

THE MICRO/MACRO CONNECTION

Conceptual Questions

20.1 The fact that liquids and solids can't be compressed tells us that the atoms/molecules of liquids and solids are essentially in contact with each other and can't be squeezed any closer together.

20.2 The fact that a gas is easily compressed tells us that the atoms/molecules are very far apart in comparison with the actual size of an atom or molecule. There's lots of empty space that can be squeezed.

20.3 The distance between molecules in air is about 10 times the distance between molecules in water. If all the linear dimensions of a substance are increased by a factor of 10, making the distances between molecules 10 times larger, the volume of the substance increases by a factor of $10^3 = 1000$ and its density (inversely proportional to volume) decreases by a factor of 1000.

20.4 (a) $\lambda \propto \frac{1}{(N/V)}$ and $\frac{N}{V} = \frac{p}{k_B T}$ so at constant temperature $\lambda \propto \frac{1}{p}$. If pressure is doubled, the mean-free path is halved to 100 nm.

(b) $\frac{N}{V} = \frac{p}{k_B T}$ so at constant pressure $\lambda \propto T$. If temperature is doubled, the mean-free path is doubled to 400 nm.

20.5 No, because there are such an unbelievably large number of collisions every second, and the force of each is unbelievably small, that even a sensitive gauge can't show a response to each and every collision.

20.6. (a) It would increase by a factor of 2. The average increase is a factor of 2; the square root of 2 squared is 2.

(b) The pressure would increase by a factor of $2^2 = 4$ because $p \propto V_{\text{rms}}^2$.

20.7. (a) The temperature will increase by a factor of 2^2 because $v_{\text{rms}}^2 = \frac{3k_B T}{m}$.

(b) C_V would not change because it only depends on the kind of gas, not on the temperature or speed of the molecules.

20.8. (a) Because they have been in contact a long time and have reached thermal equilibrium, the final temperatures are the same; however, $(v_{\text{rms}})_{\text{He}} > (v_{\text{rms}})_{\text{Ar}}$ because $v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}$ and $m_{\text{Ar}} > m_{\text{He}}$.

(b) The average kinetic energy *per atom* is the same in both gases, but the helium has fewer atoms so its thermal energy – the total kinetic energy of all atoms – is less than that of the argon.

- 20.9. (a)** You expect to find a cup of water slightly cooler than room temperature. Heat energy will not transfer spontaneously from a cooler system to a warmer system.
(b) It would violate the second law of thermodynamics.

Exercises and Problems

Exercises

Section 20.1 Molecular Speeds and Collisions

20.1. Solve: We can use the ideal-gas law in the form $pV = Nk_B T$ to determine the Loschmidt number (N/V):

$$\frac{N}{V} = \frac{p}{k_B T} = \frac{(1.013 \times 10^5 \text{ Pa})}{(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K})} = 2.69 \times 10^{25} \text{ m}^{-3}$$

20.2. Solve: The volume of the nitrogen gas is 1.0 m^3 and its temperature is 20°C or 293 K . The number of gas molecules can be found as

$$N = nN_A = \frac{pV}{RT} N_A = \frac{(1.013 \times 10^5 \text{ Pa})(1.0 \text{ m}^3)}{(8.31 \text{ J/mol K})(293 \text{ K})} (6.02 \times 10^{23} \text{ mol}^{-1}) = 2.5 \times 10^{25}$$

According to Figure 20.2, 12% of the molecules have a speed between 700 and 800 m/s, 7% between 800 and 900 m/s, and 3% between 900 and 1000 m/s. Thus, the number of molecules in the cube with a speed between 700 m/s and 1000 m/s is $(0.22)(2.51 \times 10^{25}) = 5.5 \times 10^{24}$.

20.3. Solve: Nitrogen is a diatomic molecule, so $r \approx 1.0 \times 10^{-10} \text{ m}$. We can use the ideal-gas law in the form $pV = Nk_B T$ and Equation 20.3 for the mean free path to obtain p :

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2} = \frac{k_B T}{4\sqrt{2}\pi p r^2} \Rightarrow p = \frac{k_B T}{4\sqrt{2}\pi \lambda r^2} = \frac{(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})}{4\sqrt{2}\pi(1.0 \text{ m})(1.0 \times 10^{-10} \text{ m})^2} = 0.023 \text{ Pa}$$

Assess: In Example 20.1 at STP for nitrogen, $\lambda = 1.0 \text{ m}$ must therefore require a very small pressure.

20.4. Solve: (a) Air primarily comprises diatomic molecules, so $r \approx 1.0 \times 10^{-10} \text{ m}$. Using the ideal-gas law in the form $pV = Nk_B T$, we get

$$\frac{N}{V} = \frac{p}{k_B T} = \frac{1.0 \times 10^{-10} \text{ mm of Hg} \times \frac{1.013 \times 10^5 \text{ Pa}}{760 \text{ mm of Hg}}}{(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})} = 3.30 \times 10^{12} \text{ m}^{-3}$$

(b) The mean free path is

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)(r^2)} = \frac{1}{4\sqrt{2}\pi(3.30 \times 10^{12} \text{ m}^{-3})(1.0 \times 10^{-10} \text{ m})^2} = 1.71 \times 10^6 \text{ m}$$

Assess: The pressure p in the vacuum chamber is $1.33 \times 10^{-8} \text{ Pa} = 1.32 \times 10^{-13} \text{ atm}$. A mean free path of $1.71 \times 10^6 \text{ m}$ is large but not unreasonable.

20.5. Solve: (a) The mean free path of a molecule in a gas at temperature T_1 , volume V_1 , and pressure p_1 is $\lambda_1 = 300 \text{ nm}$. We also know that

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2} \Rightarrow \lambda \propto V$$

Although $T_2 = 2T_1$, constant volume ($V_2 = V_1$) means that $\lambda_2 = \lambda_1 = 300 \text{ nm}$.

(b) For $T_2 = 2T_1$ and $p_2 = p_1$, the ideal gas equation gives

$$\frac{p_1 V_1}{N k_B T_1} = \frac{p_2 V_2}{N k_B T_2} = \frac{p_1 V_2}{N k_B (2T_1)} \Rightarrow V_2 = 2V_1$$

Because $\lambda \propto V$, $\lambda_2 = 2\lambda_1 = 2(300 \text{ nm}) = 600 \text{ nm}$.

20.6. Solve: The number density of the Ping-Pong balls inside the box is

$$\frac{N}{V} = \frac{2000}{1.0 \text{ m}^3} = 2000 \text{ m}^{-3}$$

With $r = (3.0 \text{ cm})/2 = 1.5 \text{ cm}$, the mean free path of the balls is

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)(r^2)} = 0.125 \text{ m} = 12.5 \text{ cm} \approx 13 \text{ cm}$$

20.7. Solve: The mean free path for a monatomic gas is

$$\lambda = \frac{1}{4\pi\sqrt{2}(N/V)r^2} \Rightarrow \frac{V}{N} = 4\pi\sqrt{2}\lambda r^2$$

For $\lambda = 20r$, meaning that the mean free path equals ten times the atomic diameter,

$$\frac{V}{N} = 4\pi\sqrt{2}(20r)(r^2) = \left(\frac{4\pi}{3}r^3\right)60\sqrt{2} \Rightarrow \frac{V/N}{\frac{4\pi}{3}r^3} = 60\sqrt{2} = 84.8$$

Section 20.2 Pressure in a Gas

20.8. Solve: (a) In tabular form we have

Particle	v_x (m/s)	v_y (m/s)	v_x^2 (m/s) ²	v_y^2 (m/s) ²	v^2 (m/s) ²	v (m/s)
1	20	-30	400	900	1300	36.06
2	40	70	1600	4900	6500	80.62
3	-80	20	6400	400	6800	82.46
4	30	0	900	0	900	30.00
5	40	-40	1600	1600	3200	56.57
6	-50	-20	2500	400	2900	53.85
Average	0	0			3600	56.59

The average velocity is $\vec{v}_{\text{avg}} = \vec{0} \hat{i} + \vec{0} \hat{j}$.

(b) The average speed is $v_{\text{avg}} = 57 \text{ m/s}$.

(c) The root-mean-square speed is $v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}} = \sqrt{3600 \text{ m}^2/\text{s}^2} = 60 \text{ m/s}$.

20.9. Solve: (a) The average speed is

$$\Delta v = 400 \text{ m/s} - (-400 \text{ m/s}) = 800 \text{ m/s}.$$

(b) The root-mean-square speed is

$$v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}} = \left(\frac{1}{11} \sum_{n=15}^{n=25} n^2 \right)^{\frac{1}{2}} \text{ m/s} = \left(\frac{4510}{11} \right)^{\frac{1}{2}} = 20.2 \text{ m/s}.$$

20.10. Solve: (a) The most probable speed is 4.0 m/s.

(b) The average speed is

$$v_{\text{avg}} = \frac{2 \times 2 \text{ m/s} + 4 \times 4 \text{ m/s} + 3 \times 6 \text{ m/s} + 1 \times 8 \text{ m/s}}{2 + 4 + 3 + 1} = 4.6 \text{ m/s}$$

(c) The root-mean-square speed is

$$v_{\text{rms}} = \sqrt{\frac{2 \times (2 \text{ m/s})^2 + 4 \times (4 \text{ m/s})^2 + 3 \times (6 \text{ m/s})^2 + 1 \times (8 \text{ m/s})^2}{2 + 4 + 3 + 1}} = 4.9 \text{ m/s}$$

20.11. Solve: (a) The atomic mass number of neon is 20. This means the mass of a neon atom is

$$m = 20 \text{ u} = 20(1.661 \times 10^{-27} \text{ kg}) = 3.322 \times 10^{-26} \text{ kg}$$

The pressure of the gas is

$$p = \frac{1}{3} \left(\frac{N}{V} \right) m v_{\text{rms}}^2 = \frac{1}{3} (5.00 \times 10^{25} \text{ m}^{-3}) (3.322 \times 10^{-26} \text{ kg}) (600 \text{ m/s})^2 = 19.93 \times 10^4 \text{ Pa}$$

The temperature of the gas in the container can be obtained from the ideal-gas equation in the form $pV = Nk_{\text{B}}T$:

$$T = \frac{pV}{Nk_{\text{B}}} = \frac{19.93 \times 10^4 \text{ Pa}}{(5.00 \times 10^{25} \text{ m}^{-3})(1.38 \times 10^{-23} \text{ J/K})} = 289 \text{ K}$$

(b) We already computed the pressure to be 200 kPa.

20.12. Model: Pressure is due to random collisions of gas molecules with the walls.

Solve: According to Equation 20.8, the collision rate with one wall is

$$\text{rate of collisions} = \frac{N_{\text{coll}}}{\Delta t_{\text{coll}}} = \frac{F_{\text{net}}}{2mv_x} = \frac{pA}{2mv_x}$$

where $F_{\text{net}} = pA$ is the force exerted on area A by the gas pressure. However, this equation assumed that all molecules are moving in the x -direction with constant speed. The rms speed v_{rms} is for motion in three dimensions at varying speeds. Consequently, we need to replace v_x not with $(v_x)_{\text{avg}}$, which is zero, but with

$$v_x \rightarrow \sqrt{(v_x^2)_{\text{avg}}} = \sqrt{\frac{v_{\text{rms}}^2}{3}} = \frac{v_{\text{rms}}}{\sqrt{3}}$$

With this change,

$$\text{rate of collisions} = \frac{\sqrt{3} pA}{2mv_{\text{rms}}} = \frac{\sqrt{3} (2 \times 101,300 \text{ Pa}) (0.10 \text{ m} \times 0.10 \text{ m})}{2(28 \times 1.661 \times 10^{-27} \text{ kg}) (576 \text{ m/s})} = 6.5 \times 10^{25} \text{ s}^{-1}$$

This collision rate can also be found by using the expression in Equation 20.10, making the same change in v_x , and using the ideal-gas law to determine N/V .

20.13. Visualize: We will use Equation 20.18 to find m of the atoms.

$$p = \frac{1}{3} \frac{N}{V} m v_{\text{rms}}^2$$

We are given $p = 2.0 \text{ atm} = 202,650 \text{ Pa}$, $N/V = 4.2 \times 10^{25} \text{ m}^{-3}$, and $v_{\text{rms}} = 660 \text{ m/s}$.

Solve: Solve for m .

$$m = \frac{3p}{v_{\text{rms}}^2} \frac{V}{N} = \frac{3(202,650 \text{ Pa})}{(660 \text{ m/s})^2} \frac{1}{4.2 \times 10^{25} \text{ m}^{-3}} = 3.32 \times 10^{-26} \text{ kg} = 20 \text{ u}$$

With an atomic mass of 20 the gas is likely neon.

Assess: Neon is gaseous at room temperature, so is a likely choice. As a noble gas, it also doesn't form many molecules.

Section 20.3 Temperature

20.14. Solve: Because the neon and argon atoms in the mixture are in thermal equilibrium, the temperature of each gas in the mixture must be the same. That is, using Equation 20.26,

$$m_{\text{Ar}} v_{\text{rms Ar}}^2 = m_{\text{Ne}} v_{\text{rms Ne}}^2$$

$$v_{\text{rms Ar}} = v_{\text{rms Ne}} \sqrt{\frac{m_{\text{Ne}}}{m_{\text{Ar}}}} = (400 \text{ m/s}) \sqrt{\frac{20 \text{ u}}{40 \text{ u}}} = 283 \text{ m/s}$$

20.15. Visualize: Equation 20.26 will give us v_{rms} from the temperature and mass of the particles.

$$v_{\text{rms}} = \sqrt{\frac{3k_{\text{B}}T}{m}}$$

We are given $T = 800^\circ\text{C} = 1073 \text{ K}$. The atomic mass of argon atoms is $40 \text{ u} = 6.642 \times 10^{-26} \text{ kg}$; the molecular mass of hydrogen molecules is $2 \text{ u} = 3.322 \times 10^{-27} \text{ kg}$.

Solve:

(a)

$$v_{\text{rms}} = \sqrt{\frac{3(1.38 \times 10^{-28} \text{ J/K})(1073 \text{ K})}{6.642 \times 10^{-26} \text{ kg}}} = 818 \text{ m/s}$$

(b)

$$v_{\text{rms}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(1073 \text{ K})}{3.322 \times 10^{-27} \text{ kg}}} = 3660 \text{ m/s}$$

Assess: These are fast speeds, but the temperature is high.

20.16. Visualize: Use Equation 20.26.

$$v_{\text{rms}} = \sqrt{\frac{3k_{\text{B}}T}{m}}$$

We are given $v_{\text{rms}} = 1.5 \text{ m/s}$ and $m = 28 \text{ u} = 4.650 \times 10^{-26} \text{ kg}$.

Solve: Solve the equation for T .

$$T = \frac{v_{\text{rms}}^2 m}{3k_{\text{B}}} = \frac{(1.5 \text{ m/s})^2 (4.650 \times 10^{-26} \text{ kg})}{3(1.38 \times 10^{-23} \text{ J/K})} = 2.5 \text{ mK}$$

Assess: 2.5 mK is close to absolute zero, but that's how cold it would have to be for v_{rms} to be 1.5 m/s.

20.17. Solve: The average translational kinetic energy per molecule is

$$\epsilon_{\text{avg}} = \frac{1}{2} m v_{\text{rms}}^2 = \frac{3}{2} k_{\text{B}} T \Rightarrow v_{\text{rms}} = \sqrt{\frac{3k_{\text{B}}T}{m}}$$

Since we want the v_{rms} for H_2 and N_2 to be equal,

$$\sqrt{\frac{3k_{\text{B}}T_{\text{H}_2}}{m_{\text{H}_2}}} = \sqrt{\frac{3k_{\text{B}}T}{m_{\text{N}_2}}} \Rightarrow T_{\text{H}_2} = \frac{m_{\text{H}_2}}{m_{\text{N}_2}} T_{\text{N}_2} = \left(\frac{2 \text{ u}}{28 \text{ u}} \right) (373 \text{ K}) = 27 \text{ K} = -246^\circ\text{C}$$

20.18. Solve: Solve Equation 20.18 for v_{rms} to see that if both p and V are halved then v_{rms} is also halved.

$$v_{\text{rms}} = \sqrt{\frac{3pV}{Nm}}$$

So the new rms speed will be 300 m/s.

Assess: Think microscopically; for the pressure to double when the volume is doubled the particles will have to be going a lot faster and hit the walls more often.

20.19. Solve: The formula for the root-means-square speed as a function of temperature is

$$(v_{\text{rms}})_T = \sqrt{\frac{3k_B T}{m}}$$

The ratio at 10°C and at 1000°C is

$$\frac{(v_{\text{rms}})_{100}}{(v_{\text{rms}})_{20}} = \sqrt{\frac{1273 \text{ K}}{283 \text{ K}}} = 2.12$$

Assess: Both the conversion to kelvin and the square root make the ratio smaller than one might have expected.

20.20. Solve: Reference to a periodic table of the elements shows that the mass of a cesium atom is $m = 133 \text{ u} = 2.22 \times 10^{-25} \text{ kg}$. At $T = 100 \text{ nK} = 1.0 \times 10^{-7} \text{ K}$ the rms speed is

$$v_{\text{rms}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(1.0 \times 10^{-7} \text{ K})}{2.22 \times 10^{-25} \text{ kg}}} = 4.32 \times 10^{-3} \text{ m/s} \approx 0.43 \text{ cm/s}$$

Assess: This is slow enough for us to “watch” the atoms moving about.

20.21. Solve: (a) The total translational kinetic energy of a gas is $K_{\text{micro}} = \frac{3}{2} N_A k_B T = \frac{3}{2} nRT$. For H_2 gas at STP,

$$K_{\text{micro}} = \frac{3}{2} (1.0 \text{ mol})(8.31 \text{ J/mol K})(273 \text{ K}) = 3400 \text{ J}$$

(b) For He gas at STP,

$$K_{\text{micro}} = \frac{3}{2} (1.0 \text{ mol})(8.31 \text{ J/mol K})(273 \text{ K}) = 3400 \text{ J}$$

(c) For O_2 gas at STP, $K_{\text{micro}} = 3400 \text{ J}$.

Assess: The translational kinetic energy of a gas depends on the temperature and the number of molecules but not on the molecule’s mass.

20.22. Solve: (a) The average translational kinetic energy per molecule is

$$K_{\text{avg}} = \frac{1}{2} m v_{\text{rms}}^2 = \frac{3}{2} k_B T$$

This means K_{avg} doubles if the temperature T doubles.

(b) The root-mean-square speed v_{rms} increases by a factor of $\sqrt{2}$ as the temperature doubles.

(c) The mean free path is

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2}$$

Because N/V and r do not depend on T , doubling temperature has no effect on λ .

20.23. Solve: (a) The mean free path is

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2}$$

where $r \approx 0.5 \times 10^{-10}$ m is the atomic radius for helium and N/V is the gas number density. From the ideal-gas law,

$$\frac{N}{V} = \frac{p}{kT} = \frac{0.10 \text{ atm} \times 101,300 \text{ Pa/atm}}{(1.38 \times 10^{-23} \text{ J/K})(10 \text{ K})} = 7.34 \times 10^{25} \text{ m}^{-3}$$

$$\Rightarrow \lambda = \frac{1}{4\sqrt{2}\pi(7.34 \times 10^{25} \text{ m}^{-3})(0.5 \times 10^{-10} \text{ m})^2} = 3.1 \times 10^{-7} \text{ m} = 310 \text{ nm}$$

(b) The root-mean-square speed is

$$v_{\text{rms}} = \sqrt{\frac{3k_{\text{B}}T}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(10 \text{ K})}{4 \times (1.661 \times 10^{-27} \text{ kg})}} = 250 \text{ m/s}$$

where we used $A = 4$ u as the atomic mass of helium.

(c) The average energy per atom is $\epsilon_{\text{avg}} = \frac{3}{2}k_{\text{B}}T = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(10 \text{ K}) = 2.1 \times 10^{-22} \text{ J}$.

20.24. Solve: (a) The average kinetic energy of a proton at the center of the sun is

$$K_{\text{avg}} = \frac{3}{2}k_{\text{B}}T \approx \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(2.0 \times 10^7 \text{ K}) = 4.1 \times 10^{-16} \text{ J}$$

(b) The root-mean-square speed of the proton is

$$v_{\text{rms}} = \sqrt{\frac{3k_{\text{B}}T}{m}} \approx \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(2.0 \times 10^7 \text{ K})}{1.67 \times 10^{-27} \text{ kg}}} = 7.0 \times 10^5 \text{ m/s}$$

20.25. Solve: (a) Since the hydrogen in the sun's atmosphere is monatomic, the average translational kinetic energy per atom is

$$K_{\text{avg}} = \frac{3}{2}k_{\text{B}}T = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(6000 \text{ K}) = 1.24 \times 10^{-19} \text{ J}$$

(b) The root-mean-square speed is

$$v_{\text{rms}} = \sqrt{\frac{3k_{\text{B}}T}{m_{\text{H}}}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(6000 \text{ K})}{1.67 \times 10^{-27} \text{ kg}}} = 1.22 \times 10^4 \text{ m/s}$$

Section 20.4 Thermal Energy and Specific Heat

20.26. Model: For a gas, the thermal energy is the total kinetic energy of the moving molecules.

Solve: Neon atoms have an atomic mass number $A = 14$, so the mass of each molecule is

$$m = 28 \text{ u} = 28(1.661 \times 10^{-27} \text{ kg}) = 4.51 \times 10^{-26} \text{ kg}$$

The number of molecules in the gas is

$$N = \frac{M}{m} = \frac{0.010 \text{ kg}}{4.51 \times 10^{-26} \text{ kg}} = 2.218 \times 10^{23}$$

The thermal energy is

$$E_{\text{th}} = NK_{\text{avg}} = N\left(\frac{1}{2}mv_{\text{avg}}^2\right) \Rightarrow v_{\text{avg}} = \sqrt{\frac{2E_{\text{th}}}{Nm}} = \sqrt{\frac{2(1700 \text{ J})}{(2.218 \times 10^{23})(4.51 \times 10^{-26} \text{ kg})}} = 580 \text{ m/s}$$

20.27. Model: For a gas, the thermal energy is the total kinetic energy of the moving molecules. That is, $E_{\text{th}} = K_{\text{micro}}$.

Solve: The number of atoms is

$$N = \frac{M}{m} = \frac{0.0020 \text{ kg}}{6.64 \times 10^{-27} \text{ kg}} = 3.01 \times 10^{23}$$

Because helium atoms have an atomic mass number $A = 4$, the mass of each helium atom is

$$m = 4 \text{ u} = 4(1.661 \times 10^{-27} \text{ kg}) = 6.64 \times 10^{-27} \text{ kg}$$

Thus the thermal energy of the gas is

$$E_{\text{th}} = \frac{3}{2} N k_{\text{B}} T = \frac{3}{2} N k_{\text{B}} \frac{m v_{\text{rms}}^2}{3 k_{\text{B}}} = N \frac{1}{2} m v_{\text{rms}}^2 = (3.01 \times 10^{23}) \left(\frac{1}{2} \right) (6.64 \times 10^{-27} \text{ kg}) (700 \text{ m/s})^2 = 490 \text{ J}$$

20.28. Solve: The volume of the air is $V = 6.0 \text{ m} \times 8.0 \text{ m} \times 3.0 \text{ m} = 144.0 \text{ m}^3$, the pressure $p = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$, and the temperature $T = 20^\circ\text{C} = 293 \text{ K}$. The number of moles of the gas is

$$n = \frac{pV}{RT} = 5991 \text{ mol}$$

This means the number of molecules is

$$N = n N_{\text{A}} = (5991 \text{ mol}) (6.022 \times 10^{23} \text{ mol}^{-1}) = 3.61 \times 10^{27} \text{ molecules}$$

Since air is a diatomic gas, the room's thermal energy is

$$E_{\text{th}} = N \varepsilon_{\text{avg}} = N \left(\frac{5}{2} k_{\text{B}} T \right) = 3.6 \times 10^7 \text{ J}$$

Assess: The room's thermal energy can also be obtained as follows:

$$E_{\text{th}} = n C_{\text{V}} T = (5991 \text{ mol}) (20.8 \text{ J/mol K}) (293 \text{ K}) = 3.6 \times 10^7 \text{ J}$$

20.29. Solve: (a) For a monatomic gas,

$$\Delta E_{\text{th}} = n C_{\text{V}} \Delta T = 1.0 \text{ J} = (1.0 \text{ mol}) (12.5 \text{ J/mol K}) \Delta T \Rightarrow \Delta T = 0.080^\circ\text{C} \text{ or } 0.080 \text{ K}$$

(b) For a diatomic gas,

$$1.0 \text{ J} = (1.0 \text{ mol}) (20.8 \text{ J/mol K}) \Delta T \Rightarrow \Delta T = 0.048^\circ\text{C} \text{ or K}$$

(c) For a solid,

$$1.0 \text{ J} = (1.0 \text{ mol}) (25.0 \text{ J/mol K}) \Delta T \Rightarrow \Delta T = 0.040^\circ\text{C} \text{ or K}$$

20.30. Solve: The thermal energy of a solid is

$$E_{\text{th}} = 3 N k_{\text{B}} T = 3 n R T$$

The volume of aluminum $V = 100 \text{ cm}^3 = 10^{-4} \text{ m}^3$, which means the mass is

$$M = \rho V = (2700 \text{ kg/m}^3) (10^{-4} \text{ m}^3) = 0.27 \text{ kg}$$

Because the atomic mass number of Al is 27, the number of moles is

$$n = \frac{M}{M_{\text{mol}}} = \frac{0.27 \text{ kg}}{0.027 \text{ kg/mol}} = 10 \text{ mol}$$

$$\Rightarrow E_{\text{th}} = 3(10 \text{ mols})(8.31 \text{ J/mol K})(373 \text{ K}) = 93 \text{ kJ}$$

20.31. Solve: The conservation of energy equation $(\Delta E_{\text{th}})_{\text{gas}} + (\Delta E_{\text{th}})_{\text{solid}} = 0 \text{ J}$ is

$$n_{\text{gas}} (C_{\text{V}})_{\text{gas}} (T_{\text{f}} - T)_{\text{gas}} + n_{\text{solid}} (C_{\text{V}})_{\text{solid}} (T_{\text{f}} - T_{\text{i}})_{\text{solid}} = 0 \text{ J}$$

$$\Rightarrow (1.0 \text{ mol}) (12.5 \text{ J/mol K}) (-50 \text{ K}) + (1.0 \text{ mol}) (25.0 \text{ J/mol K}) (\Delta T)_{\text{solid}} = 0 \Rightarrow (\Delta T)_{\text{solid}} = 25^\circ\text{C}$$

The temperature of the solid increases by 25°C .

20.32. Solve: (a) Nitrogen is a diatomic gas. The thermal energy of the gas at $T = 20^\circ\text{C} = 293\text{ K}$ is

$$\begin{aligned} E_{\text{th}} &= \frac{5}{2} nRT = \left(\frac{pV}{RT} \right) \left(\frac{5}{2} R \right) T \\ &= \frac{(1.013 \times 10^7 \text{ Pa})(15,000 \times 10^{-6} \text{ m}^3)}{(8.31 \text{ J/mol K})(293 \text{ K})} (20.8 \text{ J/mol K})(293 \text{ K}) = 3.80 \times 10^5 \text{ J} \end{aligned}$$

(b) The number density of the gas is

$$\frac{N}{V} = \frac{p}{k_B T} = \frac{(1.013 \times 10^7 \text{ Pa})}{(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})} = 2.51 \times 10^{27} \text{ m}^{-3}$$

Since nitrogen is a diatomic molecule, the mean free path is

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2} = \frac{1}{4\sqrt{2}\pi(2.51 \times 10^{27} \text{ m}^{-3})(1.0 \times 10^{-10} \text{ m})^2} = 2.25 \times 10^{-9} \text{ m}$$

(c) For an isothermal process, $\Delta T = 0\text{ K}$. Thus $\Delta E_{\text{th}} = 0\text{ J}$.

20.33. Visualize: At low temperatures, $C_v = \frac{3}{2}R = 12.5\text{ J/mol K}$. At room temperature and modestly hot temperatures,

$C_v = \frac{5}{2}R = 20.8\text{ J/mol K}$. At very hot temperatures, $C_v = \frac{7}{2}R = 29.1\text{ J/mol K}$.

Solve: (a) The number of moles of diatomic hydrogen gas in the rigid container is

$$\frac{0.20 \text{ g}}{2 \text{ g/mol}} = 0.10 \text{ mol}$$

The heat needed to change the temperature of the gas from 50 K to 100 K at constant volume is

$$Q = \Delta E_{\text{th}} = nC_v\Delta T = (0.10 \text{ mol})(12.5 \text{ J/mol K})(100 \text{ K} - 50 \text{ K}) = 62 \text{ J}$$

(b) To raise the temperature from 250 K to 300 K,

$$Q = \Delta E_{\text{th}} = (0.10 \text{ mol})(20.8 \text{ J/mol K})(300 \text{ K} - 250 \text{ K}) = 100 \text{ J}$$

(c) To raise the temperature from 2250 K to 2300 K, $Q = \Delta E_{\text{th}} = nC_v\Delta T = (0.10 \text{ mol})(29.1 \text{ J/mol K})(50 \text{ K}) = 150 \text{ J}$

Section 20.5 Thermal Interactions and Heat

20.34. Solve: (a) The thermal energy of a monatomic gas is

$$\begin{aligned} E_{\text{th}} &= \frac{3}{2} Nk_B T = \frac{3}{2} nRT \Rightarrow T = \frac{2}{3} \frac{E_{\text{th}}}{n} \frac{1}{R} \\ \Rightarrow T_A &= \left(\frac{2}{3} \right) \left(\frac{5000 \text{ J}}{2.0 \text{ mol}} \right) \frac{1}{(8.31 \text{ J/mol K})} = 201 \text{ K} \\ T_B &= \left(\frac{2}{3} \right) \left(\frac{8000 \text{ J}}{3.0 \text{ mol}} \right) \frac{1}{(8.31 \text{ J/mol K})} = 214 \text{ K} \end{aligned}$$

Thus, gas B has the higher initial temperature.

(b) The equilibrium condition is $(\epsilon_A)_{\text{avg}} = (\epsilon_B)_{\text{avg}} = (\epsilon_{\text{tot}})_{\text{avg}}$. This means

$$\begin{aligned} \frac{E_{\text{Af}}}{n_A} &= \frac{E_{\text{Bf}}}{n_B} = \frac{E_{\text{tot}}}{n_A + n_B} \\ \Rightarrow E_{\text{Af}} &= \frac{n_A}{n_A + n_B} E_{\text{tot}} = \left(\frac{2 \text{ mols}}{2 \text{ mols} + 3 \text{ mols}} \right) (5000 \text{ J} + 8000 \text{ J}) = 5200 \text{ J} \\ E_{\text{Bf}} &= \frac{n_B}{n_A + n_B} E_{\text{tot}} = \left(\frac{3 \text{ mols}}{5 \text{ mols}} \right) (13,000 \text{ J}) = 7800 \text{ J} \end{aligned}$$

20.35. Visualize: Because gas A starts with 9000 J of energy and transfers 1000 J to gas B then $E_{Af} = 8000\text{J}$. We are also given $n_A = 4.0\text{ mol}$ and $n_B = 3.0\text{ mol}$.

Solve:

$$E_{Af} = \frac{n_A}{n_A + n_B} E_{\text{tot}} \Rightarrow E_{\text{tot}} = \frac{n_A + n_B}{n_A} E_{Af}$$

The final total energy is the same as the initial total energy, so

$$E_{Bi} = E_{\text{tot}} - E_{Ai} = \frac{n_A + n_B}{n_A} E_{Af} - E_{Ai} = \left(\frac{4.0\text{ mol} + 3.0\text{ mol}}{4.0\text{ mol}} \right) (8000\text{ J}) - 9000\text{ J} = 5000\text{ J}$$

Assess: We can also compute $E_{Bf} = 6000\text{ J}$ showing E_{tot} is the same before and after.

Section 20.6 Irreversible Processes and the Second Law of Thermodynamics

20.36. Model: Each ball must be in one box or the other.

Visualize: There are N total balls with N_1 of them in the first box and N_2 in the second.

Solve: (a) As per the discussion in the text, the probability of finding all N balls in box #1 is $(0.5)^N$. But there are two boxes, so the probability of finding all the balls in one box (either one) is $2(0.5)^N$. But we did find this 80 times out of 10,000, so

$$2(0.5)^N = \frac{80}{10,000} = 0.008 \Rightarrow N = \log_{0.5}(0.004) = \frac{\log_{10}(0.004)}{\log_{10}(0.5)} = 7.97$$

where we used the change of base formula. But the number of balls must be an integer, so we round to $N = 8$.

(b) In equilibrium we are most likely to have $N_1 \approx N_2 \approx N/2 = 4$.

Assess: Even with only 8 balls, it is reasonably rare to find them all in only one of the two boxes.

Problems

20.37. Solve: Neon is a monatomic gas and has a radius $r \approx 5.0 \times 10^{-11}\text{ m}$. Using the ideal-gas equation,

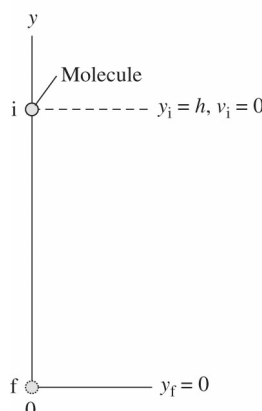
$$\frac{N}{V} = \frac{p}{k_B T} = \frac{(150)(1.013 \times 10^5\text{ Pa})}{(1.38 \times 10^{-23}\text{ J/K})(298\text{ K})} = 3.695 \times 10^{27}\text{ m}^{-3}$$

Thus, the mean free path of a neon atom is

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2} = \frac{1}{4\sqrt{2}\pi(3.695 \times 10^{27}\text{ m}^{-3})(5.0 \times 10^{-11}\text{ m})^2} = 6.09 \times 10^{-9}\text{ m}$$

Since the atomic diameter of neon is $2 \times (5.0 \times 10^{-11}\text{ m}) = 1.0 \times 10^{-10}\text{ m}$,

$$\lambda = \frac{6.09 \times 10^{-9}\text{ m}}{1.0 \times 10^{-10}\text{ m}} = 61\text{ atomic diameters}$$

20.38. Visualize:**Pictorial representation**

Solve: The average energy of an oxygen molecule at 300 K is

$$\epsilon_{\text{avg}} = \frac{E_{\text{th}}}{N} = \frac{5}{2} k_B T = \frac{5}{2} (1.38 \times 10^{-23} \text{ J/K})(300 \text{ K}) = 1.035 \times 10^{-20} \text{ J}$$

The energy conservation equation $U_{\text{gf}} + K_f = U_{\text{gi}} + K_i$ with $K_f = \epsilon_{\text{avg}}$ is

$$mgy_f + \epsilon_{\text{avg}} = mgy_i + 0 \text{ J}$$

With $y_i = h$ and $y_f = 0$ m, we have

$$mgh = 1.035 \times 10^{-20} \text{ J} \Rightarrow h = \frac{1.035 \times 10^{-20} \text{ J}}{(32 \times 1.66 \times 10^{-27} \text{ kg})(9.8 \text{ m/s}^2)} = 1.99 \times 10^4 \text{ m}$$

20.39. Solve: Mass m of a dust particle is

$$m = \rho V = \rho \left(\frac{4}{3} \pi r^3 \right) = (2500 \text{ kg/m}^3) \left[\frac{4}{3} \pi (5 \times 10^{-6} \text{ m})^3 \right] = 1.3 \times 10^{-12} \text{ kg}$$

The root-mean-square speed of the dust particles at 20°C is

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})}{1.3 \times 10^{-12} \text{ kg}}} = 9.6 \times 10^{-5} \text{ m/s}$$

20.40. Model: The analysis of these balls will be like the mean free path of molecules in a gas. The collisions between rubber balls will be as elastic as the collisions between molecules in a gas.

Visualize: We are given $\lambda = 1.8$ m and $r = 0.031$ m.

Solve: Solve the mean free path equation for N .

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2} = \frac{V}{4\sqrt{2}\pi N r^2} \Rightarrow N = \frac{V}{4\sqrt{2}\pi \lambda r^2} = \frac{(2.5 \text{ m})^3}{4\sqrt{2}\pi (1.8 \text{ m})(0.031 \text{ m})^2} = 510$$

Assess: 510 balls is a reasonable number in a box this size.

20.41. Model: Photons can scatter off of molecules. Assume the molecules are spheres.

Visualize: We are given $\lambda_{\text{photon}} \propto \frac{1}{(N/V)r^2}$ and $\lambda_0 = 500$ m. In part (b) the initial temperature is 293 K and the final temperature is 1723 K.

Solve: (a) When the volume of a sphere doubles, the radius increases by $\sqrt[3]{2}$ because $V \propto r^3$. The number density does not change, so we only look at the r dependence of λ .

$$\frac{\lambda'}{\lambda} = \frac{1/(r')^2}{1/r^2} = \frac{r^2}{(r')^2} = \frac{r^2}{(\sqrt[3]{2}r)^2} = 2^{-2/3} = 0.630 \Rightarrow \lambda' = 0.630\lambda = 0.630(500 \text{ m}) = 310 \text{ m}$$

where we used the change of base formula. But the number of balls must be an integer, so we round to $N = 7$.

(b) Assume the molecules are back to their undoubled size. So we look only at the (N/V) dependence of λ . From the ideal gas law $pV = Nk_B T$ we see that $(V/N) \propto T$ at constant pressure.

$$\frac{\lambda'}{\lambda} = \frac{1/(N/V)'}{1/(N/V)} = \frac{(N/V)}{(N/V)'} = \frac{(V/N)}{(V/N)'} = \frac{T'}{T} = \frac{1723 \text{ K}}{293 \text{ K}} = 5.88 \Rightarrow \lambda' = 5.88\lambda = 5.88(500 \text{ m}) = 2900 \text{ m}$$

Assess: We expected the mean free path to decrease in part (a) due to the larger scattering targets, and we expected the mean free path to increase in part (b) due to the increased space between target molecules.

20.42. Solve: (a) The number density is $N/V = 1 \text{ cm}^{-3} = 10^6 \text{ m}^{-3}$. Using the ideal-gas equation,

$$p = \frac{N}{V} k_B T \approx (1 \times 10^6 \text{ m}^{-3})(1.38 \times 10^{-23} \text{ J/K})(3 \text{ K}) \\ = 4 \times 10^{-17} \text{ Pa} \times \frac{1 \text{ atm}}{1.013 \times 10^5 \text{ Pa}} = 4 \times 10^{-22} \text{ atm}$$

(b) For a monatomic gas,

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(3 \text{ K})}{1.67 \times 10^{-27} \text{ kg}}} = 270 \text{ m/s}$$

(c) The thermal energy is $E_{\text{th}} = \frac{3}{2} N k_B T$, where $N = (10^6 \text{ m}^{-3})V$. Thus

$$E_{\text{th}} = 1.0 \text{ J} = \frac{3}{2} (10^6 \text{ m}^{-3}) V (1.38 \times 10^{-23} \text{ J/K})(3 \text{ K}) \Rightarrow V = 1.6 \times 10^{16} \text{ m}^3 = L^3 \\ \Rightarrow L = 2.5 \times 10^5 \text{ m}$$

20.43. Model: Assume the diatomic molecules are spheres with $r = 1.0 \times 10^{-10} \text{ m}$. Most of the air is diatomic nitrogen.

Visualize: The mass of a nitrogen molecule is $28 \text{ u} = 4.68 \times 10^{-26} \text{ kg}$.

Solve: We'll first find v_{rms} and then relate it to the temperature. Each of the two molecules must travel 25 molecular radii in 3.5 s, so the speed (which we assume to be rms) is

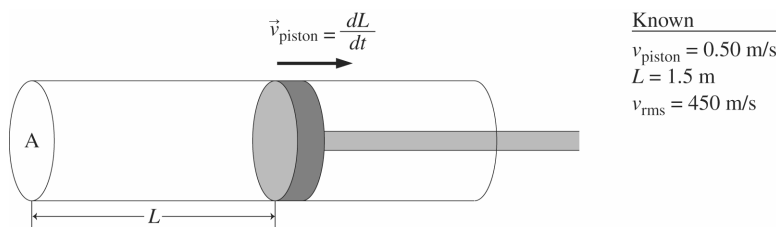
$$v_{\text{rms}} = (25r)/3.5 \text{ s} = 25(1.0 \times 10^{-10} \text{ m})/3.5 \text{ s} = 7.14 \times 10^{-10} \text{ m/s}. \\ v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} \Rightarrow T = \frac{m v_{\text{rms}}^2}{3k_B} = \frac{(4.68 \times 10^{-26} \text{ kg})(7.14 \times 10^{-10} \text{ m/s})^2}{3(1.38 \times 10^{-23} \text{ J/K})} = 5.8 \times 10^{-22} \text{ K}$$

This is obviously not a plausible temperature for air. The gases would have condensed into liquids or solids.

Assess: We cannot achieve temperatures this low in our best laboratories.

20.44. Model: Assume an ideal gas. For the isobaric expansion $p = \text{const}$.

Visualize:



Solve: (a) There are N molecules of mass m each, so the total mass of the gas is $M = Nm$.

$$v_{\text{rms}} = \sqrt{\frac{3p}{(N/V)m}} = \sqrt{\frac{3pV}{M}}$$

(b) The volume of the cylinder is the cross-sectional area A times the length L . $v_{\text{piston}} = dL/dt$.

$$v_{\text{rms}} = \sqrt{\frac{3pV}{M}} = \sqrt{\frac{3pAL}{M}} = \sqrt{\frac{3pA}{M}} L^{1/2} \Rightarrow \frac{dv_{\text{rms}}}{dt} = \frac{1}{2} \sqrt{\frac{3pA}{M}} (L^{-1/2}) \frac{dL}{dt} = \frac{1}{2} \sqrt{\frac{3pA}{ML}} v_{\text{piston}} = \frac{v_{\text{rms}} v_{\text{piston}}}{2L}$$

(c) We don't know the cross-sectional area of the cylinder, nor the pressure, nor the total mass, but we can figure out a combination of all three without knowing any one of them.

We are given $v_{\text{rms}} = 450 \text{ m/s}$. Now we plug this into the equation for the rate at which v_{rms} is changing.

$$\frac{dv_{\text{rms}}}{dt} = \frac{v_{\text{rms}} v_{\text{piston}}}{2L} \frac{(450 \text{ m/s})(0.50 \text{ m/s})}{2(1.5 \text{ m})} = 75 \text{ m/s}^2$$

Because the answer is positive the rms speed is increasing as the piston moves out.

Assess: While the units of the answer to (c) look like an acceleration no particle is accelerating at that rate.

20.45. Solve: (a) If the electron can be thought of as a point particle with zero radius, then it will collide with any gas particle that is within r of its path. Hence, the number of collisions N_{coll} is equal to the number of gas particles in a cylindrical volume of length L . The volume of a cylinder is $V_{\text{cyl}} = AL = (\pi r^2)L$. If the number density of the gas is (N/V) particles per m^3 , then the number of collisions along a trajectory of length L is

$$N_{\text{coll}} = \frac{N}{V} V_{\text{cyl}} = \frac{N}{V} (\pi r^2 L) \Rightarrow \lambda_{\text{electron}} = \frac{L}{N_{\text{coll}}} = \frac{1}{\pi(N/V)r^2}$$

Introducing a factor of $\sqrt{2}$ to account for the motion of all particles,

$$\lambda_{\text{electron}} = \frac{L}{N_{\text{coll}}} = \frac{1}{\sqrt{2}\pi(N/V)r^2}$$

(b) Assuming that most of the molecules in the accelerator are diatomic,

$$5.0 \times 10^4 \text{ m} = \frac{1}{\sqrt{2}\pi(N/V)(1.0 \times 10^{-10} \text{ m})^2} \Rightarrow N/V = 4.50 \times 10^{14} \text{ m}^{-3}$$

From the ideal-gas equation,

$$p = \frac{N}{V} k_B T = (4.50 \times 10^{14} \text{ m}^{-3})(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K}) = 1.82 \times 10^{-6} \text{ Pa} = 1.80 \times 10^{-11} \text{ atm}$$

20.46. Solve: Fluorine has atomic mass number $A=19$. Thus the root-mean-square speed of $^{238}\text{UF}_6$ is

$$v_{\text{rms}}(^{238}\text{UF}_6) = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3k_B T}{238 \text{ u} + 6 \times 19 \text{ u}}}$$

The ratio of the root-mean-square speed for the molecules of this isotope and the $^{235}\text{UF}_6$ molecules is

$$\frac{v_{\text{rms}}(^{235}\text{UF}_6)}{v_{\text{rms}}(^{238}\text{UF}_6)} = \sqrt{\frac{(238 + 6 \times 19) \text{ u}}{(235 + 6 \times 19) \text{ u}}} = \sqrt{\frac{352}{349}} = 1.0043$$

20.47. Model: Carbon dioxide is triatomic, so we'll assume the radius of the molecule (even though it is more linear than spherical) is 1.5×10^{-10} m.

Solve: $450^\circ\text{C} = 723$ K. The mass of a CO_2 molecule is $44 \text{ u} = 44(1.661 \times 10^{-27} \text{ kg}) = 7.31 \times 10^{-26} \text{ kg}$. $93 \text{ atm} = 93(101.3 \text{ kPa}) = 9.42 \times 10^6 \text{ Pa}$.

(a)

$$v_{\text{rms}} = \sqrt{\frac{3k_{\text{B}}T}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(723 \text{ K})}{7.31 \times 10^{-26} \text{ kg}}} = 640 \text{ m/s}$$

(b)

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2} = \frac{1}{4\sqrt{2}\pi\left(\frac{p}{k_{\text{B}}T}\right)r^2} = \frac{k_{\text{B}}T}{4\sqrt{2}\pi pr^2} = \frac{(1.38 \times 10^{-23} \text{ J/K})(723 \text{ K})}{4\sqrt{2}\pi(9.42 \times 10^6 \text{ Pa})(1.5 \times 10^{-10} \text{ m})^2} = 2.6 \text{ nm}$$

Assess: This very short mean free path is due to the large pressure.

20.48. Solve: The pressure on the wall with area $A = 10 \text{ cm}^2 = 10 \times 10^{-4} \text{ m}^2$ is

$$p = \frac{F}{A} = \frac{\Delta(mv)N}{A\Delta t}$$

where $K_{\text{esc}} = \frac{1}{2}mv_{\text{esc}}^2$ is the number of N_2 molecules colliding with the wall every second and $\Delta(mv)$ is the change in momentum for one collision. The mass of the nitrogen molecule is

$$m = 28 \text{ u} = 28(1.66 \times 10^{-27} \text{ kg}) = 4.648 \times 10^{-26} \text{ kg}$$

and $\Delta v = 400 \text{ m/s} - (-400 \text{ m/s}) = 800 \text{ m/s}$. Thus,

$$p = \frac{(4.648 \times 10^{-26} \text{ kg})(800 \text{ m/s})(5.0 \times 10^{23} \text{ s}^{-1})}{1.0 \times 10^{-3} \text{ m}^2} = 1.9 \times 10^4 \text{ Pa}$$

20.49. Model: Pressure is due to random collisions of gas molecules with the walls.

Solve: According to Equation 20.9, the collision rate with one wall is

$$\text{rate of collisions} = \frac{N_{\text{coll}}}{\Delta t_{\text{coll}}} = \frac{1}{2} \frac{N}{V} A v_x$$

However, this equation assumed that all molecules are moving in the x -direction with constant speed. The rms speed v_{rms} is for motion in three dimensions at varying speeds. Consequently, we need to replace v_x not with $(v_x)_{\text{avg}}$, which is zero, but with

$$v_x \rightarrow \sqrt{(v_x^2)_{\text{avg}}} = \sqrt{\frac{v_{\text{rms}}^2}{3}} = \frac{v_{\text{rms}}}{\sqrt{3}}$$

With this change,

$$\text{rate of collisions} = \frac{1}{2\sqrt{3}} \frac{N}{V} A v_{\text{rms}}$$

The molecular mass of nitrogen is $A = 28 \text{ u}$, thus the rms speed of the molecules at 20°C is

$$v_{\text{rms}} = \sqrt{\frac{3k_{\text{B}}T}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})}{28(1.661 \times 10^{-27} \text{ kg})}} = 510 \text{ m/s}$$

With $N = 0.10N_A = 6.02 \times 10^{22}$ molecules, the number density is

$$\frac{N}{V} = \frac{6.02 \times 10^{22}}{0.10 \text{ m} \times 0.10 \text{ m} \times 0.10 \text{ m}} = 6.02 \times 10^{25} \text{ m}^{-3}$$

Thus

$$\text{rate of collisions} = \frac{1}{2\sqrt{3}} (6.02 \times 10^{25} \text{ m}^{-3})(0.10 \text{ m} \times 0.10 \text{ m})(510 \text{ m/s}) = 8.9 \times 10^{25} \text{ s}^{-1}$$

20.50. Model: Assume the gas is ideal so that Equation 20.30 will apply.

$$\Delta E_{\text{th}} = nC_V \Delta T$$

Visualize: We see from the graph that $\Delta T = 200 \text{ K}$ and $\Delta E_{\text{th}} = 800 \text{ J}$. We are also given $n = 0.14 \text{ mol}$.

Solve: Solve for C_V .

$$C_V = \frac{\Delta E_{\text{th}}}{n\Delta T} = \frac{800 \text{ J}}{(0.14 \text{ mol})(200 \text{ K})} = 29 \text{ J/mol} \cdot \text{K}$$

Assess: This is about $E_{2i} = \frac{5}{2} n_2 R T_{2i}$

20.51. Solve: (a) The number of molecules of helium is

$$N_{\text{helium}} = \frac{pV}{k_B T} = \frac{(2.0 \times 1.013 \times 10^5 \text{ Pa})(100 \times 10^{-6} \text{ m}^3)}{(1.38 \times 10^{-23} \text{ J/K})(373 \text{ K})} = 3.936 \times 10^{21}$$

$$\Rightarrow n_{\text{helium}} = \frac{3.936 \times 10^{21}}{6.022 \times 10^{23} \text{ mol}^{-1}} = 6.536 \times 10^{-3} \text{ mol}$$

The initial internal energy of helium is

$$E_{\text{helium i}} = \frac{3}{2} N_{\text{helium}} k_B T = 30.4 \text{ J} \approx 30 \text{ J}$$

The number of molecules of argon is

$$N_{\text{argon}} = \frac{pV}{k_B T} = \frac{(4.0 \times 1.013 \times 10^5 \text{ Pa})(200 \times 10^{-6} \text{ m}^3)}{(1.38 \times 10^{-23} \text{ J/K})(673 \text{ K})} = 8.726 \times 10^{21}$$

$$\Rightarrow n_{\text{argon}} = \frac{8.726 \times 10^{21}}{6.022 \times 10^{23} \text{ mol}^{-1}} = 1.449 \times 10^{-2} \text{ mol}$$

The initial thermal energy of argon is

$$E_{\text{argon i}} = \frac{3}{2} N_{\text{argon}} k_B T = 121.6 \text{ J} \approx 122 \text{ J}$$

(b) The equilibrium condition for monatomic gases is

$$(\epsilon_{\text{helium f}})_{\text{avg}} = (\epsilon_{\text{argon f}})_{\text{avg}} = (\epsilon_{\text{total}})_{\text{avg}}$$

$$\Rightarrow \frac{E_{\text{helium f}}}{n_{\text{helium}}} = \frac{E_{\text{argon f}}}{n_{\text{argon}}} = \frac{E_{\text{tot}}}{n_{\text{tot}}} = \frac{(30.4 + 121.6) \text{ J}}{(6.54 \times 10^{-3} + 1.449 \times 10^{-2}) \text{ mol}} = 7228 \text{ J/mol}$$

$$\Rightarrow E_{\text{helium f}} = (7228 \text{ J/mol})n_{\text{helium}} = (7228 \text{ J/mol})(6.54 \times 10^{-3} \text{ mol}) = 47.3 \text{ J} \approx 47 \text{ J}$$

$$E_{\text{argon f}} = (7228 \text{ J/mol})n_{\text{argon}} = (7228 \text{ J/mol})(1.449 \times 10^{-2} \text{ mol}) = 104.7 \text{ J} \approx 105 \text{ J}$$

(c) The amount of heat transferred is

$$E_{\text{helium f}} - E_{\text{helium i}} = 47.3 \text{ J} - 30.4 \text{ J} = 16.9 \text{ J} \quad E_{\text{argon f}} - E_{\text{argon i}} = 104.7 \text{ J} - 121.6 \text{ J} = -16.9 \text{ J}$$

The helium gains 16.9 J of heat energy and the argon loses 16.9 J. Thus approximately 17 J are transferred from the argon to the helium.

(d) The equilibrium condition for monatomic gases is

$$(\epsilon_{\text{helium}})_{\text{avg}} = (\epsilon_{\text{argon}})_{\text{avg}} \Rightarrow \frac{E_{\text{helium f}}}{N_{\text{helium}}} = \frac{E_{\text{argon f}}}{N_{\text{argon}}} = \frac{3}{2} k_B T_f$$

Substituting the above values,

$$\frac{47.3 \text{ J}}{3.936 \times 10^{21}} = \frac{104.7 \text{ J}}{8.726 \times 10^{21}} = \frac{3}{2} (1.38 \times 10^{-23} \text{ J/K}) T_f \Rightarrow T_f = 580 \text{ K} = 307^\circ\text{C}$$

(e) The final pressure of the helium and argon are

$$p_{\text{helium f}} = \frac{N_{\text{helium}} k_B T}{V_{\text{helium}}} = \frac{(3.936 \times 10^{21})(1.38 \times 10^{-23} \text{ J/K})(580 \text{ K})}{100 \times 10^{-6} \text{ m}^3} = 3.15 \times 10^5 \text{ Pa} \approx 3.1 \text{ atm}$$

$$p_{\text{argon f}} = \frac{N_{\text{argon}} k_B T}{V_{\text{argon}}} = \frac{(8.726 \times 10^{21})(1.38 \times 10^{-23} \text{ J/K})(580 \text{ K})}{200 \times 10^{-6} \text{ m}^3} = 3.49 \times 10^5 \text{ Pa} \approx 3.4 \text{ atm}$$

20.52. Solve: (a) The number of moles of helium and oxygen is

$$n_{\text{helium}} = \frac{2.0 \text{ g}}{4.0 \text{ g/mol}} = 0.50 \text{ mol} \quad n_{\text{oxygen}} = \frac{8.0 \text{ g}}{32.0 \text{ g/mol}} = 0.25 \text{ mol}$$

Since helium is a monoatomic gas, the initial thermal energy is

$$E_{\text{helium i}} = n_{\text{helium}} \left(\frac{3}{2} R T_{\text{helium}} \right) = (0.50 \text{ mol}) \left(\frac{3}{2} \right) (8.31 \text{ J/mol K}) (300 \text{ K}) = 1870 \text{ J} \approx 1900 \text{ J}$$

Since oxygen is a diatomic gas, the initial thermal energy is

$$E_{\text{oxygen i}} = n_{\text{oxygen}} \left(\frac{5}{2} R T_{\text{oxygen}} \right) = (0.25 \text{ mol}) \left(\frac{5}{2} \right) (8.31 \text{ J/mol K}) (600 \text{ K}) = 3116 \text{ J} \approx 3100 \text{ J}$$

(b) The total initial thermal energy is

$$E_{\text{tot}} = E_{\text{helium i}} + E_{\text{oxygen i}} = 4986 \text{ J}$$

As the gases interact, they come to equilibrium at a common temperature T_f . This means

$$4986 \text{ J} = n_{\text{helium}} \left(\frac{3}{2} R T_f \right) + n_{\text{oxygen}} \left(\frac{5}{2} R T_f \right)$$

$$\Rightarrow T_f = \frac{4986 \text{ J}}{\left(\frac{1}{2} R \right) (3n_{\text{helium}} + 5n_{\text{oxygen}})} = \frac{4986 \text{ J}}{\frac{1}{2} (8.31 \text{ J/mol K}) (3 \times 0.50 \text{ mol} + 5 \times 0.25 \text{ mol})} = 436.4 \text{ K} \approx 436 \text{ K}$$

The thermal energies at the final temperature T_f are

$$E_{\text{helium f}} = n_{\text{helium}} \left(\frac{3}{2} R T_f \right) = \left(\frac{3}{2} \right) (0.50 \text{ mol}) (8.31 \text{ J/mol K}) (436.4 \text{ K}) = 2700 \text{ J}$$

$$E_{\text{oxygen f}} = n_{\text{oxygen}} \left(\frac{5}{2} R T_f \right) = \left(\frac{5}{2} \right) (0.25 \text{ mol}) (8.31 \text{ J/mol K}) (436.4 \text{ K}) = 2300 \text{ J}$$

(c) The changes in the thermal energies are

$$E_{\text{helium f}} - E_{\text{helium i}} = 2720 \text{ J} - 1870 \text{ J} = 850 \text{ J} \quad E_{\text{oxygen f}} - E_{\text{oxygen i}} = 2266 \text{ J} - 3116 \text{ J} = -850 \text{ J}$$

The helium gains energy and the oxygen loses energy.

(d) The final temperature can also be calculated as follows:

$$E_{\text{helium f}} = (n_{\text{helium}}) \frac{3}{2} R T_f \Rightarrow 2720 \text{ J} = (0.50 \text{ mol}) (1.5) (8.31 \text{ J/mol K}) T_f \Rightarrow T_f = 436.4 \text{ K} \approx 436 \text{ K}$$

20.53. Solve: The thermal energy of any ideal gas is related to the molar specific heat at constant volume by

$$E_{\text{th}} = n C_V T$$

Since $C_p = C_V + R$,

$$20.8 \text{ J/mol K} = C_V + R \Rightarrow C_V = 12.5 \text{ J/mol K}$$

The number of moles of the gas is

$$n = \frac{1.0 \times 10^{20}}{6.02 \times 10^{23} \text{ mol}^{-1}} = 1.66 \times 10^{-4} \text{ mol}$$

Thus

$$T = \frac{(1.0 \text{ J})}{(1.66 \times 10^{-4} \text{ mols})(12.5 \text{ J/mol K})} = 482 \text{ K}$$

20.54. Model: Assume an ideal gas. Hydrogen is monatomic.

Visualize: For a monatomic gas the thermal energy is $E_{\text{th}} = \frac{3}{2}nRT$.

Solve:

$$\Delta E_{\text{th}} = \frac{3}{2}nR\Delta T = \frac{3}{2}(0.50 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(270 \text{ K}) = 1700 \text{ J}$$

Assess: The units check out.

20.55. Model: Water is not diatomic, but we must count the degrees of freedom and use the equipartition theorem.

Visualize: We are given $n = 2.0 \text{ mol}$ and $T = 160^\circ\text{C} = 433 \text{ K}$.

Solve: The water molecule has three degrees of translational freedom and three rotational degrees of freedom, but the vibrational modes are not active at this temperature; this is a total of 6 degrees of freedom.

$$E_{\text{th}} = 6\left(\frac{1}{2}nRT\right) = 3nRT = 3(2.0 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(433 \text{ K}) = 22 \text{ kJ}$$

Assess: The answer is in the right range for 2.0 mol at this temperature.

20.56. Model: The heating is done at constant pressure, so we must use $Q = nC_p\Delta T$. We also learned in the previous chapter that $C_p = C_v + R$. Because the volume of both gases increases by the same amount, the same is true for temperature, so $\Delta T_{\text{diatomic}} = \Delta T_{\text{monatomic}}$.

Solve: $C_v = \frac{3}{2}$ for monatomic gases and $C_v = \frac{5}{2}$ for diatomic gases.

$$\frac{Q_{\text{diatomic}}}{Q_{\text{monatomic}}} = \frac{n((C_p)_{\text{diatomic}})\Delta T}{n((C_p)_{\text{monatomic}})\Delta T} = \frac{(C_v)_{\text{diatomic}} + R}{(C_v)_{\text{monatomic}} + R} = \frac{\frac{5}{2}R + R}{\frac{3}{2}R + R} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5}$$

20.57. Solve: Assuming that the systems are thermally isolated except from each other, the total energy for the two thermally interacting systems must remain the same. That is,

$$E_{1i} + E_{2i} = E_{1f} + E_{2f} \Rightarrow E_{1f} - E_{1i} = E_{2i} - E_{2f} = -(E_{2f} - E_{2i}) \Rightarrow \Delta E_1 = -\Delta E_2$$

No work is done on either system, so from the first law $\Delta E = Q$. Thus

$$Q_1 = -Q_2$$

That is, the heat lost by one system is gained by the other system.

20.58. Solve: (a) From Equation 20.26 $v_{\text{rms}} = \sqrt{3k_B T/m}$. For an adiabatic process

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1} \Rightarrow T_f = T_i \left(\frac{V_i}{V_f} \right)^{\gamma-1} \Rightarrow T_f = T_i (8)^{\frac{5}{3}-1} = 4T_i$$

The root-mean-square speed increases by a factor of 2 with an increase in temperature.

(b) From Equation 20.3 $\lambda = [4\sqrt{2}\pi(N/V)r^2]^{-1}$. A decrease in volume decreases the mean free path by a factor of 1/8.

(c) For an adiabatic process,

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1} \Rightarrow T_f = T_i \left(\frac{V_i}{V_f} \right)^{\gamma-1} = T_i (8)^{\frac{5}{3}-1} = 4T_i$$

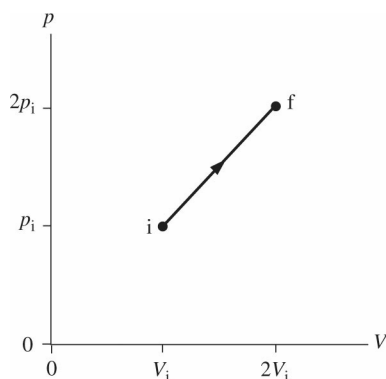
Because the decrease in volume increases T_f , the thermal energy increases by a factor of 4.

(d) The molar specific heat at constant volume is $C_V = \frac{3}{2}R$, a constant. It does not change.

20.59. Solve: (a) We are given that

$$(v_{\text{rms } i}) = \sqrt{\frac{3k_B T_i}{m}} \quad (v_{\text{rms } f}) = \sqrt{\frac{3k_B T_f}{m}} = 2(v_{\text{rms } i})$$

This means that $T_f = 4T_i$. Using the ideal-gas law, it also means that $p_f V_f = 4p_i V_i$. Since the pressure is directly proportional to the volume during the process, we have $p_i/V_i = p_f/V_f$. Combining these two equations gives $p_f = 2p_i$ and $V_f = 2V_i$.



(b) The change in thermal energy for *any* ideal gas process is related to the molar specific heat at constant volume by $\Delta E_{\text{th}} = nC_V(T_f - T_i)$. The work done on the gas is

$$W = -\int p dV = -(\text{area under the } p\text{-versus-}V \text{ graph}) = -\frac{3}{2} p_i V_i$$

The first law of thermodynamics $\Delta E_{\text{th}} = Q + W$ can be written

$$\begin{aligned} Q &= \Delta E_{\text{th}} - W = nC_V(T_f - T_i) + \frac{3}{2} p_i V_i = 3nC_V T_i + \frac{3}{2} p_i V_i \\ &= 3n\left(\frac{5}{2}R\right)T_i + \frac{3}{2} p_i V_i = \frac{15}{2} n p_i V_i + \frac{3}{2} p_i V_i = \frac{15n+3}{2} p_i V_i \end{aligned}$$

20.60. Model: We expect to get a contribution to the molar specific heat of $\frac{1}{2}R$ for each degree of freedom.

Visualize: An atom in a truly two-dimensional lattice could only move in two directions and compress or stretch molecular bonds in two dimensions.

Solve: This gives a total of four degrees of freedom so we might expect the molar specific heat of graphene to be $2R$. However, the atoms can also move out of the plane somewhat (giving another degree of freedom), and this stretches the bonds in a new way (giving another degree of freedom), so there are actually more than 4 degrees of freedom.

Assess: The actual value is $23.5 \text{ J/mol} \cdot \text{K}$, which implies there are between five and six degrees of freedom.

20.61. Visualize: We are given $v_{\text{rms}} = 1800 \text{ m/s}$. For 1.0 g of molecular hydrogen gas

$$N = 1.0 \text{ g} \left(\frac{1 \text{ mol}}{2.0 \text{ g}} \right) = 0.5 \text{ mol} \left(6.02 \times 10^{23} \frac{\text{molecules}}{\text{mol}} \right) = 3.01 \times 10^{23} \text{ molecules}$$

The mass of one molecule is $2(1.661 \times 10^{-27} \text{ kg}) = 3.322 \times 10^{-27} \text{ kg}$.

Solve:

(a)

$$e_{\text{total}} = N \left(\frac{1}{2} \right) m v_{\text{rms}}^2 = (3.01 \times 10^{23}) \left(\frac{1}{2} \right) (3.322 \times 10^{-27} \text{ kg}) (1800 \text{ m/s})^2 = 1.6 \text{ kJ}$$

(b) The temperature is given by Equation 20.25.

$$T = \frac{2}{3k_B} e_{\text{avg}} = \frac{2}{3k_B} \left(\frac{1}{2} \right) m v_{\text{rms}}^2 = \frac{3.322 \times 10^{-27} \text{ kg}}{3(1.38 \times 10^{-23} \text{ J/K})} (1800 \text{ m/s})^2 = 260 \text{ K}$$

For diatomic gases Equation 20.37 gives the thermal energy.

$$E_{\text{th}} = \frac{5}{2} n R T = \frac{5}{2} (0.50 \text{ mol}) (8.31 \text{ J/mol} \cdot \text{K}) (260 \text{ K}) = 2.7 \text{ kJ}$$

(c) There is a net loss of 700 J of energy from the system, so the new $E_{\text{th}} = 2000 \text{ J}$. Solve Equation 20.37 for T .

$$T = \frac{2}{5nR} E_{\text{th}} = \frac{2}{5(0.50 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})} (2000 \text{ J}) = 192.5 \text{ K}$$

Now Equation 20.26 gives the new rms speed.

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(192.5 \text{ K})}{3.322 \times 10^{-27} \text{ kg}}} = 1549 \text{ m/s} \approx 1550 \text{ m/s}$$

Assess: With the net loss of energy we expected v_{rms} to decrease as the temperature decreased.

20.62. Solve: (a) The escape speed is the speed with which a mass m can leave the earth's surface and escape to infinity ($r_f = \infty$) with no left over speed ($v_f = 0$). The conservation of energy equation $K_f + U_f = K_i + U_i$ is

$$0 + 0 = \frac{1}{2} m v_{\text{esc}}^2 - \frac{GM_e m}{R_e} \Rightarrow v_{\text{esc}} = \sqrt{\frac{2GM_e}{R_e}}$$

The rms speed of a gas molecule is $v_{\text{rms}} = (3k_B T/m)^{1/2}$. Equating v_{esc} and v_{rms} , and squaring both sides, the temperature at which the rms speed equals the escape speed is

$$\frac{3k_B T}{m} = \frac{2GM_e}{R_e} \Rightarrow T = m \left(\frac{2GM_e}{3k_B R_e} \right)$$

For a nitrogen molecule, with $m = 28 \text{ u}$, the temperature is

$$T = (28 \times 1.661 \times 10^{-27} \text{ kg}) \left(\frac{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{3(1.38 \times 10^{-23} \text{ J/K})(6.37 \times 10^6 \text{ m})} \right) = 141,000 \text{ K}$$

(b) For a hydrogen molecule, with $m = 2 \text{ u}$, the temperature is less by a factor of 14, or $T = 10,100 \text{ K}$.

(c) The average translational kinetic energy of a molecule is $e_{\text{avg}} = \frac{3}{2} k_B T = 6.1 \times 10^{-21} \text{ J}$ at a typical atmosphere temperature of 20°C . The kinetic energy needed to escape is $K_{\text{esc}} = \frac{1}{2} m v_{\text{esc}}^2$. For nitrogen molecules, $K_{\text{esc}} = 2.9 \times 10^{-18} \text{ J}$. Thus $e_{\text{avg}}/K_{\text{esc}} = 0.002 = 0.2\%$. Earth will retain nitrogen in its atmosphere because the molecules are moving too slowly to escape. But for hydrogen molecules, with $K_{\text{esc}} = 2.1 \times 10^{-19} \text{ J}$, the ratio is $e_{\text{avg}}/K_{\text{esc}} = 0.03 = 3\%$. Thus a large enough fraction of hydrogen molecules are moving at escape speed, or faster, to allow hydrogen to leak out of the atmosphere into space. Consequently, earth's atmosphere does not contain hydrogen.

20.63. Solve: (a) The thermal energy of a monatomic gas of n_1 moles is $E_1 = \frac{3}{2}n_1RT$. The thermal energy of a diatomic gas of n_2 moles is $E_2 = \frac{5}{2}n_2RT$. The total thermal energy of the mixture is

$$E_{\text{th}} = \frac{1}{2}(3n_1 + 5n_2)RT \Rightarrow \Delta E_{\text{th}} = \frac{1}{2}(3n_1 + 5n_2)R\Delta T$$

Comparing this expression with

$$\Delta E_{\text{th}} = n_{\text{total}} C_V \Delta T = (n_1 + n_2) C_V \Delta T$$

we get

$$C_V = \frac{(3n_1 + 5n_2)}{2(n_1 + n_2)} R$$

(b) For a diatomic gas, $n_1 \rightarrow 0$, and $C_V = \frac{5}{2}R$. For a monatomic gas, $n_2 \rightarrow 0$, and $C_V = \frac{3}{2}R$.

20.64. Solve: (a) The thermal energy is

$$E_{\text{th}} = (E_{\text{th}})_{\text{N}_2} + (E_{\text{th}})_{\text{O}_2} = \frac{5}{2}N_{\text{N}_2}k_{\text{B}}T + \frac{5}{2}N_{\text{O}_2}k_{\text{B}}T = \frac{5}{2}N_{\text{total}}k_{\text{B}}T$$

where N_{total} is the total number of molecules. The identity of the molecules makes no difference since both are diatomic. The number of molecules in the room is

$$N_{\text{total}} = \frac{pV}{k_{\text{B}}T} = \frac{(101,300 \text{ Pa})(2 \text{ m} \times 2 \text{ m} \times 2 \text{ m})}{(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K})} = 2.15 \times 10^{26}$$

The thermal energy is

$$E_{\text{th}} = \frac{5}{2}(2.15 \times 10^{26})(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K}) = 2.03 \times 10^6 \text{ J} \approx 2.0 \times 10^6 \text{ J}$$

(b) A 1 kg ball at height $y = 1 \text{ m}$ has a potential energy $U = mgy = 9.8 \text{ J}$. The ball would need 9.8 J of initial kinetic energy to reach this height. The fraction of thermal energy that would have to be conveyed to the ball is

$$\frac{9.8 \text{ J}}{2.03 \times 10^6 \text{ J}} = 4.8 \times 10^{-6}$$

(c) A temperature change ΔT corresponds to a thermal energy change $\Delta E_{\text{th}} = \frac{5}{2}N_{\text{total}}k_{\text{B}}\Delta T$. But $\frac{5}{2}N_{\text{total}}k_{\text{B}} = E_{\text{th}}/T$.

Using this, we can write

$$\Delta E_{\text{th}} = \frac{E_{\text{th}}}{T} \Delta T \Rightarrow \Delta T = \frac{\Delta E_{\text{th}}}{E_{\text{th}}} T = \frac{-9.8 \text{ J}}{2.03 \times 10^6 \text{ J}} 273 \text{ K} = -0.0013 \text{ K}$$

The room temperature would decrease by 0.0013 K or 0.0013°C.

(d) The situation with the ball at rest on the floor and in thermal equilibrium with the air is a very probable distribution of energy and thus a state with high entropy. Although energy would be conserved by removing energy from the air and transferring it to the ball, this would be a very *improbable* distribution of energy and thus a state of low entropy. The ball will not be spontaneously launched from the ground because this would require a decrease in entropy, in violation of the second law of thermodynamics.

As another way of thinking about the situation, the ball and the air are initially at the same temperature. Once even the slightest amount of energy is transferred from the air to the ball, the air's temperature will be less than that of the ball. Any further flow of energy from the air to the ball would be a situation in which heat energy is flowing from a colder object to a hotter object. This cannot happen because it would violate the second law of thermodynamics.

Challenge Problems

20.65. Solve: The thermal energy of a monatomic gas of n_1 moles is $E_1 = \frac{3}{2}n_1RT$. The thermal energy of a diatomic gas of n_2 moles is $E_2 = \frac{5}{2}n_2RT$. The total thermal energy of the mixture is

$$E_{\text{th}} = E_1 + E_2 = \frac{1}{2}(3n_1 + 5n_2)RT \Rightarrow \Delta E_{\text{th}} = \frac{1}{2}(3n_1 + 5n_2)R\Delta T$$

Comparing this expression with

$$\Delta E_{\text{th}} = n_{\text{total}} C_V \Delta T = (n_1 + n_2)C_V \Delta T$$

we get

$$(n_1 + n_2)C_V = \frac{(3n_1 + 5n_2)}{2}R$$

The requirement that the ratio of specific heats is 1.50 means

$$\gamma = 1.50 = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V} \Rightarrow C_V = 2R$$

The above equation is then

$$(n_1 + n_2)(2R) = \frac{(3n_1 + 5n_2)}{2}R \Rightarrow 4n_1 + 4n_2 = 3n_1 + 5n_2 \Rightarrow n_1 = n_2$$

Thus, monatomic and diatomic molecules need to be mixed in the ratio 1:1. Or the fraction of the molecules that are monatomic needs to be $\frac{1}{2}$.

20.66. Solve: (a) The thermal energy of a monatomic gas of n_1 moles at an initial temperature T_{1i} is $E_{1i} = \frac{3}{2}n_1RT_{1i}$. The thermal energy of a diatomic gas of n_2 moles at an initial temperature T_{2i} is $E_{2i} = \frac{5}{2}n_2RT_{2i}$. Consequently, the total initial energy is

$$E_{\text{tot}} = E_{1i} + E_{2i} = \frac{3n_1RT_{1i} + 5n_2RT_{2i}}{2} = \frac{(3n_1T_{1i} + 5n_2T_{2i})R}{2}$$

After the gases interact, they come to equilibrium with $T_{1f} = T_{2f} = T_f$. Then their total energy is

$$E_{\text{tot}} = \frac{(3n_1 + 5n_2)RT_f}{2}$$

No energy is lost, so these two expressions for E_{tot} must be equal. Thus,

$$\frac{(3n_1T_{1i} + 5n_2T_{2i})R}{2} = \frac{(3n_1 + 5n_2)RT_f}{2} \Rightarrow T_f = \frac{3n_1T_{1i} + 5n_2T_{2i}}{3n_1 + 5n_2} = \frac{2E_{\text{tot}}}{R(3n_1 + 5n_2)} = \frac{2}{R} \frac{(E_{1i} + E_{2i})}{(3n_1 + 5n_2)}$$

The thermal energies at the final temperature T_f are

$$E_{1f} = \frac{3}{2}n_1RT_f = \frac{3}{2}n_1R \frac{2}{R} \frac{E_{1i} + E_{2i}}{(3n_1 + 5n_2)} = \left(\frac{3n_1}{3n_1 + 5n_2} \right) (E_{1i} + E_{2i})$$

$$E_{2f} = \frac{5}{2}n_2RT_f = \frac{5}{2}n_2R \frac{2}{R} \frac{E_{1i} + E_{2i}}{(3n_1 + 5n_2)} = \left(\frac{5n_2}{3n_1 + 5n_2} \right) (E_{1i} + E_{2i})$$

(b) In part **(a)** we found that

$$T_f = \frac{3n_1T_{1i} + 5n_2T_{2i}}{3n_1 + 5n_2}$$

(c) 2 g of He at $T_{1i} = 300 \text{ K}$ are $n_1 = (2 \text{ g})/(4 \text{ g/mol}) = 0.50 \text{ mol}$. Oxygen has an *atomic* mass of 16, so the molecular mass of oxygen gas (O_2) is $A = 32 \text{ g/mol}$. 8 g of O_2 at $T_{2i} = 600 \text{ K}$ are $n_2 = (8 \text{ g})/(32 \text{ g/mol}) = 0.25 \text{ mol}$. The final temperature is

$$T_f = \frac{3(0.50 \text{ mol})(300 \text{ K}) + 5(0.25 \text{ mol})(600 \text{ K})}{3(0.50 \text{ mol}) + 5(0.25 \text{ mol})} = 436 \text{ K}$$

The heat flows to the two gases are found from $Q = nC_V\Delta T$. For helium,

$$Q = n_1 C_V \Delta T = (0.50 \text{ mol})(12.5 \text{ J/mol K})(436 \text{ K} - 300 \text{ K}) = 850 \text{ J}$$

For O_2 ,

$$Q = n_2 C_V \Delta T = (0.25 \text{ mol})(20.8 \text{ J/mol K})(436 \text{ K} - 600 \text{ K}) = -850 \text{ J}$$

So 850 J of heat is transferred from the oxygen to the helium.