14

FLUIDS AND ELASTICITY

Conceptual Questions

14.1. (a) The density will not change. Although the mass and volume both increase by $2^3 = 8$ times, their ratio is unchanged.

(b) The volume V has increased by $2^3 = 8$ times, but the mass has not changed, so the new density ρ' is $\rho' = m/(8V) = \rho/8$. The new density has decreased by a factor of 8.

14.2. The pressure only depends on the depth with respect to the top surface of the liquid. Since points a, b, and c are all at the same depth (i.e., with respect to point e) the pressure is the same for each, so

$$p_a = p_b = p_c$$

14.3. The pressure only depends on the depth from the surface of the liquid. Since point d is the deepest and point e the highest, then

$$p_d > p_f > p_e$$

A point halfway between point e and point b would have a pressure about the same as the pressure at point d.

14.4. (a) The pressure at the bottom of each tank is given by $p = \rho g d$, so it will be the same because the depth of the water in each tank is the same. The area of the bottom of tank A is larger than the area of the bottom of tank B. From $p = F/A$, we have $F = pA$, so the $F_A > F_B$ because the bottom area of tank A is larger.

(b) The pressure at any given depth is the same in both tanks because the water depth is the same. Since the area of the sides indicated in Figure Q14.4 is the same for each tank, the force on these sides is also the same, so $F_A = F_B$.

This makes sense. Since there's more water in tank A, the total force of the water on the bottom of tank A is larger.

14.5. A floating object displaces it weight in liquid, so placing the boat above point A will increase the depth of the body of water just enough to compensate for the boat's weight. Thus, the depth of point B is increased slightly so that it is at the same pressure as point A, and $p_A = p_B$.

While a column above point A contains the ship, it also contains less water than a similar column above point B, but both columns contain the same weight (or mass if we assume a uniform gravitational field).

14.6. Archimedes' principle tells us that the displaced liquid has the same weight as the floating object, so

$$\rho_L V_L = \rho_B V_B \quad \Rightarrow \quad \rho_B = \rho_L (V_L / V_B)$$

where the subscripts L and B refer to liquid and block, respectively. The ratio V_L/V_B is the fraction of the block that is under the liquid, so the densest block is the one for which this fraction is the largest. Thus,

$$\rho_A > \rho_C > \rho_B$$

You've heard that only 10% of an iceberg is visible above the surface of the ocean. That means 90% of the iceberg is below the surface. Therefore the density of ice is 90% the density of seawater. You can verify this by looking in Table 14.1 and finding on the Web the density of ice: $\rho_{ice} = 917 \text{ kg/m}^3$.

14.7. Archimedes' principle states that the buoyant force on an object is equal to the weight of the fluid displaced by the object. Each object displaces exactly the same amount of fluid since each is the same volume. Therefore, the buoyant force on all three objects is the same, so $F_a = F_b = F_c$. Note that the buoyant force does not depend on the mass or location of the object.

14.8. For objects that are completely submerged the buoyant force is proportional to the volume of the object. If the densities of objects A, B, and C are the same, then the objects with greater mass (A and C) must also occupy a larger volume. Thus, A and C will experience a larger buoyant force than B.
$$F_A = F_C > F_B$$

14.9. The sphere is floating in static equilibrium, so the upward buoyant force exactly equals the sphere's weight: $F_B = w$. But according to Archimedes' principle, F_B is the weight of the displaced liquid. That is, the weight of the missing water in B is exactly matched by the weight of the added ball. Thus, the total weights of both containers are equal.

14.10. The lower the velocity of fluid, the higher the pressure at a given point in the pipe (Bernoulli's principle). The pressure p in the horizontal liquid-containing pipe is $p = p_s + \rho g d$, where d is the depth of the liquid in the vertical pipes a, b, or c and p_s is the gas pressure at the liquid surface. Thus, the vertical pipes with the highest liquid level d have the lowest surface pressure p_s, or the highest gas velocity. Therefore, the gas velocities are $v_b > v_a > v_c$.

14.11. The pressure is reduced at the chimney due to the movement of the wind above (Bernoulli's principle). Thus, the air will flow in the window and out the chimney.
Prairie dogs ventilate their burrows this way; a small breeze above their mound lowers the pressure there and allows the air in the burrow to move between openings of different types or heights.

14.12. Equation 14.34 is $F/A = Y(\Delta L/L)$. The second wire is the same material, and has length $L' = 2L$ and area $A' = \pi(2r)^2 = 4\pi r^2 = 4A$. The force required to stretch it the same length $\Delta L = 1 \text{ mm}$ is
$$F' = A'Y\frac{\Delta L}{L'} = 4AY\frac{\Delta L}{2L} = 2\left(AY\frac{\Delta L}{L}\right) = 2F$$

So the force required is 4000 N.

14.13. The stress F/A on the wire is proportional to the strain $\Delta L/L$ in the elastic region. The breaking point is past the elastic limit. The elastic limit is reached at a particular strain. Since the wire is the same diameter the area A stays the same, so an equal force of 5000 N will also take the wire to its elastic limit.

Exercises and Problems

Exercises
Section 14.1 Fluids
14.1. Solve: The volume of the liquid is
$$\rho = \frac{m}{V} \quad \Rightarrow \quad V = \frac{m}{\rho} = \frac{0.055 \text{ kg}}{1100 \text{ kg/m}^3}\left(\frac{10^6 \text{ mL}}{\text{m}^3}\right) = 50 \text{ mL}$$

Assess: The liquid's density is slightly higher than that of water (1000 kg/m^3), so it is reasonable that it requires slightly less than 55 mL to get a mass of 55 g.

14.2. Model: Model the helium as an ideal gas.
Visualize: Look up the densities of helium gas and glycerin in Table 14.1. Since the cylinders have the same height, the volumes scale with the radius squared, $V_A = 4V_B$.
Solve:

$$\frac{m_B}{m_A} = \frac{V_B \rho_B}{V_A \rho_A} = \left(\frac{V_B}{V_A}\right)\frac{\rho_B}{\rho_A} = \frac{1}{4}\frac{(1260 \text{ kg/m}^3)}{0.18 \text{ kg/m}^3} = 1800$$

Assess: Helium at 1 atm pressure is not very dense.

14.3. Model: The densities of gasoline and water are given in Table 14.1.
Solve: (a) The total mass is
$$m_{\text{total}} = m_{\text{gasoline}} + m_{\text{water}} = 0.050 \text{ kg} + 0.050 \text{ kg} = 0.100 \text{ kg}$$

The total volume is
$$V_{\text{total}} = V_{\text{gasoline}} + V_{\text{water}} = \frac{m_{\text{gasoline}}}{\rho_{\text{gasoline}}} + \frac{m_{\text{water}}}{\rho_{\text{water}}} = \frac{0.050 \text{ kg}}{680 \text{ kg/m}^3} + \frac{0.050 \text{ kg}}{1000 \text{ kg/m}^3} = 1.24\times10^{-4} \text{ m}^3$$

$$\rho_{\text{avg}} = \frac{m_{\text{total}}}{V_{\text{total}}} = \frac{0.100 \text{ kg}}{1.24\times10^{-4} \text{ m}^3} = 8.1\times10^2 \text{ kg/m}^3$$
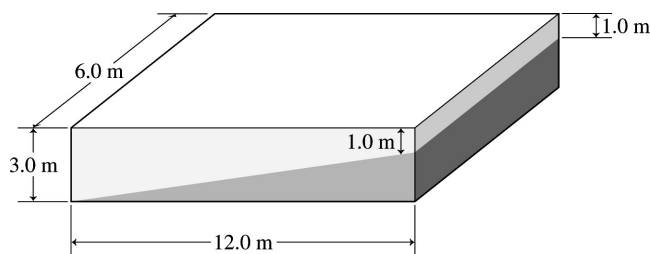
(b) The average density is calculated as follows:
$$m_{\text{total}} = m_{\text{gasoline}} + m_{\text{water}} = \rho_{\text{water}}V_{\text{water}} + \rho_{\text{gasoline}}V_{\text{gasoline}}$$

$$\rho_{\text{avg}} = \frac{\rho_{\text{water}}V_{\text{water}} + \rho_{\text{gasoline}}V_{\text{gasoline}}}{V_{\text{water}} + V_{\text{gasoline}}} = \frac{(50 \text{ cm}^3)(1000 \text{ kg/m}^3 + 680 \text{ kg/m}^3)}{100 \text{ cm}^3} = 8.4\times10^2 \text{ kg/m}^3$$

Assess: The average densities above are between those of gasoline and water, which is reasonable.

14.4. Model: The density of water is 1000 kg/m^3.
Visualize:

Solve: Volume of water in the swimming pool is
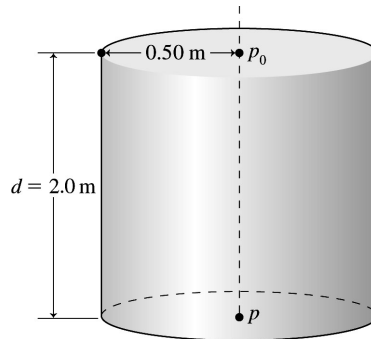$$V = 6.0 \text{ m}\times12 \text{ m}\times3.0 \text{ m} - \tfrac{1}{2}(6.0 \text{ m}\times12 \text{ m}\times2.0 \text{ m}) = 144 \text{ m}^3$$

The mass of water in the swimming pool is
$$m = \rho V = (1000 \text{ kg/m}^3)(144 \text{ m}^3) = 1.4\times10^5 \text{ kg}$$

Section 14.2 Pressure

14.5. Visualize:

Solve: The pressure at the bottom of the vat is $p = p_0 + \rho g d = 1.3$ atm. Substituting into this equation gives

$$1.013 \times 10^5 \text{ Pa} + \rho(9.8 \text{ m/s}^2)(2.0 \text{ m}) = (1.3)(1.013 \times 10^5) \text{ Pa} \Rightarrow \rho = 1550.5 \text{ kg/m}^3$$

The mass of the liquid in the vat is

$$m = \rho V = \rho \pi (0.50 \text{ m})^2 d = (1550.5 \text{ kg/m}^3)\pi(0.50 \text{ m})^2 (2.0 \text{ m}) = 2.4 \times 10^3 \text{ kg}$$

14.6. Model: The density of sea water is 1030 kg/m^3.

Solve: The pressure below sea level can be found from Equation 14.5 as follows:

$$p = p_0 + \rho g d = 1.013 \times 10^5 \text{ Pa} + (1030 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.1 \times 10^4 \text{ m})$$

$$= 1.013 \times 10^5 \text{ Pa} + 1.1103 \times 10^8 \text{ Pa} = 1.1113 \times 10^8 \text{ Pa} = 1.1 \times 10^3 \text{ atm}$$

where we have used the conversion 1 atm $= 1.013 \times 10^5$ Pa.

Assess: The pressure deep in the ocean is very large.

14.7. Model: Assume the mercury is in hydrostatic equilibrium.

Visualize: We are given the radius of the cylinder, but we don't need it. We look up in the table the density of mercury: $\rho_{\text{Hg}} = 13{,}600$ kg/m^3.

Solve:

$$p = p_0 + \rho g d \Rightarrow d = \frac{p - p_0}{\rho g} = \frac{50 \text{ kPa}}{(13{,}600 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 38 \text{ cm}$$

Assess: This is a reasonable height for a 3.0-cm-diameter tube.

14.8. Model: The density of water is 1000 kg/m^3 and the density of ethyl alcohol is 790 kg/m^3.

Solve: (a) The volume of water that has the same mass as 8.0 m^3 of ethyl alcohol is

$$V_{\text{water}} = \frac{m_{\text{water}}}{\rho_{\text{water}}} = \frac{m_{\text{alcohol}}}{\rho_{\text{water}}} = \frac{\rho_{\text{alcohol}} V_{\text{alcohol}}}{\rho_{\text{water}}} = \left(\frac{790 \text{ kg/m}^3}{1000 \text{ kg/m}^3}\right)(8.0 \text{ m}^3) = 6.3 \text{ m}^3$$
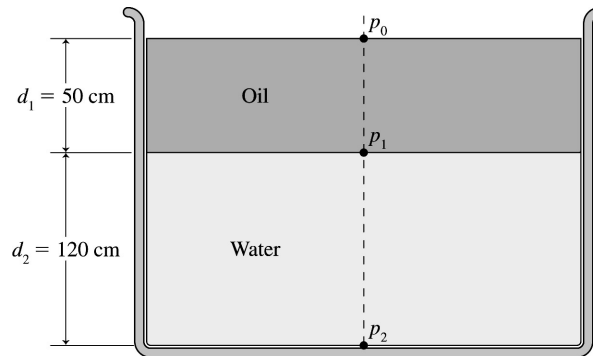
(b) The pressure at the bottom of the cubic tank is $p = p_0 + \rho_{\text{water}} g d$:

$$p = 1.013 \times 10^5 \text{ Pa} + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(6.3)^{1/3} = 1.2 \times 10^5 \text{ Pa}$$

where we have used the relation $d = (V_{\text{water}})^{1/3}$.

14.9. Model: The density of oil is $\rho_{oil} = 900$ kg/m^3 and the density of water is $\rho_{water} = 1000$ kg/m^3.

Visualize:

Solve: The pressure at the bottom of the oil layer is $p_1 = p_0 + \rho_{oil}gd_1$, and the pressure at the bottom of the water layer is
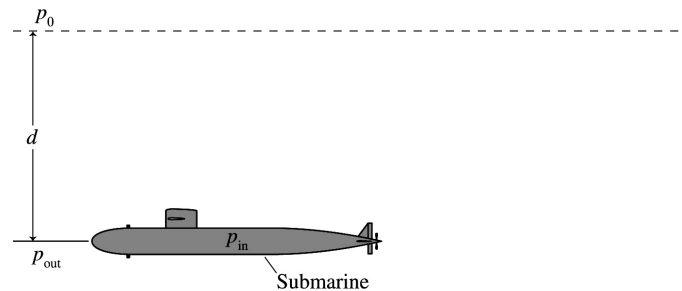
$$p_2 = p_1 + \rho_{water}gd_2 = p_0 + \rho_{oil}gd_1 + \rho_{water}gd_2$$

$$p_2 = (1.013 \times 10^5 \text{ Pa}) + (900 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.5 \text{ m}) + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1.2 \text{ m}) = 1.2 \times 10^5 \text{ Pa}$$

Assess: A pressure of 1.2×10^5 Pa $= 1.2$ atm is reasonable.

14.10. Model: The density of seawater is $\rho_{seawater} = 1030$ kg/m^2.

Visualize:

Solve: The pressure outside the submarine's window is $p_{out} = p_0 + \rho_{seawater}gd$, where d is the maximum safe depth for the window to withstand a force F. This force is $F/A = p_{out} - p_{in}$, where A is the area of the window. With $p_{in} = p_0$, we simplify the pressure equation to

$$p_{out} - p_0 = \frac{F}{A} = \rho_{seawater}gd \quad \Rightarrow \quad d = \frac{F}{A\rho_{seawater}g}$$
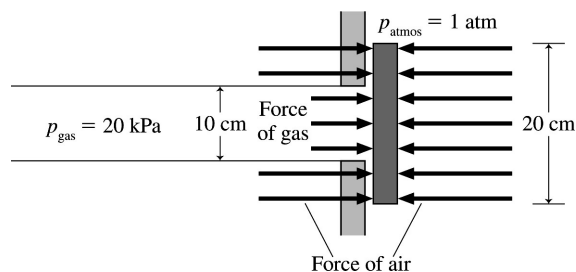
$$d = \frac{1.0 \times 10^6 \text{ N}}{\pi(0.10 \text{ m})^2(1030 \text{ kg/m}^2)(9.8 \text{ m/s}^2)} = 3.2 \text{ km}$$

Assess: A force of 1.0×10^6 N corresponds to a pressure of

$$\rho = \frac{F}{A} = \frac{1.0 \times 10^6 \text{ N}}{\pi(0.10 \text{ m})^2} = 314 \text{ atm}$$

A depth of 3 km is therefore reasonable.

14.11. Visualize:

We assume that the seal is at a radius of 5 cm. Outside the seal, atmospheric pressure presses on *both* sides of the cover and the forces cancel. Thus, only the 10-cm-diameter opening inside the seal is relevant, not the 20 cm diameter of the cover.

Solve: Within the 10 cm diameter area where the pressures differ,

$$F_{\text{to left}} = p_{\text{atmos}}A \qquad F_{\text{to right}} = p_{\text{gas}}A$$

where $A = \pi r^2 = 7.85 \times 10^{-3}$ m^2 is the area of the opening. The difference between the forces is

$$F_{\text{to left}} - F_{\text{to right}} = (p_{\text{atmos}} - p_{\text{gas}})A = (101{,}300 \text{ Pa} - 20{,}000 \text{ Pa})(7.85 \times 10^{-3} \text{ m}^2) = 0.64 \text{ kN}$$

Normally, the rubber seal exerts a 0.64 kN force to the right to balance the air pressure force. To pull the cover off, an external force must pull to the right with a force > 0.64 kN.

14.12. Model: Oil is incompressible and has a density 900 kg/m^3.

Visualize: Please refer to Figure EX14.12.

Solve: (a) The pressure at point A, which is 0.50 m below the open oil surface, is

$$p_A = p_0 + \rho_{\text{oil}}g(1.00 \text{ m} - 0.50 \text{ m}) = 101{,}300 \text{ Pa} + (900 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.50 \text{ m}) = 106 \text{ kPa}$$

(b) The pressure difference between A and B is

$$p_B - p_A = (p_0 + \rho g d_B) - (p_0 + \rho g d_A) = \rho g(d_B - d_A) = (900 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.50 \text{ m}) = 4.4 \text{ kPa}$$

Pressure depends only on depth, and C is the same depth as B. Thus $p_C - p_A = 4.4$ kPa also, even though C isn't directly under A.

Section 14.3 Measuring and Using Pressure

14.13. Model: The density of water is $\rho = 1000$ kg/m^3.

Visualize: Please refer to Figure 14.14.

Solve: From the figure and the equation for hydrostatic pressure, we have
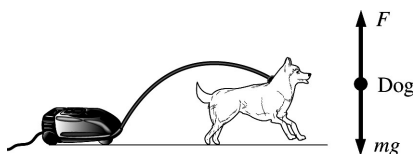
$$p_0 + \rho g h = p_{\text{atmos}}$$

Using $p_0 = 0$ atm, and $p_{\text{atmos}} = 1.013 \times 10^5$ Pa, we get

$$0 \text{ Pa} + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)h = 1.013 \times 10^5 \text{ Pa} \quad \Rightarrow \quad h = 10.3 \text{ m}$$

Assess: This large value of h is due to water having a much smaller density than mercury.

14.14. Model: Assume that the vacuum cleaner can create zero pressure.

Visualize:

Solve: The gravitational force on the dog is balanced by the force resulting from the pressure difference between the atmosphere and the vacuum $(p_{\text{hose}} = 0)$ in the hose. The force applied by the hose is

$$F = (p_{\text{atmos}} - p_{\text{hose}})A = p_{\text{atmos}}A = mg$$

$$A = \frac{(10 \text{ kg})(9.8 \text{ m/s}^2)}{1.013 \times 10^5 \text{ Pa}} = 9.7 \times 10^{-4} \text{ m}^2$$

Since $A = \pi(d/2)^2$, the diameter of the hose is $d = 2\sqrt{A/\pi} = 0.035 \text{ m} = 3.5 \text{ cm}$.

14.15. Model: Assume that the oil is incompressible. Its density is 900 kg/m^3.

Visualize: Please refer to Figure 14.16. Because the liquid is incompressible, the volume displaced in the left cylinder of the hydraulic lift is equal to the volume displaced in the right cylinder.
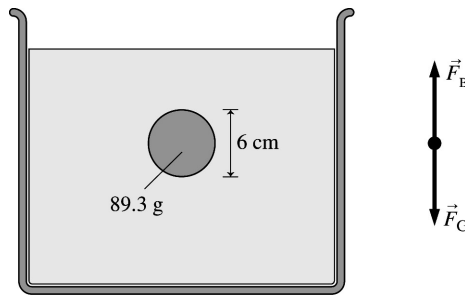
Solve: Equating the two volumes,

$$A_1 d_1 = A_2 d_2 \Rightarrow (\pi r_1^2)d_1 = (\pi r_2^2)d_2 \quad \Rightarrow \quad d_1 = \left(\frac{r_2}{r_1}\right)^2 d_2 = \left(\frac{0.040 \text{ m}}{0.010 \text{ m}}\right)^2 (0.20 \text{ m}) = 3.2 \text{ m}$$

Section 14.4 Buoyancy

14.16. Model: The buoyant force on the sphere is given by Archimedes' principle.

Visualize:

Solve: The sphere is in static equilibrium because it is neutrally buoyant. That is,

$$\Sigma F_y = F_B - F_G = 0 \text{ N} \quad \Rightarrow \quad \rho_l V_l g - m_s g = 0 \text{ N}$$

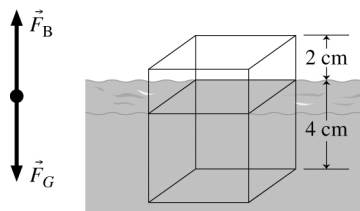The sphere displaces a volume of liquid equal to its own volume, $V_l = V_s$, so

$$\rho_l = \frac{m_s}{V_s} = \frac{m_s}{\frac{4}{3}\pi r_s^3} = \frac{0.0893 \text{ kg}}{\frac{4}{3}\pi(0.030 \text{ m})^3} = 7.9 \times 10^2 \text{ kg/m}^3$$

A density of 790 kg/m^3 in Table 14.1 identifies the liquid as ethyl alcohol.

Assess: If the density of the fluid and an object are equal, we have neutral buoyancy.

14.17. Model: The buoyant force on the block is given by Archimedes' principle.

Visualize:

V_{cyl} is the volume of the block and V_w is the volume of the water displaced by the block. Note that the volume displaced is only from the part of the block that is immersed in water.

Solve: The block is in static equilibrium, so $F_B = F_G$. The buoyant force is the weight $\rho_w V_w g$ of the displaced water. Thus

$$F_B = \rho_w V_w g = F_G = mg = \rho_{bl} V_{bl} g \quad \Rightarrow \quad \rho_w V_w = \rho_{bl} V_{bl} \quad \Rightarrow \quad \rho_{cyl} = \rho_w \frac{V_w}{V_{bl}}$$
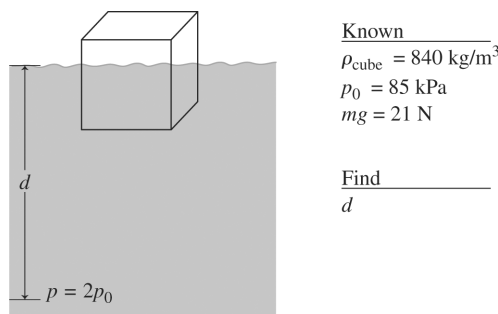
$$\rho_{bl} = (1000 \text{ kg/m}^3) \frac{A(0.040 \text{ m})}{A(0.060 \text{ m})} = 6.7 \times 10^2 \text{ kg/m}^3$$

where A is the cross-sectional area of the block.

Assess: $\rho_{cyl} < \rho_w$ for a block floating in water is an expected result.

14.18. Model: Since the cube floats in the lake 2/3 submerged, the cube's density is 2/3 of the density of the liquid in the lake.

Visualize:

Known
$\rho_{cube} = 840 \text{ kg/m}^3$
$p_0 = 85 \text{ kPa}$
$mg = 21 \text{ N}$

Find
d

Solve: Start with the pressure at a depth d in a liquid: $p = p_0 + \rho g d$. We want to know the depth at which $p = 2p_0$.

$$p = 2p_0 \Rightarrow p_0 = \rho_{liquid} g d \Rightarrow d = \frac{p_0}{\rho_{liquid} g}$$

We are given $p_0 = 85 \text{ kPa}$ but we must find ρ_{liquid} and g.

$$\rho_{liquid} = \tfrac{3}{2} \rho_{cube} = \tfrac{3}{2}(840 \text{ kg/m}^3) = 1260 \text{ kg/m}^3$$

According to the table this liquid could be glycerin. Now find g on this planet.

$$mg = 21 \text{ N} \quad \Rightarrow \quad g = \frac{21 \text{ N}}{\rho_{cube} V_{cube}} = \frac{21 \text{ N}}{(840 \text{ kg/m}^3)(0.060 \text{ m})^3} = 116 \text{ m/s}^2$$

Now put it all together.
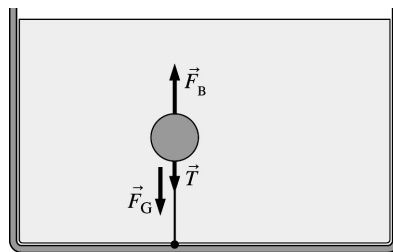
$$d = \frac{p_0}{\rho_{liquid} g} = \frac{85 \text{ kPa}}{(1260 \text{ kg/m}^3)(116 \text{ m/s}^2)} = 58 \text{ cm}$$

That is, at a depth of 58 cm the pressure in the liquid in the lake is twice the atmospheric pressure.

Assess: The acceleration of objects in free fall is quite large on this planet.

14.19. Model: The buoyant force on the sphere is given by Archimedes' principle.
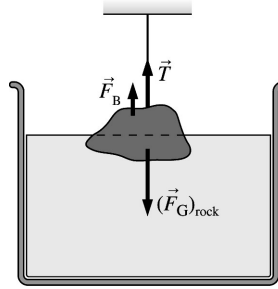
Visualize:

Solve: The sphere is in static equilibrium. The free-body diagram on the sphere shows that

$$\sum F_y = F_B - T - F_G = 0 \text{ N} \quad \Rightarrow \quad F_B = T + F_G = \frac{1}{3}F_G + F_G = \frac{4}{3}F_G$$

$$\rho_w V_{sphere} g = \frac{4}{3}\rho_{sphere} V_{sphere} g \quad \Rightarrow \quad \rho_{sphere} = \frac{3}{4}\rho_w = \frac{3}{4}(1000 \text{ kg/m}^3) = 750 \text{ kg/m}^3$$

14.20. Model: The buoyant force on the rock is given by Archimedes' principle.
Visualize:

**Solve:** Because the rock is in static equilibrium, Newton's first law gives

$$F_{net} = T + F_B - (F_G)_{rock} = 0 \text{ N}$$

$$T = \rho_{rock} V_{rock} g - \rho_{water}\left(\frac{1}{2}V_{rock}\right)g = \left(\rho_{rock} - \frac{1}{2}\rho_{water}\right)V_{rock} g = \left(\rho_{rock} - \frac{1}{2}\rho_{water}\right)\left(\frac{m_{rock} g}{\rho_{rock}}\right) = \left(1 - \frac{\rho_{water}}{2\rho_{rock}}\right)m_{rock} g$$

Using $\rho_{rock} = 4800 \text{ kg/m}^3$ and $m_{rock} = 5.0 \text{ kg}$, we get $T = 44 \text{ N}$.

14.21. Model: The buoyant force on the aluminum block is given by Archimedes' principle. The density of aluminum and ethyl alcohol are $\rho_{Al} = 2700 \text{ kg/m}^3$ and $\rho_{ethyl\ alcohol} = 790 \text{ kg/m}^3$.
Visualize:
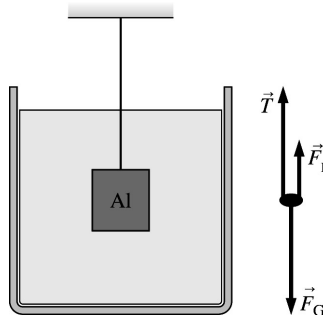
The buoyant force F_B and the tension due to the string act vertically up, and the gravitational force on the aluminum block acts vertically down. The block is submerged, so the volume of displaced fluid equals V_{Al}, the volume of the block.
Solve: The aluminum block is in static equilibrium, so

$$\sum F_y = F_B + T - F_G = 0 \text{ N} \quad \Rightarrow \quad \rho_f V_{Al} g + T - \rho_{Al} V_{Al} g = 0 \text{ N} \quad \Rightarrow \quad T = V_{Al} g(\rho_{Al} - \rho_f)$$
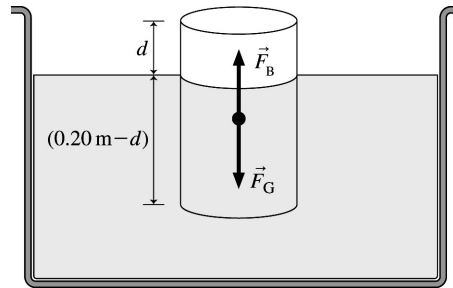
$$T = (100 \times 10^{-6} \text{ m}^3)(9.81 \text{ m/s}^2)(2700 \text{ kg/m}^3 - 790 \text{ kg/m}^3) = 1.9 \text{ N}$$

where we have used the conversion $100 \text{ cm}^3 = 100 \times (10^{-2} \text{ m})^3 = 10^{-4} \text{ m}^3$.

Assess: The gravitational force on the aluminum block is $\rho_{Al} V_{Al} g = 2.65 \text{ N}$. A similar order of magnitude for T is reasonable.

14.22. Model: The buoyant force on the steel cylinder is given by Archimedes' principle.
Visualize:

The length of the cylinder above the surface of mercury is d.
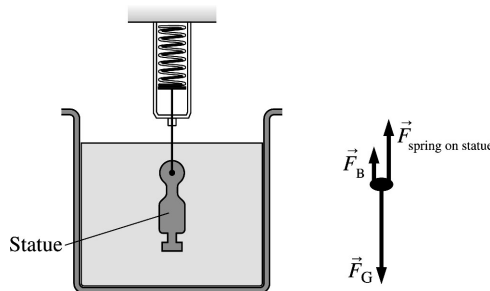**Solve:** The cylinder is in static equilibrium with $F_B = F_G$. Thus

$$F_B = \rho_{Hg} V_{Hg} g = F_G = mg = \rho_{cyl} V_{cyl} g \quad \Rightarrow \quad \rho_{Hg} V_{Hg} = \rho_{cyl} V_{cyl} \quad \Rightarrow \quad \rho_{Hg} A(0.20 \text{ m} - d) = \rho_{cyl} A(0.20 \text{ m})$$

$$d = 0.20 \text{ m} - \frac{\rho_{cyl}}{\rho_{Hg}}(0.20 \text{ m}) = (0.20 \text{ m})\left(1 - \frac{7900 \text{ kg/m}^3}{13,600 \text{ kg/m}^3}\right) = 0.084 \text{ m} = 8.4 \text{ cm}$$

That is, the length of the cylinder above the surface of the mercury is 8.4 cm.

14.23. Model: The buoyant force on the ceramic statue is given by Archimedes' principle.
**Visualize:**

Solve: The gravitational force on the statue is the 28.4 N registered on the scale in air. In water, the gravitational force on the statue is balanced by the sum of the buoyant force F_B and the spring's force on the statue; that is,

$$(F_G)_{statue} = F_B + F_{spring \text{ on statue}} \quad \Rightarrow \quad 28.4 \text{ N} = \rho_w V_{statue} g + 17.0 \text{ N} \quad \Rightarrow \quad V_{statue} = \frac{11.4 \text{ N}}{g\rho_w} = \frac{m_{statue}}{\rho_{statue}}$$

$$\rho_{statue} = \frac{(m_{statue}g)\rho_w}{(11.4 \text{ N})} = \frac{(28.4 \text{ N})(1000 \text{ kg/m}^3)}{(11.4 \text{ N})} = 2.49 \times 10^3 \text{ kg/m}^3$$

14.24. Model: The buoyant force on the sphere is given by Archimedes' principle.
Visualize:
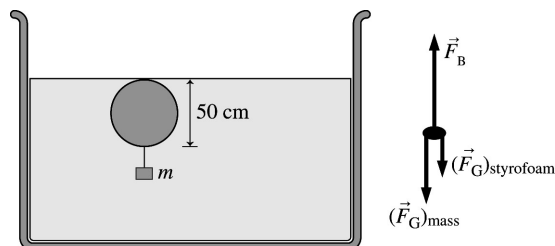
Solve: For the Styrofoam sphere and the mass not to sink, the sphere must be completely submerged and the buoyant force F_B must be equal to the sum of the gravitational force on the Styrofoam sphere and the attached mass. Neglecting the volume of the hanging mass, the volume of displaced water equals the volume of the sphere, so

$$F_B = \rho_{water} V_{water} g = (1000 \text{ kg/m}^3)\left(\frac{4\pi}{3}\right)(0.25 \text{ m})^3 (9.8 \text{ m/s}^2) = 641.4 \text{ N}$$

$$(F_G)_{Styrofoam} = \rho_{Styrofoam} V_{Styrofoam} g = (150 \text{ kg/m}^3)\left[\frac{4}{3}\pi(0.25 \text{ m})^3\right](9.8 \text{ m/s}^2) = 96.2 \text{ N}$$

Because $(F_G)_{Styrofoam} + mg = F_B$,

$$m = \frac{F_B - (F_G)_{Styrofoam}}{g} = \frac{641.4 \text{ N} - 96.2 \text{ N}}{9.8 \text{ m/s}^2} = 55.6 \text{ kg}$$

To two significant figures, the mass is 56 kg.

14.25. Model: The buoyant force is determined by Archimedes' principle. Ignore any compression the air in the beach ball may undergo as a result of submersion.
Solve: The mass of the beach ball is negligible, so the force needed to push it below the water is equal to the buoyant force.

$$F_B = p_w\left(\frac{4}{3}\pi R^3\right)g = (1000 \text{ kg/m}^3)\left[\frac{4}{3}\pi(0.30 \text{ m})^3\right](9.8 \text{ m/s}^2) = 1.1 \text{ kN}$$

Assess: It would take a 113 kg (250 lb) person to push the ball below the water. Two people together could do it. This seems about right.

Section 14.5 Fluid Dynamics
14.26. Visualize: Q is the volume flow rate, so $Q = V/\Delta t$. We are given $V = 600$ L and $v = 3.0$ m/s.
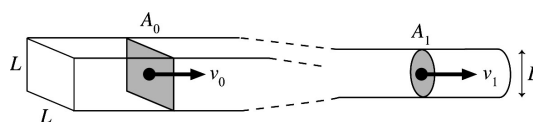**Solve:**

$$Q = vA = \frac{V}{\Delta t} \Rightarrow$$

$$\Delta t = \frac{V}{vA} = \frac{600 \text{ L}}{(3.0 \text{ m/s})\pi(0.0125 \text{ m})^2}\left(\frac{1000 \text{ mL}}{1 \text{ L}}\right)\left(\frac{1 \text{ m}^3}{1,000,000 \text{ mL}}\right) = 410 \text{ s} = 6.8 \text{ min}$$

Assess: 6.8 minutes seems like a realistic amount of time to fill a child's wading pool.

14.27. Model: Treat the water as an ideal fluid so that the flow in the tube follows the continuity equation.
Visualize:

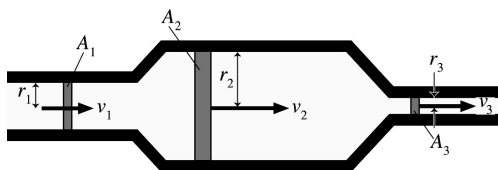**Solve:** The equation of continuity is $v_0 A_0 = v_1 A_1$, where $A_0 = L^2$ and $A_1 = \pi\left(\frac{1}{2}L\right)^2$. The above equation simplifies to

$$v_0 L^2 = v_1\pi\left(\frac{L}{2}\right)^2 \quad \Rightarrow \quad v_1 = \left(\frac{4}{\pi}\right)v_0 = 1.27 v_0$$

14.28. Model: Treat the water as an ideal fluid. The pipe itself is a flow tube, so the equation of continuity applies.
Visualize:

Note that A_1, A_2, and A_3 and v_1, v_2, and v_3 are the cross-sectional areas and the speeds in the first, second, and third segments of the pipe, respectively.

Solve: (a) The equation of continuity is

$$A_1v_1 = A_2v_2 = A_3v_3 \quad \Rightarrow \quad \pi r_1^2 v_1 = \pi r_2^2 v_2 = \pi r_3^2 v_3 \quad \Rightarrow \quad r_1^2 v_1 = r_2^2 v_2 = r_3^2 v_3$$

$$(0.0050 \text{ m})^2(4.0 \text{ m/s}) = (0.010 \text{ m})^2 v_2 = (0.0025 \text{ m})^2 v_3$$
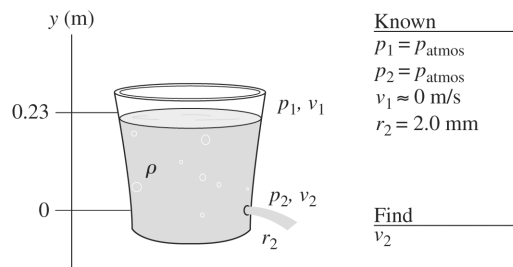
$$v_2 = \left(\frac{0.0050 \text{ m}}{0.010 \text{ m}}\right)^2 (4.0 \text{ m/s}) = 1.0 \text{ m/s} \qquad v_3 = \left(\frac{0.0050 \text{ m}}{0.0025 \text{ m}}\right)^2 (4.0 \text{ m/s}) = 16 \text{ m/s}$$

(b) The volume flow rate through the pipe is

$$Q = A_1v_1 = \pi(0.0050 \text{ m})^2(4.0 \text{ m/s}) = 3.1\times10^{-4} \text{ m}^3/\text{s}$$

14.29. Model: Treat the water as an ideal fluid obeying Bernoulli's equation. The pressure at the surface is $p_1 = p_{atmos}$ and $v_1 \approx 0$ m/s. The water discharges into air, so $p_2 = p_{atmos}$ at the exit.

Visualize:

Solve: Use Bernoulli's equation with $v_1 \approx 0$ m/s and $p_1 = p_2$ (so that term drops off both sides).

$$\rho g y_1 = \tfrac{1}{2}\rho v_2^2 \Rightarrow v_2 = \sqrt{2gy_1} = \sqrt{2(9.8 \text{ m/s}^2)(0.23 \text{ m})} = 2.1 \text{ m/s}$$

Assess: The water's exit speed from the hole is the same as if it fell 23 cm from the surface of the bucket. This is not surprising since we assumed a nonviscous liquid. We did not need to know the size of the hole at the bottom.

14.30. Model: Treat the oil as an ideal fluid obeying Bernoulli's equation. Consider the path connecting point 1 in the lower pipe with point 2 in the upper pipe a streamline.
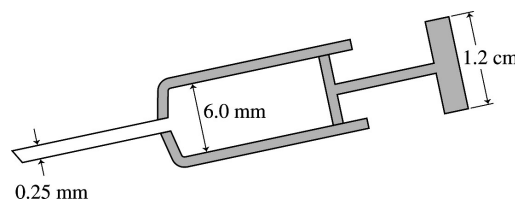
Visualize: Please refer to Figure EX14.30.

Solve: Bernoulli's equation is

$$p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 = p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 \quad \Rightarrow \quad p_2 = p_1 + \tfrac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2)$$

Using $p_1 = 200$ kPa $= 2.0\times10^5$ Pa, $\rho = 900$ kg/m^3, $y_2 - y_1 = 10.0$ m, $v_1 = 2.0$ m/s, and $v_2 = 3.0$ m/s, we get $p_2 = 1.096\times10^5$ Pa $= 110$ kPa.

14.31. Visualize:

Solve: (a) Because the patient's blood pressure is 140/100, the minimum fluid pressure needs to be 100 mm of Hg above atmospheric pressure. Since 760 mm of Hg is equivalent to 1 atm and 1 atm is equivalent to 1.013×10^5 Pa, the minimum

pressure is $100 \text{ mm} = 1.333 \times 10^4$ Pa. The excess pressure in the fluid is due to force F pushing on the internal 6.0-mm-diameter piston that presses against the liquid. Thus, the minimum force the nurse needs to apply to the syringe is

$$F = \text{ fluid pressure} \times \text{area of plunger} = (1.333 \times 10^4 \text{ Pa}) \left[\pi (0.0030 \text{ m})^2 \right] = 0.38 \text{ N}$$

(b) The flow rate is $Q = vA$, where v is the flow speed of the medicine and A is the cross-sectional area of the needle. Thus,

$$v = \frac{Q}{A} = \frac{2.0 \times 10^{-6} \text{ m}^3/2.0 \text{ s}}{\pi (0.125 \times 10^{-3} \text{ m})^2} = 20 \text{ m/s}$$

Assess: Note that the pressure in the fluid is due to F that is not dependent on the size of the plunger pad. Also note that the syringe is not drawn to scale.

Section 14.6 Elasticity
14.32. Model: The dangling mountain climber creates tensile stress in the rope.
Solve: Young's modulus for the rope is

$$Y = \frac{F/A}{\Delta L/L} = \frac{\text{stress}}{\text{strain}}$$

The tensile stress is

$$\frac{(70 \text{ kg})(9.8 \text{ m/s}^2)}{\pi (0.0050 \text{ m})^2} = 8.734 \times 10^6 \text{ Pa}$$

and the strain is $0.080 \text{ m}/50 \text{ m} = 0.00160$. Dividing the two quantities yields $Y = 5.5 \times 10^9 \text{ N/m}^2$.

14.33. Model: Turning the tuning screws on a guitar string creates tensile stress in the string.
Solve: The tensile stress in the string is given by T/A, where T is the tension in the string and A is the cross-sectional area of the string. From the definition of Young's modulus,

$$Y = \frac{T/A}{\Delta L/L} \quad \Rightarrow \quad \Delta L = \frac{T}{A}\left(\frac{L}{Y}\right)$$

Using $T = 2000 \text{ N}$, $L = 0.80 \text{ m}$, $A = \pi (0.00050 \text{ m})^2$, and $Y = 20 \times 10^{10} \text{ N/m}^2$ (from Table 14.3), we obtain $\Delta L = 0.010 \text{ m} = 1.0 \text{ cm}$.
Assess: 1.0 cm is a large stretch for a length of 80 cm, but 2000 N is a large tension.

14.34. Model: The load supported by a concrete column creates compressive stress in the concrete column.
Solve: The gravitational force on the load produces tensile stress given by F/A, where A is the cross-sectional area of the concrete column and F equals the gravitational force on the load. From the definition of Young's modulus,

$$Y = \frac{F/A}{\Delta L/L} \quad \Rightarrow \quad \Delta L = \left(\frac{F}{A}\right)\left(\frac{L}{Y}\right) = \left[\frac{(200,000 \text{ kg})(9.8 \text{ m/s}^2)}{\pi (0.25 \text{ m})^2}\right]\left(\frac{3.0 \text{ m}}{3 \times 10^{10} \text{ N/m}^2}\right) = 1 \text{ mm}$$

Assess: A compression of 1.0 mm of the concrete column by a load of approximately 200 tons is reasonable.

14.35. Model: Water is almost incompressible and it applies a volume stress.
Solve: (a) The pressure at a depth of 5000 m in the ocean is

$$p = p_0 + \rho_{\text{sea water}}g(5000 \text{ m}) = 1.013 \times 10^5 \text{ Pa} + (1030 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(5000 \text{ m}) = 5.057 \times 10^7 \text{ Pa} = 5.1 \times 10^7 \text{ Pa}$$

(b) Using the bulk modulus of water,

$$\frac{\Delta V}{V} = -\frac{p}{B} = -\frac{5.057 \times 10^7 \text{ Pa}}{0.2 \times 10^{10} \text{ Pa}} = -0.025$$

(c) The volume of a mass of water decreases from V to $0.975V$. Thus the water's density increases from ρ to $\rho/0.975$. The new density is

$$\rho_{5000 \text{ m}} = \frac{1030 \text{ kg/m}^3}{0.975} = 1056 \text{ kg/m}^3$$

14.36. Model: The aquarium creates tensile stress.
Solve: Weight of the aquarium is

$$F_G = mg = \rho_{water} V g = (1000 \text{ kg/m}^3)(10 \text{ m}^3)(9.8 \text{ m/s}^2) = 9.8 \times 10^4 \text{ N}$$

where we have used the conversion $1 \text{ L} = 10^{-3} \text{ m}^3$. The weight supported by each wood post is $\frac{1}{4}(9.8 \times 10^4 \text{ N}) =$
$2.45 \times 10^4 \text{ N}$. The cross-sectional area of each post is $A = (0.040 \text{ m})^2 = 1.6 \times 10^{-3} \text{ m}^2$. Young's modulus for the wood is

$$Y = 1 \times 10^{10} \text{ N/m}^2 = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

$$\Delta L = \frac{FL}{AY} = \frac{(2.45 \times 10^4 \text{ N})(0.80 \text{ m})}{(1.6 \times 10^{-3} \text{ m}^2)(1 \times 10^{10} \text{ N/m}^2)} = 1.23 \times 10^{-3} \text{ m} = 1 \text{ mm}$$

Assess: A compression of 1 mm due to a weight of $2.45 \times 10^4 \text{ N}$ is reasonable.

14.37. Model: The air pressure on earth applies a volume stress to the sphere, which is removed when the sphere is in outer space.
Visualize: Decreasing the pressure will increase the diameter of the sphere, so we remove the negative sign from the equation in the book so Δr will be positive as the sphere is launched into space. We are given $r_{sphere} = 2.5 \text{ m}$ and we look up in the table $B_{Al} = 7 \times 10^{10} \text{ N/m}^2$.
Solve:

$$\frac{F}{A} = p = B\frac{\Delta V}{V} \Rightarrow \Delta V = \frac{Vp}{B}$$

We also have from calculus $\Delta V = 4\pi r^2 \Delta r$. Set these expressions for ΔV equal to each other and solve for Δr.

$$\Delta r = \frac{Vp}{4\pi r^2 B} = \frac{\frac{4}{3}\pi r^3 p}{4\pi r^2 B} = \frac{rp}{3B} = \frac{(2.5 \text{ m})(101.3 \text{ kPa})}{3(7 \times 10^{10} \text{ N/m}^2)} = 1.2 \ \mu\text{m}$$

The diameter changes by twice this, increasing 2.4 μm.
Assess: The sphere expands by just a tiny amount, as expected.

Problems

14.38. Model: Assume that the oil is incompressible and its density is 900 kg/m^3.
Visualize: Please refer to Figure P14.38.
Solve: (a) The pressure at depth d in a fluid is $p = p_0 + \rho g d$. Here, pressure p_0 at the top of the fluid is due both to the atmosphere *and* to the gravitational force on the piston. That is, $p_0 = p_{atm} + (F_G)_p/A$. At point A,

$$p_A = p_{atm} + \frac{(F_G)_P}{A} + \rho g(1.00 \text{ m} - 0.30 \text{ m})$$

$$= 1.013 \times 10^5 \text{ Pa} + \frac{(10 \text{ kg})(9.8 \text{ m/s}^2)}{\pi(0.020 \text{ m})^2} + (900 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.70 \text{ m}) = 185,460 \text{ Pa}$$

$$F_A = p_A A = (185,460 \text{ Pa})\pi(0.10 \text{ m})^2 = 5.8 \text{ kN}$$

(b) In the same way,

$$p_B = p_{atm} + \frac{(F_G)_P}{A} + \rho g(1.30 \text{ m}) = 190,752 \text{ Pa} \quad \Rightarrow \quad F_B = 6.0 \text{ kN}$$

Assess: F_B is larger than F_A, because p_B is larger than p_A.

14.39. Model: We assume that there is a perfect vacuum inside the cylinders with $p = 0$ Pa. We also assume that the atmospheric pressure in the room is 1 atm.
Visualize: Please refer to Figure P14.39.

Solve: (a) The flat end of each cylinder has an area $A = \pi r^2 = \pi (0.30 \text{ m})^2 = 0.283 \text{ m}^2$. The force on each end is thus

$$F_{\text{atm}} = p_0 A = (1.013 \times 10^5 \text{ Pa})(0.283 \text{ m}^2) = 2.86 \times 10^4 \text{ N}$$

To two significant figures, the force on each end is 2.9×10^4 N.

(b) The net vertical force on the lower cylinder when it is on the verge of being pulled apart is

$$\Sigma F_y = F_{\text{atm}} - (F_G)_{\text{players}} = 0 \text{ N} \quad \Rightarrow \quad (F_G)_{\text{players}} = F_{\text{atm}} = 2.86 \times 10^4 \text{ N}$$

$$\text{number of players} = \frac{2.86 \times 10^4 \text{ N}}{(100 \text{ kg})(9.8 \text{ m/s}^2)} = 29.2$$

That is, 30 players are needed to pull the two cylinders apart.

14.40. Model: Assume that blood is incompressible.

Solve: When lying down, the pressure at both the heart and the brain is at 118 mm Hg (gauge pressure) or 118 mm Hg + 760 mm Hg = 878 mm Hg absolute pressure. Upon standing up, the pressure in the heart remains at 878 mm Hg, but the pressure in the brain is reduced according to the hydrostatic pressure equation:

$$p_H = p_B + \rho g h$$

where B and H refer to brain and heart, respectively. Inserting $h = 0.40$ m and solving for p_B gives

$$p_B = p_H - \rho g h = 878 \text{ mm Hg} - (1060 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.40 \text{ m})\left(\frac{760 \text{ mm Hg}}{101.3 \text{ kPa}}\right) = 847 \text{ mm Hg}$$

In gauge pressure, the blood pressure in the brain is $p_B = 847$ mm Hg $- 760$ mm Hg $= 87$ mm Hg.

Assess: This blood pressure is below the 90 mm Hg that can cause fainting, which explains why people whose blood vessels do not constrict to compensate for this effect feel faint when standing from a prone position.

14.41. Model: The tire flattens until the pressure force against the ground balances the upward normal force of the ground on the tire.

Solve: The area of the tire in contact with the road is $A = (0.15 \text{ m})(0.13 \text{ m}) = 0.0195 \text{ m}^2$. The normal force on each tire is

$$n = \frac{F_G}{4} = \frac{(1500 \text{ kg})(9.8 \text{ m/s}^2)}{4} = 3675 \text{ N}$$

Thus, the pressure inside each tire is

$$p_{\text{inside}} = \frac{n}{A} = \frac{3675 \text{ N}}{0.0195 \text{ m}^2} = 188,500 \text{ Pa} = 1.86 \text{ atm} \times \frac{14.7 \text{ psi}}{1 \text{ atm}} = 27 \text{ psi}$$

14.42. Model: Assume that oil is incompressible and its density is $\rho = 900 \text{ kg/m}^3$.

Visualize: Please refer to Figure P14.42.

Solve: (a) The hydraulic lift is in equilibrium and the pistons on the left and the right are at the same level. Therefore, Equation 14.10 simplifies to

$$\frac{F_{\text{left piston}}}{A_{\text{left piston}}} = \frac{F_{\text{right piston}}}{A_{\text{right piston}}} \quad \Rightarrow \quad \frac{(F_G)_{\text{student}}}{\pi (r_{\text{student}})^2} = \frac{(F_G)_{\text{elephant}}}{\pi (r_{\text{elephant}})^2}$$

$$r_{\text{student}} = \sqrt{\left(\frac{(F_G)_{\text{student}}}{(F_G)_{\text{elephant}}}\right)}(r_{\text{elephant}}) = \sqrt{\frac{(70 \text{ kg})g}{(1200 \text{ kg})g}}(1.0 \text{ m}) = 0.2415 \text{ m}$$

The diameter of the piston the student is standing on is therefore 2×0.2415 m $= 0.48$ m.

(b) With the second student added, the pistons are no longer at the same height. Therefore, Equation 15.10 gives
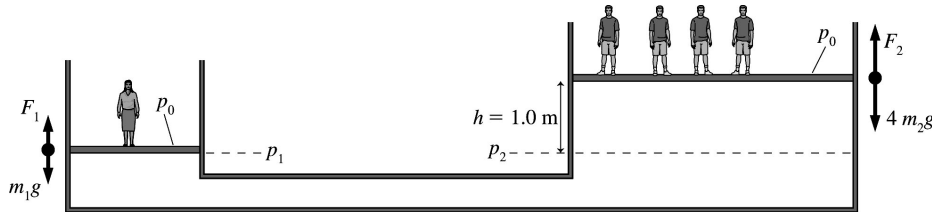
$$\frac{F_{\text{right piston}}}{A_{\text{right piston}}} = \frac{F_{\text{left piston}}}{A_{\text{left piston}}} - \rho g h$$

With $F_{\text{left piston}} = m_s g + (70 \text{ kg})g$ and $h = 0.35$ m, we can solve for the second students mass m_s:

$$m_s = \frac{m_{\text{elephant}} A_{\text{left piston}}}{A_{\text{right piston}}} - 70 \text{ kg} + \rho h A_{\text{left piston}}$$

$$= \frac{(1200 \text{ kg})(0.2415 \text{ m})^2}{(1.0 \text{ m})^2} - 70 \text{ kg} + (900 \text{ kg/m}^3)(0.35 \text{ m})\pi(0.2415 \text{ m})^2 = 58 \text{ kg}$$

14.43. Model: Assume that the oil is incompressible and its density is 900 kg/m^3.
Visualize:

Solve: The pressures p_1 and p_2 are equal. Thus,

$$p_0 + \frac{F_1}{A_1} = p_0 + \frac{F_2}{A_2} + \rho g h \quad \Rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} + \rho g h$$

With $F_1 - m_1 g$, $F_2 = 4m_2 g$, $A_1 = \pi r_1^2$, and $A_2 = \pi r_2^2$, we have

$$\frac{m_1 g}{\pi r_1^2} = \frac{4m_2 g}{\pi r_2^2} + \rho g h \quad \Rightarrow \quad r_2 = \left(\frac{4m_2 g}{\pi}\right)^{1/2}\left(\frac{m_1 g}{\pi r_1^2} - \rho g h\right)^{-1/2}$$
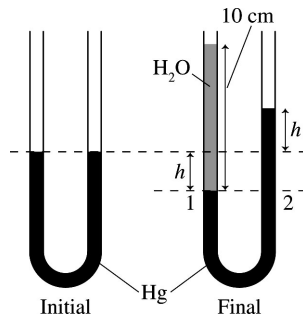
Using $m_1 = 55$ kg, $m_2 = 110$ kg, $r_1 = 0.08$ m, $\rho = 900 \text{ kg/m}^3$, and $h = 1.0$ m, the calculation yields $r_2 = 0.276$ m.
The diameter is 55 cm.
Assess: Both pistons are too small to hold the people as shown, but the ideas are correct.

14.44. Model: Water and mercury are incompressible and immiscible liquids.
Visualize:

The water in the left arm floats on top of the mercury and presses the mercury down from its initial level. Because points 1 and 2 are level with each other *and* the fluid is in static equilibrium, the pressure at these two points must be equal. If the pressures were not equal, the pressure difference would cause the fluid to flow, violating the assumption of static equilibrium.
Solve: The pressure at point 1 is due to water of depth $d_w = 10$ cm:

$$p_1 = p_{\text{atmos}} + \rho_w g d_w$$

Because mercury is incompressible, the mercury in the left arm goes down a distance h while the mercury in the right arm goes up a distance h. Thus, the pressure at point 2 is due to mercury of depth $d_{\text{Hg}} = 2h$:

$$p_2 = p_{\text{atmos}} + \rho_{\text{Hg}} g d_{\text{Hg}} = p_{\text{atmos}} + 2\rho_{\text{Hg}} g h$$

Equating p_1 and p_2 gives

$$p_{\text{atmos}} + \rho_w g d_w = p_{\text{atmos}} + 2\rho_{\text{Hg}} g h \quad \Rightarrow \quad h = \frac{1}{2}\frac{\rho_w}{\rho_{\text{Hg}}} d_w = \left(\frac{1}{2}\right)\left(\frac{1000 \text{ kg/m}^3}{13{,}600 \text{ kg/m}^3}\right)(10 \text{ cm}) = 3.7 \text{ mm}$$

The mercury in the right arm rises 3.7 mm above its initial level.

14.45. Model: Assume the liquid is incompressible.
**Visualize:**
Solve: (a) Can 2 has moved down with respect to Can 1 since the water level in Can 2 has risen. Since the total volume of water stays constant, the water level in Can 1 has fallen by the same amount. The water level is equalized in the two cans at the middle of the height change, so the change in height of the water is half the relative change in height of the cans. Can 2 has moved relative to Can 1 $(6.5 \text{ cm} - 5.0 \text{ cm}) \times 2 = 3.0 \text{ cm}$ down.

(b) The water level in Can 1 has fallen by the same amount. The new level is $5.0 - 1.5 \text{ cm} = 3.5 \text{ cm}$.
Assess: The two cans are an inexpensive method of measuring relative changes in height.

14.46. Model: Glycerin and ethyl alcohol are incompressible and do not mix.
Visualize:
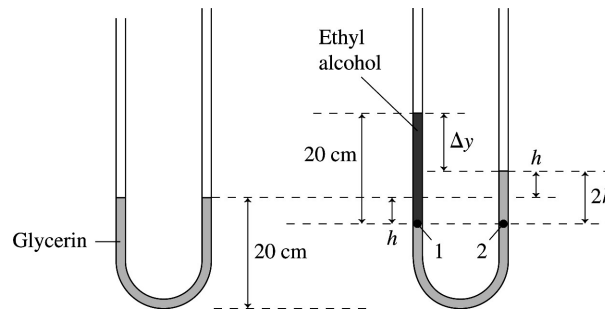
Solve: The alcohol in the left arm floats on top of the denser glycerin and presses the glycerin down distance h from its initial level. This causes the glycerin to rise distance h in the right arm. Points 1 and 2 are level with each other *and* the fluids are in static equilibrium, so the pressures at these two points must be equal:

$$p_1 = p_2 \quad \Rightarrow \quad p_0 + \rho_{\text{eth}} g d_{\text{eth}} = p_0 + \rho_{\text{gly}} g d_{\text{gly}} \quad \Rightarrow \quad \rho_{\text{eth}} g(20 \text{ cm}) = \rho_{\text{gly}} g(2h)$$

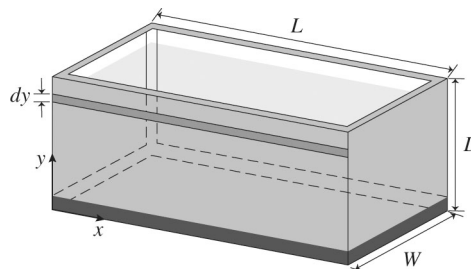$$h = \frac{1}{2}\frac{\rho_{\text{eth}}}{\rho_{\text{gly}}}(20 \text{ cm}) = \left(\frac{1}{2}\right)\left(\frac{790 \text{ kg/m}^3}{1260 \text{ kg/m}^3}\right)(20 \text{ cm}) = 6.27 \text{ cm}$$

You can see from the figure that the difference between the top surfaces of the fluids is
$$\Delta y = 20 \text{ cm} - 2h = 20 \text{ cm} - 2(6.27 \text{ cm}) = 7.46 \text{ cm} \approx 7.5 \text{ cm}$$

14.47. Model: The water is in hydrostatic equilibrium.
Visualize:

Solve: (a) The force of the liquid on the bottom is the pressure $p = \rho g D$ times the area $A = WL$. This gives
$$F = pA = \rho g DWL$$

(b) The pressure on the front panel of the aquarium can be found by integrating the force on a thin strip dy (see figure above). Using $F = pA$, we find

$$dF = \rho g y L \, dy$$

$$F = \int_0^D \rho g y L \, dy = \tfrac{1}{2}\rho g D^2 L$$

(c) For a 100-cm-long, 35-cm-wide, 40-cm-deep aquarium filled with water, the force from the liquid on the front window is
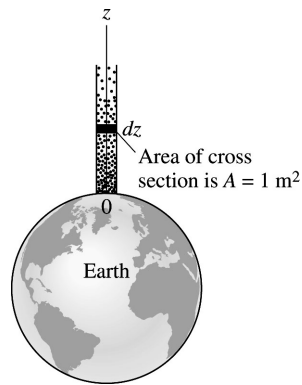
$$F = \tfrac{1}{2}(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.40 \text{ m})^2(1.0 \text{ m}) = 784 \text{ N} \approx 0.78 \text{ kN}$$

The force on the bottom is

$$F = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.40 \text{ m})(0.35 \text{ m})(1.0 \text{ m}) = 1372 \text{ N} \approx 1.4 \text{ kN}$$

Assess: The force on the bottom is significantly greater than the force on the front because the bottom must support *all* the weight of the water.

14.48. Visualize:

The figure shows a small column of air of thickness dz, of cross-sectional area $A = 1 \text{ m}^2$, and of density $\rho(z)$. The column is at a height z above the surface of the earth.

Solve: (a) The atmospheric pressure at sea level is 1.013×10^5 Pa. That is, the weight of the air column with a 1 m^2 cross section is 1.013×10^5 N. Consider the weight of a 1 m^2 slice of thickness dz at a height z. This slice has volume $dV = A \, dz = (1 \text{ m}^2) dz$, so its weight is $dw = (\rho \, dV)g = \rho g (1 \text{ m}^2) dz = \rho_0 e^{-z/z_0} g (1 \text{ m}^2) dz$. The total weight of the 1 m^2 column is found by adding all the dw. Integrating from $z = 0$ to $z = \infty$,

$$w = \int_0^\infty \rho_0 g (1 \text{ m}^2) e^{-z/z_0} dz$$

$$= \left[-\rho_0 g (1 \text{ m}^2) z_0 \right]\left[e^{-z/z_0} \right]_0^\infty$$

$$= \rho_0 g (1 \text{ m}^2) z_0$$

Because $w = 101{,}300 \text{ N} = \rho_0 g(1 \text{ m}^2) z_0$,

$$z_0 = \frac{101{,}300 \text{ N}}{(1.28 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1.0 \text{ m}^2)} = 8.1 \times 10^3 \text{ m}$$
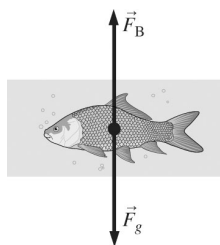
(b) Using the density at sea level from Table 14.1,

$$\rho = (1.28 \text{ kg/m}^3) e^{-z/(8.08\times10^3 \text{ m})} = (1.28 \text{ kg/m}^3) e^{-1600 \text{ m}/(8.08\times10^3 \text{ m})} = 1.05 \text{ kg/m}^3$$

This is 82% of ρ_0.

14.49. Model: Ignore the change in water density over lengths comparable to the size of the fish.
Visualize:

Solve: The buoyant force is the force of gravity on the volume of water displaced by the fish times, or
$$F_B = \rho_w(V_F + V_b)g$$
where the subscripts w, F, and b indicate water, fish, and bladder, respectively. The force due to gravity on the fish is
$$F_G = \rho_F V_F g + \rho_{air} V_b g$$
For neutral buoyancy, the two forces must have equal magnitude. Equating them and solving for the V_b gives
$$\rho_w(V_F + V_b)g = \rho_F V_F g + \rho_{air} V_b g$$
$$V_b = \frac{\rho_w - \rho_F}{\rho_{air} - \rho_w}V_F = \frac{1000\ \text{kg/m}^3 - 1080\ \text{kg/m}^3}{1.19\ \text{kg/m}^3 - 1000\ \text{kg/m}^3}V_F = 0.0801V_F$$
so the fish needs to increase its volume by 8.01%.
Assess: The result of 8% is reasonable because the fish is about 8% denser than water and the air density is much less than the water density.

14.50. Model: The buoyant force on the cylinder is given by Archimedes' principle.
Visualize:
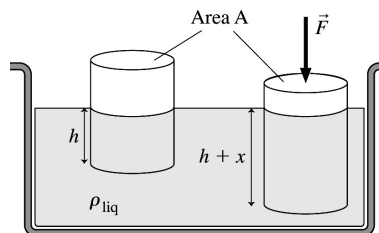
Solve: (a) Initially, as it floats, the cylinder is in static equilibrium, with the buoyant force balancing the gravitational force on the cylinder. The volume of displaced liquid is Ah, so
$$F_B = \rho_{liq}(Ah)g = F_G$$
Force F pushes the cylinder down distance x, so the submerged length is $h + x$ and the volume of displaced liquid is $A(h + x)$. The cylinder is again in equilibrium, but now the buoyant force balances both the gravitational force and force F. Thus
$$F_B = \rho_{liq}[A(h+x)]g = F_G + F$$
Since $\rho_{liq}(Ah)g = F_G$, we're left with
$$F = \rho_{liq}Agx$$
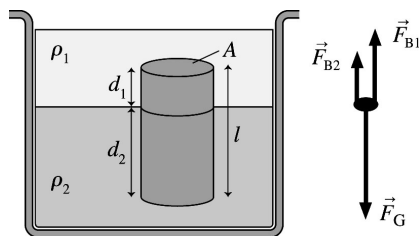**(b)** The amount of work dW done by force F to push the cylinder from x to $x + dx$ is $dW = Fdx = (\rho_{liq}Agx)dx$. To push the cylinder from $x_i = 0\ \text{m}$ to $x_f = 10\ \text{cm} = 0.10\ \text{m}$ requires work
$$W = \int_{x_i}^{x_f} Fdx = \rho_{liq}\,Ag\int_{x_i}^{x_f} xdx = \tfrac{1}{2}\rho_{liq}Ag(x_i^2 - x_f^2)$$
$$= \tfrac{1}{2}(1000\ \text{kg/m}^3)\pi(0.020\ \text{m})^2(9.8\ \text{m/s}^2)(0.10\ \text{m})^2 = 0.62\ \text{J}$$

14.51. Model: The buoyant force on the cylinder is given by Archimedes' principle.
Visualize:

Let d_1 be the length of the cylinder in the less-dense liquid with density ρ_1, and d_2 be the length of the cylinder in the more-dense liquid with density p_2.

Solve: The cylinder is in static equilibrium, so
$$\Sigma F_y = F_{B1} + F_{B2} - F_G = 0 \text{ N} \quad \Rightarrow \quad \rho_1(Ad_1)g + \rho_2(Ad_2)g = \rho A(d_1 + d_2)g$$

$$\frac{\rho_1}{\rho_2}d_1 + d_2 = \frac{\rho}{\rho_2}(d_1 + d_2)$$

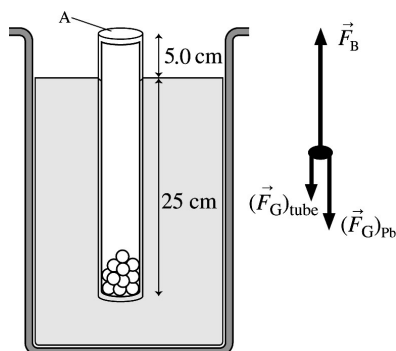Since $l = d_1 + d_2 \quad \Rightarrow \quad d_1 = l - d_2$, we can simplify the above equation to obtain

$$d_2 = \left(\frac{\rho - \rho_1}{\rho_2 - \rho_1}\right)l$$

That is, the fraction of the cylinder in the more dense liquid is $f = (\rho - \rho_1)/(\rho_2 - \rho_1)$.

Assess: As expected $f = 0$ when ρ is equal to ρ_1, and $f = 1$ when $\rho = \rho_2$.

14.52. Model: The buoyant force on the cylinder is given by Archimedes' principle.
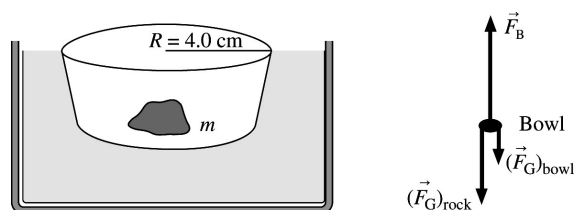**Visualize:**

Solve: The tube is in static equilibrium, so
$$\Sigma F_y = F_B - (F_G)_{\text{tube}} - (F_G)_{\text{Pb}} = 0 \text{ N} \quad \Rightarrow \quad \rho_{\text{liquid}} A(0.25 \text{ m})g = (0.030 \text{ kg})g + (0.250 \text{ kg})g$$

$$\rho_{\text{liquid}} = \frac{(0.280 \text{ kg})}{A(0.25 \text{ m})} = \frac{(0.280 \text{ kg})}{\pi(0.020 \text{ m})^2(0.25 \text{ m})} = 8.9 \times 10^2 \text{ kg/m}^3$$

Assess: This is a reasonable value for a liquid.

14.53. Model: Archimedes' principle determines the buoyant force.
Visualize:

Solve: The plastic hemisphere will hold the most weight when its rim is at the surface of the water. The buoyant force balances the gravitational force on the bowl and rock.

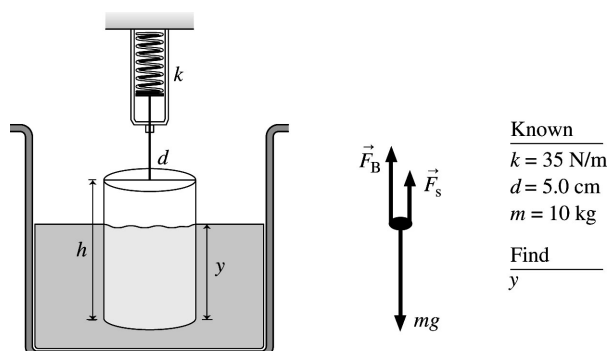$$\Sigma F_y = F_B - (F_G)_{\text{rock}} - (F_G)_{\text{bowl}} = 0 \text{ N}$$

Thus

$$mg = p_w V_{\text{bowl}} g - m_{\text{bowl}} g = (1000 \text{ kg/m}^3)\left(\frac{1}{2}\right)\left[\frac{4}{3}\pi(0.040 \text{ m})^3\right](9.8 \text{ m/s}^2) - (0.021 \text{ kg})(9.8 \text{ m/s}^2) \Rightarrow m = 0.16 \text{ kg}$$

Assess: Putting a rock as big as 160 g in an 8-cm-diameter bowl before it sinks is reasonable.

14.54. Model: The buoyant force is determined by Archimedes' principle. The spring is ideal.
Visualize:

Known
k = 35 N/m
d = 5.0 cm
m = 10 kg

Find
y

Solve: The spring is stretched by the same amount that the cylinder is submerged. The buoyant force and spring force balance the gravitational force on the cylinder.

$$\Sigma F_y = F_B + F_S - mg = 0 \text{ N}$$
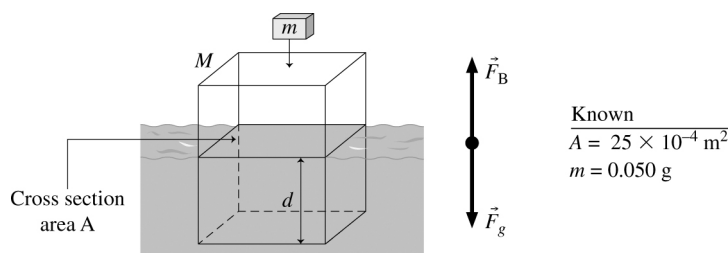
$$p_w Ayg + ky = mg$$

$$y = \frac{mg}{p_w Ag + k} = \frac{(1.0 \text{ kg})(9.8 \text{ m/s}^2)}{(1000 \text{ kg/m}^3)\pi(0.025 \text{ m})^2(9.8 \text{ m/s}^2) + 35 \text{ N/m}}$$

$$= 0.181 \text{ m} = 18 \text{ cm}$$

Assess: This is difficult to assess because we don't know the height h of the cylinder and can't calculate it without the density of the metal material.

14.55. Model: The buoyant force is determined by Archimedes' principle, and the boat and masses are floating in static equilibrium.
**Visualize:**

Solve: For the boat to float, the buoyant force must have the same magnitude as the force due to gravity. When the boat sinks to a depth d, it displaces a volume Ad of liquid. Thus

$$F_B = F_g \quad \Rightarrow \quad \rho A d g = (M + m)g$$

where ρ is the density of the liquid, $\rho A d g$ is the weight of the displaced liquid, M is the mass of the empty boat, and m is the added mass. Rearranging this expression gives the submerged depth after n blocks are added.

$$d = \frac{1}{\rho A} m + \frac{M}{\rho A}$$

If we plot the depth d versus mass m, the graph should be a straight line with slope $1/\rho A$ and y-intercept $M/\rho A$.
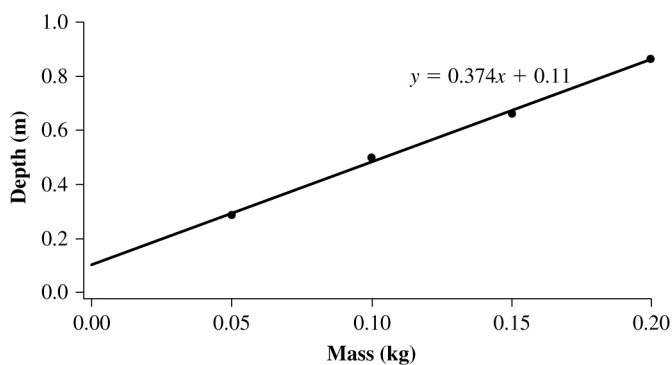
The linear fit is quite good. From the slope of the line, we have

$$\rho = \frac{1}{A \times \text{slope}} = \frac{1}{(25 \times 10^{-4} \text{ m}^2)(0.374 \text{ m/kg})} = 1067 \text{ kg/m}^3 = 1.1 \times 10^3 \text{ kg/m}^3$$

From the y-intercept, we get

$$M = \rho A \times \text{intercept} = (1070 \text{ kg/m}^3)(25 \times 10^{-4} \text{ m}^2)(0.11) = 0.29 \text{ kg} = 290 \text{ g}$$

Assess: The mass of the boat is around 6 times greater than the mass of the blocks, which is not unreasonable. The density of the liquid is slightly greater than the density of water, which is also a reasonable result.

14.56. Model: The buoyant force on the can is given by Archimedes' principle.
Visualize:
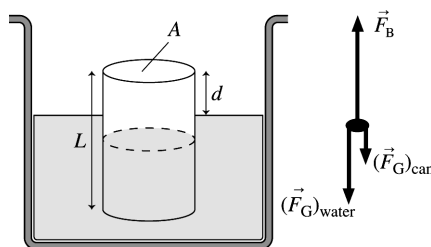
The length of the can above the water level is d, the length of the can is L, and the cross-sectional area of the can is A.
Solve: The can is in static equilibrium, so
$$\Sigma F_y = F_B - (F_G)_{can} - (F_G)_{water} = 0 \text{ N} \tfrac{1}{2}\rho_{water} A(L-d)g = (0.020 \text{ kg})g + m_{water}g$$
The mass of the water in the can is
$$m_{water} = \rho_{water}\left(\frac{V_{can}}{2}\right) = (1000 \text{ kg / m}^3)\frac{355 \times 10^{-6} \text{ m}^3}{2} = 0.1775 \text{ kg}$$
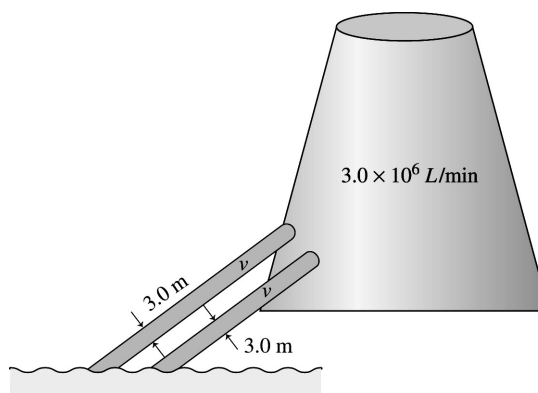
$$\rho_{water} A(L-d) = 0.020 \text{ kg} + 0.1775 \text{ kg} = 0.1975 \text{ kg} \quad \Rightarrow \quad d - L = -\frac{0.1975 \text{ kg}}{\rho_{water} A} = 0.0654 \text{ m}$$

Because $V_{can} = \pi(0.031 \text{ m})^2 L = 355 \times 10^{-6} \text{ m}^3$, $L = 0.1176 \text{ m}$. Using this value of L, we get $d = 0.0522 \text{ m} \approx 5.2 \text{ cm}$.
Assess: $d/L = 5.22 \text{ cm} / 11.76 \text{ cm} = 0.444$, thus 44.4% of the length of the can is above the water surface. This is reasonable.

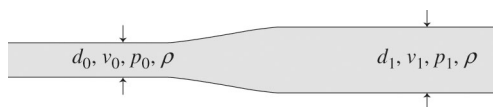**14.57. Model:** The two pipes are identical.
Visualize:

Solve: The water speed is the same in both pipes. The flow rate is
$$Q = 3.0 \times 10^6 \text{ L/min} = 2(vA)$$

$$v = \frac{Q}{2A} = \frac{(3.0 \times 10^6)(10^{-3})\left(\dfrac{1}{60}\right) \text{ m}^3/\text{s}}{2\pi(1.5 \text{ m})^2} = 3.5 \text{ m/s}$$

14.58. Model: Treat the liquid as an ideal liquid obeying Bernoulli's equation (Equation 14.27).
Visualize:

d_0, v_0, p_0, ρ d_1, v_1, p_1, ρ

Solve: (a) Bernoulli's equation gives

$$p_0 + \tfrac{1}{2}\rho v_0^2 + \rho g y_0 = p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1$$

Because the pipe expands equally in the positive and negative y directions, the contributions from the gravitational-potential-energy term cancel by symmetry, so Bernoulli's equation reduces to

$$p_0 + \tfrac{1}{2}\rho v_0^2 = p_1 + \tfrac{1}{2}\rho v_1^2$$

Expressing p_1 in terms of p_0 gives

$$p_1 = p_0 + \tfrac{1}{2}\rho(v_0^2 - v_1^2)$$

Finally, we can express the velocity v_1 in terms of v_0 using the continuity equation (Equation 14.17):

$$v_0 A_0 = v_1 A_1 \quad \Rightarrow \quad v_1 = v_0 \frac{d_0^2}{d_1^2}$$

Inserting this into the expression for p_0 gives

$$p_1 = p_0 + \tfrac{1}{2}\rho v_0^2 \left(1 - \frac{d_0^4}{d_1^4}\right)$$

(b) Inserting the given values gives

$$p_1 = 50\ \text{kPa} + \tfrac{1}{2}(1000\ \text{kg/m}^3)(10.0\ \text{m/s})^2 \left[1 - \left(\frac{16.8\ \text{cm}}{20.0\ \text{cm}}\right)^4\right] = 50\ \text{kPa} + 25.1\ \text{kPa} = 75\ \text{kPa}$$

Assess: We find that the pressure increases where the velocity has decreased, which is in accordance with Bernoulli's equation.

14.59. Model: Treat the sap as incompressible and assume sap is almost entirely made of water.
Solve: To replace the water lost to the atmosphere, the vessels must supply 110 g/h of sap. The sap provided by each vessel is $\rho v \pi (d/2)^2$, so for $N = 2000$ sap vessels, the flow rate in each vessel is

$$N\rho v \pi \left(\frac{d}{2}\right)^2 = 0.110\ \text{kg/h} \quad \Rightarrow \quad v = \frac{4(0.110\ \text{kg/h})(1\ \text{h}/3600\ \text{s})}{\pi(1040\ \text{kg/m}^3)(2000)(100 \times 10^{-6}\ \text{m})^2} = 1.87 \times 10^{-3}\ \text{m/s} = 1.87\ \text{mm/s}$$

Assess: The motion of the sap would be fast enough to observe with the naked eye.

14.60. Model: Treat the water as an ideal fluid obeying Bernoulli's equation. A streamline begins in the bigger size pipe and ends at the exit of the narrower pipe.
Visualize: Please see Figure P14.60. Let point 1 be beneath the standing column and point 2 be where the water exits the pipe.
Solve: (a) The pressure of the water as it exits into the air is $p_2 = p_{\text{atmos}}$.
(b) Bernoulli's equation, Equation 14.26, relates the pressure, water speed, and heights at points 1 and 2:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \Rightarrow \quad p_1 - p_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$

From the continuity equation,

$$v_1 A_1 = v_2 A_2 = (4\ \text{m/s})(5 \times 10^{-4}\ \text{m}^2) \quad \Rightarrow \quad v_1(10 \times 10^{-4}\ \text{m}^2) = 20 \times 10^{-4}\ \text{m}^3/\text{s} \quad \Rightarrow \quad v_1 = 2.0\ \text{m/s}$$

Substituting into Bernoulli's equation,

$$p_1 - p_2 = p_1 - p_{\text{atmos}} = \tfrac{1}{2}(1000\ \text{kg/m}^3)[(4.0\ \text{m/s})^2 - (2.0\ \text{m/s})^2] + (1000\ \text{kg/m}^3)(9.8\ \text{m/s})(4.0\ \text{m})$$

$$= 6000\ \text{Pa} + 39{,}200\ \text{Pa} = 45\ \text{kPa}$$

But $p_1 - p_2 = \rho g h$, where h is the height of the standing water column. Thus

$$h = \frac{45 \times 10^3 \text{ Pa}}{(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 4.6 \text{ m}$$

14.61. Model: Air is an ideal gas and obeys Boyle's law.
Visualize: Please refer to Figure P14.61. The quantity h is the length of the air column when the mercury fills the cylinder to the top. A is the cross-sectional area of the cylinder.
**Solve:** For the column of air, Boyle's law is $p_0 V_0 = p_1 V_1$, where p_0 and V_0 are the pressure and volume before any mercury is poured, and p_1 and V_1 are the pressure and volume when mercury fills the cylinder above the air. Using $p_1 = p_0 + \rho_{\text{Hg}} g (1.0 \text{ m} - h)$, Boyle's law becomes

$$p_0 V_0 = [p_0 + \rho_{\text{Hg}} g (1.0 \text{ m} - h)] V_1 \quad \Rightarrow \quad p_0 A (1.0 \text{ m}) = [p_0 + \rho_{\text{Hg}} g (1.0 \text{ m} - h)] A h$$

$$p_0 (1.0 \text{ m} - h) = \rho_{\text{Hg}} g (1.0 \text{ m} - h) h \quad \Rightarrow \quad h = \frac{p_0}{\rho_{\text{Hg}} g} = \frac{1.013 \times 10^5 \text{ Pa}}{(13{,}600 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 0.76 \text{ m} = 76 \text{ cm}$$

14.62. Model: Treat the water as an ideal fluid obeying Bernoulli's equation. A streamline begins at the faucet and continues down the stream.
Visualize:
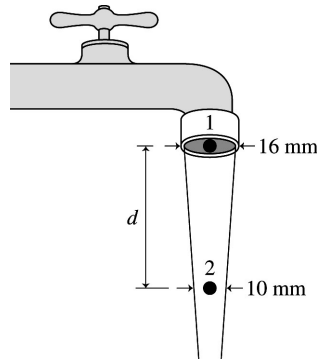
The pressure at point 1 is p_1 and the pressure at point 2 is p_2. Both p_1 and p_2 are atmospheric pressure. The velocity and the area at point 1 are v_1 and A_1 and they are v_2 and A_2 at point 2. Let d be the distance of point 2 below point 1.
Solve: The flow rate is

$$Q = v_1 A = \frac{2.0 \times 1000 \times 10^{-6} \text{ m}^3}{10 \text{ s}} = 2.0 \times 10^{-4} \text{ m}^3/\text{s} \quad \Rightarrow \quad v_1 = \frac{2.0 \times 10^{-4} \text{ m}^3/\text{s}}{\pi (0.0080 \text{ m})^2} = 1.0 \text{ m/s}$$

Bernoulli's equation at points 1 and 2 is

$$p_1 + \tfrac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2} \rho v_2^2 + \rho g y_2 \quad \Rightarrow \quad \rho g d = \tfrac{1}{2} \rho (v_2^2 - v_1^2)$$

From the continuity equation,
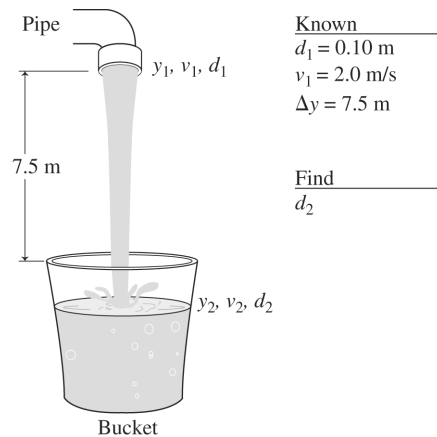
$$v_1 A_1 = v_2 A_2 \quad \Rightarrow \quad (1.0 \text{ m/s}) \pi (8.0 \times 10^{-3} \text{ m})^2 = v_2 \pi (5.0 \times 10^{-3} \text{ m})^2 \quad \Rightarrow \quad v_2 = 2.56 \text{ m/s}$$

Going back to Bernoulli's equation, we have

$$g d = \tfrac{1}{2} [(2.56 \text{ m/s})^2 - (1.0 \text{ m/s})^2] \quad \Rightarrow \quad d = 0.283 \text{ m} \approx 28 \text{ cm}$$

14.63. Model: Treat the water as an ideal fluid. Assume that it is in free fall between the opening of the pipe and the bucket.
Visualize:

Known
$d_1 = 0.10$ m
$v_1 = 2.0$ m/s
$\Delta y = 7.5$ m

Find
d_2

Solve: First find the speed of the water as it hits the bucket. Use a kinematic equation.

$$v_2^2 = v_1^2 + 2g\Delta y \Rightarrow v_2 = \sqrt{v_1^2 + 2g\Delta y}$$

Now use the equation of continuity.

$$v_1 A_1 = v_2 A_2 \Rightarrow \frac{A_2}{A_1} = \frac{v_1}{v_2} = \frac{v_1}{\sqrt{v_1^2 + 2g\Delta y}} = \frac{2.0 \text{ m/s}}{\sqrt{(2.0 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(7.5 \text{ m})}} = 0.1628$$
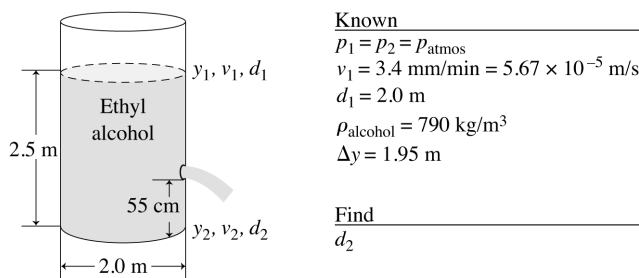
Now use $A = 4\pi r^2$.

$$\frac{d_2}{d_1} = \frac{2r_2}{2r_1} = \frac{\sqrt{A_2/\pi}}{\sqrt{A_1/\pi}} = \sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{v_1}{v_2}} = \sqrt{0.1628} = 0.40$$

That is, $d_2 = 0.40d_1 = 0.40(10 \text{ cm}) = 4.0$ cm.

Assess: We know from experience the stream gets narrower as it falls, and this seems like a reasonable answer.

14.64. Model: Treat the alcohol as an ideal fluid. The pressure above the alcohol in the cylinder is atmospheric pressure, as is the pressure when it comes out the hole.
Visualize:

Known
$p_1 = p_2 = p_{atmos}$
$v_1 = 3.4$ mm/min $= 5.67 \times 10^{-5}$ m/s
$d_1 = 2.0$ m
$\rho_{alcohol} = 790$ kg/m^3
$\Delta y = 1.95$ m

Find
d_2

Solve: Use Bernoulli's equation to find the speed the alcohol exits the hole.

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

The pressures are the same and so drop out of the equation. We can then cancel ρ.

$$v_2^2 = v_1^2 + 2g\Delta y \Rightarrow v_2 = \sqrt{v_1^2 + 2g\Delta y}$$

Now use the equation of continuity.

$$v_1 A_1 = v_2 A_2 \Rightarrow \frac{A_2}{A_1} = \frac{v_1}{v_2} = \frac{v_1}{\sqrt{v_1^2 + 2g\Delta y}} = \frac{5.67 \times 10^{-5} \text{ m/s}}{\sqrt{(5.67 \times 10^{-5} \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(1.95 \text{ m})}} = 9.166 \times 10^{-6}$$

Now use $A = 4\pi r^2$.

$$\frac{d_2}{d_1} = \frac{2r_2}{2r_1} = \frac{\sqrt{A_2/\pi}}{\sqrt{A_1/\pi}} = \sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{v_1}{v_2}} = \sqrt{9.166 \times 10^{-6}} = 0.00303$$

That is, $d_2 = 0.00303 d_1 = 0.00303(2.0 \text{ m}) = 6.1 \text{ mm}$.

Assess: This is a reasonable sized hole. The density of alcohol canceled, so the answer would be the same for any ideal fluid.

14.65. Model: Treat the air as an ideal fluid obeying Bernoulli's equation.
Solve: (a) The pressure above the roof is lower due to the higher velocity of the air.
(b) Bernoulli's equation, with $y_{\text{inside}} \approx y_{\text{outside}}$, is

$$p_{\text{inside}} = p_{\text{outside}} + \frac{1}{2}\rho_{\text{air}}v^2 \quad \Rightarrow \quad \Delta p = \frac{1}{2}\rho_{\text{air}}v^2 = \frac{1}{2}(1.28 \text{ kg/m}^3)\left(\frac{130 \times 1000 \text{ m}}{3600 \text{ s}}\right)^2 = 835 \text{ Pa}$$

The pressure difference is 0.83 kPa.
(c) The force on the roof is $(\Delta p)A = (835 \text{ Pa})(6.0 \text{ m} \times 15.0 \text{ m}) = 7.5 \times 10^4 \text{ N}$. The roof will blow up, because pressure inside the house is greater than pressure on the top of the roof.

14.66. Model: The ideal fluid obeys Bernoulli's equation.
Visualize: Please refer to Figure P14.66. There is a streamline connecting point 1 in the wider pipe on the left with point 2 in the narrower pipe on the right. The air speeds at points 1 and 2 are v_1 and v_2 and the cross-sectional area of the pipes at these points are A_1 and A_2. Points 1 and 2 are at the same height, so $y_1 = y_2$.
Solve: The volume flow rate is $Q = A_1 v_1 = A_2 v_2 = 1200 \times 10^{-6} \text{ m}^3/\text{s}$. Thus

$$v_2 = \frac{1200 \times 10^{-6} \text{ m}^3/\text{s}}{\pi(0.0020 \text{ m})^2} = 95.49 \text{ m/s} \qquad v_1 = \frac{1200 \times 10^{-6} \text{ m}^3/\text{s}}{\pi(0.010 \text{ m})^2} = 3.82 \text{ m/s}$$

Now we can use Bernoulli's equation to connect points 1 and 2:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1) = \frac{1}{2}(1.28 \text{ kg/m}^3)[(95.49 \text{ m/s})^2 - (3.82 \text{ m/s})^2] + 0 \text{ Pa} = 5.83 \text{ kPa}$$

Because the pressure above the mercury surface in the right tube is p_2 and in the left tube is p_1, the difference in the pressures p_1 and p_2 is $\rho_{\text{Hg}}gh$. That is,

$$p_1 - p_2 = 5.83 \text{ kPa} = \rho_{\text{Hg}}gh \quad \Rightarrow \quad h = \frac{5.83 \times 10^3 \text{ Pa}}{(13,600 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 4.4 \text{ cm}$$

14.67. Model: The ideal fluid (i.e., air) obeys Bernoulli's equation.
Visualize: Please refer to Figure P14.67. There is a streamline connecting points 1 and 2. The air speeds at points 1 and 2 are v_1 and v_2, and the cross-sectional areas of the pipes at these points are A_1 and A_2. Points 1 and 2 are at the same height, so $y_1 = y_2$.
Solve: (a) The height of the mercury is 10 cm. So, the pressure at point 2 is larger than at point 1 by

$$\rho_{\text{Hg}}g(0.10 \text{ m}) = (13,600 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.10 \text{ m}) = 13,328 \text{ Pa} \quad \Rightarrow \quad p_2 = p_1 + 13,328 \text{ Pa}$$

Using Bernoulli's equation,

$$p_1 + \tfrac{1}{2}\rho_{air}v_1^2 + \rho_{air}gy_1 = p_2 + \tfrac{1}{2}\rho_{air}v_2^2 + \rho_{air}gy_2 \quad \Rightarrow \quad p_2 - p_1 = \tfrac{1}{2}\rho_{air}(v_1^2 - v_2^2)$$

$$v_1^2 - v_2^2 = \frac{2(p_2 - p_1)}{\rho_{air}} = \frac{2(13{,}328 \text{ Pa})}{(1.28 \text{ kg/m}^3)} = 20{,}825 \text{ m}^2/\text{s}^2$$

From the continuity equation, we can obtain another equation connecting v_1 and v_2:

$$A_1v_1 = A_2v_2 \quad \Rightarrow \quad v_1 = \frac{A_2}{A_1}v_2 = \frac{\pi(0.0050 \text{ m})^2}{\pi(0.0010 \text{ m})^2}v_2 = 25v_2$$

Substituting $v_1 = 25v_2$ in the Bernoulli equation, we get

$$(25\,v_2)^2 - v_2^2 = 20{,}825 \text{ m}^2/\text{s}^2 \quad \Rightarrow \quad v_2 = 5.78 \text{ m/s}$$

Thus $v_2 = 5.8$ m/s and $v_1 = 25v_2 = 1.4 \times 10^2$ m/s.

(b) The volume flow rate $A_2v_2 = \pi(0.0050 \text{ m})^2 (5.78 \text{ m/s}) = 4.5 \times 10^{-3} \text{ m}^3/\text{s}$.

14.68. Model: Treat the water as an ideal fluid that obeys Bernoulli's equation. There is a streamline from the top of the water to the hole.

Visualize: The top of the water (at $y_1 = h$) and the hole (at $y_2 = y$) are at atmospheric pressure. The speed of the water at the top is zero because the tank is kept filled.
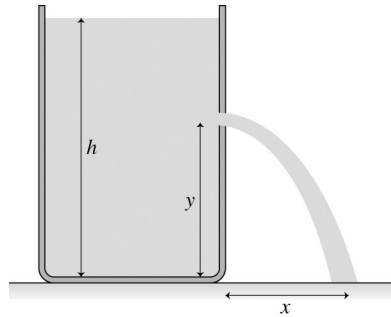
Solve: (a) Bernoulli's equation connecting the two points gives

$$0 + \rho gh = \tfrac{1}{2}\rho v^2 + \rho gy \quad \Rightarrow \quad v = \sqrt{2g(h - y)}$$

(b) For a particle shot horizontally from a height y with speed v, the range can be found using kinematic equations. For the y-motion, using $t_0 = 0$ s, we have

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 \quad \Rightarrow \quad 0 \text{ m} = y + (0 \text{ m/s})t_1 + \tfrac{1}{2}(-g)t_1^2 \quad \Rightarrow \quad t_1 = \sqrt{2y/g}$$

For the x-motion,

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 \quad \Rightarrow \quad x = 0 \text{ m} + vt_1 + 0 \text{ m} \quad \Rightarrow \quad x = v\sqrt{2y/g}$$

(c) Combining the results of (a) and (b), we obtain

$$x = \sqrt{2g(h - y)}\sqrt{2y/g} = \sqrt{4y(h - y)}$$

To find the maximum range relative to the vertical height,

$$\frac{dx}{dy} = 0 \quad \Rightarrow \quad \frac{1}{\sqrt{4y(h - y)}}[4(h - y) - 4y] = 0 \quad \Rightarrow \quad y = \frac{h}{2}$$

With $y = \tfrac{1}{2}h$, the maximum range is

$$x_{max} = \sqrt{4\left(\frac{h}{2}\right)\left(h - \frac{h}{2}\right)} = h$$

14.69. Model: Treat the water as an ideal fluid that obeys Bernoulli's equation. There is a streamline from the top of the water to the hole.

Visualize: The top of the water and the hole are at atmospheric pressure. The speed of the water at the top is not zero because the tank is emptying.
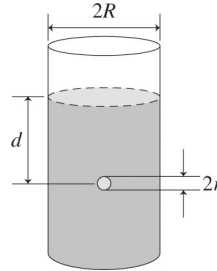
Solve: (a) Bernoulli's equation connecting the two points gives

$$p_{atm} + \rho gh + \tfrac{1}{2}\rho v_{top}^2 = p_{atm} + \tfrac{1}{2}\rho v_{hole}^2 + \rho g(h-d)$$

The continuity equation relates the flow rates at the surface and at the hole:

$$v_{top}A_{top} = v_{hole}A_{hole} \quad \Rightarrow \quad v_{top} = v_{hole}\left(\frac{r}{R}\right)^2$$

Inserting this into Bernoulli's equation and solving for v_{hole} gives

$$v_{hole} = \sqrt{\frac{2gd}{1-(r/R)^4}}$$

The volume flow rate Q through the hole is

$$Q = Av_{hole} = \pi r^2 \sqrt{\frac{2gd}{1-(r/R)^4}}$$

(b) Inserting the given values, we find that the water level will initial at the following rate:

$$v_{top} = v_{hole}\left(\frac{r}{R}\right)^2 = \left(\frac{r}{R}\right)^2 \sqrt{\frac{2gd}{1-(r/R)^4}} = \left(\frac{2.0\times10^{-3}\text{ m}}{1.0\text{ m}}\right)^2 \sqrt{\frac{2(9.8\text{ m/s}^2)(1.0\text{ m})}{1-[(2.0\times10^{-3}\text{ m})/(1.0\text{ m})]^4}} = 1.77\times10^{-5}\text{ m/s}$$

which converts to 1.1 mm/min.

Assess: For the given dimensions, the result seems reasonable.

14.70. Model: The weight of the person creates tensile stress in the disk cartilage.

Solve: The cross-sectional area of each disk of cartilage is πr^2. Inserting this into Equation 14.34 gives

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$
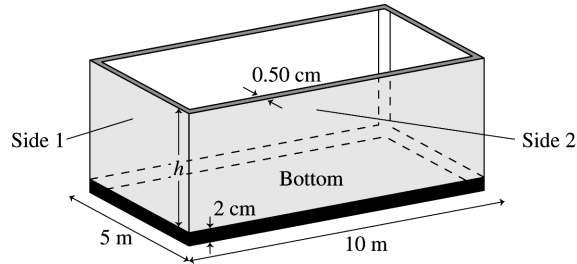
$$\Delta L = \frac{FL}{YA} = \frac{mgL}{YA} = \frac{(33\text{ kg})(9.8\text{ m/s}^2)(0.0050\text{ m})}{(1.0\times10^6\text{ N/m}^2)\pi(0.020\text{ m})^2} = 1.3\text{ mm}$$

Assess: The cartilage experiences a significant compressive axial strain (~25%).

Challenge Problems

14.71. Model: The buoyant force on the boat is given by Archimedes' principle.
Visualize:

The minimum height of the boat that will enable the boat to float in perfectly calm water is h.
Solve: The boat barely floats if the water comes completely to the top of the sides. In this case, the volume of displaced water is the volume of the boat. Archimedes' principle in equation form is $F_B = \rho_w V_{boat} g$. For the boat to float $F_B = (F_G)_{boat}$. Let us first calculate the gravitational force on the boat:

$$(F_G)_{boat} = (F_G)_{bottom} + 2(F_G)_{side\ 1} + 2(F_G)_{side\ 2}, \text{ where}$$

$$(F_G)_{bottom} = \rho_{steel} V_{bottom} g = (7900 \text{ kg/m}^3)(5.0 \times 10 \times 0.020 \text{ m}^3)g = 7900g \text{ N}$$

$$(F_G)_{side\ 1} = \rho_{steel} V_{side\ 1} g = (7900 \text{ kg/m}^3)(5.0 \times h \times 0.0050 \text{ m}^3)g = 197.5gh \text{ N}$$

$$(F_G)_{side\ 2} = \rho_{steel} V_{side\ 2} g = (7900 \text{ kg/m}^3)(10 \times h \times 0.0050 \text{ m}^3)g = 395gh$$

$$(F_G)_{boat} = [7900g + 2(197.5gh) + 2(395gh)] \text{ N} = (7900 + 1185h)g \text{ N}$$
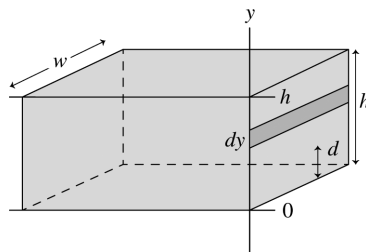
Going back to the Archimedes' equation and remembering that h is in meters, we obtain

$$\rho_w V_{boat} g = (7900 + 1185h)g \text{ N} \implies (1000)[10 \times 5.0 \times (h + 0.020)] = 7900 + 1185h$$

$$h = 14 \text{ cm}$$

14.72. Model: Treat the fluid as ideal.
Visualize:

Solve: (a) The pressure changes with depth, so we must integrate to find the total force.

$$F = \int dF = \int p \, dA = \int p(w \, dy) = w \int_0^h p_0 + \tfrac{1}{2}\rho g (h - y) \, dy$$

$$= w \left[p_0 y + \rho g h y - \tfrac{1}{2}\rho g y^2 \right]_0^h = w \left[p_0 h + \tfrac{1}{2}\rho g h^2 \right] = wh \left(p_0 + \tfrac{1}{2}\rho g h \right)$$

If the top of the tank is open to the air then $p_0 = p_{atmos}$.

(b) To produce no torque, the spring must be in the middle horizontally. To find the height d above the bottom where the spring must be attached to produce no torque we want the "center of force."
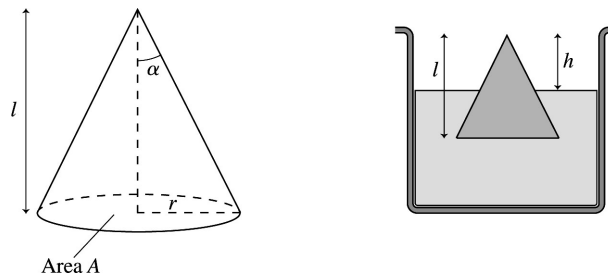
$$d = \frac{1}{F}\int ydF = \frac{w\int_0^h p_0 y + \rho gy(h-y)dy}{w\left[p_0 h + \frac{1}{2}\rho gh^2\right]} = \frac{\left[\frac{1}{2}p_0 y^2 + \frac{1}{2}\rho ghy^2 - \frac{1}{3}\rho gy^3\right]_0^h}{\left[p_0 h + \frac{1}{2}\rho gh^2\right]}$$

$$= \frac{\left[\frac{1}{2}p_0 h^2 + \frac{1}{2}\rho gh^3 - \frac{1}{3}\rho gh^3\right]}{\left[p_0 h + \frac{1}{2}\rho gh^2\right]} = \frac{\frac{1}{2}p_0 h^2 + \frac{1}{6}\rho gh^3}{p_0 h + \frac{1}{2}\rho gh^2} = \frac{\frac{1}{2}p_0 h + \frac{1}{6}\rho gh^2}{p_0 + \frac{1}{2}\rho gh}$$

Assess: We expected the answer to (a) to depend on w, but not the answer to (b). We did not need the length of the tank.

14.73. Model: The buoyant force on the cone is given by Archimedes' principle.
Visualize:

Area A

Solve: It may seem like we need the formula for the volume of a cone. You can use that formula if you know it, but it isn't essential. The volume is clearly the area of the base multiplied by the height multiplied by some constant. That is, the cone shown above has $V = aAl$ where a is some constant. But the radius of the base is $r = l\tan\alpha$, where α is the angle of the apex of the cone, and $A = \pi r^2$, making A proportional to l^2. Thus the volume of a cone of height l is $V = cl^3$, where c is a constant. Because the cone is floating in static equilibrium, we must have $F_B = F_G$. The cone's density is ρ_0, so the gravitational force on it is $F_G = \rho_0 Vg = \rho_0 cl^3 g$. The buoyant force is the gravitational force on the displaced fluid. The volume of displaced fluid is the full volume of the cone minus the volume of the cone of height h above the water, or $V_{disp} = cl^3 - ch^3$. Thus $F_B = \rho_f V_{disp} g = \rho_f c(l^3 - h^3)g$, and the equilibrium condition is

$$F_B = F_G \quad\Rightarrow\quad \rho_f c(l^3 - h^3)g = \rho_0 cl^3 g \quad\Rightarrow\quad \rho_f\left(1 - \frac{h^3}{l^3}\right) = \rho_0 \quad\Rightarrow\quad \frac{h}{l} = \left(1 - \frac{\rho_0}{\rho_f}\right)^{1/3}$$

14.74. Model: The grinding wheel is a uniform disk. We will use the model of kinetic friction and hydrostatics.
Visualize: Please refer to Figure CP14.74.
Solve: This is a three-part problem. First find the desired angular acceleration, then use that to find the force applied by each brake pad, then finally the needed oil pressure. The angular acceleration required to stop the wheel is found using rotational kinematics.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 \text{ rad/s} - 900\times 2\pi\times(1/60) \text{ rad/s}}{5.0 \text{ s}} = -18.85 \text{ rad/s}^2$$

Each brake pad applies a frictional force $f_k = \mu_k n$ to the wheel. The normal force is equal to the force applied by the piston by Newton's third law. Rotational dynamics can be used to find the magnitude of the force. f_k is applied 12 cm from the rotation axis on both sides of the disk.

$$\tau_{net} = I\alpha$$

$$-2f_k(0.12 \text{ m}) = \left(\frac{1}{2}MR^2\right)\alpha$$

$$n = \frac{MR^2\alpha}{-4\mu_k(0.12 \text{ m})} = \frac{(15 \text{ kg})(0.13 \text{ m})^2(-18.85 \text{ rad/s}^2)}{-4(0.60)(0.12 \text{ m})} = 16.6 \text{ N}$$

The oil pressure required to generate this much force at each brake pad is
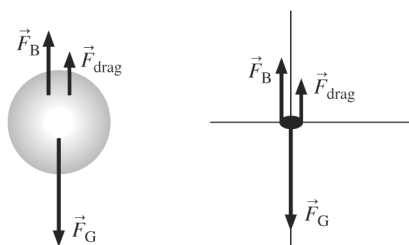
$$p = \frac{F}{A} = \frac{16.6 \text{ N}}{\pi(0.010 \text{ m})^2} = 53 \text{ kPa}$$

relative to atmospheric pressure. The absolute pressure is $53 \text{ kPa} + 101.3 \text{ kPa} = 154.3 \text{ kPa} = 1.5 \times 10^5$ Pa to two significant figures.

Assess: The required oil pressure is about half an atmosphere, which is quite reasonable.

14.75. Model: The volume of the fluid displaced is the same as the volume of the sphere: $V_f = V$.

Visualize:

Solve: (a) Use Newton's 2nd law with the acceleration equal to zero at terminal speed. Solve for the terminal speed.

$$\Sigma F_y = F_B + F_{drag} - F_G = ma_y = 0$$

$$\rho_f V_f g + 6\pi\eta Rv_{term} - mg = 0$$

$$\rho_f\left(\frac{4}{3}\pi R^3\right)g + 6\pi\eta Rv_{term} - \left(\frac{4}{3}\pi R^3\right)\rho g = 0$$

$$6\pi\eta Rv_{term} = \left(\frac{4}{3}\pi R^3\right)\rho g - \rho_f\left(\frac{4}{3}\pi R^3\right)g = \frac{4}{3}\pi R^3 g(\rho - \rho_f)$$

$$v_{term} = \frac{\frac{4}{3}\pi R^3 g(\rho - \rho_f)}{6\pi\eta R} = \frac{2R^2 g}{9\eta}(\rho - \rho_f)$$

(b)

$$\Sigma F_y = F_B + F_{drag} - F_G = ma_y$$

$$\rho_f Vg + b(-v_y) - mg = m\frac{dv_y}{dt}$$

The sign on v_y is negative because it is going down and the drag force must be up. Divide both sides by m and rearrange terms.

$$\frac{\rho_f Vg}{m} - g - \frac{b}{m}v_y = \frac{dv_y}{dt}$$

$$\left(\frac{\rho_f}{\rho} - 1\right)g - \frac{b}{m}v_y = \frac{dv_y}{dt}$$

Use some substitutions $A = \left(\dfrac{\rho_f}{\rho} - 1 \right) g$ and $B = \dfrac{-b}{m}$ for simplicity. Also note that $A/B = v_{term}$.

Multiply both sides by dt. Then integrate.

$$A + Bv_y = \frac{dv_y}{dt}$$

$$dt = \frac{dv_y}{A + Bv_y}$$

$$\int_0^t dt = \int_0^v \frac{dv_y}{A + Bv_y}$$

Evaluate both integrals at the upper and lower limits. Then solve for v_y.

$$t = \left[\frac{1}{B} \ln(A + Bv_y) \right]_0^v = \frac{1}{B} \ln\left(\frac{A + Bv_y}{A} \right)$$

$$e^{Bt} = \frac{A + Bv_y}{A} = 1 + \frac{B}{A} v_y$$

$$v_y = \frac{A}{B}(e^{Bt} - 1) = \frac{A}{B}(1 - e^{Bt}) = -v_{term}(1 - e^{Bt})$$

where the terminal speed is positive and v_y is negative due to the downward direction. Substitute back for A and B.

$$v_y(t) = -\frac{2R^2 g}{9\eta}(\rho - \rho_f)\left(1 - e^{-\frac{b}{m}t} \right) = -\frac{2R^2 g}{9\eta}(\rho - \rho_f)\left(1 - e^{-\frac{9\eta}{2r^2\rho}t} \right)$$

(c) Plug the data into the answer to part (a):

$$v_{term} = \frac{2R^2 g}{9\eta}(\rho - \rho_f) = \frac{2(1.5\text{ mm})^2 (9.8\text{ m/s}^2)}{9(1.0 \times 10^{-3}\text{ Pa} \cdot \text{s})}(2700\text{ kg/m}^3 - 1000\text{ kg/m}^3) = 8.3\text{ m/s}$$

Solve for t in the answer to part (b) with $v_y = (0.90)v_{term}$.

$$v_y = v_{term}\left(1 - e^{-\frac{9\eta}{2R^2\rho}t} \right) \Rightarrow \frac{v_y}{v_{term}} = 0.90 = \left(1 - e^{-\frac{9\eta}{2R^2\rho}t} \right) \Rightarrow 0.10 = e^{-\frac{9\eta}{2R^2\rho}t}$$

$$\frac{-9\eta}{2R^2\rho}t = \ln(0.10) \Rightarrow$$

$$t = \frac{2R^2\rho}{-9\eta}\ln(0.10) = \frac{2(0.0015\text{ m})^2 (2700\text{ kg/m}^3)}{-9(1.0 \times 10^{-3}\text{ Pa} \cdot \text{s})}\ln(0.10) = 3.1\text{ s}$$

Assess: We are glad to see that the answer to part (b) tends to the answer to part (a) in the limit of very large t.