19

WORK, HEAT, AND THE FIRST LAW OF THERMODYNAMICS

Conceptual Questions

19.1. No. There is not a transfer of energy due to a temperature difference. When the shuttle re-enters the atmosphere there is friction between the molecules of the atmosphere and the shuttle. The mechanical energy of the shuttle is transformed into thermal energy. The initial kinetic energy of the shuttle decreases.

19.2. **(a)** Temperature is a property of a system related to the thermal energy per molecule.
(b) Heat is the energy transferred between the system and the environment. Heat may cause the thermal energy to change, but that doesn't mean that heat and thermal energy are the same thing.
(c) Thermal energy is a state variable and property of the system. It continues to exist even if the system is isolated and not interacting with the environment.

19.3. Greater than. $Q = n_A C_V \Delta T_A = n_B C_P \Delta T_B = 10$ J and $n_A = n_B \Rightarrow C_V \Delta T_A = C_P \Delta T_B$. Since $C_P > C_V \Rightarrow$ $\Delta T_A > \Delta T_B$. But $T_{Ai} = T_{Bi} \Rightarrow T_{Af} > T_{Bf}$.

19.4. To use the least amount of heat energy you should heat the gas at constant volume. Then $nC_V \Delta T < nC_P \Delta T \Rightarrow C_V < C_P$.

19.5. In a constant-volume process all heat input is used to increase the temperature of the gas. But in a constant-pressure process, since some work is done, some energy leaves the system as work. So more heat is needed to produce the same ΔT: $C_P > C_V$.

19.6. **(a)** Lower.
**(b)** No. The temperature and thermal energy decrease because of the work done by the gas on the surroundings.

19.7. Even though the initial and final positions on the diagram are the same for both paths, the work done is not equal. The work done on the gas is the negative of the area under the pV curve V_i and V_f. There is more area under the A path than under the B path, so $W_A > W_B$.

19.8. **(a)** There is one particular temperature corresponding to point *i* and another corresponding to point *f*, and the ΔT between the two will be the same no matter which path is taken.
(b) $W_A + Q_A = W_B + Q_B$. The area under process A is larger than the area under B, so W_A is more negative than W_B, so Q_A has to be more positive than Q_B to maintain the equality. Therefore, more heat energy is added for process A than for process B.

19.9. **(a)** (i) $\Delta T < 0$ because the initial temperature is $> 0°C$ and the final temperature is $0°C$. (ii) $W = 0$ because the volume stays constant. (iii) $Q < 0$ because energy is transferred due to contact with the cold ice.

(b)
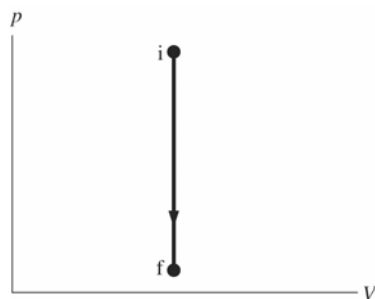
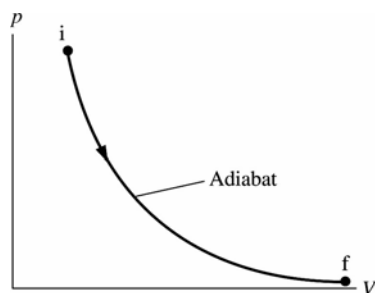**19.10.** **(a)** (i) $\Delta T < 0$ because the initial temperature is $> 0°C$ and the final temperature is $0°C$. (ii) $W > 0$ because the volume decreases. (iii) $Q < 0$ because energy is transferred due to contact with the cold ice.

(b)
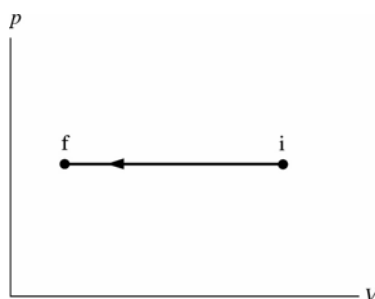
19.11. **(a)** (i) $\Delta T < 0$ because the gas expands adiabatically. (ii) $W < 0$ because the volume increases. (iii) $Q = 0$ because the system is thermally insulated.

(b)

Exercises and Problems

Exercises

Section 19.1 It's All About Energy

Section 19.2 Work in Ideal-Gas Processes

19.1. **Model:** Assume the gas is ideal. The work done on a gas is the negative of the area under the pV curve.

Visualize: The gas expands, so we expect the work done on the gas by the environment to be negative.

Solve: The work done on the gas is

$$W = -\int_{V_i}^{V_f} pdV = -(\text{area under the } pV \text{ curve})$$

$$= -[(400\,\text{kPa})(300 \times 10^{-6}\,\text{m}^3 - 100 \times 10^{-6}\,\text{m}^3)] = -80\,\text{J}$$

Assess: The area under the curve is positive because $V_f > V_i$. Thus, the expanding gas does positive work on the environment, so the environment does negative work on the gas.

19.2. Model: Assume the gas is ideal. The work done on a gas is the negative of the area under the pV curve.
Visualize: The gas is undergoing compression, so we expect the work done on it to be positive.
Solve: The work done on the gas is the negative of the area under the pV curve from V_i to V_f, which is the area of the rectangle and triangle. We have

$$W = -\int_{V_i}^{V_f} p\, dV = -[-(200\times10^{-6}\ \text{m}^3)(200\times10^3\ \text{Pa}) - \tfrac{1}{2}(200\times10^{-6}\ \text{m}^3)(200\times10^3\ \text{Pa})] = 60\ \text{J}$$

where the inner negative signs come from the fact that $V_f < V_i$. The work done on the gas by the environment is thus $W = 60$ J.
Assess: The environment does positive work on the gas to compress it.

19.3. Model: Assume the gas is ideal. The work done on a gas is the negative of the area under the pV curve.
Solve: The area under the curve is a trapezoid of width ΔV, and the work done on gas in this case is average height of the trapezoid multiplied by the width.

$$W = -p_{ave}\Delta V = -p_{ave}(V_f - V_i)$$

where $p_{ave} = (300\ \text{kPa} + 100\ \text{kPa})/2 = 200\ \text{kPa}$. Substituting into this equation,

$$80\ \text{J} = -(200\times10^3\ \text{Pa})(V_1 - 3V_1) \quad 2V_1 = \frac{80\ \text{J}}{200\ \text{kPa}} \Rightarrow V_i = 2.0\times10^{-4}\ \text{m}^3 = 200\ \text{cm}^3$$

Assess: The work done to compress a gas is positive.

19.4. Model: Helium is an ideal gas that undergoes isobaric and isothermal processes.
Solve: (a) Since the pressure ($p_i = p_f = p$) is constant the work done is

$$W_{\text{on gas}} = -p\Delta V = -p(V_f - V_i) = -\frac{nRT_i}{V_i}(V_f - V_i)$$

$$= -(0.10\ \text{mol})(8.31\ \text{J/mol K})(573\ \text{K})\frac{(1000\ \text{cm}^3 - 2000\ \text{cm}^3)}{2000\ \text{cm}^3} = 240\ \text{J}$$

(b) For compression at a constant temperature,
$$W_{\text{on gas}} = -nRT\ \ln(V_f/V_i)$$

$$= -(0.10\ \text{mol})(8.31\ \text{J/mol K})(573\ \text{K})\ln\left(\frac{1000\times10^{-6}\ \text{m}^3}{2000\times10^{-6}\ \text{m}^3}\right) = 330\ \text{J}$$

19.5. Model: This is an isothermal process. The work done is positive for a compression.
Solve: For an isothermal process,

$$W = -nRT\ln(V_f/V_i)$$

For the first process,

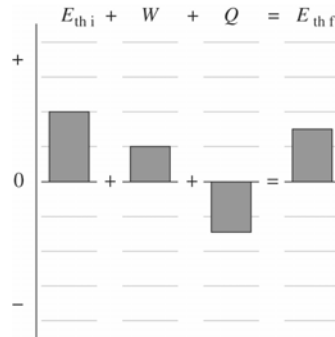$$W = 500\ \text{J} = -nRT\ln\left(\tfrac{1}{2}\right) \Rightarrow nRT = 721.35\ \text{J}$$

For the second process,

$$W = -nRT\ln\left(\tfrac{1}{10}\right) \Rightarrow W = -(721.35\ \text{J})\ln\left(\tfrac{1}{10}\right) = 16.6\ \text{kJ}$$

Section 19.3 Heat

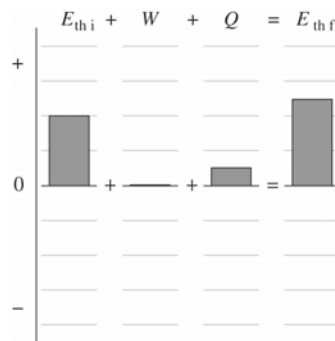**Section 19.4 The First Law of Thermodynamics**
19.6. Visualize:

Solve: Because this is an isobaric process $W = -\int p\,dV = -p(V_f - V_i)$. Since V_f is less than V_i, W is positive. That is, the gas contracts. Since the final point is on a lower isotherm than the initial point, $T_f < T_i$. In other words, the thermal energy decreases. For this to happen, the heat energy transferred from the gas must be larger than the work done on the gas.

19.7. Visualize:
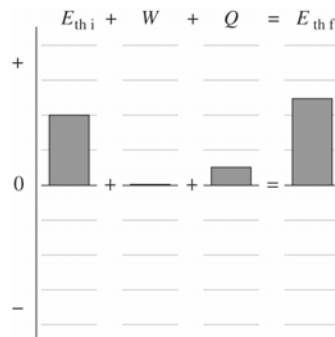
Solve: Because $W = -\int p\,dV$ and this is an isochoric process, $W = 0$ J. The final point is on a higher isotherm than the initial point, so $T_f > T_i$. Heat energy is thus transferred into the gas $(Q > 0)$ and the thermal energy of the gas increases $(E_{th\,f} > E_{th\,i})$ as the temperature increases.

19.8. Visualize:

Solve: This is an adiabatic process of gas compression so no heat energy is transferred between the gas and the environment. That is, $Q = 0$ J. According to the first law of thermodynamics, the work done on a gas in an adiabatic process goes entirely to changing the thermal energy of the gas. The work W is positive because the gas is compressed.

19.9. Visualize:
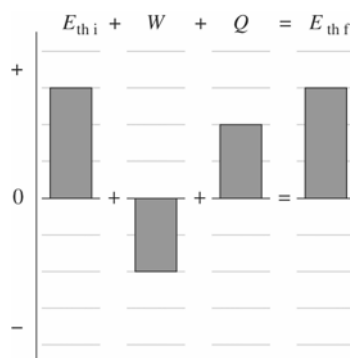
Solve: Because the process is isothermal, $\Delta E_{\text{th}} = E_{\text{th f}} - E_{\text{th i}} = 0$ J. According to the first law of thermodynamics, $\Delta E_{\text{th}} = W + Q$. This can be satisfied only if $W = -Q$. W is negative because the gas is expanding, hence Q is positive. That is, heat energy is added to the gas.

19.10. Solve: This is an isobaric process. $W > 0$ because the gas is compressed, which transfers energy into the system. Also, 100 J of heat energy is transferred out of the gas. The first law of thermodynamics gives

$$\Delta E_{\text{th}} = W + Q = -p\Delta V + Q = -(4.0 \times 10^5 \text{ Pa})(200 - 600) \times 10^{-6} \text{ m}^3 - 100 \text{ J} = 60 \text{ J}$$

Thermal energy increases by 60 J.

19.11. Solve: The first law of thermodynamics gives

$$\Delta E_{\text{th}} = W + Q \Rightarrow -200 \text{ J} = 500 \text{ J} + Q = Q \Rightarrow Q = -700 \text{ J}$$

The negative sign means a transfer of energy from the system to the environment.
Assess: Because $W > 0$ energy is transferred into the system, Q must be less than zero and larger in magnitude than W, so $E_{\text{th f}} < E_{\text{th i}}$.

Section 19.5 Thermal Properties of Matter
19.12. Model: The addition of heat to the aluminum increases its thermal energy and its temperature. Assume that the specific heat of the copper stays constant over the temperature range involved.
Visualize: The temperature change is $\Delta T = 150°C - (-50°C) = 200°C$.

Solve: The heat needed to change an object's temperature is $Q = Mc\Delta T$. The density of copper is 8920 kg/m³, so the mass of the copper sphere is

$$M = \rho_{\text{Cu}}V = (8920 \text{ kg/m}^3)\tfrac{4}{3}\pi(0.030 \text{ m})^3 = 1.0 \text{ kg}$$

The specific heat of copper from the table is $c_{\text{Cu}} = 385$ J/kg K, so

$$Q = (1.0 \text{ kg})(385 \text{ J/kg K})(200 \text{ K}) = 78 \text{ kJ}$$

Thus, the energy required to heat the Cu sphere is 78 kJ.
Assess: The positive sign of Q means the system (i.e., Cu sphere) gains energy.

19.13. Model: The spinning paddle wheel does work and changes the water's thermal energy and its temperature.
Solve: (a) $Q = 0$. No energy is transferred between the system and the environment because of a difference in temperature.
(b) The temperature change is $\Delta T = T_f - T_i = 25°C - 21°C = 4$ K. The mass of the water is

$$M = (200 \times 10^{-6} \text{m}^3)(1000 \text{ kg/m}^3) = 0.20 \text{ kg}$$

The work done is

$$W = \Delta E_{th} = Mc_{water}\Delta T = (0.20 \text{ kg})(4190 \text{ J/kg K})(4 \text{ K}) = 3352 \text{ J} \approx 3.4 \text{ kJ}$$

19.14. Model: Heating the mercury changes its thermal energy and its temperature.
Solve: (a) The heat needed to change the mercury's temperature is

$$Q = Mc_{Hg}\Delta T \implies \Delta T = \frac{Q}{Mc_{Hg}} = \frac{100 \text{ J}}{(0.020 \text{ kg})(140 \text{ J/kg K})} = 35.7 \text{ K} \approx 36°C$$

(b) The amount of heat required to raise the temperature of the same amount of water by the same amount is

$$Q = Mc_{water}\Delta T = (0.020 \text{ kg})(4190 \text{ J/kg K})(35.7 \text{ K}) = 3000 \text{ J}$$

Assess: Q is directly proportional to c_{water} and the specific heat for water is much higher than the specific heat for mercury. This explains why $Q_{water} > Q_{mercury}$.

19.15. Model: Heating the mercury at its boiling point changes its thermal energy without a change in temperature.
Solve: The mass of the mercury is $M = 20$ g $= 2.0 \times 10^{-2}$ kg, the specific heat $c_{mercury} = 140$ J/kg K, the boiling point $T_b = 357°C$, and the heat of vaporization $L_v = 2.96 \times 10^5$ J/kg. The heat required for the mercury to change to the vapor phase is the sum of two steps. The first step is

$$Q_1 = Mc_{mercury}\Delta T = (2.0 \times 10^{-2} \text{ kg})(140 \text{ J/kg K})(357°C - 20°C) = 940 \text{ J}$$

The second step is

$$Q_2 = ML_V = (2.0 \times 10^{-2} \text{ kg})(2.96 \times 10^5 \text{ J/kg}) = 5920 \text{ J}$$

The total heat needed is $940 \text{ J} + 5920 \text{ J} = 6860 \text{ J}$ or, to two significant figures, 6.9 kJ.

19.16. Model: Changing liquid ethyl alcohol at 20°C to gaseous ethyl alcohol at its boiling point $(T_b = 78°C)$ requires two steps: raising the temperature to T_b and then vaporizing the liquid at T_b to a gas at T_b.
Solve: The total heat is the sum of these two contributions, so

$$Q = Q_1 + Q_2 \implies 1000 \text{ J} = Mc_{EA}(T_f - T_i) + ML_v$$
$$1000 \text{ J} = M(2400 \text{ J/kg K})(78°C - 20°C) + M(8.79 \times 10^5 \text{ J/kg})$$
$$M = \frac{1000 \text{ J}}{1,018,200 \text{ J/kg}} = 9.8 \times 10^{-4} \text{ kg} = 0.98 \text{ g}$$

The maximum mass of 20°C ethyl alcohol you can vaporize with 1000 J of heat is 0.98 g.

19.17. Model: Assume sweat is mostly water, and estimate the latent heat of vaporization for water at 35°C by its value at 100°C. Furthermore assume that all the energy required to heat and vaporize the sweat comes from the body (as opposed to the air surrounding the body).
Solve: To vaporize 30 g of 100°C water every minute requires a power

$$P_1 = \frac{Q_2}{(60 \text{ s})} = \frac{ML_v}{(60 \text{ s})} = \frac{(0.030 \text{ kg})(22.6 \times 10^5 \text{ J/kg})}{60 \text{ s}} = 1130 \text{ W}$$

Thus, to two significant figures, the total power that can be exhausted by perspiring is approximately 1100 J/s.

Assess: This is the power consumed by twelve 100-W light bulbs! The result is so high because water has a large heat of vaporization.

19.18. Model: Assume the heating is 100% efficient.
Visualize: The temperature change is $\Delta T = 35°C - 20°C = 15°C.$ For **changes** in temperature we can use either °C or K. The specific heat of aluminum from the table is $c_{Al} = 900$ J/kg K.
Solve: Recall that $P = \Delta E/\Delta t \Rightarrow \Delta E = P\Delta t.$ Now look at the thermal energy change of a system.

$$\Delta E = cM\Delta T \Rightarrow m = \frac{\Delta E}{c\Delta T} = \frac{P\Delta t}{c\Delta T} = \frac{(2.5 \text{ kW})(30 \text{ s})}{(900 \text{ J/kg} \cdot \text{K})(15 \text{ K})} = 5.6 \text{ kg}$$

Assess: It would be easier to find a scale to ascertain the mass, but this method would give a reasonable answer. The units all check out.

19.19. Model: Each car's kinetic energy is transformed into thermal energy.
Solve: For each car,

$$K = \tfrac{1}{2}Mv^2 = \Delta E_{th} = Mc_{car}\Delta T \Rightarrow \Delta T = \frac{v^2}{2c_{car}}$$

Assume $c_{car} = c_{iron}.$ The speed of the car is

$$v = 80 \text{ km/h} = \frac{80 \times 1000 \text{ m}}{3600 \text{ s}} = 22.22 \text{ m/s} \Rightarrow \Delta T = \frac{(22.22 \text{ m/s})^2}{2(449 \text{ J/kg K})} = 0.55°C$$

Assess: Notice the answer is independent of the car's mass.

19.20. Model: Heating the material increases its thermal energy.
Visualize: Heat raises the temperature of the substance from $-40°C$ to $-20°C,$ at which temperature a solid-to-liquid phase change occurs. From $-20°C,$ heat raises the liquid's temperature up to $40°C.$ Boiling occurs at $40°C$ where all of the liquid is converted to the vapor phase.
Solve: (a) In the solid phase,

$$\Delta Q = Mc\Delta T \Rightarrow c = \left(\frac{\Delta Q}{\Delta T}\right)\frac{1}{M} = \left(\frac{20 \text{ kJ}}{20 \text{ K}}\right)\left(\frac{1}{0.50 \text{ kg}}\right) = 2.0 \text{ kJ/kg K}$$

(b) In the liquid phase,

$$c = \frac{1}{M}\left(\frac{\Delta Q}{\Delta T}\right) = \left(\frac{1}{0.50 \text{ kg}}\right)\left(\frac{80 \text{ kJ}}{60 \text{ K}}\right) = 2.7 \text{ kJ/kg K}$$

(c) The melting point $T_m = -20°C$ and the boiling point $T_b = +40°C.$
(d) The heat of fusion is

$$L_f = \frac{Q}{M} = \frac{20,000 \text{ J}}{0.50 \text{ kg}} = 4.0 \times 10^4 \text{ J/kg}$$

The heat of vaporization is

$$L_v = \frac{Q}{M} = \frac{60,000 \text{ J}}{0.50 \text{ kg}} = 1.2 \times 10^5 \text{ J/kg}$$

Section 19.6 Calorimetry
19.21. Model: We have a thermal interaction between the copper pellets and the water.
Solve: The conservation of energy equation $Q_{Cu} + W_w = 0$ is

$$M_{Cu} c_{Cu} (T_f - 300°C) + M_w c_w (T_f - 20°C) = 0 \text{ J}$$

Solving this equation for the final temperature T_f gives

$$T_f = \frac{M_{Cu}c_{Cu}(300°C) + M_w c_w(20°C)}{M_{Cu}c_{Cu} + M_w c_w}$$

$$= \frac{(0.030 \text{ kg})(385 \text{ J/kg K})(300°C) + (0.10 \text{ kg})(4190 \text{ J/kg K})(20°C)}{(0.030 \text{ kg})(385 \text{ J/kg K}) + (0.10 \text{ kg})(4190 \text{ J/kg K})} = 28°C$$

The final temperature of the water and the copper is 28°C.

19.22. Model: We have a thermal interaction between the aluminum pan and the water.
Solve: The conservation of energy equation $Q_{Al} + Q_{water} = 0$ J gives

$$M_{Al}c_{Al}[T_f - (T_i)_{Al}] + M_{water}c_{water}[T_f - (T_i)_{water}]$$

The pan and water reach a common final temperature $T_f = 24.0°C$

$$(0.750 \text{ kg})(900 \text{ J/kg K})[24.0°C - (T_i)_{Al}] + (10.0 \times 10^{-3} \text{ m}^3)(1000 \text{ kg/m}^3)(4190 \text{ J/kg K})(24.0°C - 20.0°C)$$
$$= (675.0 \text{ J/K})[24.0°C - (T_i)_{Al}] + 167,600 \text{ J} = 0 \text{ J}$$
$$(T_i)_{Al} = 272°C = [(272)(9/5) + 32]°F = 522°F$$

19.23. Model: We have a thermal interaction between the thermometer and the water.
Solve: The conservation of energy equation $Q_{thermo} + Q_{water} = 0$ J gives

$$M_{thermo}c_{thermo}[T_f - (T_i)_{thermo}] + M_{water}c_{water}[T_f - (T_i)_{water}] = 0 \text{ J}$$

The thermometer slightly cools the water until both have the same final temperature $T_f = 71.2°C$. Thus

$$0 \text{ J} = (0.0500 \text{ kg})(750 \text{ J/kg K})(71.2°C - 20.0°C) + (200 \times 10^{-6} \text{ m}^3)(1000 \text{ kg/m}^3)(4190 \text{ J/kg K})[71.2°C - (T_i)_{water}]$$
$$= 1920 \text{ J} + 838 \text{ (J/K)}[71.2°C - (T_i)_{water}]$$

$$(T_i)_{water} = 73.5°C$$

Assess: The thermometer reads $71.2°C$ for a real temperature of $73.5°C$. This is reasonable.

19.24. Model: We have a thermal interaction between the metal sphere and the mercury.
Solve: The conservation of energy equation $Q_{metal} + Q_{Hg} = 0$ J gives

$$M_{metal}c_{metal}[T_f - (T_i)_{metal}] + M_{Hg}c_{Hg}[T_f - (T_i)_{Hg}] = 0 \text{ J}$$

The metal and mercury reach a common final temperature $T_f = 99.0°C$. Thus

$$(0.500 \text{ kg})c_{metal}(99.0°C - 300°C) + (300 \times 10^{-6} \text{ m}^3)(13,600 \text{ kg/m}^3)(140 \text{ J/kg K})(99.0°C - 20.0°C) = 0 \text{ J}$$

We find that $c_{metal} = 449$ J/kg K. The metal is iron.

19.25. Model: We have a thermal interaction between the iron block and the water.
Solve: The mass of the iron block is $\rho V = (7870 \text{ kg/m}^3)(65 \times 10^{-6} \text{ m}^3) = 0.512$ kg. The iron will heat all the water to $100°C$ and then cause a phase transformation of a mass M_s of water into steam. The conservation of energy equation $Q_{Fe} + Q_w = 0$ J gives

$$M_{Fe}c_{Fe}[T_f - (T_i)_{Fe}] + M_w c_w[T_f - (T_i)_w] + M_s(L_v)_w = 0 \text{ J}$$

The final temperature of the remaining water and the iron block is 100°C. Solving for M_s gives

$$M_s = \frac{-M_{Fe}c_{Fe}[T_f - (T_i)_{Fe}] - M_w c_w[T_f - (T_i)_w]}{(L_v)_w}$$

$$= \frac{-(0.512 \text{ kg})(449 \text{ J/kg K})(100°C - 800°C) - (0.200 \text{ kg})(4190 \text{ J/kg K})(100°C - 20°C)}{22.6 \times 10^5 \text{ J/kg}}$$

$$= 4.15 \times 10^{-2} \text{ kg}$$

Thus, the fraction f of the initial water mass that boils away is $f = (4.15 \times 10^{-2} \text{ kg})/(0.200 \text{ kg}) = 21\%$.

Section 19.7 The Specific Heats of Gases

19.26. Model: Use the models of isochoric and isobaric heating. Note that the change in temperature on the Kelvin scale is the same as the change in temperature on the Celsius scale.

Solve: (a) The atomic mass number of argon is 40. That is, $M_{mol} = 40$ g/mol. The number of moles of argon gas in the container is

$$n = \frac{M}{M_{mol}} = \frac{1.0 \text{ g}}{40 \text{ g/mol}} = 0.025 \text{ mol}$$

The amount of heat is

$$Q = nC_V\Delta T = (0.025 \text{ mol})(12.5 \text{ J/mol K})(100°C) = 31.25 \text{ J} \approx 31 \text{ J}$$

(b) For the isobaric process $Q = nC_P\Delta T$ becomes

$$31.25 \text{ J} = (0.025 \text{ mol})(20.8 \text{ J/mol K})\Delta T \implies \Delta T = 60°C$$

19.27. Model: The heating processes are isobaric and isochoric. O_2 is a diatomic ideal gas.

Solve: (a) The number of moles of oxygen is

$$n = \frac{M}{M_{mol}} = \frac{1.0 \text{ g}}{32 \text{ g/mol}} = 0.03125 \text{ mol}$$

For the isobaric process,

$$Q = nC_p\Delta T = (0.03125 \text{ mol})(29.2 \text{ J/mol K})(100°C) = 91.2 \text{ J} \approx 91 \text{ J}$$

(b) For the isochoric process,

$$Q = nC_V\Delta T = 91.2 \text{ J} = (0.03125 \text{ mol})(20.9 \text{ J/mol K})\Delta T \Rightarrow \Delta T = 140°C$$

19.28. Model: The gas is an ideal gas that is subjected to an adiabatic process.

Solve: (a) For an adiabatic process,

$$p_f V_f^\gamma = p_i V_i^\gamma \implies \frac{p_f}{p_i} = \left(\frac{V_i}{V_f}\right)^\gamma \implies 2.5 = (2.0)^\gamma \implies \gamma = \frac{\ln(2.5)}{\ln(2.0)} = 1.3$$

(b) Equation 19.36 for an adiabatic process is

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1} \implies \frac{T_f}{T_i} = \left(\frac{V_i}{V_f}\right)^{\gamma-1} = (2.0)^{1.32-1} = (2.0)^{0.32} = 1.3$$

19.29. Model: We assume the gas is an ideal gas and $\gamma = 1.40$ for a diatomic gas.

Solve: Using the ideal-gas law,

$$V_i = \frac{nRT_i}{p_i} = \frac{(0.10 \text{ mol})(8.31 \text{ J/mol K})(423 \text{ K})}{(3 \times 1.013 \times 10^5 \text{ Pa})} = 1.157 \times 10^{-3} \text{ m}^3$$

(a) For an adiabatic process,

$$p_i V_i^\gamma = p_f V_f^\gamma$$

$$V_f = V_i \left(\frac{p_i}{p_f}\right)^{1/\gamma} = (1.157 \times 10^{-3} \text{ m}^3)\left(\frac{p_i}{0.5 \, p_i}\right)^{1/1.40} = 1.9 \times 10^{-3} \text{ m}^3$$

(b) To find the final temperature, we use the ideal-gas law once again as follows:

$$T_f = T_i \frac{p_f}{p_i} \frac{V_f}{V_i} = (423 \text{ K})\left(\frac{0.5 \, p_i}{p_i}\right)\left(\frac{1.90 \times 10^{-3} \text{ m}^3}{1.157 \times 10^{-3} \text{ m}^3}\right) = 346.9 \text{ K} \approx 74°C$$

19.30. Model: O_2 gas has $\gamma = 1.40$ and is an ideal gas.

Solve: (a) For an adiabatic process, pV^γ remains a constant. That is,

$$p_i V_i^\gamma = p_f V_f^\gamma \implies p_f = p_i \left(\frac{V_i}{V_f}\right)^\gamma = (3.0 \text{ atm})\left(\frac{V_i}{2V_i}\right)^{1.40} = (3.0 \text{ atm})\left(\frac{1}{2}\right)^{1.40} = 1.14 \text{ atm} \approx 1.1 \text{ atm}$$

(b) Using the ideal-gas law, the final temperature of the gas is calculated as follows:

$$\frac{p_i V_i}{T_i} = \frac{p_f V_f}{T_f} \implies T_f = T_i \frac{p_f}{p_i} \frac{V_f}{V_i} = (423 \text{ K})\left(\frac{1.14 \text{ atm}}{3.0 \text{ atm}}\right)\left(\frac{2V_i}{V_i}\right) = 321.5 \text{ K} \approx 48°C$$

19.31. Model: The two processes are isochoric and isobaric.
Solve: Process A is isochoric which means

$$T_f / T_i = p_f / p_i \implies T_f = T_i (p_f / p_i) = T_i (1 \text{ atm}/3 \text{ atm}) = \tfrac{1}{3}T_i$$

From the ideal-gas equation,

$$T_i = \frac{p_i V_i}{nR} = \frac{(3 \times 1.013 \times 10^5 \text{ Pa})(2000 \times 10^{-6} \text{ m}^3)}{(0.10 \text{ mol})(8.31 \text{ J/mol K})} = 731.4 \text{ K} \implies T_f = \tfrac{1}{3}T_i = 243.8 \text{ K}$$

$$T_f - T_i = -487.6 \text{ K}$$

Thus, the heat required for process A is

$$Q_A = nC_V \Delta T = (0.10 \text{ mol})(20.8 \text{ J/mol K})(-487.6 \text{ K}) = -1000 \text{ J}$$

Process B is isobaric which means

$$T_f / V_f = T_i / V_i \implies T_f = T_i (V_f / V_i) = T_i (3000 \text{ cm}^3 / 1000 \text{ cm}^3) = 3T_i$$

From the ideal-gas equation,

$$T_i = \frac{p_i V_i}{nR} = \frac{(2 \times 1.013 \times 10^5 \text{ Pa})(1000 \times 10^{-6} \text{ m}^3)}{(0.10 \text{ mol})(8.31 \text{ J/mol K})} = 243.8 \text{ K}$$

$$T_f = 3T_i = 731.4 \text{ K} \implies T_f - T_i = 487.6 \text{ K}$$

Thus, heat required for process B is

$$Q_B = nC_P \Delta T = (0.10 \text{ mol})(29.1 \text{ J/mol K})(487.6 \text{ K}) = 1400 \text{ J}$$

Assess: Heat is transferred out of the gas in process A, but transferred into the gas in process B.

19.32. Model: The gas is assumed to be an ideal gas that is subjected to isobaric and isochoric processes.
Solve: (a) The initial conditions are $p_1 = 3.0 \text{ atm} = 304{,}000 \text{ Pa}$, $V_1 = 100 \text{ cm}^3 = 1.0 \times 10^{-4} \text{ m}^3$, and $T_1 = 100°C = 373 \text{ K}$. The number of moles of gas is

$$n = \frac{p_1 V_1}{RT_1} = \frac{(304{,}000 \text{ Pa})(1.0 \times 10^{-4} \text{ m}^3)}{(8.31 \text{ J/mol K})(373 \text{ K})} = 9.81 \times 10^{-3} \text{ mol}$$

At point 2 we have $p_2 = p_1 = 3.0 \text{ atm}$ and $V_2 = 300 \text{ cm}^3 = 3V_1$. This is an isobaric process, so

$$\frac{V_2}{T_2} = \frac{V_1}{T_1} \implies T_2 = \frac{V_2}{V_1}T_1 = 3(373 \text{ K}) = 1119 \text{ K}$$

The gas is heated to raise the temperature from T_1 to T_2. The amount of heat required is

$$Q = nC_P \Delta T = (9.81 \times 10^{-3} \text{ mol})(20.8 \text{ J/mol K})(1119 \text{ K} - 373 \text{ K}) = 0.15 \text{ kJ}$$

This amount of heat is *added* during process $1 \to 2$.
(b) Point 3 returns to $T_3 = 100°C = 373 \text{ K}$. This is an isochoric process, so

$$Q = nC_V \Delta T = (9.81 \times 10^{-3} \text{ mol})(12.5 \text{ J/mol K})(373 \text{ K} - 1119 \text{ K}) = -91 \text{ J}$$

This amount of heat is *removed* during process $2 \to 3$.

Section 19.8 Heat-Transfer Mechanisms

19.33. Model: To determine the material we will solve for *k* in Equation 19.45:

$$\frac{Q}{\Delta t} = k\frac{A}{L}\Delta T$$

We are given $L = 0.20$ m and $\Delta T = 100$ K. We compute $A = \pi r^2 = \pi(0.010\,\text{m})^2 = 3.14 \times 10^{24}\,\text{m}^2$.

We also convert the heat conduction to watts.

$$4.5 \times 10^4\,\text{J/h}\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 12.5\,\text{W}$$

Solve: Solve the equation for *k*.

$$k = \left(\frac{Q}{\Delta t}\right)\frac{L}{A}\frac{1}{\Delta T} = (12.5\,\text{W})\left(\frac{0.20\,\text{m}}{3.14 \times 10^{24}\,\text{m}^2}\right)\left(\frac{1}{100\,\text{K}}\right) = 80\,\text{W/m}\cdot\text{K}$$

Look this up in Table 19.5; the value corresponds to iron.

Assess: We are grateful that our answer was one of the entries in the table.

19.34. Model: We are asked for the heat-loss rate which is given by Equation 19.45:

$$\frac{Q}{\Delta t} = k\frac{A}{L}\Delta T$$

We are given $A = 10\,\text{m} \times 14\,\text{m} = 140\,\text{m}^2$, $L = 0.12$ m, and $-T = 22°\text{C} - 5°\text{C} = 17°\text{C} = 17$ K. We look up the thermal conductivity of concrete in Table 19.5: $k = 0.8\,\text{W/m}\cdot\text{K}$.
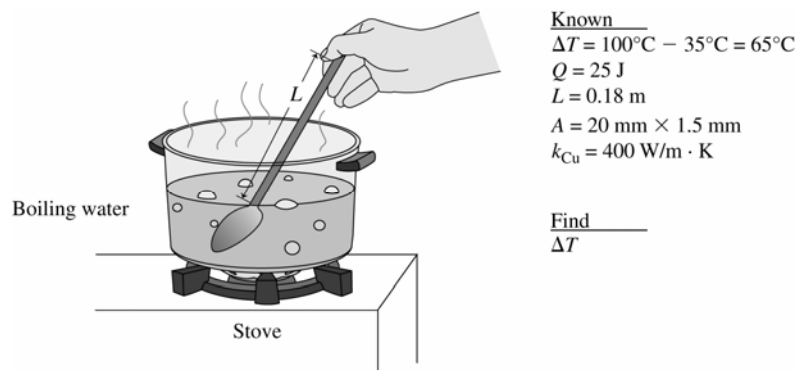
Solve:

$$\frac{Q}{\Delta t} = k\frac{A}{L}\Delta T = (0.8\,\text{W/m}\cdot\text{K})\left(\frac{140\,\text{m}^2}{0.12\,\text{m}}\right)(17\,\text{K}) = 16\,\text{kW}$$

Assess: The answer is in a reasonable range. The heat loss could be reduced with thicker concrete or adding a layer of a different material.

19.35. Model: Assume you are at sea level so the boiling water is at $100°\text{C}$.

Visualize:

Known
$\Delta T = 100°\text{C} - 35°\text{C} = 65°\text{C}$
$Q = 25$ J
$L = 0.18$ m
$A = 20\text{ mm} \times 1.5\text{ mm}$
$k_{Cu} = 400\,\text{W/m}\cdot\text{K}$

Find
ΔT

Solve: Use the conduction heat transfer formula.

$$\frac{Q}{\Delta t} = k\frac{A}{L}\Delta T \Rightarrow \Delta t = \frac{QL}{kA\Delta T} = \frac{(25\text{ J})(0.18\text{ m})}{(400\,\text{W/m}\cdot\text{K})(20\text{ mm} \times 1.5\text{ mm})(65°\text{C})} = 5.8\text{ s}$$

Assess: This seems like a reasonable amount of time.

19.36. Model: Assume the lead sphere is an ideal radiator with $e = 1$. Also assume that the highest temperature the solid lead sphere can have is the melting temperature of lead.

Visualize: Use Equation 19.46. First look up the melting temperature of lead in Table 19.3: $T_m = 328°C = 601$ K.

Then compute the surface area of the sphere: $A = 4\pi R^2 = 4\pi(0.050 \text{ m})^2 = 0.0314 \text{ m}^2$

Solve:

$$\frac{Q}{\Delta t} = e\sigma A T^4 = (1)(5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4)(0.0314 \text{ m}^2)(601 \text{ K})^4 = 230 \text{ W}$$

Assess: If the sphere were larger it could radiate more power without melting.

19.37. Model: We will ignore the bottom of the head and model it with just the cylindrical sides and top, all covered in skin. As instructed, assume an emissivity of $e = 0.95$.

Visualize: The area of the cylinder is

$$A = A_{side} + A_{top} = 2\pi rh + \pi r^2 = 2\pi(0.10 \text{ m})(0.20 \text{ m}) + \pi(0.10 \text{ m})^2 = 0.157 \text{ m}^2.$$

Solve: Use Equation 19.47.

$$e\sigma A(T^4 - T_0^4) = (0.95)(5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4)(0.157 \text{ m}^2)[(308 \text{ K})^4 - (278 \text{ K})^4] = 26 \text{ W}$$

Assess: This is a significant amount of the heat lost by the body. Wearing a hat can help a lot.

Problems

19.38. Model: There are various steps to the problem. We must (1) raise the temperature of the ice to 0°C, (2) melt the ice to liquid water at 0°C, (3) raise the water temperature to 100°C, (4) boil the water to produce steam at 100°C, and (5) raise the temperature of the steam to 200°C.

Solve: The heat needed for each step is

$$Q_1 = Mc_{ice}\Delta T_{ice} = (0.0050 \text{ kg})(2090 \text{ J/kg K})(20 \text{ K}) = 209 \text{ J}$$

$$Q_2 = ML_f = (0.0050 \text{ kg})(3.333 10^5 \text{ J/kg}) = 1665 \text{ J}$$

$$Q_3 = Mc_{water}\Delta T_{water} = (0.0050 \text{ kg})(4190 \text{ J/kg K})(100 \text{ K}) = 2095 \text{ J}$$

$$Q_4 = ML_v = (0.0050 \text{ kg})(22.63 10^5 \text{ J/kg}) = 11,300 \text{ J}$$

$$Q_5 = Mc_{steam}\Delta T_{steam} = (0.0050 \text{ kg})(2009 \text{ J/kg K})(100 \text{ K}) = 1005 \text{ J}$$

The total heat is

$$Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 = 16 \text{ kJ}$$

Assess: It is interesting to note which step required the most heat: The boiling to change the liquid to the gas is by far the largest contributor to the total.

19.39. Solve: The area of the garden pond is $A = \pi(2.5 \text{ m})^2 = 19.635 \text{ m}^2$ and its volume is $V = A(0.30 \text{ m}) = 5.891 \text{ m}^3$. The mass of water in the pond is

$$M = \rho V = (1000 \text{ kg/m}^3)(19.635 \text{ m}^3) = 5891 \text{ kg}$$

The water absorbs all the solar power, which is

$$(400 \text{ w/m}^2)(19.635 \text{ m}^2) = 7854 \text{ W}$$

This power is used to raise the temperature of the water. That is,

$$Q = (7854 \text{ W})\Delta t = Mc_{water}\Delta T = (5891 \text{ kg})(4190 \text{ J/kg K})(10 \text{ K}) \implies \Delta t = 31,425 \text{ s} \approx 8.7 \text{ h}$$

19.40. Model: Heating the water and the kettle raises the temperature of the water to the boiling point and raises the temperature of the kettle to 100°C.

Solve: The amount of heat energy from the electric stove's output in 3 minutes is

$$Q = (2000 \text{ W})(3.0 \text{ min} \times 60 \text{ s/min}) = 3.6 \times 10^5 \text{ J}$$

This heat energy heats the kettle and brings the water to a boil. Thus,

$$Q = M_\text{water} \, c_\text{water} \Delta T + M_\text{kettle} \, c_\text{kettle} \Delta T$$

Substituting the given values into this equation,

$$3.6 \times 10^5 \text{ J} = M_\text{water}(4190 \text{ J/kg K})(100°\text{C} - 20°\text{C}) + (0.750 \text{ kg})(449 \text{ J/kg K})(100°\text{C} - 20°\text{C})$$
$$M_\text{water} = 0.994 \text{ kg}$$

The volume of water in the kettle is

$$V = \frac{M_\text{water}}{\rho_\text{water}} = \frac{0.994 \text{ kg}}{1000 \text{ kg/m}^3} = 0.994 \times 10^{-3} \text{ m}^3 = 994 \text{ cm}^3 \approx 990 \text{ cm}^3$$

Assess: $1 \text{ L} = 10^3 \text{ cm}^3$, so $V \approx 1 \text{ L}$. This is a reasonable volume of water.

19.41. Model: Model evaporation in the same manner as for Exercise 19.17.
Solve: At 12 breaths/min, we exhale every minute $M = (12)(25 \times 10^{-6} \text{ kg}) = 300 \times 10^{-6} \text{ kg}$ of water. To vaporize $300 \text{ } \mu\text{g}$ of water at 35°C every minute requires an approximate power of

$$P_2 = \frac{Q_2}{(60 \text{ s})} = \frac{M L_\text{v}}{(60 \text{ s})} = \frac{(300 \times 10^{-6} \text{ kg})(24 \times 10^5 \text{ J/kg})}{60 \text{ s}} = 12 \text{ J/s}$$

Assess: This is much less than the energy lost from the body by radiation.

19.42. Model: There are three interacting systems: metal, aluminum, and water.
Solve: The metal, aluminum container, and water form a closed system, so $Q_\text{m} + Q_\text{Al} + Q_\text{w} = 0 \text{ J}$, where Q_m is the heat transferred to the metal sample. This equation can be written as

$$M_\text{m} c_\text{m} \Delta T_\text{m} + M_\text{Al} c_\text{Al} \Delta T_\text{Al} + M_\text{w} c_\text{w} \Delta T = 0 \text{ J}$$

Substituting in the given values gives

$$(0.512 \text{ kg}) c_\text{m}(351 \text{ K} - 288 \text{ K}) + (0.100 \text{ kg})(900 \text{ J/kg K})(351 \text{ K} - 371 \text{ K})$$
$$+ (0.325 \text{ kg})(4190 \text{ J/kg K})(351 \text{ K} - 371 \text{ K}) = 0 \text{ J}$$
$$c_\text{m} = 900 \text{ J/kg K}$$

From Table 19.2, we see that this is the specific heat of aluminum.

19.43. Model: Heating the water raises its thermal energy and its temperature.
Solve: A 5.0 kW heater has power $P = 5000 \text{ W}$. That is, it supplies heat energy at the rate 5000 J/s. The heat supplied in time Δt is $Q = 5000 \Delta t \text{ J}$. The temperature increase is $\Delta T_\text{C} = (5/9) \Delta T_\text{F} = (5/9)(75°) \, 41.67°\text{C}$. Thus

$$Q = 5000 \Delta t \text{ J} = M c_\text{w} \Delta T = (150 \text{ kg})(4190 \text{ J/kg K})(41.67°\text{C}) \implies \Delta t = 5283 \text{ s} \approx 87 \text{ min}$$

Assess: A time of ~1.5 hours to heat 40 gallons of water is reasonable.

19.44. Model: Do *not* assume the c of diamond is constant like we do for most solids.
Visualize: See this as an integration where the little amounts of Q (i.e., dQ) are added up to give the total Q.
Solve: Use the conduction heat transfer formula. The c goes in the integrand because it changes.

$$Q = \int dQ = M \int c \, dT = (3.5 \text{ carat}) \left(\frac{0.20 \text{ kg}}{\text{carat}} \right) \int_{200 \text{ K}}^{600 \text{ K}} (2.8T - 350 \text{ J/kg K}) dT$$

$$= (0.70) \left[\frac{2.8}{2} T^2 - 350T \right]_{200}^{600} = (0.70)[294{,}000 - (-14{,}000)] = 220 \text{ kJ}$$

It is messy and lengthy to track all the S.I. units through every step of an integration, but we used S.I. units on every quantity, so they come out right in the end.

Assess: This is a lot of energy, but it is also a big diamond and a large temperature range.

19.45 Model: We need to turn the lava from a liquid to a solid. We'll first cool it to its melting temperature and then change its phase at the constant temperature. Once it is solid we do not need to cool it any further with additional water.

Visualize: Our strategy will be to first find the mass of lava we need to solidify each minute, then how much energy (removal) it would take to do that, and then how much cold water will remove that much energy.

Solve: The volume of lava we need to solidify each minute is the cross-sectional area times the velocity times 1 min.

$$V = A \times L = A \times v\Delta t = (400 \text{ m})(0.35 \text{ m})(1.0 \text{ m/min})(1 \text{ min}) = 140 \text{ m}^3$$

The mass of this volume of lava is

$$M = \rho V = (2500 \text{ kg/m}^3)(140 \text{ m}^3) = 350{,}000 \text{ kg}$$

The required heat removal from this much lava is

$$Q = Mc\Delta T + ML_f = M(c\Delta T + L_f) = (350{,}000 \text{ kg})((1100 \text{ J/kg} \cdot \text{K})(400 \text{ K}) + 4.0 \times 10^5 \text{ J/kg})$$
$$= 2.94 \times 10^{11} \text{ J}$$

Wow! We'll need a lot of water to absorb that much energy each minute. The water absorbs energy as it warms up to the boiling temperature and also as it boils/vaporizes. We'll use an equation very similar to the one above, but solve for M. Look up the specific heat and heat of vaporization for water: $c_w = 4190 \text{ J/kg} \cdot \text{K}$, $(L_v)_w = 22.6 \times 10^5 \text{ J/kg}$.

$$Q = M(c\Delta T + L_v) \implies$$

$$M = \frac{Q}{c\Delta T + L_v} = \frac{2.94 \times 10^{11} \text{J}}{(4190 \text{ J/kg} \cdot \text{K})(80 \text{ K}) + 22.6 \times 10^5 \text{ J/kg}} = 1.1 \times 10^5 \text{ kg}$$

Since 1 L of water has a mass of 1 kg, the number of liters of water needed to cool the lava is also 1.1×10^5 L every minute.

Assess: The emergency and its solution outlined in this problem really happened in 1973 in Iceland. Search the web for Eldfell 1973.

19.46. Model: Assume that your body heats the air in an isobaric process, and model air as consisting essentially of nitrogen.

Solve: (a) Use the ideal gas to find the moles of air in a $V_i = 3.0 \text{ L} = 3.0 \times 10^{-3} \text{ m}^3$ volume at the given pressure:

$$n = \frac{p_i V_i}{R T_i}$$

where $T_i = 0°\text{C} = 273 \text{ K}$ and $p_i = 1 \text{ atm} = 101.3 \text{ kPa}$. Use Equation 19.24 to find the energy required to change the gas temperature:

$$Q = nC_P \Delta T = \frac{p_i V_i C_P \Delta T}{R T_i} = \frac{(101.3 \text{ kPa})(3.0 \times 10^{-3} \text{ m}^3)(29.1 \text{ J/mol K})(37 \text{ K})}{(8.31 \text{ J/mol K})(273 \text{ K})} \approx 140 \text{ J}$$

(b) Take the ratio of the ideal gas law at the initial and final temperature to find the volume increase:

$$\frac{p_i V_i}{p_f V_f} = \frac{nRT_1}{nRT_2} \implies V_f = V_i \frac{T_f}{T_i} = (3.0 \text{ L})\frac{310 \text{ K}}{273 \text{ K}} = 3.4 \text{ L}$$

so the change in volume is 0.4 L.

Assess: The volume change is about 13%, which is not inconsiderable.

19.47. Model: There are two interacting systems: coffee (i.e., water) and ice. Changing the coffee temperature from $90°\text{C}$ to $60°\text{C}$ requires four steps: (1) raise the temperature of ice from $-20°\text{C}$ to $0°\text{C}$, (2) change ice at $0°\text{C}$ to water at $0°\text{C}$, (c) raise the water temperature from $0°\text{C}$ to $60°\text{C}$, and (4) lower the coffee temperature from $90°\text{C}$ to $60°\text{C}$.

Solve: For the closed coffee-ice system,

$$Q = Q_{ice} + Q_{coffee} = (Q_1 + Q_2 + Q_3) + (Q_4) = 0 \text{ J}$$
$$Q_1 = M_{ice}c_{ice}\Delta T = M_{ice}(2090 \text{ J/kg K})(20 \text{ K}) = M_{ice}(41,800 \text{ J/kg})$$
$$Q_2 = M_{ice}L_f = M_{ice}(330,000 \text{ J/kg})$$
$$Q_3 = M_{ice}c_{water}\Delta T = M_{ice}(4190 \text{ J/kg K})(60 \text{ K}) = M_{ice}(251,400 \text{ J/kg})$$
$$Q_4 = M_{coffee}c_{coffee}\Delta T = (300 \times 10^{-6} \text{ m}^3)(1000 \text{ kg/m}^3)(4190 \text{ J/kg K})(-30 \text{ K}) = -37,000 \text{ J}$$

The $Q = 0 \text{ J}$ equation thus becomes

$$M_{ice}(41,800 + 330,000 + 251,400) \text{ J/kg} - 37,710 \text{ J} = 0 \text{ J} \Rightarrow M_{ice} = 0.061 \text{ kg} = 61 \text{ g}$$

Assess: 61 g is the mass of approximately 1 ice cube.

19.48. Model: There are two interacting systems: the nuclear reactor and the water. The heat generated by the nuclear reactor is used to raise the water temperature.
Solve: For the closed reactor-water system, energy conservation per second requires

$$Q = Q_{reactor} + Q_{water} = 0 \text{ J}$$

The heat from the reactor in $\Delta t = 1 \text{ s}$ is $Q_{reactor} = -2000 \text{ MJ} = -2.0 \times 10^9 \text{ J}$ and the heat absorbed by the water is

$$Q_{water} = m_{water}c_{water}\Delta T = m_{water}(4190 \text{ J/kg K})(12 \text{ K})$$

$$-2.0 \times 10^9 \text{ J} + m_{water}(4190 \text{ J/kg K})(12 \text{ K}) = 0 \text{ J} \Rightarrow m_{water} = 3.98 \times 10^4 \text{ kg}$$

Each second, 3.98×10^4 kg of water is needed to remove heat from the nuclear reactor. Thus, the water flow per minute is

$$\left(3.98 \times 10^4 \text{ } \frac{\text{kg}}{\text{s}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right)\left(\frac{1 \text{ m}^3}{1000 \text{ kg}}\right)\left(\frac{1 \text{ L}}{10^{-3} \text{ m}^3}\right) = 2.4 \times 10^6 \text{ L/min}$$

Thus, 2.4×10^6 L of cooling water must flow through the reactor each minute.

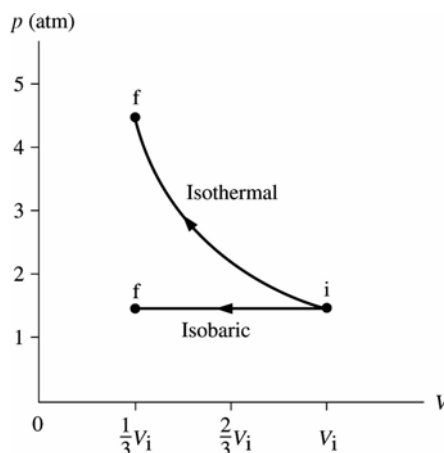**19.49. Model:** These are isothermal and isobaric ideal-gas processes.
Solve: (a) The work done at constant temperature is

$$W = -\int_{V_i}^{V_f} p \, dV = -\int_{V_i}^{V_f} \frac{nRT}{V} \, dV = -nRT(\ln V_f - \ln V_i) = -nRT \ln(V_f/V_i)$$

$$= -(2.0 \text{ mol})(8.31 \text{ J/mol K})(303 \text{ K})\ln\left(\tfrac{1}{3}\right) = 5.5 \text{ kJ}$$

(b) The work done at constant pressure is

$$W = -\int_{V_i}^{V_f} p \, dV = -p(V_f - V_i) = -p\left(\frac{V_i}{3} - V_i\right) = \frac{2}{3}pV_i$$

$$= \frac{2}{3}nRT = \frac{2}{3}(2.0 \text{ mol})(8.31 \text{ J/mol K})(303 \text{ K}) = 3.4 \text{ kJ}$$

(c) For an isothermal process in which $V_f = \frac{1}{3}V_i$, the pressure changes to $p_f = 3p_i = 4.5$ atm.

19.50. Model: The gas is an ideal gas, and its thermal energy is the total kinetic energy of the moving molecules.
Visualize: Please refer to Figure P19.51.
Solve: (a) The piston is floating in static equilibrium, so the downward force of gravity on the piston's mass must exactly balance the upward force of the gas, $F_{gas} = pA$ where $A = \pi r^2$ is the area of the face of the piston. Since the upper part of the cylinder is evacuated, there is no gas pressure force pushing downward. Thus,

$$M_{piston}g = pA \Rightarrow p = \frac{M_{piston}g}{A} = \frac{\rho_{Cu}V_{piston}g}{A} = \rho_{Cu}gh = (8920 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.040 \text{ m}) = 3500 \text{ Pa}$$

(b) The gas volume is $V_1 = \pi r^2 L = \pi(0.030)^2(0.20 \text{ m}) = 5.65 \times 10^{-4} \text{ m}^3$. The number of moles is

$$n = \frac{p_1 V_1}{RT_1} = \frac{(3500 \text{ Pa})(5.65 \times 10^{-4} \text{ m}^3)}{(8.31 \text{ J/mol K})(293 \text{ K})} = 8.12 \times 10^{-4} \text{ mol}$$

The number of molecules is

$$N = nN_A = (8.12 \times 10^{-4} \text{ mol})(6.02 \times 10^{23} \text{ mol}^{-1}) = 4.9 \times 10^{20}$$

(c) The pressure in the gas is determined simply by the weight of the piston. That will not change as heat is added, so the heating takes place at constant pressure with $Q = nC_P\Delta T$. The temperature increase is

$$\Delta T = \frac{Q}{nC_P} = \frac{2.0 \text{ J}}{(8.12 \times 10^{-4} \text{ mol})(29.1 \text{ J/mol K})} = 85 \text{ K}$$

This raises the gas temperature to $T_2 = T_1 + \Delta T = 378 \text{ K} = 105°C$. To two significant figures, this is 110°C.

(d) Noting that the volume of a cylinder is $V = \pi r^2 L$ and that r doesn't change, the ideal-gas relationship for an isobaric process is

$$\frac{V_2}{T_2} = \frac{V_1}{T_1} \Rightarrow \frac{L_2}{T_2} = \frac{L_1}{T_1} \Rightarrow L_2 = \frac{T_2}{T_1}L_1 = \left(\frac{378 \text{ K}}{293 \text{ K}}\right)20 \text{ cm} = 26 \text{ cm}$$

(e) The work done by the gas is $W_{gas} = F_{gas}\Delta y$. The force exerted on the piston by the gas is

$$F_{gas} = pA = p\pi r^2 = 9.90 \text{ N}$$

This force is applied through $\Delta y = 5.8 \text{ cm} = 0.058 \text{ m}$, so the work done is

$$W_{gas} = (9.90 \text{ N})(0.058 \text{ m}) = 0.57 \text{ J}$$

Thus, 0.57 J is the work done by the gas on the piston. The work done on the gas is -0.57 J.

19.51. Model: Assume an ideal gas. Because this is done slowly it is an isothermal process, so $\Delta T = 0$.
Visualize: In such a process $\Delta E_{th} = 0$. We are given $m = 560$ kg, $L = 1.20$ m, $r = 12.5$ cm, and $p_1 = 1.0$ atm.
Solve: The work done in an isothermal process is $W = -p_i V_i \ln(V_f / V_i)$. But because $p_i V_i = p_f V_f$, $V_f / V_i = p_i / p_f$, so

$$W = -p_i V_i \ln\left(\frac{p_i}{p_f}\right)$$

We will need the final pressure from the weight of the slab. The area of all four cylinders is $A = 4(\pi r^2)$.

$$p_f = F/A + p_i = F/A + 1.0 \text{ atm} = mg/4(\pi r^2) + 1.0 \text{ atm}$$
$$= (560 \text{ kg})(9.8 \text{ m/s}^2)/4\pi(0.125 \text{ m})^2 + 1.0 \text{ atm} = 279.5 \text{ kPa} + 101.3 \text{ kPa} = 129.3 \text{ kPa}$$

We also need to compute the initial volume of the 4 cylinders.

$$V_i = (4)\pi r^2 L = (4)\pi(0.125 \text{ m})^2(1.20 \text{ m}) = 0.2356 \text{ m}^3$$

Now we can compute the work done on the gas in the four cylinders.

$$W = -p_i V_i \ln\left(\frac{p_i}{p_f}\right) = -(101.3 \text{ kPa})(0.2356 \text{ m}^3)\ln\left(\frac{101.3 \text{ kPa}}{129.3 \text{ kPa}}\right) = 5.8 \text{ kJ}$$

Assess: This is a moderate amount of work done on the gas.

19.52. Model: Assume an ideal gas.
Visualize: The initial temperature of the gas is $150°C = 423$ K.
Solve: (a) The work done in an ideal gas process is

$$W = -\int p\, dV = -\int_{V_1}^{V_2} (cV^{1/2})dV = -c\left[\tfrac{2}{3}V^{3/2}\right]_{V_1}^{V_2} = \tfrac{2}{3}c\left(V_1^{3/2} - V_2^{3/2}\right)$$

(b) We will need to solve for the constant c from the ideal gas law.

$$p_1 V_1 = nRT_1 \Rightarrow p_1 = \frac{nRT_1}{V_1} = cV_1^{1/2} \Rightarrow c = \frac{nRT_1}{V_1^{3/2}}$$

Now we plug this into the formula for work done on the gas.

$$W = \tfrac{2}{3}c\left(V_1^{3/2} - V_2^{3/2}\right) = \tfrac{2}{3}\left(\frac{nRT_1}{V_1^{3/2}}\right)\left(V_1^{3/2} - V_2^{3/2}\right) = \tfrac{2}{3}nRT_1\left(1 - \frac{V_2^{3/2}}{V_1^{3/2}}\right)$$

$$= \tfrac{2}{3}(0.033 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(423 \text{ K})\left(1 - \frac{\left(200 \text{ cm}^3\right)^{3/2}}{\left(300 \text{ cm}^2\right)^{3/2}}\right) = 35 \text{ J}$$

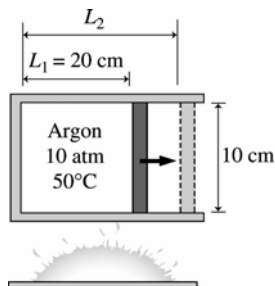**(c)** Use the ideal gas law to find T_2.

$$T_2 = \frac{p_2 V_2}{nR} = \frac{\left(cV_2^{1/2}\right)V_2}{nR} = \frac{cV_2^{3/2}}{nR} = \frac{\left(\frac{nRT_1}{V_1^{3/2}}\right)V_2^{3/2}}{nR} = T_1\left(\frac{V_2}{V_1}\right)^{3/2} = (423 \text{ K})\left(\frac{200 \text{ cm}^3}{300 \text{ cm}^3}\right)^{3/2} = 230 \text{ K}$$

Convert this back to Celsius and the final temperature is $-43°C$.
Assess: This is a moderate amount of work done on the gas.

19.53. Model: This is an isobaric process.
Visualize:

Solve: (a) The initial conditions are $p_1 = 10$ atm $= 1.013 \times 10^6$ Pa, $T_1 = 50°C = 323$ K, $V_1 = \pi r^2 L_1 = \pi (0.050\ \text{m})^2$

$(0.20\ \text{m}) = 1.57 \times 10^{-3}$ m^3. The gas is heated at a constant pressure, so heat and temperature change are related by

$Q = nC_p \Delta T$. From the ideal gas law, the number of moles of gas is

$$n = \frac{p_1 V_1}{RT_1} = \frac{(1.013 \times 10^6\ \text{Pa})(1.57 \times 10^{-3}\ \text{m}^3)}{(8.31\ \text{J/mol K})(323\ \text{K})} = 0.593\ \text{mol}$$

The temperature change due to the addition of $Q = 2500$ J of heat is thus

$$\Delta T = \frac{Q}{nC_P} = \frac{2500\ \text{J}}{(0.593\ \text{mol})(20.8\ \text{J/mol K})} = 203\ \text{K}$$

The final temperature is $T_2 = T_1 + \Delta T = 526$ K $= 253°C$, or $250°C$ to two significant figures.

(b) Noting that the volume of a cylinder is $V = \pi r^2 L$ and that r doesn't change, the ideal gas relationship for an isobaric process is

$$\frac{V_2}{T_2} = \frac{V_1}{T_1} \ \Rightarrow\ \frac{L_2}{T_2} = \frac{L_1}{T_1} \ \Rightarrow\ L_2 = \frac{T_2}{T_1} L_1 = \frac{526\ \text{K}}{323\ \text{K}}(20\ \text{cm}) = 33\ \text{cm}$$

19.54. Model: The process in part (a) is isochoric and the process in part (b) is isobaric.
Solve: (a) Initially $V_1 = (0.20\ \text{m})^3 = 0.0080$ m$^3 = 8.0$ L and $T_1 = 293$ K. Helium has an atomic mass number $A = 4$,

so 3 g of helium is $n = M/M_{\text{mol}} = 0.75$ mole of helium. We can find the initial pressure from the ideal-gas law:

$$p_1 = \frac{nRT_1}{V_1} = \frac{(0.75\ \text{mol})(8.31\ \text{J/mol K})(293\ \text{K})}{0.0080\ \text{m}^3} = 228\ \text{kPa} = 2.25\ \text{atm}$$

Heating the gas will raise its temperature. A constant-volume process has $Q = nC_V \Delta T$, so

$$\Delta T = \frac{Q}{nC_V} = \frac{1000\ \text{J}}{(0.75\ \text{mol})(12.5\ \text{J/mol K})} = 107\ \text{K}$$

This raises the final temperature to $T_2 = T_1 + \Delta T = 400$ K. Because the process is isochoric,

$$\frac{p_2}{T_2} = \frac{p_1}{T_1} \ \Rightarrow\ p_2 = \frac{T_2}{T_1} p_1 = \frac{400\ \text{K}}{293\ \text{K}}(2.25\ \text{atm}) = 3.1\ \text{atm}$$
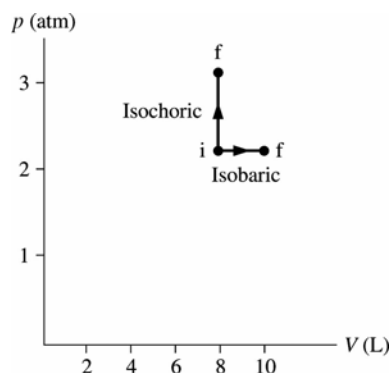
(b) The initial conditions are the same as part a, but now $Q = nC_P \Delta T$. Thus,

$$\Delta T = \frac{Q}{nC_P} = \frac{1000\ \text{J}}{(0.75\ \text{mol})(20.8\ \text{J/mol K})} = 64.1\ \text{K}$$

Now the final temperature is $T_2 = T_1 + \Delta T = 357$ K. Because the process is isobaric,

$$\frac{V_2}{T_2} = \frac{V_1}{T_1} \ \Rightarrow\ V_2 = \frac{T_2}{T_1} V_1 = \frac{357\ \text{K}}{293\ \text{K}}(0.0080\ \text{m}^3) = 0.0097\ \text{m}^3 = 9.7\ \text{L}$$

(c)

19.55. Model: Assume the air and the nitrogen are ideal gases. For the piston to be in equilibrium the forces on it must sum to zero. Also assume that the temperature doesn't change in the second part.

Solve: The area is $A = \pi r^2 = \pi (0.040 \text{ m})^2 = 5.0 \times 10^{-3} \text{ m}^2$. $V_1 = Ah_1 = (5.0 \times 10^{-3} \text{ m}^2)(0.26 \text{ m}) = 0.0013 \text{ m}^3$.

(a)

$$F_{\text{net}} = p_{\text{in}}A - mg - p_{\text{above}}A = 0 \text{ N}$$

$$p_{\text{in}} = \frac{mg + p_{\text{above}}A}{A} = \frac{(5.1 \text{ kg})(9.8 \text{ m/s}^2) + (100 \text{ kPa})(5.0 \times 10^{-3} \text{ m}^2)}{5.0 \times 10^{-3} \text{ m}^2} = 110 \text{ kPa}$$

(b) Since $T_1 = T_2$, then $p_1 V_1 = p_2 V_2$ where p_1 is p_{in} from above, and p_2 is the inside pressure after the new weight is added. Compute p_2 exactly as above replacing 5.1 kg with $5.1 \text{ kg} + 3.5 \text{ kg} = 8.6 \text{ kg}$.

$$p_2 = \frac{mg + p_{\text{above}}A}{A} = \frac{(8.6 \text{ kg})(9.8 \text{ m/s}^2) + (100 \text{ kPa})(5.0 \times 10^{-3} \text{ m}^2)}{5.0 \times 10^{-3} \text{ m}^2} = 117 \text{ kPa}$$

$$V_2 = \frac{p_1 V_1}{p_2} = \frac{(110 \text{ kPa})(0.0013 \text{ m}^3)}{117 \text{ kPa}} = 1.22 \times 10^{-3} \text{ m}^3$$

$$h_2 = \frac{V_2}{A} = \frac{1.22 \times 10^{-3} \text{ m}^3}{5.0 \times 10^{-3} \text{ m}^2} = 0.2447 \text{ m} \approx 24 \text{ cm}$$

Assess: The result seems to be a reasonable number; we expected the piston to be a little lower than before due to the increased mass on it.

19.56. Model: This is an isothermal process.

Solve: (a) The final temperature is $T_2 = T_1$ because the process is isothermal.

(b) The work done on the gas is

$$W = -\int_{V_1}^{V_2} p\,dV = -\int_{V_1}^{V_2} \frac{nRT_1}{V}\,dV = -nRT_1 \ln \frac{V_2}{V_1} = -nRT_1 \ln 2$$

(c) From the first law of thermodynamics $\Delta E_{\text{th}} = W + Q = 0 \text{ J}$ because $\Delta T = 0 \text{ K}$. Thus, the heat energy transferred to the gas is $Q = -W = nRT_1 \ln 2$.

19.57. Model: The gas is an ideal gas and it goes through an isobaric and an isochoric process.

Solve: (a) The initial conditions are $p_1 = 3.0\,\text{atm} = 304{,}000\,\text{Pa}$ and $T_1 = 293\,\text{K}$. Nitrogen has a molar mass $M_{mol} = 28\,\text{g/mol}$, so 5.0 g of nitrogen gas has $n = M/M_{mol} = 0.1786\,\text{mol}$. From this, we can find the initial volume:

$$V_1 = \frac{nRT_1}{p_1} = \frac{(0.1786\text{ mol})(8.31\text{ J/mol K})(293\text{ K})}{304{,}000\text{ Pa}} = 1.430 \times 10^{-3}\text{ m}^3 \approx 1400\text{ cm}^3$$

The volume triples, so $V_2 = 3V_1 = 4300\,\text{cm}^3$. The expansion is isobaric $(p_2 = p_1 = 3.0\text{ atm})$, so

$$\frac{V_2}{T_2} = \frac{V_1}{T_1} \quad \Rightarrow \quad T_2 = \frac{V_2}{V_1}T_1 = (3)293\text{ K} = 879\text{ K} \approx 610°\text{C}$$

(b) The process is isobaric, so

$$Q = nC_P\Delta T = (0.1786\text{ mol})(29.1\text{ J/mol K})(879\text{ K} - 293\text{ K}) = 3000\text{ J}$$

(c) The pressure is decreased at constant volume $(V_3 = V_2 = 4290\text{ cm}^3)$ until the original temperature is reached $(T_3 = T_1 = 293\text{ K})$. For an isochoric process,
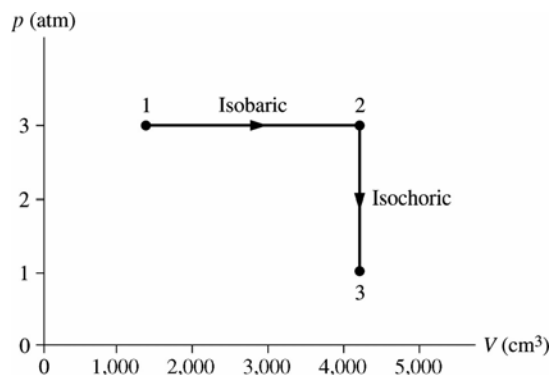
$$\frac{p_3}{T_3} = \frac{p_2}{T_2} \quad \Rightarrow \quad p_3 = \frac{T_3}{T_2}p_2 = \frac{293\text{ K}}{879\text{ K}}(3.0\text{ atm}) = 1.0\text{ atm}$$

(d) The process is isochoric, so

$$Q = nC_V\Delta T = (0.1786\text{ mol})(20.8\text{ J/mol K})(293\text{ K} - 879\text{ K}) = -2200\text{ J}$$

So, 2200 J of heat was removed to decrease the pressure.

(e)

19.58. Model: We have an adiabatic and an isothermal process.

Solve: For the adiabatic process, no heat is added or removed. That is $Q = 0\,\text{J}$.

Isothermal processes occur at a fixed temperature, so $\Delta T = 0\,\text{K}$. Thus $\Delta E_{th} = 0\,\text{J}$, and the first law of thermodynamics gives

$$Q = -W = nRT\ln(V_f/V_i)$$

The temperature T can be obtained from the ideal-gas equation as follows:

$$p_iV_i = nRT \quad \Rightarrow \quad T = \frac{p_iV_i}{nR} = \frac{(1.013 \times 10^5\text{ Pa})(3000 \times 10^{-6}\text{ m}^3)}{(0.10\text{ mol})(8.31\text{ J/mol K})} = 366\text{ K}$$

Substituting into the equation for Q we get

$$Q = (0.10\text{ mol})(8.31\text{ J/mol K})(366\text{ K})\ln\left(\frac{1000 \times 10^{-6}\text{ m}^3}{3000 \times 10^{-6}\text{ m}^3}\right) = -330\text{ J}$$

That is, 330 J of heat energy is removed from the gas.

19.59. Model: There is a thermal interaction between the iron, assumed to be initially at room temperature $(20°C)$, and the liquid nitrogen. The boiling point of liquid nitrogen is $-196°C = 77$ K.

Solve: The piece of iron has mass $M_{iron} = 197$ g $= 0.197$ kg and volume $V_{iron} = M_{iron}/\rho_{iron} = (0.197$ kg$)/$ $(7870$ kg/m$^3) = 25 \times 10^{-6}$ m$^3 = 25$ cm$^3 = 25$ mL. The heat lost by the iron is

$$Q_{iron} = M_{iron}c_{iron}\Delta T = (0.197 \text{ kg})(449 \text{ J/kg K})(77 \text{ K } - 293 \text{ K}) = -1.911 \times 10^4 \text{ J}$$

This heat causes mass M of the liquid nitrogen to boil. Energy conservation requires

$$Q_{iron} + Q_{N2} = Q_{iron} + ML_f = 0 \Rightarrow M = -\frac{Q_{iron}}{L_f} = \frac{1.911 \times 10^4 \text{ J}}{1.99 \times 10^5 \text{ J/kg}} = 0.0960 \text{ kg}$$

The volume of liquid nitrogen boiled away is thus

$$V_{boil} = \frac{M}{\rho_{N2}} = \frac{0.0960 \text{ kg}}{810 \text{ kg/m}^3} = 1.19 \times 10^{-4} \text{ m}^3 = 119 \text{ mL}$$

Now the volume of nitrogen gas (at 77 K) is 1500 mL before the iron is dropped in. The volume of the piece of iron excludes 25 mL of gas, so the initial gas volume, when the lid is sealed and the liquid starts to boil, is $V_1 = 1475$ mL. The pressure is $p_1 = 1.0$ atm and the temperature is $T_1 = 77$ K. Thus the number of moles of nitrogen gas is

$$n_1 = \frac{p_1 V_1}{RT_1} = \frac{(101,300 \text{ Pa})(1.475 \times 10^{-3} \text{ m}^3)}{(8.31 \text{ J/mol K})(77 \text{ K})} = 0.234 \text{ mol}$$

119 mL of liquid boils away, so the gas volume increases to $V_2 = V_1 = V_{boil} = 1475$ mL $+ 119$ mL $= 1594$ mL. The temperature is still $T_2 = 77$ K, but the number of moles of gas has been increased by the liquid that boiled. The number of moles that boiled away is

$$n_{boil} = \frac{96.0 \text{ g}}{28 \text{ mol/g}} = 3.429 \text{ mol}$$

Thus the number of moles of nitrogen gas increases to $n_2 = n_1 + n_{boil} = 0.234$ mol $+ 3.429$ mol $= 3.663$ mol. Consequently, the gas pressure increases to

$$p_2 = \frac{n_2 RT_2}{V_2} = \frac{(3.663 \text{ mol})(8.31 \text{ J/mol K})(77 \text{ K})}{1.594 \times 10^{-3} \text{ m}^3} = 1.470 \times 10^6 \text{ Pa} = 15 \text{ atm}$$

Assess: Don't try this! The large pressure increase could cause a flask of liquid nitrogen to explode, leading to serious injuries.

19.60. Model: Model the gas as ideal and the compressions as adiabatic processes because they are rapid.
Solve: The gas temperature changes as the cylinder volume changes. Applying Equation 19.38 gives

$$TV^{\gamma-1} = T_i V_i^{\gamma-1} \quad \Rightarrow \quad TL^{\gamma-1} = T_i L_i^{\gamma-1} = \text{constant } C$$

The volume of the gas is $V = AL$, but since the cross section area of the cylinder doesn't change, we need only consider the cylinder lengths L. The constant C depends on the initial conditions, but is the same for each push. Note that the data show how far the piston is pushed. Thus the length is

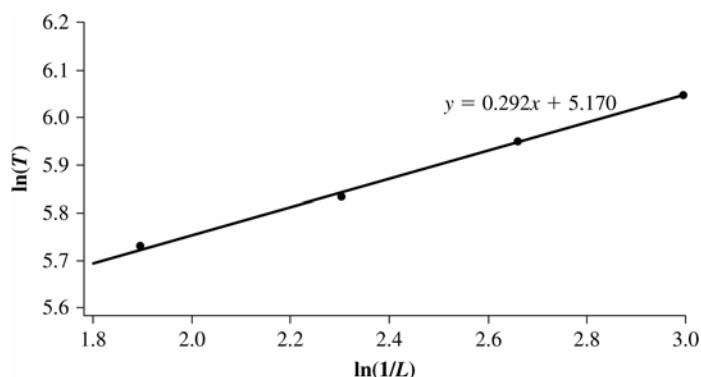$$L = 20 \text{ cm} - \text{push distance}$$

Rearranging the earlier equation gives

$$TL^{\gamma-1} = C \quad \Rightarrow \quad T = \frac{C}{L^{\gamma-1}} = C\left(\frac{1}{L}\right)^{\gamma-1}$$

Taking natural logarithms of both sides of the result gives us

$$\ln T = \ln\left[C\left(\frac{1}{L}\right)^{\gamma-1}\right] = (\gamma-1)\ln(1/L) + \ln C$$

If we plot $\ln T$ versus $\ln(1/L)$, the graph should be a straight line with slope $\gamma - 1$. The plot is shown below, with T converted to K and L to m. The linear fit is quite good. The experimental slope of 0.292 gives $\gamma = 1.29$.

Assess: Monatomic ideal gases have $\gamma = 1.67$ and diatomic gases have $\gamma = 1.40$, suggesting that γ decreases as the basic particles of the gas become more complex and therefore have more degrees of freedom. Thus $\gamma = 1.29$ for triatomic CO_2 seems reasonable.

19.61. Model: Assume that the gas is an ideal gas. A diatomic gas has $\gamma = 1.40$.
Solve: (a) For container A,

$$V_{iA} = \frac{nRT_{Ai}}{p_{Ai}} = \frac{(0.10 \text{ mol})(8.31 \text{ J/mol K})(300 \text{ K})}{3.0 \times 1.013 \times 10^5 \text{ Pa}} = 8.20 \times 10^{-4} \text{ m}^3$$

For an isothermal process $p_{Af}V_{Af} = p_{Ai}V_{Ai}$. This means $T_{Af} = T_{Ai} = 300 \text{ K}$ and

$$V_{Af} = V_{Ai}(p_{Ai}/p_{Af}) = (8.20 \times 10^{-4} \text{ m}^3)(3.0 \text{ atm}/1.0 \text{ atm}) = 2.5 \times 10^{-3} \text{ m}^3$$
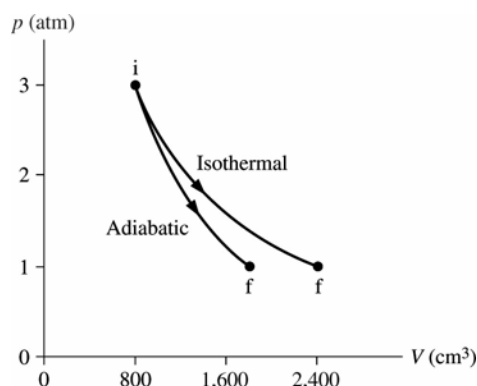
The gas in container B starts with the same initial volume. For an adiabatic process,

$$p_{Bf}V_{Bf}^{\gamma} = p_{Bi}V_{Bi}^{\gamma} \implies V_{Bf} = V_{Bi}\left(\frac{p_{Bi}}{p_{Bf}}\right)^{1/\gamma} = (8.20 \times 10^{-4} \text{ m}^3)\left(\frac{3.0 \text{ atm}}{1.0 \text{ atm}}\right)^{1/1.40} = 1.8 \times 10^{-3} \text{ m}^3$$

The final temperature T_{Bf} can now be obtained by using the ideal-gas equation:

$$T_{Bf} = T_{iB}\frac{p_{Bf}}{p_{Bi}}\frac{V_{Bf}}{V_{Bi}} = (300 \text{ K})\left(\frac{1.0 \text{ atm}}{3.0 \text{ atm}}\right)\left(\frac{1.80 \times 10^{-3} \text{ m}^3}{8.20 \times 10^{-4} \text{ m}^3}\right) = 220 \text{ K}$$

(b)

19.62. Model: Assume that the gas is an ideal gas and that the work, heat, and thermal energy are connected by the first law of thermodynamics.

Solve: (a) For point 1, $V_1 = 1000 \text{ cm}^3 = 1.0 \times 10^{-3} \text{ m}^3$, $T_1 = 133°C = 406 \text{ K}$, and the number of moles is

$$n = \frac{M}{M_{\text{mol}}} = \left(\frac{120 \times 10^{-3} \text{ g}}{4 \text{ g/mol}} \right) = 0.030 \text{ mol}$$

Thus, the pressure p_1 is

$$p_1 = \frac{nRT_1}{V_1} = 1.012 \times 10^5 \text{ Pa} = 1.0 \text{ atm}$$

The process $1 \rightarrow 2$ is isochoric $(V_2 = V_1)$ and $p_2 = 5p_1 = 5.0 \text{ atm}$. Thus,

$$T_2 = T_1(p_2/p_1) = (406 \text{ K})(5) = 2030 \text{ K} = 1757°C$$

The process $2 \rightarrow 3$ is isothermal $(T_2 = T_3)$, so

$$V_3 = V_2(p_2/p_3) = V_2(p_2/p_1) = 5V_2 = 5000 \text{ cm}^3$$

	p (atm)	T(°C)	V(cm^3)
Point 1	1.0	133	1000
Point 2	5.0	1757	1000
Point 3	1.0	1757	5000

(b) The work $W_{1 \rightarrow 2} = 0$ J because it is an isochoric process. The work in process $2 \rightarrow 3$ can be found using Equation 19.14 as follows:

$$W_{2 \rightarrow 3} = -nRT_2 \ln(V_3/V_2) = -(0.030 \text{ mol})(8.31 \text{ J/mol K})(2030 \text{ K}) \ln(5) = -815 \text{ J}$$

The work in the isobaric process $3 \rightarrow 1$ is

$$W_{3 \rightarrow 1} = -p(V_f - V_i) = -(1.012 \times 10^5 \text{ Pa})(1.0 \times 10^{-3} \text{ m}^3 - 5.0 \times 10^{-3} \text{ m}^3) = 405 \text{ J}$$

(c) The heat transferred in process $1 \rightarrow 2$ is

$$Q_{1 \rightarrow 2} = nC_V \Delta T = (0.030 \text{ mol})(12.5 \text{ J/mol K})(2030 \text{ K} - 406 \text{ K}) = 609 \text{ J}$$

The heat transferred in the isothermal process $2 \rightarrow 3$ is $Q_{2 \rightarrow 3} = -W_{2 \rightarrow 3} = 815 \text{ J}$. The heat transferred in the isobaric process $3 \rightarrow 1$ is

$$Q_{3 \rightarrow 1} = nC_P \Delta T = (0.030 \text{ mol})(20.8 \text{ J/mol K})(406 \text{ K} - 2030 \text{ K}) = -1.01 \text{ kJ}$$

19.63. Model: Model the gas as an ideal gas.
Solve: Because the two gases have the same initial conditions, the ideal-gas law tells us that they have the same number of moles. Apply Equation 19.24 for constant pressure to find the heat energy required by the first process: $Q = nC_P \Delta T_1$. Insert this into Equation 19.24 for a constant volume process to find the temperature increase:

$$Q = nC_V \Delta T_2 \quad \Rightarrow \quad \Delta T_2 = \frac{Q}{nC_V} = \frac{nC_P \Delta T_1}{nC_V} = \frac{C_P}{C_V} \Delta T_1 = \gamma \Delta T_1$$

For a diatomic gas, $\gamma = 1.40$ (see Equation 19.34), so the temperature change for the second container of gas is

$$\Delta T_2 = 1.40(20°C) = 28°C$$

Assess: Since $C_V < C_P$, the temperature change at constant volume is greater than the change at constant pressure.

19.64. Model: The nitrogen gas is assumed to be an ideal gas that is subjected to an adiabatic process.
Solve: (a) The number of moles in 14.0 g of N_2 gas is

$$n = \frac{M}{M_{\text{mol}}} = \frac{14.0 \text{ g}}{28 \text{ g/mol}} = 0.50 \text{ mol}$$

At $T_i = 273$ K and $p_i = 1.0$ atm $= 1.013 \times 10^5$ Pa, the gas has a volume

$$V_i = \frac{nRT_i}{p_i} = 0.0112 \text{ m}^3 = 11.2 \text{ L}$$

For an adiabatic process that compresses to a pressure $p_f = 20$ atm, we can use Equations 19.36 and 19.37 as follows:

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1} \quad \Rightarrow \quad \left(\frac{T_f}{T_i}\right) = \left(\frac{V_i}{V_f}\right)^{\gamma-1} \qquad p_f V_f^{\gamma} = p_i V_i^{\gamma} \quad \Rightarrow \quad \left(\frac{p_f}{p_i}\right)^{\frac{1}{\gamma}} = \frac{V_i}{V_f}$$

Combining the above two equations yields

$$(T_f / T_i) = (p_f / p_i)^{\frac{\gamma-1}{\gamma}} \quad \Rightarrow \quad T_f = T_i (20)^{\frac{1.4-1.0}{1.4}} = T_i (2.3535) = 643 \text{ K}$$
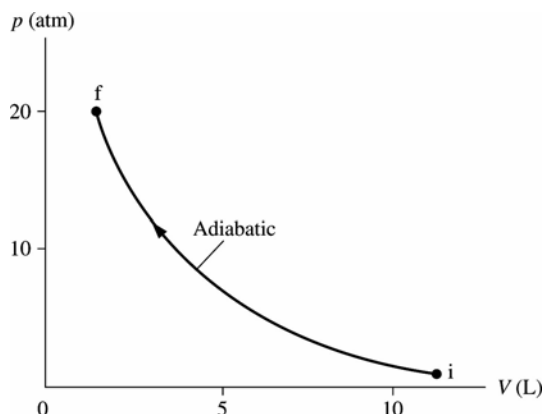
(b) The work done on the gas is

$$W = \Delta E_{th} = nC_V(T_f - T_i) = (0.50 \text{ mol})(20.8 \text{ J/mol K})(643 \text{ K} - 273 \text{ K}) = 3.8 \text{ kJ}$$

(c) The heat input to the gas is $Q = 0$ J.

(d) From the above equation,

$$\frac{V_i}{V_f} = \left(\frac{p_f}{p_i}\right)^{\frac{1}{\gamma}} = (20)^{\frac{1}{1.4}} = 8.5 = \frac{V_{max}}{V_{min}}$$

(e)

19.65. Model: The gas is assumed to be an ideal gas that is subjected to an isochoric process.
Solve: (a) The number of moles in 14.0 g of N_2 gas is

$$n = \frac{M}{M_{mol}} = \frac{14.0 \text{ g}}{28 \text{ g/mol}} = 0.50 \text{ mol}$$

At $T_i = 273$ K and $p_i = 1.0$ atm $= 1.013 \times 10^5$ Pa, the gas has a volume

$$V_i = \frac{nRT_i}{p_i} = 0.0112 \text{ m}^3 = 11.2 \text{ L}$$

For an isochoric process $(V_i = V_f)$,

$$\frac{T_f}{T_i} = \frac{p_f}{p_i} = \frac{20 \text{ atm}}{1 \text{ atm}} = 20 \quad \Rightarrow \quad T_f = 20(273 \text{ K}) = 5.5 \text{ kK}$$

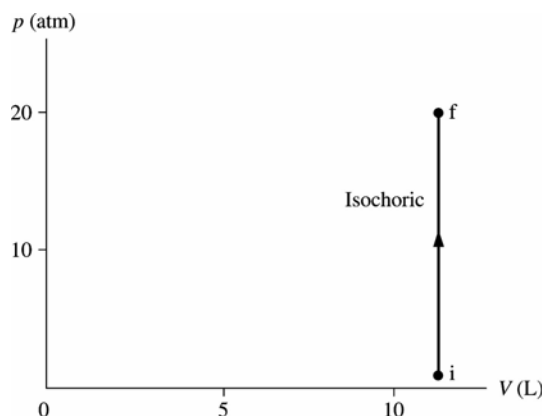**(b)** The work done on the gas is $W = -p\Delta V = 0$ J.

(c) The heat input to the gas is

$$Q = nC_V(T_f - T_i) = (0.50 \text{ mol})(20.8 \text{ J/mol K})(5460 \text{ K} - 273 \text{ K}) = 5.4 \times 10^4 \text{ J}$$

(d) The pressure ratio is

$$\frac{p_{max}}{p_{min}} = \frac{p_f}{p_i} = \frac{20 \text{ atm}}{1 \text{ atm}} = 20$$

(e)

19.66. Model: Assume an ideal gas. The process is isobaric. Assume the temperature of the environment is constant.

Visualize: We are given $\Delta T = T_{gas} - T_{env} = (\Delta T)_0 e^{-t/\tau} \Rightarrow T_{gas} = T_{env} + (\Delta T)_0 e^{-t/\tau}$.

Solve: (a) We'll start with the ideal gas law: $p\Delta V_{gas} = nR\Delta T_{gas}$.

$$p\Delta V_{gas} = nR\Delta T_{gas} = nR(T_{env} + (\Delta T)_0 e^{-t/\tau} - T_{env} - (\Delta T)_0) = nR(\Delta T)_0 (e^{-t/\tau} - 1)$$

The work done in an isobaric process is $W = -p\Delta V$. We'll take the derivative with respect to t to find the rate at which the environment does work on the gas.

$$\frac{dW}{dt} = \frac{d}{dt}(-p\Delta V) = \frac{d}{dt}(-nR(\Delta T)_0(e^{-t/\tau} - 1)) = \frac{nR(\Delta T)_0}{\tau} e^{-t/\tau}$$

(b) Now we plug our values into the formula above.

$$P = \frac{dW}{dt} = \frac{nR(\Delta T)_0}{\tau} e^{-t/\tau} = \frac{(0.15 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(12 \text{ K})}{60 \text{ s}}(1) = 0.25 \text{ W}$$

Assess: This is a reasonable amount of power.

19.67. Model: The air is assumed to be an ideal gas. Because the air is compressed without time to exchange heat with its surroundings, the compression is an adiabatic process.

Solve: The initial pressure of air in the mountains behind Los Angeles is $p_i = 60 \times 10^3$ Pa at $T_i = 273$ K. The pressure of this air when it is carried down to the elevation near sea level is $p_f = 100 \times 10^3$ Pa. The adiabatic compression of a gas leads to an increase in temperature according to Equations 19.37 and 19.38, which are

$$p_f V_f^\gamma = p_i V_i^\gamma \quad \Rightarrow \quad \left(\frac{p_f}{p_i}\right)^{\frac{1}{\gamma}} = \frac{V_i}{V_f} \qquad T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1} \quad \Rightarrow \quad \left(\frac{T_f}{T_i}\right) = \left(\frac{V_i}{V_f}\right)^{\gamma-1}$$

Combining these two equations,

$$(T_f/T_i) = (p_f/p_i)^{\frac{\gamma-1}{\gamma}} \quad \Rightarrow \quad T_f = T_i\left(\frac{100 \times 10^3 \text{ Pa}}{60 \times 10^3 \text{ Pa}}\right)^{\frac{1.4-1}{1.4}} = (273 \text{ K})\left(\frac{5}{3}\right)^{0.286} = 316 \text{ K} = 43°\text{C} = 109°\text{F}$$

19.68. Model: Assume the collector has emissivity $e = 1$. We will use Equation 19.47 and divide both sides by A to get intensity in W/m^2. At the equilibrium temperature the collector will absorb $800\ W/m^2$ on the sun side and must radiate the same amount from the same side. $T_0 = 20°C = 293\ K$.

Solve:

$$\frac{Q/\Delta t}{A} = e\sigma(T^4 - T_0^4)$$

Solve this for T.

$$\frac{Q/\Delta t}{Ae\sigma} = T^4 - T_0^4$$

$$T = \sqrt[4]{\frac{Q/\Delta t}{Ae\sigma} + T_0^4} = \sqrt[4]{\frac{800\ W/m^2}{(1.0)(5.67 \times 10^{-8}\ W/m^2 \cdot K^4)} + (293\ K)^4} = 383\ K = 110°C$$

Assess: $110°C$ is hot enough to boil water, so this seems feasible as a hot water heater, at least during the 5 hours a day the collector gets $800\ W/m^2$. You can store the hot water in an insulated tank for use at other times of the day.

19.69. Model: This is a problem about conduction of the heat from inside the box to outside. 100 W of heat is generated by the light bulb inside the box, so in equilibrium that is how much must be conducted away through the sides of the box.

Visualize: We are given $L = 0.012\ m$, $Q/\Delta t = 100\ W$ and the thermal conductivity of concrete $k = 0.8\ W/m\,K$. We compute $A = 6(0.20\ m \times 0.20\ m) = 0.24\ m^2$. $T_C = 20°C = 293\ K$.

Solve: Use Equation 19.45 for the rate of heat transfer by conduction.

$$\frac{Q}{\Delta t} = k\frac{A}{L}(T_H - T_C)$$

Solve for T_H.

$$\left(\frac{Q}{\Delta t}\right)\frac{L}{kA} = (T_H - T_C)$$

$$T_H = \left(\frac{Q}{\Delta t}\right)\frac{L}{kA} + T_C = (100\ W)\frac{0.012\ m}{(0.8\ W/m\,K)(0.24\ m^2)} + 293\ K = 299\ K = 26°C$$

Assess: We expected T_H to be a few degrees hotter than T_C, which it is.

19.70. Model: Ignore any radiation and assume all of the heat is transferred by conduction through the compound rod from the hot end to the cold end.

Visualize: Look up the thermal conductivities of the two metals in Table 19.5: $k_{Cu} = 400\ W/m\,K$ and $k_{Fe} = 80\ W/m\,K$. $T_H = 373\ K$ and $T_C = 273\ K$. Equation 19.45 will be applied to each rod and all of the heat must go through both rods at the same rate.

$$\left(\frac{Q}{\Delta t}\right)_{Cu} = \left(\frac{Q}{\Delta t}\right)_{Fe}$$

Solve: Since A and L are the same for the two rods, then

$$k_{Cu}\Delta T_{Cu} = k_{Fe}\Delta T_{Fe}$$

Label the temperature at the joining point in the middle with a subscript M.

$$k_{Cu}(T_H - T_M) = k_{Fe}(T_M - T_C)$$

Solve for T_M.

$$k_{Cu}T_H - k_{Cu}T_M = k_{Fe}T_M - k_{Fe}T_C$$
$$k_{Cu}T_H + k_{Fe}T_C = k_{Fe}T_M + k_{Cu}T_M$$
$$(k_{Fe} + k_{Cu})T_M = k_{Cu}T_H + k_{Fe}T_C$$
$$T_M = \frac{k_{Cu}T_H + k_{Fe}T_C}{k_{Cu} + k_{Fe}}$$

$$T_M = \frac{(400 \text{ W/m K})(373 \text{ K}) + (80 \text{ W/m K})(273 \text{ K})}{400 \text{ W/m K} + 80 \text{ W/m K}} = 356 \text{ K} = 83°\text{C}$$

Assess: Had we not arrived at an answer between $100°\text{C}$ and $0°\text{C}$ we would have been very worried. Furthermore, because the conductivity of copper is greater than the conductivity of iron we expected the answer to be above $50°\text{C}$.

19.71. Model: Assume the sun and star both have the same emissivity. $P/P_{sun} = 1.5(M/M_{sun})^{3.5}$.

Visualize: The power the sun and star emit are given by $P = Q/\Delta T = e\sigma A T^4$. The unsubscripted variables refer to Regulus A.

Solve:

$$\frac{P}{P_{sun}} = \frac{e\sigma A T^4}{e\sigma A_{sun} T_{sun}^4} = \frac{(\pi R^2)}{(\pi R_{sun}^2)}\frac{T^4}{T_{sun}^4} = \frac{R^2 T^4}{R_{sun}^2 T_{sun}^4} = 1.5\left(\frac{M}{M_{sun}}\right)^{3.5} \Rightarrow$$

$$T^4 = T_{sun}^4 \left(\frac{R_{sun}}{R}\right)^2 (1.5)\left(\frac{M}{M_{sun}}\right)^{3.5} \Rightarrow$$

$$T = T_{sun}\sqrt[4]{\left(\frac{R_{sun}}{R}\right)^2 (1.5)\left(\frac{M}{M_{sun}}\right)^{3.5}} = (5800 \text{ K})\sqrt[4]{\left(\frac{R_{sun}}{3.1R_{sun}}\right)^2 (1.5)\left(\frac{3.8M_{sun}}{M_{sun}}\right)^{3.5}} = 12000 \text{ K}$$

Assess: This is about the right temperature for a bluish star (blue stars are hotter than red or yellow stars).

19.72. Model: Assume deep space has a temperature of 0 K; then the satellite will absorb no energy while it radiates.

Visualize: The mass of the satellite is

$$M = \rho V = \rho 4\pi R^2 \Delta R = (8.92 \text{ g/cm}^3)(4\pi)(100 \text{ cm})^2(0.20 \text{ cm}) = 224{,}000 \text{ g}$$

Solve: The energy loss is related to temperature via specific heat.

$$dQ = cmdT$$

But the rate at which energy is radiated is $dQ/dt = -e\sigma A T^4 = cm \cdot dT/dt$, so

$$cm\frac{dT}{dt} = -e\sigma A T^4$$

Separate variables and integrate.

$$\int_{T_i}^{T_f} \frac{1}{T^4}dT = \frac{-e\sigma A}{cm}\int_{t_i}^{t_f} dt \Rightarrow -\frac{1}{3}[T_f^{-3} - T_f^{-3}] = \frac{-e\sigma A}{cm}\Delta t \Rightarrow T_f^{-3} - T_f^{-3} = \frac{3e\sigma A}{cm}\Delta t \Rightarrow$$

$$T_f = \left(\frac{3e\sigma A}{cm}\Delta t + T_f^{-3}\right)^{-1/3}$$

$$= \left(\frac{3(0.75)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(4\pi(1.0 \text{ m})^2)}{(385 \text{ J/kg}\cdot\text{kg})(224 \text{ kg})}(2700 \text{ s}) + (323 \text{ K})^{-3}\right)^{-1/3} = 232 \text{ K} = -41°\text{C}$$

Assess: This is a very wide swing in temperature every orbit.

19.73. Model: Refer to Example 19.11. Assume the surface of the earth is an ideal radiator with $e = 1$.

Visualize: Only half the earth faces the sun at a time, so the earth intercepts 1370 W/m^2 over a cross section of $\pi R_e^2 = \pi (6.37 \times 10^6 \text{ m})^2 = 1.275 \times 10^{14} \text{ m}^2$. If the earth's surface absorbs 70% of the incident power then the total power absorbed is $(0.70)(1370 \text{ W/m}^2)(1.275 \times 10^{14} \text{ m}^2) = 1.222 \times 10^{17}$ W. This much power must also be radiated away from the earth in equilibrium.

Solve: The power is radiated from the whole spherical surface of the earth: $A = 4\pi R_e^2$. Use Equation 19.46 to find the temperature of the earth.

$$T = \left[\frac{Q/\Delta t}{e\sigma(4\pi R_e^2)} \right]^{1/4} = \left[\frac{1.222 \times 10^{17} \text{ W}}{(1)(5.67 \times 10^{-8} \text{ W/m}^2 \text{ K})4\pi(6.37 \times 10^6 \text{ m})^2} \right]^{1/4} = 255 \text{ K} = -18°C$$

Assess: Without a moderate greenhouse effect the earth would be too cold for mammal life. On the other hand, when the greenhouse effect gets out of hand, such as on Venus, it can be too hot for life.

19.74. Solve: (a) 50 J of work are done on a gas to compress it to one-third of its original volume at a constant temperature of 77°C. How many moles of the gas are in the sample?
(b) The number of moles is

$$n = \frac{50 \text{ J}}{-(8.31 \text{ J/mol K})(350 \text{ K})\left(\ln \frac{1}{3} \right)} = 0.0156 \text{ mol}$$

19.75. Solve: (a) A heated 500 g iron slug is dropped into a 200 cm^3 pool of mercury at 15°C. If the mercury temperature rises to 90°C, what was the initial temperature of the iron slug?
(b) The initial temperature was 217°C.

19.76. Solve: (a) A diatomic gas is adiabatically compressed from 1 atm pressure to 10 atm pressure. What is the compression ratio V_{max}/V_{min}?
(b) The ratio is $V_{max}/V_{min} = 10^{1/1.4} = 5.18$.

Challenge Problems

19.77. Model: There are three interacting systems: aluminum, copper, and ethyl alcohol.
Solve: The aluminum, copper, and alcohol form a closed system, so $Q = Q_{Al} + Q_{cu} + Q_{eth} = 0$ J. The mass of the alcohol is

$$M_{eth} = \rho V = (790 \text{ kg/m}^3)(50 \times 10^{-6} \text{ m}^3) = 0.0395 \text{ kg}$$

Expressed in terms of specific heats and using the fact that $\Delta T = T_f - T_i$, the $Q = 0$ J condition is

$$M_{Al} c_{Al} \Delta T_{Al} + M_{Cu} c_{Cu} \Delta T_{Cu} + M_{eth} c_{eth} \Delta T_{eth} = 0 \text{ J}$$

Substituting the appropriate values into this expression gives

$$(0.010 \text{ kg})(900 \text{ J/kg K})(298 \text{ K} - 473 \text{ K}) + (0.020 \text{ kg})(385 \text{ J/kg K})(298 \text{ K} - T)$$
$$+ (0.0395 \text{ kg})(2400 \text{ J/kg K})(298 \text{ K} - 288 \text{ K}) = -1575 \text{ J} + (7.7 \text{ J/K})(298 - T) + 948 \text{ J} = 0 \text{ J}$$
$$T = 216.6 \text{ K} = -56.4°C \approx -56°C$$

19.78. Model: We have two interacting systems: the water and the gas. For the closed system comprising water and gas to come to equilibrium, heat is transferred from one interacting system to the other.
Solve: Energy conservation requires that

$$Q_{air} + Q_{water} = 0 \text{ J} \quad \Rightarrow \quad n_{gas} C_V [T_f - (T_i)_{gas}] + m_{water} c [T_f - (T_i)_{water}] = 0 \text{ J}$$

Using the ideal-gas law,

$$(T_i)_{gas} = \frac{p_{gas}V_{gas}}{n_{gas}R} = \frac{(10 \times 1.013 \times 10^5 \text{ Pa})(4000 \times 10^{-6} \text{ m}^3)}{(0.40 \text{ mol})(8.31 \text{ J/mol K})} = 1219 \text{ K}$$

The energy conservation equation with $(T_i)_{water} = 293$ K becomes

$$(0.40 \text{ mol})(12.5 \text{ J/mol K})(T - 1219 \text{ K}) + (20 \times 10^{-3} \text{ kg})(4190 \text{ J/kg K})(T - 293 \text{ K}) = 0 \text{ J} \quad \Rightarrow \quad T_f = 345 \text{ K}$$

We can now use the ideal-gas equation to find the final gas pressure.

$$\frac{p_iV_i}{T_i} = \frac{p_fV_f}{T_f} \quad \Rightarrow \quad p_f = p_i\frac{T_f}{T_i} = \left(\frac{345 \text{ K}}{1219 \text{ K}}\right)(10 \text{ atm}) = 2.8 \text{ atm}$$

19.79. Assume the gas to be an ideal gas.
Solve: The work done on the gas is the negative of the area under the *p*-versus-*V* graph, that is

$$W = -\text{area under curve} = -50.7 \text{ J}$$

The change in thermal energy is

$$\Delta E_{th} = nC_V\Delta T = nC_V(T_f - T_i)$$

Using the ideal-gas law to calculate the initial and final temperatures,

$$T_i = \frac{p_iV_i}{nR} = \frac{(4.0 \times 1.013 \times 10^5 \text{ Pa})(100 \times 10^{-6} \text{ m}^3)}{(0.015 \text{ mol})(8.31 \text{ J/mol K})} = 325 \text{ K}$$

$$T_f = \frac{p_fV_f}{nR} = \frac{(1.013 \times 10^5 \text{ Pa})(300 \times 10^{-6} \text{ m}^3)}{(0.015 \text{ mol})(8.31 \text{ J/mol K})} = 244 \text{ K}$$

$$\Delta E_{th} = (0.015 \text{ mol})(12.5 \text{ J/mol K})(244 \text{ K} - 325 \text{ K}) = -15.2 \text{ J}$$

From the first law of thermodynamics,

$$\Delta E_{th} = Q + W \quad \Rightarrow \quad Q = \Delta E_{th} - W = -15.2 \text{ J} - (-50.7 \text{ J}) \approx 36 \text{ J}$$

That is, 36 J of heat energy is transferred to the gas.

19.80. Model: The air is assumed to be an ideal diatomic gas that is subjected to an adiabatic process.
Solve: The air admitted into the cylinder at $T_0 = 30°C = 303$ K and $p_0 = 1 \text{ atm} = 1.013 \times 10^5$ Pa has a volume $V_0 = 600 \times 10^{-6}$ m^3 and contains

$$n = \frac{p_0V_0}{RT_0} = 0.024 \text{ mol}$$

Using Equation 19.33 and the fact that $Q = 0$ J for an adiabatic process,

$$\Delta E_{th} = Q + W = nC_V\Delta T \quad \Rightarrow \quad W = nC_VT$$

$$400 \text{ J} = (0.024 \text{ mol})(20.8 \text{ J/mol K})(T_f - 303 \text{ K}) \quad \Rightarrow \quad T_f = 1100 \text{ K}$$

For an adiabatic process Equation 19.36 is

$$T_fV_f^{\gamma-1} = T_0V_0^{\gamma-1} \quad \Rightarrow \quad V_f = V_0\left(\frac{T_0}{T_f}\right)^{\frac{1}{\gamma-1}} = (6.0 \times 10^{-4} \text{ m}^3)\left(\frac{303 \text{ K}}{1100 \text{ K}}\right)^{\frac{1}{1.4-1}} = 2.39 \times 10^{-5} \text{ m}^3 \approx 24 \text{ cm}^3$$

Assess: Note that *W* is positive because the environment does work on the gas.

19.81. Model: Assume the gas is ideal. The process is *not* adiabatic, despite Equation 19.35, because the exponent given in the problem is not equal to γ. The gas is identified as diatomic which means that $\gamma = 1.40$, but the exponent is

explicitly given as 2, so while we are given $pV^2 = \text{constant}$, it is not true that $pV^\gamma = \text{constant}$. We will use the fact that the gas is diatomic to deduce $C_V = \frac{5}{2}R = 20.8 \text{ J/mol} \cdot \text{K}$, although we won't specifically need $\gamma = 1.40$.

Solve: We are given $n = 0.020 \text{ mol}$, $T_i = 293 \text{ K}$, $V_i = 1500 \text{ cm}^3$, and $V_f = 500 \text{ cm}^3$. The strategy to find Q will be to use the first law $Q = \Delta E_{th} - W$. We will use $E_{th} = nC_V \Delta T$ and $W = -\int pdV$.

$$\Delta E_{th} = nC_V \Delta T = (0.020 \text{ mol})(20.8 \text{ J/mol K})(879 \text{ K} - 293 \text{ K}) = 243.8 \text{ J}$$

To do the $W = -\int pdV$ integral we need to know what the constant is in $pV^2 = \text{constant}$. Use the ideal gas law to compute p_i.

$$p_i = \frac{nRT_i}{V_i} = \frac{(0.020 \text{ mol})(8.31 \text{ J/mol K})(293 \text{ K})}{1.5 \times 10^{-3} \text{ m}^3} = 32,460 \text{ Pa}$$

$$\text{constant} = p_i V_i^2 = (32,460 \text{ Pa})(1.5 \times 10^{-3} \text{ m}^3)^2 = 0.0730 \text{ Pa} \cdot \text{m}^6$$

$$W = -\int pdV = 2\int_{V_i}^{V_f} \frac{\text{constant}}{V^2}dV = -(0.0730 \text{ Pa} \cdot \text{m}^6)\int_{0.0015\text{m}^3}^{0.00050\text{m}^3} \frac{dV}{V^2} = (0.0730 \text{ Pa} \cdot \text{m}^6)\left[\frac{1}{V}\right]_{0.0015\text{m}^3}^{0.00050\text{m}^3} = 97.4 \text{ J}$$

$$Q = \Delta E_{th} - W = 243.8 \text{ J} - 97.4 \text{ J} = 146.4 \text{ J} \approx 150 \text{ J}$$

Assess: Both Q and W are positive because heat was added to the system and work was done on the system.

19.82. Model: Assume the gas is ideal.

Solve: Apply the first law of thermodynamics, $\Delta E_{th} = W + Q = nC_V \Delta T$. Consider adding a small amount of heat energy, so that the gas expands by an infinitesimal amount. This process can be considered isobaric, so we can use Equation 19.11, from which we see that the infinitesimal work done on the gas is

$$dW = -pdV \quad \Rightarrow \quad W = -\int_{V_i}^{V_f} pdV$$

The gas pressure is simply the force of the spring times the piston area, or $p = kx/A$. Inserting this into the expression for work and integrating gives

$$W = -\int_{V_i}^{V_f} \frac{kx}{A}dV = -\int_{x_i}^{x_f} kxdx = \frac{k}{2}\left(x_i^2 - x_f^2\right)$$

where we have used $dV = Adx$. Now, use the ideal gas law to find the temperature change. The initial temperature is $T_1 = 300 \text{ K}$. The final temperature is

$$T_f = \frac{p_f V_f}{nR} = \frac{p_f V_f}{p_i V_i}T_i$$

Thus, the temperature change is

$$\Delta T = T_f - T_i = T_i\left(\frac{p_f V_f}{p_i V_i} - 1\right)$$

The last piece needed is the number of moles n, which may be found from the ideal gas law:

$$n = \frac{p_i V_i}{RT_i}$$

Inserting these components into the first-law expression gives

$$W + Q = nC_V \Delta T \quad \Rightarrow \quad Q = \left(\frac{p_i V_i}{RT_i}\right)C_V T_i\left(\frac{p_f V_f}{p_i V_i} - 1\right) - \frac{k}{2}\left(x_i^2 - x_f^2\right) = \frac{C_V p_i V_i}{R}\left(\frac{p_f V_f}{p_i V_i} - 1\right) - \frac{k}{2}\left(x_i^2 - x_f^2\right)$$

Because the gas in monatomic, $C_v = 12.5$ J/mol K (see Table 19.4). From the problem statement, we know that

$$p_i = \frac{F_{spring}}{A} = \frac{kx_i}{A}, \quad p_f = \frac{F_{spring}}{A} = \frac{kx_f}{A}, \quad V_i = LA, \quad V_f = (L + 6.0\,\text{cm})A$$

with $x_i = 20$ cm, $x_f = 8.0$ cm, and $A = 8.0$ cm^2. Inserting these values into the expression for the heat transferred gives

$$Q = \frac{C_V k x_i L}{R}\left[\frac{x_f(L + 6.0\,\text{cm})}{x_i L} - 1\right] - \frac{k}{2}\left(x_i^2 - x_f^2\right)$$

$$= \frac{(12.5\,\text{J/mol K})(2000\,\text{N/m})(0.020\,\text{m})(0.100\,\text{m})}{(8.3\,\text{J/mol K})}\left[\frac{(0.080\,\text{m})(0.160\,\text{m})}{(0.020\,\text{m})(0.100\,\text{m})} - 1\right] - \frac{(2000\,\text{N/m})}{2}\left[(0.020\,\text{m})^2 - (0.080\,\text{m})^2\right]$$

$$= 39\,\text{J}$$