17

SUPERPOSITION

Conceptual Questions

17.1. **(a)** When a string is fixed at both ends the number of antinodes is the mode (m-value) of the standing wave, so $m = 4$. **(b)** When the frequency is doubled the wavelength is halved. This halving of the wavelength will increase the number of antinodes to eight.

17.2. When the string is flat most of the segments of the string (except at the nodes) are moving (quite rapidly at the antinodes), so the energy is in the kinetic energy of the moving string.

17.3. **(a)** $m = 2$ **(b)** horizontally, because sound is a longitudinal wave **(c)** 0 cm, 16 cm, 32 cm **(d)** 8 cm and 24 cm because the pressure antinodes occur where there are displacement nodes.

17.4. The speed of sound in air is related to the temperature by $v = \sqrt{\gamma RT/M}$, so speed increases with increasing temperature. Some organ pipes are open at both ends and some are open-closed. However, in both cases the frequency increases with increasing speed. So frequency increases as speed increases, and speed increases as temperature increases. As a result, as the air in the organ pipe warms up, the frequency of the fundamental and hence all the harmonics will increase slightly.

17.5. The sound you hear is the vibration of the glass; it is set into motion by the disturbance of the flowing, sloshing liquid. The disturbances are of many frequencies but the natural frequency of the glass resonates and is amplified by the glass while other frequencies are quickly damped out. The reason the pitch rises as the glass fills is that the natural frequency of the glass changes; as the glass fills the resonance frequency rises.

17.6. The length of the flute is the same regardless of the gas it is filled with; consequently, the fundamental wavelength is the same too. But because helium atoms have a smaller atomic mass than air molecules, helium atoms move faster on average at the same temperature. So the speed of sound is greater in helium than in air at the same temperature. In $v = \lambda f$ if λ is unchanged but v is increased, then f must increase also; this is perceived as a higher pitch. The same explanation applies to the voices of people who have inhaled a breath of helium.

17.7. Use $v = \sqrt{T_s/\mu}$ and $v = \lambda f$. Assume the linear mass density is the same in all parts of this question. Use primes for new quantities. **(a)** $\dfrac{T'}{T_0} = \dfrac{\mu v'^2}{\mu v_0^2} = \dfrac{\mu(\lambda f')^2}{\mu(\lambda f_0)^2} = \dfrac{f'^2}{f_0^2} = \dfrac{(2f_0)^2}{f_0^2} = 4.$ Changing the tension is how one tunes a guitar; it would be difficult to increase the tension by a factor of 4 without breaking the string, so only small changes are made this way.
(b) $\dfrac{L'}{L_0} = \dfrac{\lambda'}{\lambda_0} = \dfrac{v/f'}{v/f_0} = \dfrac{v/(2f_0)}{v/(f_0)} = \dfrac{1}{2}$ so the length must be decreased by a factor of 2. Guitar players effectively decrease the length by putting their fingers down on the string at various frets to create a node there.

17.8. **(a)** 4 mm **(b)** 0 mm **(c)** −4 mm. This is easily seen by adding the contributions from Wave 1 and Wave 2 at each point.

17.9. At point a, two crests are arriving at the same place at the same time from in-phase sources, so it is a point of constructive interference. At point b, a crest from source 2 is arriving at the same time as a trough is arriving from source 1, so it is a point of destructive interference. At point c, two troughs are arriving at the same place at the same time from in-phase sources, so it is a point of constructive interference.

17.10. Pulling out the tuning valve lengthens the instrument and the wavelength of the standing waves. This lowers the frequency of the sound produced. By pulling out the valve, she decreased the frequency of the beat from 5 Hz to 3 Hz. If she were playing at a frequency below the 440 Hz tuning fork, the beat frequency would have increased because the difference between her frequency and the tuning fork frequency would have increased. However if she were playing above the tuning fork frequency, reducing her frequency would bring her closer to the frequency of the tuning fork, which would decrease the beat frequency. Since the beat frequency did, in fact, decrease, she must have been playing above the tuning fork frequency, at 445 Hz.

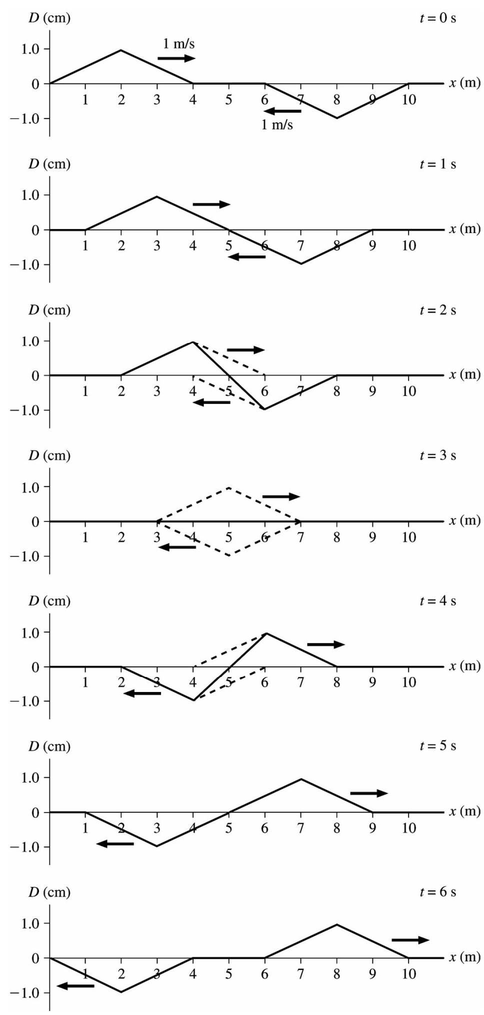
Exercises and Problems

Exercises

Section 17.1 The Principle of Superposition

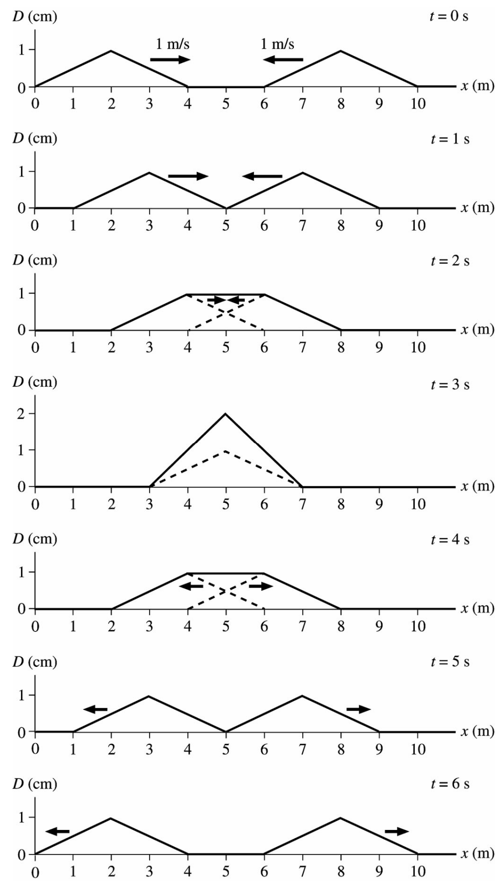
17.1. Model: The principle of superposition comes into play whenever the waves overlap.

Visualize:



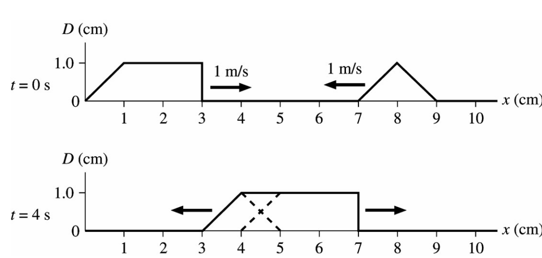
The snapshot graph at $t = 1.0$ s differs from the graph $t = 0.0$ s in that the left wave has moved to the right by 1.0 m and the right wave has moved to the left by 1.0 m. This is because the distance covered by each wave in 1.0 s is 1.0 m. The snapshot graphs at $t = 2.0$, 3.0, and 4.0 s are a superposition of the left and the right moving waves. The overlapping parts of the two waves are shown by the dotted lines.

17.2. Model: The principle of superposition comes into play whenever the waves overlap.
Visualize:



The graph at $t = 1.0$ s differs from the graph at $t = 0.0$ s in that the left wave has moved to the right by 1.0 m and the right wave has moved to the left by 1.0 m. This is because the distance covered by the wave pulse in 1.0 s is 1.0 m. The snapshot graphs at $t = 2.0$, 3.0, and 4.0 s are a superposition of the left- and the right-moving waves. The overlapping parts of the two waves are shown by the dotted lines.

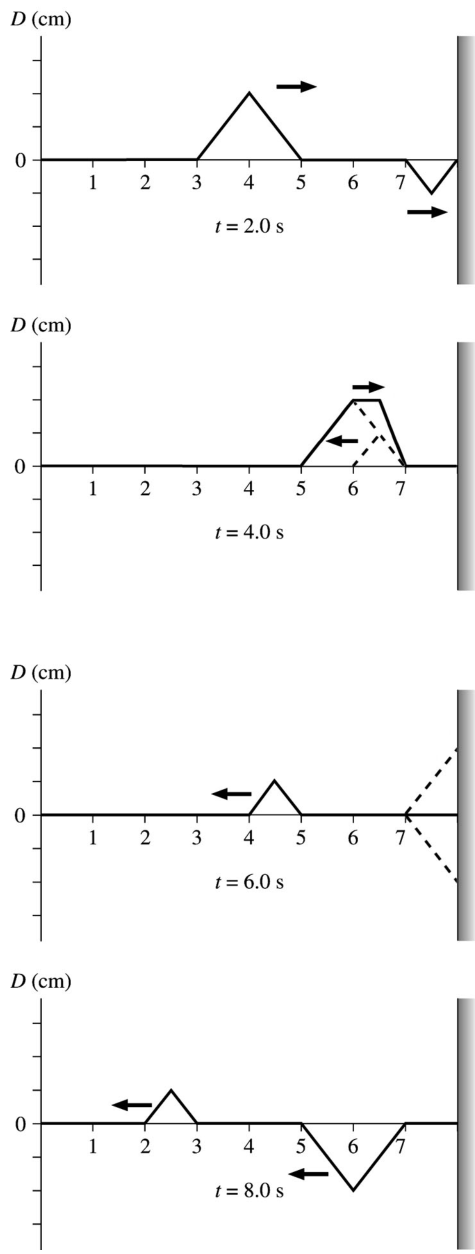
17.3. Model: The principle of superposition comes into play whenever the waves overlap.
Solve: As graphically illustrated in the figure below, the snapshot graph was taken at $t = 4$ s.



Section 17.2 Standing Waves

Section 17.3 Standing Waves on a String

17.4. Model: A wave pulse reflected from the string-wall boundary is inverted and its amplitude is unchanged.
Visualize:



The graph at $t = 2$ s differs from the graph at $t = 0$ s in that both waves have moved to the right by 2 m. This is because the distance covered by the wave pulse in 2 s is 2 m. The shorter pulse wave encounters the boundary wall at 2 s and is inverted upon reflection. This reflected pulse wave overlaps with the broader pulse wave, as shown in the snapshot graph at $t = 4$ s. At $t = 6$ s, only half of the broad pulse is reflected and hence inverted; the shorter pulse wave continues to move to the left with a speed of 1 m/s. Finally, at $t = 8$ both the reflected pulse waves are inverted and they are both moving to the left.

17.5. Model: Reflections at the string boundaries cause a standing wave on the string.

Solve: The figure indicates 5/2 wavelengths on the string. Hence $\lambda = \frac{2}{5}(50\text{ cm}) = 20\text{ cm} = 0.20\text{ m}$. Thus

$$v = \lambda f = (0.20\text{ m})(100\text{ Hz}) = 20\text{ m/s}$$

17.6. Model: Reflections at both ends of the string cause the formation of a standing wave.

Solve: The figure indicates two full wavelengths on the string. Thus, the wavelength of the standing wave is $\lambda = \frac{1}{2}(2.0\text{ m}) = 1.0\text{ m}$. The frequency of the standing wave is

$$f = \frac{v}{\lambda} = \frac{40\text{ m/s}}{1.0\text{ m}} = 40\text{ Hz}$$

17.7. Model: Reflections at the string boundaries cause a standing wave on the string.

Solve: (a) When the frequency is doubled to 200 Hz the wavelength is halved $(\lambda' = \frac{1}{2}\lambda_0)$. This halving of the wavelength will increase the number of antinodes from 4 to 8.

(b) Increasing the tension by a factor of 4 means

$$v = \sqrt{\frac{T}{\mu}} \quad \Rightarrow \quad v' = \sqrt{\frac{T'}{\mu}} = \sqrt{\frac{4T}{\mu}} = 2v$$

For the string to continue to oscillate as a standing wave with four antinodes means $\lambda' = \lambda_0$. Hence,

$$v' = 2v \quad \Rightarrow \quad f'\lambda' = 2f_0\lambda_0 \quad \Rightarrow \quad f'\lambda_0 = 2f_0\lambda_0 \quad \Rightarrow \quad f' = 2f_0 = 200\text{ Hz}$$

That is, the new frequency is 200 Hz, twice the original frequency.

17.8. Model: A string fixed at both ends supports standing waves.

Solve: (a) A standing wave can exist on the string only if its wavelength is

$$\lambda_m = \frac{2L}{m} \qquad m = 1, 2, 3, \ldots$$

The three longest wavelengths for standing waves will therefore correspond to $m = 1, 2,$ and 3. Thus,

$$\lambda_1 = \frac{2(2.4\text{ m})}{1} = 4.8\text{ m} \quad \lambda_2 = \frac{2(2.4\text{ m})}{2} = 2.4\text{ m} \quad \lambda_3 = \frac{2(2.4\text{ m})}{3} = 1.6\text{ m}$$

(b) Because the wave speed on the string is unchanged from one *m* value to the other,

$$f_2\lambda_2 = f_3\lambda_3 \quad \Rightarrow \quad f_3 = \frac{f_2\lambda_2}{\lambda_3} = \frac{(50\text{ Hz})(2.4\text{ m})}{1.6\text{ m}} = 75\text{ Hz}$$

17.9. Model: A string fixed at both ends supports standing waves.

Solve: (a) We have $f_\text{a} = 36\text{ Hz} = mf_1$ where f_1 is the fundamental frequency that corresponds to $m = 1$. The next successive frequency is $f_\text{b} = 48\text{ Hz} = (m+1)f_1$. Thus,

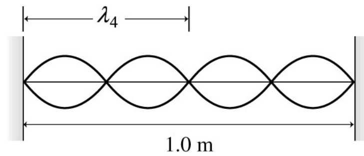
$$\frac{f_\text{b}}{f_\text{a}} = \frac{(m+1)f_1}{mf_1} = \frac{m+1}{m} = \frac{48\text{ Hz}}{36\text{ Hz}} = \frac{4}{3} \quad \Rightarrow \quad m+1 = \frac{4m}{3} \quad \Rightarrow \quad m = 3 \quad \Rightarrow \quad f_1 = \frac{36\text{ Hz}}{3} = 12\text{ Hz}$$

The wave speed is

$$v = \lambda_1 f_1 = \frac{2L}{1}f_1 = (2.0\text{ m})(12\text{ Hz}) = 24\text{ m/s}$$

(b) The frequency of the fourth harmonic is 48 Hz. For $m = 4$, the wavelength is

$$\lambda_m = \frac{2L}{m} = \frac{2(1.0\text{ m})}{4} = \frac{1}{2}\text{ m}$$



17.10. Model: Assume the violin strings have the same tension and same length (and therefore the same fundamental wavelength).

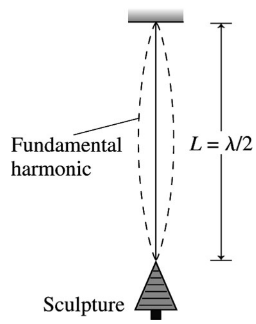
Solve: Use $\mu = m/L$ and $\mu = T_s/v^2$.

$$\frac{m_A}{m_E} = \frac{\mu_A L_A}{\mu_E L_E} = \frac{\mu_A}{\mu_E} = \frac{T_A/v_A^2}{T_E/v_E^2} = \frac{v_E^2}{v_A^2} = \frac{(\lambda_E f_E)^2}{(\lambda_A f_A)^2} = \frac{f_E^2}{f_A^2} = \frac{(659\text{ Hz})^2}{(440\text{ Hz})^2} = 2.24$$

Assess: We know the A-string is thicker than the E-string, and it seems reasonable to be 2.24 times as massive.

17.11. Model: For the stretched wire vibrating at its fundamental frequency, the wavelength of the standing wave is $\lambda_1 = 2L$.

Visualize:



Solve: The wave speed on the steel wire is

$$v_{\text{wire}} = f\lambda = f(2L) = (80\text{ Hz})(2\times 0.90\text{ m}) = 144\text{ m/s}$$

and is also equal to $\sqrt{T_S/\mu}$, where

$$\mu = \frac{\text{mass}}{\text{length}} = \frac{5.0\times10^{-3}\text{ kg}}{0.90\text{ m}} = 5.555\times10^{-3}\text{ kg/m}$$

The tension T_S in the wire equals the weight of the sculpture or Mg. Thus,

$$v_{\text{wire}} = \sqrt{\frac{Mg}{\mu}} \quad\Rightarrow\quad M = \frac{\mu v_{\text{wire}}^2}{g} = \frac{(5.555\times10^{-3}\text{ kg/m})(144\text{ m/s})^2}{9.8\text{ m/s}^2} = 12\text{ kg}$$

17.12. Model: The laser light forms a standing wave inside the cavity.

Solve: The wavelength of the laser beam is

$$\lambda_m = \frac{2L}{m} \quad\Rightarrow\quad \lambda_{100,000} = \frac{2(0.5300\text{ m})}{100,000} = 10.60\,\mu\text{m}$$

The frequency is

$$f_{100,000} = \frac{c}{\lambda_{100,000}} = \frac{2.9979\times10^8\text{ m/s}}{10.60\times10^{-6}\text{ m}} = 2.828\times10^{13}\text{ Hz}$$

17.13. Model: The microwave forms a standing wave between the two reflectors.
Solve: **(a)** There are reflectors at both ends, so the electromagnetic standing wave acts just like the standing wave on a string that is tied at both ends. The frequencies of the standing waves are

$$f_m = m\frac{v_{\text{light}}}{2L} = m\frac{c}{2L} = m\frac{3.0\times10^8 \text{ m/s}}{2(0.10 \text{ m})} = m(1.5\times10^9 \text{ Hz}) = 1.5m \text{ GHz}$$

where we have used the fact that electromagnetic waves of all frequencies travel at the speed of light c. The generator can produce standing waves at any frequency between 10 GHz and 20 GHz. These are

m	f_m(GHz)
7	10.5
8	12.0
9	13.5
10	15.0
11	16.5
12	18.0
13	19.5

Section 17.4 Standing Sound Waves and Musical Acoustics
17.14. Solve: **(a)** For the open-open tube, the two open ends exhibit antinodes of a standing wave. The possible wavelengths for this case are

$$\lambda_m = \frac{2L}{m} \qquad m = 1, 2, 3, \ldots$$

The three longest wavelengths are

$$\lambda_1 = \frac{2(1.21 \text{ m})}{1} = 2.42 \text{ m} \qquad \lambda_2 = \frac{2(1.21 \text{ m})}{2} = 1.21 \text{ m} \qquad \lambda_3 = \frac{2(1.21 \text{ m})}{3} = 0.807 \text{ m}$$

(b) In the case of an open-closed tube,

$$\lambda_m = \frac{4L}{m} \qquad m = 1, 3, 5, \ldots$$

The three longest wavelengths are

$$\lambda_1 = \frac{4(1.21 \text{ m})}{1} = 4.84 \text{ m} \qquad \lambda_2 = \frac{4(1.21 \text{ m})}{3} = 1.61 \text{ m} \qquad \lambda_3 = \frac{4(1.21 \text{ m})}{5} = 0.968 \text{ m}$$

17.15. Model: We have an open-open tube that forms standing sound waves.
Solve: The gas molecules at the ends of the tube exhibit maximum displacement, making antinodes at the ends. There is another antinode in the middle of the tube. Thus, this is the $m = 2$ mode and the wavelength of the standing wave is equal to the length of the tube, that is, $\lambda = 0.80$ m. Since the frequency $f = 500$ Hz, the speed of sound in this case is $v = f\lambda = (500 \text{ Hz})(0.80 \text{ m}) = 400$ m/s.
Assess: The experiment yields a reasonable value for the speed of sound.

17.16. Solve: For the open-open tube, the fundamental frequency of the standing wave is $f_1 = 1500$ Hz when the tube is filled with helium gas at 0°C. Using $\lambda_m = 2L/m$,

$$f_{1\text{ helium}} = \frac{v_{\text{helium}}}{\lambda_1} = \frac{970 \text{ m/s}}{2L}$$

Similarly, when the tube is filled with air,

$$f_{1\text{ air}} = \frac{v_{\text{air}}}{\lambda_1} = \frac{331 \text{ m/s}}{2L} \Rightarrow \frac{f_{1\text{ air}}}{f_{1\text{ helium}}} = \frac{331 \text{ m/s}}{970 \text{ m/s}} \Rightarrow f_{1\text{ air}} = \left(\frac{331 \text{ m/s}}{970 \text{ m/s}}\right)(1500 \text{ Hz}) = 512 \text{ Hz}$$

Assess: Note that the length of the tube is one-half the wavelength whether the tube is filled with helium or air.

17.17. Solve: For the open-closed tube, Equation 17.18 gives the possible frequencies:

$$f_m = m\frac{v}{4L} = mf_1 \quad \Rightarrow \quad L = \frac{v}{4f_1}$$

for a fundamental frequency of $f_1 = 250\text{ Hz}$ and a speed of sound of $v = 350\text{ m/s}$, the length of the vocal tract must be approximately

$$L = \frac{350\text{ m/s}}{4(250\text{ Hz})} = 35\text{ cm}$$

Assess: A distance of about 35 cm from the mouth to the diaphragm seems reasonable for most people.

17.18. Model: Reflections at the string boundaries cause a standing wave on a stretched string.
Solve: Because the vibrating section of the string is 1.9 m long, the two ends of this vibrating wire are fixed, and the string is vibrating in the fundamental harmonic. The wavelength is

$$\lambda_m = \frac{2L}{m} \quad \Rightarrow \quad \lambda_1 = 2L = 2(1.90\text{ m}) = 3.80\text{ m}$$

The wave speed along the string is $v = f_1\lambda_1 = (27.5\text{ Hz})(3.80\text{ m}) = 104.5\text{ m/s}$. The tension in the wire can be found as follows:

$$v = \sqrt{\frac{T_S}{\mu}} \quad \Rightarrow \quad T_S = \mu v^2 = \left(\frac{\text{mass}}{\text{length}}\right)v^2 = \left(\frac{0.400\text{ kg}}{2.00\text{ m}}\right)(104.5\text{ m/s})^2 = 2180\text{ N}$$

17.19. Model: Assume the violin strings have the same tension and same length (and therefore the same fundamental wavelength).
Solve: First find the fundamental frequency at the cool room temperature (where the speed of sound is 343 m/s).

$$f = \frac{v}{4L} = \frac{343\text{ m/s}}{4(1.2\text{ m})} = 71.46\text{ Hz}$$

Now use ratios to see what the increased temperature does to the frequency.

$$\frac{f_{\text{hot}}}{f_{\text{cool}}} = \frac{v_{\text{hot}}/4L}{v_{\text{cool}}/4L} = \frac{v_{\text{hot}}}{v_{\text{cool}}} = \frac{352\text{ m/s}}{343\text{ m/s}} = 1.026 \Rightarrow f_{\text{hot}} = 1.026(71.46\text{ Hz}) = 73.3\text{ Hz}$$

The warmer air temperature makes the fundamental frequency increase, and we see that it increased by 73.3 Hz – 71.46 Hz = 1.9 Hz.
Assess: 1.9 Hz is not a lot, but musicians can hear this difference. If two bass clarinets play together, one cool and one warmed then they will hear a beat frequency of almost 2 Hz. This is why musicians "warm up" their instruments before tuning with the orchestra.

17.20. Model: A string fixed at both ends forms standing waves.
Solve: A simple string sounds the fundamental frequency $f_1 = v/(2L)$. Initially, when the string is of length $L_A = 30\text{ cm}$, the note has the frequency $f_{1A} = v/(2L_A)$. For a different length, $f_{1B} = v/(2L_B)$. Taking the ratio of each side of these two equations gives

$$\frac{f_{1A}}{f_{1B}} = \frac{v/(2L_A)}{v/(2L_B)} = \frac{L_B}{L_A} \quad \Rightarrow \quad L_B = \frac{f_{1A}}{f_{1B}}L_A$$

We know that the second frequency is desired to be $f_{1B} = 523\text{ Hz}$. The string length must be

$$L_B = \frac{440\text{ Hz}}{523\text{ Hz}}(30\text{ cm}) = 25.2\text{ cm}$$

The question is not how long the string must be, but where must the violinist place his finger. The full string is 30 cm long, so the violinist must place his finger 4.8 cm from the end.
Assess: A fingering distance of 4.8 cm from the end is reasonable.

17.21. Model: In a rod in which a longitudinal standing wave can be created, the standing wave is equivalent to a sound standing wave in an open-open tube. Both ends of the rod are antinodes, and the rod is vibrating in the fundamental mode.

Solve: Since the rod is in the fundamental mode, $\lambda_1 = 2L = 2(2.0 \text{ m}) = 4.0 \text{ m}$. Using the speed of sound in aluminum, the frequency is

$$f_1 = \frac{v_{\text{Al}}}{\lambda_1} = \frac{6420 \text{ m/s}}{4.0 \text{ m}} = 1605 \text{ Hz} \approx 1.6 \text{ kHz}$$

Section 17.5 Interference in One Dimension

Section 17.6 The Mathematics of Interference

17.22. Model: Interference occurs according to the difference between the phases $(\Delta\phi)$ of the two waves.

Solve: (a) A separation of 20 cm between the speakers leads to maximum intensity on the *x*-axis, but a separation of 60 cm leads to zero intensity. That is, the waves are in phase when $(\Delta x)_1 = 20 \text{ cm}$ but out of phase when $(\Delta x)_2 = 60 \text{ cm}$. Thus,

$$(\Delta x)_2 - (\Delta x)_1 = \frac{\lambda}{2} \quad \Rightarrow \quad \lambda = 2(60 \text{ cm} - 20 \text{ cm}) = 80 \text{ cm}$$

(b) If the distance between the speakers continues to increase, the intensity will again be a maximum when the separation between the speakers that produced a maximum has increased by one wavelength. That is, when the separation between the speakers is $20 \text{ cm} + 80 \text{ cm} = 100 \text{ cm}$.

17.23. Model: The interference of two waves depends on the difference between the phases $(\Delta\phi)$ of the two waves.

Solve: (a) Because the speakers are in phase, $\Delta\phi_0 = 0 \text{ rad}$. Let *d* represent the path-length difference. Using $m = 0$ for the smallest *d* and the condition for destructive interference, we get

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = 2\left(m + \tfrac{1}{2}\right)\pi \text{ rad}, \quad m = 0, 1, 2, 3, \ldots$$

$$2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = \pi \text{ rad} \quad \Rightarrow \quad 2\pi\frac{d}{\lambda} + 0 \text{ rad} = \pi \text{ rad} \quad \Rightarrow \quad d = \frac{\lambda}{2} = \frac{1}{2}\left(\frac{v}{f}\right) = \frac{1}{2}\left(\frac{343 \text{ m/s}}{686 \text{ Hz}}\right) = 25 \text{ cm}$$

(b) When the speakers are out of phase, $\Delta\phi_0 = \pi$. Using $m = 1$ for the smallest *d* and the condition for constructive interference, we get

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = 2m\pi, \quad m = 0, 1, 2, 3, \ldots$$

$$2\pi\frac{d}{\lambda} + \pi = 2\pi \quad \Rightarrow \quad d = \frac{\lambda}{2} = \frac{1}{2}\left(\frac{v}{f}\right) = \frac{1}{2}\left(\frac{343 \text{ m/s}}{686 \text{ Hz}}\right) = 25 \text{ cm}$$

17.24. Visualize: Examine just one side of the headphones, since it works the same on both sides.

Solve: To produce maximum destructive interference the delayed wave needs to be π rad out of phase with the incoming wave; this corresponds to ½ of a period.

$$\Delta t = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2(110 \text{ Hz})} = 0.45 \text{ ms}$$

Assess: Since *v* does not appear the answer is independent of it; we are glad noise-canceling headphones do not need to be readjusted every time the temperature (and therefore the speed) changes. We also did not need to know the distance (1.8 m) to the buzzing sound. 4.5 ms is doable with modern electronics.

17.25. Model: Reflection is maximized if the two reflected waves interfere constructively.
Solve: The film thickness that causes constructive interference at wavelength λ is given by Equation 17.32:

$$\lambda_C = \frac{2nd}{m} \quad \Rightarrow \quad d = \frac{\lambda_C m}{2n} = \frac{(600 \times 10^{-9}\ \text{m})(1)}{(2)(1.39)} = 216\ \text{nm}$$

where we have used $m = 1$ to calculate the thinnest film.
Assess: The film thickness is much less than the wavelength of visible light. The above formula is applicable because $n_{\text{air}} < n_{\text{film}} < n_{\text{glass}}$.

17.26. Model: Reflection is maximized if the two reflected waves interfere constructively.
Solve: The film thickness that causes constructive interference at wavelength λ is given by Equation 17.32:

$$\lambda_C = \frac{2nd}{m} \quad \Rightarrow \quad d = \frac{\lambda_C m}{2n} = \frac{(500 \times 10^{-9}\ \text{m})(1)}{(2)(1.25)} = 200\ \text{nm}$$

where we have used $m = 1$ to calculate the thinnest film.
Assess: The film thickness is much less than the wavelength of visible light. The above formula is applicable because $n_{\text{air}} < n_{\text{oil}} < n_{\text{water}}$.

Section 17.7 Interference in Two and Three Dimensions

17.27. Solve: (a) The circular wave fronts emitted by the two sources show that the two sources are in phase because the wave fronts of each source have moved the same distance from their sources.
(b) Let us label the top source as 1 and the bottom source as 2. Since the sources are in phase, $\Delta\phi_0 = 0$ rad. For the point *P*, $r_1 = 3\lambda$ and $r_2 = 4\lambda$. Thus, $\Delta r = r_2 - r_1 = 4\lambda - 3\lambda = \lambda$. The phase difference is

$$\Delta\phi = \frac{2\pi\Delta r}{\lambda} = \frac{2\pi(\lambda)}{\lambda} = 2\pi$$

This corresponds to constructive interference.
For the point *Q*, $r_1 = \frac{7}{2}\lambda$ and $r_2 = 2\lambda$. The phase difference is

$$\Delta\phi = \frac{2\pi\Delta r}{\lambda} = \frac{2\pi\left(\frac{3}{2}\lambda\right)}{\lambda} = 3\pi$$

This corresponds to destructive interference.
For the point *R*, $r_1 = \frac{5}{2}\lambda$ and $r_2 = \frac{7}{2}\lambda$. The phase difference is

$$\Delta\phi = \frac{2\pi(\lambda)}{\lambda} = 2\pi$$

This corresponds to constructive interference.

	r_1	r_2	Δr	C/D
P	3λ	4λ	λ	C
Q	$\frac{7}{2}\lambda$	2λ	$\frac{3}{2}\lambda$	D
R	$\frac{5}{2}\lambda$	$\frac{7}{2}\lambda$	λ	C

17.28. Solve: (a) The circular wave fronts emitted by the two sources indicate the sources are out of phase. This is because the wave fronts of each source have not moved the same distance from their sources.
(b) Let us label the top source as 1 and the bottom source as 2. Because the wave front closest to source 2 has moved only half of a wavelength, whereas the wave front of source 1 has moved one wavelength, the phase difference between the sources is $\Delta\phi_0 = \pi$. For the point *P*, $r_1 = 2\lambda$ and $r_2 = 3\lambda$. The phase difference is

$$\Delta\phi = \frac{2\pi\Delta r}{\lambda} + \Delta\phi_0 = \frac{2\pi(3\lambda - 2\lambda)}{\lambda} + \pi = 3\pi$$

This corresponds to destructive interference.

For the point Q, $r_1 = 3\lambda$ and $r_2 = \frac{3}{2}\lambda$. The phase difference is

$$\Delta\phi = \frac{2\pi\left(\frac{3}{2}\lambda\right)}{\lambda} + \pi = 4\pi$$

This corresponds to constructive interference.

For the point R, $r_1 = \frac{5}{2}\lambda$ and $r_2 = 3\lambda$. The phase difference is

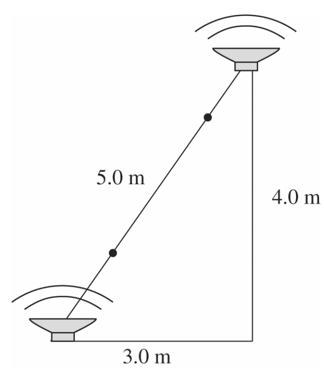
$$\Delta\phi = \frac{2\pi\left(\frac{1}{2}\lambda\right)}{\lambda} + \pi = 2\pi$$

This corresponds to constructive interference.

	r_1	r_2	Δr	C/D
P	2λ	3λ	λ	D
Q	3λ	$\frac{3}{2}\lambda$	$\frac{3}{2}\lambda$	C
R	$\frac{5}{2}\lambda$	3λ	$\frac{1}{2}\lambda$	C

Assess: Note that it is not r_1 or r_2 that matters, but the difference Δr between them.

17.29. Visualize:



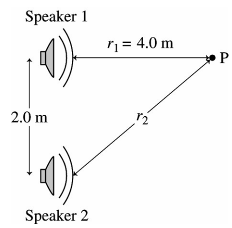
Solve: The final distance between speakers is given by the Pythagorean theorem: 5 m. The distance between spots of constructive interference is $\frac{3}{4}(5.0\text{ m}) - \frac{1}{4}(5.0\text{ m}) = (5.0\text{ m})/2$. For the longest wavelength possible in this situation this distance must be half a wavelength.

$$\frac{1}{2}\lambda = \frac{5.0\text{ m}}{2} \Rightarrow \lambda = 5.0\text{ m}$$

Assess: This is a typical wavelength for a low-pitch sound.

17.30. Model: The two speakers are identical, and so they are emitting circular waves in phase. The overlap of these waves causes interference.

Visualize:



Solve: From the geometry of the figure,

$$r_2 = \sqrt{r_1^2 + (2.0 \text{ m})^2} = \sqrt{(4.0 \text{ m})^2 + (2.0 \text{ m})^2} = 4.472 \text{ m}$$

so $\Delta r = r_2 - r_1 = 4.472 \text{ m} - 4.0 \text{ m} = 0.472 \text{ m}$. The phase difference between the sources is $\Delta\phi_0 = 0$ rad and the wavelength of the sound waves is

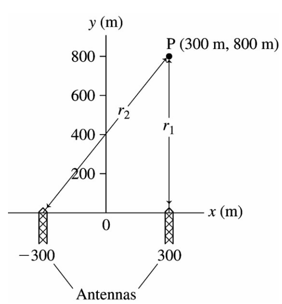
$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{1800 \text{ Hz}} = 0.1889 \text{ m}$$

Thus, the phase difference of the waves at the point 4.0 m in front of one source is

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = \frac{2\pi(0.472 \text{ m})}{0.1889 \text{ m}} + 0 \text{ rad } = 5\pi \text{ rad} = \tfrac{5}{2}(2\pi \text{ rad})$$

This is a half-integer multiple of 2π rad, so the interference is perfect destructive.

17.31. Model: The two radio antennas are emitting out-of-phase, circular waves. The overlap of these waves causes interference.
Visualize:



Solve: From the geometry of the figure, $r_1 = 800$ m and

$$r_2 = \sqrt{(800 \text{ m})^2 + (600 \text{ m})^2} = 1000 \text{ m}$$

So, $\Delta r = r_2 - r_1 = 200$ m and $\Delta\phi_0 = \pi$ rad. The wavelength of the waves is

$$\lambda = \frac{c}{f} = \frac{3.0\times10^8 \text{ m/s}}{3.0\times10^6 \text{ Hz}} = 100 \text{ m}$$

Thus, the phase difference of the waves at the point (300 m, 800 m) is

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = \frac{2\pi(200 \text{ m})}{100 \text{ m}} + \pi \text{ rad} = 5\pi \text{ rad} = \tfrac{5}{2}(2\pi \text{ rad})$$

This is a half-integer multiple of 2π rad, so the interference is perfect destructive.

Section 17.8 Beats

17.32. Solve: The beat frequency is

$$f_{\text{beat}} = f_1 - f_2 \quad \Rightarrow \quad 3 \text{ Hz} = f_1 - 200 \text{ Hz} \quad \Rightarrow \quad f_1 = 203 \text{ Hz}$$

f_1 is larger than f_2 because the increased tension increases the wave speed and hence the frequency.

17.33. Solve: The flute player's initial frequency is either $523 \text{ Hz} + 4 \text{ Hz} = 527 \text{ Hz}$ or $523 \text{ Hz} - 4 \text{ Hz} = 519 \text{ Hz}$. Since she matches the tuning fork's frequency by lengthening her flute, she is increasing the wavelength of the standing wave in the flute. A wavelength increase means a decrease of frequency because $v = f\lambda$. Thus, her initial frequency was 527 Hz.

17.34. Solve: The beat frequency is the difference of the two gong frequencies: 155 Hz – 151 Hz = 4.0 Hz. The number of beats heard in 2.5 s is $(4.0 \text{ Hz})(2.5 \text{ s}) = (4.0 \text{ beats/s})(2.5 \text{ s}) = 10 \text{ beats}$.
Assess: This would give a vibrating quality to the music.

17.35. Solve: The beat frequency is

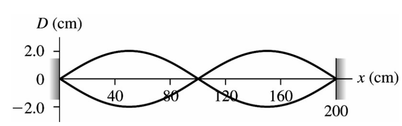
$$f_{\text{beat}} = f_1 - f_2 = 100 \text{ MHz}$$

where we have used the fact that the $\lambda_1 < \lambda_2$ so $f_1 > f_2$. The frequency of emitter 1 is $f_1 = c/\lambda_1$, where $\lambda_1 = 1.250 \times 10^{-2}$ m. The wavelength of emitter 2 is

$$\lambda_2 = c/f_2 = \frac{c}{f_1 - 100 \text{ MHz}} = \frac{c}{c/\lambda_1 - 100 \text{ MHz}} = \frac{(3.00 \times 10^8 \text{ m/s})}{(3.00 \times 10^8 \text{ m/s})/(1.250 \times 10^{-2} \text{ m}) - 100 \text{ MHz}} = 1.26 \text{ cm}.$$

Problems

17.36. Model: The wavelength of the standing wave on a string vibrating at its second-harmonic frequency is equal to the string's length.
Visualize:



Solve: The length of the string $L = 2.0$ m, so $\lambda = L = 2.0$ m. This means the wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2.0 \text{ m}} = \pi \text{ rad/m}$$

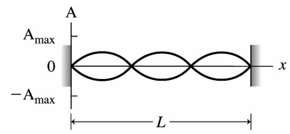
According to Equation 17.5, the displacement of a medium when two sinusoidal waves superpose to give a standing wave is $D(x,t) = A(x)\cos\omega t$, where $A(x) = 2a\sin kx = A_{\max}\sin kx$. The amplitude function gives the amplitude of oscillation from point to point in the medium. For $x = 10$ cm,

$$A(x = 10 \text{ cm}) = (2.0 \text{ cm})\sin[(\pi \text{ rad/m})(0.10 \text{ m})] = 0.62 \text{ cm}$$

Similarly, $A(x = 20 \text{ cm}) = 1.2$ cm, $A(x = 30 \text{ cm}) = 1.6$ cm, $A(x = 40 \text{ cm}) = 1.9$ cm, and $A(x = 50 \text{ cm}) = 2.0$ cm.
Assess: Consistent with the above figure, the amplitude of oscillation is a maximum at $x = 0.50$ m.

17.37. Model: The wavelength of the standing wave on a string is $\lambda_m = 2L/m$, where $m = 1, 2, 3, \ldots$. We assume that 30 cm is the first place from the left end of the string where $A = A_{\max}/2$.

Visualize:



Solve: The amplitude of oscillation on the string is $A(x) = A_{\max} \sin kx$. Since the string is vibrating in the third harmonic, the wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(2L/3)} = 3\frac{\pi}{L}$$

Substituting into the equation for the amplitude gives

$$\frac{1}{2}A_{\max} = A_{\max} \sin\left[\frac{3\pi}{L}(0.30 \text{ m})\right] \Rightarrow \sin\left[\frac{3\pi}{L}(0.30 \text{ m})\right] = \frac{1}{2} \Rightarrow \frac{3\pi}{L}(0.30 \text{ m}) = \frac{\pi}{6} \text{ rad} \Rightarrow L = 5.4 \text{ m}$$

17.38. Model: Model the tendon as a string. The wavelength of the standing wave on a string vibrating at its fundamental frequency is equal to $2L$.
Solve: The linear density of the string is $\mu = \rho\sigma$, where $\rho = 1100 \text{ kg/m}^3$ and $\sigma = 90 \text{ mm}^2$. The velocity of waves on this string is $v = \sqrt{T/\mu} = \sqrt{T/(\rho\sigma)}$. The allowed wavelengths for standing waves are $\lambda = 2L/m$ (Equation 17.13), and the fundamental frequency corresponds to $m = 1$ (see Equation 17.15). Thus, the fundamental frequency is

$$f_{m=1} = \frac{mv}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\rho\sigma}} = \frac{1}{2(0.20 \text{ m})}\sqrt{\frac{(500 \text{ N})}{(1100 \text{ kg/m}^3)(90\times10^{-6} \text{ m}^2)}} = 177.7 \text{ Hz} \approx 180 \text{ Hz}$$

17.39. Model: Model the spider's silk as a string. The wavelength of the standing wave on a string vibrating at its equal to $2L$.
Solve: The linear density of the string is $\mu = \rho\sigma$, where $\rho = 1300 \text{ kg/m}^3$ and $\sigma = \pi r^2 = \pi(10\times10^{-6} \text{ m})^2 =$ $3.14\times10^{-10} \text{ m}^2$. To have a fundamental frequency (i.e., $m = 1$) at 100 Hz, Equation 17.14 gives

$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\rho\sigma}} \Rightarrow T = 4L^2 f_1^2 \rho\sigma = 4(0.12 \text{ m})^2(100 \text{ Hz})^2(1300 \text{ kg/m}^3)(3.14\times10^{-10} \text{ m}^2) = 2.4\times10^{-4} \text{ N}$$

Assess: This is a very small force, but remember that the spider web is *very* thin.

17.40. Model: The wave on a stretched string with both ends fixed is a standing wave. For a vibration at its fundamental frequency, $\lambda = 2L$.
Solve: The wavelength of the wave reaching your ear is $39.1 \text{ cm} = 0.391 \text{ m}$, so the frequency of the sound wave is

$$f = \frac{v_{\text{air}}}{\lambda} = \frac{344 \text{ m/s}}{0.391 \text{ m}} = 879.8 \text{ Hz}$$

This is also the frequency emitted by the wave on the string. Thus,

$$879.8 \text{ Hz} = \frac{v_{\text{string}}}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T_S}{\mu}} = \frac{1}{\lambda}\sqrt{\frac{150 \text{ N}}{0.0006 \text{ kg/m}}} \Rightarrow \lambda = 0.568 \text{ m}$$

$$L = \tfrac{1}{2}\lambda = 0.284 \text{ m} = 28.4 \text{ cm}$$

17.41. Model: The wave on a stretched string with both ends fixed is a standing wave.
Solve: We must distinguish between the sound wave in the air and the wave on the string. The listener hears a sound wave of wavelength $\lambda_{\text{sound}} = 40 \text{ cm} = 0.40 \text{ m}$. Thus, the frequency is

$$f = \frac{v_{\text{sound}}}{\lambda_{\text{sound}}} = \frac{343 \text{ m/s}}{0.40 \text{ m}} = 857.5 \text{ Hz}$$

The violin string oscillates at the same frequency, because each oscillation of the string causes one oscillation of the air. But the *wavelength* of the standing wave on the string is very different because the wave speed on the string is not the same as the wave speed in air. Bowing a string produces sound at the string's fundamental frequency, so the wavelength of the string is

$$\lambda_{\text{string}} = \lambda_1 = 2L = 0.60 \text{ m} \quad \Rightarrow \quad v_{\text{string}} = \lambda_{\text{string}} f = (0.60 \text{ m})(857.5 \text{ Hz}) = 514.5 \text{ m/s}$$

The tension is the string is found as follows:

$$v_{\text{string}} = \sqrt{\frac{T_S}{\mu}} \quad \Rightarrow \quad T_S = \mu(v_{\text{string}})^2 = (0.001 \text{ kg/m})(514.5 \text{ m/s})^2 = 260 \text{ N}$$

17.42. Model: The steel wire is under tension and it vibrates with three antinodes.
Solve: When the spring is stretched 8.0 cm, the standing wave on the wire has three antinodes. This means $\lambda_3 = \frac{2}{3}L$ and the tension T_S in the wire is $T_S = k(0.080 \text{ m})$, where *k* is the spring constant. For this tension,

$$v_{\text{wire}} = \sqrt{\frac{T_S}{\mu}} \quad \Rightarrow \quad f\lambda_3 = \sqrt{\frac{T_S}{\mu}} \quad \Rightarrow \quad f = \frac{3}{2L}\sqrt{\frac{k(0.08 \text{ m})}{\mu}}$$

We will let the stretching of the spring be Δx when the standing wave on the wire displays two antinodes. This means $\lambda_2 = L$ and $T_S' = kx$. For the tension T_S',

$$v_{\text{wire}}' = \sqrt{\frac{T_S'}{\mu}} \quad \Rightarrow \quad f\lambda_2 = \sqrt{\frac{T_S'}{\mu}} \quad \Rightarrow \quad f = \frac{1}{L}\sqrt{\frac{k\Delta x}{\mu}}$$

The frequency *f* is the same in the above two situations because the wire is driven by the same oscillating magnetic field. Now, equating the two frequency equations gives

$$\frac{1}{L}\sqrt{\frac{k\Delta x}{\mu}} = \frac{3}{2L}\sqrt{\frac{k(0.080 \text{ m})}{\mu}} \quad \Rightarrow \quad \Delta x = 0.18 \text{ m} = 18 \text{ cm}$$

17.43. Model: The wave on a stretched string with both ends fixed is a standing wave. Assume the pulley is frictionless, and ignore the effect on the string tension that is due to the mass of the string itself.
Solve: The linear density of the string is

$$\mu = \frac{5.00\times10^{-3} \text{ kg}}{2.50 \text{ m}} = 2.00\times10^{-3} \text{ kg/m}$$

The length of the vibrating part of the string is $L = 2.00$ m and the tension on it is $T = Mg$, where $M = 4.00$ kg is the mass of the hanging weight. Applying Equation 17.14 gives

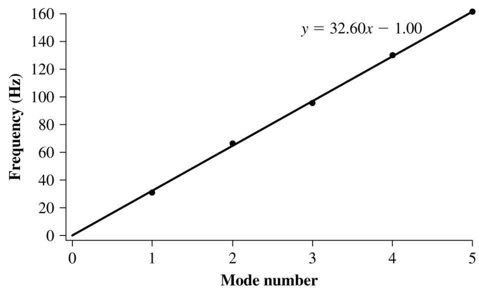
$$f_m = \frac{mv}{2L} = \frac{m}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2L}\sqrt{\frac{Mg}{\mu}} \cdot m$$

Thus a plot of the frequency f_m versus mode number *m* should give a straight line with slope given by

$$\text{slope} = \frac{1}{2L}\sqrt{\frac{Mg}{\mu}}$$

Therefore

$$g = \frac{\mu(2L\times\text{slope})^2}{M}$$

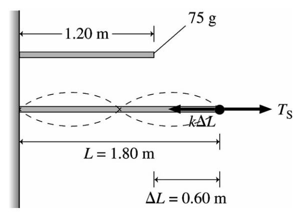


The linear fit is quite good. The experimental slope of 32.60 Hz gives

$$g = \frac{(2.00 \times 10^{-3}\ \text{kg/m})[2(2.00\ \text{m})(32.60\ \text{Hz})]^2}{4.00\ \text{kg}} = 8.50\ \text{m/s}^2$$

Assess: An acceleration due to gravity of 8.50 m/s^2 is close to that of the earth, so this might be an inviting place for astronauts to land.

17.44. Model: The stretched bungee cord that forms a standing wave with two antinodes is vibrating at the second harmonic frequency.
Visualize:



Solve: Because the vibrating cord has two antinodes, $\lambda_2 = L = 1.80$ m. The wave speed on the cord is

$$v_{\text{cord}} = f\lambda = (20\ \text{Hz})(1.80\ \text{m}) = 36\ \text{m/s}$$

The linear density of the cord is $v_{\text{cord}} = \sqrt{T_S/\mu}$. The tension T_S in the cord is equal to $k\Delta L$, where k is the bungee's spring constant and ΔL is the 0.60 m the bungee has been stretched. The linear density has to be calculated at the stretched length of 1.8 m where it is now vibrating. Thus,

$$v_{\text{cord}} = \sqrt{\frac{T_S}{\mu}} = \sqrt{\frac{k\Delta L}{m/L}} \Rightarrow k = \frac{mv_{\text{cord}}^2}{L\Delta L} = \frac{(0.075\ \text{kg})(36\ \text{m/s})^2}{(1.80\ \text{m})(0.60\ \text{m})} = 90\ \text{N/m}$$

17.45. Model: The wave on a stretched string with both ends fixed is a standing wave.
Solve: Use the fact that the wave speed v is $v = \sqrt{T/\mu}$ and apply Equation 17.14 to express both the fundamental frequency and the second-harmonic frequency:

$$f_1 = \frac{1}{2L}\sqrt{\frac{T_0}{\mu}}, \quad f_2 = \frac{2}{2L}\sqrt{\frac{T_0'}{\mu}}$$

For the second-harmonic frequency to be the same as the fundamental, we must have $f_1 = f_2$, which gives

$$\frac{1}{2L}\sqrt{\frac{T_0}{\mu}} = \frac{2}{2L}\sqrt{\frac{T_0'}{\mu}} \quad \Rightarrow \quad T_0' = \frac{T_0}{4}$$

Thus, the tension must be one-fourth its original value.
Assess: In the second case, the frequency of the fundamental decreased, which means the tension should have decreased, just as we found.

17.46. Visualize: Use primed quantities for when the sphere is submerged. We are given $f_5' = f_3$ and $M = 1.5$ kg. We also know the density of water is $\rho = 1000\ \text{kg/m}^3$. In the third mode before the sphere is submerged $L = \frac{3}{2}\lambda \Rightarrow \lambda = \frac{2}{3}L$. Likewise, after the sphere is submerged $L = \frac{5}{2}\lambda' \Rightarrow \lambda' = \frac{2}{5}L$. The tension in the string before the sphere is submerged is $T_s = Mg$, but after the sphere is submerged, according to Archimedes' principle, it is reduced by the weight of the water displaced by the sphere: $T_s' = Mg - \rho Vg$, where $V = \frac{4}{3}\pi R^3$.

Solve: We are looking for R so solve $T_s' = Mg - \rho Vg$ for ρVg and later we will isolate R from that.

$$\rho Vg = Mg - T_s'$$

Solve $v' = \sqrt{T_s/\mu}$ for T_s'. Also substitute for V.

$$\rho\left(\frac{4}{3}\right)\pi R^3 g = Mg - \mu v'^2$$

Now use $v' = \lambda' f'$.

$$\rho\left(\frac{4}{3}\right)\pi R^3 g = Mg - \mu(\lambda' f_5')^2$$

Recall that $f_5' = f_3$ and $\lambda' = \frac{2}{5}L$.

$$\rho\left(\frac{4}{3}\right)\pi R^3 g = Mg - \mu\left(\frac{2}{5}Lf_3\right)^2$$

Substitute $f_3 = v/\lambda$.

$$\rho\left(\frac{4}{3}\right)\pi R^3 g = Mg - \mu\left(\frac{2}{5}L\frac{v}{\lambda}\right)^2$$

Now use $\lambda = \frac{2}{3}L$ and $v = \sqrt{T_s/\mu}$.

$$\rho\left(\frac{4}{3}\right)\pi R^3 g = Mg - \mu\left(\frac{2}{5}L\frac{\sqrt{T_s/\mu}}{\frac{2}{3}L}\right)^2$$

The 2's, μ' s, and L's cancel.

$$\rho\left(\frac{4}{3}\right)\pi R^3 g = Mg - \left(\frac{3}{5}\sqrt{T_s}\right)^2$$

Recall that $T_s = Mg$.

$$\rho\left(\frac{4}{3}\right)\pi R^3 g = Mg - \left(\frac{3}{5}\right)^2 Mg$$

Cancel g and factor M out on the right side.

$$\rho\left(\frac{4}{3}\right)\pi R^3 = M\left[1 - \left(\frac{3}{5}\right)^2\right] = M\left(1 - \frac{9}{25}\right) = M\left(\frac{16}{25}\right)$$

Now solve for R.

$$R^3 = \left(\frac{3}{4}\right)\left(\frac{16}{25}\right)\frac{M}{\pi\rho}$$

$$R = \sqrt[3]{\frac{12}{25}\frac{M}{\pi\rho}} = \sqrt[3]{\frac{12}{25}\frac{(1.5\ \text{kg})}{\pi(1000\ \text{kg/m}^3)}} = 6.1\ \text{cm}$$

Assess: The density of the sphere turns out to be about 1.5 times the density of water, which means it sinks and is in a reasonable range for densities.

17.47. Model: Model the piano string as a damped harmonic oscillator.
Visualize: Recall from the chapter on damped harmonic oscillators that the amplitude decreases as $A(t) = A_0 e^{-t/2\tau}$. And use $I \propto A^2$.
Solve: We are told the intensity is proportional to the amplitude squared.

$$\Delta\beta = \beta_f - \beta_i = (10\text{ dB})(\log(I_f/I_0) - \log(I_i/I_0)) = (10\text{ dB})\log\left(\frac{I_f/I_0}{I_i/I_0}\right)$$

$$= (10\text{ dB})\log\left(\frac{I_f}{I_i}\right) = (10\text{ dB})\log\left(\frac{A_f}{A_i}\right)^2 = (20\text{ dB})\log\left(\frac{A_f}{A_i}\right) = (20\text{ dB})\log(e^{-t/2\tau})$$

Solve for τ.

$$10^{\Delta\beta/(20\text{ dB})} = e^{-t/2\tau} \Rightarrow \ln(10^{\Delta\beta/(20\text{ dB})}) = -t/2\tau \Rightarrow$$

$$\tau = \frac{-t}{2\ln(10^{\Delta\beta/(20\text{ dB})})} = \frac{-t}{2\left(\frac{\Delta\beta}{20\text{ dB}}\right)\ln(10)} = \frac{-(1.0\text{ s})}{2\left(\frac{-8\text{ dB}}{20\text{ dB}}\right)\ln(10)} = 0.54\text{ s}$$

Assess: This seems reasonable for a piano string.

17.48. Visualize: First compute the length L of the wire from the Pythagorean theorem. $L = 2.0\sqrt{2}$ m. Now $\mu = M/L = (0.075\text{ kg})/(2.0\sqrt{2}\text{ m}) = 0.02652\text{ kg/m}$. Also, in the fundamental mode $\lambda = 2L$; here $\lambda = 4{,}0\sqrt{2}\text{ m} = 5.657\text{ m}$.
Solve: To find the tension in the cable, recall that the net torque about any point in the system must be zero in a static situation. Taking the sum of the torques about the point where the horizontal bar joins the wall gives

$$\Sigma\tau = \frac{mgd}{2} - T_s d\sin\theta + Mgd = 0\text{ N} \Rightarrow \quad T_s = \frac{mg/2 + Mg}{\sin\theta} = \frac{(4.0\text{ kg})/2 + 8.0\text{ kg}}{\sin(45°)}(9.8\text{ m/s}^2) = 138.6\text{ N}$$

Use these values for λ, μ, and T_s to find f.

$$f = \frac{v}{\lambda} = \frac{\sqrt{T_s/\mu}}{\lambda} = \frac{\sqrt{(138.6\text{ N})/(0.02652\text{ kg/m})}}{5.657\text{ m}} = 13\text{ Hz}$$

Assess: This seems like a reasonable frequency for a mechanical system like this.

17.49. Model: Assume that the spring provides the same tension to both strings and also acts as a fixed point for the end of each string so an integral number of half wavelengths fit in each string.
Visualize: Use a subscript L for the left string and R for the right string. From the assumption above we know $(T_s)_L = (T_s)_R = T_s$. We also know $f_L = f_R = f$ and $L_L = L_R = L$. Notice from the diagram that $\lambda_L = L$ and $\lambda_R = \frac{2}{3}L$.
Solve: From $v = \lambda f$ and $v = \sqrt{T_s/\mu}$ eliminate v and solve for μ: $\mu = T/(\lambda f)^2$. Take the ratio of the linear densities to find μ_R:

$$\frac{\mu_R}{\mu_L} = \frac{(T_s)_R/(\lambda_R f_R)^2}{(T_s)_L/(\lambda_L f_L)^2} = \frac{T_s/(\frac{2}{3}Lf)^2}{T_s/(Lf)^2} = \frac{1}{\left(\frac{2}{3}\right)^2} = \frac{9}{4} \quad \Rightarrow \quad \mu_R = \mu_L\frac{9}{4} = \frac{9}{4}(5.0\text{ g/m}) = 11\text{ g/m}$$

Assess: We expect a slower wave speed in the right string to correspond to a larger mass density.

17.50. Model: Assume equal temperament.
Visualize: Use a subscript L for the left string and R for the right string. From the assumption above we know $(T_s)_L = (T_s)_R = T_s$. We also know $f_L = f_R = f$ and $L_L = L_R = L$. Notice from the diagram that $\lambda_L = L$ and $\lambda_R = \frac{2}{3}L$.
Solve: **(a)** Use the information given and that after 12 notes the frequency is doubled.

$$f_{n+1} = rf_n \Rightarrow f_{n+12} = r^{12} f_n = 2f_n \Rightarrow r = 2^{1/12} = \sqrt[12]{2}$$

(b) The A-sharp just above the tuning frequency will be

$$f_{A\#} = 2^{1/12} f_A = 2^{1/12}(440 \text{ Hz}) = 466 \text{ Hz}$$

Assess: Historically there have been other tuning systems where not all half-step intervals were exactly the same; this made music sound different in different keys.

17.51. Model: The fundamental wavelength of an open-open tube is $2L$ and that of an open-closed tube is $4L$.
Solve: We are given that

$$f_{1\text{ open-closed}} = f_{3\text{ open-open}} = 3f_{1\text{ open-open}}$$

$$\frac{v_{\text{air}}}{\lambda_{1\text{ open-closed}}} = 3\frac{v_{\text{air}}}{\lambda_{1\text{ open-open}}} \quad \Rightarrow \quad \frac{1}{4L_{\text{open-closed}}} = \frac{3}{2L_{\text{open-open}}}$$

$$L_{\text{open-closed}} = \frac{2L_{\text{open-open}}}{12} = \frac{2(78.0 \text{ cm})}{12} = 13.0 \text{ cm}$$

17.52. Model: Model the vocal tract as an open-closed tube that forms standing waves.
Solve: The standing-wave frequencies in an open-closed tube are proportional to the wave speed (see Equation 17.18). Therefore the new frequencies will be

$$f_{m_1} = (270 \text{ Hz})\frac{750 \text{ m/s}}{350 \text{ m/s}} = 580 \text{ Hz}, \quad f_{m_2} = (2300 \text{ Hz})\frac{750 \text{ m/s}}{350 \text{ m/s}} = 4.9 \text{ kHz}$$

Assess: The sound becomes higher in pitch with the helium-oxygen mixture, just as is often observed in demonstrations of this effect.

17.53. Model: Particles of the medium at the nodes of a standing wave have zero displacement.
Solve: The cork dust settles at the nodes of the sound wave where there is no motion of the air molecules. The separation between the centers of two adjacent piles is $\frac{1}{2}\lambda$. Thus,

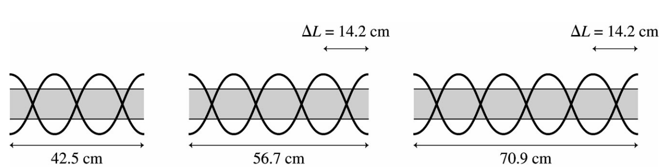
$$\frac{123 \text{ cm}}{3} = \frac{\lambda}{2} \quad \Rightarrow \quad \lambda = 82.0 \text{ cm}$$

Because the piston is driven at a frequency of 400 Hz, the speed of the sound wave in oxygen is

$$v = f\lambda = (400 \text{ Hz})(0.820 \text{ m}) = 328 \text{ m/s}$$

Assess: A speed of 328 m/s in oxygen is close to the speed of sound in air, which is 343 m/s at 20°C.

17.54. Model: The nodes of a standing wave are spaced $\lambda/2$ apart. Assume there are no standing-wave modes between those given.
Visualize:



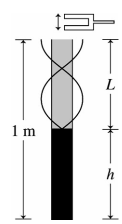
Solve: The wavelength of the *m*th mode of an open-open tube is $\lambda_m = 2L/m$. Or, equivalently, the length of the tube that generates the *m*th mode is $L = m(\lambda/2)$. Here λ is the same for all modes because the frequency of the tuning fork is unchanged. Increasing the length of the tube to go from mode *m* to mode $m+1$ requires a length change

$$\Delta L = (m+1)(\lambda/2) - m\lambda/2 = \lambda/2$$

That is, lengthening the tube by $\lambda/2$ adds an additional antinode and creates the next standing wave. This is consistent with the idea that the nodes of a standing wave are spaced $\lambda/2$ apart. This tube is first increased $\Delta L = 56.7\text{ cm} - 42.5\text{ cm} = 14.2\text{ cm}$, then by $\Delta L = 70.9\text{ cm} - 56.7\text{ cm} = 14.2\text{ cm}$. Thus $\lambda/2 = 14.2\text{ cm}$ and thus $\lambda = 28.4\text{ cm} = 0.284\text{ m}$. Therefore the frequency of the tuning fork is

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{0.284\text{ m}} = 1208\text{ Hz} \approx 12.1\text{ kHz}$$

17.55. Model: The open-closed tube forms standing waves.
Visualize:



Solve: When the air column length *L* is the proper length for a 580 Hz standing wave, a standing-wave resonance will be created and the sound will be loud. From Equation 17.18, the standing-wave frequencies of an open-closed tube are $f_m = m(v/4L)$, where *v* is the speed of sound in air and *m* is an *odd* integer: $m = 1, 3, 5, \ldots$. The frequency is fixed at 580 Hz, but as the length *L* changes, 580 Hz standing waves will occur for different values of *m*. The length that causes the *m*th standing-wave mode to be at 580 Hz is

$$L = \frac{m(343\text{ m/s})}{(4)(580\text{ Hz})}$$

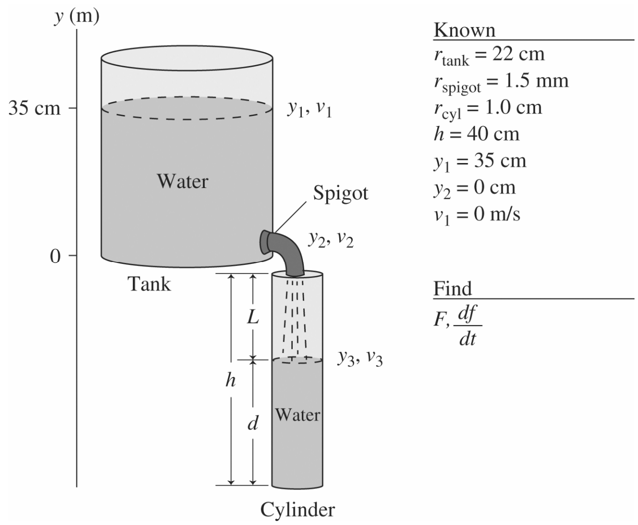
We can place the values of *L*, and corresponding values of $h = 1.0\text{ m} - L$, in a table:

m	L	$h = 1.0\text{ m} - L$
1	0.148 m	$0.852\text{ m} = 85.2\text{ cm}$
3	0.444 m	$0.556\text{ m} = 55.6\text{ cm}$
5	0.739 m	$0.261\text{ m} = 26.1\text{ cm}$
7	1.035 m	*h* can't be negative

So water heights of 26, 56, and 85 cm will cause a standing-wave resonance at 580 Hz. The figure shows the $m = 3$ standing wave at $h = 56$ cm.

17.56. Model: Assume the water is an ideal fluid that obeys Bernoulli's equation. Assume room temperature so the speed of sound in air is $v_s = 343$ m/s. The cylinder filling with water will act like a column of air open at one end and closed at the other.

Visualize:



Solve: **(a)** Use Bernoulli's equation to find the speed of the water exiting the spigot at the bottom of the tank.

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

Both pressures are atmospheric so they subtract from both sides. The speed of the water at the top of the tank is zero.

$$\rho g y_1 = \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \Rightarrow v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.8\ \text{m/s}^2)(0.35\ \text{m})} = 2.62\ \text{m/s}$$

We'll also use the equation of continuity.

$$v_2 A_2 = v_3 A \Rightarrow v_3 = v_2 \frac{A_2}{A_3} = v_2 \frac{r_{\text{spigot}}^2}{r_{\text{cyl}}^2} = (2.62\ \text{m/s})\frac{(1.5\ \text{mm})^2}{(1.0\ \text{cm})^2} = 0.0589\ \text{m/s}$$

Now we recall the frequency of an open-closed pipe of length *l*. Refer to the diagram above and note $d = v_3 t$.

$$f = \frac{v_s}{4l} = \frac{v_s}{4(h-d)} = \frac{v_s}{4(h - v_3 t)}$$

Evaluate this at $t = 4.0$ s.

$$f = \frac{v_s}{4(h - v_3 t)} = \frac{343\ \text{m/s}}{4(0.40\ \text{m} - (0.0589\ \text{m/s})(4.0\ \text{s}))} = 522\ \text{Hz}$$

Which we report to two significant figures as 520 Hz.
(b) Now we take the derivative of the expression for frequency to find the rate at which the frequency is changing.

$$\frac{df}{dt} = \frac{d}{dt}\left(\frac{v_s}{4}(h - v_3 t)^{-1}\right) = -\frac{v_s}{4}(h - v_3 t)^{-2}(-v_3) = \frac{v_s v_3}{4(h - v_3 t)^2}$$

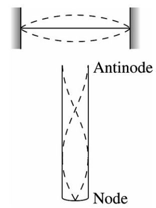
Evaluate this at $t = 4.0$ s.

$$\frac{df}{dt} = \frac{v_s v_3}{4(h - v_3 t)^2} = \frac{(343\ \text{m/s})(0.0589\ \text{m/s})}{4(0.40\ \text{m} - (0.0589\ \text{m/s})(4.0\ \text{s}))^2} = 190\ \text{Hz/s}$$

Assess: We did not need the radius of the tank except to know it was big enough so that $v_1 \approx 0$.

17.57. Model: A stretched wire, which is fixed at both ends, creates a standing wave whose fundamental frequency is $f_{1\ \text{wire}}$. The second vibrational mode of an open-closed tube is $f_{3\ \text{open-closed}}$. These two frequencies are equal because the wire's vibrations generate the sound wave in the open-closed tube.

Visualize:



Solve: The frequency in the tube is

$$f_{3\text{ open-closed}} = \frac{3v_{\text{air}}}{4L_{\text{tube}}} = \frac{3(340\text{ m/s})}{4(0.85\text{ cm})} = 300\text{ Hz}$$

$$f_{1\text{ wire}} = 300\text{ Hz} = \frac{v_{\text{wire}}}{2L_{\text{wire}}} = \frac{1}{2L_{\text{wire}}}\sqrt{\frac{T_S}{\mu}}$$

$$T_S = (300\text{ Hz})^2(2L_{\text{wire}})^2\mu = (300\text{ Hz})^2(2\times 0.25\text{ m})^2(0.020\text{ kg/m}) = 450\text{ N}$$

17.58. Model: Model the tunnel as an open-closed tube.

Visualize: We are given $v = 335$ m/s. We would like to use $f_m = m\dfrac{v}{4L}(m = \text{odd})$ to find *L*, but we need to know m first. Since m takes on only odd values for the open-closed tube the next resonance after m is $m+2$. We are given $f_m = 4.5$ Hz and $f_{m+2} = 6.3$ Hz.

Solve:

$$\frac{f_{m+2}}{f_m} = \frac{(m+2)\frac{v}{4L}}{(m)\frac{v}{4L}} = \frac{m+2}{m} \quad\Rightarrow\quad m\left(\frac{f_{m+2}}{f_m}\right) = m+2 \quad\Rightarrow\quad m\left(\frac{f_{m+2}}{f_m} - 1\right) = 2 \quad\Rightarrow\quad m = \frac{2}{\frac{f_{m+2}}{f_m} - 1} = \frac{2}{\frac{6.3\text{ Hz}}{4.5\text{ Hz}} - 1} = 5$$

Now that we know m we can find the length *L* of the tunnel.

$$f_m = m\frac{v}{4L} \quad\Rightarrow\quad L = m\frac{v}{4f_m} = (5)\frac{335\text{ m/s}}{4(4.5\text{ Hz})} = 93\text{ m}$$

Assess: 93 m seems like a reasonable length for a tunnel.

17.59. Model: The amplitude is determined by the interference of the two waves.

Solve: For interference in one dimension, where the speakers are separated by a distance Δx, the amplitude of the net wave is $A = 2a\cos\left(\frac{1}{2}\Delta\phi\right)$, where a is the amplitude of each wave and $\Delta\phi = 2\pi\Delta x/\lambda + \Delta\phi_0$ is the phase difference between the two waves. The speakers are emitting identical waves so they have identical phase constants, so $\Delta\phi_0 = 0$. Thus,

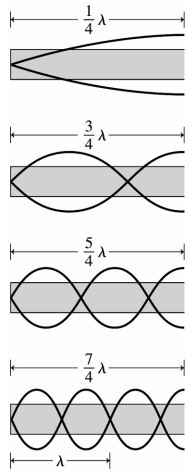
$$A = 1.5a = 2a\cos\left(\frac{\pi\Delta x}{\lambda}\right) \quad\Rightarrow\quad \Delta x = \frac{\lambda}{\pi}\cos^{-1}\left(\frac{1.5}{2}\right)$$

The wavelength of a 1000 Hz tone is $\lambda = v_{\text{sound}}/f = 0.343$ m. Thus the separation must be

$$\Delta x = \frac{0.343\text{ m}}{\pi}\cos^{-1}(0.75) = 0.0789\text{ m} \approx 7.9\text{ cm}$$

It is essential to note that the argument of the arccosine is in radians, *not* in degrees.

17.60. Model: A standing wave in an open-closed tube must have a node at the closed end of the tube and an antinode at the open end.
Visualize:



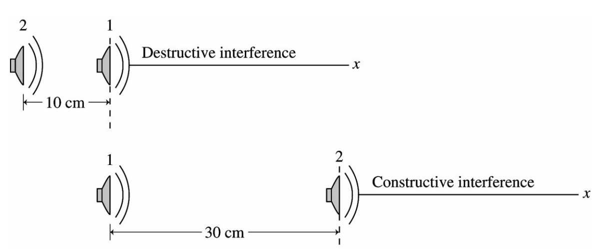
Solve: We first draw a series of pictures showing all the possible standing waves. By examination, we see that the first standing wave mode is $\frac{1}{4}$ of a wavelength, so the tube's length is $L = \frac{1}{4}\lambda$. The next mode is $\frac{3}{4}$ of a wavelength. The tube's length hasn't changed, so in this mode $L = \frac{3}{4}\lambda$. The next mode is now slightly more than a wavelength: $L = \frac{5}{4}\lambda$. The next mode is $\frac{7}{4}$ of a wavelength, so $L = \frac{7}{4}\lambda$. We see that there is a pattern. The length of the tube and the possible standing-wave wavelengths are related by

$$L = \frac{m\lambda}{4} \text{ where } m = 1, 3, 5, 7, \ldots = \text{odd integers}$$

Solving for λ, we find that the wavelengths and frequencies of standing waves in an open-closed tube are

$$\begin{cases} \lambda_m = \dfrac{4L}{m} \\ f_m = \dfrac{v}{\lambda_m} = m\dfrac{v}{4L} \end{cases} \quad m = 1, 3, 5, 7, \ldots = \text{ odd integers}$$

17.61. Model: Constructive or destructive interference occurs according to the phases of the two waves.
Visualize:



Solve: (a) To go from destructive to constructive interference requires moving the speaker $\Delta x = \frac{1}{2}\lambda$, which is equivalent to a phase change of π rad. Since $\Delta x = 40$ cm, we find $\lambda = 80$ cm.

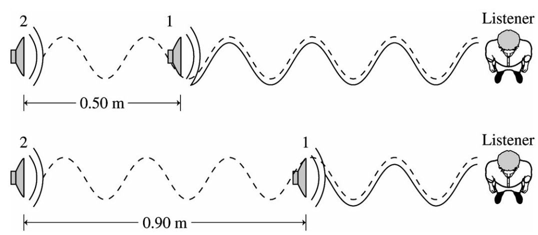
(b) Destructive interference at $\Delta x = 10$ cm requires

$$2\pi\frac{\Delta x}{\lambda}+\Delta\phi_0 = 2\pi\left(\frac{10\text{ cm}}{80\text{ cm}}\right)\text{ rad}+\Delta\phi_0 = \pi\text{ rad} \quad\Rightarrow\quad \Delta\phi_0 = \frac{3\pi}{4}\text{ rad}$$

(c) When side by side, with $\Delta x = 0$, the phase difference is $\Delta\phi = \Delta\phi_0 = 3\pi/4$ rad. The amplitude of the superposition of the two waves is

$$a = \left|2a\cos\left(\frac{\Delta\phi}{2}\right)\right| = \left|2a\cos\frac{3\pi}{8}\right| = 0.77a$$

17.62. Model: Interference occurs according to the difference between the phases of the two waves.
Visualize:



Solve: (a) The phase difference between the sound waves from the two speakers is

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda}+\Delta\phi_0$$

We have a maximum intensity when $\Delta x = 0.50$ m and $\Delta x = 0.90$ m. This means

$$2\pi\frac{(0.50\text{ m})}{\lambda}+\Delta\phi_0 = 2m\pi\text{ rad} \qquad 2\pi\left(\frac{0.90\text{ m}}{\lambda}\right)+\Delta\phi_0 = 2(m+1)\pi\text{ rad}$$

Taking the difference of these two equations gives

$$2\pi\left(\frac{0.40\text{ m}}{\lambda}\right) = 2\pi \quad\Rightarrow\quad \lambda = 0.40\text{ m} \quad\Rightarrow\quad f = \frac{v_{\text{sound}}}{\lambda} = \frac{340\text{ m/s}}{0.40\text{ m}} = 850\text{ Hz}$$

(b) Using again the equations that correspond to constructive interference, we find

$$2\pi\left(\frac{0.50\text{ m}}{0.40\text{ m}}\right)+\Delta\phi_0 = 2m\pi\text{ rad} \quad\Rightarrow\quad \Delta\phi_0 = \phi_{20}-\phi_{10} = -\frac{\pi}{2}\text{rad}$$

We have taken $m = 1$ in the last equation. This is because we always specify phase constants in the range $-\pi$ rad to π rad (or 0 rad to 2π rad). $m = 1$ gives $-\frac{1}{2}\pi$ rad (or equivalently, $m = 2$ will give $\frac{3}{2}\pi$ rad).

17.63. Model: Reflection is maximized for constructive interference of the two reflected waves, but minimized for destructive interference.
Solve: (a) Constructive interference of the reflected waves occurs for wavelengths given by Equation 17.32:

$$\lambda_m = \frac{2nd}{m} = \frac{2(1.42)(500\text{ nm})}{m} = \frac{(1420\text{ nm})}{m}$$

Thus, $\lambda_1 = 1420$ nm, $\lambda_2 = \frac{1}{2}(1420\text{ nm}) = 710$ nm, $\lambda_3 = 473$ nm, $\lambda_4 = 355$ nm, …. Only the wavelength of 473 nm is in the visible range.
(b) For destructive interference of the reflected waves,

$$\lambda = \frac{2nd}{m-\frac{1}{2}} = \frac{2(1.42)(500\text{ nm})}{m-\frac{1}{2}} = \frac{1420\text{ nm}}{m-\frac{1}{2}}$$

Thus, $\lambda_1 = 2\times1420 \text{ nm} = 2840 \text{ nm}$, $\lambda_2 = \frac{2}{3}(1420 \text{ nm}) = 947 \text{ nm}$, $\lambda_3 = 568 \text{ nm}$, $\lambda_4 = 406 \text{ nm}, \ldots$

The wavelengths of 406 nm and 568 nm are in the visible range.
(c) Beyond the limits 430 nm and 690 nm the eye's sensitivity drops to about 1 percent of its maximum value. The reflected light is enhanced in blue (473 nm). The transmitted light at mostly 568 nm will be yellowish green.

17.64. Model: Reflection is minimized when the two reflected waves interfere destructively.
Solve: Equation 17.23 gives the condition for perfect destructive interference between the two waves:

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = 2\left(m+\tfrac{1}{2}\right)\pi \text{ rad}$$

The wavelength of the sound is

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{1200 \text{ Hz}} = 0.2858 \text{ m}$$

Let d be the separation between the mesh and the wall. Substituting $\Delta\phi_0 = 0$ rad, $\Delta x = 2d$, $m = 0$, and the above value for the wavelength,

$$\frac{2\pi(2d)}{0.2858 \text{ m}} + 0 \text{ rad} = \pi \text{ rad} \quad\Rightarrow\quad d = \frac{0.2858 \text{ m}}{4} = 0.0715 \text{ m} \approx 7.2 \text{ cm}$$

17.65. Model: A light wave that reflects from a boundary at which the index of refraction increases has a phase shift of π rad.
Solve: (a) Because $n_{\text{film}} > n_{\text{air}}$, the wave reflected from the outer surface of the film (called 1) is inverted due to the phase shift of π rad. The second reflected wave does not go through any phase shift of π rad because the index of refraction decreases at the boundary where this wave is reflected, which is on the inside of the soap film. We can write for the phases

$$\phi_1 = kx_1 + \phi_{10} + \pi \text{ rad} \qquad \phi_2 = kx_2 + \phi_{20} + 0 \text{ rad}$$

$$\Delta\phi = \phi_2 - \phi_1 = k(x_2 - x_1) + (\phi_{20} - \phi_{10}) - \pi \text{ rad} = k\Delta x + \Delta\phi_0 - \pi \text{ rad} = k\Delta x - \pi \text{ rad}$$

$\Delta\phi_0 = 0$ rad because the sources are identical. For constructive interference,

$$\Delta\phi = 2m\pi \text{ rad} \quad\Rightarrow\quad k\Delta x - \pi \text{ rad} = 2m\pi \text{ rad} \quad\Rightarrow\quad \left(\frac{2\pi}{\lambda_{\text{film}}}\right)(2d) = (2m+1)\pi \text{ rad}$$

$$\lambda_{\text{film}} = \frac{\lambda_C}{n} = \frac{2d}{m+\frac{1}{2}} \quad\Rightarrow\quad \lambda_C = \frac{2nd}{m+\frac{1}{2}} = \frac{2.66d}{m+\frac{1}{2}} \qquad m = 0, 1, 2, 3, \ldots$$

(b) For $m = 0$ the wavelength for constructive interference is

$$\lambda_C = \frac{(2.66)(390 \text{ nm})}{\left(\frac{1}{2}\right)} = 2075 \text{ nm}$$

For $m = 1$ and 2, $\lambda_C = 692 \text{ nm (\sim red)}$ and $\lambda_C = 415 \text{ nm (\sim violet)}$. Red and violet together give a purplish color.

17.66. Model: We do not assume the m is the same value for the constructive interference with the violet light as for the destructive interference with the red light. Assume that the wavelengths given are the wavelengths in the film.
Visualize: We are given $\lambda_D = 640 \text{ nm (red)}$ and $\lambda_C = 400 \text{ nm (violet)}$.
Solve: For constructive and destructive interference in a thin film we have, respectively,

$$\lambda_C = \frac{2nd}{m_C} \qquad \lambda_D = \frac{2nd}{m_D - \frac{1}{2}}$$

We have two equations but three unknowns: $(n,\ m_C,\ m_D)$, so we may have to try to guess the *m*s or find them by trial and error. A good way is to use ratios to see if we are guided to what the *m*s are.

$$\frac{\lambda_D}{\lambda_C}=\frac{\dfrac{2nd}{m_D-\frac{1}{2}}}{\dfrac{2nd}{m_C}}=\frac{m_C}{m_D-\frac{1}{2}}=\frac{640\ \text{nm}}{400\ \text{nm}}=1.6$$

We seek two small whole numbers m_C and m_D such that

$$\frac{m_C}{m_D-\frac{1}{2}}=1.6=\frac{8}{5}$$

Trial and error with small whole numbers suggests that $m_C=4$ and $m_D=3$. Plug $m_C=4$ into the first equation to find *n*.

$$\lambda_C=\frac{2nd}{m_C}\Rightarrow n=\frac{\lambda_C m_C}{2d}=\frac{(400\ \text{nm})(4)}{2(560\ \text{nm})}=1.4$$

Assess: This is a reasonable index of refraction, and less than most types of glass.

17.67. Model: We do not assume the *m* is the same value for the constructive interference with the violet light as for the destructive interference with the red light. Assume that the wavelengths given are the wavelengths in the film.
Visualize: We are given $d=295$ nm.
Solve: For constructive interference in a thin film we have

$$\lambda_C=\frac{2nd}{m}=\frac{2\left(\dfrac{30.0\ \text{nm}^{1/2}}{\sqrt{\lambda_C}}\right)(295\ \text{nm})}{m}\Rightarrow\lambda_C^{3/2}=\frac{2(30.0\ \text{nm}^{1/2})(295\ \text{nm})}{m}$$

$$\Rightarrow\lambda_C=\left(\frac{2(30.0\ \text{nm}^{1/2})(295\ \text{nm})}{m}\right)^{2/3}$$

Now try cases for different orders.

$$m=1\qquad\lambda_C=\left(\frac{2(30.0\ \text{nm}^{1/2})(295\ \text{nm})}{1}\right)^{2/3}=679\ \text{nm}$$

$$m=2\qquad\lambda_C=\left(\frac{2(30.0\ \text{nm}^{1/2})(295\ \text{nm})}{2}\right)^{2/3}=428\ \text{nm}$$

$$m=3\qquad\lambda_C=\left(\frac{2(30.0\ \text{nm}^{1/2})(295\ \text{nm})}{3}\right)^{2/3}=326\ \text{nm}$$

Only the first two are in the visible range, so the answers are 679 nm and 428 nm.
Assess: The indices of refraction are greater than 1 and less than *n* for glass over the visible range.

17.68. Model: The changing sound intensity is due to the interference of two overlapped sound waves.
Solve: Minimum intensity implies destructive interference. Destructive interference occurs where the path length difference for the two waves is $\Delta r=\left(m+\frac{1}{2}\right)\lambda.$ We assume $\Delta\phi_0=0$ rad for two speakers playing "exactly the same" tone. The wavelength of the sound is $\lambda=v_{\text{sound}}/f=(343\ \text{m/s})/686\ \text{Hz}=0.500\ \text{m}.$ Consider a point that is a distance *x* in front of the top speaker. Let r_1 be the distance from the top speaker to the point and r_2 the distance from the bottom speaker to the point. We have

$$r_1=x\qquad r_2=\sqrt{x^2+(3.0\ \text{m})^2}$$

Destructive interference occurs at distances x such that

$$\Delta r = \sqrt{x^2 + 9.0\ \text{m}^2} - x = \left(m + \tfrac{1}{2}\right)\lambda$$

To solve for x, isolate the square root on one side of the equation and then square:

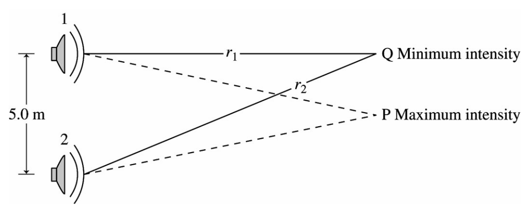
$$x^2 + 9.0\ \text{m} = \left[x + \left(m + \tfrac{1}{2}\right)\lambda\right]^2 = x^2 + 2\left(m + \tfrac{1}{2}\right)\lambda x + \left(m + \tfrac{1}{2}\right)^2\lambda^2 \quad \Rightarrow \quad x = \frac{9.0\ \text{m} - \left(m + \tfrac{1}{2}\right)^2\lambda^2}{2\left(m + \tfrac{1}{2}\right)\lambda}$$

Evaluating x for different values of m:

m	x (m)
0	17.88
1	5.62
2	2.98
3	1.79

Because you start at $x = 2.5$ m and walk *away* from the speakers, you will only hear minima for values $x > 2.5$ m. Thus, to correct significant figures, minima will occur at distances of 3.0 m, 5.6 m, and 18 m.

17.69. Model: The changing sound intensity is due to the interference of two overlapped sound waves.
Visualize: The listener moving relative to the speakers changes the phase difference between the waves.

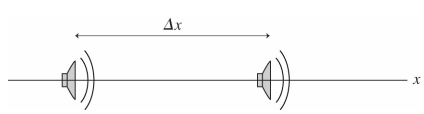


Solve: Initially when you are at P, equidistant from the speakers, you hear a sound of maximum intensity. This implies that the two speakers are in phase $(\Delta\phi_0 = 0)$. However, upon moving to Q you hear a minimum of sound intensity, which implies that the path length difference from the two speakers to Q is $\lambda/2$. Thus,

$$\tfrac{1}{2}\lambda = \Delta r = r_2 - r_1 = \sqrt{(r_1)^2 + (5.0\ \text{m})^2} - r_1 = \sqrt{(12.0\ \text{m})^2 + (5.0\ \text{m})^2} - 12.0\ \text{m} = 1.0\ \text{m}$$

$$\lambda = 2.0\ \text{m} \quad \Rightarrow \quad f = \frac{v}{\lambda} = \frac{340\ \text{m/s}}{2.0\ \text{m}} = 170\ \text{Hz}$$

17.70. Model: The amplitude is determined by the interference of the two waves.
Visualize:



Solve: The amplitude of the sound wave is $A = \left|2a\cos\left(\tfrac{1}{2}\Delta\phi\right)\right|$. We are to find the minimum ratio $\Delta x/\lambda$ at which $A = a$. Because the speakers are identical, we may take $\Delta\phi_0 = 0$.

$$a = 2a\cos\left(\frac{\Delta\phi}{2}\right) \quad \Rightarrow \quad \Delta\phi = 2\cos^{-1}\left(\tfrac{1}{2}\right) = 2\pi\frac{\Delta x}{\lambda} \quad \Rightarrow \quad \frac{\Delta x}{\lambda} = \frac{1}{\pi}\cos^{-1}\left(\tfrac{1}{2}\right) = \frac{1}{3}$$

17.71. Model: The amplitude is determined by the interference of the three waves.

Solve: (a) We have three identical loudspeakers as sources. Δr between speakers 1 and 2 is 1.0 m and $\lambda = 2.0$ m. Thus $\Delta r = \frac{1}{2}\lambda$, which gives perfect destructive interference for in-phase sources. That is, the interference of the waves from loudspeakers 1 and 2 is perfect destructive, leaving only the contribution due to speaker 3. Thus the amplitude is a.

(b) If loudspeaker 2 is moved to the left by one-half of a wavelength or 1.0 m, then all three waves will reach you in phase. The amplitude of the superposed waves will therefore be maximum and equal to $A = 3a$.

(c) The maximum intensity is $I_{\text{max}} = CA^2 = 9Ca^2$. The ratio of the intensity to the intensity of a single speaker is

$$\frac{I_{\text{max}}}{I_{\text{single speaker}}} = \frac{9Ca^2}{Ca^2} = 9$$

17.72. Model: The superposition of two slightly different frequencies gives rise to beats.

Solve: The third harmonic of note A and the second harmonic of note E are

$$f_{3\text{A}} = 3f_{1\text{A}} = 3(440 \text{ Hz}) = 1320 \text{ Hz} \qquad f_{2\text{E}} = 2f_{1\text{E}} = 2(659 \text{ Hz}) = 1318 \text{ Hz}$$

The beat frequency is therefore

$$f_{3\text{A}} - f_{2\text{E}} = 1320 \text{ Hz} - 1318 \text{ Hz} = 2 \text{ Hz}$$

(b) The beat frequency between the first harmonics is

$$f_{1\text{E}} - f_{1\text{A}} = 659 \text{ Hz} - 440 \text{ Hz} = 219 \text{ Hz}$$

The beat frequency between the second harmonics is

$$f_{2\text{E}} - f_{2\text{A}} = 1318 \text{ Hz} - 880 \text{ Hz} = 438 \text{ Hz}$$

The beat frequency between $f_{3\text{A}}$ and $f_{2\text{E}}$ is 2 Hz. Thus, the tuner must listen for a beat frequency of 2 Hz.

(c) If the beat frequency is 4 Hz, then the second harmonic frequency of the E string is

$$f_{2\text{E}} = 1320 \text{ Hz} - 4 \text{ Hz} = 1316 \text{ Hz} \quad \Rightarrow \quad f_{1\text{E}} = \tfrac{1}{2}(1316 \text{ Hz}) = 658 \text{ Hz}$$

Note that the second harmonic frequency of the E string could also be

$$f_{2\text{E}} = 1320 \text{ Hz} + 4 \text{ Hz} = 1324 \text{ Hz} \quad \Rightarrow \quad f_{1\text{E}} = 662 \text{ Hz}$$

This higher frequency can be ruled out because the tuner started with low tension in the E string and we know that

$$v_{\text{string}} = \lambda f = \sqrt{\frac{T}{\mu}} \quad \Rightarrow \quad f \propto \sqrt{T}$$

17.73. Model: The superposition of two slightly different frequencies creates beats.

Solve: (a) The wavelength of the sound initially created by the flutist is

$$\lambda = \frac{342 \text{ m/s}}{440 \text{ Hz}} = 0.77727 \text{ m}$$

When the speed of sound inside her flute has increased due to the warming up of the air, the new frequency of the A note is

$$f' = \frac{346 \text{ m/s}}{0.77727 \text{ m}} = 445 \text{ Hz}$$

Thus the flutist will hear beats at the following frequency:

$$f' - f = 445 \text{ Hz} - 440 \text{ Hz} = 5 \text{ beats/s}$$

Note that the wavelength of the A note is determined by the length of the flute rather than the temperature of air or the increased sound speed.

(b) The initial length of the flute is $L = \frac{1}{2}\lambda = \frac{1}{2}(0.77727\text{ m}) = 0.3886\text{ m}$. The new length to eliminate beats needs to be

$$L' = \frac{\lambda'}{2} = \frac{1}{2}\left(\frac{v'}{f}\right) = \frac{1}{2}\left(\frac{346\text{ m/s}}{440\text{ Hz}}\right) = 0.3932\text{ m}$$

Thus, she will have to extend the "tuning joint" of her flute by

$$0.3932\text{ m} - 0.3886\text{ m} = 0.0046\text{ m} = 4.6\text{ mm}$$

17.74. Model: Assume the dropped box is in free fall.
Visualize: Since the speed of the dropped box changes then the perceived frequency is also a function of the speed, and therefore of the time falling. So we must integrate:

$$\#\text{beats} = \int f_{\text{beat}}\,dt$$

Solve: To find the limits of integration we first compute the time for the box to free fall 20 m. $\Delta y = v_0 t + \frac{1}{2}gt^2$.

With $v_0 = 0$ and $t_0 = 0$ we get $t_\text{f} = \sqrt{2\Delta y/g} = \sqrt{2(20\text{ m})/(9.8\text{ m/s}^2)} = 2.02\text{ s}$.
Use the Doppler effect formula for a receding source. The beat frequency is the difference between the perceived frequencies from the two boxes. The speed of the falling box is $v_\text{s} = v_0 t + gt = gt$.

$$f_{\text{beat}} = |f_- - f_0| = \left|\frac{f_0}{1+v_\text{s}/v} - f_0\right| = f_0\left|\frac{1}{1+v_\text{s}/v} - 1\right| = f_0\left|\frac{-v_\text{s}}{v+v_\text{s}}\right| = f_0\left(\frac{gt}{v+gt}\right)$$

Now we integrate.

$$\#\text{beats} = \int_0^{t_\text{f}} f_{\text{beat}}\,dt = \int_0^{t_\text{f}} f_0\left(\frac{gt}{v+gt}\right)dt = f_0 g\int_0^{t_\text{f}}\left(\frac{t}{v+gt}\right)dt$$

Use a u-substitution or integral tables.

$$\#\text{beats} = f_0 g\int_0^{t_\text{f}}\left(\frac{t}{v+gt}\right)dt = f_0 g\left[\frac{t}{g} - \frac{v}{g^2}\ln(v+gt)\right]_0^{t_\text{f}} = f_0\left[t - \frac{v}{g}\ln(v+gt)\right]_0^{t_\text{f}}$$

$$= f_0\left[t_\text{f} - \frac{v}{g}\ln(v+gt_\text{f}) + \frac{v}{g}\ln v\right] = f_0\left[t_\text{f} - \frac{v}{g}\ln\left(\frac{v+gt_\text{f}}{v}\right)\right]$$

$$= (440\text{ Hz})\left[2.02\text{ s} - \frac{(343\text{ m/s})}{(9.8\text{ m/s}^2)}\ln\left(\frac{343\text{ m/s} + (9.8\text{ m/s}^2)(2.02\text{ s})}{343\text{ m/s}}\right)\right] = 24.7$$

This would round to 25 under usual significant figure rules, but we round down because we don't quite get to the 25th beat. So you hear 24 beats.
Assess: The beats would get faster the farther it falls. It seems reasonable to hear 24 beats in about 2 seconds.

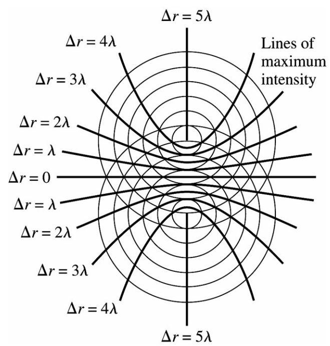
17.75. Model: The frequency of the loudspeaker's sound in the back of the pick-up truck is Doppler shifted. As the truck moves away from you, the frequency of the sound emitted by its speaker is decreased.
Solve: Because you hear 8 beats per second as the truck drives away from you, the frequency of the sound from the speaker in the pick-up truck is $f_- = 400\text{ Hz} - 8\text{ Hz} = 392\text{ Hz}$. This frequency is

$$f_- = \frac{f_0}{1+v_\text{S}/v} \quad\Rightarrow\quad 1 + \frac{v_\text{S}}{343\text{ m/s}} = \frac{400\text{ Hz}}{392\text{ Hz}} = 1.020408 \quad\Rightarrow\quad v_\text{S} = 7.0\text{ m/s}$$

That is, the velocity of the source v_S and hence the pick-up truck is 7.0 m/s.

Challenge Problems

17.76. Model: The two radio antennas are sources of in-phase, circular waves. The overlap of these waves causes interference.
Visualize:



Solve: Maxima occur along lines such that the path difference to the two antennas is $\Delta r = m\lambda$. The $750 \text{ MHz} = 7.50\times10^8 \text{ Hz}$ wave has a wavelength $\lambda = c/f = 0.40 \text{ m}$. Thus, the antenna spacing $d = 2.0 \text{ m}$ is exactly 5λ. The maximum possible intensity is on the line connecting the antennas, where $\Delta r = d = 5\lambda$. So this is a line of maximum intensity. Similarly, the line that bisects the two antennas is the $\Delta r = 0$ line of maximum intensity. In between, in each of the four quadrants, are four lines of maximum intensity with $\Delta r = \lambda$, 2λ, 3λ, and 4λ. Although we have drawn a fairly accurate picture, you do *not* need to know precisely where these lines are located to know that you *have* to cross them if you walk all the way around the antennas. Thus, you will cross 20 lines where $\Delta r = m\lambda$ and will detect 20 maxima.

17.77. Model: The microphone will detect a loud sound only if there is a standing-wave resonance in the tube. The sound frequency does not change, but changing the length of the tube can create a standing wave.
Solve: The standing-wave condition is

$$f = 280 \text{ Hz } = m\frac{v}{2L} \qquad m = 1, 2, 3, \ldots$$

where L is the total length of the tube. When the slide is extended a distance s, the tube has two straight sides, each of length $s + 80$ cm, plus a semicircular bend of length $\frac{1}{2}(2\pi r)$. The radius is $r = \frac{1}{2}(10 \text{ cm}) = 5.0 \text{ cm}$. The tube's total length is

$$L = 2(s + 80 \text{ cm}) + \tfrac{1}{2}(2\pi \times 5.0 \text{ cm}) = 175.7 \text{ cm } + 2s = 1.757 \text{ m} + 2s$$

A standing-wave resonance will be created if

$$[L = 1.757 + 2s] = \left[m\frac{v}{2f} = m\frac{343 \text{ m/s}}{2(280 \text{ Hz})} = 0.6125m\right]$$

$$s = 0.3063m - 0.8785 \text{ meters}$$

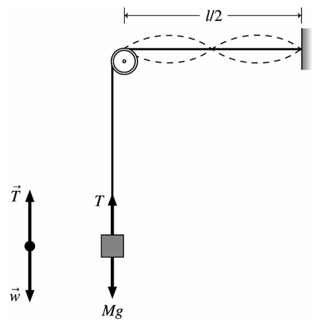
We can tabulate the different extensions s that correspond to standing-wave modes $m = 1$, $m = 2$, $m = 3$, and so on.

m	s
1	-0.572 m
2	-0.266 m
3	$0.040 \text{ m} = 4.0 \text{ cm}$
4	$0.347 \text{ m} = 34.7 \text{ cm}$
5	$0.653 \text{ m} = 65.3 \text{ cm}$
6	1.959 m

Physically, the extension must be greater than 0 cm and less than 80 cm. Thus, the three slide extensions that create a standing-wave resonance at 280 Hz are 4.0, 35, and 65 cm to two significant figures.

17.78. Model: The stretched wire is vibrating at its second harmonic frequency.

Visualize: Let l be the full length of the wire, and L be the vibrating length of the wire. That is, $L = \left(\frac{1}{2}\right)l$.



Solve: The wave speed on a stretched wire is

$$v_{\text{wire}} = \sqrt{\frac{T_s}{\mu}} = f\lambda$$

The frequency $f = 100$ Hz and the wavelength $\lambda = \frac{1}{2}l$ because it is a second harmonic wave. The tension $T_s = (1.25\text{ kg})g$ because the hanging mass is in static equilibrium and $\mu = 1.00\times10^{-3}$ kg/m. Substituting in these values,

$$\sqrt{\frac{(1.25\text{ kg})g}{(1.00\times10^{-3}\text{ kg/m})}} = (100\text{ Hz})\frac{l}{2} \quad\Rightarrow\quad g = (100\text{ Hz})^2\left(\frac{l}{2}\right)^2\frac{(1.00\times10^{-3}\text{ kg/m})}{(1.25\text{ kg})} = (2.00\text{ m}^{-1}\text{s}^{-2})l^2$$

To find l we can use the equation for the time period of a simple pendulum:

$$T = 2\pi\sqrt{\frac{l}{g}} \quad\Rightarrow\quad l = \frac{T^2}{4\pi^2}g = \frac{(314\text{ s}/100)^2}{4\pi^2}g = (0.250\text{ s}^2)g$$

Substituting this expression for l into the equation for g, we get

$$g = (2.000\text{ m}^{-1}\text{s}^{-2})(0.025\text{ s}^2)^2 g^2 \quad\Rightarrow\quad (0.125\text{ m}^{-1}\text{s}^2)g^2 - g = 0$$
$$\left[(0.125\text{ m}^{-1}\text{s}^2)g - 1\right]g = 0 \quad\Rightarrow\quad g = 8.00\text{ m/s}^2$$

Assess: A value of 8.0 m/s^2 is reasonable for the information given in the problem.

17.79. Model: Assume that the extra kilogram doesn't stretch the wire longer (so L stays the same) nor thinner (so μ stays the same). Also assume that, because the wire is thin, its own weight is negligible, so T_s is constant throughout the wire and is equal to Mg.

Visualize: The wire is fixed at both ends so, in the second harmonic, $L = \lambda$. We are given $f_2 = 200$ Hz, $f_2' = 245$ Hz, and $M' = M + 1.0$ kg. Apply $v = \lambda f$ and $v = \sqrt{T_s/\mu}$.

Solve: Taking the ratio of the frequencies allows gives

$$\frac{f_2'}{f_2} = \frac{v'/\lambda}{v/\lambda} = \frac{\sqrt{T_s'/\mu}}{\sqrt{T_s/\mu}} = \frac{\sqrt{M'g}}{\sqrt{Mg}} = \frac{\sqrt{(M+1.0\text{ kg})g}}{\sqrt{Mg}} = \sqrt{\frac{M+1.0\text{ kg}}{M}}$$

$$\left(\frac{f_2'}{f_2}\right)^2 = \frac{M+1.0\text{ kg}}{M}$$

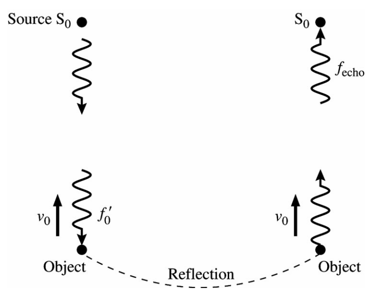
$$M\left[\left(\frac{f_2'}{f_2}\right)^2 - 1\right] = 1.0 \text{ kg}$$

$$M = \frac{1.0 \text{ kg}}{\left(\frac{f_2'}{f_2}\right)^2 - 1} = \frac{1.0 \text{ kg}}{\left(\frac{245 \text{ Hz}}{200 \text{ Hz}}\right)^2 - 1} = 2.0 \text{ kg}$$

Assess: We did not expect M to be really huge or **(a)** it would have broken the wire, and **(b)** adding one more kilogram wouldn't have made as big a difference in f_2 as it did.

17.80. Model: The frequency is Doppler shifted to higher values for a detector moving toward the source. The frequency is also shifted to higher values for a source moving toward the detector.

Visualize:



Solve: **(a)** We will derive the formula in two steps. First, the object acts like a moving detector and "observes" a frequency that is given by $f_0' = f_0(1 + v_0/v)$. Second, as this moving object reflects (or acts as a "source" of ultrasound waves), the frequency f_{echo} as observed by the original source S_o is $f_{echo} = f_0'(1 - v_0/v)^{-1}$. Combining these two equations gives

$$f_{echo} = \frac{f'_0}{1 - v_0/v} = \frac{f_0(1 + v_0/v)}{1 - v_0/v} = \frac{v + v_0}{v - v_0} f_0$$

(b) If $v_0 \ll v$, then

$$f_{echo} = f_0\left(1 + \frac{v_0}{v}\right)\left(1 - \frac{v_0}{v}\right)^{-1} = f_0\left(1 + \frac{v_0}{v}\right)\left(1 + \frac{v_0}{v} + \ldots\right) = f_0\left(1 + \frac{2v_0}{v} + \ldots\right)$$

$$f_{beat} = f_{echo} - f_0 \approx \frac{2v_0}{v} f_0$$

(c) Using part **(b)** for the beat frequency,

$$65 \text{ Hz} = \left(\frac{2v_0}{1540 \text{ m/s}}\right)(2.40 \times 10^6 \text{ Hz}) \quad \Rightarrow \quad v_0 = 2.09 \text{ cm/s}$$

(d) Assuming the heart rate is 90 beats per minute the angular frequency is

$$\omega = 2\pi f = 2\pi(1.5 \text{ beats/s}) = 9.425 \text{ rad/s}$$

Using $v_0 = v_{max} = \omega A$,

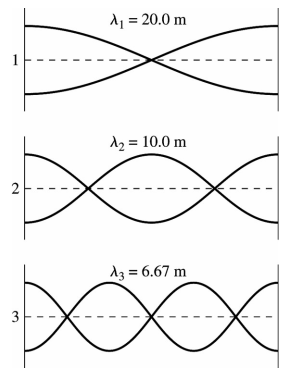
$$A = \frac{v_0}{\omega} = \frac{2.09 \text{ cm/s}}{90 \text{ min}^{-1}} = \frac{2.09 \text{ cm/s}}{9.425 \text{ rad/s}} = 2.2 \text{ mm}$$

17.81. Solve: **(a)** The wavelengths of the standing-wave modes are

$$\lambda_m = \frac{2L}{m} \quad m = 1, 2, 3, \ldots$$

$$\lambda_1 = \frac{2(10.0 \text{ m})}{1} = 20.0 \text{ m} \qquad \lambda_2 = \frac{2(10.0 \text{ m})}{2} = 10.0 \text{ m} \qquad \lambda_3 = \frac{2(10.0 \text{ m})}{3} = 6.67 \text{ m}$$

The depth of the pool is 5.0 m. Clearly the standing waves with λ_2 and λ_3 are "deep water waves" because the 5.0 m depth is larger than one-quarter of the wavelength. The wave with λ_1 barely qualifies to be a deep water standing wave.



(b) The wave speed for the first standing-wave mode is

$$v_1 = \sqrt{\frac{g\lambda_1}{2\pi}} = \sqrt{\frac{(9.8 \text{ m/s}^2)(20.0 \text{ m})}{2\pi}} = 5.6 \text{ m/s}$$

Likewise, $v_2 = 4.0$ m/s and $v_3 = 3.2$ m/s.

(c) We have

$$v = \sqrt{\frac{g\lambda_m}{2\pi}} = f_m\lambda_m \quad \Rightarrow \quad f_m = \sqrt{\frac{g}{2\pi\lambda_m}} = \sqrt{\frac{mg}{4\pi L}}$$

Note that *m* is the mode and not the mass.

(d) The period of oscillation for the first standing-wave mode is calculated as follows

$$f_1 = \sqrt{\frac{(1)(9.8 \text{ m/s}^2)}{4\pi(10.0 \text{ m})}} = 0.279 \text{ Hz} \Rightarrow T_1 = \frac{1}{f_1} = 3.6 \text{ s}$$

Likewise, $T_2 = 2.5$ s and $T_3 = 2.1$ s.