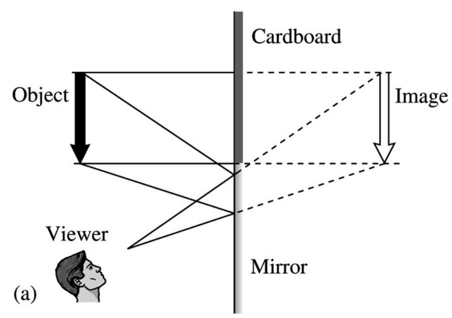
34

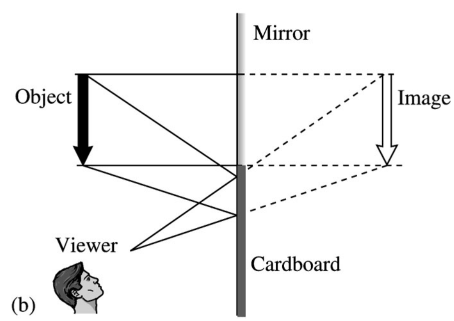
RAY OPTICS

Conceptual Questions

34.1. The shape of a small pinhole does not materially affect the image in a pinhole camera. There might be minor differences in quality of focus, but the image on the film will be generally the same whether the pinhole is round or square.

34.2. Light is scattered off all points of the pencil and into all directions of space. If light directed toward the mirror is reflected into your eye, you see the image of the pencil. **(a)** As part **(a)** of the figure shows, if the top half of the mirror is covered, light scattered from the pencil and reflected off the mirror can enter your eye and you will see the image of the pencil. **(b)** As part **(b)** of the figure shows, if the bottom half of the mirror is covered, light scattered from the pencil cannot be reflected off the mirror in such a manner that it enters your eye. You cannot see the image of the pencil.





34.3. The speed of light in a medium is a function of the index of refraction of that medium ($v = c/n$). If you use optic fiber at the center (where the light would have a shorter travel distance) with a larger index of refraction, you will slow the light down and it will get to the end of the fiber closer to the time the other light gets there.

34.4. The beam bends away from the normal as it goes from medium 2 to medium 1, so $n_1 < n_2$.

34.5. The rays bend toward the normal when they go from the air into the water in the tank, so the rays appear to come from a farther distance.

34.6. **(a)** Two rays that emanate from the point on the object and cross at the image point are required to locate the image point. **(b)** An infinite number! All rays from the object point that strike the lens will converge to the image point.

34.7. You will still see the entire image, but it will be dimmer as less light passes through the lens.

34.8. The image is real, so the object distance is greater than *f*. Solve the thin lens equation for s'.

$$s' = \frac{sf}{s-f}$$

If $s = 2f$, then $s' = 2f$ and the image will be the same size as the object. If $s > 2f$, then the image will be smaller than the object, and this is not the case, so *s* must be less than 2*f*. Therefore *s* is between *f* and 2*f*.

34.9. While the law of refraction depends on the index of the medium, the law of reflection does not. Under water the angle of incidence will still equal the angle of reflection. There is no reason for a ray to travel a different path in water than in air in this case. Hence, the sun's rays will be focused the same distance from the mirror. Of course the preceding analysis is not true for light passing through a lens that might be immersed in either water or air; in that case the index of refraction matters. But with a mirror the water doesn't change anything. Note that this analysis also relies on the fact that the rays from the Sun are essentially parallel when they reach Earth, so although they may be refracted at the water's surface, they remain parallel in the water.

34.10. To get a magnified image that is upright the mirror must be concave and the object must be inside the focal length. The text indicates that such an image is virtual and behind the mirror.

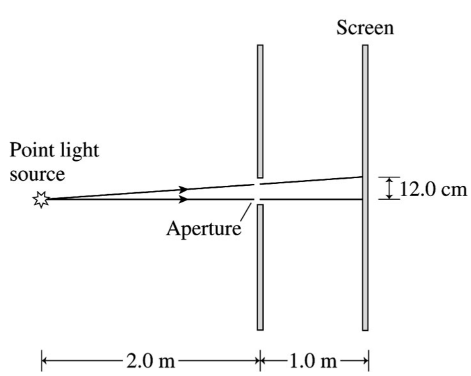
34.11. The spoon acts like a concave mirror and the image is inverted when $s > f$. A careful ray tracing diagram will convince you of this (see Example 34.15). It should be noted that this is only true for $s > f$; when the object is closer to the spoon than the focal distance, $s < f$, and the image is upright. Magnifying mirrors, such as make-up mirrors are concave like this and have a large focal length so that your face is within the focal length of the mirror; the image is virtual and behind the mirror. If you put the spoon very close to your eye you will notice this.

Exercises and Problems

Exercises

Section 34.1 The Ray Model of Light

34.1. **Model:** Light rays travel in straight lines. The light source is a point source.
Visualize:



Solve: Let w be the width of the aperture. Then from the geometry of the figure,

$$\frac{w}{2.0\text{ m}} = \frac{12.0\text{ cm}}{2.0\text{ m} + 1.0\text{ m}} \Rightarrow w = 8.0\text{ cm}$$

34.2. Model: Light rays travel in straight lines.
Solve: **(a)** The time is

$$t = \frac{\Delta x}{c} = \frac{1.0\text{ m}}{3.0\times10^8\text{ m/s}} = 3.3\times10^{-9}\text{ s} = 3.3\text{ ns}$$

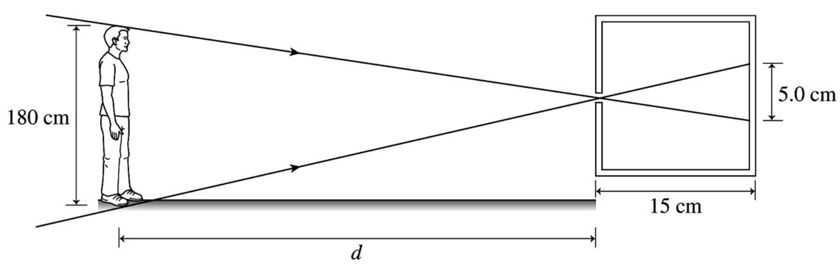
(b) The refractive indices for water, glass, and cubic zirconia are 1.33, 1.50, and 1.96, respectively. In a time of 3.33 ns, light will travel the following distance in water:

$$\Delta x_{\text{water}} = v_{\text{water}} t = \left(\frac{c}{n_{\text{water}}}\right) t = \left(\frac{3.0\times10^8\text{ m/s}}{1.33}\right)(3.33\times10^{-9}\text{ s}) = 0.75\text{ m}$$

Likewise, the distances traveled in the glass and cubic zirconia are $\Delta x_{\text{glass}} = 0.67\text{ m}$ and $\Delta x_{\text{cubic zirconia}} = 0.46\text{ m}$.

Assess: The higher the refractive index of a medium, the slower the speed of light and hence smaller the distance it travels in that medium in a given time.

34.3. Model: Light rays travel in straight lines.
Visualize: Note the similar triangles in this figure.



Solve:

$$\frac{15\text{ cm}}{5.0\text{ cm}} = \frac{d}{180\text{ cm}} \Rightarrow d = \frac{15\text{ cm}}{5.0\text{ cm}}(180\text{ cm}) = 540\text{ cm} = 5.4\text{ m}$$

Assess: This is a typical distance for photographs of people.

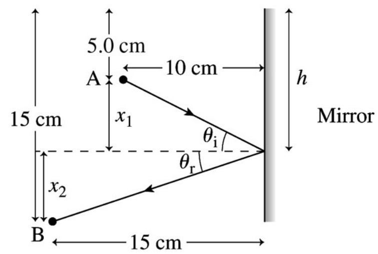
34.4. Model: Light rays travel in straight lines.
Solve: Let t_{glass}, t_{oil}, and t_{plastic} be the times light takes to pass through the layers of glass, oil, and plastic, respectively. The time to traverse the glass is

$$t_{\text{glass}} = \frac{\Delta x}{v_{\text{glass}}} = \frac{\Delta x}{c/n_{\text{glass}}} = \frac{\Delta x\, n_{\text{glass}}}{c} = \frac{(1.0\times10^{-2}\text{ m})(1.50)}{3.0\times10^8\text{ m/s}} = 0.050\text{ ns}$$

Likewise, $t_{\text{oil}} = 0.243\text{ ns}$ and $t_{\text{plastic}} = 0.106\text{ ns}$. Thus, $t_{\text{total}} = t_{\text{glass}} + t_{\text{oil}} + t_{\text{plastic}} = 0.050\text{ ns} + 0.243\text{ ns} + 0.106\text{ ns} = 0.399\text{ ns} \approx 0.40\text{ ns}$.
Assess: The small time is due to the high value of the speed of light.

Section 34.2 Reflection

34.5. Model: Light rays travel in straight lines and follow the law of reflection.
Visualize:



Solve: We are asked to obtain the distance $h = x_1 + 5.0$ cm. From the geometry of the diagram,

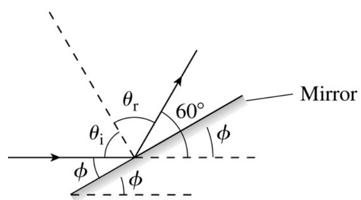
$$\tan\theta_i = \frac{x_1}{10 \text{ cm}} \qquad \tan\theta_r = \frac{x_2}{15 \text{ cm}} \qquad x_1 + x_2 = 10 \text{ cm}$$

Because $\theta_r = \theta_i$, we have

$$\frac{x_1}{10 \text{ cm}} = \frac{x_2}{15 \text{ cm}} \Rightarrow x_1 = \frac{10 \text{ cm} - x_1}{15 \text{ cm}}(10 \text{ cm}) = \frac{100 \text{ cm}^2}{15 \text{ cm}} - \frac{10 \text{ cm}}{15 \text{ cm}}x_1 \Rightarrow x_1 = 4.0 \text{ cm}$$

Thus, the ray strikes a distance 9.0 cm below the top edge of the mirror.

34.6. Model: Use the ray model of light.
Visualize:



According to the law of reflection, $\theta_r = \theta_i$.
Solve: From the geometry of the diagram,

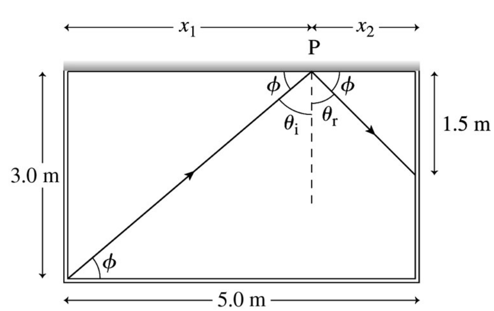
$$\theta_i + \phi = 90° \qquad \theta_r + (60° - \phi) = 90°$$

Using the law of reflection, we get

$$90° - \phi = 90° - (60° - \phi) \Rightarrow \phi = 30°$$

Assess: The above result leads to a general result for plane mirrors: If a plane mirror makes an angle ϕ relative to the incident ray, the reflected ray makes an angle of 2ϕ with respect to the incident ray.

34.7. Model: Light rays travel in straight lines and follow the law of reflection.
Visualize:



To determine the angle ϕ, we must know the point P on the mirror where the ray is incident. P is a distance x_2 from the far wall and a horizontal distance x_1 from the laser source. The angle of incidence θ_i is equal to the angle of reflection θ_r.

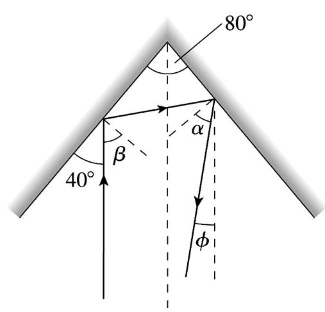
Solve: From the geometry of the diagram,

$$\tan\phi = \frac{1.5 \text{ m}}{x_2} = \frac{3.0 \text{ m}}{x_1} \qquad x_1 + x_2 = 5.0 \text{ m}$$

$$\frac{1.5 \text{ m}}{5.0 \text{ m} - x_1} = \frac{3.0 \text{ m}}{x_1} \quad \Rightarrow \quad (1.5 \text{ m})x_1 = 15 \text{ m}^2 - (3.0 \text{ m})x_1 \quad \Rightarrow \quad x_1 = \frac{10}{3.0} \text{ m}$$

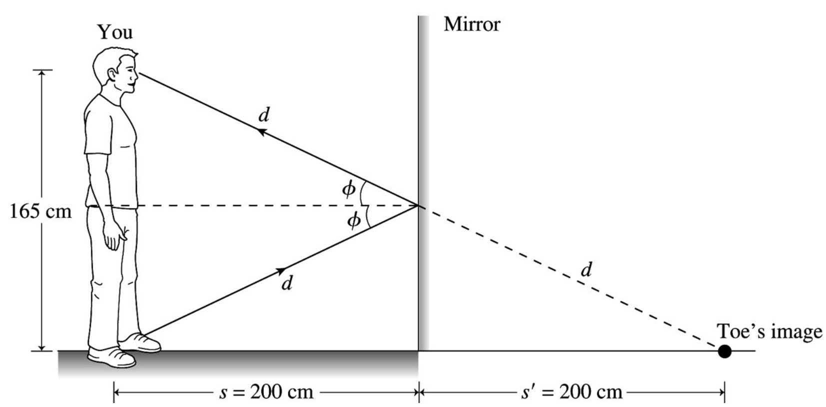
$$\tan\phi = \frac{3.0 \text{ m}}{x_1} = \frac{9.0}{10} \quad \Rightarrow \quad \phi = 42°$$

34.8. Model: Treat the laser beam as a ray and use the ray model of light.
Visualize:



Solve: From the geometry of the mirrors and the rays, $\beta = 50°$, $\alpha = 30°$, and $\phi = 20°$.

34.9. Model: Use the ray model of light and the law of reflection.
Visualize:



We only need one ray of light that leaves your toes and reflects into your eye.
Solve: From the geometry of the diagram, the distance from your eye to your toes' image is

$$2d = \sqrt{(400 \text{ cm})^2 + (165 \text{ cm})^2} = 433 \text{ cm}$$

Assess: The light appears to come from your toes' image.

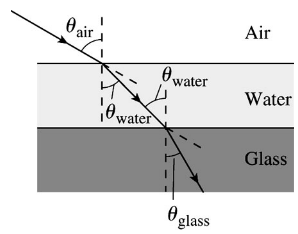
Section 34.3 Refraction

34.10. Model: Represent the laser beam with a single ray and use the ray model of light.
Solve: Using Snell's law at the air-water boundary,

$$n_{\text{air}}\sin\theta_{\text{air}} = n_{\text{liquid}}\sin\theta_{\text{liquid}} \quad\Rightarrow\quad n_{\text{liquid}} = n_{\text{air}}\frac{\sin\theta_{\text{air}}}{\sin\theta_{\text{liquid}}} = 1.00\left(\frac{\sin 53°}{\sin 35°}\right) = 1.4$$

Assess: As expected, n_{liquid} is larger than n_{air}.

34.11. Model: Use the ray model of light and Snell's law.
Visualize:



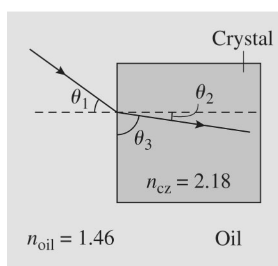
Solve: According to Snell's law for the air-water and water-glass boundaries,

$$n_{\text{air}}\sin\theta_{\text{air}} = n_{\text{water}}\sin\theta_{\text{water}} \qquad n_{\text{water}}\sin\theta_{\text{water}} = n_{\text{glass}}\sin\theta_{\text{glass}}$$

From these two equations, we have

$$n_{\text{air}}\sin\theta_{\text{air}} = n_{\text{glass}}\sin\theta_{\text{glass}} \quad\Rightarrow\quad \sin\theta_{\text{glass}} = \frac{n_{\text{air}}}{n_{\text{glass}}}\sin\theta_{\text{air}} = \left(\frac{1.00}{1.50}\right)\sin 60° \quad\Rightarrow\quad \theta_{\text{glass}} = \sin^{-1}\left(\frac{\sin 60°}{1.50}\right) = 35°$$

34.12. Model: Use the ray model of light and Snell's law.
Visualize: See the figure below. Note that the angle we need to find is θ_3.



Solve: To angle θ_3 is given by $\theta_3 = \pi/2 - \theta_2$. To find θ_2, apply Snell's law at the interface between the oil and the cubic zirconium:

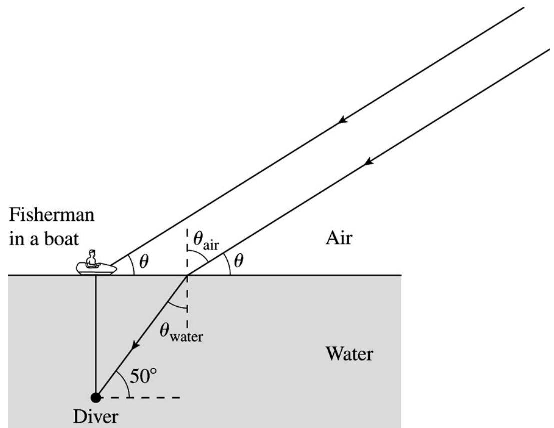
$$n_{\text{oil}}\sin\theta_1 = n_{\text{CZ}}\sin\theta_2 \quad\Rightarrow\quad \theta_2 = \sin^{-1}\left(\frac{n_{\text{oil}}}{n_{\text{CZ}}}\sin\theta_1\right)$$

From these two equations, we have

$$\theta_3 = 90° - \sin^{-1}\left(\frac{n_{\text{oil}}}{n_{\text{CZ}}}\sin\theta_1\right) = 90° - \sin^{-1}\left(\frac{1.46}{2.18}\sin 25°\right) = 74°$$

Assess: We have found that the ray is deflected toward the normal in the crystal, which it should be because $n_{\text{CZ}} > n_{\text{oil}}$.

34.13. Model: Use the ray model of light. The sun is a point source of light.
Visualize:



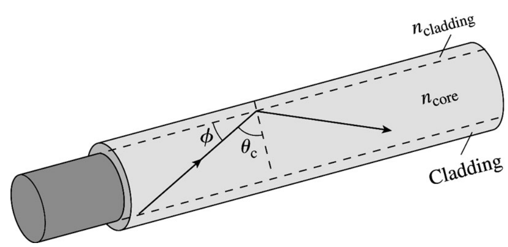
A ray that arrives at the diver $50°$ above horizontal refracted into the water at $\theta_{water} = 40°$.

Solve: Using Snell's law at the water-air boundary

$$n_{air}\sin\theta_{air} = n_{water}\sin\theta_{water} \quad\Rightarrow\quad \sin\theta_{air} = \frac{n_{water}}{n_{air}}\sin\theta_{water} = \left(\frac{1.33}{1.0}\right)\sin 40° \quad\Rightarrow\quad \theta_{air} 58.7°$$

Thus the height above the horizon is $\theta = 90° - \theta_{air} = 31.3° \approx 31°$. Because the sun is far away from the fisherman (and the diver), the fisherman will see the sun at the same angle of $31°$ above the horizon.

34.14. Model: Use the ray model of light. For an angle of incidence greater than the critical angle, the ray of light undergoes total internal reflection.
Visualize:



Solve: The critical angle of incidence is given by Equation 34.9:

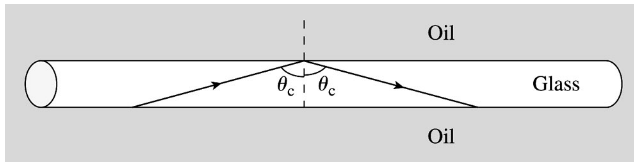
$$\theta_c = \sin^{-1}\left(\frac{n_{cladding}}{n_{core}}\right) = \sin^{-1}\left(\frac{1.48}{1.60}\right) = 67.7°$$

Thus, the maximum angle a light ray can make with the wall of the core to remain inside the fiber is $90.0° - 67.7° = 23.3°$.

Assess: We can have total internal reflection because $n_{core} > n_{cladding}$.

34.15. Model: Use the ray model of light. For an angle of incidence greater than the critical angle, the ray of light undergoes total internal reflection.

Visualize:

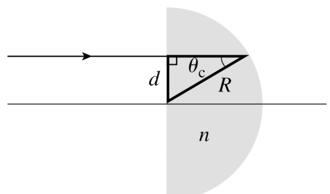


Solve: The critical angle of incidence is given by Equation 34.9:

$$\theta_c = \sin^{-1}\left(\frac{n_{oil}}{n_{glass}}\right) = \sin^{-1}\left(\frac{1.46}{1.50}\right) = 76.7°$$

Assess: The critical angle exists because $n_{oil} < n_{glass}$.

34.16. Model: Assume the glass hemisphere is in air with $n_2 = 1.0$.
Visualize: Note the right triangle with side d and hypotenuse R.



Solve: For the critical angle

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1.0}{n} = \frac{1}{n}$$

But we also have from the right triangle

$$\sin\theta_c = \frac{\text{opp}}{\text{hyp}} = \frac{d}{R}$$

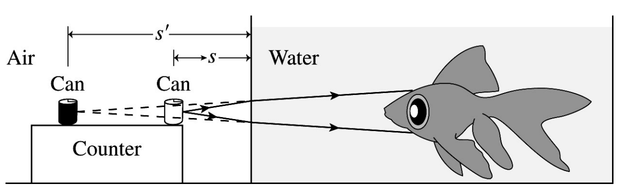
Set the two expressions for $\sin\theta_c$ equal to each other and solve for d.

$$\frac{1}{n} = \frac{d}{R} \Rightarrow d = \frac{R}{n}$$

Assess: We expect d to scale with R and be inversely proportional to n.

Section 34.4 Image Formation by Refraction at a Plane Surface

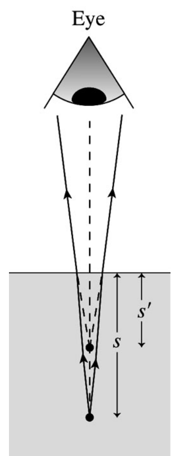
34.17. Model: Represent the can as a point source and use the ray model of light.
Visualize:



Paraxial rays from the can refract into the water and enter into the fish's eye.
Solve: The object distance from the edge of the aquarium is s. From the water side, the can appears to be at an image distance $s' = 30$ cm. Using Equation 34.13,

$$s' = \frac{n_2}{n_1}s = \frac{n_{water}}{n_{air}}s = \left(\frac{1.33}{1.0}\right)s \quad \Rightarrow \quad s = \frac{30\text{ cm}}{1.33} = 23\text{ cm}$$

34.18. Model: Represent the beetle as a point source and use the ray model of light.
Visualize:



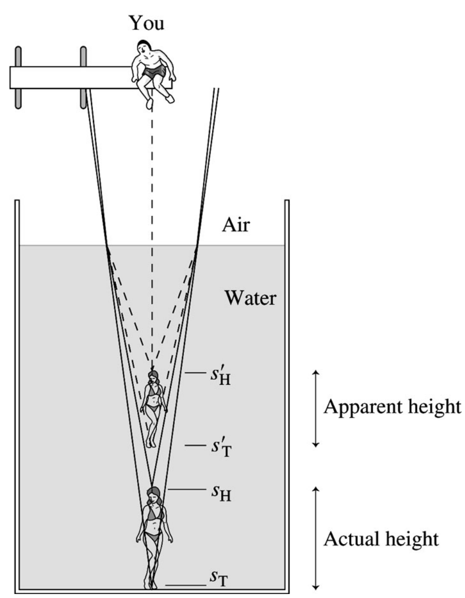
Paraxial rays from the beetle refract into the air and then enter into the observer's eye. The rays in the air when extended into the plastic appear to be coming from the beetle at a shallower location, a distance s' from the plastic-air boundary.

Solve: The actual object distance is s and the image distance is $s' = 2.0$ cm. Using Equation 34.13,

$$s' = \frac{n_2}{n_1}s = \frac{n_{\text{air}}}{n_{\text{plastic}}}s \quad \Rightarrow \quad 2.0\text{ cm} = \frac{1.0}{1.59}s \quad \Rightarrow \quad s = 3.2\text{ cm}$$

Assess: The beetle is much deeper in the plastic than it appears to be.

34.19. Model: Represent the diver's head and toes as point sources. Use the ray model of light.
Visualize:



Paraxial rays from the head and the toes of the diver refract into the air and then enter into your eyes. When these refracted rays are extended into the water, the head and the toes appear elevated toward you.

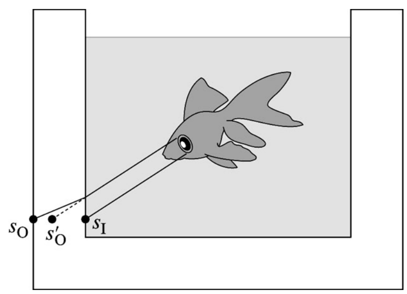
Solve: Using Equation 34.13,

$$s'_T = \frac{n_2}{n_1}s_T = \frac{n_{air}}{n_{water}}s_T \qquad s'_H = \frac{n_{air}}{n_{water}}s_H$$

Subtracting the two equations, her apparent height is

$$s'_H - s'_T = \frac{n_{air}}{n_{water}}(s_H - s_T) = \frac{1.00}{1.33}(150 \text{ cm}) = 113 \text{ cm}$$

34.20. Model: Represent the aquarium's wall as a point source, and use the ray model of light.
Visualize:



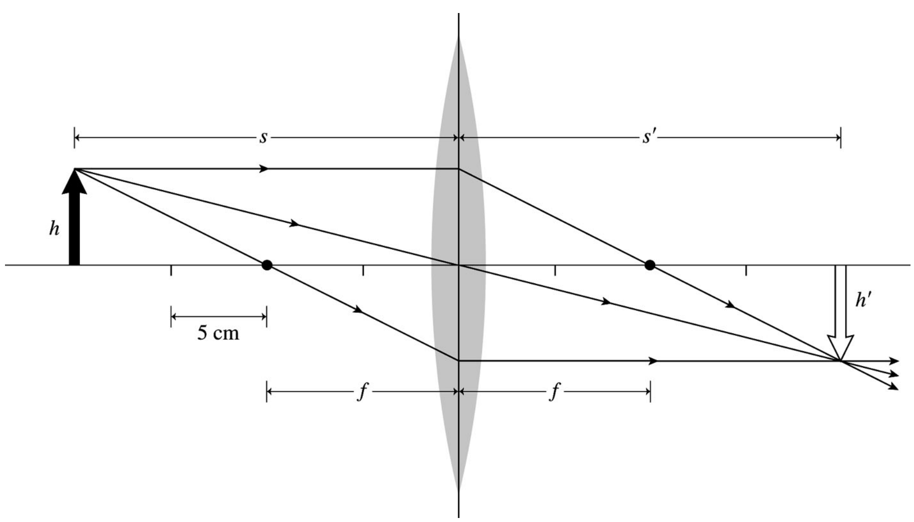
Paraxial rays from the outer edge (O) are refracted into the water and then enter into the fish's eye. When extended into the wall, these rays will appear to be coming from O′ rather from O. The point on the inside edge (I) of the wall will not change its apparent location.
Solve: We are given that $s_O - s_I = 4.00$ mm and $s'_O - s'_I = 3.50$ mm. Using Equation 34.13,

$$s'_O = \frac{n_{water}}{n_{wall}}s_O \qquad s'_I = \frac{n_{water}}{n_{wall}}s_I$$

$$s'_O - s'_I = \frac{n_{water}}{n_{wall}}(s_O - s_I) \quad \Rightarrow \quad 3.50 \text{ mm} = \frac{1.33}{n_{wall}}(4.00 \text{ mm}) \quad \Rightarrow \quad n_{wall} = (1.33)\left(\frac{4.00 \text{ mm}}{3.50 \text{ mm}}\right) = 1.52$$

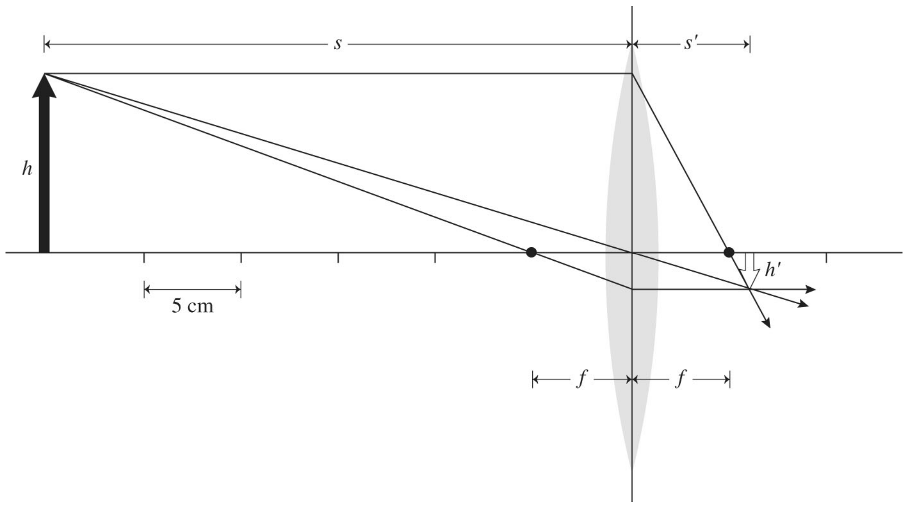
Section 34.5 Thin Lenses: Ray Tracing

34.21. Model: Use ray tracing to locate the image.
Solve:



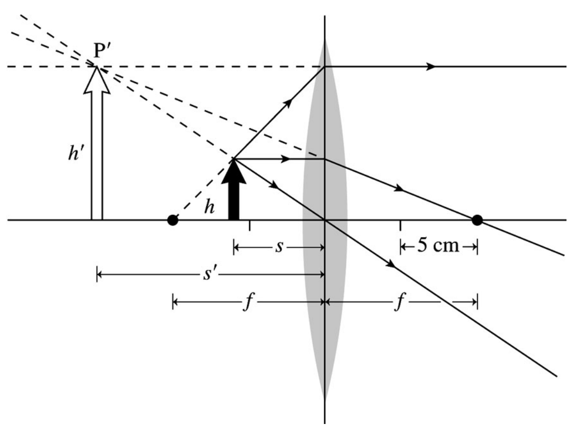
The figure shows the ray-tracing diagram using the steps of Tactics Box 34.2. You can see from the diagram that the image is in the plane where the three special rays converge. The image is inverted and is located at s′ = 20 cm behind the converging lens.

34.22. Model: Use ray tracing to locate the image.
Solve:



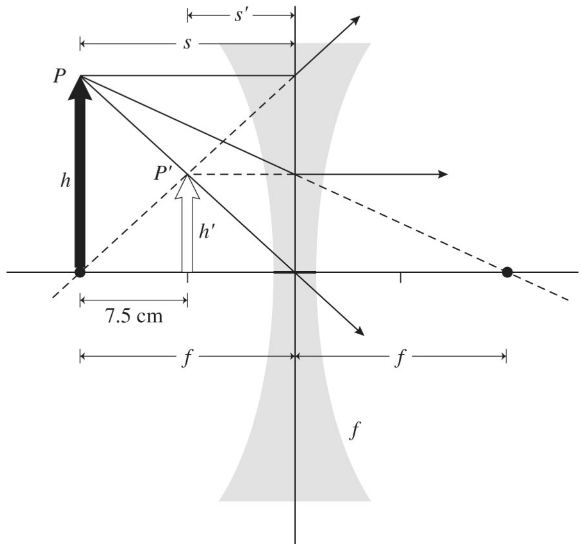
The figure shows the ray-tracing diagram using the steps of Tactics Box 34.2. You can see from the diagram that the image is in the plane where the three special rays converge. The image is located at $s' = 6$ cm to the right of the converging lens, and is inverted.

34.23. Model: Use ray tracing to locate the image.
Solve:



The figure shows the ray-tracing diagram using the steps of Tactics Box 34.2. You can see that the rays after refraction do not converge at a point on the refraction side of the lens. On the other hand, the three special rays, when extrapolated backward toward the incidence-side of the lens, meet at P′, which is 15 cm from the lens. That is, $s' = -15$ cm. Thus, the image is in front of the lens and is upright.

34.24. Model: Use ray tracing to locate the image.
Solve:



The figure shows the ray-tracing diagram using the steps of Tactics Box 34.3. The three rays after refraction do not converge at a point, but they appear to come from P′. P′ is 7.5 cm from the diverging lens, so $s' = -7.5$ cm. Thus, the image is in front of the lens and is upright.

Section 34.6 Thin Lenses: Refraction Theory

34.25. Model: Assume the biconvex lens is a thin lens.
Solve: If the object is on the left, then the first surface has $R_1 = +24\text{ cm}$ (convex toward the object) and the second surface has $R_2 = -40\text{ cm}$ (concave toward the object). The index of refraction of glass is n = 1.50, so the lens maker's equation gives

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50-1)\left(\frac{1}{24\text{ cm}} - \frac{1}{-40\text{ cm}}\right) \quad \Rightarrow \quad f = 30\text{ cm}$$

Assess: This is a converging lens since $f > 0$.

34.26. Model: Assume the planoconvex lens is a thin lens.
Solve: If the object is on the left, then the first surface has $R_1 = \infty\text{ cm}$ and the second surface has $R_2 = -40\text{ cm}$ (concave toward the object). The index of refraction of polystyrene plastic is 1.59, so the lens maker's equation gives

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.59-1)\left(\frac{1}{\infty} - \frac{1}{-40\text{ cm}}\right) \quad \Rightarrow \quad f = 68\text{ cm}$$

Assess: This is a converging lens since $f > 0$.

34.27. Model: Assume the biconcave lens is a thin lens.
Solve: If the object is on the left, then the first surface has $R_1 = -40$ cm (concave toward the object) and the second surface has $R_2 = +40$ cm (convex toward the object). The index of refraction of glass is 1.50, so the lens maker's equation gives

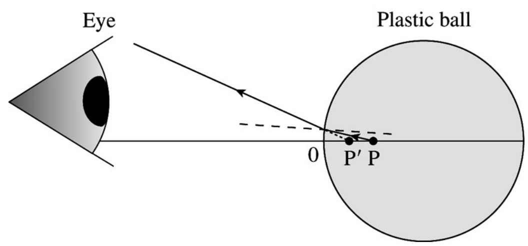
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50-1)\left(\frac{1}{-40\text{ cm}} - \frac{1}{+40\text{ cm}}\right) = (0.50)\left(-\frac{1}{20\text{ cm}}\right) \quad \Rightarrow \quad f = -40\text{ cm}$$

34.28. Model: Assume the meniscus lens is a thin lens.
Solve: If the object is on the left, then the first surface has $R_1 = -40$ cm (concave toward the object) and the second surface has $R_2 = -30$ cm (concave toward the object). The index of refraction of polystyrene plastic is 1.59, so the lens maker's equation gives

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.59-1)\left(\frac{1}{-40\text{ cm}} - \frac{1}{-30\text{ cm}}\right) \Rightarrow f = 2.0\text{ m}$$

Assess: This is a converging lens since $f > 0$.

34.29. Model: Model the bubble as a point source and consider the paraxial rays that refract from the plastic into the air. The edge of the plastic is a spherical refracting surface.
Visualize:



Solve: The bubble is at P, a distance of 2.0 cm from the surface. So, s = 2.0 cm. A ray from P after refracting from the plastic-air boundary bends away from the normal axis and enters the eye. This ray appears to come from P′, so the image of P is at P′ and is a virtual image. Because P faces the concave side of the refracting surface, $R = -4.0$ cm. Furthermore, $n_1 = 1.59$ and $n_2 = 1.00$. Using Equation 34.20,

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1.59}{2.0\text{ cm}} + \frac{1.00}{s'} = \frac{1.00-1.59}{-4.0\text{ cm}} = +\frac{0.59}{4.0\text{ cm}} = 0.1475\text{ cm}^{-1}$$

$$\frac{1}{s'} = 0.1475\text{ cm}^{-1} - 0.795\text{ cm}^{-1} \Rightarrow s' = 1.5\text{ cm}$$

That is, the bubble appears 1.5 cm beneath the surface.

34.30. Model: The water is a spherical refracting surface. Consider the paraxial rays that refract from the air into the water.
Solve: If the cat's face is 20 cm from the edge of the bowl, then s = +20 cm. The spherical fish bowl surface has $R = +25$ cm, because it is the convex surface that is toward the object. Also $n_1 = 1.00$ (air) and $n_2 = 1.33$ (water). Using Equation 34.20,

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \Rightarrow \frac{1.00}{20\text{ cm}} + \frac{1.33}{s'} = \frac{1.33-1.00}{25\text{ cm}} = \frac{0.33}{25\text{ cm}} = 0.0132\text{ cm}^{-1}$$

$$\frac{1.33}{s'} = (0.0132 - 0.050)\text{ cm}^{-1} \Rightarrow s' = -36\text{ cm}$$

This is a virtual image located 36 cm outside the fishbowl. The fish, inside the bowl, sees the virtual image. That is, the fish sees the cat's face 36 cm from the bowl.

34.31. Model: Assume the lens is thin.
Visualize:

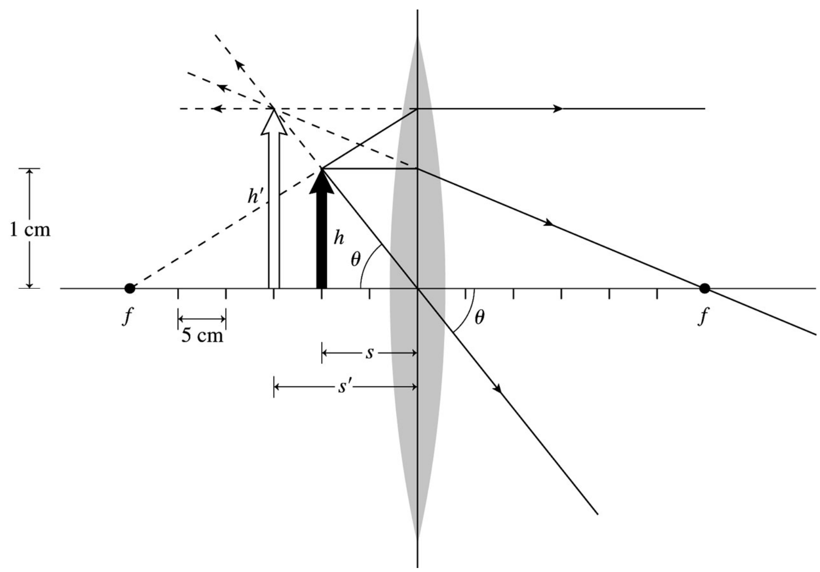
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow s'=\frac{fs}{s-f}$$

Solve:

$$s'=\frac{fs}{s-f}=\frac{(20\text{ cm})(60\text{ cm})}{60\text{ cm}-20\text{ cm}}=30\text{ cm}$$

The magnification is $m=-s'/s=-30\text{ cm}/60\text{ cm}=-0.50$. This means the image is inverted and has a height of 0.50 cm.
Assess: Ray tracing will confirm these results.

34.32. Model: Use ray tracing to locate the image. Assume that the converging lens is a thin lens.
Solve: (a)



The figure shows the ray-tracing diagram made using the steps of Tactics Box 34.2. The three special rays that experience refraction do not converge at a point. Instead they appear to come from a point that is 15 cm on the same side as the object itself. Thus $s'=-15$ cm. The image is upright and has a height of $h'=1.5$ cm.
(b) Using the thin-lens formula,

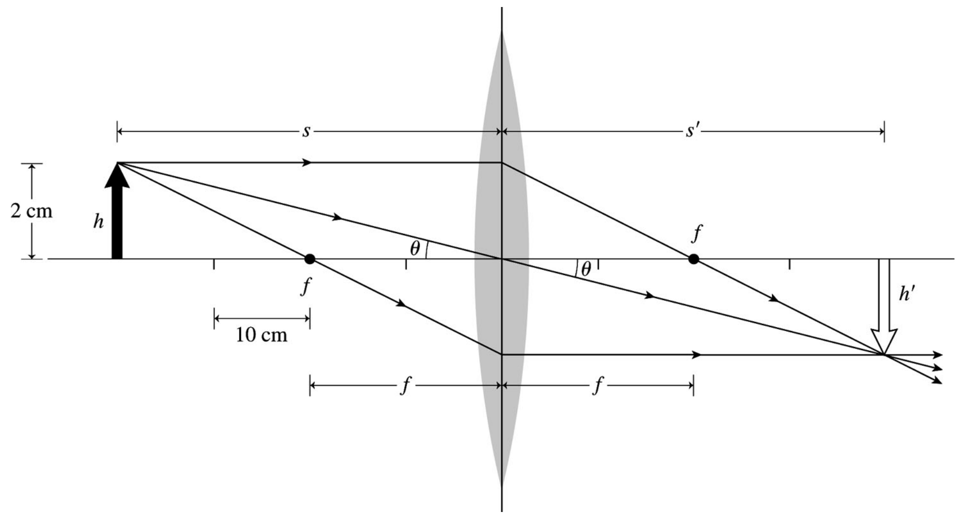
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\quad\Rightarrow\quad\frac{1}{10\text{ cm}}+\frac{1}{s'}=\frac{1}{30\text{ cm}}\quad\Rightarrow\quad\frac{1}{s'}=-\frac{1}{15\text{ m}}\quad\Rightarrow\quad s'=-15\text{ cm}$$

The image height is obtained from

$$m=-\frac{s'}{s}=-\frac{-15\text{ cm}}{10\text{ cm}}=+1.5$$

The image is upright and 1.5 times the object, that is, 1.5 cm high. These values agree with those obtained in part **(a)**.

34.33. Model: Assume that the converging lens is a thin lens. Use ray tracing to locate the image.
Solve: (a)



The figure shows the ray-tracing diagram made using the steps of Tactics Box 34.2. The three rays after refraction converge to give an image at $s' = 40$ cm. The height of the image is $h' = 2.0$ cm.
(b) Using the thin-lens formula,

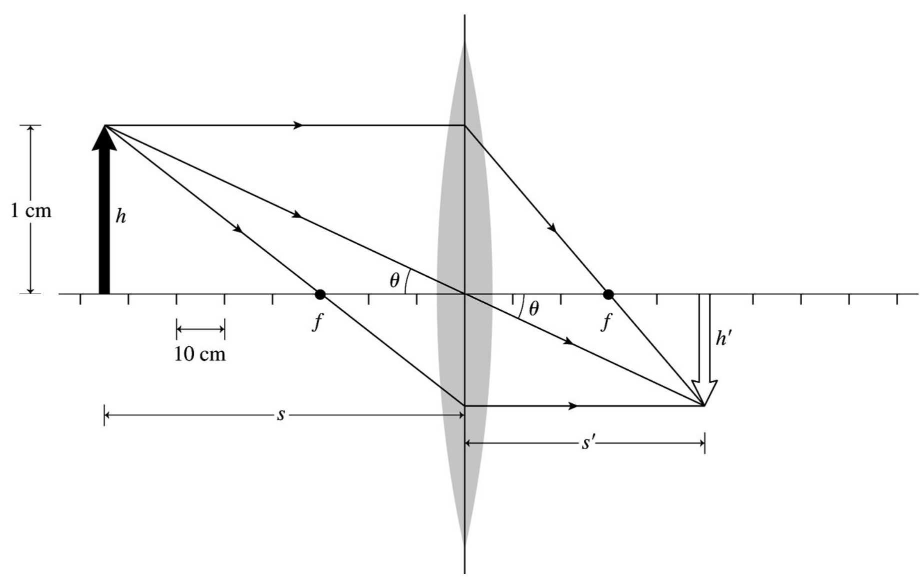
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \quad\Rightarrow\quad \frac{1}{40\text{ cm}}+\frac{1}{s'}=\frac{1}{20\text{ cm}} \quad\Rightarrow\quad \frac{1}{s'}=\frac{1}{40\text{ cm}} \quad\Rightarrow\quad s'=40\text{ cm}$$

The image height is obtained from

$$m=-\frac{s'}{s}=-\frac{40\text{ cm}}{40\text{ cm}}=-1.0$$

The image is inverted and as tall as the object; that is, $h' = 2.0$ cm. The values for h' and s' obtained in parts **(a)** and **(b)** agree.

34.34. Model: Use ray tracing to locate the image. Assume the converging lens is a thin lens.
Solve: (a)



The figure shows the ray-tracing diagram made using the steps of Tactics Box 34.2. After refraction, the three special rays converge and give an image 50 cm away from the converging lens. Thus, $s' = +50$ cm. The image is inverted and its height is 0.65 cm.

(b) Using the thin-lens formula,

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{75\text{ cm}}+\frac{1}{s'}=\frac{1}{30\text{ cm}} \Rightarrow \frac{1}{s'}=\frac{1}{50\text{ cm}} \Rightarrow s'=50\text{ cm}$$

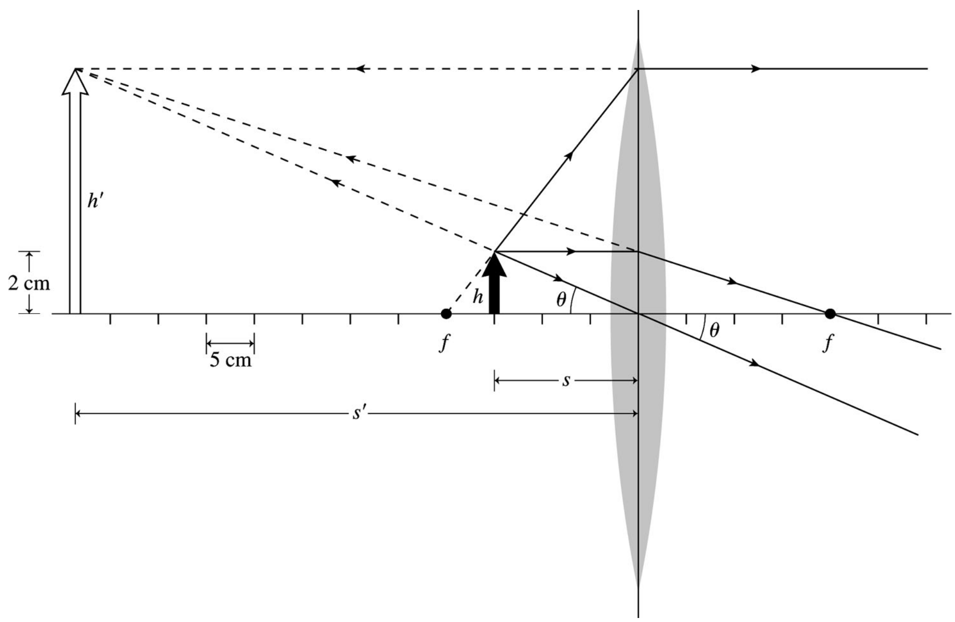
The image height is obtained from

$$m=-\frac{s'}{s}=-\frac{50\text{ cm}}{75\text{ cm}}=-\frac{2}{3}$$

The image height is $h'=|m|h=|-2/3|(1.0\text{ cm})=0.67\text{ cm}$. These results agree with those obtained in part **(a)**.

34.35. Model: Use ray tracing to locate the image. Assume that the converging lens is a thin lens.

Solve: (a)



The figure shows the ray-tracing diagram made using the steps of Tactics Box 34.2. The three special rays after refracting do not converge. Instead the rays appear to come from a point that is 60 cm on the same side of the lens as the object, so $s' = -60$ cm. The image is upright and has a height of 8.0 cm.

(b) Using the thin-lens formula,

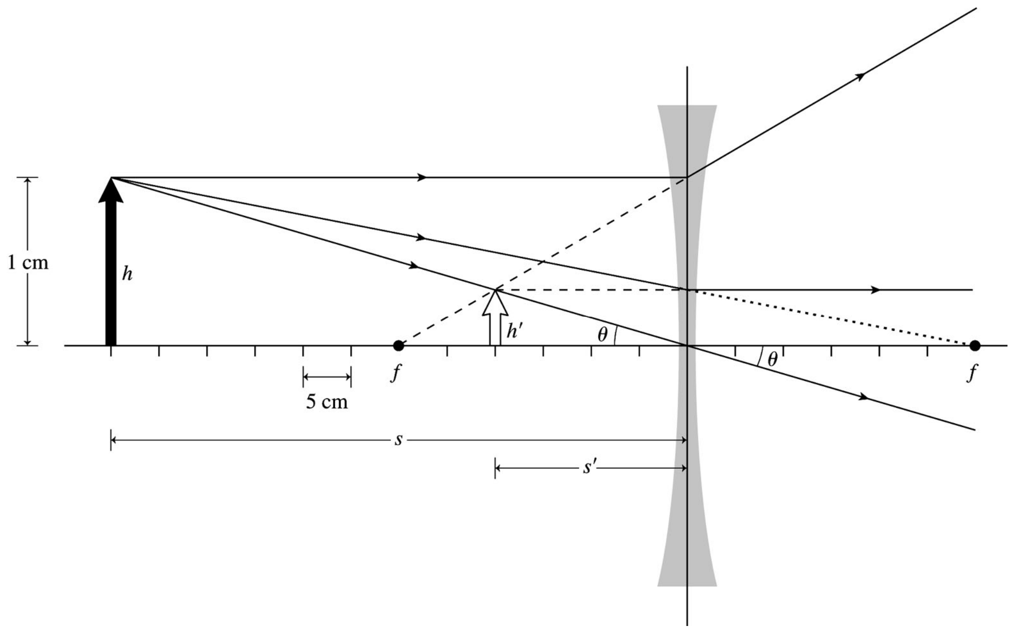
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{15\text{ cm}}+\frac{1}{s'}=\frac{1}{20\text{ cm}} \Rightarrow \frac{1}{s'}=-\frac{1}{60\text{ cm}} \Rightarrow s'=-60\text{ cm}$$

The image height is obtained from

$$m=-\frac{s'}{s}=-\frac{-60\text{ cm}}{15\text{ cm}}=+4.0$$

Thus, the image is 4.0 times larger than the object or $h'=|m|h=4.0h=4.0(2.0\text{ cm})=8.0\text{ cm}$. The image is upright. These values agree with those obtained in part **(a)**.

34.36. Model: Use ray tracing to locate the image. Assume the diverging lens is a thin lens.
Solve: (a)



The figure shows the ray-tracing diagram made using the steps of Tactics Box 34.3. After refraction from the diverging lens, the three special rays do not converge. However, the rays appear to originate at a point that is 20 cm on the same side as the object. So $s' = -20$ cm. The image is upright and has a height of 0.3 cm.

(b) The thin-lens formula gives

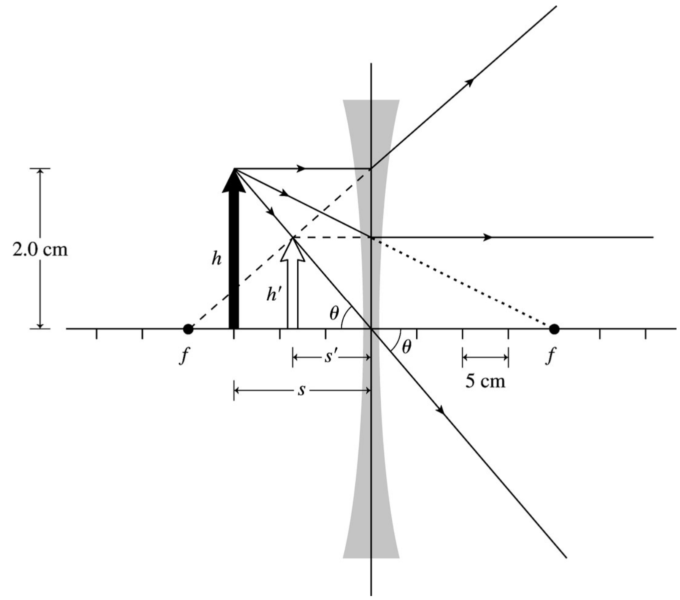
$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-30\text{ cm}} - \frac{1}{60\text{ cm}} = -\frac{1}{20\text{ cm}} \quad \Rightarrow \quad s' = -20\text{ cm}$$

The image height is obtained from

$$m = -\frac{s'}{s} = -\frac{-20\text{ cm}}{60\text{ cm}} = \frac{1}{3} = 0.33$$

Thus, $h' = |m|h = (0.33)(1.0\text{ cm}) = 0.33\text{ cm}$, and the image is upright because m is positive. These values for s' and h' agree with those obtained in part **(a)**.

34.37. Model: Use ray tracing to locate the image. Assume the diverging lens is a thin lens.
Solve: (a)



The figure shows the ray-tracing diagram made using the steps of Tactics Box 34.3. After refraction, the three special rays do not converge. The rays, on the other hand, appear to originate from a point that is 8.5 cm on the same side of the lens as the object. So $s' = 8.4$ cm. The image is upright and has a height of 1.1 cm.
(b) Using the thin-lens formula, we find

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{15\text{ cm}}+\frac{1}{s'}=\frac{1}{-20\text{ cm}} \Rightarrow \frac{1}{s'}=-\frac{7}{60\text{ cm}} \Rightarrow s'=-\frac{60}{7}\text{ cm}=-8.6\text{ cm}$$

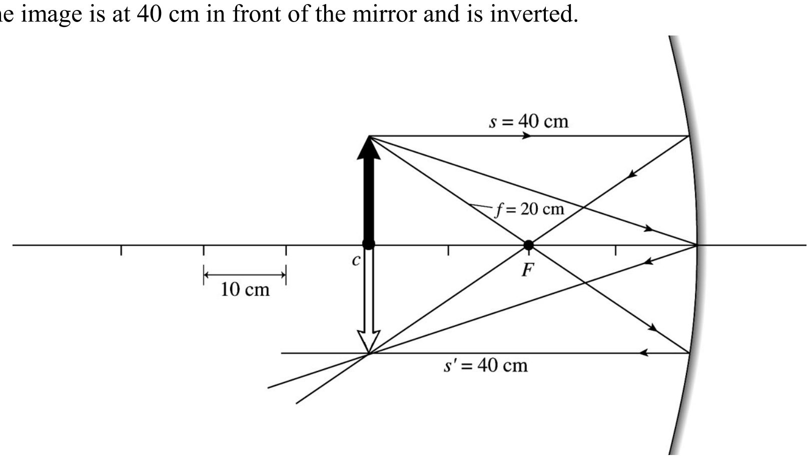
The image height is obtained from

$$m=-\frac{s'}{s}=-\frac{(-60/7\text{ cm})}{15\text{ cm}}=+\frac{4}{7}=0.57$$

Thus, the image is 0.57 times larger than the object, or $h'=|m|h=(0.57)(2.0\text{ cm})=1.1\text{ cm}$. The image is upright because m is positive. These values agree, within measurement accuracy, with those obtained in part **(a)**.

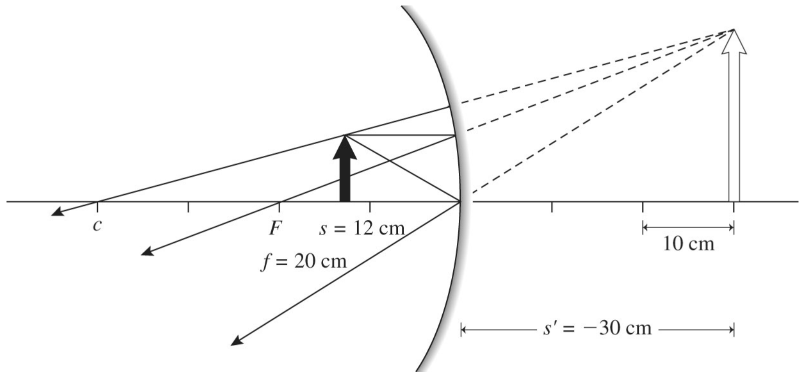
Section 34.7 Image Formation with Spherical Mirrors

34.38. Solve: The image is at 40 cm in front of the mirror and is inverted.



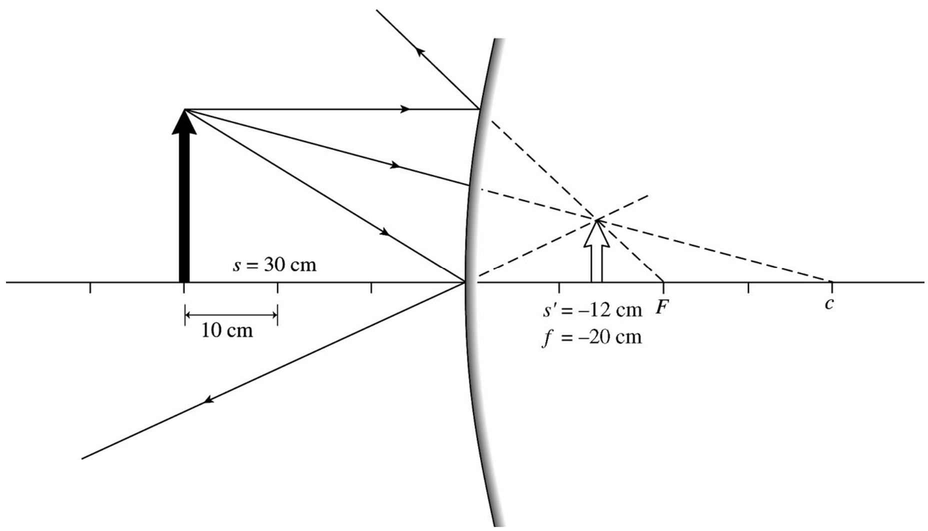
Assess: When the object is outside the focal length we get an inverted image.

34.39. Solve: The image is at −30 cm; that is, it is 30 cm behind the mirror and is upright.



Assess: When the object is within the focal length we get a magnified upright image.

34.40. Solve: The image is at −12 cm; that is, it is 12 cm behind the mirror and is upright.



Assess: We expected an upright virtual image from the convex mirror.

34.41. Visualize: Refer to Figure 34.51.
Solve: Apply the thin-lens equation, with $f = 60$ cm, $s = 20$ cm, and $h = 1.0$ cm:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \quad \Rightarrow \quad s'=\frac{fs}{s-f}=\frac{(60 \text{ cm})(20 \text{ cm})}{20 \text{ cm}-60 \text{ cm}}=-30 \text{ cm}$$

The negative sign means the image is behind the mirror, so it is a virtual image. The magnification is $m = -s'/s = 30 \text{ cm}/20 \text{ cm} = 1.5$. This means the image is upright and has a height of $h' = |m|h = |1.5|(1.0 \text{ cm}) = 1.5 \text{ cm}$.

Assess: Ray tracing will confirm these results.

34.42. Visualize: Refer to Figure 34.49.
Solve: Apply the thin-lens equation with $f = -60$ cm, $s = 20$ cm, and $h = 1.0$ cm.

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \quad \Rightarrow \quad s'=\frac{fs}{s-f}=\frac{(-60 \text{ cm})(20 \text{ cm})}{20 \text{ cm}+60 \text{ cm}}=-15 \text{ cm}$$

The negative sign means the image is behind the mirror; it is a virtual image. The magnification is $m = -s'/s = -(-15\text{ cm})/(20\text{ cm}) = 0.75$. This means the image is upright and has a height of $h' = |m|h = (0.75)(1.0\text{ cm}) = 0.75\text{ cm}$.

Assess: Ray tracing will confirm these results.

Problems

34.43. Model: The speed of light in a material is determined by the refractive index as $v = c/n$.

Solve: To acquire data from memory, a total time of only 2.0 ns is allowed. This time includes 0.5 ns that the memory unit takes to process a request. Thus, the *travel time* for an infrared light pulse from the central processing unit to the memory unit and back is 1.5 ns. Let d be the distance between the central processing unit and the memory unit. The refractive index of silicon for infrared light is $n_{\text{Si}} = 3.5$. Then,

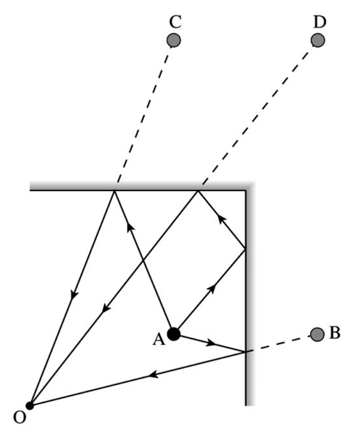
$$1.5\text{ ns} = \frac{2d}{v_{\text{Si}}} = \frac{2d}{c/n_{\text{Si}}} = \frac{2dn_{\text{Si}}}{c} \Rightarrow d = \frac{(1.5\text{ ns})c}{2n_{\text{Si}}} = \frac{(1.5\times10^{-9}\text{ s})(3.0\times10^{8}\text{ m/s})}{2(3.5)} \Rightarrow d = 6.4\text{ cm}$$

34.44. Model: Treat the red ball as a point source and use the ray model of light.

Solve: (a) Using the law of reflection, we can obtain 3 images of the red ball.

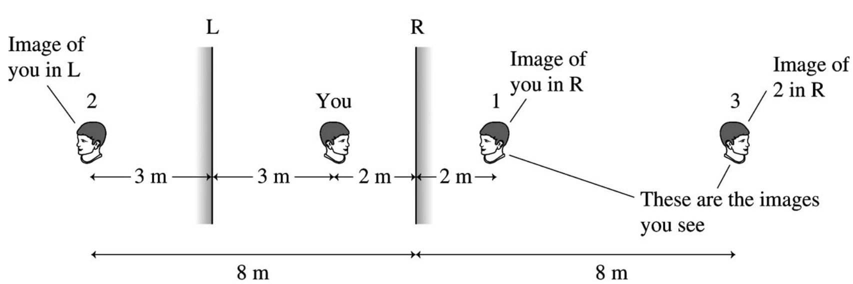
(b) The images of the ball are located at B, C, and D. Relative to the intersection point of the two mirrors, the coordinates of B, C, and D are B(+1.0 m, −2.0 m), C(−1.0 m, +2.0 m), and D(+1.0 m, +2.0 m).

(c)



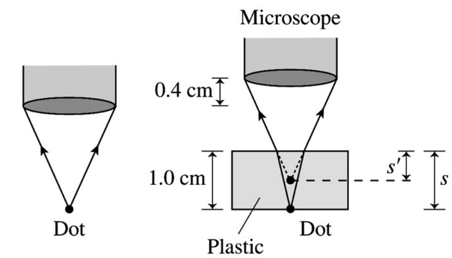
34.45. Model: For a mirror, the image distance behind the mirror equals the object's distance in front of the mirror.

Visualize:



Solve: Your face is 2.0 from the mirror into which you are looking. The image of your face (image 1) is 2.0 m behind the mirror, or 4.0 m away. Behind you, the image of the back of your head (image 2) is 3.0 m behind the mirror on the other wall. You can't see this image because you're looking to the right. However, the reflected rays that appear to come from image 2 (a virtual image) act just like the rays from an object—that is, just as the rays would if the back of your head were really at the position of image 2. These rays reflect from the mirror 2.0 m in front of you into which you're staring and form an image (image 3) 8.0 m behind the mirror. This is the image of the back of your head that you see in the mirror in front of you. Since you're 2.0 m from the mirror, the image of the back of your head is 10 m away.

34.46. Model: Use the ray model of light.
Visualize:

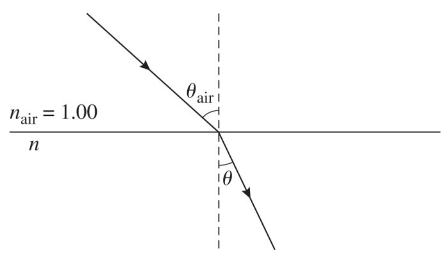


Solve: When the plastic is in place, the microscope focuses on the virtual image of the dot. From the figure, we note that $s = 1.0\text{ cm}$ and $s' = 1.0\text{ cm} - 0.4\text{ cm} = 0.6\text{ cm}$. The rays are paraxial, and the object and image distances are measured relative to the plastic-air boundary. Using Equation 34.13,

$$s' = \frac{n_{\text{air}}}{n_{\text{plastic}}} s \quad \Rightarrow \quad 0.60\text{ cm} = \frac{1.00}{n_{\text{plastic}}}(1.00\text{ cm}) \quad \Rightarrow \quad n_{\text{plastic}} = \frac{1.00\text{ cm}}{0.60\text{ cm}} = 1.7$$

Assess: This value seems reasonable because it is fairly close to the value for polystyrene plastic $(n = 1.59)$.

34.47. Model: Use the ray model of light.
Visualize:



Solve: (a) We are given that $\theta_{\text{air}} = 2\theta$. Insert this into Snell's law (Equation 34.3) and solve for θ_{air}:

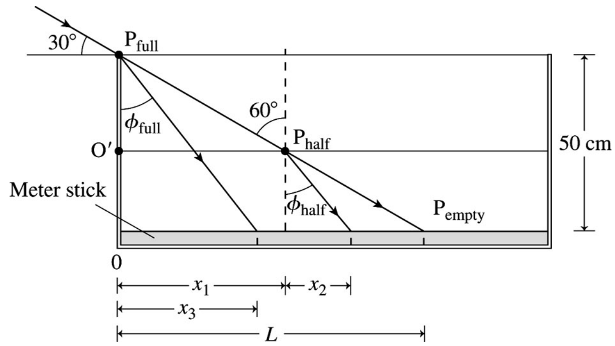
$$n_{\text{air}} \sin\theta_{\text{air}} = n\sin\theta = n\sin\tfrac{\theta_{\text{air}}}{2} \quad \Rightarrow \quad \cos\tfrac{\theta_{\text{air}}}{2} = \frac{n}{2n_{\text{air}}} \quad \Rightarrow \quad \theta_{\text{air}} = 2\cos^{-1}\left(\frac{n}{2n_{\text{air}}}\right)$$

In the second step, we have used the trigonometric identity $\sin 2u = 2\sin u \cos u$.
(b) For light incident on glass, we have

$$\theta_{\text{air}} = 2\cos^{-1}\left(\frac{n}{2n_{\text{air}}}\right) = 2\cos\left(\frac{1.50}{2(1.00)}\right) = 82.8°$$

Assess: This angle is less than 90° and so is physically reasonable.

34.48. Model: Use the ray model of light and the law of refraction.
Visualize:



Solve: **(a)** The ray of light strikes the meter stick at P_{empty}, which is a distance L from the zero mark of the meter stick. So,

$$\tan 60° = \frac{L}{50\text{ cm}} \quad \Rightarrow \quad L = (50\text{ cm})\tan 60° = 87\text{ cm}$$

(b) The ray of light refracts at P_{half} and strikes the meter stick a distance $x_1 + x_2$ from the zero of the meter stick. We can find x_1 from the triangle $P_{full}P_{half}O'$:

$$\tan 60° = \frac{x_1}{25\text{ cm}} \quad \Rightarrow \quad x_1 = (25\text{ cm})\tan 60° = 43.30\text{ cm}$$

We also have $x_2 = (25\text{ cm})\tan\phi_{half}$. Using Snell's law,

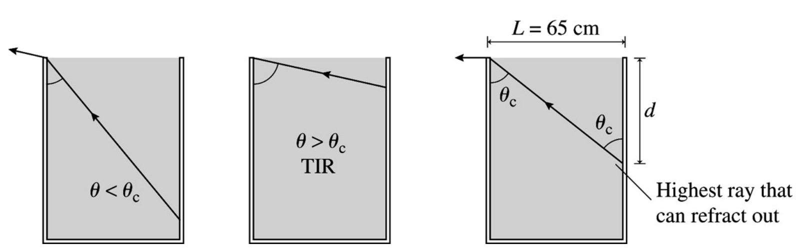
$$n_{air}\sin 60° = n_{water}\sin\phi_{half} \quad \Rightarrow \quad \phi_{half} = \sin^{-1}\left(\frac{\sin 60°}{1.33}\right) = 40.63°$$

$$x_2 = (25\text{ cm})\tan 40.63° = 21.45\text{ cm} \quad \Rightarrow \quad x_1 + x_2 = 43.30\text{ cm} + 21.45\text{ cm} = 65\text{ cm}$$

(c) The ray of light experiences refraction at P_{full} and the angle of refraction is the same as in part **(b)**. We get

$$\tan\phi_{full} = \frac{x_3}{50\text{ cm}} \quad \Rightarrow \quad x_3 = (50\text{ cm})\tan 40.63° = 43\text{ cm}$$

34.49. Model: Use the ray model of light. Light undergoes total internal reflection if it is incident on a boundary at an angle greater than the critical angle.
Visualize:



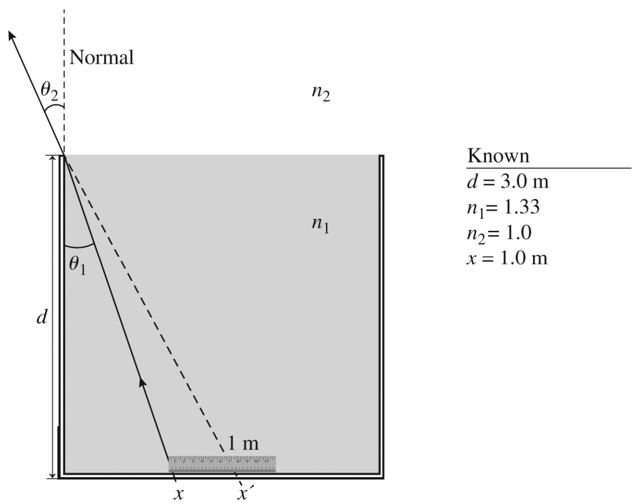
Solve: **(a)** To reach your eye, a light ray must refract through the *top* surface of the water and into the air. You can see in the figure that rays coming from the bottom of the tank are incident on the top surface at fairly small angles, but rays from the marks near the top of the tank are incident at very large angles—greater than the critical angle. These rays undergo total internal reflection in the water and do not exit into the air where they can be seen. Thus you can see the marks from the bottom of the tank upward.

(b) The highest point you can see is the one from which the ray reaches the top surface at the critical angle θ_c. For a water-air boundary, the critical angle is $\theta_c = \sin^{-1}(1/1.33) = 48.75°$. You can see from the figure that the depth of this point is such that

$$\frac{L}{d} = \tan\theta_c \quad \Rightarrow \quad d = \frac{L}{\tan\theta_c} = \frac{65\text{ cm}}{\tan(48.75°)} = 57\text{ cm}$$

Since the marks are every 10 cm, the high mark you can see is the one at 60 cm.

34.50. Model: Assume pool is filled to the top and there is air above the water.
Visualize:



We compute the apparent position (call this x') of the left end of the stick (really at position x) by finding θ_2 and then extending the line down to the bottom of the pool.
Solve: First find θ_1 from the right triangle.

$$\tan\theta_1 = \frac{x}{d} \Rightarrow \theta_1 = \tan^{-1}\frac{x}{d}$$

With $n_2 = 1.0$, Snell's law gives

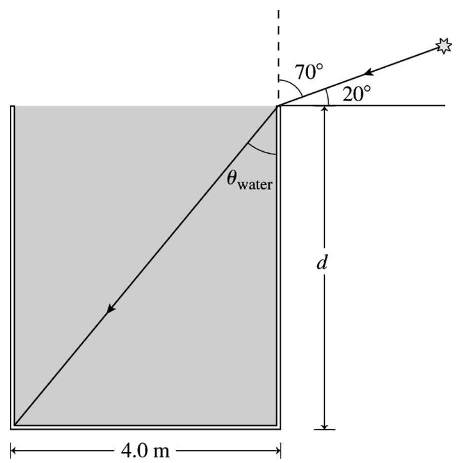
$$\theta_2 = \sin^{-1}(n_1 \sin\theta_1) = \sin^{-1}\left(n_1 \sin\left(\tan^{-1}\frac{x}{d}\right)\right)$$

Now draw that line back down to the bottom of the pool to find x'.

$$\tan\theta_2 = \frac{x'}{d} \Rightarrow x' = d\tan\theta_2 = d\left(\tan\left(\sin^{-1}\left(n_1 \sin\left(\tan^{-1}\frac{x}{d}\right)\right)\right)\right)$$

For the left end of the meter stick $x = 1.0$ m; plugging in to the equation above gives $x' = 1.39$ m.
Repeat for the right end of the meter stick with $x = 2.0$ m. The equation then gives $x' = 3.28$ m.
The length of the meter stick will appear to be 3.28 m – 1.39 m = 1.9 m long.
Assess: We expected the meter stick to look longer under water.

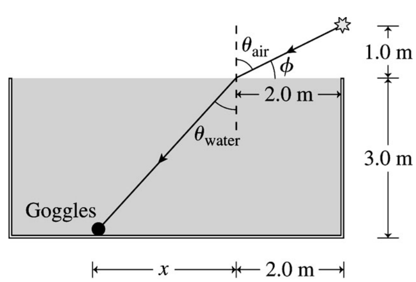
34.51. Model: Use the ray model of light and the law of refraction. Assume the sun is a point source of light.
Visualize:



When the bottom of the pool becomes completely shaded, a ray of light that is incident at the top edge of the swimming pool does not reach the bottom of the pool after refraction.
Solve: The depth of the swimming pool is $d = 4.0 \text{ m}/\tan\theta_{water}$. We will find the angle by using Snell's law. We have

$$n_{water}\sin\theta_{water} = n_{air}\sin 70° \quad\Rightarrow\quad \theta_{water} = \sin^{-1}\left(\frac{\sin 70°}{1.33}\right) = 44.95° \quad\Rightarrow\quad d = \frac{4.0 \text{ m}}{\tan 44.95°} = 4.0 \text{ m}$$

34.52. Model: Use the ray model of light and the law of refraction. Assume that the laser beam is a ray of light.
Visualize:



The laser beam enters the water 2.0 m from the edge, undergoes refraction, and illuminates the goggles. The ray of light from the goggles then retraces its path and enters your eyes.
Solve: From the geometry of the diagram,

$$\tan\phi = \frac{1.0 \text{ m}}{2.0 \text{ m}} \quad\Rightarrow\quad \phi = \tan^{-1}(0.50) = 26.57° \quad\Rightarrow\quad \theta_{air} = 90° - 26.57° = 63.43°$$

Snell's law at the air-water boundary is $n_{air}\sin\theta_{air} = n_{water}\sin\theta_{water}$. Using the above result gives

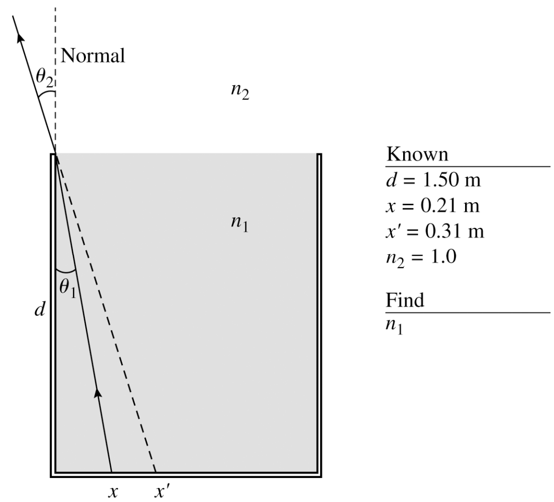
$$(1.00)\sin 63.43° = 1.33\sin\theta_{water} \quad\Rightarrow\quad \theta_{water} = \sin^{-1}\left(\frac{\sin 63.43°}{1.33}\right) = 42.26°$$

Taking advantage of the geometry in the diagram again, we have

$$\frac{x}{3.0 \text{ m}} = \tan\theta_{water} \quad\Rightarrow\quad x = (3.0 \text{ m})\tan 42.26° = 2.73 \text{ m}$$

The distance of the goggles from the edge of the pool is 2.73 m + 2.0 m = 4.73 m ≈ 4.7 m.

34.53. Model: Assume pool is filled to the top and there is air with $n_2 = 1.0$ above the water.
Visualize: Note the right triangle with side d and hypotenuse R.



Solve: Find both angles from their respective right triangles.

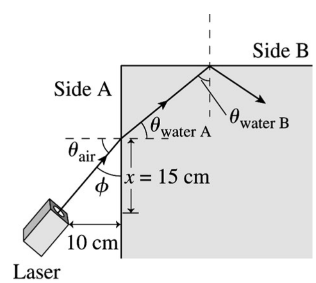
$$\theta_1 = \tan^{-1}\frac{x}{d} \qquad \theta_2 = \tan^{-1}\frac{x'}{d}$$

With $n_2 = 1.0$, Snell's law $(n_1 \sin\theta_1 = n_2 \sin\theta_2)$ gives

$$n_1 = \frac{\sin\theta_2}{\sin\theta_1} = \frac{\sin\left(\tan^{-1}\frac{x'}{d}\right)}{\sin\left(\tan^{-1}\frac{x}{d}\right)} = \frac{\sin\left(\tan^{-1}\frac{0.31\text{ m}}{1.50\text{ m}}\right)}{\sin\left(\tan^{-1}\frac{0.21\text{ m}}{1.50\text{ m}}\right)} = 1.46$$

Assess: This is in the range of typical indices of refraction.

34.54. Model: Use the ray model of light and the law of refraction. Assume that the laser beam is a ray of light.
Visualize:



Solve: **(a)** From the geometry of the diagram at side A, we have

$$\tan\phi = \frac{10\text{ cm}}{15\text{ cm}} \Rightarrow \phi = \tan^{-1}\left(\frac{10}{15}\right) \Rightarrow \phi = 33.69°$$

This means the angle of incidence at side A is $\theta_{\text{air}} = 90° - 33.69° = 56.31°$. Using Snell's law at side A gives

$$n_{\text{air}} \sin\theta_{\text{air}} = n_{\text{water}} \sin\theta_{\text{water A}} \Rightarrow \theta_{\text{water A}} = \sin^{-1}\left(\frac{1.00 \sin(56.31°)}{1.33}\right) = 38.73°$$

This ray of light now strikes side B. The angle of incidence at this water-air boundary is $\theta_{\text{water B}} = 90° - \theta_{\text{water A}} = 51.3°$. The critical angle for the water-air boundary is

$$\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) = \sin^{-1}\left(\frac{1.0}{1.33}\right) = 48.8°$$

Because the angle $\theta_{\text{water B}} > \theta_c$, the ray will experience total internal reflection.

(b) We will now repeat the above calculation with $x = 25$ cm. From the geometry of the diagram at side A, $\phi = 21.80°$ and $\theta_{\text{air}} = 68.20°$. Using Snell's law at the air-water boundary, $\theta_{\text{water A}} = 44.28°$ and $\theta_{\text{water B}} = 45.72°$. Because $\theta_{\text{water B}} < \theta_c$, the ray will be refracted into the air. The angle of refraction is calculated as follows:

$$n_{\text{air}} \sin\theta_{\text{air B}} = n_{\text{water}} \sin\theta_{\text{water B}} \quad \Rightarrow \quad \theta_{\text{air B}} = \sin^{-1}\left(\frac{1.33\sin(45.72°)}{1.00}\right) = 72°$$

(c) Using the critical angle for the water-air boundary found in part **(a)**, $\theta_{\text{water A}} = 90° - 48.75° = 41.25°$. According to Snell's law,

$$n_{\text{air}} \sin\theta_{\text{air}} = n_{\text{water}} \sin\theta_{\text{water A}} \Rightarrow \theta_{\text{air}} = \sin^{-1}\left(\frac{1.33\sin 41.25°}{1.0}\right) = 61.27°$$

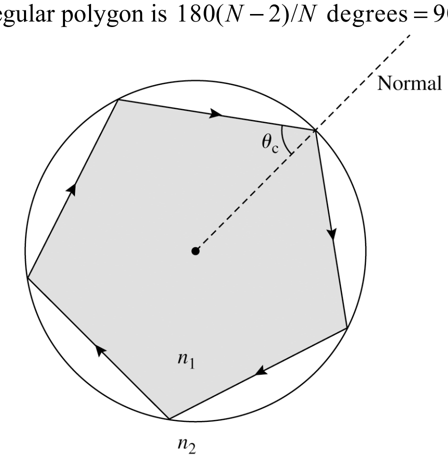
$$\phi = 90° - 61.27° = 28.73°$$

The minimum value of x for which the laser beam passes through side B and emerges into the air is calculated as follows:

$$\tan\phi = \frac{10 \text{ cm}}{x} \quad \Rightarrow \quad x = \frac{10 \text{ cm}}{\tan 28.73°} = 18.2 \text{ cm} \approx 18 \text{ cm}$$

34.55. Model: Assume the cylinder is surrounded with air with $n_2 = 1.0$.

Visualize: One interior angle in a regular polygon is $180(N-2)/N$ degrees $= 90° - 180°/N$.



Solve: The critical angle would have to be less than half the interior angle.

$$\theta_c = \sin^{-1}\frac{1}{n_1} < 90° - \frac{180°}{N} \Rightarrow N > \frac{180°}{90° - \sin^{-1}(1/n_1)}$$

So in each case the minimum possible value of N is the smallest integer that satisfies the above inequality.

(a) For water $n_1 = 1.33$ and

$$N > \frac{180°}{90° - \sin^{-1}(1/n_1)} = \frac{180°}{90° - \sin^{-1}(1/1.33)} = 4.36 \Rightarrow N = 5$$

(b) For polystyrene $n_1 = 1.59$ and

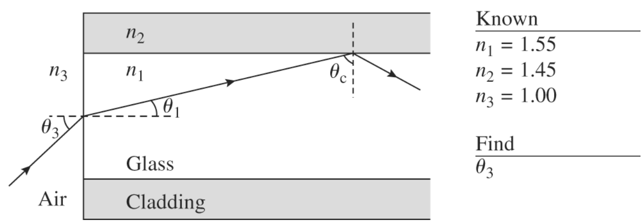
$$N > \frac{180°}{90° - \sin^{-1}(1/n_1)} = \frac{180°}{90° - \sin^{-1}(1/1.59)} = 3.53 \Rightarrow N = 4$$

(c) For cubic zirconia $n_1 = 2.18$ and

$$N > \frac{180°}{90° - \sin^{-1}(1/n_1)} = \frac{180°}{90° - \sin^{-1}(1/2.18)} = 2.87 \Rightarrow N = 3$$

Assess: It makes sense that the higher indices of refraction allow polygons with smaller *N*.

34.56. Model: Assume the end of the fiber is in air with $n_3 = 1.0$.
Visualize:



Solve: The critical angle between the glass fiber and the cladding is

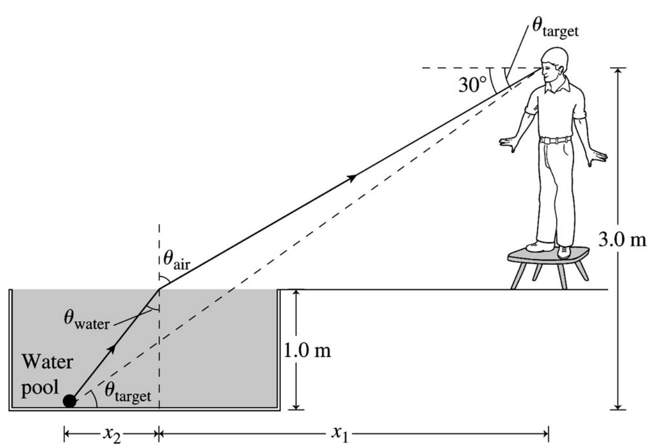
$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Note the right triangle in the figure: $\theta_1 = 90° - \theta_c$. Use Snell's law to find θ_3, the angle of the acceptance cone.

$$n_1 \sin\theta_1 = n_3 \sin\theta_3 \Rightarrow \theta_3 = \sin^{-1}(n_1 \sin\theta_1) = \sin^{-1}(n_1 \sin(90° - \theta_c)) = \sin^{-1}(n_1 \sin(90° - \sin^{-1}(n_2/n_1)))$$
$$= \sin^{-1}((1.55)\sin(90° - \sin^{-1}(1.45/1.55))) = 33.2°$$

Assess: This seems like a reasonable acceptance cone angle. The angles in the figure would be the same no matter where the ray entered on the entrance face.

34.57. Model: Use the ray model of light. Assume that the target is a point source of light.
Visualize:



Solve: From the geometry of the figure with $\theta_{\text{air}} = 60°$, we find

$$\tan\theta_{\text{air}} = \frac{x_1}{2.0\text{ m}} \quad\Rightarrow\quad x_1 = (2.0\text{ m})(\tan 60°) = 3.464\text{ m}$$

Let us find the horizontal distance x_2 by applying Snell's law to the air-water boundary. We have

$$n_{\text{water}}\sin\theta_{\text{water}} = n_{\text{air}}\sin\theta_{\text{air}} \quad\Rightarrow\quad \theta_{\text{water}} = \sin^{-1}\left(\frac{\sin 60°}{1.33}\right) = 40.63°$$

Using the geometry of the diagram,

$$\frac{x_2}{1.0\text{ m}} = \tan\theta_{\text{water}} \quad\Rightarrow\quad x_2 = (1.0\text{ m})\tan 40.63° = 0.858\text{ m}$$

To determine θ_{target}, we note that

$$\tan\theta_{\text{target}} = \frac{3.0\text{ m}}{x_1 + x_2} = \frac{3.0\text{ m}}{3.464\text{ m} + 0.858\text{ m}} = 0.6941 \quad\Rightarrow\quad \theta_{\text{target}} = 35°$$

34.58. Model: Use the ray model of light.
Solve: **(a)** Using Snell's law at the air-glass boundary, with ϕ being the angle of refraction inside the prism,

$$n_{\text{air}}\sin\beta = n\sin\phi \quad\Rightarrow\quad \sin\beta = n\sin\phi$$

Considering the triangle made by the apex angle and the refracted ray,

$$(90° - \phi) + (90° - \phi) + \alpha = 180° \quad\Rightarrow\quad \phi = \tfrac{1}{2}\alpha$$

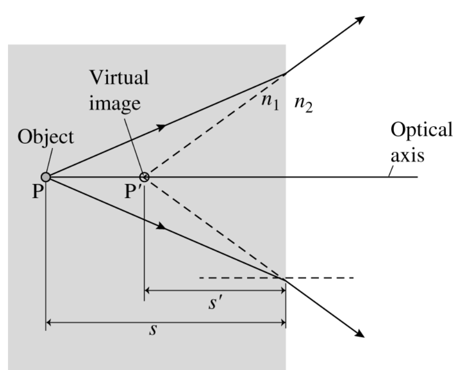
Thus

$$\sin\beta = n\sin\left(\tfrac{\alpha}{2}\right) \quad\Rightarrow\quad \beta = \sin^{-1}\left[n\sin\left(\tfrac{\alpha}{2}\right)\right]$$

(b) For an equilateral triangle, $\alpha = 60.0°$. Using the above expression, we obtain

$$n = \frac{\sin\beta}{\sin\left(\tfrac{1}{2}\alpha\right)} = \frac{\sin(52.2°)}{\sin(30.0°)} = 1.58$$

34.59. Model: Assume the shark tank is next to the air with $n_3 = 1.0$. Ignore the glass walls of the tank.
Visualize:



Solve: The text demonstrates that the (virtual) image distance is

$$s' = \frac{n_2}{n_1}s$$

Both s and s' are changing in time; to see the relationship take the derivative of both sides of the equation above.

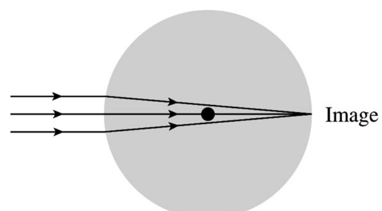
$$\frac{ds'}{dt} = \frac{n_2}{n_1}\frac{ds}{dt}$$

The left side is the apparent speed and the last factor on the right is the shark's actual speed. Solve for the actual speed, which we seek.

$$v = \frac{ds}{dt} = \frac{n_1}{n_2}\frac{ds'}{dt} = \frac{n_1}{n_2}v' = \frac{1.33}{1.00}(2.0\text{ m/s}) = 2.7\text{ m/s}$$

Assess: We anticipated that the actual speed would be greater than the apparent speed because the shark is farther away than it looks, but if the motion continues to the glass wall the object and image would both arrive together. We did not need the length of the shark with this method of solution.

34.60. Model: Use the ray model of light. The surface is a spherically refracting surface.
Visualize:



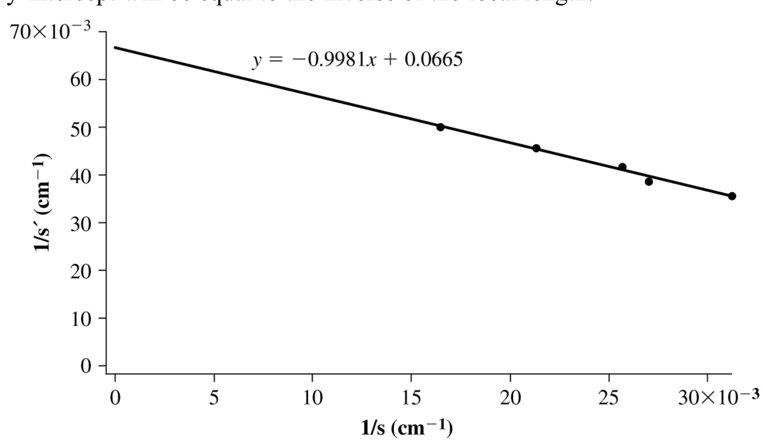
Solve: Because the rays are parallel, $s = \infty$. The rays come to focus on the rear surface of the sphere, so $s' = 2R$, where R is the radius of curvature of the sphere. Equation 34.20 gives

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \quad\Rightarrow\quad \frac{1}{\infty} + \frac{n}{2R} = \frac{n-1}{R} \quad\Rightarrow\quad n = 2.00$$

34.61. Model: Use the ray model of light and assume the lens is thin.
Solve: Applying the thin-lens equation (Equation 34.25) to the data, we get

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad\Rightarrow\quad \frac{1}{s'} = \frac{1}{f} - \frac{1}{s}$$

If we plot the inverse of the image distance s' versus the inverse of the object distance s, the graph should be a straight line and the y-intercept will be equal to the inverse of the focal length.



The linear fit is very good. The y-intercept of 0.0665 cm^{-1} gives a focal length of $f = 1/(0.0665\text{ cm}^{-1}) = 15.0\text{ cm}$.
Assess: We can check our result by using a data point to calculate f: $1/f = 1/(20\text{ cm}) + 1/(61\text{ cm})$, which gives $f = 15.1\text{ cm}$, which is very close to the graphical value.

34.62. Solve: Apply the mirror equation (Equation 34.27) with $s' = +1.2$ m (image is real and on same side as object) and $f = R/2 = (30 \text{ cm})/2 = 15$ cm (mirror is concave toward object):

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \Rightarrow \quad s = \frac{fs'}{s' - f} = \frac{(15 \text{ cm})(120 \text{ cm})}{120 \text{ cm} - 15 \text{ cm}} = 17 \text{ cm}$$

Thus, the lamp should be 17 cm from the mirror.
Assess: Ray tracing will confirm the results.

34.63. Visualize: Refer to Figure 34.51.
Solve: We are given that $s = +1.2$ m. For an upright image, $m > 0$, so $m = +1.5$. Equation 34.29 then gives the image distance: $s' = -ms = -(1.5)(1.2 \text{ cm}) = -1.8$ cm. This is a virtual image that is located on the opposite side of the mirror from the object. The focal length may be found by using Equation 34.27:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \Rightarrow \quad f = \frac{ss'}{s + s'} = \frac{(1.2 \text{ cm})(-1.8 \text{ cm})}{1.2 \text{ cm} - 1.8 \text{ cm}} = 3.6 \text{ cm}$$

The focal length is positive, so the mirror is concave toward the object.
Assess: Ray tracing will confirm the results.

34.64. Model: The cornea acts as a convex mirror.
Visualize: The magnitude of the magnification of the image is $m = s'/s \Rightarrow s' = ms$.
Solve: Combine the mirror equation, the magnification equation, and the equation relating f to the radius of curvature. First the mirror equation:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow f = \frac{ss'}{s + s'}$$

Put these in the equation for R and simplify.

$$f = \frac{R}{2} \Rightarrow R = 2f = 2\frac{ss'}{s + s'} = 2\frac{s(ms)}{s + ms} = 2s\frac{m}{1 + m} = 2(0.075 \text{ m})\frac{0.049}{1.049} = 7.0 \text{ mm}$$

Assess: The answer is a realistic value.

34.65. Model: The source is in air with $n_2 = 1.0$.
Visualize: We are given $f = R/2 = 4.0 \text{ cm}/2 = 2.0$ cm.
Solve: From inside the glass the object will appear to be farther than 2.0 cm away from the surface; it will appear to be $n_1/n_2 = 1.33$ times farther away. So, for the mirror equation, we set $s = 4.0 \text{ cm} + (1.33)(2.0 \text{ cm}) = 6.66$ cm.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = \frac{sf}{s - f} = \frac{(6.66 \text{ cm})(2.0 \text{ cm})}{6.66 \text{ cm} - 2.0 \text{ cm}} = 2.8 \text{ cm}$$

Assess: Because the answer is still in the glass we do not have to re-correct as the rays leave the glass into the air.

34.66. Model: Assume the lens to be a thin lens.
Solve: Because we want to form an image of the candle on the wall, we need a converging lens. We have $s + s' = 200$ cm. Using the thin-lens formula,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{s} + \frac{1}{200 \text{ cm} - s} = \frac{1}{32 \text{ cm}} \quad \Rightarrow \quad s^2 - (200 \text{ cm})s + 6400 \text{ cm}^2 = 0 \quad \Rightarrow$$

The two solutions to this equation are $s = 160$ cm and 40 cm. When $s = 160$ cm, then $s' = 200 \text{ cm} - 160 \text{ cm} = 40$ cm. The magnification is

$$m = -\frac{s'}{s} = -\frac{40 \text{ cm}}{160 \text{ cm}} = -0.25$$

so the image is inverted and its height is (2.0 cm)(0.25) = 0.50 cm. When s = 40 cm, then s' = 200 cm – 40 cm = 160 cm. The magnification is

$$m = -\frac{s'}{s} = -\frac{160\ \text{cm}}{40\ \text{cm}} = -4.0$$

so the image is again inverted and its height is (2.0 cm)(4.0) = 8.0 cm.

34.67. Model: The ball is dropped from rest and is in free fall.
Visualize: Call the distance dropped d. We are given $d + s = 3.0\ \text{m}$ and $\Delta t = 0.65\ \text{s}$.
Solve: Use the equation for R with $s' = s$.

$$f = \frac{R}{2} \Rightarrow R = 2f = 2\frac{ss'}{s+s'} = 2\frac{s^2}{s+s} = s = 3.0\ \text{m} - d = 3.0\ \text{m} - \frac{1}{2}g(\Delta t)^2 = 3.0\ \text{m} - \frac{1}{2}(9.8\ \text{m/s}^2)(0.65)^2 = 93\ \text{cm}$$

Assess: This sounds like a reasonable radius of curvature for a concave mirror.

34.68. Model: The metamaterial lens is thin and we can use the lens maker's equation.
Visualize: Call the distance dropped d. We are given $d + s = 3.0\ \text{m}$ and $\Delta t = 0.65\ \text{s}$.
Solve: Use the lens maker's equation with $R_2 = \infty$ to find the focal length.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n-1)\left(\frac{1}{R_1}\right) = (-1.25-1)\left(\frac{1}{15\ \text{cm}}\right) \Rightarrow f = -6.67\ \text{cm}$$

Now use the thin lens equation to find the image distance.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = \frac{sf}{s-f} = \frac{(12\ \text{cm})(-6.67\ \text{cm})}{12\ \text{cm} + 6.67\ \text{cm}} = -4.3\ \text{cm}$$

(a) Because of the negative sign, the image will be virtual.
(b) The image will be 4.3 cm in front of the lens.
Assess: It's a diverging lens that is thicker in the center.

34.69. Model: Assume a thin lens.
Solve: The image and object distance must sum to 3.0 m: $s' + s$ = 3.0 m. The magnification $m = \pm 2 = -s'/s$. From these two equations, the image distance is determined to be

$$\left.\begin{array}{r} s + s' = 3.0\ \text{m} \\ \pm 2 = -\dfrac{s'}{s} \end{array}\right\} s \mp 2s = 3.0\ \text{m} \quad \Rightarrow \quad s = \frac{3.0\ \text{m}}{1 \mp 2} = -3.0\ \text{m or } 1.0\ \text{m}$$

A positive magnification leads to an unphysical object distance of –3.0 m, so the magnification must be negative. Thus, s = 1.0 m, s' = 2.0 m, and the focal length is

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \Rightarrow \quad f = \frac{ss'}{s+s'} = \frac{(1.0\ \text{m})(2.0\ \text{m})}{1.0\ \text{m} + 2.0\ \text{m}} = 0.67\ \text{m}$$

Thus, the lens should be 1.0 m from the bulb and have a focal length of 67 cm.
Assess: Only a converging lens can form an image on the wall since the image must be real. This agrees with our positive focal length.

34.70. Model: Assume the projector lens is a thin lens.
Solve: **(a)** The absolute value of the magnification of the lens is

$$|m| = \left|\frac{h'}{h}\right| = \left|\frac{98\ \text{cm}}{2.0\ \text{cm}}\right| = 49$$

Because the projector forms a real image of a real object, the image will be inverted. Thus,

$$m = -49 = -\frac{s'}{s} \quad \Rightarrow \quad s' = 49s$$

We also have

$$s + s' = 300 \text{ cm} \quad \Rightarrow \quad s + 49s = 300 \text{ cm} \quad \Rightarrow \quad s = 6.0 \text{ cm} \quad \Rightarrow \quad s' = 294 \text{ cm}$$

Using these values of s and s', we can find the focal length of the lens:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{6.0 \text{ cm}} + \frac{1}{294 \text{ cm}} \quad \Rightarrow \quad f = 5.9 \text{ cm}$$

(b) From part **(a)** the lens should be 6.0 cm from the slide.

34.71. Model: The lens is thin and we can use the lens maker's equation.
Visualize: We are given $s = 50.0$ cm, $R_1 = 15.0$ cm.
Solve: Use the lens maker's equation with $R_2 = \infty$ to find the focal length before the adjustment.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n-1)\left(\frac{1}{R_1}\right) = (1.500-1)\left(\frac{1}{15 \text{ cm}}\right) \Rightarrow f = 30.0 \text{ cm}$$

Now use the thin lens equation to find the image distance before the adjustment.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = \frac{sf}{s-f} = \frac{(50.0 \text{ cm})(30.0 \text{ cm})}{50.0 \text{ cm} - 30.0 \text{ cm}} = 75.0 \text{ cm}$$

Now we adjust the index of refraction to make $s' = 75.0 \text{ cm} - 5.0 \text{ cm} = 70.0$ cm and reverse the steps we went through. Solve the thin lens equation for f.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow f = \frac{ss'}{s+s'} = \frac{(50.0 \text{ cm})(70.0 \text{ cm})}{50.0 \text{ cm} + 70.0 \text{ cm}} = 29.1667 \text{ cm}$$

Solve the lens maker's equation for n and insert the value for f just obtained.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1}\right) \Rightarrow \frac{1/f}{1/R_1} = n-1 \Rightarrow n = \frac{R_1}{f} + 1 = \frac{15.0 \text{ cm}}{29.1667 \text{ cm}} + 1 = 1.514$$

The amount the index of refraction must be increased is $\Delta n = 1.514 - 1.500 = 0.014$.
Assess: The needed increase is small, so the electro-optic material can probably handle it.

34.72. Model: The lens is thin and we can use the lens maker's equation. $n_{\text{glass}} = 1.5$.
Visualize: We are given $R_1 = R_2 = 3.0$ cm and $s = d + R = d + 3.0 \text{ cm} \Rightarrow d = s - 3.0$ cm.
Solve: Use the thin lens equation with $s = s'$ and the lens maker's equation with $R_1 = R_2 = R$.

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{s} = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n-1)\left(\frac{2}{R}\right) \Rightarrow \frac{1}{s} = (n-1)\frac{1}{R} \Rightarrow s = \frac{1}{n-1}R = \frac{1}{1.5-1}(3.0 \text{ cm}) = 6.0 \text{ cm}$$

Now $d = s - 3.0 \text{ cm} = 6.0 \text{ cm} - 3.0 \text{ cm} = 3.0$ cm.
Assess: The distance from the center is twice the radius in this thin lens approximation.

34.73. Visualize: The lens must be a converging lens for this scenario to happen, so we expect f to be positive. In the first case the upright image is virtual $(s_2' < 0)$ and the object must be closer to the lens than the focal point. The lens is then moved backward past the focal point and the image becomes real $(s_2' > 0)$.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \Rightarrow \quad f = \frac{ss'}{s+s'}$$

We are given $s_1 = 10$cm and $m_1 = 2$.

Solve: Since the first image is virtual, $s' < 0$. We are told the first magnification is $m_1 = 2 = -s_2'/s_1 \Rightarrow s_1' = -20$ cm. We can now find the focal length of the lens:

$$f = \frac{s_1 s_1'}{s_1 + s_1'} = \frac{(10 \text{ cm})(-20 \text{ cm})}{10 \text{ cm} - 20 \text{ cm}} = 20 \text{ cm}$$

After the lens is moved, $m_2 = -2 = -s_2'/s_2$. Start with the thin lens equation again.

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f}$$

Replace s_2' with $-m_2 s_2$.

$$\frac{1}{s_2} + \frac{1}{-m_2 s_2} = \frac{1}{f}$$

Now solve for s_2.

$$\frac{-m_2 s_2 + s_2}{s_2(-m_2 s_2)} = \frac{1}{f} \quad \Rightarrow \quad \frac{-m_2 s_2 + s_2}{-m_2 s_2^2} = \frac{1}{f} \quad \Rightarrow \quad \frac{m_2 - 1}{m_2 s_2} = \frac{1}{f}$$

Multiply both sides by $f s_2$.

$$s_2 = f\left(\frac{m_2 - 1}{m_2}\right) = (20 \text{ cm})\left(\frac{-2-1}{-2}\right) = 30 \text{ cm}$$

The distance the lens moved is $s_2 - s_1 = 30 \text{ cm} - 10 \text{ cm} = 20 \text{ cm}$.

Assess: We knew s_2 needed to be bigger than f; it is, and is in a reasonable range. The final answer for the distance the lens moved also seems reasonable.

34.74. Model: Assume the symmetric converging lens is a thin lens.
Solve: Because the lens forms a real image on the screen of a real object, the image is inverted. Thus, $m = -2 = -s'/s$. Also,

$$s + s' = 60 \text{ cm} \quad \Rightarrow \quad s + 2s = 60 \text{ cm} \quad \Rightarrow \quad s = 20 \text{ cm} \quad \Rightarrow \quad s' = 40 \text{ cm}$$

We can use the thin-lens formula to determine the radius of curvature of the symmetric converging lens ($|R_1| = |R_2|$) as follows:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Using $R_1 = +|R|$ (convex toward the object), $R_2 = -|R|$ (concave toward the object), and $n = 1.59$, we find

$$\frac{1}{20 \text{ cm}} + \frac{1}{40 \text{ cm}} = (1.59-1)\left(\frac{1}{|R|} - \frac{1}{-|R|}\right) \quad \Rightarrow \quad \frac{3}{40 \text{ cm}} = \frac{1.18}{|R|} \quad \Rightarrow \quad |R| = 15.7 \text{ cm} \approx 16 \text{ cm}$$

34.75. Model: The lens is thin.
Visualize: We are given $f = 200 \text{ mm} = 0.20 \text{ m}$ and $s = 100 \text{ m}$. Set the positive direction as toward the lens (the direction the rhino is running). We are given $ds/dt = 5.0$ m/s.
Solve: Solve the thin lens equation for s'.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = \frac{sf}{s-f}$$

With f a constant, take the derivative with respect to t of both sides. Use the quotient rule.

$$\frac{ds'}{dt} = \frac{(s-f)f\frac{ds}{dt} - sf\frac{ds}{dt}}{(s-f)^2} = \frac{(s-f-s)f\frac{ds}{dt}}{(s-f)^2} = \frac{-f^2}{(s-f)^2}\frac{ds}{dt} = \frac{-(0.20 \text{ m})^2}{(100 \text{ m} - 0.20 \text{ m})^2}(5.0 \text{ m/s}) = -20\ \mu\text{m/s}$$

The speed of the image of the rhino is $20\ \mu\text{m/s}$ and the negative sign says it is moving away from the lens.
Assess: The image is moving suitably slowly.

34.76. Visualize: We are given $f = R/2 = 40\text{ cm}/2 = 20\text{ cm}$. We are also given $m = -s'/s = 3$.
Solve: Solve the thin lens equation for s'.

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \quad \Rightarrow \quad s'=\frac{fs}{s-f}$$

Plug this into the magnification equation, $s = -s'/m$:

$$s=-\frac{1}{m}\frac{fs}{s-f}$$

Cancel an s from the numerator of each side, and multiply both sides by $s-f$;

$$s-f=-\frac{f}{m} \quad \Rightarrow \quad s=f-\frac{f}{m}=f\left(1-\frac{1}{m}\right)=(20\text{ cm})\left(1-\frac{1}{3}\right)=13\text{ cm}$$

Assess: This answer is within the focal length of the concave mirror as we expect for an upright, magnified, virtual image.

34.77. Visualize: We are given $h = 2.0\text{ cm}$ and $h' = 1.0\text{ cm}$, so we know $m = 0.5$. We are also given $s + s' = 150\text{ cm}$. That, plus $m = -s'/s = 0.50$, gives $s' = 50\text{ cm}$.
Solve: Solve $m = -s'/s$ for s.

$$s=\frac{-s'}{m}$$

Plug this result into the thin-lens equation.

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \quad \Rightarrow \quad f=\frac{ss'}{s+s'}$$

$$f=\frac{ss'}{s+s'}=\frac{\left(\frac{-s'}{m}\right)(s')}{\frac{-s'}{m}+s'}=\frac{\left(\frac{-s'}{m}\right)(s')}{s'\left(1-\frac{1}{m}\right)}$$

Cancel one s' and distribute the m in the denominator.

$$f=\frac{-s'}{m\left(1-\frac{1}{m}\right)}=\frac{-s'}{m-1}=\frac{50\text{ cm}}{0.5-1}=-100\text{ cm}$$

Assess: The negative value for f tells us this is a convex mirror.

34.78. Visualize: First concentrate on the optic axis and the ray parallel to it. Geometry says if parallel lines are both cut by a diagonal (in this case the line through the center of curvature and normal to the mirror at the point of incidence) the interior angles are equal; so $\phi = \theta_\text{i}$. The law of reflection says that $\theta_\text{i} = \theta_\text{r}$, so we conclude $\phi = \theta_\text{r}$. Now concentrate on the triangle whose sides are R, a, and b. Because two of the angles are equal then it is isosceles; therefore $b = a$. Apply the law of cosines to this triangle.
Solve:

$$b^2 = a^2 + R^2 - 2aR\cos\phi$$

Because $a = b$, they drop out.

$$R^2 = 2aR\cos\phi \quad \Rightarrow \quad R = 2a\cos\phi$$

We want to know how big a is in terms of R, so solve for a.

$$a=\frac{R}{2\cos\phi}$$

If $\phi \ll 1$ then $\cos\phi \approx 1$, so in the limit of small ϕ, $a = R/2$, and then since $f = R - a$ it must also be that

$$f = \frac{R}{2}$$

Assess: Many textbooks forget to stress that $f = R/2$ only in the limit of small ϕ, *i.e.*, for paraxial rays.

Challenge Problems

34.79. Model: Use the ray model of light and assume the lens is a thin lens.
Visualize: Please refer to Figure 34.41.
Solve: Let n_1 be the refractive index of the fluid and n_2 the refractive index of the lens. The lens consists of two spherical surfaces having radii of curvature R_1 and R_2 and the lens thickness $t \to 0$. For the refraction from the surface with radius R_1, we use Equation 34.20:

$$\frac{n_1}{s_1} + \frac{n_2}{s_1'} = \frac{n_2 - n_1}{R_1}$$

For the refraction from surface with radius R_2,

$$\frac{n_2}{-s_1'} + \frac{n_1}{s_2'} = \frac{n_1 - n_2}{R_2}$$

A negative sign is used with s_1' because the image formed by the first surface of the lens is a virtual image. This virtual image is the object for the second surface. Adding the two equations gives

$$\frac{n_1}{s_1} + \frac{n_1}{s_2'} = n_2 - n_1\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \Rightarrow \quad \frac{1}{s_1} + \frac{1}{s_2'} = \frac{1}{f} = \frac{(n_2 - n_1)}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(b) In air, $R_1 = +40$ cm (convex toward the object), $R_2 = -40$ cm (concave toward the object), $n_1 = 1.00$, and $n_2 = 1.50$. So,

$$\frac{1}{f} = \left(\frac{1.50 - 1.00}{1.00}\right)\left(\frac{1}{40\text{ cm}} - \frac{1}{-40\text{ cm}}\right) \quad \Rightarrow \quad f = 40\text{ cm}$$

In water, $n_1 = 1.33$ and $n_2 = 1.50$. So,

$$\frac{1}{f} = \left(\frac{1.50 - 1.33}{1.33}\right)\left(\frac{1}{40\text{ cm}} - \frac{1}{-40\text{ cm}}\right) \quad \Rightarrow \quad f = 1.6\text{ m}$$

34.80. Model: Use the ray model of light.
Solve: (a) The time (t) is the time to travel from A to the interface (t_1) and from the interface to B (t_2). That is,

$$t = t_1 + t_2 = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{d_1}{c/n_1} + \frac{d_2}{c/n_2} = \frac{n_1 d_1}{c} + \frac{n_2 d_2}{c} = \frac{n_1}{c}\sqrt{x^2 + a^2} + \frac{n_2}{c}\sqrt{(w-x)^2 + b^2}$$

(b) Because t depends on x and there is only one value of x for which the light travels from A to B in the least possible amount of time, we have

$$\frac{dt}{dx} = 0 = \frac{n_1 x}{c\sqrt{x^2 + a^2}} - \frac{n_2(w-x)}{c\sqrt{(w-x)^2 + b^2}}$$

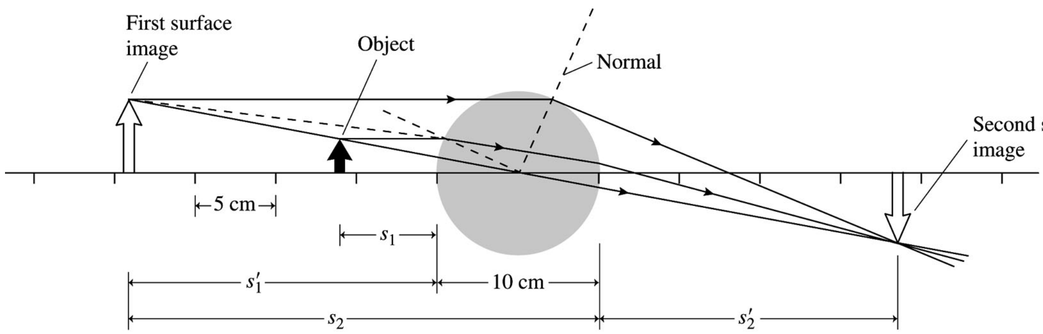
The solution (hard to do!) would give x_{min}.
(c) From the geometry of the figure,

$$\frac{x}{\sqrt{x^2 + a^2}} = \frac{x}{d_1} = \sin\theta_1 \qquad \frac{w-x}{\sqrt{(w-x)^2 + b^2}} = \frac{w-x}{d_2} = \sin\theta_2$$

Thus, the condition of part **(b)** becomes

$$\frac{n_1}{c}\sin\theta_1 - \frac{n_2}{c}\sin\theta_2 = 0 \quad \Rightarrow \quad n_1\sin\theta_1 = n_2\sin\theta_2$$

34.81. Model: Assume the ray model of light. The ball is *not* a thin lens. However, the image due to refraction from the first surface is the object for the second surface.
Visualize:



Solve: **(a)** For refraction from the first surface, $R = +5.0$ cm (convex toward the object). Thus,

$$\frac{n_1}{s_1} + \frac{n_2}{s_1'} = \frac{n_2 - n_1}{R} \quad \Rightarrow \quad \frac{1.0}{6.0\text{ cm}} + \frac{1.50}{s_1'} = \frac{0.50}{5.0\text{ cm}} \quad \Rightarrow \quad \frac{1.50}{s_1'} = -\frac{1}{15\text{ cm}} \quad \Rightarrow \quad s_1' = -22.5\text{ cm}$$

The image is virtual (to the left of the surface) and upright. For refraction from the second surface, $s_2 = 22.5\text{ cm} + 10\text{ cm} = 32.5\text{ cm}$ and $\text{R} = -5.0\text{ cm}$ (concave toward the object). Thus,

$$\frac{1.50}{32.5\text{ cm}} + \frac{1.0}{s_2'} = \frac{1.0 - 1.50}{-5.0\text{ cm}} = \frac{1}{10\text{ cm}} \quad \Rightarrow \quad \frac{1}{s_2'} = \frac{1}{10\text{ cm}} - \frac{1.50}{32.5\text{ cm}} \quad \Rightarrow \quad s_2' = 18.6\text{ cm}$$

The image is 19 cm from the right edge of the ball and thus 24 cm from the center.
(b) The ray diagram showing the formation of the image is shown above.
(c) Using the thin-lens equation,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{6.0\text{ cm} + 5.0\text{ cm}} + \frac{1}{18.6\text{ cm} + 5.0\text{ cm}} = \frac{1}{f} \quad \Rightarrow \quad f = 7.5\text{ cm}$$

34.82. Visualize: The lateral magnification is $m = -s'/s$.
Solve: We need to solve the thin-lens equation for s'.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{s'} = \frac{1}{f} - \frac{1}{s} \quad \Rightarrow \quad s' = \frac{1}{\dfrac{1}{f} - \dfrac{1}{s}} = \frac{fs}{s-f}$$

Now insert this expression for s' into the expression for m:

$$m = -\frac{s'}{s} = -\frac{\dfrac{fs}{s-f}}{s} = -\frac{f}{s-f}$$

Now define the longitudinal magnification as the rate of change of s' with respect to s.

$$M = \frac{ds'}{ds} = \frac{d}{ds}(s') = \frac{d}{ds}\left(\frac{fs}{s-f}\right)$$

Use the quotient rule of differentiation.

$$M = \frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{fs}{s-f}\right) = \frac{(s-f)f - fs}{(s-f)^2} = \frac{-f^2}{(s-f)^2}$$

This last result is equal to $-m^2$, so $M = -m^2$.

Assess: For further reading, see “Longitudinal Magnification Drawing Mistake” by Héctor Rabal, Nelly Cam, and Marcelo Trivi in *The Physics Teacher*, vol. 42, January 2004, pp. 31–33, but note that the fourth equation is missing several minus signs.