FERMILAB
ENGINEERING NOTE

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SUBJECT: Phys 2C 2/18

NAME:

DATE: REVISION DATE:

① EM Waves: general properties

② Poynting Vector, Intensity, Pressure

③ Polarization of EM Waves

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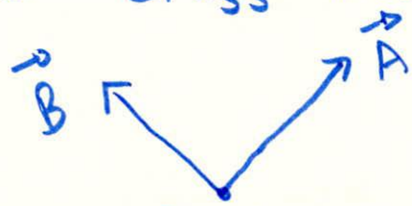
① Review Cross-Product



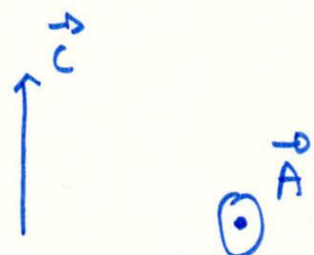
$\vec{A} \times \vec{B}$ ⊙ (out of the page)

$\vec{B} \times \vec{A}$ ⊗ (into the page)

Cyclic for orthogonal unit vectors
$$\begin{cases} \hat{x} \times \hat{y} = \hat{z} \\ \hat{y} \times \hat{z} = \hat{x} \\ \hat{z} \times \hat{x} = \hat{y} \end{cases}$$

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

$\vec{A} \times \vec{B} = \vec{C}$



What direction is $\vec{B}$?

(Note: In general there is no unique answer but here we're always dealing w/ 3 orthogonal vectors, so there is)

$\longrightarrow \vec{B}$

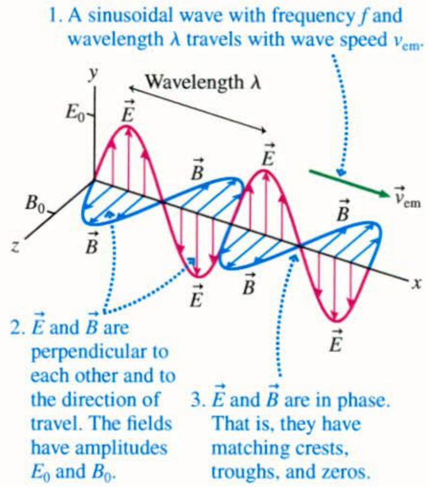
EM waves: 3 important directions (vectors
- Direction light is traveling
- $\vec{E}$ field direction
- $\vec{B}$ field direction

$\boxed{\text{Animations}}$

$\boxed{\vec{E} \times \vec{B} = (\text{dir of wave})}$

Properties of EM Waves

1. Maxwell's equations predict the existence of sinusoidal electromagnetic waves that travel through empty space independent of any charges or currents.
2. The waves are transverse waves, with $\vec{E}$ and $\vec{B}$ perpendicular to the direction of propagation $\vec{v}_{em}$.
3. $\vec{E}$ and $\vec{B}$ are perpendicular to each other in a manner such that $\vec{E} \times \vec{B}$ is in the direction of $\vec{v}_{em}$.
4. All electromagnetic waves, regardless of frequency or wavelength, travel in vacuum at speed $v_{em} = 1/\sqrt{\epsilon_0 \mu_0} = c$, the speed of light.
5. The field strengths are related by $E = cB$ at every point on the wave.
6. $\vec{E}$ and $\vec{B}$ are in phase.



$\vec{E} \times \vec{B} \propto (+\hat{y})$

An EM wave is moving in the +y direction. At a certain time/position in space, the electric field is pointing in the -z direction. What is the direction of the magnetic field at the same time/position?

$\vec{E} \propto (-\hat{z})$

$\hat{y}$ $\hat{x}$ $\cdot\,\hat{z}$ $\otimes \vec{E}$

$\uparrow$ wave

A) +x direction
B) -x direction
C) Neither of these

$(-\hat{z}) \times (?) = +\hat{y}$

$\leftarrow \vec{B}$

$\vec{B} \propto -\hat{x}$

$$\vec{E} = E_0 \cos[ky - \omega t + \phi_0](-\hat{z})$$
$$\vec{B} = B_0 \cos[ky - \omega t + \phi_0](-\hat{x})$$

(also, write down a possible equation for the electric field)

Which of the following could describe the B-field of an EM wave traveling in the +z direction? $\frac{\omega}{k} = c = 3\times10^8$ m/s

A) $\vec{B}(x,y,z,t) = (1.00\text{ nT})\cos\left[\underbrace{(1.26\times10^7\text{ m}^{-1})}_{k}z - \underbrace{(3.77\times10^{15}\text{ s}^{-1})}_{\omega}t\right](+\hat{y})$

B) $\vec{B}(x,y,z,t) = (1.00\text{ nT})\cos\left[(3.77\times10^7\text{ m}^{-1})z - (1.26\times10^{15}\text{ s}^{-1})t\right](-\hat{y})$

~~C)~~ $\vec{B}(x,y,z,t) = (1.00\text{ nT})\cos\left[\cancel{(1.26\times10^7\text{ m}^{-1})y + (3.77\times10^{15}\text{ s}^{-1})t}\right](+\hat{z})$

~~D)~~ $\vec{B}(x,y,z,t) = (1.00\text{ nT})\cos\left[\cancel{(3.77\times10^7\text{ m}^{-1})y - (1.26\times10^{15}\text{ s}^{-1})t}\right](-\hat{z})$

~~E)~~ $\vec{B}(x,y,z,t) = (1.00\text{ nT})\cos\left[(1.26\times10^7\text{ m}^{-1})z - (3.77\times10^{15}\text{ s}^{-1})t\right]\cancel{(+\hat{z})}$

$\vec{B}$ field must be $\perp$ to $\hat{z}$

$$\vec{E} = E_0\cos\left[(1.26\times10^7\text{ m}^{-1})z - (3.77\times10^{15}\text{ s}^{-1})t\right](+\hat{x})$$

$$E_0 = cB_0 = (3\times10^8\text{ m/s})(1.00\text{ nT}) = 0.300\text{ V/m}$$

(also, given this magnetic field, what would the electric field be?)

STOP TO THINK 31.5 The amplitude of the oscillating electric field at your cell phone is 4.0 μV/m when you are 10 km east of the broadcast antenna. What is the electric field amplitude when you are 20 km east of the antenna?

a. 1.0 μV/m
b. 2.0 μV/m
c. 4.0 μV/m
d. There's not enough information to tell.

$I \propto \frac{1}{R^2}$

$I \propto A^2$

$A \propto \sqrt{I}$

$$\frac{I_2}{I_1} = \frac{R_1^2}{R_2^2}$$

$$\frac{A_2}{A_1} = \sqrt{\frac{I_2}{I_1}} = \frac{R_1}{R_2} = \frac{10\text{ km}}{20\text{ km}} = \frac{1}{2}$$

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② Poynting Vector : $\vec{S} = \dfrac{\vec{E} \times \vec{B}}{\mu_0}$ $\qquad \dfrac{\text{Power}}{\text{Area}}$

$$S_{max} = \frac{E_0 B_0}{\mu_0} = \frac{E_0^2}{c\mu_0} = c\epsilon_0 E_0^2$$

but $\vec{E}$, $\vec{B}$ oscillate! And S oscillates from 0 to S_{max}

time average : $I = \dfrac{P}{A} = S_{avg} = \dfrac{E_0 B_0}{2\mu_0} = \dfrac{E_0^2}{2c\mu_0} = \dfrac{c\epsilon_0 E_0^2}{2}$

$\langle \sin^2\theta \rangle = \frac{1}{2}$
$= \langle \cos^2\theta \rangle$

intensity of wave

radiation pressure $\quad P_{rad} = \dfrac{I}{c}$

★ EM waves carry both energy and momentum! Momentum is typically small but energy can be big

③ Polarization

Two main facts

ⓐ unpolarized light of intensity I_0

Polarizer

polarized

$$\boxed{I_f = \tfrac{1}{2} I_0}$$

ⓑ Polarized light I_0

polarized

$$\boxed{I_f = I_0 \cos^2\theta}$$

Animations!

Suppose you have three polarizers: one that will polarize along the y-axis, one along the x-axis, and one along an axis that is 45 degrees with respect to both the y- and x-axes, arranged in some order.

Can initially unpolarized light pass through all three polarizers?

A) Yes, definitely (no matter the order)
B) No, definitely not (no matter the order)
C) Maybe — depends on the order

Suppose you have three polarizers: one that will polarize along the y-axis, one along the x-axis, and one along an axis that is 45 degrees with respect to both the y- and x-axes, arranged in some order.

$I_f = (\frac{1}{2}I_0)(\cos^2 45°)(\cos^2 45°)$
$= \frac{1}{8}I_0$

If the initial intensity of the unpolarized light is I_0, what its the maximum possible final intensity after the light has gone through all 3 polarizers?

A) $(0.125)I_0$
B) $(0.25)I_0$
C) $(0.50)I_0$
D) None of the above

Suppose you have three polarizers: one that will polarize along the y-axis, one along the x-axis, and one along an axis that is 45 degrees with respect to both the y- and x-axes, arranged in some order.

If the initial intensity of polarized light (along the y-axis) is I_0, what its the maximum possible final intensity after the light has gone through all 3 polarizers?

A) $(0.125)I_0$
B) $(0.25)I_0$
C) $(0.50)I_0$
D) None of the above