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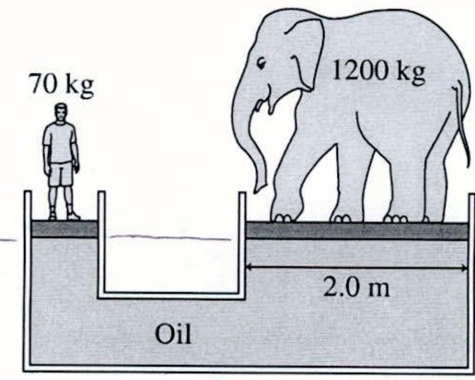
1. (15 points, 5 points each): A 70 kg student shown in the figure below balances a 1200 kg elephant on a hydraulic lift containing oil. Assume the oil is incompressible and its density is $\rho = 900\ \text{kg/m}^3$.



same height h

(a) What is the *diameter* of the piston the student is standing on?

(b) When a second student joins the first, the piston sinks 35 cm. What is the second student's mass?

(c) The first student steps off the piston. How much does the piston rise?

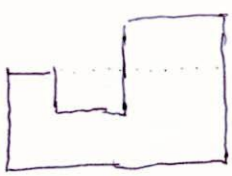
(a) same height $\Rightarrow$ $P_1 = P_2$ $\quad \pi r_1^2 = A_1 = A_2 \frac{F_1}{F_2} = \pi (1\ m)^2 \frac{(70\ kg)\,g}{(1200\ kg)\,g}$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$r_1 = 0.24\ m$

Assuming height of right platform is essentially constant $\boxed{\text{diameter} = 0.48\ m}$

(b)



h

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} + \rho g h$$

$$(m_1 + m_2)\cancel{g} = \frac{A_1}{A_2} m_{elephant} \cancel{g} + \rho \cancel{g} h$$

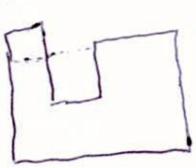
$$m_2 = \frac{\pi (0.24\ m)^2}{\pi (1\ m)^2} (1200\ kg) + \left(900 \frac{kg}{m^3}\right)(0.35\ m)(\pi)(0.24\ m)^2$$

$$\boxed{m_2 = 56\ kg}$$

(can be 56-58 kg depending on rounding)

After student steps off, piston rises above platform

(c)



h

$$\frac{F_1}{A_1} + \rho g h = \frac{F_2}{A_2} \Rightarrow h = \frac{m_{elephant}}{\rho A_2} - \frac{m_2}{\rho A_1} = \frac{1200\ kg}{\left(900 \frac{kg}{m^3}\right)\pi (1\ m)^2} - \frac{56\ kg}{\left(900 \frac{kg}{m^3}\right)\pi (0.24)^2}$$

$$\boxed{h = 8\ cm}$$

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2. (15 points, 5 points each): At a distance D away from a fire alarm, you hear the alarm sounding at an intensity level of 92.0 dB. Assume sound is emitted from the source isotropically in three dimensions.

   (a) At what distance R should you stand from the alarm in order for the sound intensity level to drop to 83.0 dB?

   (b) Suppose there are now two identical fire alarms. You're standing exactly in between them, such that the distance between you and each fire alarm is R. What sound intensity level do you observe now in dB?

   (c) You begin walking away from one fire alarm and toward the other alarm at a speed of 1.0 m/s. Due to the Doppler effect, you notice a beat frequency of 2.0 Hz. Assuming both alarms are emitting sound at the same frequency, what is the frequency of each alarm (the frequency emitted by the sources)?

(a) A decrease in intensity level of 9.0 dB is a "halving" in absolute intensity 3 times

$$I_f = \left(\frac{1}{2}\right)^3 I_i = \frac{1}{8} I_i$$

Also, $I \propto \frac{1}{A} \propto \frac{1}{r^2}$ so $\frac{I_f}{I_i} = \left(\frac{r_i}{r_f}\right)^2 \Rightarrow \boxed{r_f = \sqrt{8}\, D = (2.8)\, D}$

(b) Two fire alarms $\Rightarrow$ double in absolute intensity

$\Rightarrow + 3.0\,\text{dB}$ so $\boxed{86.0\ \text{dB}}$

(c) $f' = f\left(\frac{v \pm v_r}{v \pm \cancel{v_s}^{0}}\right) \Rightarrow |\Delta f'| = 2f\frac{v_r}{v}$

$f = |\Delta f'| \frac{v}{2v_r} = (2\ \text{Hz}) \frac{343\ \text{m/s}}{2(1\ \text{m/s})}$

+ for the one you're walking toward

− for the one you're walking away from

$\boxed{f = 343\ \text{Hz}}$

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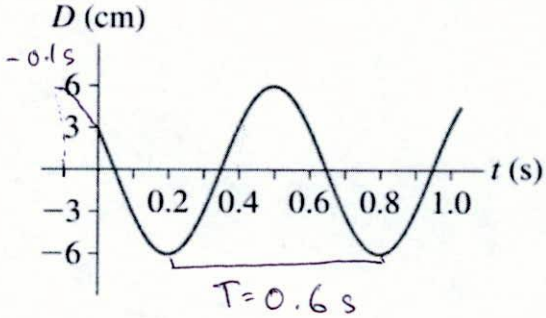
3. (15 points, 5 points each): A transverse wave travels to the left in the $-\hat{x}$ direction with speed 2.0 m/s. Consider the following history graph of the wave at $x = 0$ m:



(a) What is the frequency (in Hz) and the wavelength (in m) of this wave?

(b) Give an equation for this wave of the form $D(x, t) = A\cos[\ldots]$. $D(x,t) = A\cos(kx \pm \omega t + \phi_0)$ moving to left

(c) Draw a snapshot graph for this wave at $t = 0.20$ s.

(a) $f = \frac{1}{T} = \frac{1}{0.6s} = \boxed{1.67\ Hz}$ $\quad v = \lambda f \Rightarrow \lambda = \frac{v}{f} = \frac{2.0\ m/s}{1.67\ Hz} = \boxed{1.2\ m}$

(b) $\omega = 2\pi f = \frac{10\pi}{3}\frac{rad}{s}$, $A = 6m$, $k = \frac{2\pi}{\lambda} = \frac{5\pi}{3}\frac{rad}{m}$

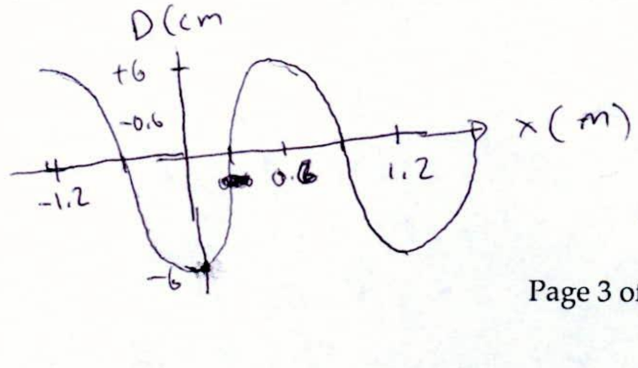
$D(x=0,t) = A\cos(\omega t + \phi_0)$ $\quad$ @ $t=0$

$D(x=0,t=0) = 3cm = (6\ cm)\cos(\phi_0)$

$\Rightarrow \phi_0 = \pi/3$

$$\boxed{D(x,t) = (6\ m)\cos\left[\left(\frac{5\pi}{3}\frac{rad}{m}\right)x + \left(\frac{10\pi}{3}\frac{rad}{s}\right)t + \pi/3\right]}$$

(c) $D(x, t=0.2s) = (6m)\cos\left[\left(\frac{5\pi}{3}\frac{rad}{m}\right)x + \pi\right]$



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(12 points, 3 points each): 4 Multiple-choice questions / fill-in-the-blanks on various topics.

Directions for multiple-choice questions: COMPLETELY FILL IN THE SQUARE for the answer.
Directions for fill-in-the-blank questions: Your answer should be entirely in the boxed region. Include the number of significant figures ("sig. figs.") requested in the problem.

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4. Two guitar strings (A and E) are under the same tension and produce standing waves of the same wavelength. The frequency of the A string is 110.00 Hz, and the frequency of the E string is 82.41 Hz. Which of the following best describes how the linear mass densities of the two strings sompare with one another? $v = \lambda f = \sqrt{T/\mu} \Rightarrow \mu \propto \frac{1}{f^2}$ $\quad \frac{\mu_A}{\mu_E} = \left(\frac{f_E}{f_A}\right)^2 = \left(\frac{82.41\ \text{Hz}}{110.00\ \text{Hz}}\right)^2$

   - ☐ The A string has a linear mass density that is 16% higher than that of the E string.
   - ☐ The A string has a linear mass density that is 33% higher than that of the E string. $\frac{\mu_A}{\mu_E} = 0.56$
   - ☐ The A string has a linear mass density that is 16% lower than that of the E string.
   - ☐ The A string has a linear mass density that is 33% lower than that of the E string.
   - ■ The A string has a linear mass density that is 44% lower than that of the E string.

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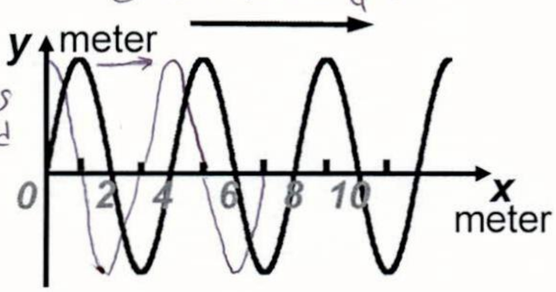
5. A transverse wave is traveling to the right with velocity 2 m/s and wavelength 4 m (shown below at $t = 0$ s). Of the following ($A > 0$), which equation describes the wave at $t = 1.5$ s?

$v = 2$ m/s
$\lambda = 4$ m
$T = 2$ s

@ $t = 1.5t = \frac{3}{4}T$

crest moves forword

$\Delta x = 3$ m



   - ☐ $+A\sin(kx)$
   - ☐ $-A\sin(kx)$
   - ■ $+A\cos(kx)$
   - ☐ $-A\cos(kx)$
   - ☐ None of the above

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$V_1 A_1 = V_2 A_2 \Rightarrow V_2 > V_1$

$P_2 < P_1$

6. Water flows through a horizontal tapered pipe. At the wide end its speed is 4.0 m/s. The difference in pressure between the two ends is 4.5 × 10^3 Pa. The speed of the water at the narrow end is:

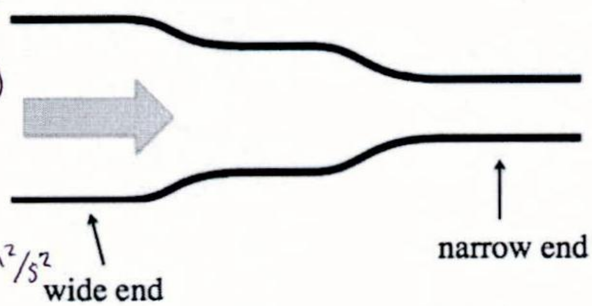
- ☐ 2.6 cm/s
- ☐ 3.4 cm/s
- ☐ 4.0 cm/s
- ☐ 4.5 cm/s
- ■ 5.0 cm/s

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$

$\frac{2(P_1 - P_2)}{\rho} = v_2^2 - v_1^2$

$\frac{2(4.5\times10^3 Pa)}{1000\ kg/m^3} = 9\ m^2/s^2$



$v_2^2 = 9\ m^2/s^2 + (4\ m/s)^2 \Rightarrow v_2 = 5\ m/s$

7. A light wave has a 670 nm wavelength in air. Its wavelength in a transparent solid is 420 nm. What is the speed of light to 2 sig. figs. in this solid?

$c = \lambda_{air} f$; $v_{solid} = \lambda_{solid} f$

$= c\frac{\lambda_{solid}}{\lambda_{air}} = (3\times10^8\ m/s)\frac{420\ nm}{670\ nm}$

$\boxed{1.9\times10^8\ m/s}$