} \left(\frac{0.41 \times 10^6 \text{ m/s}}{1.41 \times 10^6 \text{ m/s}} \right) = 16.2^\circ$$

31.3. Model: Use the Galilean transformation of fields.

Visualize: We are given $\vec{v}_{BA} = 2.0 \times 10^6 \hat{i} \text{ m/s}$, $B_B = 1.0 \hat{j} \text{ T}$, and $\vec{E}_B = 1.0 \times 10^6 \hat{k} \text{ V/m}$.

Solve: Equation 31.10 gives the Galilean transformation equations for the electric and magnetic fields in different frames:

$$\vec{E}_B = \vec{E}_A + \vec{v}_{BA} \times \vec{B}_A \quad \vec{B}_B = \vec{B}_A - \frac{1}{c^2} \vec{v}_{BA} \times \vec{E}_A$$

The electric and magnetic fields viewed from earth are

$$\vec{E}_A = 1.0 \times 10^6 \hat{k} \text{ V/m} - (2.0 \times 10^6 \hat{i} \text{ m/s}) \times (1.0 \hat{j} \text{ T}) = -(1.0 \times 10^6 \text{ V/m}) \hat{k}$$

$$\vec{B}_A = 1.0 \hat{j} \text{ T} + \frac{1}{c^2} (2.0 \times 10^6 \hat{i} \text{ m/s}) \times (1.0 \times 10^6 \hat{k} \text{ V/m}) = 1.0 \hat{j} \text{ T} - \frac{2.0 \times 10^{12} \text{ V/m}}{(3.0 \times 10^8 \text{ m/s})^2} \hat{j} = 0.99998 \hat{j} \text{ T}$$

Assess: Although $B_A < B_V$, you need five significant figures of accuracy to tell the difference between them.

31.4. Model: Use the Galilean transformation of fields.

Solve: Equation 31.10 gives the Galilean transformation equations for the electric and magnetic fields in different frames:

$$\vec{E}_B = \vec{E}_A + \vec{v}_{BA} \times \vec{B}_A \quad \vec{B}_B = \vec{B}_A - \frac{1}{c^2} \vec{v}_{BA} \times \vec{E}_A$$

In a region of space where $\vec{B} = \vec{0}$, $\vec{E}_B = \vec{E}_A = -1.0 \times 10^6 \hat{k} \text{ V/m}$. The magnetic field is

$$\vec{B}_B = \vec{0} - \frac{1}{c^2} (1.0 \times 10^6 \hat{i} \text{ m/s}) \times (-1.0 \times 10^6 \hat{k} \text{ V/m}) = -\frac{1.0 \times 10^{12}}{(3.0 \times 10^8)^2} \hat{j} \text{ T} = -1.11 \times 10^{-5} \hat{j} \text{ T}$$

31.5. Model: Use the Galilean transformation of fields.

Visualize: We are given $\vec{v}_{BA} = 1.0 \times 10^6 \hat{i} \text{ m/s}$, $\vec{B}_A = 0.50 \hat{k} \text{ T}$, and $\vec{E}_A = \left(\frac{1}{\sqrt{2}} \hat{i} + \frac{1}{\sqrt{2}} \hat{j} \right) \times 10^6 \text{ V/m}$.

Solve: Equation 31.10 gives the Galilean transformation equation for the electric field in different frames:

$$\vec{E}_B = \vec{E}_A + \vec{v}_{BA} \times \vec{B}_A \quad \vec{B}_B = \vec{B}_A - \frac{1}{c^2} \vec{v}_{BA} \times \vec{E}_A$$

The electric field from the moving rocket is

$$\vec{E}' = (\hat{i} + \hat{j}) 0.707 \times 10^6 \text{ V/m} + (1.0 \times 10^6 \hat{i} \text{ m/s}) \times (0.50 \hat{k} \text{ T}) = (0.707 \times 10^6 \hat{i} + 0.207 \times 10^6 \hat{j}) \text{ V/m}$$

$$\theta = \tan^{-1} \left(\frac{0.207 \times 10^6 \text{ V/m}}{0.707 \times 10^6 \text{ V/m}} \right) = 16.3^\circ \text{ above the } x\text{-axis}$$

Section 31.2 The Field Laws Thus Far

Section 31.3 The Displacement Current

31.6. Model: The net magnetic flux over a closed surface is zero.

Solve: Because we can't enclose a "net pole" within a surface, $\Phi_m = \oint \vec{B} \cdot d\vec{A} = 0$. Since the magnetic field is uniform over each face of the box, the total magnetic flux around the box is

$$(1 \text{ cm} \times 2 \text{ cm})(3 \text{ T} - 2 \text{ T} - 1 \text{ T} + 3 \text{ T}) + (1 \text{ cm} \times 1 \text{ cm})(-2 \text{ T}) + \Phi_{\text{unknown}} = 0$$

$$\Rightarrow \Phi_{\text{unknown}} = \int \vec{B}_{\text{unknown}} \cdot d\vec{A} = -0.0004 \text{ T m}^2 \Rightarrow B_{\text{unknown}} \cos \theta = -4 \text{ T}$$

The angle θ must be 180° . Because θ is the angle between \vec{B} and the outward normal of $d\vec{A}$, the field \vec{B} is directed into the face and has a magnitude of 4 T.

31.7. Solve: The units of $\epsilon_0(d\Phi_e/dt)$ are

$$\frac{\text{C}^2}{\text{N m}^2} \times \frac{(\text{N/C})(\text{m}^2)}{\text{s}} = \frac{\text{C}}{\text{s}} = \text{A}$$

31.8. Solve: The displacement current is defined as $I_{\text{disp}} = \epsilon_0(d\Phi_e/dt)$. The electric flux inside a capacitor with plate area A is $\Phi_e = EA$. The electric field inside a capacitor is $E = \eta/\epsilon_0 = (Q/A)/\epsilon_0$, and thus the electric flux is

$$\Phi_e = EA = \frac{Q}{\epsilon_0} = \frac{CV_C}{\epsilon_0}$$

where V_C is the capacitor voltage. The capacitance C is constant, hence the displacement current is

$$I_{\text{disp}} = \epsilon_0 \frac{d\Phi_e}{dt} = \epsilon_0 \frac{d}{dt} \left(\frac{CV_C}{\epsilon_0} \right) = C \frac{dV_C}{dt}$$

31.9. Model: Use the results of Exercise EX31.8.

Solve:

$$I_{\text{disp}} = C \frac{dV_C}{dt} \Rightarrow C = \frac{I_{\text{disp}}}{\frac{dV_C}{dt}} = \frac{1.0 \text{ A}}{1.0 \times 10^6 \text{ V/s}} = 1.0 \mu\text{F}$$

Assess: This is a typical capacitance.

31.10. Model: The displacement current is numerically equal to the current in the wires leading to and from the capacitor.

Solve: The process of charging increases the charge on the plates of a parallel-plate capacitor. The charge Q on a capacitor plate at time t is $Q = CV_C$, where V_C is the voltage across the capacitor plates. Taking the derivative,

$$I_{\text{disp}} = I = \frac{dQ}{dt} = C \frac{dV_C}{dt} = \frac{\epsilon_0 A}{d} \frac{dV_C}{dt} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2) \pi (0.025 \text{ m})^2}{0.50 \times 10^{-3} \text{ m}} (500,000 \text{ V/s}) = 17 \mu\text{A}$$

31.11. Model: The electric field inside a parallel-plate capacitor is uniform. As the capacitor is charged, the changing electric field induces a magnetic field.

Visualize: The induced magnetic field lines are circles concentric with the capacitor. Please refer to Figure 31.17.

Solve: (a) The Ampere-Maxwell law is

$$\oint \vec{B} \cdot d\vec{s} = \epsilon_0 \mu_0 \frac{d\Phi_e}{dt} = \epsilon_0 \mu_0 A \frac{dE}{dt}$$

where EA is the electric flux through the circle of radius r . The magnetic field is everywhere tangent to the circle of radius r , so the integral of $\vec{B} \cdot d\vec{s}$ around the circle is simply $B(2\pi r)$. The Ampere-Maxwell law becomes

$$2\pi r B = \epsilon_0 \mu_0 (\pi r^2) \frac{dE}{dt} \Rightarrow B = \epsilon_0 \mu_0 \frac{r}{2} \frac{dE}{dt}$$

On the axis, $r = 0 \text{ m}$, so $B = 0 \text{ T}$.

(b) At $r = 3.0 \text{ cm}$,

$$B = \frac{1}{(3.0 \times 10^8 \text{ m/s})^2} \left(\frac{0.030 \text{ m}}{2} \right) (1.0 \times 10^6 \text{ V/m s}) = 1.67 \times 10^{-13} \text{ T}$$

(c) For $r > 5.0 \text{ cm}$, the electric flux Φ_e is the flux through a 10-cm-diameter circle because $E = 0 \text{ V/m}$ outside the capacitor plates. The Ampere-Maxwell law is

$$\begin{aligned} B(2\pi r) &= \epsilon_0 \mu_0 \pi R^2 \frac{dE}{dt} \\ \Rightarrow B &= \epsilon_0 \mu_0 \frac{R^2}{2r} \frac{dE}{dt} = \frac{1}{(3.0 \times 10^8 \text{ m/s})^2} \frac{(0.050 \text{ m})^2}{2(0.07 \text{ m})} (1.0 \times 10^6 \text{ V/m s}) = 1.98 \times 10^{-13} \text{ T} \end{aligned}$$

Section 31.5 Electromagnetic Waves**31.12. Model:** The electric and magnetic field amplitudes of an electromagnetic wave are related.**Solve:** Using Equation 34.29,

$$B_0 = \frac{E_0}{c} = \frac{10 \text{ V/m}}{3 \times 10^8 \text{ m/s}} = 3.3 \times 10^{-8} \text{ Vs/m}^2 = 3.3 \times 10^{-8} \text{ T}$$

31.13. Model: The electric and magnetic field amplitudes of an electromagnetic wave are related.**Solve:**

$$E_0 = cB_0 = (3.0 \times 10^8 \text{ m/s})(2.0 \times 10^{-3} \text{ T}) = 6.0 \times 10^5 \text{ V/m}$$

31.14. Model: Electromagnetic waves are sinusoidal.**Solve: (a)** The magnetic field is $B_z = B_0 \sin(kx - \omega t)$, where $B_0 = 3.00 \mu\text{T}$ and $k = 1.00 \times 10^7 \text{ m}^{-1}$. The wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.00 \times 10^7 \text{ m}^{-1}} = 6.28 \times 10^{-7} \text{ m} = 628 \text{ nm}$$

(b) The frequency is

$$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{6.28 \times 10^{-7} \text{ m}} = 4.77 \times 10^{14} \text{ Hz}$$

(c) The electric field amplitude is

$$E_0 = cB_0 = (3 \times 10^8 \text{ m/s})(3.00 \times 10^{-6} \text{ T}) = 900 \text{ V/m}$$

31.15. Model: Electromagnetic waves are sinusoidal.**Solve: (a)** The electric field is $E_y = E_0 \cos(kx - \omega t)$, where $E_0 = 20.0 \text{ V/m}$ and $k = 6.28 \times 10^8 \text{ m}^{-1}$. The wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{6.28 \times 10^8 \text{ m}^{-1}} = 1.00 \times 10^{-8} \text{ m} = 10.0 \text{ nm}$$

(b) The frequency is

$$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{1.00 \times 10^{-8} \text{ m}} = 3.00 \times 10^{16} \text{ Hz}$$

(c) The magnetic field amplitude is

$$B_0 = \frac{E_0}{v_{\text{em}}} = \frac{20.0 \text{ V/m}}{3.0 \times 10^8 \text{ m/s}} = 6.67 \times 10^{-8} \text{ T}$$

Section 31.6 Properties of Electromagnetic Waves**31.16. Model:** The electric and magnetic field amplitudes of an electromagnetic wave are related to each other.**Solve: (a)**

$$B_0 = \frac{1}{c} E_0 = \frac{100 \text{ V/m}}{3.0 \times 10^8 \text{ m/s}} = 3.33 \times 10^{-7} \text{ T}$$

(b) From Equation 31.40, the intensity of an electromagnetic wave is

$$I = \frac{c\epsilon_0}{2} E_0^2 = \frac{(3 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)}{2} (100 \text{ V/m})^2 = 13.3 \text{ W/m}^2$$

31.17. Model: \vec{E} and \vec{B} are perpendicular to each other and $\vec{E} \times \vec{B}$ is in the direction of \vec{v}_{em} .

Solve: At any point, the Poynting vector $\vec{S} \equiv \mu_0^{-1} \vec{E} \times \vec{B}$ points in the direction that an electromagnetic wave is traveling. We have $\vec{B} = B\hat{i}$ and $\vec{S} = -S\hat{j}$. So,

$$-S\hat{j} = \frac{1}{\mu_0} \vec{E} \times B\hat{i} \Rightarrow \vec{E} = -E\hat{k}$$

The electric field points in the negative z -direction.

31.18. Model: A radio signal is an electromagnetic wave.

Solve: From Equation 31.40, the intensity of an electromagnetic wave is

$$I = \frac{c\epsilon_0}{2} E_0^2 = \frac{(3 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)}{2} (300 \times 10^{-6} \text{ V/m})^2 = 1.195 \times 10^{-10} \text{ W/m}^2$$

31.19. Model: The laser beam is an electromagnetic plane wave.

Solve: First compute the intensity of the beam.

$$I = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{1.0 \times 10^{-3} \text{ W}}{\pi (0.00050 \text{ m})^2} = 1270 \text{ W/m}^2$$

Relate this intensity to the electric field amplitude:

$$E_0 = \sqrt{\frac{2I}{c\epsilon_0}} = \sqrt{\frac{2(1270 \text{ W/m}^2)}{(3.00 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)}} = 978 \text{ V/m} \approx 980 \text{ V/m}$$

Use the electric field amplitude to get the magnetic field amplitude:

$$B_0 = \frac{E_0}{c} = \frac{978 \text{ V/m}}{3.0 \times 10^8 \text{ m/s}} = 3.26 \text{ T} \approx 3.3 \text{ T}$$

Assess: This is a sizable electric field, comparable to the electric field near a charged glass or plastic rod.

31.20. Model: A radio wave is an electromagnetic wave.

Solve: (a) The energy transported per second by the radio wave is 25 kW, or $25 \times 10^3 \text{ J/s}$. This energy is carried uniformly in all directions. From Equation 31.40, the light intensity is

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{25 \times 10^3 \text{ W}}{4\pi (30 \times 10^3 \text{ m})^2} = 2.2 \times 10^{-6} \text{ W/m}^2$$

(b) Using Equation 31.40 again,

$$I = \frac{c\epsilon_0}{2} E_0^2 \Rightarrow 2.2 \times 10^{-6} \text{ W/m}^2 = \frac{(3 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)}{2} E_0^2 \Rightarrow E_0 = 0.041 \text{ V/m}$$

31.21. Model: The laser beam is an electromagnetic plane wave. Assume that the energy is uniformly distributed over the diameter of the laser beam.

Solve: (a) Using Equation 31.40, the light intensity is

$$I = \frac{P}{A} = \frac{c\epsilon_0}{2} E_0^2 \Rightarrow \frac{200 \times 10^6 \text{ W}}{\pi (1.0 \times 10^{-6} \text{ m})^2} = \frac{(3 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)}{2} E_0^2 \Rightarrow E_0 = 2.2 \times 10^{11} \text{ V/m}$$

(b) The electric field between the proton and the electron is

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \frac{(9 \times 10^9 \text{ N m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})}{(0.053 \times 10^{-9} \text{ m})^2} = 5.1 \times 10^{11} \text{ V/m}$$

The ratio of the laser beam's electric field to this field is

$$\frac{2.2 \times 10^{11} \text{ V/m}}{5.1 \times 10^{11} \text{ V/m}} = 0.43$$

The laser beam's electric field is approximately half the electric field that keeps the electron in its orbit.

31.22. Model: An object gains momentum when it absorbs electromagnetic waves.

Solve: The radiation force on an object that absorbs all the light is

$$F = \frac{P}{c} = \frac{1000 \text{ W}}{3.0 \times 10^8 \text{ m/s}} = 3.3 \times 10^{-6} \text{ N}$$

Assess: The force is independent of the size of the beam and the wavelength.

31.23. Solve: Substitute $E = cB$ in the equation for intensity to get it in terms of B_0 .

$$I = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{E_0^2}{2c\mu_0} = \frac{(cB_0)^2}{2c\mu_0} = \frac{cB_0^2}{2\mu_0}$$

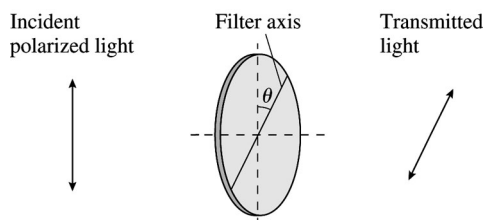
Then solve for r .

$$\begin{aligned} \frac{P}{\pi r^2} &= \frac{cB_0^2}{2\mu_0} \Rightarrow r^2 = \frac{2\mu_0 P}{\pi c B_0^2} \Rightarrow \\ r &= \sqrt{\frac{2\mu_0 P}{\pi c B_0^2}} = \sqrt{\frac{2(4\pi \times 10^{-7} \text{ T m/A})(10 \text{ W})}{\pi(3.0 \times 10^8 \text{ m/s})(1.0 \mu\text{T})^2}} = 16 \text{ cm} \end{aligned}$$

Section 31.7 Polarization

31.24. Model: Use Malus's law for the polarized light.

Visualize:



Solve: From Equation 31.45, the relationship between the incident and the transmitted polarized light is $I_{\text{transmitted}} = I_0 \cos^2 \theta$ where θ is the angle between the electric field and the axis of the filter. Therefore,

$$0.25 I_0 = I_0 \cos^2 \theta \Rightarrow \cos \theta = 0.50 \Rightarrow \theta = 60^\circ$$

Assess: Note that θ is the angle between the electric field and the axis of the filter.

31.25. Model: A polarized radio wave is an electromagnetic wave.

Solve: (a)

$$B_0 = \frac{E_0}{c} = \frac{1000 \text{ V/m}}{3.0 \times 10^8 \text{ m/s}} = 3.33 \times 10^{-6} \text{ T}$$

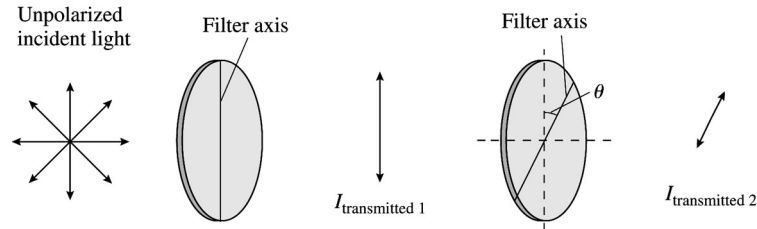
(b) Likewise,

$$B = \frac{E}{c} = \frac{500 \text{ V/m}}{3.0 \times 10^8 \text{ m/s}} = 1.67 \times 10^{-6} \text{ T}$$

The direction of the wave is into the page ($-\hat{k}$ direction) and \vec{E} is down ($-\hat{j}$ direction). Using the right-hand rule and $\vec{S} = \mu_0^{-1} \vec{E} \times \vec{B}$, \vec{B} is to the left ($-\hat{i}$ direction).

31.26. Model: Use Malus's law for polarized light.

Visualize:



Solve: For unpolarized light, the electric field vector varies randomly through all possible values of θ . Because the *average* value of $\cos^2 \theta$ is $\frac{1}{2}$, the intensity transmitted by a polarizing filter is $I_{\text{transmitted}} = \frac{1}{2} I_0$. On the other hand, for polarized light $I_{\text{transmitted}} = I_0 \cos^2 \theta$. Therefore,

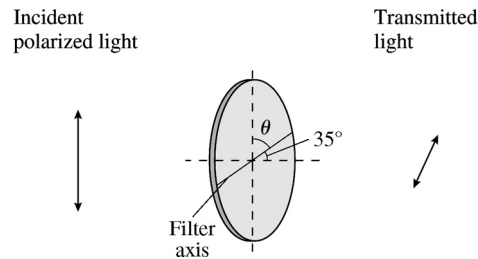
$$I_{\text{transmitted } 2} = I_{\text{transmitted } 1} \cos^2 \theta = \frac{1}{2} I_0 \cos^2 \theta = \frac{1}{2} (350 \text{ W/m}^2) \cos^2 \theta = 131 \text{ W/m}^2 \Rightarrow$$

$$\cos^2 \theta = \frac{131 \text{ W/m}^2}{\frac{1}{2} (350 \text{ W/m}^2)} \Rightarrow \theta = \cos^{-1} \sqrt{\frac{131 \text{ W/m}^2}{\frac{1}{2} (350 \text{ W/m}^2)}} = 30^\circ$$

Assess: Note that any particular wave has a clear polarization. It is only in a “sea” of waves that the resultant wave has no polarization.

31.27. Model: Use Malus's law for the polarized light.

Visualize:



Solve: We can use Equation 31.45 for transmitted power as well as intensity if the area of the beam does not change. The transmitted polarized light is

$$P_{\text{transmitted}} = P_0 \cos^2 \theta = (200 \text{ mW}) \cos^2 (90^\circ - 35^\circ) = 66 \text{ mW}$$

Assess: Note that θ is the angle between the electric field and the axis of the filter, which is $90^\circ - 35^\circ = 55^\circ$.

Problems

31.28. Model: Assume the electric and magnetic fields are uniform.

Solve: The force on the electron, which is the sum of the electric and magnetic forces, is

$$\vec{F} = \vec{F}_e + \vec{F}_m = -F \cos 30^\circ \hat{i} + F \sin 30^\circ \hat{j} = (-2.77 \hat{i} + 1.60 \hat{j}) \times 10^{-13} \text{ N}$$

Since \vec{v} points out of the page, the magnetic force is $\vec{F}_m = q\vec{v} \times \vec{B} = -1.60 \times 10^{-13} \hat{j}$ N. Thus

$$\vec{F}_e = q\vec{E} = \vec{F} - \vec{F}_m = -2.77 \times 10^{-13} \hat{i} \text{ N} + 2(1.60 \times 10^{-13} \hat{j} \text{ N}) \Rightarrow \vec{E} = \vec{F}_e / e = (-1.73 \times 10^6 \hat{i} + 2.0 \times 10^6 \hat{j}) \text{ V/m}$$

Assess: The force due to the electric field does not depend on the velocity.

31.29. Model: Electric and magnetic fields exert forces on charged particles. Assume the fields are uniform.

Visualize: The electric field is in the direction of the positive y -axis and the magnetic field is in the direction of the negative z -axis.

Solve: Substituting into the Lorentz force law,

$$\begin{aligned}\vec{F}_{\text{net}} &= q(\vec{E} + \vec{v} \times \vec{B}) = (1.6 \times 10^{-19} \text{ C})(-1.0 \times 10^6 \hat{i} \text{ V/m} + (1.0 \times 10^7 \hat{i} \text{ m/s}) \times (-0.10 \hat{k} \text{ T})) \\ &= 1.6 \times 10^{-13}(-\hat{i} + \hat{j}) \text{ N} \\ \Rightarrow F_{\text{net}} &= \sqrt{(1.6)^2 + (1.6)^2} \times 10^{-13} \text{ N} = 2.3 \times 10^{-13} \text{ N} \quad \theta = \tan^{-1} \left(\frac{1.6 \times 10^{-13} \text{ N}}{1.6 \times 10^{-13} \text{ N}} \right) = 45^\circ\end{aligned}$$

The angle θ is measured counterclockwise from the vertical.

31.30. Model: Assume that the electric and magnetic fields are uniform fields.

Visualize: The magnetic force on the positive proton by the right-hand rule is directed *upward*. So that the proton is undeflected, we must apply an electric field to cause an electric force directed downward. That is, the electric field must point *downward*.

Solve: For the proton to not deflect,

$$F_B = F_E \Rightarrow e|\vec{v} \times \vec{B}| = eE \Rightarrow E = vB \sin 90^\circ = (2.0 \times 10^7 \text{ m/s})(0.010 \text{ T}) = 2.0 \times 10^5 \text{ V/m}$$

So the strength of the electric field must be $2.0 \times 10^5 \text{ V/m}$ and the direction must be down.

31.31. Model: Use the Galilean transformation of fields. Assume that the electric and magnetic fields are uniform inside the capacitor.

Visualize: The laboratory frame is the A frame and the proton's frame is the B frame.

Solve: (a) The electric field is directed downward, and thus the electric force on the proton is downward. The magnetic field \vec{B} is oriented so that the force on the proton is directed upward. Use of the right-hand rule tells us that the magnetic field is directed into the page. The magnitude of the magnetic field is obtained from setting the magnetic force equal to the electric force, yielding the equation $evB = eE$. Solving for B ,

$$B = \frac{E}{v} = \frac{1.0 \times 10^5 \text{ V/m}}{1.0 \times 10^6 \text{ m/s}} = 0.10 \text{ T}$$

Thus $\vec{B} = (0.10 \text{ T, into page})$.

(b) In the B frame, the magnetic and electric fields are

$$\begin{aligned}\vec{B}_B &= \vec{B}_A - \frac{1}{c^2} \vec{v}_{BA} \times \vec{E}_A = -0.10 \hat{k} \text{ T} - \frac{(1.0 \times 10^6 \hat{j} \text{ m/s}) \times (1.0 \times 10^5 \hat{i} \text{ V/m})}{(3.0 \times 10^8 \text{ m/s})^2} \approx -0.10 \hat{k} \text{ T} \\ \vec{E}_B &= \vec{E}_A + \vec{v}_{BA} \times \vec{B}_A = 1.0 \times 10^5 \hat{i} \frac{\text{V}}{\text{m}} + (1.0 \times 10^6 \hat{j} \text{ m/s}) \times (-0.10 \hat{k} \text{ T}) = 0 \frac{\text{V}}{\text{m}}\end{aligned}$$

(c) There is no electric force in the proton's frame because $E_B = 0$, and there is no magnetic force because the proton is at rest in the B frame.

31.32. Model: Electric and magnetic fields exert forces on charged particles. Assume the fields are uniform.

Solve: Substituting into the Lorentz force law,

$$\begin{aligned}\vec{F}_{\text{net}} &= q(\vec{E} + \vec{v} \times \vec{B}) = (-1.6 \times 10^{-19} \text{ C}) \left[2.0 \times 10^5 (\hat{i} - \hat{j}) \frac{\text{V}}{\text{m}} + (5.0 \times 10^6 \hat{i} \text{ m/s}) \times (-0.10 \hat{k} \text{ T}) \right] \\ &= -(3.2 \times 10^{-14}) (\hat{i} - \hat{j}) \text{ N} - (8.0 \times 10^{-14} \hat{j}) \text{ N} = (-3.2 \times 10^{-14} \hat{i} - 4.8 \times 10^{-14} \hat{j}) \text{ N}\end{aligned}$$

31.33. Model: Use the Galilean transformation of fields. Assume that the wire is infinite.

Visualize: The laboratory frame is frame W (wire) and the circular loop's frame is frame L.

Solve: (a) From Equation 23.17, the electric field \vec{E} due to a wire at rest is

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{r} = \left(\frac{\lambda}{2\pi\epsilon_0 r}, \text{away from wire} \right)$$

The magnetic field \vec{B} at a point on the loop is zero because there are no moving charges. These are the fields measured in frame L.

(b) In frame L, $\vec{E}_L = \vec{E}_W + \vec{v}_{LW} \times \vec{B}_W = \vec{E}_W$ because $\vec{B}_W = 0$ T. The magnetic field in frame L is

$$\vec{B}_L = \vec{B}_W - \frac{1}{c^2} \vec{v}_{LW} \times \vec{E}_W = 0 \text{ T} - \frac{1}{c^2} \vec{v}_{LW} \times \hat{r} \frac{\lambda}{2\pi\epsilon_0 r} = \left(\frac{1}{c^2 \epsilon_0} \frac{v\lambda}{2\pi r}, \text{into the page at the top} \right)$$

(c) Consider a segment of the wire of length Δx with charge $\Delta Q = \lambda \Delta x$. The experimenter in the loop's frame sees a charge on this segment passing by him/her with a velocity v to the left. Thus,

$$I = \frac{\Delta Q}{\Delta t} = \frac{\lambda \Delta x}{\Delta t} = \lambda v$$

(d) The magnetic field in the loop's frame is due to current I :

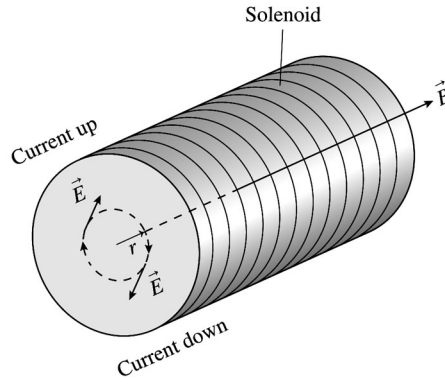
$$B_L = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 \lambda v}{2\pi r} = \frac{\mu_0 \lambda v}{2\pi r} \times \frac{\epsilon_0}{\epsilon_0} = \frac{1}{\epsilon_0 c^2} \frac{\lambda v}{2\pi r}$$

Since the current I is moving to the left, the right-hand rule states the direction of B_L is into the page on the top. The electric field \vec{E}_L which is due to the charge on the wire would be equal to \vec{E}_W .

(e) As we see, the results in parts (b) and (d) are the same.

31.34. Model: Use Faraday's law of electric induction and assume that the magnetic field inside the solenoid is uniform.

Visualize:



Equation 31.16 for Faraday's law is

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt} \left[\int \vec{B} \cdot d\vec{A} \right]$$

To solve this equation, choose a *clockwise* direction around a circle of radius r as the closed curve. The electric field vectors, as the figure shows, are everywhere tangent to the curve. The line integral of \vec{E} then is

$$\oint \vec{E} \cdot d\vec{s} = E(2\pi r)$$

To do the surface integral, we need to know the sign of the flux or the integral $\int \vec{B} \cdot d\vec{A}$. Curl your right fingers around the circle in the clockwise direction. Your thumb points to the right, which is along the same direction as the magnetic field \vec{B} . That is, $\int \vec{B} \cdot d\vec{A} = B\pi r^2$ is positive.

Solve: (a) Since $B = 10.0 \text{ T} + (2.0 \text{ T})\sin[2\pi(10 \text{ Hz})t]$, Faraday's law simplifies to

$$E(2\pi r) = -\pi r^2 \frac{dB}{dt} = -\pi r^2 (2.0 \text{ T})[2\pi(10 \text{ Hz})]\cos[2\pi(10 \text{ Hz})t] \Rightarrow E = -\left(20.0 \frac{\text{V}}{\text{m}^2}\right)\pi r \cos[2\pi(10 \text{ Hz})t]$$

The field strength is maximum when the cosine function is equal to -1 . Hence at $r = 1.5 \text{ cm}$,

$$E_{\max} = \left(20.0 \frac{\text{V}}{\text{m}^2}\right)\pi(0.015 \text{ m}) = 0.94 \text{ V/m}$$

(b) E is maximum when $\cos[2\pi(10 \text{ Hz})t] = -1$ which means when $\sin[2\pi(10 \text{ Hz})t] = 0$. Under this condition,

$$B = (10.0 \text{ T}) + (2.0 \text{ T}) \sin[2\pi(10 \text{ Hz})t] = 10.0 \text{ T}$$

That is, $B = 10.0 \text{ T}$ at the instant E has a maximum value of 0.94 V/m .

31.35. Model: Assume a uniform electric field inside the capacitor. The displacement current I_{disp} between the capacitor plates is numerically equal to the current I in the wires leading to and from the capacitor.

Solve: (a) From Equation 28.34, the voltage across the capacitor that develops during charging is $V_C = \mathcal{E}(1 - e^{-t/\tau})$, where $\tau = RC$ and $\mathcal{E} = 25 \text{ V}$ is the battery emf. Since $Q = CV_C$, we have

$$I_{\text{disp}} = I = \frac{dQ}{dt} = C \frac{dV_C}{dt} = \frac{C\mathcal{E}}{\tau} e^{-t/\tau} \Rightarrow I_{\text{disp max}} = \frac{C\mathcal{E}}{\tau} = \frac{C\mathcal{E}}{RC} = \frac{\mathcal{E}}{R} = \frac{25 \text{ V}}{150 \Omega} = 0.17 \text{ A}$$

The maximum electric flux through the capacitor plates is

$$\Phi_{\max} = E_{\max} A = \frac{V_{\max}}{d} A = \frac{\mathcal{E} C d}{d \epsilon_0} = \frac{\mathcal{E} C}{\epsilon_0} = \frac{(25 \text{ V})(2.5 \times 10^{-12} \text{ F})}{8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2} = 7.1 \text{ V m}$$

(b) As a function of time, the flux is

$$\begin{aligned} \Phi &= \frac{V_C C}{\epsilon_0} = \frac{C}{\epsilon_0} \mathcal{E}(1 - e^{-t/\tau}) = \Phi_{\max}(1 - e^{-t/\tau}) \\ \Rightarrow \Phi &= (7.1 \text{ V m}) \left[1 - e^{-(0.50 \times 10^{-9} \text{ s})/(2.5 \times 10^{-12} \text{ F} \times 150 \Omega)} \right] = 5.2 \text{ V m} \end{aligned}$$

Likewise, from part (a) the displacement current is

$$I_{\text{disp}} = \frac{C\mathcal{E}}{\tau} e^{-t/\tau} = (0.167 \text{ A}) e^{-(0.50 \times 10^{-9} \text{ s})/(2.5 \times 10^{-12} \text{ F} \times 150 \Omega)} = 0.044 \text{ A}$$

31.36. Model: Use Equation 31.20 for the definition of the displacement current.

Solve: The current in a conductor arises from the electric field E in the conductor. From Equation 27.17,

$$J = \frac{I}{A} = \sigma E \Rightarrow \frac{dI}{dt} = \frac{d}{dt}(\sigma EA) = \sigma \frac{d}{dt}(EA) = \sigma \frac{d}{dt}\Phi_e = \sigma \frac{I_{\text{disp}}}{\epsilon_0}$$

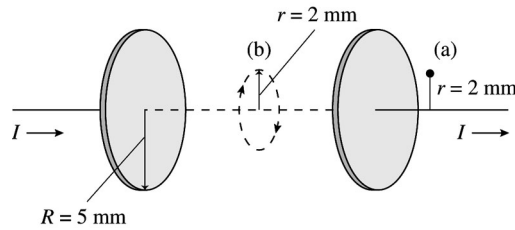
where $\Phi_e = EA$ is the electric flux through the wire and, by definition, $I_{\text{disp}} = \epsilon_0 d\Phi_e/dt$. Thus $I_{\text{disp}} = (\epsilon_0/\sigma) dI/dt$.

(b) Using the value for the conductivity of copper wire from Table 27.2,

$$I_{\text{disp}} = \frac{\epsilon_0}{\sigma} \frac{dI}{dt} = \frac{8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2}{6.0 \times 10^7 \Omega^{-1} \text{ m}^{-1}} (1.0 \times 10^6 \text{ A/s}) = 1.5 \times 10^{-13} \text{ A}$$

31.37. Model: Assume the electric field inside the capacitor is uniform and use the Ampere-Maxwell law.

Visualize:



Solve: (a) For a current-carrying wire, Example 29.3 yields an equation for the magnetic field strength:

$$B = \frac{\mu_0 I_{\text{wire}}}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{ T m/A})(10 \text{ A})}{(2\pi)(2.0 \times 10^{-3} \text{ m})} = 1.0 \times 10^{-3} \text{ T}$$

(b) Example 31.3 found that the induced magnetic field inside a charging capacitor is

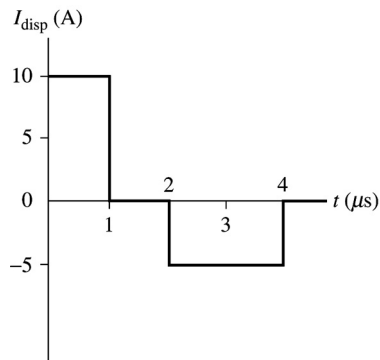
$$B = \frac{\mu_0}{2\pi} \frac{r}{R^2} \frac{dQ}{dt} = \frac{\mu_0}{2\pi} \frac{r}{R^2} I_{\text{wire}}$$

where we used $I_{\text{wire}} = dQ/dt$ as the actual current in the wire leading to the capacitor. At $r = 2.0 \text{ mm}$,

$$B = \frac{(4\pi \times 10^{-7} \text{ T m/A})(2.0 \times 10^{-3} \text{ m})}{2\pi (5.0 \times 10^{-3} \text{ m})^2} (10 \text{ A}) = 1.60 \times 10^{-4} \text{ T}$$

31.38. Model: The displacement current through a capacitor is the same as the current in the connecting wires.

Solve:



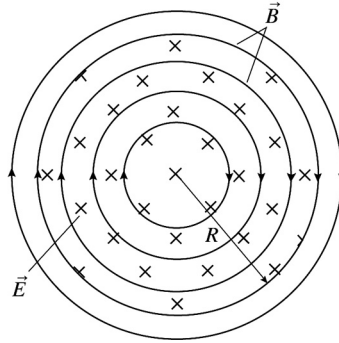
We have

$$I_{\text{disp}} = \epsilon_0 \frac{d\Phi_{\text{e}}}{dt} = \epsilon_0 \frac{d}{dt} \left(\frac{CV_c}{\epsilon_0} \right) = C \frac{dV_c}{dt}$$

The displacement current is the slope of the V_c vs. t curve times C .

31.39. Model: Assume that the electric field inside the cylinder is uniform. Use the Ampere-Maxwell law to obtain the induced magnetic field.

Visualize:



Solve: (a) The electric flux through the entire cylinder is $\Phi = \int \vec{E} \cdot d\vec{A}$. For a closed curve of radius R , assuming a clockwise direction around the ring, the right-hand rule places the thumb into the page, which is the same direction as that of the electric field \vec{E} . Thus, we will take the flux Φ to have a positive sign:

$$\Phi = E(\pi R^2) = \left(1.0 \times 10^8 t^2 \frac{\text{V}}{\text{m}}\right) [\pi (3.0 \times 10^{-3} \text{ m})^2] = (2.8 \times 10^3 t^2) \text{ Vm}$$

(b) From the Ampere-Maxwell law, with $I_{\text{through}} = 0$,

$$\oint \vec{B} \cdot d\vec{s} = \epsilon_0 \mu_0 \frac{d\Phi_e}{dt} = \epsilon_0 \mu_0 \frac{d\Phi_e}{dt}$$

Because Φ_e is positive and the field strength increases with time, $d\Phi_e/dt$ is positive. The line integral is therefore positive, which means that B is in the clockwise direction. This reasoning applies to both the $r < R$ and $r > R$ regions. Thus the field lines are clockwise, and they are shown in the figure.

(c) We can now make use of the Ampere-Maxwell law to find the magnitude of B for $r < R$. Based upon parts (a) and (b),

$$B(2\pi r) = \epsilon_0 \mu_0 \frac{dE}{dt} (\pi r^2) \Rightarrow B = \epsilon_0 \mu_0 \left(\frac{r}{2}\right) \frac{dE}{dt} = \frac{r}{2c^2} \left[\left(1.0 \times 10^8 \frac{\text{V}}{\text{m}}\right) 2t \right] = (1.11 \times 10^{-9} rt) \text{ T}$$

where r is in m and t is in s.

At $r = 2 \text{ mm}$ and $t = 2.0 \text{ s}$,

$$B = (1.11 \times 10^{-9})(2.0 \times 10^{-3})(2.0) \text{ T} = 4.4 \times 10^{-12} \text{ T}$$

(d) For $r > R$, the flux is confined to a circle of radius R . Thus

$$B(2\pi r) = \epsilon_0 \mu_0 \left(\frac{dE}{dt}\right) \pi R^2 \Rightarrow B = \frac{R^2}{2c^2 r} \left[\left(1.0 \times 10^8 \frac{\text{V}}{\text{m}}\right) 2t \right] = \left(1.00 \times 10^{-14} \frac{t}{r}\right) \text{ T}$$

where r is in m and t is in s.

At $r = 4.0 \text{ mm}$ and $t = 2.0 \text{ s}$,

$$B = \frac{1.00 \times 10^{-14} (2.0)}{(4.0 \times 10^{-3})} \text{ T} = 5.0 \times 10^{-12} \text{ T}$$

31.40. Model: There is no net magnetic flux through the ring: $\oint \vec{B} \cdot d\vec{s} = 0$.

Solve: The left side of the Ampère-Maxwell law is zero, so the right side must also be zero.

$$\begin{aligned}\mu_0 I_{\text{through}} &= -\epsilon_0 \mu_0 \frac{d\Phi_E}{dt} = -\epsilon_0 \mu_0 \frac{d(\vec{E} \cdot \vec{A})}{dt} = -\epsilon_0 \mu_0 A \frac{dE}{dt} \Rightarrow \\ \frac{dE}{dt} &= \frac{-I_{\text{through}}}{\epsilon_0 A} = \frac{-I_{\text{through}}}{\epsilon_0 \pi r^2} = \frac{-15 \text{ A}}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) \pi (1.0 \text{ cm})^2} = -5.4 \times 10^{15} \text{ N/C} \cdot \text{s}\end{aligned}$$

(a) So the magnitude is $5.4 \times 10^{15} \text{ N/C} \cdot \text{s}$.

(b) The negative sign tells us the direction of the electric field is opposite the direction of the current.

Assess: The rate of change of the electric field is large and it would be difficult to keep it up for a long time in a steady way.

31.41. Model: The displacement current through a capacitor is the same as the current in the wires.

Solve: From Chapter 29, a discharging capacitor has circuit current

$$I = I_0 e^{-t/\tau} = \frac{(\Delta V_c)_0}{R} e^{-t/\tau}$$

Identifying $\tau = 2.0 \mu\text{s} = RC$, we get $R = 2.0 \Omega$. Thus

$$I_0 = \frac{(\Delta V_c)_0}{R} \Rightarrow (\Delta V_c)_0 = I_0 R = (10 \text{ A})(2.0 \Omega) = 20 \text{ V}$$

31.42. Model: \vec{E} and \vec{B} are perpendicular in electromagnetic waves, and their magnitudes are related.

Solve: (a) Since $\vec{E} \perp \vec{B}$, the dot product must be zero.

$$\begin{aligned}\vec{E} \cdot \vec{B} &= 0 = (200)(7.3) + (300)(-7.3) + (-50)a \\ \Rightarrow a &= -14.6\end{aligned}$$

Since B_0 multiplies the complete vector \vec{B} it does not affect the calculation for a . Requiring $|\vec{E}| = c|\vec{B}|$ and squaring yields

$$\begin{aligned}((200)^2 + (300)^2 + (-50)^2)(\text{V/m})^2 &= B_0^2 (3.0 \times 10^8 \text{ m/s})^2 ((7.3)^2 + (-7.3)^2 + (-14.6)^2)(\mu\text{T})^2 \\ \Rightarrow B_0 &= 6.8 \times 10^{-2}\end{aligned}$$

(b) The Poynting vector is

$$\begin{aligned}\vec{S} &= \mu_0^{-1} \vec{E} \times \vec{B} \\ &= \mu_0^{-1} B_0 (10^{-6}) \left\{ \begin{aligned} &[(300)(-14.6) - (-7.3)(-50)]\hat{i} \\ &+ [(-50)(7.3) - (-14.6)(200)]\hat{j} \\ &+ [(200)(-7.3) - (7.3)(300)]\hat{k} \end{aligned} \right\} \\ &= \mu_0^{-1} B_0 (10^{-3}) [-4.75\hat{i} + 2.56\hat{j} - 3.65\hat{k}] = -260\hat{i} + 140\hat{j} - 200\hat{k} \text{ W/m}^2\end{aligned}$$

Assess: A quick check yields $\vec{E} \cdot \vec{S} = 0$ and $\vec{B} \cdot \vec{S} = 0$.

31.43. Solve: (a) The electric field energy density is $u_E = \frac{1}{2} \epsilon_0 E^2$ and the magnetic field energy density is

$u_B = (1/2 \mu_0) B^2$. In an electromagnetic wave, the fields are related by $E = cB$. Using this and the fact that $c^2 = 1/(\epsilon_0 \mu_0)$, we find

$$u_E = \frac{\epsilon_0}{2} E^2 = \frac{\epsilon_0}{2} (cB)^2 = \frac{c^2 \epsilon_0}{2} B^2 = \frac{\epsilon_0}{2 \epsilon_0 \mu_0} B^2 = \frac{1}{2 \mu_0} B^2 = u_B$$

(b) Since the energy density is equally divided between the electric field and the magnetic field, the total energy density in an electromagnetic wave is $u_{\text{tot}} = u_E + u_B = 2u_E$. We also know that the wave intensity is $I = \frac{1}{2}c\epsilon_0 E_0^2$.

Thus

$$u_{\text{tot}} = 2u_E = \epsilon_0 E_0^2 = \frac{2}{c} \left(\frac{1}{2} c \epsilon_0 E_0^2 \right) = \frac{2I}{c} = \frac{2(1000 \text{ W/m}^2)}{3.0 \times 10^8 \text{ m/s}} = 6.67 \times 10^{-6} \text{ J/m}^3$$

31.44. Model: Sunlight is an electromagnetic wave.

Solve: (a) The sun's energy is transported by the electromagnetic waves in all directions. From Equation 31.40, the light intensity is

$$I = \frac{P}{A} \Rightarrow P = IA = (1360 \text{ W/m}^2)(4\pi R_{\text{sun-earth}}^2) = (1360 \text{ W/m}^2)4\pi(1.50 \times 10^{11} \text{ m})^2 = 3.85 \times 10^{26} \text{ W}$$

(b) The intensity of sunlight at Mars is

$$I = \frac{P}{A} = \frac{(1360 \text{ W/m}^2)(4\pi R_{\text{earth}}^2)}{4\pi R_{\text{Mars}}^2} = (1360 \text{ W/m}^2) \left(\frac{1.50 \times 10^{11} \text{ m}}{2.28 \times 10^{11} \text{ m}} \right)^2 = 589 \text{ W/m}^2$$

31.45. Model: Light is an electromagnetic wave.

Solve: 100 W is the energy transported per second by the electromagnetic light wave. This energy is carried in all directions. The light intensity is given by Equation 31.40:

$$\begin{aligned} I &= \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{c\epsilon_0}{2} E_0^2 \\ \Rightarrow \frac{100 \text{ W}}{4\pi(3.5 \times 10^{-2} \text{ m})^2} &= \frac{(3 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)}{2} E_0^2 \Rightarrow E_0 = 2212 \frac{\text{V}}{\text{m}} \approx 2200 \frac{\text{V}}{\text{m}} \\ \Rightarrow B_0 &= \frac{E_0}{c} = \left(\frac{2212 \text{ V/m}}{3.0 \times 10^8 \text{ m/s}} \right) \approx 7.4 \times 10^{-6} \text{ T} \end{aligned}$$

31.46. Model: The wave is sinusoidal.

Visualize: $B_0 = E_0/c$.

Solve: Start with the solution of the wave equation and take the partial derivative with respect to t . The derivative is maximum when the cosine is 1.

$$E_y = E_0 \sin(2\pi(x/\lambda - ft)) \Rightarrow \frac{dE_y}{dt} = -2\pi f E_0 \cos(2\pi(x/\lambda - ft)) \Rightarrow E_0 = \frac{(dE_y/dt)_{\text{max}}}{2\pi f}$$

Put this result into the equation for B_0 .

$$B_0 = \frac{E_0}{c} = \frac{(dE_y/dt)_{\text{max}}}{2\pi f c} = \frac{4.5 \times 10^{11} \text{ (V/m)/s}}{2\pi(450 \times 10^6 \text{ Hz})(3.0 \times 10^8 \text{ m/s})} = 0.53 \mu\text{T}$$

Assess: This is a fairly small magnetic field.

31.47. Model: Radio waves are electromagnetic waves.

Solve: (a) The radio transmitter is radiating energy in all directions at the rate of 21 J per second. The signal intensity received at the earth is given by Equation 31.40:

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{21 \text{ W}}{4\pi(4.5 \times 10^{12} \text{ m})^2} = 8.3 \times 10^{-26} \text{ W/m}^2$$

(b) Using Equation 31.40 again,

$$I = \frac{c\epsilon_0}{2} E_0^2 \Rightarrow E_0 = \sqrt{\frac{2I}{c\epsilon_0}} = \sqrt{\frac{2(8.3 \times 10^{-26} \text{ W/m}^2)}{(3.0 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)}} = 7.9 \times 10^{-12} \frac{\text{V}}{\text{m}}$$

31.48. Model: Radio waves are electromagnetic waves. Assume that the transmitter unit radiates in all directions.

Solve: The transmitting unit radiates energy in all directions at the rate of 250 mJ per second. From the equation in the text, the signal intensity at a distance of 42 m is

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{250 \times 10^{-3} \text{ W}}{4\pi (42 \text{ m})^2} = 1.13 \times 10^{-5} \text{ W/m}^2$$

Using the equation for intensity,

$$I = \frac{c\epsilon_0}{2} E_0^2 \Rightarrow E_0 = \sqrt{\frac{2I}{c\epsilon_0}} = \sqrt{\frac{2(1.13 \times 10^{-5} \text{ W/m}^2)}{(3.0 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)}} = 0.092 \frac{\text{V}}{\text{m}}$$

A few steps before 42 m, the field strength was 0.100 V/m and the door opened.

Assess: 42 m seems like a typical distance for garage transmitters to fail.

31.49. Model: The laser beam is an electromagnetic wave.

Solve: The maximum intensity of the laser beam is determined by the maximum electric field strength in air. Thus the maximum power delivered by the beam is

$$\begin{aligned} P &= IA = \frac{c\epsilon_0}{2} E_0^2 A \\ &= \frac{(3 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)}{2} (3.0 \times 10^6 \text{ V/m})^2 \pi (0.050 \text{ m})^2 \\ &= 9.4 \times 10^7 \text{ W} \end{aligned}$$

31.50. Model: The laser beam is an electromagnetic plane wave.

Visualize: Power is energy/time: $P = 2.5 \text{ mJ}/10 \text{ ns} = 2.5 \times 10^5 \text{ W}$. The beam has a circular cross section with radius $r = (0.85 \text{ mm})/2 = 0.425 \text{ mm}$.

Solve:

$$\begin{aligned} I &= \frac{P}{A} = \frac{P}{\pi r^2} = \frac{E_0^2}{2c\mu_0} \Rightarrow \\ E_0 &= \sqrt{\frac{2P}{c\epsilon_0 \pi r^2}} = \sqrt{\frac{2(2.5 \times 10^5 \text{ W})}{(3.0 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2) \pi (0.425 \text{ mm})^2}} = 1.8 \times 10^7 \text{ V/m} \end{aligned}$$

Assess: This is a very large field strength that accompanies powerful laser bursts.

31.51. Model: The earth is a complete absorber of sunlight. An object gains momentum when it absorbs electromagnetic waves.

Solve: (a) The radiation pressure on an object that absorbs all the light is

$$p_{\text{rad}} = \frac{I}{c} = \frac{1360 \text{ W/m}^2}{3.0 \times 10^8 \text{ m/s}} = 4.533 \times 10^{-6} \text{ Pa}$$

Seen from the sun, the earth is a circle of radius R_{earth} and area $A = \pi R_{\text{earth}}^2$. The pressure exerts a force on this area

$$F_{\text{rad}} = p_{\text{rad}} A = p_{\text{rad}} (\pi R_{\text{earth}}^2) = (4.533 \times 10^{-6} \text{ Pa}) \pi (6.37 \times 10^6 \text{ m})^2 = 5.78 \times 10^8 \text{ N}$$

(b) The sun's gravitational force on the earth is

$$\begin{aligned} F_{\text{grav}} &= \frac{GM_{\text{sun}} M_{\text{earth}}}{R_{\text{sun-earth}}^2} = \frac{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})(5.98 \times 10^{24} \text{ kg})}{(1.50 \times 10^{11} \text{ m})^2} = 3.53 \times 10^{22} \text{ N} \\ \Rightarrow \frac{F_{\text{rad}}}{F_{\text{grav}}} &= \frac{5.78 \times 10^8 \text{ N}}{3.53 \times 10^{22} \text{ N}} = 1.64 \times 10^{-14} \end{aligned}$$

That is, F_{rad} is $1.64 \times 10^{-12} \%$ of F_{grav} .

31.52. Visualize: Use subscript 1 for the 27 MHz waves and subscript 2 for the 2.4 GHz waves. Also recall that for EM waves $c = \lambda f$.

Solve: $d \propto \lambda^{1/2}$.

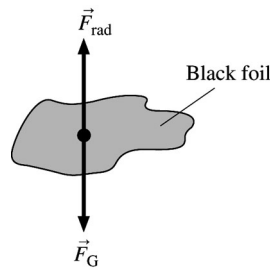
$$\frac{d_2}{d_1} = \left(\frac{\lambda_2}{\lambda_1} \right)^{1/2} = \left(\frac{c/f_2}{c/f_1} \right)^{1/2} = \sqrt{\frac{f_1}{f_2}}$$

$$d_2 = d_1 \sqrt{\frac{f_1}{f_2}} = (14 \text{ cm}) \sqrt{\frac{27 \text{ MHz}}{2.4 \text{ GHz}}} = 132 \text{ cm} \approx 1.3 \text{ m}$$

Assess: 1.3 m is farther than most of us are thick, so the 2.4 GHz waves go through people fairly well.

31.53. Model: Assume that the black foil absorbs the laser light completely. Use the particle model for the foil.

Visualize:



For the foil to levitate, the radiation-pressure force must be equal to the gravitational force on the foil.

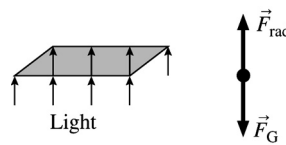
Solve: (a) Using Equation 31.42,

$$F_{\text{rad}} = p_{\text{rad}} A = \frac{I}{c} A = \frac{P}{c} \Rightarrow P = c F_{\text{rad}} = c F_G = cmg$$

(b) $(3.0 \times 10^8 \text{ m/s})(25 \times 10^{-6} \text{ kg})(9.8 \text{ m/s}^2) = 73.5 \text{ W}$

31.54. Model: Assume that the black paper absorbs the light completely. Use the particle model for the paper.

Visualize:



For the black paper to be suspended, the radiation-pressure force must be equal to the gravitational force on the paper.

Solve: From Equation 31.42, $F_{\text{rad}} = p_{\text{rad}} A = IA/c$. Hence,

$$I = \frac{c}{A} F_{\text{rad}} = \frac{c}{A} F_G = \frac{(3.0 \times 10^8 \text{ m/s})(1.0 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2)}{(8.5 \text{ inch} \times 11 \text{ inch})(2.54 \times 10^{-2} \text{ m/inch})^2} = 4.9 \times 10^7 \text{ W/m}^2$$

31.55. Model: Assume that the block absorbs the laser light completely. Use the particle model for the block.

Solve: From Equation 31.42,

$$F_{\text{rad}} = p_{\text{rad}} A = \frac{P}{c} = \frac{25 \times 10^6 \text{ W}}{3.0 \times 10^8 \text{ m/s}} = 0.0833 \text{ N}$$

Applying Newton's second law,

$$F_{\text{rad}} = ma = (100 \text{ kg})a \Rightarrow a = \frac{F_{\text{rad}}}{100 \text{ kg}} = \frac{0.0833 \text{ N}}{100 \text{ kg}} = 8.33 \times 10^{-4} \text{ m/s}^2$$

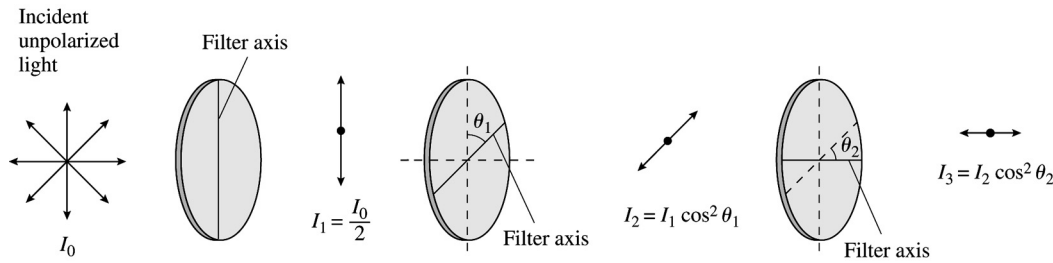
From kinematics,

$$v_f^2 = v_i^2 + 2a(s_f - s_i) = 0 \text{ m}^2/\text{s}^2 + 2(8.33 \times 10^{-4} \text{ m/s}^2)(100 \text{ m}) \Rightarrow v_f = 0.408 \text{ m/s} \approx 0.41 \text{ m/s}$$

Assess: This does not seem like a promising method for launching satellites.

31.56. Model: Use Malus's law for the polarized light.

Visualize:



Solve: For unpolarized light, the electric field vector varies randomly through all possible values of θ . Because the average value of $\cos^2 \theta$ is $\frac{1}{2}$, the intensity transmitted by a polarizing filter when the incident light is unpolarized is

$I_1 = \frac{1}{2} I_0$. For polarized light, $I_{\text{transmitted}} = I_0 \cos^2 \theta$. Therefore,

$$\begin{aligned} I_2 &= I_1 \cos^2 45^\circ & I_3 &= I_2 \cos^2 45^\circ \\ \Rightarrow I_3 &= (I_1 \cos^2 45^\circ) \cos^2 45^\circ = \frac{1}{2} I_0 (\cos^4 45^\circ) = \frac{1}{8} I_0 \end{aligned}$$

31.57. Model: The light is initially unpolarized (or randomly polarized).

Visualize: The first polarizer lets through exactly $\frac{1}{2}$ of the light: $(I_{\text{transmitted}})_1 = \frac{1}{2} I_0$.

Solve: The second polarizer lets through an amount determined by the angle between the two polarizers.

$$(I_{\text{transmitted}})_2 = I_0/10 = (I_{\text{transmitted}})_1 \cos^2 \theta = \frac{1}{2} I_0 \cos^2 \theta \Rightarrow \cos^2 \theta = \frac{I_0/10}{I_0/2} \Rightarrow \theta = \cos^{-1} \frac{1}{\sqrt{5}} = 63^\circ$$

Assess: This is a reasonable answer.

31.58. Model: Use Malus's law for the polarized light.

Solve: For unpolarized light, the electric field vector varies randomly through all possible values of θ . Because the average value of $\cos^2 \theta$ is $\frac{1}{2}$, the intensity transmitted by the first polarizing filter is $I_1 = \frac{1}{2} I_0$. For polarized light,

$I_{\text{transmitted}} = I_0 \cos^2 \theta$. For the second filter the transmitted intensity is $I_2 = I_1 \cos^2 \theta = \frac{1}{2} I_0 \cos^2 \theta$. Similarly, $I_3 = I_2 \cos^2 \theta = \frac{1}{2} I_0 (\cos^2 \theta)^2$, and so on. Thus,

$$I_7 = I_1 (\cos^2 \theta)^{7-1} = \frac{1}{2} I_0 (\cos^2 \theta)^6 = \frac{1}{2} I_0 (\cos^2 15^\circ)^6 = 0.33 I_0$$

Challenge Problems

31.59. Model: The microwave beam is an electromagnetic wave. The water does not lose heat during the process.

Solve: The rate of energy transfer from the beam to the cube is

$$\begin{aligned} P &= (0.80)IA = (0.80)\frac{C\epsilon_0}{2}E_0^2 A \\ &= (0.80)\frac{(3 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2)}{2}(11 \times 10^3 \text{ V/m})^2(0.10 \text{ m})^2 = 1.29 \text{ kW} \end{aligned}$$

The amount of energy required to raise the temperature by 50 °C is

$$\Delta E = mc\Delta T = (0.10 \text{ m})^3(1000 \text{ kg/m}^3)(4186 \text{ J/kg/}^\circ\text{C})(50 \text{ }^\circ\text{C}) = 2.09 \times 10^5 \text{ J}$$

The time required for the water to absorb this much energy from the microwave beam is

$$\Delta t = \frac{\Delta E}{P} = \frac{2.09 \times 10^5 \text{ J}}{1.29 \times 10^3 \text{ W}} = 162 \text{ s} \approx 160 \text{ s}$$

Assess: Raising 1 kg of water by 50°C in a microwave oven takes around 2–3 minutes, so this is reasonable.

31.60. Model: Use the particle model for the astronaut.

Solve: According to Newton's third law, the force of the radiation on the astronaut is equal to the momentum delivered by the radiation. For this force we have

$$F = p_{\text{rad}}A = \frac{P}{c} = \frac{1000 \text{ W}}{3.0 \times 10^8 \text{ m/s}} = 3.333 \times 10^{-6} \text{ N}$$

Using Newton's second law, the acceleration of the astronaut is

$$a = \frac{3.333 \times 10^{-6} \text{ N}}{80 \text{ kg}} = 4.167 \times 10^{-8} \text{ m/s}^2$$

Using $v_f = v_i + a(t_f - t_i)$ and a time equal to the lifetime of the batteries,

$$v_f = 0 \text{ m/s} + (4.167 \times 10^{-8} \text{ m/s}^2)(3600 \text{ s}) = 1.500 \times 10^{-4} \text{ m/s}$$

The distance traveled in the first hour is calculated as follows:

$$\begin{aligned} v_f^2 - v_i^2 &= 2a(\Delta s)_{\text{first hour}} \\ \Rightarrow (1.500 \times 10^{-4} \text{ m/s})^2 - (0 \text{ m/s})^2 &= 2(4.167 \times 10^{-8} \text{ m/s}^2)(\Delta s)_{\text{first hour}} \Rightarrow (\Delta s)_{\text{first hour}} = 0.270 \text{ m} \end{aligned}$$

This means the astronaut must cover a distance of $5.0 \text{ m} - 0.27 \text{ m} = 4.73 \text{ m}$ in a time of 9 hours. The acceleration is zero during this time. The time it will take the astronaut to reach the space capsule is

$$\Delta t = \frac{4.73 \text{ m}}{1.500 \times 10^{-4} \text{ m/s}} = 31,533 \text{ s} = 8.76 \text{ hours} \approx 8.8 \text{ h}$$

Because this time is less than 9 hours, the astronaut is able to make it safely to the space capsule.

31.61. Model: Assume that the electric and magnetic fields are uniform fields.

Solve: Substituting into the Lorentz force law,

$$\begin{aligned} \vec{F} &= q(\vec{E} + \vec{v} \times \vec{B}) \Rightarrow (9.6 \times 10^{-14} \text{ N})(\hat{i} - \hat{k}) = -(1.60 \times 10^{-19} \text{ C})[\vec{E} + (5.0 \times 10^6 \hat{i} \text{ m/s}) \times (0.10 \hat{j} \text{ T})] \\ \Rightarrow \vec{E} &= -(6.0 \times 10^5 \text{ N/C})(\hat{i} - \hat{k}) - (5.0 \times 10^5 \text{ N/C})\hat{k} = (-6.0 \times 10^5 \hat{i} + 1.0 \times 10^5 \hat{k}) \text{ V/m} \end{aligned}$$

31.62. Model: The radar beam is an electromagnetic wave. There is no absorption of the beam by the atmosphere.

Solve: The intensity at the airplane is

$$I = \frac{P_{\text{source}}}{4\pi r^2} = \frac{(150 \times 10^3 \text{ W})}{4\pi(30 \times 10^3 \text{ m})^2} = 1.33 \times 10^{-5} \text{ W/m}^2$$

The airplane then acts as a source of microwaves with power

$$P = IA = (1.33 \times 10^{-5} \text{ W/m}^2)(31 \text{ m}^2) = 4.11 \times 10^{-4} \text{ W}$$

The intensity of the reflected radar beam at the airport is thus

$$\begin{aligned} I_{\text{airport}} &= \frac{P}{4\pi r^2} = \frac{c\epsilon_0}{2} E_0^2 \Rightarrow E_0 = \sqrt{\frac{2P}{4\pi\epsilon_0 cr^2}} \\ &= \sqrt{\frac{2(4.11 \times 10^{-4} \text{ W})}{4\pi(8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2)(3 \times 10^8 \text{ m/s})(30 \times 10^3 \text{ m})^2}} \\ &= 5.2 \mu\text{V/m} \end{aligned}$$

31.63. Model: Dust particles absorb sunlight completely.

Solve: Let R be the radius of a dust grain and ρ its density. The area of cross section of the dust particles is πR^2 , its mass is $m = \rho\left(\frac{4\pi}{3}R^3\right)$. Letting d be the distance of the dust grain from the sun, the gravitational force on the dust grain is

$$F_G = \frac{GM_S m}{d^2}$$

On the other hand, the pressure on the grain due to the sun's electromagnetic radiation is

$$p_{\text{rad}} = \frac{I}{c} = \frac{P_{\text{sun}}}{(4\pi d^2)c} = \frac{F_{\text{rad}}}{A} = \frac{F_{\text{rad}}}{\pi R^2} \Rightarrow F_{\text{rad}} = \frac{R^2}{4d^2} \frac{P_{\text{sun}}}{c}$$

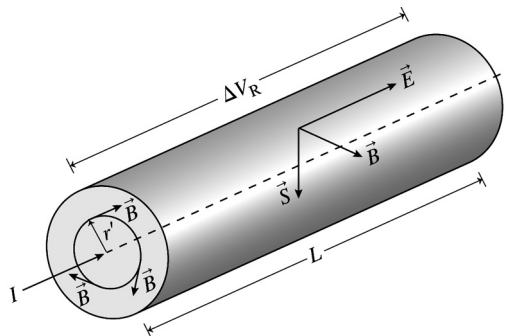
For the dust particles to remain in the solar system over long periods of time, $F_{\text{rad}} < F_G$. Hence,

$$\begin{aligned} \frac{R^2}{4d^2} \frac{P_{\text{sun}}}{c} &< \frac{GM_S m}{d^2} = \frac{GM_S \rho \frac{4\pi R^3}{3}}{d^2} \Rightarrow R > \frac{3 P_{\text{sun}}}{16\pi c GM_S \rho} \\ \Rightarrow R &> \frac{3(3.9 \times 10^{26} \text{ W})}{16\pi(3.0 \times 10^8 \text{ m/s})(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})(2000 \text{ kg/m}^3)} = 2.9 \times 10^{-7} \text{ m} \\ &\Rightarrow \text{diameter } D = 2R < 0.58 \mu\text{m} \end{aligned}$$

For $D > 0.58 \mu\text{m}$, the radiation pressure force is less than the sun's gravitational force, so the particle can orbit the sun. However, for $D < 0.58 \mu\text{m}$, radiation pressure force is greater than the sun's gravitational force, so the particle is gradually pushed out of the solar system. Thus, the diameter of the dust particle that can remain in the solar system for a very long period of time is $0.58 \mu\text{m}$.

31.64. Model: Use Ampere's law and assume that the current through the resistor is uniform.

Visualize:



Solve: (a) The electric field E in the resistor is

$$E = \frac{\Delta V_R}{L} = \frac{IR}{L} = E_{\text{surface}}$$

For the magnetic field, we use the Ampere-Maxwell law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} = \mu_0 I \Rightarrow B(2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r} = B_{\text{surface}}$$

(b) The Poynting vector $\vec{S} = \mu_0^{-1} \vec{E} \times \vec{B}$ is directed into the curved surface. That is, into the wire. The magnitude is

$$S = \frac{1}{\mu_0} EB = \frac{1}{\mu_0} \frac{IR}{L} \frac{\mu_0 I}{2\pi r} = \frac{I^2 R}{2\pi r L}$$

(c) The flux over the surface of the resistor is

$$\int \vec{S} \cdot d\vec{A} = \int_{\text{faces}} \vec{S} \cdot d\vec{A} + \int_{\text{wall}} \vec{S} \cdot d\vec{A} = 0 - SA_{\text{wall}} = \frac{I^2 R}{2\pi r L} (2\pi r L) = -I^2 R$$

The Poynting vector is the power per unit area carried by electric and magnetic fields. Thus the Poynting flux (SA_{wall}) is electromagnetic power directed into the resistor. It matches the power dissipated by the resistor ($I^2 R$), which is what we expect from energy conservation.