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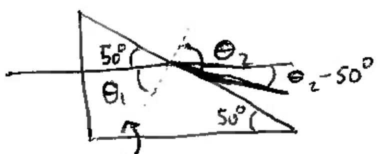
1. (15 points, 5 points each): A ray of light is perpendicular to the face *ab* of a glass prism ($n = 1.52$).

(a) If the prism is immersed in air and $\phi =$ [redacted] 50°, what is the angle of the emerging ray with respect to the horizontal?

(b) If the prism is immersed in air ($n = 1.00$), find the largest value for the angle ϕ so that the ray is totally internally reflected at face *ac*.

(c) Repeat part (b) for the case when the prism is immersed in water ($n = 1.33$).

(a) $n_2 = 1.00$



$n_1 = 1.52$

$\theta_1 = 90° - \phi = 40°$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1.52 \sin(40°) = \cancel{n_2}^{1} \sin\theta_2 \Rightarrow \theta_2 = 77.7°$

w.r.t. horizontal $\boxed{\theta_2 - 50° = 27.7°}$

(b) $\theta_2 = 90°$ for total internal reflection (critical angle)

$n_1 \sin(90° - \phi) = n_2 \sin$ [redacted] $(90°)$

$1.52 \cos\phi = 1 \Rightarrow \boxed{\phi_{max} = 48.9°}$

(c) $n_2 = 1.33$ $\Rightarrow 1.52 \cos\phi = 1.33 \Rightarrow \boxed{\phi_{max} = 29.0°}$

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2. (15 points, 5 points each): Sunlight that reaches the Earth's surface has intensity 1300 $\mathrm{W/m^2}$. Suppose the sun is directly overhead, and the light is traveling in the $-\hat{z}$ direction. We filter this sunlight using a polarizer that accomplishes two things. First, it polarizes the sunlight so that only the $\pm\hat{y}$ component of the electric field survives and, second, it filters out all colors except for a narrow range centered at 540.0 nm (green light). As a result of the filter, the intensity of light that gets through is only 1.00 $\mathrm{W/m^2}$. This problem focuses on the light that gets through.

   (a) Suppose the electric field after the filter can be expressed via the equation

   $$\vec{E}(z,t) = E_0 \cos(kz \pm \omega t)\hat{y}$$

   Determine the constants $E_0 > 0$ and ω. Also, should the plus sign or the minus sign be chosen inside the cosine?

   (b) At $z = 0$ and $t = 0$, the electric field is $\vec{E} = E_0\hat{y}$. Find the corresponding magnetic field (magnitude and direction).

   (c) What would happen to the intensity of the light if we pass the filtered light through another polarizer, in the $(\hat{x} + \hat{y})/\sqrt{2}$ direction?

(a) $I = 1.00 \frac{W}{m^2} = \frac{E_o^2}{2c\mu_o} \Rightarrow E_o = \sqrt{2(3\times10^8 \,m/s)(4\pi\times10^{-7}\,\frac{H}{m})(1\,\frac{W}{m^2})}$

$\boxed{E_o = 27.5\,\frac{V}{m}}$

$\omega = ck = \frac{2\pi c}{\lambda} = \frac{2\pi(3\times10^8\,m/s)}{(540\times10^{-4}\,m)} = \boxed{3.49\times10^{15}\,rad/s}$

It should be $\boxed{\cos(kz+\omega t)}$ if wave moves in $-\hat{z}$ dir.

(b) $B_o = \frac{E_o}{c} = \boxed{91.5\,nT}$ and points in the $\boxed{+\hat{x}\text{ dir.}}$

$\vec{E}$ $\vec{B}$ y x

$\otimes\ \vec{S} \propto \vec{E}\times\vec{B}$ $\odot\ \hat{z}$

(c) $I_f = I_o \cos^2(45^\circ) = \frac{1}{2} I_o$

Intensity would drop to $\boxed{0.500\,\frac{W}{m^2}}$

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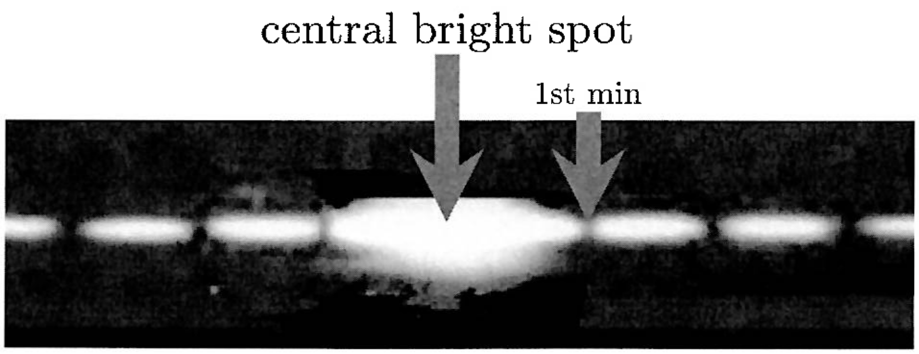
3. (10 points, 5 points each): Either a double-slit interference or single-slit diffraction pattern is shown below (you are to determine which it is). The light has wavelength $\lambda = 575$ nm, and the distance shown in the photograph is 15.0 cm (the left and right ends of the photo are at the centers of bright spots). The screen is 4.00 m from the slit(s) that produced this pattern. The central bright spot and 1st minimum are indicated.



(a) Is this a diffraction pattern from a single slit or an interference pattern from a double slit? Explain.

(b) Solve for either the slit width (if single-slit diffraction) or the slit separation (if double-slit interference).

(a) The central bright spot is twice the width of the other spots, indicating a $\boxed{\text{single-slit diffraction pattern}}$

(b) Width of 1 bright spot (not central)

is $\Delta y = \dfrac{15.0\,\text{cm}}{7} = 2.143$ cm

Using small angle approx, $a \dfrac{\Delta y}{L} = \lambda \Rightarrow a = \dfrac{\lambda L}{\Delta y}$

$= \dfrac{(575\,\text{nm})(4\,\text{m})}{21.43\,\text{mm}}$

$$\boxed{a = 107\ \mu\text{m}}$$

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(12 points, 3 points each): 4 Multiple-choice questions / fill-in-the-blanks on various topics.

Directions for multiple-choice questions: COMPLETELY FILL IN THE SQUARE for the answer. Directions for fill-in-the-blank questions: Your answer should be entirely in the boxed region. Include the number of significant figures ("sig. figs.") requested in the problem.

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4. Unpolarized light of original intensity I_0 passes through two ideal polarizing filters having their polarizing axes oriented as shown below. You want to adjust the angle ϕ so that the intensity at point P is equal to $I_0/3$. If the original light is unpolarized, what should ϕ be?

☐ 25°
■ 35°
☐ 45°
☐ 55°
☐ 65°

$\frac{1}{2} \cancel{I_0} \cos^2\phi = \frac{1}{3} \cancel{I_0}$

$\Rightarrow \cos\phi = \sqrt{\frac{2}{3}}$

$\phi \simeq 35^\circ$

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5. Coherent laser light is incident on a double slit, producing an interference pattern on a screen far away. Which of the following things will make the bright spots closer together?

$\frac{d\,\Delta y}{L} = \lambda$

☐ Moving the screen farther away from the double slit.
☐ Decreasing the distance between the two slits.
■ Doing the entire experiment under water (but using the same laser, with the same frequency).
☐ Changing the laser light to one with a lower frequency.

Smaller wavelength
$\Rightarrow$ smaller Δy

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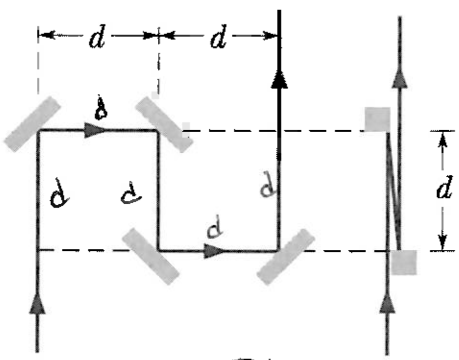
6. The figure below shows two light rays that are initially exactly in phase and that reflect from several glass surfaces. Neglect the slight slant in the path of the light in the second arrangement. In terms of the wavelength λ, what is the smallest value of d that will give destructive interference between the outgoing rays?



- ☐ $d = 2\lambda$
- ☐ $d = \lambda$
- ☐ $d = \lambda/2$
- ☐ $d = \lambda/3$
- ☒ $d = \lambda/4$

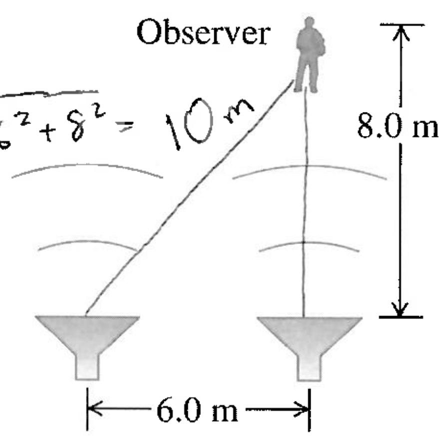
Smallest d giving destructive int. has path-length diff. equal to half a wavelength

$5d$ vs. $3d \Rightarrow \Delta r = 2d = \frac{1}{2}\lambda \Rightarrow d = \frac{\lambda}{4}$

7. Two speakers are emitting identical sound waves in phase with one another. An observer is walking upward and away from the right speaker and notices that the sound waves first cancel each other out at the position shown. What is the wavelength of the sound waves emitted by the speakers to 2 sig. figs.?

$\Delta r = 2\,m = \frac{\lambda}{2}$ (dest. int.)

$\Rightarrow \lambda$



$\sqrt{6^2+8^2} = 10\,m$

$$\boxed{\lambda = 4.0\ m}$$