21

HEAT ENGINES AND REFRIGERATORS

Conceptual Questions

21.1. $W_s = -W$.

(a) $W < 0,\ W_s > 0$. Work is done by the system; the area under the curve is positive.

(b) $W > 0,\ W_s < 0$. Work is done on the system to compress it to a smaller volume.

(c) $W > 0,\ W_s < 0$. More work is done on the system than by the system.

21.2. $W_3 > W_1 = W_2 > W_4$. The amount of work done by the gas is the area inside the closed cycle loop (traversed in a clockwise direction).

21.3. No. $\eta = \dfrac{W_{\text{out}}}{Q_{\text{H}}} = \dfrac{\text{what you get}}{\text{what you had to pay}}$. You cannot get out more than you put in.

21.4. **(a)** 1
(b) 3
(c) 3
(d) 2
In stage 1 the volume is fixed (so no work is done) but the temperature increases, so heat was added. In stage 2 on the isotherm work is done by the gas. In stage 3 work is done on the gas and heat is removed.

21.5. $\eta_3 > \eta_1 = \eta_2 > \eta_4$. $\eta = \dfrac{W_{\text{out}}}{Q_{\text{H}}}$ $\eta_1 = \dfrac{4}{10}$ $\eta_2 = \dfrac{40}{100}$ $\eta_3 = \dfrac{6}{10}$ $\eta_4 = \dfrac{6}{100}$

21.6. The thermal efficiency is larger for engine 1; the same amount of heat is added per cycle in both engines, but the cycle for engine 1 has a larger W_{out} due to the larger enclosed area: $\eta = \dfrac{W_{\text{out}}}{Q_{\text{H}}}$.

21.7. It is an isothermal process with equal amounts of heat added to the system and work done by the system.

21.8. **(a)** No; cannot have $15\ \text{J}_{\text{out}} > 10\ \text{J}_{\text{in}}$.

(b) Yes, this is a heat engine with $\eta = \dfrac{W_{\text{out}}}{Q_{\text{H}}} = \dfrac{4}{10} = 0.4$, which is less than $\eta_{\text{Carnot}} = 0.5$.

(c) No, it isn't possible to have $\eta > \eta_{\text{Carnot}}$: $\eta = \dfrac{W_{\text{out}}}{Q_{\text{H}}} = \dfrac{6}{10} = 0.6$, but $\eta_{\text{Carnot}} = 0.5$.

21.9. (a) No, the purpose of a refrigerator is to remove heat from the cold reservoir, and this diagram dumps heat into the cold reservoir.
(b) Yes, which is less than $K_{\text{Carnot}} = 1$.
(c) No, $K = \dfrac{Q_C}{W_{\text{in}}} = \dfrac{20}{10} = 2$, which is less than $K_{\text{Carnot}} = 1$.

21.10. No. The first law of thermodynamics (energy conservation) for a refrigerator or air conditioner requires $Q_H = Q_C + W$. There are no perfect refrigerators (second law), so $W > 0$ (work must be done by the compressor) and thus $Q_H > Q_C$. Because the "air conditioner" exhausts more heat into the room than it extracts from the room, the net effect is to increase the room temperature, not decrease it.

21.11. Yes, the first law says that energy is conserved, so we will never get more work out of the heat engine than heat energy is transferred to the system. In fact, the second law (informal statement #4) says that there are no perfect heat engines with $\eta = 1$, so there is always some waste heat exhausted to the cold reservoir.

Exercises and Problems

Exercises

Section 21.1 Turning Heat into Work

Section 21.2 Heat Engines and Refrigerators

21.1. Solve: During each cycle, the work done by the engine is $W_{\text{out}} = 200$ J and the engine exhausts $Q_C = 400$ J of heat energy. By conservation of energy,

$$Q_H = W_{\text{out}} + Q_C = 200 \text{ J} + 400 \text{ J} = 600 \text{ J}$$

Thus, the efficiency of the engine is

$$\eta = \frac{W_{\text{out}}}{Q_H} = \frac{400 \text{ J}}{600 \text{ J}} = 0.33$$

21.2. Solve: (a) The engine has a thermal efficiency of $\eta = 40\% = 0.40$ and a work output of 100 J per cycle. The heat input is calculated as follows:

$$\eta = \frac{W_{\text{out}}}{Q_H} \quad \Rightarrow \quad 0.40 = \frac{100 \text{ J}}{Q_H} \quad \Rightarrow \quad Q_H = 250 \text{ J}$$

(b) Because $W_{\text{out}} = Q_H - Q_C$, the heat exhausted is

$$Q_C = Q_H - W_{\text{out}} = 250 \text{ J} - 100 \text{ J} = 150 \text{ J}$$

21.3. Solve: (a) During each cycle, the heat transferred into the engine is $Q_H = 55$ kJ, and the heat exhausted is $Q_C = 40$ kJ. The thermal efficiency of the heat engine is

$$\eta = 1 - \frac{Q_C}{Q_H} = 1 - \frac{40 \text{ kJ}}{55 \text{ kJ}} = 0.27 = 27\%$$

(b) The work done by the engine per cycle is

$$W_{\text{out}} = Q_H - Q_C = 55 \text{ kJ} - 40 \text{ kJ} = 15 \text{ kJ}$$

21.4. Solve: (a) The heat extracted from the cold reservoir is calculated as follows:

$$K = \frac{Q_C}{W_{\text{in}}} \quad \Rightarrow \quad 4.0 = \frac{Q_C}{50 \text{ J}} \quad \Rightarrow \quad Q_C = 200 \text{ J}$$

(b) The heat exhausted to the hot reservoir is

$$Q_H = Q_C + W_{\text{in}} = 200 \text{ J} + 50 \text{ J} = 250 \text{ J}$$

21.5. Solve: The coefficient of performance of the refrigerator is

$$K=\frac{Q_C}{W_{in}}=\frac{Q_H-W_{in}}{W_{in}}=\frac{600\text{ J}-200\text{ J}}{200\text{ J}}=2.0$$

21.6. Solve: The amount of heat discharged per second is calculated as follows:

$$\eta=\frac{W_{out}}{Q_H}=\frac{W_{out}}{Q_C+W_{out}}\quad\Rightarrow\quad Q_C=W_{out}\left(\frac{1}{\eta}-1\right)=(900\text{ MW})\left(\frac{1}{0.32}-1\right)=1.913\times10^9\text{ W}$$

That is, each second the electric power plant discharges 1.913×10^9 J of energy into the ocean. Since a typical American house needs 2.0×10^4 J of energy per second for heating, the number of houses that could be heated with the waste heat is $(1.913\times10^9\text{ J})/(2.0\times10^4\text{ J})=96{,}000$.

21.7. Model: Assume that the car engine follows a closed cycle.
Solve: **(a)** Since 2400 rpm is 40 cycles per second, the work output of the car engine per cycle is

$$W_{out}=\left(500\ \frac{\text{kJ}}{\text{s}}\right)\left(\frac{1\text{ s}}{40\text{ cycles}}\right)=12.5\frac{\text{kJ}}{\text{cycle}}\approx13\frac{\text{kJ}}{\text{cycle}}$$

(b) The heat input per cycle is calculated as follows:

$$\eta=\frac{W_{out}}{Q_H}\quad\Rightarrow\quad Q_H=\frac{12.5\text{ kJ}}{0.20}=62.5\text{ kJ}$$

The heat exhausted per cycle is

$$Q_C=Q_H-W_{in}=62.5\text{ kJ}-12.5\text{ kJ}=50\text{ kJ}$$

21.8. Solve: The amount of heat removed from the water in cooling it down in 1 hour is $Q_C=m_{water}c_{water}\Delta T$. The mass of the water is

$$m_{water}=\rho_{water}V_{water}=(1000\text{ kg/m}^3)(1\text{ L})=(100\text{ kg/m}^3)(10^{-3}\text{ m}^3)=1.0\text{ kg}$$
$$Q_C=(1.0\text{ kg})(4190\text{ J/kg K})(20°\text{C}-5°\text{C})=6.285\times10^4\text{ J}$$

The rate of heat removal from the refrigerator is

$$Q_C=\frac{6.285\times10^4\text{ J}}{3600\text{ s}}=17.46\text{ J/s}$$

The refrigerator does work $W=8.0\ \ \text{W}=8.0\text{ J/s}$ to remove this heat. Thus the performance coefficient of the refrigerator is

$$K=\frac{17.46\text{ J/s}}{8.0\text{ J/s}}=2.2$$

Section 21.3 Ideal-Gas Heat Engines

Section 21.4 Ideal-Gas Refrigerators

21.9. Model: Process A is adiabatic, process B is isochoric, and process C is isothermal.
Solve: Process A is adiabatic, so $Q=0$ J. Work W_s is positive as the gas expands. Since $Q=W_s+\Delta E_{th}=0$ J, ΔE_{th} must be negative. The temperature falls during an adiabatic expansion. Process B is isochoric. No work is done $(W_s=0\text{ J})$, and Q is positive as heat energy is added to raise the temperature (ΔE_{th} positive). Process C is isothermal so $\Delta T=0$ and $\Delta E_{th}=0$ J. The gas is compressed, so W_s is negative. $Q=W_s$ for an isothermal process, so Q is negative. Heat energy is withdrawn during the compression to keep the temperature constant.

	ΔE_{th}	W_s	Q
A	−	+	0
B	+	0	+
C	0	−	−

21.10. Model: Process A is isochoric, process B is adiabatic, process C is isothermal, and process D is isobaric.
Solve: Process A is isochoric, so the increase in pressure increases the temperature and hence the thermal energy. Because $\Delta E_{th} = Q - W_s$ and $W_s = 0$ J, Q increases for process A. Process B is adiabatic, so Q = 0 J. W_s is positive because of the increase in volume. Since $Q = 0\text{ J} = W_s + \Delta E_{th}$, ΔE_{th} is negative for process B. Process C is isothermal, so T is constant and hence $\Delta E_{th} = 0$ J. The work done W_s is positive because the gas expands. Because $Q = W_s + \Delta E_{th}$, Q is positive for process B. Process D is isobaric, so the decrease in volume leads to a decrease in temperature and hence a decrease in the thermal energy. Due to the decrease in volume, W_s is negative. Because $Q = W_s + \Delta E_{th}$, Q also decreases for process D.

	ΔE_{th}	W_s	Q
A	+	0	+
B	−	+	0
C	0	+	+
D	−	−	−

21.11. Model: The work done by the gas per cycle is the area inside the closed p-versus-V curve.
Solve: The area inside the triangle is

$$W_{out} = \tfrac{1}{2}(3\text{ atm} - 1\text{ atm})(600\times10^{-6}\text{ m}^3 - 200\times10^{-6}\text{ m}^3)$$
$$= \tfrac{1}{2}\left(2\text{ atm}\times\frac{1.013\times10^5\text{ Pa}}{1\text{ atm}}\right)(400\times10^{-6}\text{ m}^3) = 40\text{ J}$$

21.12. Solve: The work done by the gas per cycle is the area enclosed within the pV curve. We have

$$60\text{ J} = \tfrac{1}{2}(p_{max} - 100\text{ kPa})(800\text{ cm}^3 - 200\text{ cm}^3) \Rightarrow \frac{2(60\text{ J})}{600\times10^{-6}\text{ m}^3} = p_{max} - 1.0\times10^5\text{ Pa}$$
$$p_{max} = 3.0\times10^5\text{ Pa} = 300\text{ kPa}$$

21.13. Model: The heat engine follows a closed cycle, which consists of four individual processes.
Solve: (a) The work done by the heat engine per cycle is the area enclosed by the p-versus-V graph. We get

$$W_{out} = (400\text{ kPa} - 100\text{ kPa})(100\times10^{-6}\text{ m}^3) = 30\text{ J}$$

The heat energy leaving the engine is $Q_C = 90\text{ J} + 25\text{ J} = 115\text{ J}$. The heat input is calculated as follows:

$$W_{out} = Q_H - Q_C \Rightarrow Q_H = Q_C + W_{out} = 115\text{ J} + 30\text{ J} = 145\text{ J} \approx 0.15\text{ kJ}$$

(b) The thermal efficiency of the engine is

$$\eta = \frac{W_{out}}{Q_H} = \frac{30\text{ J}}{145\text{ J}} = 0.21$$

21.14. Model: The heat engine follows a closed cycle, starting and ending in the original state. The cycle consists of three individual processes.

Solve: (a) The work done by the heat engine per cycle is the area enclosed by the *p*-versus-*V* graph. We get

$$W_{\text{out}} = \tfrac{1}{2}(200\text{ kPa})(100\times10^{-6}\text{ m}^3) = 10\text{ J}$$

The heat energy leaving the engine is $Q_C = 114$ J. Because $W_{\text{out}} = Q_H - Q_C$, the heat energy exhausted is

$$Q_H = W_{\text{out}} + Q_C = 10\text{ J} + 114\text{ J} = 124\text{ J}$$

(b) The thermal efficiency of the engine is

$$\eta = \frac{W_{\text{out}}}{Q_H} = \frac{10\text{ J}}{124\text{ J}} = 0.081$$

Assess: Practical engines have thermal efficiencies in the range $\eta \approx 0.1 - 0.4$ and this one isn't quite there.

21.15. Model: The heat engine follows a closed cycle.

Solve: (a) The work done by the gas per cycle is the area inside the closed *p*-versus-*V* curve. We get

$$W_{\text{out}} = \tfrac{1}{2}(300\text{ kPa} - 100\text{ kPa})(600\text{ cm}^3 - 200\text{ cm}^3) = \tfrac{1}{2}(200\times10^3\text{ Pa})(400\times10^{-6}\text{ m}^3) = 40\text{ J}$$

The heat exhausted is $Q_C = 180\text{ J} + 100\text{ J} = 280$ J. The thermal efficiency of the engine is

$$\eta = \frac{W_{\text{out}}}{Q_H} = \frac{W_{\text{out}}}{Q_C + W_{\text{out}}} = \frac{40\text{ J}}{280\text{ J} + 40\text{ J}} = 0.13$$

(b) The heat extracted from the hot reservoir is $Q_H = Q_C + W_{\text{out}} = 320$ J.

21.16. Model: The heat engine follows a closed cycle.

Solve: The work done by the gas per cycle is the area inside the closed *p*-versus-*V* curve. We get

$$W_{\text{out}} = \tfrac{1}{2}(300\text{ kPa} - 100\text{ kPa})(600\text{ cm}^3 - 300\text{ cm}^3) = \tfrac{1}{2}(200\times10^3\text{ Pa})(300\times10^{-6}\text{ m}^3) = 30\text{ J}$$

Because $W_{\text{out}} = Q_H - Q_C$, the heat exhausted is

$$Q_C = Q_H - W_{\text{out}} = (225\text{ J } + 90\text{ J}) - 30\text{ J} = 315\text{ J} - 30\text{ J} = 285\text{ J}$$

21.17. Model: The Brayton cycle involves two adiabatic processes and two isobaric processes. The adiabatic processes involve compression and expansion through the turbine.

Solve: The thermal efficiency for the Brayton cycle is $\eta_B = 1 - r_p^{(1-\gamma)/\gamma}$, where $\gamma = C_P/C_V$ and r_p is the pressure ratio. For a diatomic gas $\gamma = 1.4$. For an adiabatic process,

$$p_1V_1^\gamma = p_2V_2^\gamma \quad \Rightarrow \quad p_2/p_1 = (V_1/V_2)^\gamma$$

Because the volume is halved, $V_2 = \tfrac{1}{2}V_1$ so

$$r_p = p_2/p_1 = (2)^\gamma = 2^{1.4} = 2.639$$

The efficiency is

$$\eta_B = 1 - (2.639)^{-0.4/1.4} = 0.24$$

21.18. Model: The gas is an ideal monatomic gas.
Visualize: For a monatomic gas $\gamma = 1.67$.
Solve:

$$\eta_B = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}} \Rightarrow 1 - \eta_B = \frac{1}{r_p^{(\gamma-1)/\gamma}} \Rightarrow r_p^{(\gamma-1)/\gamma} = \frac{1}{1-\eta_B} \Rightarrow$$

$$r_p = \left(\frac{1}{1-\eta_B}\right)^{\gamma/(\gamma-1)} = \left(\frac{1}{1-0.50}\right)^{1.67/(1.67-1)} = \left(\frac{1}{0.50}\right)^{1.67/(0.67)} = 2^{1.67/(0.67)} = 5.6$$

Assess: A higher pressure ratio corresponds to 50% efficiency if the gas is diatomic, as shown in the graph in the text.

21.19. Model: The 115 W of electrical power is 115 J of energy each second.
Visualize: Since the refrigerator is 95% efficient at converting electric energy to work, $W_{in} = 0.95(115\text{ J}) = 109.3\text{ J}$
Solve: (a)

$$K = \frac{Q_C}{W_{in}} \Rightarrow Q_C = K \cdot W_{in} = 6.0(109.3\text{ J}) = 656\text{ J}$$

The rate at which heat energy is removed is (reported to two significant figures) 660 J/s or 660 W.
(b)

$$Q_H = Q_C + W_{in} = 656\text{ J} + 109.3\text{ J} = 765\text{ J}$$

The rate at which heat energy is exhausted into the room is 760 J/s or 760 W.
Assess: In a typical house, the refrigerator isn't running all day and all night, but when it is it can exhaust this much heat energy into the room every second.

21.20. Model: The refrigerator follows a closed cycle.
Solve: (a) The net work done *on* the refrigerator in one cycle is

$$W_{in} = -W_s = -78\text{ J} + 119\text{ J} = 41\text{ J}$$

This is the work needed to push the heat from the cold reservoir up to the hot reservoir. The heat exhausted to the hot reservoir is $Q_H = 105\text{ J}$. From the first law of thermodynamics, the heat extracted from the cold reservoir is

$$W_{in} + Q_C = Q_H \quad \Rightarrow \quad Q_C = Q_H - W_{in} = 105\text{ J} - 41\text{ J} = 64\text{ J}$$

(b) The coefficient of performance is

$$K = \frac{Q_C}{W_{in}} = \frac{64\text{ J}}{41\text{ J}} = 1.6$$

Assess: This is a reasonable value for the coefficient of performance for a refrigerator.

21.21. Visualize: If we do this problem on a "per-second" basis then in one second $Q_C = (1\text{ s})(5.0 \times 10^5\text{ J/min})$ $(1\text{ min}/60\text{ s}) = 8.33 \times 10^3\text{ J}$. $Q_H = (1\text{ s})(8.0 \times 10^5\text{ J/min})(1\text{ min}/60\text{ s}) = 13.33 \times 10^3\text{ J}$.
Solve: (a) Again, in one second

$$W_{in} = Q_H - Q_C = 13.33 \times 10^3\text{ J} - 8.33 \times 10^3\text{ J} = 5.0 \times 10^3\text{ J}$$

Since this is per second, the power required by the compressor is $P = 5.0\text{ kW}$.
(b) The coefficient of performance is

$$K = \frac{Q_C}{W_{in}} = \frac{8.33 \times 10^3\text{ J}}{5.0 \times 10^3\text{ J}} = 1.7$$

Assess: The result is typical for air conditioners.

Section 21.5 The Limits of Efficiency

Section 21.6 The Carnot Cycle

21.22. Model: The efficiency of a Carnot engine (η_{Carnot}) depends only on the temperatures of the hot and cold reservoirs. On the other hand, the thermal efficiency (η) of a heat engine depends on the heats Q_H and Q_C.

Solve: (a) According to the first law of thermodynamics, $Q_H = W_{\text{out}} + Q_C$. For engine **(a)**, $Q_H = 500\text{ J}$, $Q_C = 200\text{ J}$ and $W_{\text{out}} = 300\text{ J}$, so the first law of thermodynamics is obeyed. For engine **(b)**, $Q_H = 500\text{ J}$, $Q_C = 200\text{ J}$ and $W_{\text{out}} = 200\text{ J}$, so the first law is violated. For engine **(c)** $Q_H = 300\text{ J}$, $Q_C = 200\text{ J}$ and $W_{\text{out}} = 100\text{ J}$, so the first law of thermodynamics is obeyed.

(b) For the three heat engines, the maximum or Carnot efficiency is

$$\eta_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{300\text{ K}}{600\text{ K}} = 0.50$$

Engine **(a)** has

$$\eta = 1 - \frac{Q_C}{Q_H} = \frac{W_{\text{out}}}{Q_H} = \frac{300\text{ J}}{500\text{ J}} = 0.60$$

This is larger than η_{Carnot}, thus violating the second law of thermodynamics. For engine **(b)**,

$$\eta = \frac{W_{\text{out}}}{Q_H} = \frac{200\text{ J}}{500\text{ J}} = 0.40 < \eta_{\text{Carnot}}$$

so the second law is obeyed. Engine **(c)** has a thermal efficiency of

$$\eta = \frac{100\text{ J}}{300\text{ J}} = 0.33 < \eta_{\text{Carnot}}$$

so the second law of thermodynamics is obeyed.

21.23. Model: For a refrigerator $Q_H = Q_C + W_{\text{in}}$, and the coefficient of performance and the Carnot coefficient of performance are

$$K = \frac{Q_C}{W_{\text{in}}}, \quad K_{\text{Carnot}} = \frac{T_C}{T_H - T_C}$$

Solve: (a) For refrigerator **(a)** $Q_H = Q_C + W_{\text{in}}$ $(60\text{ J} = 40\text{ J} + 20\text{ J})$, so the first law of thermodynamics is obeyed. For refrigerator **(b)** $50\text{ J} = 40\text{ J} + 10\text{ J}$, so the first law of thermodynamics is obeyed. For the refrigerator **(c)** $40\text{ J} \neq 30\text{ J} + 20\text{ J}$, so the first law of thermodynamics is violated.

(b) For the three refrigerators, the maximum coefficient of performance is

$$K_{\text{Carnot}} = \frac{T_C}{T_H - T_C} = \frac{300\text{ K}}{400\text{ K} - 300\text{ K}} = 3$$

For refrigerator **(a)**,

$$K = \frac{Q_C}{W_{\text{in}}} = \frac{40\text{ J}}{20\text{ J}} = 2 < K_{\text{Carnot}}$$

so the second law of thermodynamics is obeyed. For refrigerator **(b)**,

$$K = \frac{Q_C}{W_{\text{in}}} = \frac{40\text{ J}}{10\text{ J}} = 4 > K_{\text{Carnot}}$$

so the second law of thermodynamics is violated. For refrigerator **(c)**,

$$K = \frac{30\text{ J}}{20\text{ J}} = 1.5 < K_{\text{Carnot}}$$

so the second law is obeyed.

21.24. Model: The efficiency of a Carnot engine depends only on the absolute temperatures of the hot and cold reservoirs.

Solve: The efficiency of a Carnot engine is

$$\eta_{\text{Carnot}} = 1 - \frac{T_C}{T_H} \quad \Rightarrow \quad 0.60 = 1 - \frac{T_C}{(427 + 273)\text{ K}} \quad \Rightarrow \quad T_C = 280\text{ K} = 7\,°\text{C}$$

Assess: A "real" engine would need a lower temperature than 7°C to provide 60% efficiency because no real engine can match the Carnot efficiency.

21.25. Model: The efficiency of an ideal engine (or Carnot engine) depends only on the temperatures of the hot and cold reservoirs.

Solve: (a) The engine's thermal efficiency is

$$\eta = \frac{W_{\text{out}}}{Q_H} = \frac{W_{\text{out}}}{Q_C + W_{\text{out}}} = \frac{10\text{ J}}{15\text{ J} + 10\text{ J}} = 0.40 = 40\%$$

(b) The efficiency of a Carnot engine is $\eta_{\text{Carnot}} = 1 - T_C/T_H$. The minimum temperature in the hot reservoir is found as follows:

$$0.40 = 1 - \frac{293\text{ K}}{T_H} \quad \Rightarrow \quad T_H = 488\text{ K} = 215\,°\text{C}$$

This is the minimum possible temperature. In a real engine, the hot-reservoir temperature would be higher than 215°C because no real engine can match the Carnot efficiency.

21.26. Model: Assume that the heat engine follows a closed cycle.

Solve: (a) The engine's thermal efficiency is

$$\eta = \frac{W_{\text{out}}}{Q_H} = \frac{W_{\text{out}}}{Q_C + W_{\text{out}}} = \frac{200\text{ J}}{600\text{ J} + 200\text{ J}} = 0.25 = 25\%$$

(b) The thermal efficiency of a Carnot engine is $\eta_{\text{Carnot}} = 1 - T_C/T_H$. For this to be 25%,

$$0.25 = 1 - \frac{T_C}{(400 + 273)\text{ K}} \quad \Rightarrow \quad T_C = 504.8\text{ K} = 232\,°\text{C}$$

21.27. Solve: (a) The efficiency of the Carnot engine is

$$\eta_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{300\text{ K}}{500\text{ K}} = 0.40 = 40\%$$

(b) An engine with power output of 1000 W does $W_{\text{out}} = 1000\text{ J}$ of work during each $\Delta t = 1\text{ s}$. A Carnot engine has a heat input that is

$$Q_{\text{in}} = \frac{W_{\text{out}}}{\eta_{\text{Carnot}}} = \frac{1000\text{ J}}{0.40} = 2500\text{ J}$$

during each $\Delta t = 1\text{ s}$. The *rate* of heat input is $2500\text{ J/s} = 2500\text{ W}$.

(c) $W_{\text{out}} = Q_{\text{in}} - |Q_{\text{out}}|$, so the heat output during $\Delta t = 1\text{ s}$ is $|Q_{\text{out}}| = Q_{\text{in}} - W_{\text{out}} = 1500\text{ J}$. The *rate* of heat output is thus $1500\text{ J/s} = 1500\text{ W}$.

21.28. Model: We will use Equation 21.27 for the efficiency of a Carnot engine.

$$\eta_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$$

We are given $T_H = 673\text{ K}$ and the original efficiency $\eta_{\text{Carnot}} = 0.40$.

Solve: First solve for T_C.

$$T_C = T_H(1-\eta_{Carnot}) = (673\text{ K})(1-0.40) = 404\text{ K}$$

Solve for T_C' again with $\eta'_{Carnot} = 0.60$.

$$T_C' = T_H(1-\eta'_{Carnot}) = (673\text{ K})(1-0.60) = 269\text{ K}$$

The difference of these T_C values is 135 K, so the temperature of the cold reservoir should be decreased by 135 °C to raise the efficiency from 40% to 60%.
Assess: We expected to have to lower T_C by quite a bit to get the better efficiency.

21.29. Model: The ideal gas in the Carnot engine follows a closed cycle in four steps. During the isothermal expansion at temperature T_H, heat Q_H is transferred from the hot reservoir into the gas. During the isothermal compression at T_C, heat Q_C is removed from the gas. No heat is transferred during the remaining two adiabatic steps.
Solve: The thermal efficiency of the Carnot engine is

$$\eta_{Carnot} = 1-\frac{T_C}{T_H} = \frac{W_{out}}{Q_H} \quad\Rightarrow\quad 1-\frac{323\text{ K}}{573\text{ K}} = \frac{W_{out}}{1000\text{ J}} \quad\Rightarrow\quad W_{out} = 436\text{ J}$$

Using $Q_H = Q_C + W_{out}$, we obtain

$$Q_{isothermal} = Q_C = Q_H - W_{out} = 1000\text{ J} - 436\text{ J} \approx 0.56\text{ kJ}$$

21.30. Model: The maximum possible efficiency for a heat engine is provided by the Carnot engine.
Solve: The maximum efficiency is

$$\eta_{max} = \eta_{Carnot} = 1-\frac{T_C}{T_H} = 1-\frac{(273+20)\text{ K}}{(273+600)\text{ K}} = 0.6644$$

Because the heat engine is running at only 30% of the maximum efficiency, $\eta = (0.30)\eta_{max} = 0.1993$. The amount of heat that must be extracted is

$$Q_H = \frac{W_{out}}{\eta} = \frac{1000\text{ J}}{0.1993} = 5.0\text{ kJ}$$

21.31. Model: We are given $T_H = 773$ K and $T_C = 273$ K, therefore (by Equation 21.27) the Carnot efficiency is $n_{Carnot} = 1-\frac{273\text{ K}}{773\text{ K}} = 0.647$ We are also given $\eta = 0.60(\eta_{Carnot})$.
Solve: Rearrange Equation 21.5: $Q_H = W_{out}(1-\eta)$. W_{out} is the same for both engines, so it cancels.

$$\frac{Q_H}{(Q_H)_{Carnot}} = \frac{W_{out}(1-\eta)}{W_{out}(1-\eta_{Carnot})} = \frac{1-0.60(\eta_{Carnot})}{1-\eta_{Carnot}} = \frac{1-0.60(0.647)}{1-0.647} = 1.7$$

Assess: This engine requires 1.7 times as much heat energy during each cycle as a Carnot engine to do the same amount of work.

21.32. Model: The coefficient of performance of a Carnot refrigerator depends only on the temperatures of the cold and hot reservoirs.
Solve: (a) The Carnot performance coefficient of a refrigerator is

$$K_{Carnot} = \frac{T_C}{T_H - T_C} = \frac{(-20+273)\text{ K}}{(20+273)\text{ K} - (-20+273)\text{ K}} = 6.325 \approx 6.3$$

(b) The rate at which work is done on the refrigerator is found as follows:

$$K = \frac{Q_C}{W_{in}} \Rightarrow W_{in} = \frac{Q_C}{K} = \frac{200\text{ J/s}}{6.325} = 32\text{ J/s} = 32\text{ W}$$

(c) The heat exhausted to the hot side per second is

$$Q_H = Q_C + W_{in} = 200\ \text{J/s} + 32\ \text{J/s} = 232\ \text{J/s} \approx 0.23\ \text{kW}$$

21.33. Model: The minimum possible value of T_C occurs with a Carnot refrigerator.
Solve: (a) For the refrigerator, the coefficient of performance is

$$K = \frac{Q_C}{W_{in}} \Rightarrow Q_C = KW_{in} = (5.0)(10\ \text{J}) = 50\ \text{J}$$

The heat energy exhausted per cycle is

$$Q_H = Q_C + W_{in} = 50\ \text{J} + 10\ \text{J} = 60\ \text{J}$$

(b) If the hot-reservoir temperature is $27°\text{C} = 300\ \text{K}$, the lowest possible temperature of the cold reservoir can be obtained as follows:

$$K_{Carnot} = \frac{T_C}{T_H - T_C} \Rightarrow 5.0 = \frac{T_C}{300\ \text{K} - T_C} \Rightarrow T_C = 250\ \text{K} = -23\,°\text{C}$$

21.34. Model: Equation 21.27 gives $\eta_{Carnot} = 1 - \frac{T_C}{T_H}$. We are given $\eta_{Carnot} = 1/3$.

Solve:

$$\eta_{Carnot} = 1 - \frac{T_C}{T_H} = \frac{1}{3} \Rightarrow \frac{T_C}{T_H} = \frac{2}{3} \Rightarrow T_H = \frac{3}{2}T_C$$

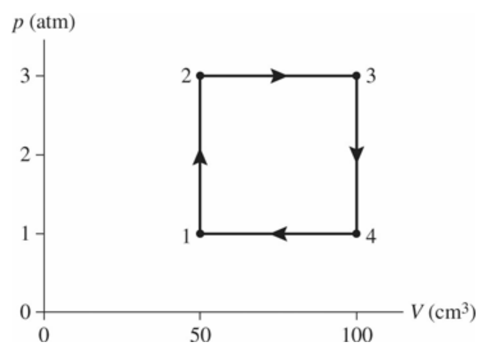
Equation 21.28 gives the coefficient of performance for the Carnot refrigerator.

$$K_{Carnot} = \frac{T_C}{T_H - T_C} = \frac{T_C}{\frac{3}{2}T_C - T_C} = \frac{1}{\frac{3}{2} - 1} = 2$$

Assess: This result is in the ballpark for coefficients of performance.

Problems

21.35. Model: The heat engine follows a closed cycle, starting and ending in the original state.
Visualize: The figure indicates the following seven steps. First, the pin is inserted when the heat engine has the initial conditions. Second, heat is turned on and the pressure increases at constant volume from 1 to 3 atm. Third, the pin is removed. The flame continues to heat the gas and the volume increases at constant pressure from $50\ \text{cm}^3$ to $100\ \text{cm}^3$. Fourth, the pin is inserted and some of the weights are removed. Fifth, the container is placed on ice and the gas cools at constant volume to a pressure of 1 atm. Sixth, with the container still on ice, the pin is removed. The gas continues to cool at constant pressure to a volume of $50\ \text{cm}^3$. Seventh, with no ice or flame, the pin is inserted back in and the weights returned bringing the engine back to the initial conditions and ready to start over.
Solve: (a)



(b) The work done per cycle is the area inside the curve:

$$W_{\text{out}} = (\Delta p)(\Delta V) = (2\times 101{,}300\ \text{Pa})(50\times 10^{-6}\ \text{m}^3) = 10\ \text{J}$$

(c) Heat energy is input during processes $1 \to 2$ and $2 \to 3$, so $Q_{\text{H}} = Q_{12} + Q_{23}$. This is a diatomic gas, with $C_{\text{V}} = \frac{3}{2}R$ and $C_{\text{P}} = \frac{5}{2}R$. The number of moles of gas is

$$n = \frac{p_1 V_1}{RT_1} = \frac{(101{,}300\ \text{Pa})(50\times 10^{-6}\ \text{m}^3)}{(8.31\ \text{J/mol K})(293\ \text{K})} = 0.00208\ \text{mol}$$

Process $1 \to 2$ is isochoric, so $T_2 = (p_2/p_1)T_1 = 3T_1 = 879\ \text{K}$. Process $2 \to 3$ is isobaric, so $T_3 = (V_3/V_2)T_2 = 2T_2 = 1758\ \text{K}$. Thus

$$Q_{12} = nC_{\text{V}}\Delta T = \tfrac{5}{2}nR(T_2 - T_1) = \tfrac{5}{2}(0.00208\ \text{mol})(8.31\ \text{J/mol K})(586\ \text{K}) = 25.32\ \text{J}$$

Similarly,

$$Q_{23} = nC_{\text{P}}\Delta T = \tfrac{7}{2}nR(T_3 - T_2) = \tfrac{7}{2}(0.00208\ \text{mol})(8.31\ \text{J/mol K})(879\ \text{K}) = 53.18\ \text{J}$$

Thus $Q_{\text{H}} = 25.32\ \text{J} + 53.18\ \text{J} = 78.50\ \text{J}$ and the engine's efficiency is

$$\eta = \frac{W_{\text{out}}}{Q_{\text{H}}} = \frac{10.13\ \text{J}}{78.50\ \text{J}} = 0.13 = 13,$$

21.36. Solve: The work done by the engine is equal to the change in the gravitational potential energy. Thus,

$$W_{\text{out}} = \Delta U_{\text{grav}} = mgh = (2000\ \text{kg})(9.8\ \text{m/s}^2)(30\ \text{m}) = 588{,}000\ \text{J}$$

The efficiency of this engine is

$$\eta = 0.40\ \eta_{\text{Carnot}} = 0.40\left(1 - \frac{T_{\text{C}}}{T_{\text{H}}}\right) = 0.40\left(1 - \frac{(273+20)\ \text{K}}{(273+2000)\ \text{K}}\right) = 0.3484$$

The amount of heat energy transferred is calculated as follows:

$$\eta = \frac{W_{\text{out}}}{Q_{\text{H}}} \quad\Rightarrow\quad Q_{\text{H}} = \frac{W_{\text{out}}}{\eta} = \frac{588{,}000\ \text{J}}{0.3484} = 1.7\times 10^6\ \text{J}$$

21.37. Model: Assume an ideal spring for which $U = (1/2)k(\Delta x)^2$.

Visualize: We are given $T_{\text{C}} = 293\ \text{K}$, $T_{\text{H}} = 473\ \text{K}$, $Q_{\text{H}} = 63\ \text{J}$ each second. For the spring we are given $\Delta x = 0.22\ \text{m}$, $\Delta t = 0.50\ \text{s}$.

Solve:

$$\eta = 0.50\eta_{\text{Carnot}} = 0.50\left(1 - \frac{T_{\text{C}}}{T_{\text{H}}}\right) = \frac{W_{\text{out}}}{Q_{\text{H}}} = \frac{W_{\text{out}}/\Delta t}{Q_{\text{H}}/\Delta t}$$

$$\Rightarrow \frac{W_{\text{out}}}{\Delta t} = 0.50\left(1 - \frac{T_{\text{C}}}{T_{\text{H}}}\right)\left(\frac{Q_{\text{H}}}{\Delta t}\right) = 0.50\left(1 - \frac{293\ \text{K}}{473\ \text{K}}\right)(63\ \text{W}) = 12\ \text{W}$$

The spring is compressed in 0.50 s, so the energy it takes to compress it is $U = (12\ \text{W})(0.50\ \text{s}) = 6.0\ \text{J}$. The potential energy in a compressed spring is

$$U = \tfrac{1}{2}k(\Delta x)^2 \Rightarrow k = \frac{2U}{(\Delta x)^2} = \frac{2(6.0\ \text{J})}{(0.22\ \text{m})^2} = 250\ \text{N/m}$$

Assess: This is a typical value for a spring constant.

21.38. Solve: An adiabatic process has $Q = 0$ and thus, from the first law, $W_s = -\Delta E_{th}$. For any ideal-gas process, $\Delta E_{th} = nC_V \Delta T$, so $W_s = -nC_V \Delta T$. We can use the ideal-gas law to find

$$T = \frac{pV}{nR} \quad \Rightarrow \quad \Delta T = \frac{\Delta(pV)}{nR} = \frac{(pV)_f - (pV)_i}{nR} = \frac{p_f V_f - p_i V_i}{nR}$$

Consequently, the work is

$$W_s = -nC_V \Delta T = -nC_V \left(\frac{p_f V_f - p_i V_i}{nR} \right) = -\frac{C_V}{R}(p_f V_f - p_i V_i)$$

Because $C_P = C_V + R$, we can use the specific heat ratio γ to find

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = \frac{C_V/R + 1}{C_V/R} \quad \Rightarrow \quad \frac{C_V}{R} = \frac{1}{\gamma - 1}$$

With this, the work done in an adiabatic process is

$$W_s = -\frac{C_V}{R}(p_f V_f - p_i V_i) = -\frac{1}{\gamma - 1}(p_f V_f - p_i V_i) = \frac{p_f V_f - p_i V_i}{1 - \gamma}$$

21.39. Model: We are given $T_H = 323\text{ K}$ and $T_C = 253\text{ K}$. See Figure 21.11.

Solve: Every second, the refrigerator must draw enough heat from the cold reservoir to compensate for the heat lost through the stainless-steel panel. Therefore, the heat transferred from the cold reservoir to the system is

$$Q_C = k\frac{A}{L}\Delta T \Delta t = (14\text{ W/m K})\frac{(0.40\text{ m})(0.40\text{ m})}{0.010\text{ m}}[25\,°\text{C} - (-20\,°\text{C})](1.0\text{ s}) = 10{,}080\text{ J}.$$

Using the coefficient of performance for a Carnot refrigerator, we can find the energy required to operate for one second:

$$K_{\text{Carnot}} = \frac{T_C}{T_H - T_C} = \frac{Q_C}{W_{in}} \quad \Rightarrow \quad W_{in} = \frac{Q_C(T_H - T_C)}{T_C} = \frac{(10{,}080\text{ J})(70\text{ K})}{253\text{ K}} = 2.8\text{ kJ}$$

The power required is therefore $P = W_{in}/\Delta t = (2.8\text{ kJ})(1.0\text{ s}) = 2.8\text{ kW}$.

Assess: This is much more power than is required for the Brayton-cycle refrigerator of Example 21.3, which shows why refrigerators are insulated with more than simple steel doors.

21.40. Solve: For any heat engine, $\eta = 1 - Q_C/Q_H$. For a Carnot heat engine, $\eta_{\text{Carnot}} = 1 - T_C/T_H$. Thus a property of the Carnot cycle is that $Q_C/Q_H = T_C/T_H$. Consequently, the coefficient of performance of a Carnot refrigerator is

$$K_{\text{Carnot}} = \frac{Q_C}{W_{in}} = \frac{Q_C}{Q_H - Q_C} = \frac{Q_C/Q_H}{1 - Q_C/Q_H} = \frac{T_C/T_H}{1 - T_C/T_H} = \frac{T_C}{T_H - T_C}$$

21.41. Model: We are given $T_H = 298\text{ K}$ and $T_C = 273\text{ K}$. See Figure 21.11.

Solve: $Q_C = mL_f = (10\text{ kg})(3.33 \times 10^5\text{ J/kg}) = 3.33 \times 10^6\text{ J}$.

(a) For a Carnot cycle $\eta_{\text{Carnot}} = 1 - \dfrac{T_C}{T_H}$ but that must also equal $\eta = 1 - \dfrac{Q_C}{Q_H}$, so $\dfrac{Q_C}{Q_H} = \dfrac{T_C}{T_H}$.

$$Q_H = Q_C\frac{T_H}{T_C} = (3.33 \times 10^6\text{ J})\left(\frac{298\text{ K}}{273\text{ K}}\right) = 3.6 \times 10^6\text{ J}$$

(b) $W_{in} = Q_H - Q_C = 3.63 \times 10^6\text{ J} - 3.33 \times 10^6\text{ J} = 0.30 \times 10^6\text{ J} = 3.0 \times 10^5\text{ J}$

Assess: This is a reasonable amount of work to freeze 10 kg of water.

21.42. Model: Assume all the energy removed from the freezer came from the 3.0 L of water.

Visualize: We are given $T_C = 252\text{ K}$, $T_H = 298\text{ K}$, $W_{in} = 200\text{ J}$ each second.

Solve: We compute the rate at which heat energy is removed from inside the freezer.

$$K = 0.30K_{Carnot} = 0.30\left(\frac{T_C}{T_H - T_C}\right) = \frac{Q_C}{W_{in}} = \frac{Q_C/\Delta t}{W_{in}/\Delta t}$$

$$\Rightarrow \frac{Q_C}{\Delta t} = 0.30\left(\frac{T_C}{T_H - T_C}\right)\left(\frac{W_{in}}{\Delta t}\right) = 0.30\left(\frac{251\text{ K}}{298\text{ K} - 251\text{ K}}\right)(200\text{ W}) = 320\text{ W}$$

This is the amount of heat removed from the freezer each second. Now we figure the total amount of energy that needs to be removed from the 3.0 L of water to cool it from $20°\text{C}$ to $0°$ and then freeze it.

$$Q = cm\Delta T + mL_f = (4190\text{ J/kg}\cdot\text{K})(3.0\text{ kg})(20\text{ K}) + (3.0\text{ kg})(3.3\times 10^5\text{ J/kg}) = 1.24\times 10^6\text{ J}$$

The time it will take will be

$$\Delta t = \frac{Q}{Q_C/\Delta t} = \frac{1.24\times 10^6\text{ J}}{320\text{ J/s}} = 3900\text{ s} = 65\text{ min}$$

Assess: This seems about right to freeze 3.0 L of water in the freezer.

21.43. Model: We will use the Carnot engine to find the maximum possible efficiency of a floating power plant.

Solve: The efficiency of a Carnot engine is

$$\eta_{max} = \eta_{Carnot} = 1 - \frac{T_C}{T_H} = 1 - \frac{(273+5)\text{ K}}{(273+30)\text{ K}} = 0.0825 \approx 8.3\%$$

21.44. Solve: From the thermal efficiency of the Carnot engine, we can find the work done each cycle:

$$\eta_{Carnot} = 1 - \frac{T_C}{T_H} = \frac{W_{out}}{Q_H} \quad\Rightarrow\quad W_{out} = \left(1 - \frac{273\text{ K}}{455\text{ K}}\right)(25\text{ J}) = 10\text{ J}$$

The work required to lift a 10 kg mass 10 m is $W = Fd = (10\text{ kg})(9.8\text{ m/s}^2)(10\text{ m})$ = 980 J. At 10 J/cycle, the Carnot engine will have to cycle 98 times to do this work.

21.45. Model: The work output by the engine is the work input into the refrigerator. Both operate at maximum Carnot efficiency.

Visualize: Our final goal is $(Q_H)_{ref}/(Q_H)_{eng}$, the ratio of heat energy exhausted to the room to the heat energy used by the heat engine. For the engine we are given $(T_C)_{eng} = 278\text{ K}$, $(T_H)_{eng} = 773\text{ K}$, and for the refrigerator $(T_C)_{ref} = 268\text{ K}$, $(T_H)_{ref} = 298\text{ K}$.

Solve: First we concentrate on the engine. We want the work output of the engine.

$$\eta_{Carnot} = 1 - \frac{(T_C)_{eng}}{(T_H)_{eng}} = \frac{(W_{out})_{eng}}{(Q_H)_{eng}} \Rightarrow (W_{out})_{eng} = (Q_H)_{eng}\eta_{Carnot}$$

This becomes the work input to the refrigerator.

$$K_{Carnot} = \frac{(T_C)_{ref}}{(T_H)_{ref} - (T_C)_{ref}} = \frac{(Q_C)_{ref}}{(W_{in})_{ref}} \Rightarrow (Q_C)_{ref} = (W_{in})_{ref}K_{Carnot}$$

We next find the heat exhausted by the refrigerator, $(Q_H)_{ref}$.

$$\begin{aligned}(Q_H)_{ref} &= (Q_C)_{ref} + (W_{in})_{ref} = (W_{in})_{ref}K_{Carnot} + (W_{in})_{ref} = (W_{in})_{ref}(1+K_{Carnot}) \\ &= (W_{out})_{eng}(1+K_{Carnot}) = (Q_H)_{eng}\,\eta_{Carnot}(1+K_{Carnot})\end{aligned}$$

Now we find our end goal.

$$\frac{(Q_{\rm H})_{\rm ref}}{(Q_{\rm H})_{\rm eng}} = \eta_{\rm Carnot}(1+K_{\rm Carnot}) = \left(1-\frac{(T_{\rm C})_{\rm eng}}{(T_{\rm H})_{\rm eng}}\right)\left(1+\frac{(T_{\rm C})_{\rm ref}}{(T_{\rm H})_{\rm ref}-(T_{\rm C})_{\rm ref}}\right)$$

$$= \left(1-\frac{(T_{\rm C})_{\rm eng}}{(T_{\rm H})_{\rm eng}}\right)\left(\frac{(T_{\rm H})_{\rm ref}}{(T_{\rm H})_{\rm ref}-(T_{\rm C})_{\rm ref}}\right) = \left(1-\frac{278\ \text{K}}{773\ \text{K}}\right)\left(\frac{298\ \text{K}}{298\ \text{K}-268\ \text{K}}\right) = 6.4$$

So the refrigerator exhausts 6.4 J into the room for each joule of heat energy used by the heat engine.
Assess: Since these are the best possible efficiencies, any real system would be less efficient.

21.46. Solve: (a) Q_1 is given as 1000 J. Using the energy transfer equation for the heat engine,

$$Q_{\rm H} = Q_{\rm C} + W_{\rm out} \quad\Rightarrow\quad Q_1 = Q_2 + W_{\rm out} \quad\Rightarrow\quad Q_2 = Q_1 - W_{\rm out}$$

The thermal efficiency of a Carnot engine is

$$\eta = 1-\frac{T_{\rm C}}{T_{\rm H}} = 1-\frac{300\ \text{K}}{600\ \text{K}} = 0.50 = \frac{W_{\rm out}}{Q_1}$$

$$Q_2 = Q_1 - \eta Q_1 = Q_1(1-\eta) = (1000\ \text{J})(1-0.50) = 500\ \text{J}$$

To determine Q_3 and Q_4, we turn our attention to the Carnot refrigerator, which is driven by the output of the heat engine with $W_{\rm in} = W_{\rm out}$. The coefficient of performance is

$$K = \frac{T_{\rm C}}{T_{\rm H}-T_{\rm C}} = \frac{400\ \text{K}}{500\ \text{K}-400\ \text{K}} = 4.0 = \frac{Q_{\rm C}}{W_{\rm in}} = \frac{Q_4}{W_{\rm out}} = \frac{Q_4}{\eta Q_1}$$

$$Q_4 = K\eta Q_1 = (4.0)(0.50)(1000\ \text{J}) = 2000\ \text{J}$$

Using now the energy transfer equation $W_{\rm in} + Q_4 = Q_3$, we have

$$Q_3 = W_{\rm out} + Q_4 = \eta Q_1 + Q_4 = (0.50)(1000\ \text{J}) + 2000\ \text{J} = 2500\ \text{J}$$

(b) From part **(a)** $Q_3 = 2500$ J and $Q_1 = 1000$ J, so $Q_3 > Q_1$.
(c) Although $Q_1 = 1000$ J and $Q_3 = 2500$ J, the two devices together do not violate the second law of thermodynamics. This is because the hot and cold reservoirs are different for the heat engine and the refrigerator.

21.47. Solve: The work done by the Carnot engine powers the refrigerator, so $(W_{\rm out})_{\rm Carnot\,Eng} = (W_{\rm in})_{\rm Refrigerator}$. We are given that $T_{\rm H} = 350$ K and $T_{\rm C} = 250$ K for both the Carnot engine and the refrigerator and $(Q_{\rm H})_{\rm Carnot\ Eng} =$ 10.0 J for the Carnot engine. The work done by the Carnot engine is

$$\eta = 1-\frac{T_{\rm C}}{T_{\rm H}} = \frac{(W_{\rm out})_{\rm Carnot\,Eng}}{(Q_{\rm H})_{\rm Carnot\,Eng}} \quad\Rightarrow\quad (W_{\rm out})_{\rm Carnot\,Eng} = \left(1-\frac{250\ \text{K}}{350\ \text{K}}\right)(10.0\ \text{J}) = 2.857\ \text{J}$$

The heat extracted from the cold reservoir by the refrigerator may be found from its coefficient of performance:

$$K = \frac{Q_{\rm C}}{(W_{\rm in})_{\rm Refrigerator}} = \frac{Q_{\rm C}}{(W_{\rm out})_{\rm Carnot\ Eng}} \quad\Rightarrow\quad Q_{\rm C} = (2.00)(2.857\ \text{J}) = 5.713\ \text{J}$$

The heat exhausted by the refrigerator to the hot reservoir may be found from the first law of thermodynamics:

$$(W_{\rm in})_{\rm Refrigerator} + Q_{\rm C} = Q_{\rm H} \quad\Rightarrow\quad Q_{\rm H} = 2.857\ \text{J} + 5.713\ \text{J} = 8.57\ \text{J}$$

Assess: The work done on the refrigerator is less than the heat exhausted to the hot reservoir, as expected.

21.48. Model: A heat pump is a refrigerator that is cooling the already cold outdoors and warming the indoors with its exhaust heat.

Solve: (a) The coefficient of performance for this heat pump is $K = 5.0 = Q_C/W_{in}$, where Q_C is the amount of heat removed from the cold reservoir. Q_H is the amount of heat exhausted into the hot reservoir. $Q_H = Q_C + W_{in}$, where W_{in} is the amount of work done on the heat pump. We have

$$Q_C = 5.0W_{in} \quad \Rightarrow \quad Q_H = 5.0W_{in} + W_{in} = 6.0W_{in}$$

If the heat pump is to deliver 15 kJ of heat per second to the house, then

$$Q_H = 15\text{ kJ} = 6.0W_{in} \quad \Rightarrow \quad W_{in} = \frac{15\text{ kJ}}{6.0} = 2.5\text{ kJ}$$

In other words, 2.5 kW of electric power is used by the heat pump to deliver 15 kJ/s of heat energy to the house.

(b) The monthly heating cost in the house using an electric heater is

$$\left(\frac{15\text{ kJ}}{\text{s}}\right)(200\text{ h})\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{1\$}{40\text{ MJ}}\right) = \$270$$

The monthly heating cost in the house using a heat pump is

$$\left(\frac{2.5\text{ kJ}}{\text{s}}\right)(200\text{ h})\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{1\$}{40\text{ MJ}}\right) = \$45$$

21.49. Model: The useful work done by the heat engine changes the kinetic energy of the car; ignore the drag and friction forces.

Visualize: For the engine we are given $T_H = 1773\text{ K}$, $T_C = 293\text{ K}$.

Solve: First we compute the efficiency

$$\eta = 0.30\eta_{Carnot} = 0.30\left(1 - \frac{T_C}{T_H}\right) = 0.30\left(1 - \frac{293\text{ K}}{1773\text{ K}}\right) = 0.25$$

Use the work-kinetic energy theorem: $W = \Delta K$ and compute the gas needed to change the kinetic energy of the car.

$$m_{gas} = \frac{\Delta K_{car}}{\eta(Q_{gas}/m_{gas})} = \frac{\frac{1}{2}mv^2}{\eta(Q_{gas}/m_{gas})} = \frac{\frac{1}{2}(1500\text{ kg})(30\text{ m/s})^2}{(0.25)(47\text{ kJ/g})} = 57\text{ g}$$

Assess: The efficiency η is not very high, but gasoline has a lot of energy per gram.

21.50. Model: All of the Q_C output by the power plant goes directly into heating homes. Assume the homes are heated steadily for 6 months each year.

Visualize: For the engine we are given $T_H = 723\text{ K}$, $T_C = 303\text{ K}$. The 1.0 MW power plant supplies 1.0 MJ each second.

Solve: First we compute the efficiency

$$\eta = 0.65\eta_{Carnot} = 0.65\left(1 - \frac{T_C}{T_H}\right) = 0.65\left(1 - \frac{303\text{ K}}{723\text{ K}}\right) = 0.378$$

Now we compute the energy needed by a home in one second—for each second for the six months of heating.

$$\frac{70\text{ GJ}}{0.5\text{ y}}\left(\frac{1\text{ y}}{365\text{ d}}\right)\left(\frac{1\text{ d}}{24\text{ h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 4440\text{ J/s}$$

Q_C from the power plant will be used to heat the homes. We compute Q_C for one second.

$$Q_C = Q_H - W_{out} = \frac{W_{out}}{Q_H} - W_{out} = W_{out}\left(\frac{1}{\eta} - 1\right) = (1.0\text{ MJ})\left(\frac{1}{0.378} - 1\right) = 1.65\text{ MJ}$$

$$\#\text{homes} = \frac{Q_C\text{ per second}}{(\text{energy needed})/\text{home/second}} = \frac{1.65\text{ MJ/s}}{4440\text{ J/s}} = 370\text{ homes}$$

Assess: This sounds like a reasonable number. Of course there would be many inefficiencies in getting the heat to the homes.

21.51. Model: The power plant is to be treated as a heat engine.

Solve: (a) Every hour 300 metric tons or 3×10^5 kg of coal is burnt. The volume of coal is

$$\left(\frac{3\times10^5\ \text{kg}}{1\ \text{h}}\right)\left(\frac{\text{m}^3}{1500\ \text{kg}}\right)(24\ \text{h}) = 4800\ \text{m}^3$$

The height of the room will be 48 m.

(b) The thermal efficiency of the power plant is

$$\eta = \frac{W_{\text{out}}}{Q_{\text{H}}} = \frac{7.50\times10^8\ \text{J/s}}{\left(\frac{3\times10^5\ \text{kg}}{1\ \text{h}}\right)\left(\frac{28\times10^6\ \text{J}}{\text{kg}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)} = \frac{7.50\times10^8\ \text{J}}{2.333\times10^9\ \text{J}} = 0.32 = 32\%$$

Assess: An efficiency of 32% is typical of power plants.

21.52. Model: The power plant is treated as a heat engine.

Solve: We are given that $T_{\text{H}} = 300°\text{C} = 573\ \text{K}$ and $T_{\text{C}} = 25°\text{C} = 298\ \text{K}$.

(a) The maximum possible thermal efficiency of the power plant is

$$\eta_{\text{max}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} = 1 - \frac{298\ \text{K}}{573\ \text{K}} = 0.48 = 48\%$$

(b) In one second, the plant generates $W_{\text{out}} = 1000\times10^6\ \text{J}$ of work and $Q_{\text{H}} = 3000\times10^6\ \text{J}$ of heat energy to replace the energy taken from the hot reservoir to heat the water. The plant's actual efficiency is

$$\eta = \frac{W_{\text{out}}}{Q_{\text{H}}} = \frac{1000\times10^6\ \text{J}}{3000\times10^6\ \text{J}} 0.33 = 33\%$$

(c) Because $Q_{\text{H}} = Q_{\text{C}} + W_{\text{out}}$,

$$Q_{\text{C}} = Q_{\text{H}} - W_{\text{out}} \quad\Rightarrow\quad Q_{\text{C}} = 3.0\times10^9\ \text{J/s} - 1.0\times10^9\ \text{J/s} = 2.0\times10^9\ \text{J/s}$$

The mass of water that flows per second through the condenser is

$$m = \left(1.2\times10^8\ \frac{\text{L}}{\text{h}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right)\left(\frac{10^{-3}\ \text{m}^3}{1\ \text{L}}\right)\left(\frac{1000\ \text{kg}}{\text{m}^3}\right)(1\ \text{s}) = 3.333\times10^4\ \text{kg}$$

The change in the temperature as $Q_{\text{C}} = 2.0\times10^9\ \text{J}$ of heat is transferred to $m = 3.333\times10^4\ \text{kg}$ of water is

$$Q_{\text{C}} = mc\Delta T \quad\Rightarrow\quad 2.0\times10^9\ \text{J} = (3.333\times10^4\ \text{kg})(4186\ \text{J/kg K})\Delta T \quad\Rightarrow\quad \Delta T = 14\ °\text{C}$$

The exit temperature is $18°\text{C} + 14°\text{C} = 32°\text{C}$.

Assess: A temperature increase of 14 °C would certainly affect wildlife in and around the river!

21.53. Model: The power plant is treated as a heat engine.

Solve: The mass of water per second that flows through the plant every second is

$$m = \left(1.0\times10^8\ \frac{\text{L}}{\text{h}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right)\left(\frac{10^{-3}\ \text{m}^3}{1\ \text{L}}\right)\left(\frac{1000\ \text{kg}}{\text{m}^3}\right) = 2.778\times10^4\ \text{kg/s}$$

The amount of heat transferred per second to the cooling water is thus

$$Q_{\text{C}} = mc\Delta T = (2.778\times10^4\ \text{kg/s})(4186\ \text{J/kg K})(27°\text{C} - 16°\text{C}) = 1.279\times10^9\ \text{J/s}$$

The amount of heat per second input into the power plant is

$$Q_{\text{H}} = W_{\text{out}} + Q_{\text{C}} = 0.750\times10^9\ \text{J/s} + 1.279\times10^9\ \text{J/s} = 2.029\times10^9\ \text{J/s}$$

Finally, the power plant's thermal efficiency is

$$\eta = \frac{W_{\text{out}}}{Q_{\text{H}}} = \frac{0.750\times10^9\ \text{J/s}}{2.029\times10^9\ \text{J/s}} = 0.37 = 37\%$$

21.54. Model: The varying steam temperature clues us in that we will use a derivative. We are changing T_{H}.

Visualize: For the generator we are given $T_{\text{H}} = 573\ \text{K}$, $T_{\text{C}} = 293\ \text{K}$ and $d\eta/dT_{\text{H}} = 3.5\times10^{-4}\ \text{K}^{-1}$ at $T_{\text{H}} = 573\ \text{K}$. We seek $\varepsilon = \eta/\eta_{\text{Carnot}} \Rightarrow \eta = \varepsilon\eta_{\text{Carnot}}$.

Solve:

$$\eta = \varepsilon\eta_{\text{Carnot}} = \varepsilon\left(1 - \frac{T_{\text{C}}}{T_{\text{H}}}\right) = \varepsilon\left(1 - T_{\text{C}}\cdot T_{\text{H}}^{-1}\right)$$

Take the derivative of η with respect to T_{H}.

$$\frac{d\eta}{dT_{\text{H}}} = \varepsilon\left(0 + T_{\text{C}}T_{\text{H}}^{-2}\right) = \varepsilon\frac{T_{\text{C}}}{T_{\text{H}}^2} \quad\Rightarrow\quad \varepsilon = \left(\frac{d\eta}{dT_{\text{H}}}\right)\left(\frac{T_{\text{H}}^2}{T_{\text{C}}}\right)$$

Evaluate at the given hot temperature.

$$\varepsilon\Big|_{T_{\text{H}}=573\ \text{K}} = \left(\frac{d\eta}{dT_{\text{H}}}\right)\left(\frac{T_{\text{H}}^2}{T_{\text{C}}}\right)\Bigg|_{T_{\text{H}}=573\ \text{K}} = (3.5\times10^{-4}\ \text{K}^{-1})\frac{(573\ \text{K})^2}{293\ \text{K}} = 0.39$$

So the ratio of the efficiency to the Carnot efficiency is 0.39.

Assess: This is a typical efficiency and the units work out.

21.55. Model: The heat engine follows a closed cycle with process $1\to2$ and process $3\to4$ being isochoric and process $2\to3$ and process $4\to1$ being isobaric. For a monatomic gas, $C_{\text{V}} = \frac{3}{2}R$ and $C_{\text{P}} = \frac{5}{2}R$.

Visualize: Please refer to Figure P21.55.

Solve: (a) The first law of thermodynamics is $Q = \Delta E_{\text{th}} + W_{\text{S}}$. For the isochoric process $1\to2$, $W_{\text{S}\ 1\to2} = 0\ \text{J}$. Thus,

$$Q_{1\to2} = 3750\ \text{J} = \Delta E_{\text{th}} = nC_{\text{V}}\Delta T$$

$$\Delta T = \frac{3750\ \text{J}}{nC_{\text{V}}} = \frac{3750\ \text{J}}{(1.0\ \text{mol})\left(\frac{3}{2}R\right)} = \frac{3750\ \text{J}}{(1.0\ \text{mol})\left(\frac{3}{2}\right)(8.31\ \text{J/mol K})} = 301\ \text{K}$$

$$T_2 - T_1 = 300.8\ \text{K} \quad\Rightarrow\quad T_2 = 300.8\ \text{K} + 300\ \text{K} = 601\ \text{K}$$

To find volume V_2,

$$V_2 = V_1 = \frac{nRT_1}{p_1} = \frac{(1.0\ \text{mol})(8.31\ \text{J/mol K})(300\ \text{K})}{3.0\times10^5\ \text{Pa}} = 8.31\times10^{-3}\ \text{m}^3$$

The pressure p_2 can be obtained from the isochoric condition as follows:

$$\frac{p_2}{T_2} = \frac{p_1}{T_1} \Rightarrow p_2 = \frac{T_2}{T_1}p_1 = \left(\frac{601\ \text{K}}{300\ \text{K}}\right)(3.00\times10^5\ \text{Pa}) = 6.01\times10^5\ \text{Pa}$$

With the above values of p_2, V_2, and T_2, we can now obtain p_3, V_3, and T_3. We have

$$V_3 = 2V_2 = 1.662\times10^{-2}\ \text{m}^3$$

$$p_3 = p_2 = 6.01\times10^5\ \text{Pa}$$

$$\frac{T_3}{V_3} = \frac{T_2}{V_2} \quad\Rightarrow\quad T_3 = \frac{V_3}{V_2}T_2 = 2T_2 = 1202\ \text{K}$$

For the isobaric process $2 \to 3$,

$$Q_{2\to3} = nC_P\Delta T = (1.0 \text{ mol})\left(\tfrac{5}{2}R\right)(T_3 - T_2) = (1.0 \text{ mol})\left(\tfrac{5}{2}\right)(8.31 \text{ J/mol K})(601 \text{ K}) = 12{,}480 \text{ J}$$

$$W_{S\ 2\to3} = p_3(V_3 - V_2) = (6.01\times10^5 \text{ Pa})(8.31\times10^{-3} \text{ m}^3) = 4990 \text{ J}$$

$$\Delta E_{th} = Q_{2\to3} - W_{S\ 2\to3} = 12{,}480 \text{ J} - 4990 \text{ J} = 7490 \text{ J}$$

We are now able to obtain p_4, V_4, and T_4. We have

$$V_4 = V_3 = 1.662\times10^{-2} \text{ m}^3$$

$$p_4 = p_1 = 3.00\times10^5 \text{ Pa}$$

$$\frac{T_4}{p_4} = \frac{T_3}{p_3} \quad\Rightarrow\quad T_4 = \frac{p_4}{p_3}T_3 = \left(\frac{3.00\times10^5 \text{ Pa}}{6.01\times10^5 \text{ Pa}}\right)(1202 \text{ K}) = 600 \text{ K}$$

For isochoric process $3 \to 4$,

$$Q_{3\to4} = nC_V\Delta T = (1.0 \text{ mol})\left(\tfrac{3}{2}R\right)(T_4 - T_3) = (1.0 \text{ mol})\left(\tfrac{3}{2}\right)(8.31 \text{ J/mol K})(-602) = -7500 \text{ J}$$

$$W_{S\ 3\to4} = 0 \text{ J} \quad\Rightarrow\quad \Delta E_{th} = Q_{3\to4} - W_{S\ 3\to4} = -7500 \text{ J}$$

For isobaric process $4 \to 1$,

$$Q_{4\to1} = nC_P\Delta T = (1.0 \text{ mol})\tfrac{5}{2}(8.31 \text{ J/mol K})(300 \text{ K} - 600 \text{ K}) = -6230 \text{ J}$$

$$W_{S\ 4\to1} = p_4(V_1 - V_4) = (3.00\times10^5 \text{ Pa})\times(8.31\times10^{-3} \text{ m}^3 - 1.662\times10^{-2} \text{ m}^3) = -2490 \text{ J}$$

$$\Delta E_{th} = Q_{4\to1} - W_{S\ 4\to1} = -6230 \text{ J} - (-2490 \text{ J}) = -3740 \text{ J}$$

	W_S (J)	Q (kJ)	ΔE_{th} (kJ)
$1 \to 2$	0	3750	3750
$2 \to 3$	4990	12,480	7490
$3 \to 4$	0	–7500	–7500
$4 \to 1$	–2490	–6230	–3740
Net	2500	2500	0

(b) The thermal efficiency of this heat engine is

$$\eta = \frac{W_{out}}{Q_H} = \frac{W_{out}}{Q_{1\to2} + Q_{2\to3}} = \frac{2500 \text{ J}}{3750 \text{ J} + 12{,}480 \text{ J}} = 0.15 = 15\%$$

Assess: Note that more than two significant figures are retained in part **(a)** because the results are intermediate. For a closed cycle, as expected, $(W_s)_{net} = Q_{net}$ and $(\Delta E_{th})_{net} = 0$ J

21.56. Model: The heat engine follows a closed cycle. For a diatomic gas, $C_V = \frac{5}{2}R$ and $C_P = \frac{7}{2}R$.

Visualize: Please refer to Figure P21.56.

Solve: (a) Since $T_1 = 293$ K, the number of moles of the gas is

$$n = \frac{p_1V_1}{RT_1} = \frac{(0.5\times1.013\times10^5 \text{ Pa})(10\times10^{-6} \text{ m}^3)}{(8.31 \text{ J/mol K})(293 \text{ K})} = 2.08\times10^{-4} \text{ mol}$$

At point 2, $V_2 = 4V_1$ and $p_2 = 3p_1$. The temperature is calculated as follows:

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \quad\Rightarrow\quad T_2 = \frac{p_2}{p_1}\frac{V_2}{V_1}T_1 = (3)(4)(293 \text{ K}) = 3516 \text{ K}$$

At point 3, $V_3 = V_2 = 4V_1$ and $p_3 = p_1$. The temperature is calculated as before:

$$T_3 = \frac{p_3}{p_1}\frac{V_3}{V_1}T_1 = (1)(4)(293 \text{ K}) = 1172 \text{ K}$$

For process $1 \to 2$, the work done is the area under the *p*-versus-*V* curve. That is,

$$W_s = (0.5\text{ atm})(40\text{ cm}^3 - 10\text{ cm}^3) + \tfrac{1}{2}(1.5\text{ atm} - 0.5\text{ atm})(40\text{ cm}^3 - 10\text{ cm}^3)$$
$$= (30\times10^{-6}\text{ m}^3)(1\text{ atm})\left(\frac{1.013\times10^5\text{ Pa}}{1\text{ atm}}\right) = 3.04\text{ J}$$

The change in the thermal energy is

$$\Delta E_{th} = nC_V\Delta T = (2.08\times10^{-4}\text{ mol})\tfrac{5}{2}(8.31\text{ J/mol K})(3516\text{ K} - 293\text{ K}) = 13.93\text{ J}$$

The heat is $Q = W_s + \Delta E_{th} = 16.97$ J. For process $2 \to 3$, the work done is $W_s = 0$ J and

$$Q = \Delta E_{th} = nC_V\Delta T = n\left(\tfrac{5}{2}R\right)(T_3 - T_2)$$
$$= (2.08\times10^{-4}\text{ mol})\tfrac{5}{2}(8.31\text{ J/mol K})(1172\text{ K} - 3516\text{ K}) = -10.13\text{ J}$$

For process $3 \to 1$,

$$W_s = (0.5\text{ atm})(10\text{ cm}^3 - 40\text{ cm}^3) = (0.5\times1.013\times10^5\text{ Pa})(-30\times10^{-6}\text{ m}^3) = -1.52\text{ J}$$
$$\Delta E_{th} = nC_V\Delta T = (2.08\times10^{-4}\text{ mol})\tfrac{5}{2}(8.31\text{ J/mol K})(293\text{ K} - 1172\text{ K}) = -3.80\text{ J}$$

The heat is $Q = \Delta E_{th} + W_s = -5.32$ J.

	W_s (J)	Q (J)	ΔE_{th}
$1 \to 2$	3.04	16.97	13.93
$2 \to 3$	0	−10.13	−10.13
$3 \to 1$	−1.52	−5.32	−3.80
Net	1.52	1.52	0

(b) The efficiency of the engine is

$$\eta = \frac{W_{net}}{Q_H} = \frac{1.52\text{ J}}{16.97\text{ J}} = 0.090 = 9.0\%$$

(c) The power output of the engine is

$$500\left(\frac{\text{revolutions}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{W_{net}}{\text{revolution}}\right) = \left(\frac{500}{60}\right)(1.52\text{ J/s}) = 13\text{ W}$$

Assess: Note that more than two significant figures are retained in part **(a)** because the results are intermediate. For a closed cycle, as expected, $(W_s)_{net} = Q_{net}$ and $(\Delta E_{th})_{net} = 0$ J.

21.57. Model: For the closed cycle, process $1 \to 2$ is isothermal, process $2 \to 3$ is isobaric, and process $3 \to 1$ is isochoric.

Visualize: Please refer to Figure P21.57.

Solve: **(a)** We first need to find the conditions at points 1, 2, and 3. We can then use that information to find W_S and Q for each of the three processes that make up this cycle. Using the ideal-gas equation the number of moles of the gas is

$$n = \frac{p_1V_1}{RT_1} = \frac{(1.013\times10^5\text{ Pa})(600\times10^{-6}\text{m}^3)}{(8.31\text{ J/mol K})(300\text{ K})} = 0.0244\text{ mol}$$

We are given that $\gamma = 1.25$, which means this is not a monatomic or a diatomic gas. The specific heats are

$$C_V = \frac{R}{1-\gamma} = 4R \qquad C_P = C_V + R = 5R$$

At point 2, process $1 \to 2$ is isothermal, so we can find the pressure p_2 as follows:

$$p_1V_1 = p_2V_2 \quad \Rightarrow \quad p_2 = \frac{V_1}{V_2} p_1 = \frac{6.00\times10^{-4}\ \text{m}^3}{2.00\times10^{-4}\ \text{m}^3} p_1 = 3p_1 = 3\ \text{atm} = 3.039\times10^5\ \text{Pa}$$

At point 3, process $2 \to 3$ is isobaric, so we can find the temperature T_3 as follows:

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \quad \Rightarrow \quad T_3 = \frac{V_3}{V_2} T_2 = \frac{6.00\times10^{-4}\ \text{m}^3}{2.00\times10^{-4}\ \text{m}^3} T_2 = 3T_2 = 900\ \text{K}$$

Point	P (Pa)	V (m^3)	T (K)
1	$1.0\ \text{atm} = 1.013\times10^5$	6.00×10^{-4}	300
2	$3.0\ \text{atm} = 3.039\times10^5$	2.00×10^{-4}	300
3	$3.0\ \text{atm} = 3.039\times10^5$	6.00×10^{-4}	900

Process $1 \to 2$ is isothermal:

$$(W_\text{S})_{12} = p_1V_1 \ln(V_2/V_1) = -66.8\ \text{J} \quad Q_{12} = (W_\text{S})_{12} = -66.8\ \text{J}$$

Process $2 \to 3$ is isobaric:

$$(W_\text{S})_{23} = p_2\Delta V = p_2(V_3 - V_2) = 121.6\ \text{J} \quad Q_{23} = nC_\text{P}\Delta T = nC_\text{P}(T_3 - T_2) = 608.3\ \text{J}$$

Process $3 \to 1$ is isochoric:

$$(W_\text{S})_{31} = 0\ \text{J} \quad Q_{31} = nC_\text{V}\Delta T = nC_\text{V}(T_1 - T_3) = -486.7\ \text{J}$$

We find that

$$(W_\text{S})_\text{cycle} = -66.8\ \text{J} + 121.6\ \text{J} + 0\ \text{J} = 54.8\ \text{J} \quad Q_\text{cycle} = -66.8\ \text{J} + 608.3\ \text{J} - 486.7\ \text{J} = 54.8\ \text{J}$$

These are equal, as they should be. Knowing that the work done is $W_\text{out} = (W_\text{S})_\text{cycle} = 54.8$ J/cycle, an engine operating at 20 cycles/s has a power output of

$$P_\text{out} = \left(\frac{54.8\ \text{J}}{\text{cycle}}\right)\left(\frac{20\ \text{cycle}}{\text{s}}\right) = 1096\ \frac{\text{J}}{\text{s}} = 1096\ \text{W} \approx 1.10\ \text{kW}$$

(b) Only Q_{23} is positive, so $Q_\text{in} = Q_{23} = 608\ \text{J}$. Thus, the thermal efficiency is

$$\eta = \frac{W_\text{out}}{Q_\text{in}} = \frac{54.8\ \text{J}}{608.3\ \text{J}} = 0.0901 = 9.01\%$$

21.58. Model: For the closed cycle of the heat engine, process $1 \to 2$ is isobaric, process $2 \to 3$ is isochoric, and process $3 \to 1$ is adiabatic. $C_\text{V} = \frac{3}{2}R$ and $C_\text{P} = \frac{5}{2}R$ for a monatomic gas, so $\gamma = 5/3$.

Visualize: Please refer to Figure P21.58.

Solve: (a) We can use the adiabat $3 \to 1$ to calculate p_1 as follows:

$$p_1V_1^\gamma = p_3V_3^\gamma \quad \Rightarrow \quad p_1 = p_3\left(\frac{V_3}{V_1}\right)^\gamma = (100\ \text{kPa})\left(\frac{600\ \text{cm}^3}{100\ \text{cm}^3}\right)^{5/3} = 1981\ \text{kPa}$$

T_1 can be determined by taking the ratio of the ideal-gas equation applied to points 1 and 2. This gives

$$\frac{p_1V_1}{p_2V_2} = \frac{T_1}{T_2}$$

$$T_1 = T_2\frac{V_1}{V_2} = (600\ \text{K})\frac{100\ \text{cm}^3}{600\ \text{cm}^3} = 100\ \text{K}$$

where we have used the fact that $p_1 = p_2$. Applying the same strategy at point 3 gives

$$\frac{p_2V_2}{p_3V_3} = \frac{T_2}{T_3}$$

$$T_3 = T_2\frac{p_3}{p_2} = (600\text{ K})\left(\frac{100\text{ kPa}}{1981\text{ kPa}}\right) = 30.3\text{ K}$$

where we have used the fact that $V_2 = V_3$. Before we calculate the work and heat exchanged for each cycle, we need to know the number of moles. This may be calculated by applying the ideal gas law at any point on the cycle:

$$n = \frac{p_1V_1}{RT_1} = \frac{(1981\text{ kPa})(100\times10^{-6}\text{ m}^3)}{(8.31\text{ J/mol K})(100\text{ K})} = 0.238\text{ mol}$$

Now we can calculate W_s, Q, and ΔE_{th} for the three processes involved in the cycle. For process $1\to2$,

$$\Delta E_{th,1\to2} = nC_V(T_2 - T_1) = n\left(\tfrac{3}{2}R\right)(T_2 - T_1) = 1.486\text{ kJ}$$

$$Q_{1\to2} = nC_P(T_2 - T_1) = n\left(\tfrac{5}{2}R\right)(T_2 - T_1) = 2.476\text{ kJ}$$

The work done $W_{s\ 1\to2}$ is the area under the p-versus-V graph. We have

$$W_{s\ 1\to2} = (1981\text{ kPa})(600\times10^{-6}\text{ m}^3 - 100\times10^{-6}\text{ m}^3) = 0.991\text{ kJ}$$

For process $2\to3$, $W_{s\ 2\to3} = 0$ J and

$$\Delta E_{th,2\to3} = Q_{2\to3} = nC_V(T_3 - T_2) = n\left(\tfrac{3}{2}R\right)(T_3 - T_2) = -1.693\text{ kJ}$$

For process $3\to1$, $Q_{3\to1} = 0$ J and

$$\Delta E_{th,3\to1} = nC_V(T_1 - T_3) = n\left(\tfrac{3}{2}R\right)(T_1 - T_3) = 0.2072\text{ kJ}$$

Because $\Delta E_{th} = W + Q$ and $W = -W_s$, $W_{s\ 3\to1} = -\Delta E_{th,3\to1} = -0.2072$ kJ for process $3\to1$.

	W_s (kJ)	Q (kJ)	ΔE_{th} (kJ)
$1\to2$	0.991	2.476	1.486
$2\to3$	0	−1.693	−1.693
$3\to1$	−0.207	0	0.207
Net	0.783	0.783	0

(b) The thermal efficiency of the engine is

$$\eta = \frac{W_{out}}{Q_H} = \frac{783\text{ J}}{2476\text{ J}} = 0.32 = 32\%$$

Assess: Note that more than two significant figures are retained in part **(a)** because the results are intermediate. As expected for a closed cycle, $(W_s)_{net} = Q_{net}$ and $(\Delta E_{th})_{net} = 0$ J.

21.59. Model: For the closed cycle of the heat engine, process $1\to2$ is isochoric, process $2\to3$ is adiabatic, and process $3\to1$ is isothermal. For a diatomic gas $C_V = \frac{5}{2}R$ and $\gamma = \frac{7}{5}$.

Solve: **(a)** From the graph $V_2 = 1000\text{ cm}^3$.

The pressure p_2 lies on the adiabat from $2\to3$. We can find the pressure as follows:

$$p_2V_2^\gamma = p_3V_3^\gamma \quad\Rightarrow\quad p_2 = p_3\left(\frac{V_3}{V_2}\right)^\gamma = (1.00\times10^5\text{ Pa})\left(\frac{4000\text{ cm}^3}{1000\text{ cm}^3}\right)^{7/5} = 6.964\times10^5\text{ Pa} \approx 696\text{ kPa}$$

The temperature T_2 can be obtained from the ideal-gas equation relating points 1 and 2:

$$\frac{p_1V_1}{T_1}=\frac{p_2V_2}{T_2} \quad\Rightarrow\quad T_2=T_1\frac{p_2}{p_1}\frac{V_2}{V_1}=(300\text{ K})\left(\frac{6.964\times10^5\text{ Pa}}{4.00\times10^5\text{ Pa}}\right)(1)=522.3\text{ K}\approx522\text{ K}$$

(b) The number of moles of the gas is

$$n=\frac{p_1V_1}{RT_1}=\frac{(4.00\times10^5\text{ Pa})(1.00\times10^{-3}\text{ m}^3)}{(8.31\text{ J/mol K})(300\text{ K})}=0.1604\text{ mol}$$

For isochoric process $1\to2$, $W_s=0$ J and

$$Q=\Delta E_{th}=nC_V\Delta T=n\left(\tfrac{5}{2}R\right)\Delta T=741.1\text{ J}$$

For adiabatic process $2\to3$, $Q=0$ J and

$$\Delta E_{th}=nC_V\Delta T=n\left(\tfrac{5}{2}R\right)(T_3-T_2)=-741.1\text{ J}$$

Using the first law of thermodynamics, $\Delta E_{th}=W_s+Q$, which means $W_s=-\Delta E_{th}=+741.1$ J. W_s can also be determined from

$$W_s=\frac{p_3V_3-p_2V_2}{1-\gamma}=\frac{nR(T_3-T_2)}{1-\gamma}=\frac{\left(\tfrac{4}{3}\text{ J/K}\right)(300\text{ K}-522.3\text{ K})}{\left(-\tfrac{2}{5}\right)}=741.1\text{ J}$$

For isothermal process $3\to1$, $\Delta E_{th}=0$ J and

$$W_s=nRT_1\ln\frac{V_1}{V_3}=-554.5\text{ J}$$

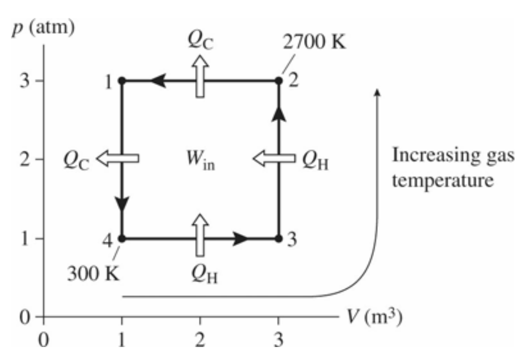
Using the first law of thermodynamics, $\Delta E_{th}=-W_s+Q$, $Q=W_s=-554.5$ J.

	ΔE_{th} (J)	W_s (J)	Q (J)
$1\to2$	741.1	0	741.1
$2\to3$	−741.1	741.1	0
$3\to1$	0	−554.5	−554.5
Net	0	186.6	186.6

(c) The work per cycle is 187 J and the thermal efficiency is

$$\eta=\frac{W_s}{Q_H}=\frac{186.6\text{ J}}{741.1\text{ J}}=0.25=25\%$$

21.60. Model: Processes $2\to1$ and $4\to3$ are isobaric. Processes $3\to2$ and $1\to4$ are isochoric.
Visualize:



Solve: (a) Except in an adiabatic process, heat must be transferred into the gas to raise its temperature. Thus heat is transferred in during processes $4\to3$ and $3\to2$. This is the reverse of the heat engine in Example 21.2.

(b) Heat flows from hot to cold. Since heat energy is transferred into the gas during processes $4 \to 3$ and $3 \to 2$, which end with the gas at temperature 2700 K, the reservoir temperature must be $T > 2700$ K. This is the hot reservoir, so the heat transferred is Q_H. Similarly, heat energy is transferred out of the gas during processes $2 \to 1$ and $1 \to 4$. This requires that the reservoir temperature be $T < 300$ K. This is the cold reservoir, and the energy transferred during these two processes is Q_C.

(c) The heat energies were calculated in Example 21.2, but now they have the opposite signs.

$$Q_H = Q_{43} + Q_{32} = 7.09\times10^5 \text{ J} + 15.19\times10^5 \text{ J} = 22.28\times10^5 \text{ J}$$

$$Q_C = Q_{21} + Q_{14} = 21.27\times10^5 \text{ J} + 5.06\times10^5 \text{ J} = 26.33\times10^5 \text{ J}$$

(d) For a counterclockwise cycle in the pV diagram, the work is W_{in}. Its value is the area inside the curve, which is $W_{in} = (\Delta p)(\Delta V) = (2\times101{,}300 \text{ Pa})(2 \text{ m}^3) = 4.05\times10^5$ J. Note that $W_{in} = Q_C - Q_H$, as expected from energy conservation.

(e) No. A refrigerator uses work input to transfer heat energy from the cold reservoir to the hot reservoir. This device uses work input to transfer heat energy from the hot reservoir to the cold reservoir.

21.61. Model: Process $1 \to 2$ of the cycle is isochoric, process $2 \to 3$ is isothermal, and process $3 \to 1$ is isobaric. For a monatomic gas, $C_V = \frac{3}{2}R$ and $C_P = \frac{5}{2}$ R.

Visualize: Please refer to Figure P21.61.

Solve: (a) At point 1, the pressure $p_1 = 1 \text{ atm} = 1.013\times10^5$ Pa and the volume $V_1 = 1000\times10^{-6} \text{ m}^3 = 1\times10^{-3} \text{ m}^3$. The number of moles is

$$n = \frac{0.120 \text{ g}}{4 \text{ g/mol}} = 0.03 \text{ mol}$$

Using the ideal-gas law,

$$T_1 = \frac{p_1V_1}{nR} = \frac{(1.013\times10^5 \text{ Pa})(1.0\times10^{-3} \text{ m}^3)}{(0.030 \text{ mol})(8.31 \text{ J/mol K})} = 406 \text{ K} \approx 0.4 \text{ kK}$$

At point 2, the pressure $p_2 = 5 \text{ atm} = 5.06\times10^5$ Pa and $V_2 = 1\times10^{-3} \text{ m}^3$. The temperature is

$$T_2 = \frac{p_2V_2}{nR} = \frac{(5.06\times10^5 \text{ Pa})(1.0\times10^{-3} \text{ m}^3)}{(0.030 \text{ mol})(8.31 \text{ J/mol K})} = 2030 \text{ K} \approx 2 \text{ kK}$$

At point 3, the pressure is $p_3 = 1 \text{ atm} = 1.013\times10^5$ Pa and the temperature is $T_3 = T_2 = 2030$ K. The volume is

$$V_3 = V_2\frac{p_2}{p_3} = (1\times10^{-3} \text{ m}^3)\left(\frac{5 \text{ atm}}{1 \text{ atm}}\right) = 5\times10^{-3} \text{ m}^3$$

(b) For the isochoric process $1 \to 2$, $W_{1\to2} = 0$ J and

$$Q_{1\to2} = nC_V\Delta T = (0.030 \text{ mol})\left(\tfrac{3}{2}R\right)(2030 \text{ K} - 406 \text{ K}) = 607 \text{ J}$$

For the isothermal process $2 \to 3$, $\Delta E_{\text{th } 2\to3} = 0$ J and

$$Q_{2\to3} = W_{2\to3} = nRT_2\ln\frac{V_3}{V_2} = (0.030 \text{ mol})(8.31 \text{ J/mol K})(2030 \text{ K})\ln\left(\frac{5.0\times10^{-3} \text{ m}^3}{1.0\times10^{-3} \text{ m}^3}\right) = 815 \text{ J}$$

For the isobaric process $3 \to 1$,

$$W_{3\to1} = p_3\Delta V = (1.013\times10^5 \text{ Pa})(1.0\times10^{-3} \text{ m}^3 - 5.0\times10^{-3} \text{ m}^3) = -405 \text{ J}$$

$$Q_{3\to1} = nC_P\Delta T = (0.030 \text{ mol})\left(\tfrac{5}{2}\right)(8.31 \text{ J/mol K})(406 \text{ K} - 2030 \text{ K}) = -1012 \text{ J}$$

The total work done is $W_{net} = W_{1\to2} + W_{2\to3} + W_{3\to1} = 410$ J. The total heat input is $Q_H = Q_{1\to2} + Q_{2\to3} = 1422$ J. The thermal efficiency of the engine is

$$\eta = \frac{W_{\text{net}}}{Q_{\text{H}}} = \frac{410\ \text{J}}{1422\ \text{J}} = 29,$$

(c) The maximum possible efficiency of a heat engine that operates between $T_{\max}$ and $T_{\min}$ is

$$\eta_{\max} = 1 - \frac{T_{\min}}{T_{\max}} = 1 - \frac{406\ \text{K}}{2030\ \text{K}} = 80,$$

Assess: The actual efficiency of an engine is less than the maximum possible efficiency.

21.62. Model: The process $2 \to 3$ of the heat engine cycle is isochoric and the process $3 \to 1$ is isobaric. For a monatomic gas $C_{\text{V}} = \frac{3}{2}R$ and $C_{\text{P}} = \frac{5}{2}R$.

Solve: **(a)** The three temperatures are

$$T_1 = \frac{p_1V_1}{nR} = \frac{(4.0\times10^5\ \text{Pa})(0.025\ \text{m}^3)}{(2.0\ \text{mol})(8.31\ \text{J/mol K})} = 601.7\ \text{K} \approx 0.60\ \text{kK}$$

$$T_2 = \frac{p_2V_2}{nR} = \frac{(6.0\times10^5\ \text{Pa})(0.050\ \text{m}^3)}{(2.0\ \text{mol})(8.31\ \text{J/mol K})} = 1805.1\ \text{K} \approx 1.8\ \text{kK}$$

$$T_3 = \frac{p_3V_3}{nR} = \frac{(4.0\times10^5\ \text{Pa})(0.050\ \text{m}^3)}{(2.0\ \text{mol})(8.31\ \text{J/mol K})} = 1203.4\ \text{K} \approx 1.2\ \text{kK}$$

(b) For process $1 \to 2$, the work done is the area under the p-versus-V graph. The work and the change in internal energy are

$$\begin{aligned} W_{\text{s}} &= \tfrac{1}{2}(6.0\times10^5\ \text{Pa} - 4.0\times10^5\ \text{Pa})(0.050\ \text{m}^3 - 0.025\ \text{m}^3) + (4.0\times10^5\ \text{Pa})(0.050\ \text{m}^3 - 0.025\ \text{m}^3) \\ &= 1.25\times10^4\ \text{J} \\ \Delta E_{\text{th}} &= nC_{\text{V}}\Delta T = (2.0\ \text{mol})\left(\tfrac{3}{2}R\right)(T_2 - T_1) \\ &= (2.0\ \text{mol})\left(\tfrac{3}{2}\right)(8.31\ \text{J/mol K})(1805.1\ \text{K} - 601.7\ \text{K}) = 3.00\times10^4\ \text{J} \end{aligned}$$

The heat input is $Q = W_{\text{s}} + \Delta E_{\text{th}} = 4.25\times10^4\ \text{J}$. For isochoric process $2 \to 3$, $W_{\text{s}} = 0\ \text{J}$ and

$$Q = \Delta E_{\text{th}} = nC_{\text{V}}\Delta T = (2.0\ \text{mol})\tfrac{3}{2}(8.31\ \text{J/mol K})(1203.4\ \text{K} - 1805.1\ \text{K}) = -1.50\times10^4\ \text{J}$$

For isobaric process $3 \to 1$, the work done is the area under the p-versus-V curve. Hence,

$$W_{\text{s}} = (4.0\times10^5\ \text{Pa})(0.025\ \text{m}^3 - 0.050\ \text{m}^3) = -1.0\times10^4\ \text{J}$$

$$\Delta E_{\text{th}} = nC_{\text{V}}\Delta T = n\left(\tfrac{3}{2}R\right)(T_1 - T_3) = (2.0\ \text{mol})\tfrac{3}{2}(8.31\ \text{J/mol K})(601.7\ \text{K} - 1203.4\ \text{K}) = -1.5\times10^4\ \text{J}$$

The heat input is $Q = W_{\text{S}} + \Delta E_{\text{th}} = -2.50\times10^4\ \text{J}$.

	ΔE_{th} (J)	W_{S} (J)	Q (J)
$1 \to 2$	3.0×10^4	1.25×10^4	4.25×10^4
$2 \to 3$	-1.5×10^4	0	-1.50×10^4
$3 \to 1$	-1.5×10^4	-1.0×10^4	-2.50×10^4
Net	0	2.5×10^3	2.5×10^3

(c) The thermal efficiency is

$$\eta = \frac{W_{\text{net}}}{Q_{\text{H}}} = \frac{2.5\times10^3\ \text{J}}{4.25\times10^4\ \text{J}} = 5.9,$$

21.63. Model: The closed cycle in this heat engine includes adiabatic process 1 → 2, isobaric process 2 → 3, and isochoric process 3 → 1. For a diatomic gas, $C_V = \frac{5}{2}R, C_P = \frac{7}{2}R,$ and $\gamma = \frac{7}{5} = 1.4.$

Visualize: Please refer to Figure P21.63.

Solve: (a) We can find the temperature T_2 from the ideal-gas equation as follows:

$$T_2 = \frac{p_2V_2}{nR} = \frac{(4.0\times10^5\ \text{Pa})(1.0\times10^{-3}\ \text{m}^3)}{(0.020\ \text{mol})(8.31\ \text{J/mol K})} = 2407\ \text{K} \approx 2.4\ \text{kK}$$

We can use the equation $p_2V_2^\gamma = p_1V_1^\gamma$ to find V_1,

$$V_1 = V_2\left(\frac{p_2}{p_1}\right)^{1/\gamma} = (1.0\times10^{-3}\ \text{m}^3)\left(\frac{4.0\times10^5\ \text{Pa}}{1.0\times10^5\ \text{Pa}}\right)^{1/1.4} = 2.692\times10^{-3}\ \text{m}^3$$

The ideal-gas equation can now be used to find T_1,

$$T_1 = \frac{p_1V_1}{nR} = \frac{(1.0\times10^5\ \text{Pa})(2.692\times10^{-3}\ \text{m}^3)}{(0.020\ \text{mol})(8.31\ \text{J/mol K})} = 1620\ \text{K} \approx 1.6\ \text{kK}$$

At point 3, $V_3 = V_1$ so we have

$$T_3 = \frac{p_3V_3}{nR} = \frac{(4\times10^5\ \text{Pa})(2.692\times10^{-3}\ \text{m}^3)}{(0.020\ \text{mol})(8.31\ \text{J/mol K})} = 6479\ \text{K} \approx 6.5\ \text{kK}$$

(b) For adiabatic process 1 → 2, $Q = 0$ J, $\Delta E_{th} = -W_s$, and

$$W_s = \frac{p_2V_2 - p_1V_1}{1-\gamma} = \frac{nR(T_2 - T_1)}{1-\gamma} = \frac{(0.020\ \text{mol})(8.31\ \text{J/mol K})(2407\ \text{K} - 1620\ \text{K})}{(1-1.4)} = -327.0\ \text{J}$$

For isobaric process 2 → 3,

$$Q = nC_P\Delta T = n\left(\tfrac{7}{2}R\right)(\Delta T) = (0.020\ \text{mol})\tfrac{7}{2}(8.31\ \text{J/mol K})(6479\ \text{K} - 2407\ \text{K}) = 2369\ \text{J}$$

$$\Delta E_{th} = nC_V\Delta T = n\left(\tfrac{5}{2}R\right)\Delta T = 1692\ \text{J}$$

The work done is the area under the p-versus-V graph. Hence,

$$W_s = (4.0\times10^5\ \text{Pa})(2.692\times10^{-3}\ \text{m}^3 - 1.0\times10^{-3}\ \text{m}^3) = 677\ \text{J}$$

For isochoric process 3 → 1, $W_s = 0$ J and

$$\Delta E_{th} = Q = nC_V\Delta T = (0.020\ \text{mol})\left(\tfrac{5}{2}\right)(8.31\ \text{J/mol K})(1620\ \text{K} - 6479\ \text{K}) = -2019\ \text{J}$$

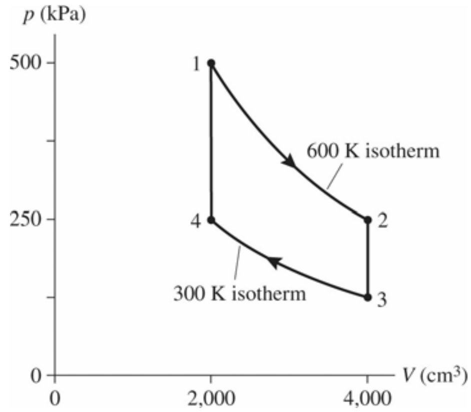
	ΔE_{th} (J)	W_S (J)	Q (J)
1 → 2	327	−327	0
2 → 3	1692	677	2369
3 → 1	−2019	0	−2019
Net	0	350	350

(c) The engine's thermal efficiency is

$$\eta = \frac{W_{net}}{Q_H} = \frac{350\ \text{J}}{2369\ \text{J}} = 0.15 = 15,$$

21.64. Model: The closed cycle of the heat engine involves the following four processes: isothermal expansion, isochoric cooling, isothermal compression, and isochoric heating. For a monatomic gas $C_V = \frac{3}{2}R$.

Visualize:



Solve: Using the ideal-gas law,

$$p_1 = \frac{nRT_1}{V_1} = \frac{(0.20\text{ mol})(8.31\text{ J/mol K})(600\text{ K})}{2.0\times 10^{-3}\text{ m}^3} = 4.986\times 10^5\text{ Pa}$$

At point 2, because of the isothermal conditions, $T_2 = T_1 = 600\text{ K}$ and

$$p_2 = p_1\frac{V_1}{V_2} = (4.986\times 10^5\text{ Pa})\left(\frac{2.0\times 10^{-3}\text{ m}^3}{4.0\times 10^{-3}\text{ m}^3}\right) = 2.493\times 10^5\text{ Pa}$$

At point 3, because it is an isochoric process, $V_3 = V_2 = 4000\text{ cm}^3$ and

$$p_3 = p_2\frac{T_3}{T_2} = (2.493\times 10^5\text{ Pa})\left(\frac{300\text{ K}}{600\text{ K}}\right) = 1.247\times 10^5\text{ Pa}$$

Likewise at point 4, $T_4 = T_3 = 300\text{ K}$ and

$$p_4 = p_3\frac{V_3}{V_4} = (1.247\times 10^5\text{ Pa})\left(\frac{4.0\times 10^{-3}\text{ m}^3}{2.0\times 10^{-3}\text{ m}^3}\right) = 2.493\times 10^5\text{ Pa}$$

Let us now calculate $W_{net} = W_{1\to 2} + W_{2\to 3} + W_{3\to 4} + W_{4\to 1}$. For the isothermal processes,

$$W_{1\to 2} = nRT_1\ln\frac{V_2}{V_1} = (0.20\text{ mol})(8.31\text{ J/mol K})(600\text{ K})\ln(2) = 691.2\text{ J}$$

$$W_{3\to 4} = nRT_3\ln\frac{V_4}{V_3} = (0.20\text{ mol})(8.31\text{ J/mol K})(300\text{ K})\ln\left(\tfrac{1}{2}\right) = -345.6\text{ J}$$

For the isochoric processes, $W_{2\to 3} = W_{4\to 1} = 0\text{ J}$. Thus, the work done per cycle is $W_{net} = 345.6\text{ J} \approx 350\text{ J}$. Because $Q = W_S + \Delta E_{th}$,

$$Q_{1\to 2} = W_{1\to 2} + (\Delta E_{th})_{1\to 2} = 691.2\text{ J} + 0\text{ J} = 691.2\text{ J}$$

For the first isochoric process,

$$Q_{2\to 3} = nC_V\Delta T = (0.20\text{ mol})\left(\tfrac{3}{2}R\right)(T_3 - T_2)$$
$$= (0.20\text{ mol})\tfrac{3}{2}(8.31\text{ J/mol K})(300\text{ K} - 600\text{ K}) = -747.9\text{ K}$$

For the second isothermal process

$$Q_{3\to 4} = W_{3\to 4} + (\Delta E_{th})_{3\to 4} = -345.6\text{ J} + 0\text{ J} = -345.6\text{ J}$$

For the second isochoric process,

$$Q_{4\to 1} = nC_V\Delta T = n\left(\tfrac{3}{2}R\right)(T_1 - T_4)$$
$$= (0.20\text{ mol})\left(\tfrac{3}{2}\right)(8.31\text{ J/mol K})(600\text{ K} - 300\text{ K}) = 747.9\text{ K}$$

Thus, $Q_H = Q_{1\to 2} + Q_{4\to 1} = 1439.1\text{ J}$. The thermal efficiency of the engine is

$$\eta = \frac{W_{net}}{Q_H} = \frac{345.6\text{ J}}{1439.1\text{ J}} = 0.24 = 24,$$

21.65. Solve: (a) If you wish to build a Carnot engine that is 80% efficient and exhausts heat into a cold reservoir at 0°C, what temperature (in °C) must the hot reservoir be?
(b)

$$0.80 = 1 - \frac{(0°\text{C} + 273)}{(T_H + 273)} \quad\Rightarrow\quad \frac{273}{T_H + 273} = 0.20 \quad\Rightarrow\quad T_H = 1.1\times 10^3\ °\text{C}$$

21.66. Solve: (a) A refrigerator with a coefficient of performance of 4.0 exhausts 100 J of heat in each cycle. What work is required each cycle and how much heat is removed each cycle from the cold reservoir?
(b) We have $4.0 = Q_C/W_{in} \Rightarrow Q_C = 4W_{in}$. This means

$$Q_H = Q_C + W_{in} = 4W_{in} + W_{in} = 5W_{in} \quad\Rightarrow\quad W_{in} = \frac{Q_H}{5} = \frac{100\text{ J}}{5} = 20\text{ J}$$

Hence, $Q_C = Q_H - W_{in} = 100\text{ J} - 20\text{ J} = 80\text{ J}$.

21.67. Solve: (a) A heat engine operates at 20% efficiency and produces 20 J of work in each cycle. What is the net heat extracted from the hot reservoir and the net heat exhausted in each cycle?
(b) We have $0.20 = 1 - Q_C/Q_H$. Using the first law of thermodynamics,

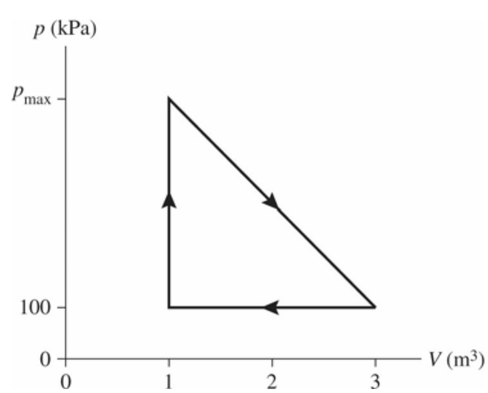
$$W_{out} = Q_H - Q_C = 20\text{ J} \quad\Rightarrow\quad Q_C = Q_H - 20\text{ J}$$

Substituting into the definition of efficiency,

$$0.20 = 1 - \frac{Q_H - 20\text{ J}}{Q_H} = 1 - 1 + \frac{20\text{ J}}{Q_H} = \frac{20\text{ J}}{Q_H} \quad\Rightarrow\quad Q_H = \frac{20\text{ J}}{0.20} = 100\text{ J}$$

The heat exhausted is $Q_C = Q_H - 20\text{ J} = 100\text{ J} - 20\text{ J} = 80\text{ J}$.

21.68. Solve: (a)



In this heat engine, 400 kJ of work is done each cycle. What is the maximum pressure?

(b)

$$\frac{1}{2}(p_{\max} - 1.0\times10^5\ \text{Pa})(2.0\ \text{m}^3) = 4.0\times10^5\ \text{J} \quad \Rightarrow \quad p_{\max} = 5.0\times10^5\ \text{Pa} = 500\ \text{kPa}$$

Challenge Problems

21.69. Solve: The mass of the water is

$$(100\times10^{-3}\ \text{L})\left(\frac{10^{-3}\ \text{m}^3}{1\ \text{L}}\right)\left(\frac{1000\ \text{kg}}{\text{m}^3}\right) = 0.100\ \text{kg}$$

The heat energy is removed from the water in three steps: (1) cooling from +15 °C to 0 °C, (2) freezing at 0 °C, and (3) cooling from 0 °C to −15 °C. The three heat energies are

$$Q_1 = mc\Delta T = (0.100\ \text{kg})(4186\ \text{J/kg K})(15\ \text{K}) = 6279\ \text{J}$$

$$Q_2 = mL_\text{f} = (0.100\ \text{kg})(3.33\times10^5\ \text{J/kg}) = 33{,}300\ \text{J}$$

$$Q_3 = mc\Delta T = (0.100\ \text{kg})(2090\ \text{J/kg K})(15\ \text{K}) = 3135\ \text{J}$$

$$Q_\text{C} = Q_1 + Q_2 + Q_3 = 42{,}714\ \text{J}$$

Using the performance coefficient,

$$K = \frac{Q_\text{C}}{W_\text{in}} \quad \Rightarrow \quad 4.0 = \frac{42{,}714\ \text{J}}{W_\text{in}} \quad \Rightarrow \quad W_\text{in} = \frac{42{,}714\ \text{J}}{4.0} = 10{,}679\ \text{J}$$

The heat exhausted into the room is thus

$$Q_\text{H} = Q_\text{C} + W_\text{in} = 42{,}714\ \text{J} + 10{,}679\ \text{J} = 5.3\times10^4\ \text{J}$$

21.70. Model: System 1 undergoes an isochoric process and system 2 undergoes an isobaric process.
Solve: (a) Heat will flow from system 1 to system 2 because system 1 is hotter. Because there is no heat input from (or loss to) the outside world, we have $Q_1 + Q_2 = 0$ J. Heat Q_1, which is negative, will change the temperature of system 1. Heat Q_2 will both change the temperature of system 2 *and* do work by lifting the piston. But these *consequences* of heat flow don't change the fact that $Q_1 + Q_2 = 0$ J. System 1 undergoes constant volume cooling from $T_{1\text{i}} = 600$ K to T_f. System 2, whose pressure is controlled by the weight of the piston, undergoes constant pressure heating from $T_{2\text{i}} = 300$ K to T_f. Thus,

$$Q_1 + Q_2 = 0\ \text{J} = n_1 C_\text{V}(T_\text{f} - T_{1\text{i}}) + n_2 C_\text{P}(T_\text{f} - T_{2\text{i}}) = n_1\left(\tfrac{3}{2}R\right)(T_\text{f} - T_{1\text{i}}) + n_2\left(\tfrac{5}{2}R\right)(T_\text{f} - T_{2\text{i}})$$

Solving this equation for T_f gives

$$T_\text{f} = \frac{3n_1T_{1\text{i}} + 5n_2T_{2\text{i}}}{3n_1 + 5n_2} = \frac{3(0.060\ \text{mol})(600\ \text{K}) + 5(0.030\ \text{mol})(300\ \text{K})}{3(0.060\ \text{mol}) + 5(0.030\ \text{mol})} = 464\ \text{K}$$

(b) Knowing T_f, we can compute the heat transferred from system 1 to system 2:

$$Q_2 = n_2 C_\text{P}(T_\text{f} - T_{2\text{i}}) = n_2\left(\tfrac{5}{2}R\right)(T_\text{f} - T_{2\text{i}}) = 102\ \text{J}$$

(c) The change of thermal energy in system 2 is

$$\Delta E_\text{th} = n_2 C_\text{V}\Delta T = n_2\left(\tfrac{3}{2}R\right)(T_\text{f} - T_{2\text{i}}) = \tfrac{3}{5}Q_2 = 61.2\ \text{J}$$

According to the first law of thermodynamics, $Q_2 = W_\text{S} + \Delta E_\text{th}$. Thus, the work done by system 2 is $W_\text{S} = Q - \Delta E_\text{th} =$ $102.0\ \text{J} - 61.2\ \text{J} = 40.8$ J. The work is done to lift the weight of the cylinder and the air above it by a height Δy. The weight of the air is $w_\text{air} = pA = p\pi r^2 = (101.3\times10\ \text{N/m}^2)\pi(0.050\ \text{m})^2 = 795.6$ N. Therefore,

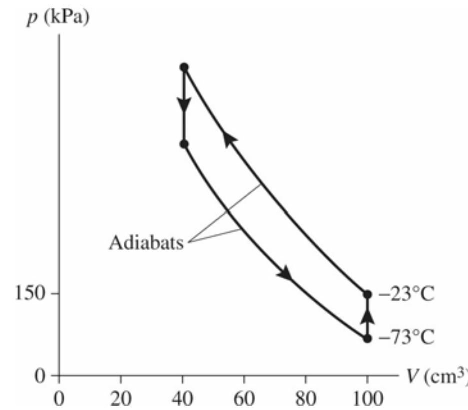
$$W_\text{s} = (w_\text{cyl} + w_\text{air})\Delta y \quad \Rightarrow \quad \Delta y = \frac{W_\text{s}}{(w_\text{cyl} + w_\text{air})} = \frac{40.8\ \text{J}}{(2.0\ \text{kg})(9.8\ \text{m/s}^2) + 795.6\ \text{N}} = 0.050\ \text{m}$$

(d) The fraction of heat converted to work is

$$\frac{W_s}{Q_2} = \frac{40.8\text{ J}}{102.0\text{ J}} = 0.40 = 40\%$$

21.71. Model: Call 1 the point at –73 °C, 2 the point at –23 °C, etc. For the closed cycle of the refrigerator, process $1 \to 2$ is isochoric, process $2 \to 3$ is adiabatic, process $3 \to 4$ is isochoric, and process $4 \to 1$ is adiabatic. For a monatomic gas $C_V = \frac{3}{2}R$ and $\gamma = \frac{5}{3}$.

Visualize: Please refer to the figure below.



Solve: (a) The number of moles of gas may be found by applying the ideal-gas equation to point 2. The result is

$$n = \frac{p_2V_2}{RT_2} = \frac{(150\text{ kPa})(1.00\times10^{-4}\text{ m}^3)}{(8.31\text{ J/mol K})(250\text{ K})} = 7.22\times10^{-3}\text{ mol}$$

The temperatures at points 3 and 4 may be found using Table 21.1:

$$T_3 = T_2\left(\frac{V_2}{V_3}\right)^{\gamma-1} = (250\text{ K})\left(\frac{100\text{ cm}^3}{40\text{ cm}^3}\right)^{2/3} = 461\text{ K}$$

$$T_4 = T_1\left(\frac{V_1}{V_4}\right)^{\gamma-1} = (200\text{ K})\left(\frac{100\text{ cm}^3}{40\text{ cm}^3}\right)^{2/3} = 368\text{ K}$$

Only the adiabatic segments do work, so the total work done by the system is

$$W_S = W_{S,\,2\to3} + W_{S,\,4\to1} = -nC_v(\Delta T_{2\to3} + \Delta T_{4\to1}) = -n\left(\tfrac{3}{2}R\right)(T_3 - T_2 + T_1 - T_4)$$
$$= -(7.22\times10^{-3}\text{ mol})\left(\tfrac{3}{2}\right)(8.31\text{ J/mol K})(461\text{ K} - 250\text{ K} + 200\text{ K} - 368\text{ K}) = -3.87\text{ J}$$

Thus, the work done on the system is $W_{in} = -W_S = -(3.87\text{ J}) = 3.87\text{ J}$. During the adiabatic segments, no heat is exchanged with the heat reservoirs, so heat is exchanged only during the isochoric segments. For a refrigerator, the heat exchanged with the cold reservoir is the heat that is put into the system (i.e., > 0), which occurs in segment $1 \to 2$. With the help of Table 21.1, this is

$$Q_C = nC_V\Delta T_{1\to2} = n\left(\tfrac{3}{2}R\right)(T_2 - T_1) = (7.22\times10^{-3}\text{ mol})\left(\tfrac{3}{2}\right)(8.31\text{ J/mol K})(250\text{ K} - 200\text{ K}) = 4.50\text{ J}$$

Thus, the coefficient of performance is

$$K = \frac{Q_C}{W_{in}} = \frac{4.50\text{ J}}{3.87\text{ J}} = 1.16 \approx 1.2$$

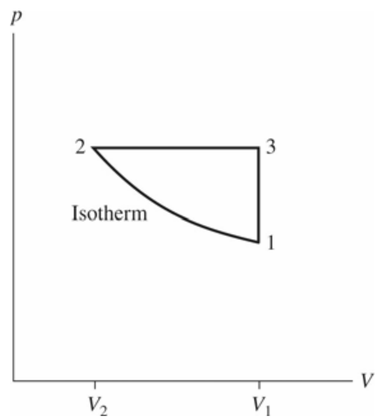
(b) Since each cycle takes 1/60 s, the power needed to run the refrigerator is

$$P = \frac{W_{\text{in}}}{\frac{1}{60}\ \text{s}} = 230\ \text{W}$$

Assess: This coefficient of performance is fairly typical.

21.72. Model: For the closed cycle of the heat engine, process $1 \to 2$ is isothermal, process $2 \to 3$ is isobaric, and process $3 \to 1$ is isochoric. For a diatomic gas $C_V = \frac{5}{2}R$ and $\gamma = \frac{7}{5}$.

Visualize: Please refer to the figure below.



Solve: (a) Begin by expressing the pressure, volume, and temperature in terms of the pressure, volume, and temperature at point 1. In the isothermal expansion $1 \to 2$, the volume is halved so the pressure must double (ideal gas equation). Therefore $p_2 = 2p_1$. Because $2 \to 3$ is isobaric, $p_3 = p_2 = 2p_1$. We are given that $V_2 = V_1/2$ and that $V_3 = V_1$. Finally, we know that $T_2 = T_1$ because they are on the same isotherm, and the ideal gas equation gives

$$T_3 = \frac{p_3V_3}{nR} = \frac{2p_1V_1}{nR} = 2T_1$$

The table below summarizes:

p_1	V_1	T_1
$p_2 = 2p_1$	$V_2 = V_1/2$	$T_2 = T_1$
$p_3 = 2p_1$	$V_3 = V_1$	$T_3 = 2T_1$

With the help of Table 21.1, we can find expressions for the work and heat for each segment in the cycle. The results are given in the table below.

	W_S	Q
$1 \to 2$	$nRT_1 \ln(V_2/V_1) = -nRT_1 \ln 2$	$-nRT_1 \ln 2$
$2 \to 3$	$p_2(V_1 - V_2) = (2p_1)(V_1/2) = p_1V_1 = nRT_1$	$nC_P(T_3 - T_2) = \frac{7}{2}nRT_1$
$3 \to 1$	0	$nC_V(T_1 - T_3) = -\frac{5}{2}nRT_1$

The heat transferred from the hot reservoir into the heat engine (> 0) is done in the isochoric segment $2 \to 3$. The total work done by the system is

$$W_{\text{out}} = -nRT_1 \ln 2 + nRT_1 = nRT_1(1 - \ln 2)$$

The thermal efficiency is therefore

$$\eta = \frac{W_{\text{out}}}{Q_{\text{H}}} = \frac{nRT_1(1-\ln 2)}{\frac{7}{2}nRT_1} = 0.088 = 8.8,$$

(b) The thermal efficiency of a Carnot engine operating between T_1 and T_3 is

$$\eta_{\text{Carnot}} = 1 - \frac{T_{\text{C}}}{T_H} = 1 - \frac{T_1}{T_3} = 0.5 = 50,$$

Assess: The efficiency is much less than the Carnot efficiency.

21.73. Model: Process $1 \to 2$ and process $3 \to 4$ are adiabatic, and process $2 \to 3$ and process $4 \to 1$ are isochoric.
Visualize: Please refer to Figure CP21.73.
Solve: (a) For adiabatic process $1 \to 2$, $Q_{12} = 0\text{ J}$ and

$$W_{12} = \frac{p_2V_2 - p_1V_1}{1-\gamma} = \frac{nR(T_2 - T_1)}{1-\gamma}$$

For isochoric process $2 \to 3$, $W_{23} = 0\text{ J}$ and $Q_{23} = nC_{\text{V}}(T_3 - T_2)$. For adiabatic process $3 \to 4$, $Q_{34} = 0\text{ J}$ and

$$W_{34} = \frac{p_4V_4 - p_3V_3}{1-\gamma} = \frac{nR(T_4 - T_3)}{1-\gamma}$$

For isochoric process $4 \to 1$, $W_{41} = 0\text{ J}$ and $Q_{41} = nC_{\text{V}}(T_1 - T_4)$. The work done per cycle is

$$W_{\text{net}} = W_{12} + W_{23} + W_{34} + W_{41} = \frac{nR(T_2 - T_1)}{1-\gamma} + 0\text{ J} + \frac{nR(T_4 - T_3)}{1-\gamma} + 0\text{ J} = \frac{nR}{1-\gamma}(T_2 - T_1 + T_4 - T_3)$$

(b) The thermal efficiency of the heat engine is

$$\eta = \frac{W_{\text{out}}}{Q_{\text{H}}} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}} = 1 - \frac{|Q_{41}|}{|Q_{23}|} = 1 - \frac{nC_{\text{V}}(T_4 - T_1)}{nC_{\text{V}}(T_3 - T_2)}$$

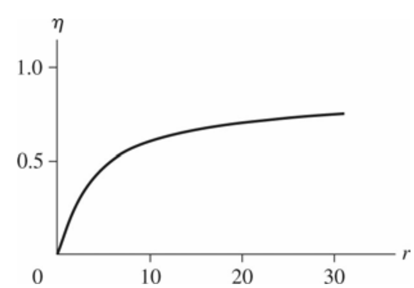
The last step follows from the fact that $T_3 > T_2$ and $T_4 > T_1$. We will now simplify this expression further as follows:

$$pV^{\gamma} = pVV^{\gamma-1} = nRTV^{\gamma-1} \quad \Rightarrow \quad nRT_1V_1^{\gamma-1} = nRT_2V_2^{\gamma-1} \quad \Rightarrow \quad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1r^{\gamma-1}$$

Similarly, $T_3 = T_4r^{\gamma-1}$. The equation for thermal efficiency now becomes

$$\eta = 1 - \frac{T_4 - T_1}{T_4r^{\gamma-1} - T_1r^{\gamma-1}} = 1 - \frac{1}{r^{\gamma-1}}$$

(c)



21.74. Model: For the Diesel cycle, process $1 \to 2$ is an adiabatic compression, process $2 \to 3$ is an isobaric expansion, process $3 \to 4$ is an adiabatic expansion, and process $4 \to 1$ is isochoric.
Visualize: Please refer to CP19.72.
Solve: **(a)** It will be useful to do some calculations using the compression ratio, which is

$$r = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{V_1}{V_2} = \frac{1050 \text{ cm}^3}{50 \text{ cm}^3} = 21$$

The number of moles of gas is

$$n = \frac{p_1 V_1}{RT_1} = \frac{(1.013\times10^5 \text{ Pa})(1050\times10^{-6} \text{ m}^3)}{(8.31 \text{ J/mol K})[(25+273) \text{ K}]} = 0.0430 \text{ mol}$$

For an adiabatic process,

$$p_1 V_1^\gamma = p_2 V_2^\gamma \quad\Rightarrow\quad p_2 = \left(\frac{V_1}{V_2}\right)^\gamma p_1 = r^\gamma p_1 = 21^{1.40}\times 1 \text{ atm} = 71.0 \text{ atm} = 7.19\times10^6 \text{ Pa}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \quad\Rightarrow\quad T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1 = r^{\gamma-1} T_1 = 21^{0.41}\times 298 \text{ K} = 1007 \text{ K}$$

Process $2 \to 3$ is an isobaric heating with $Q = 1000$ J. Constant pressure heating obeys

$$Q = nC_P \Delta T \quad\Rightarrow\quad \Delta T = \frac{Q}{nC_P}$$

The gas has a specific heat ratio $\gamma = 1.40 = 7/5$, thus $C_V = \frac{5}{2}R$ and $C_P = \frac{7}{2}R$. Knowing C_p, we can calculate first $\Delta T = 800$ K and then $T_3 = T_2 + \Delta T = 1807$ K. Finally, for an isobaric process we have

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \quad\Rightarrow\quad V_3 = \frac{T_3}{T_2}V_2 = \frac{1807 \text{ K}}{1007 \text{ K}}(50\times10^{-6} \text{ m}^3) = 89.7\times10^{-6} \text{ m}^3$$

Process $3 \to 4$ is an adiabatic expansion to $V_4 = V_1$. Thus,

$$p_3 V_3^\gamma = p_4 V_4^\gamma \quad\Rightarrow\quad p_4 = \left(\frac{V_3}{V_4}\right)^\gamma p_3 = \left(\frac{89.7\times10^{-6} \text{ m}^3}{1050\times10^{-6} \text{ m}^3}\right)^{1.4} (7.19\times10^6 \text{ Pa}) = 2.30\times10^5 \text{ Pa} = 2.27 \text{ atm}$$

$$T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1} \quad\Rightarrow\quad T_4 = \left(\frac{V_3}{V_4}\right)^{\gamma-1} T_3 = \left(\frac{89.7\times10^{-6} \text{ m}^3}{1050\times10^{-6} \text{ m}^3}\right)^{0.4} (1807 \text{ K}) = 675 \text{ K}$$

Point	P	V	T
1	$1.00 \text{ atm} = 1.013\times10^5 \text{ Pa}$	$1050\times10^{-6} \text{ m}^3$	$298 \text{ K} = 25°\text{C}$
2	$71.0 \text{ atm} = 7.19\times10^6 \text{ Pa}$	$50.0\times10^{-6} \text{ m}^3$	$1007 \text{ K} = 734°\text{C}$
3	$71.0 \text{ atm} = 7.19\times10^6 \text{ Pa}$	$89.7\times10^{-6} \text{ m}^3$	$1807 \text{ K} = 1534°\text{C}$
4	$2.27 \text{ atm} = 2.30\times10^5 \text{ Pa}$	$1050\times10^{-6} \text{ m}^3$	$675 \text{ K} = 402°\text{C}$

(b) For adiabatic process $1 \to 2$,

$$(W_s)_{12} = \frac{p_2 V_2 - p_1 V_1}{1-\gamma} = -633 \text{ J}$$

For isobaric process $2 \to 3$,

$$(W_s)_{23} = p_2 \Delta V = p_2(V_3 - V_2) = 285 \text{ J}$$

For adiabatic process $3 \to 4$,

$$(W_s)_{34} = \frac{p_4V_4 - p_3V_3}{1-\gamma} = 1009\text{ J}$$

For isochoric process $4 \to 1$, $(W_s)_{41} = 0\text{ J}$. Thus,

$$(W_s)_{\text{cycle}} = W_{\text{out}} = (W_s)_{12} + (W_s)_{23} + (W_s)_{34} + (W_s)_{41} = 661\text{ J}$$

(c) The efficiency is

$$\eta = \frac{W_{\text{out}}}{Q_H} = \frac{661\text{ J}}{1000\text{ J}} = 66.1\% \approx 66\%$$

(d) The power output of one cylinder is

$$\left(\frac{661\text{ J}}{\text{cycle}}\right)\left(\frac{2400\text{ cycle}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ sec}}\right) = 26{,}440\ \frac{\text{J}}{\text{s}} = 26.4\text{ kW}$$

For an 8-cylinder engine the power will be 211 kW or 283 horsepower.