

Name:

PID:

1. (10 points, 5 points each): The magnetic field of a linearly-polarized EM wave is described by the following equation:

$$\vec{B}(y,t) = (1 \mu\text{T}) \cos(ky + \omega t) \hat{z}$$

The wavelength of the wave is 540 nm (green light).

- (a) What is the frequency of this wave, in Hertz?
(b) Find the amplitude of the electric field. Also, find the direction of the electric field at $x = 0$ and $t = 0$.

a) $f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{540 \times 10^{-9} \text{ m}} = \boxed{5.6 \times 10^{14} \text{ Hz}}$

b) $\vec{B}(y,t) = B_0 \cos(ky + \omega t) \hat{z}$

$$E_0 = cB_0 = (3 \times 10^8 \text{ m/s})(10^{-6} \text{ T}) = \boxed{300 \text{ V/m}}$$

$$\vec{E}(0,0) = E_0$$

from $ky + \omega t$ we see wave is travelling in $-\hat{y}$ direction

$\vec{E} \times \vec{B}$ points in the direction of the wave ($-\hat{y}$)

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ E_x & E_y & E_z \\ 0 & 0 & B_0 \end{vmatrix} = E_y B_0 \hat{x} - E_x B_0 \hat{y} + 0 \hat{z}$$

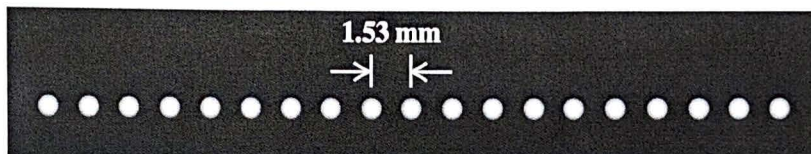
clearly $E_y = 0$
 $E_x = E_0$ gives us a vector in $-\hat{y}$ direction

$$\boxed{+\hat{x}}$$

Name:

PID:

2. (15 points, 5 points each): A double-slit experiment yields the pattern of closely spaced bright and dark fringes shown below. Only the central portion of the pattern is shown in the figure. The bright spots are equally spaced at 1.53 mm center to center (except for the missing spots) on a screen 2.50 m from the slits. The light source was a He-Ne laser producing a wavelength of 632.8 nm.



- (a) How far apart are the two slits?
 (b) The path-length differences of light emerging from the two slits for two consecutive bright spots are $(\Delta r)_m$ and $(\Delta r)_{m+1}$. What is $|(\Delta r)_{m+1} - (\Delta r)_m|$?
 (c) Describe three different changes we could make to the setup so that the bright spots appear closer together. Explain why the bright spots become closer for each change.

a)

$$\tan \theta \approx \theta = \frac{\Delta y}{L}$$

$$\theta = \frac{\lambda}{d} \quad \text{from double-slit formula}$$

$$d = \frac{\lambda L}{\Delta y} = \frac{(632.8 \times 10^{-9} \text{ m})(2.5)}{(1.53 \times 10^{-3})} = \boxed{1.03 \text{ mm}}$$

b) Two consecutive bright spots have

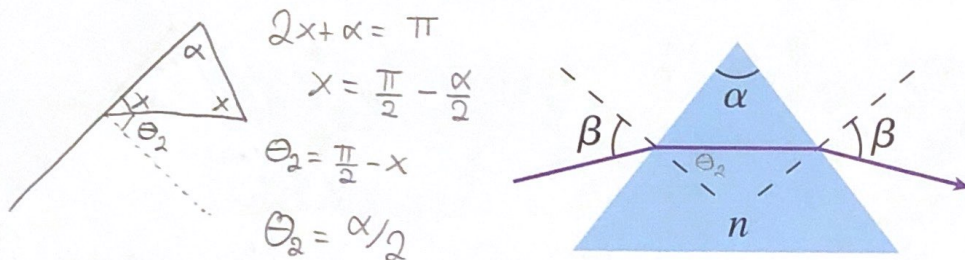
$$|(\Delta r)_m - (\Delta r)_{m+1}| = \lambda \quad \text{since } (\Delta r)_m = m\lambda$$

- c) $\Delta y = \frac{\lambda L}{d}$
- 1) decrease λ or increase f of laser
 - 2) Move screen closer to slits
 - 3) Submerge apparatus in material $n > 1$ (e.g. water)
 - 4) ~~decrease~~ increase d (slit spacing)

Name:

PID:

3. (10 points, 5 points each): There's one angle of incidence β onto a prism for which the light inside an isosceles prism (with apex angle α) travels parallel to the base and emerges at angle β . Assume the index of refraction of the glass prism is n , and the prism is in air with index of refraction $n_{\text{air}} = 1$.



- (a) Find an expression for β in terms of the prism's apex angle α and index of refraction n .
 (b) We measure $\beta = 52.2^\circ$ for a prism shaped like an equilateral triangle. What is the prism's index of refraction n ?

a) $\sin(\beta) = n \sin(\alpha/2)$

$$\beta = \sin^{-1}[n \sin(\alpha/2)]$$

b) $\alpha = 60^\circ$

$$n = \frac{\sin(52.2)}{\sin(30)} = 1.58$$

Name:

PID:

(12 points, 3 points each): 4 Multiple-choice questions / fill-in-the-blanks on various topics.

Directions for multiple-choice questions: COMPLETELY FILL IN THE SQUARE for the answer.

Directions for fill-in-the-blank questions: Your answer should be entirely in the boxed region. Include the number of significant figures ("sig. figs.") requested in the problem.

4. Unpolarized light with electric field amplitude E_0 passes first through a polarizer with its polarization axis vertical, then through one with its axis 35° to the vertical. What is the electric field amplitude after the second polarizer?

- ☐ $0.34E_0$
☒ $0.58E_0$
☐ $0.67E_0$
☐ $0.75E_0$
☐ $0.82E_0$

$$I_{\text{trans}} = I \cos^2 \theta$$

$$I_1 = \frac{1}{2} \epsilon_0 E_0^2$$

$$I_2 = \frac{I_1}{2}$$

$$I_3 = I_2 \cos^2(35^\circ)$$

$$I_3 = 0.336 I_1$$

$$\frac{I_3}{I_1} = \frac{E_3^2}{E_0^2} = 0.336$$

$$E_3 = \sqrt{0.336} E_0$$

5. Which of the following is a possible equation for the magnetic field of a light wave traveling in the $(-\hat{y})$ -direction, with a wavelength in the visible part of the EM spectrum, which has its electric field parallel to the \hat{x} -axis?

- ☒ $\vec{B}(x, y, z, t) = (1.00 \text{ nT}) \cos[(1.26 \times 10^7 \text{ m}^{-1})y + (3.77 \times 10^{15} \text{ s}^{-1})t](+\hat{z})$
☐ $\vec{B}(x, y, z, t) = (1.00 \text{ nT}) \cos[(3.77 \times 10^7 \text{ m}^{-1})z - (1.26 \times 10^{15} \text{ s}^{-1})t](-\hat{y})$
☐ $\vec{B}(x, y, z, t) = (1.00 \text{ nT}) \cos[(3.77 \times 10^7 \text{ m}^{-1})y - (1.26 \times 10^{15} \text{ s}^{-1})t](+\hat{z})$
☐ $\vec{B}(x, y, z, t) = (1.00 \text{ nT}) \cos[(1.26 \times 10^7 \text{ m}^{-1})z + (3.77 \times 10^{15} \text{ s}^{-1})t](+\hat{y})$
☐ $\vec{B}(x, y, z, t) = (1.00 \text{ nT}) \cos[(1.26 \times 10^7 \text{ m}^{-1})z + (3.77 \times 10^{15} \text{ s}^{-1})t](+\hat{z})$

$$\hat{x} \times \hat{z} = -\hat{y}$$

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k}$$

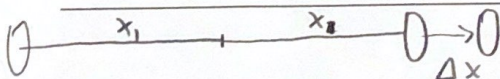
$$\lambda = 498 \text{ nm}$$

6. Two speakers are emitting sound in phase with one another, and you are standing exactly halfway between the speakers. You hear sound of wavelength 1.00 m. How far would someone need to move one of the speakers directly away from you in order for you to hear destructive interference? Give the smallest possible distance the speaker has to be moved. (2 sig. fig. answer)

$\lambda = 1 \text{ m}$

destructive $\rightarrow \Delta x = \lambda/2$

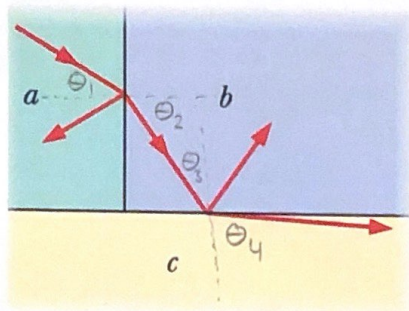
0.50 m



Name:

PID:

7. The figure below shows the paths of light that result from a light ray which starts in medium a . At each interface, both reflection and refraction occur. Which material (a , b , or c) has the smallest speed of light in the medium?



$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1} \Rightarrow \text{Smaller angle larger index}$$

- ☒ Material a
☐ Material b
☐ Material c

$$\theta_1 < \theta_2 \Rightarrow n_a > n_b > n_c$$

$$\theta_3 < \theta_4$$

$$V_i = \frac{c}{n_i} \Rightarrow V_a < V_b < V_c$$