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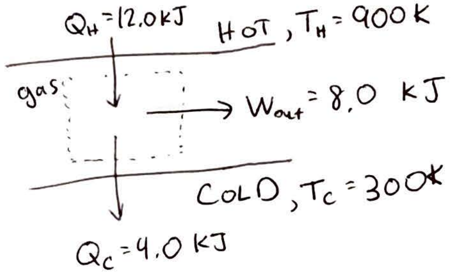
1. (10 points, 5 points each): A Carnot engine with 1.00 mol of Helium as the working substance operates between the temperatures 300 K and 900 K. In one cycle, 12.0 kJ of heat is delivered to the gas.

   (a) Draw an energy transfer diagram, showing what happens to every 12.0 kJ of heat that is extracted from the high-temperature reservoir. What is the efficiency of this engine?

   (b) Draw the pV diagram for this process. If volume of the gas is V_0 at the beginning of the isothermal expansion step, what is the volume of the gas at the end of this step? Your answer should be a number times V_0.

(a)

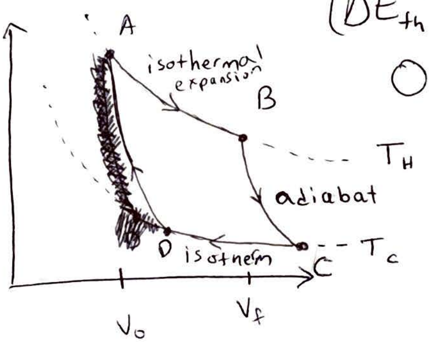


$$\boxed{\eta_{Carnot} = 1 - \frac{T_c}{T_H} = 1 - \frac{300}{900} = \frac{2}{3}}$$

$$\frac{2}{3} = \frac{W_{out}}{Q_H}$$

$$Q_H = 12.0\,kJ \Rightarrow \begin{cases} W_{out} = 8.0\,k \\ Q_c = 4.0\,k \end{cases}$$

(b)



$$(\Delta E_{th})^{AB} = (-W_{by})^{AB} + (Q)^{AB}$$

$$0 = -(W_{by})^{AB} + Q^{AB}$$

$$Q^{AB} = Q_H = 12.0\,kJ = (W_{by})^{A}$$

$$W_{by,isotherm} = nRT \ln\left(\frac{V_f}{V_0}\right)$$

$$\Rightarrow (12.0\,kJ) = (1.00\,mol)\left(8.31\,\frac{J}{mol \cdot K}\right)(900\,K) \ln\left(\frac{V_f}{V_0}\right)$$

$$\ln\left(\frac{V_f}{V_0}\right) = 1.604$$

$$\boxed{V_f = 4.98\,V_0}$$

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2. (15 points, 5 points each): A refrigerator using a diatomic gas as the working substance (with rotational but not vibrational degrees of freedoms) operates via a 3-step cycle. From initial pressure p_0 and volume V_0 (point A on a pV diagram), the gas undergoes an adiabatic expansion to quadruple its volme (point B), followed by an isochoric process to return to pressure p_0 (point C), and finally an isobaric process to return to the starting point.

$f = 5$

$\gamma = 7/5$

$C_v = \frac{5}{2}R$

$C_p = \frac{7}{2}R$

(a) Draw this process on a pV diagram. Label the three corners of your cycle "A," "B," and "C." For each of the three steps (AB, BC, and CA), say whether heat is transferred to the gas, heat is transferred away from the gas, or there is no heat transfer.

(b) What is the temperature of the gas at point B? Express your answer as a number times T_0, where T_0 is defined to be the temperature at point A.

(c) What is the coefficient of performance (COP) of this fridge? You may use the following formula, which gives the work done by a gas in an adiabatic expansion:

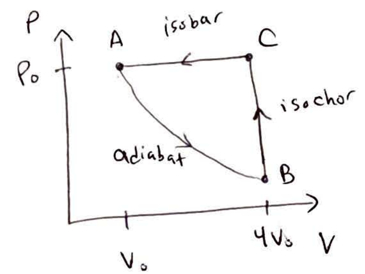
$$W_{\text{by,adiabat}} = \frac{p_i V_i - p_f V_f}{\gamma - 1}$$

$\boxed{Q_{CA} = n C_p \Delta T < 0}$

away from gas

Since $\Delta T < 0$ (jumping to lower isotherm)

(a)

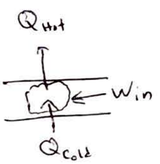


No heat transfer

$\boxed{Q_{AB} = 0 \text{ (adiabat)}}$

$\boxed{Q_{BC} > 0}$ isochoric heating $\Delta E_{th} > 0$ since $\Delta T > 0$, $W = 0$

To gas



(b) $T_0 V_0^{\gamma - 1} = T_f (4V_0)^{\gamma - 1} \Rightarrow \boxed{T_f = T_0 \left(\frac{1}{4}\right)^{0.40} = 0.574\, T_0}$

(c) $Q_{cold} = Q_{BC} = n C_v \Delta T = \frac{5}{2} nR[4T_0 - 0.574 T_0] = 8.56\, nRT_0$
$= 8.56\, p_0 V_0$

$W_{in} = 3 p_0 V_0 - W^{(A \to B)}_{by, adiabat} = 3 p_0 V_0 - \left(\frac{p_0 V_0 - (0.1436\, p_0)(4V_0)}{1.40 - 1} \right) = 1.936\, p_0 V_0$

[note: $p_f = p_0 \left(\frac{V_0}{V_f}\right)^{\gamma} = p_0 \left(\frac{1}{4}\right)^{1.4} = 0.1436\, p_0$]

$\Rightarrow \boxed{COP = \frac{Q_{cold}}{W_{in}} = \frac{8.56}{1.936} = 4.42}$

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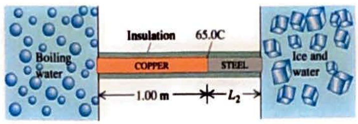
3. (10 points, 5 points each) A long rod, insulated to prevent heat loss along its sides, is in perfect thermal contact with boiling water at one end and with an ice-water mixture at the other. The rod consists of a 1.00 m section of copper (one end in boiling water) joined end-to-end to a length L_2 of steel (one end in the ice-water mixture). Both sections of the rod have the same cross-sectional area:



The temperature of the copper-steel junction is 65.0° C after a steady state has been set up (i.e., the heat transfer rate is the same through the copper and the steel). The thermal conductivities for copper and steel are $k_{Cu} = 401\ \mathrm{W/(m{\cdot}K)}$ and $k_{steel} = 46\ \mathrm{W/(m{\cdot}K)}$, respectively.

(a) What is the length L_2 of the steel section?

(b) Suppose we remove the steel section, so that it just just the 1.00 m copper rod connecting the two ends. Would the energy per unit time transferred through the rod increase or decease? By what factor? Explain.

(a) (P_{cond} same for both rods) $\Rightarrow K_{Cu} \frac{\cancel{A}}{L_1} (\Delta T)_{Cu} = K_{Steel} \frac{\cancel{A}}{L_2} (\Delta T)_{steel}$

$$L_2 = (1.00\ m) \left| \frac{\Delta T_{steel}}{\Delta T_{Cu}} \right| \frac{K_{steel}}{K_{Cu}} = \left(\frac{65.0}{35.0}\right)\left(\frac{46}{401}\right) m$$

$$\boxed{L_2 = 0.21\ m}$$

(b) The power would increase:

$$P^{(new)}_{cond} = K_{Cu} \frac{A}{L_1} (100°C) = \frac{100}{35}\left[K_{Cu} \frac{A}{L_1} (35°C)\right] = \frac{100}{35} P^{(old)}_{cond}$$

i.e. the ΔT <u>increases</u> so P_{cond} <u>increases</u>

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(12 points, 3 points each): 4 Multiple-choice questions / fill-in-the-blanks on various topics.

Directions for multiple-choice questions: COMPLETELY FILL IN THE SQUARE for the answer. Directions for fill-in-the-blank questions: Your answer should be entirely in the boxed region. Include the number of significant figures ("sig. figs.") requested in the problem.

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4. It costs $300.00 each summer to operate a home air conditioner with a COP of 2.0. How much would it cost to cool the home in the summer after upgrading to a model with a COP of 4.0? Assume the same amount of heat is to be extracted from the house. (express your answer to the nearest dollar)

$COP = K = \frac{Q_C}{W_{in}}$

If K doubles and Q_C remains the same, W_{in} must half

$\boxed{\$150}$

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5. A car's internal combustion engine can be modeled as a heat engine operating between a combustion temperature of 1500°C and an air temperature of 20°C with 30% of the Carnot efficiency. The heat of combustion of gasoline is 47 kJ/g. What mass of gasoline is burned to accelerate a 1500 kg car from rest to a speed of 30 m/s?

- ☐ 1 gram
- ☐ 7 grams
- ☐ 20 grams
- ■ 60 grams

$\eta_{actual} = 0.3\eta_{Carnot} = (0.3)\left[1 - \frac{293}{1773}\right] = 0.25$

$W_{out} = \frac{1}{2}mv^2 = \frac{1}{2}(1500\text{ kg})(30\text{ m/s})^2 = 675\text{ KJ}$

$W_{out} = \eta_{actual} Q_H \Rightarrow Q_H = 2700\text{ KJ} \Rightarrow 57\text{ g}$ of gasoline

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6. What is the root-mean-square speed v_{rms} of oxygen molecules in your car tires? Note O_2 has a molar mass of 32.0 g/mol. Take the absolute pressure of gas in your tires to be 3.8 atm and take the temperature to be 40°C. $= 313\text{ K} = T$

- ■ 500 m/s
- ☐ 900 m/s
- ☐ 1300 m/s
- ☐ 1700 m/s
- ☐ 2100 m/s

$\frac{3}{2}KT = \frac{1}{2}mv_{rms}^2 \Rightarrow v_{rms} = \sqrt{\frac{3K_BT}{m_{O_2}}}$

$m_{O_2} = \frac{32.0\text{g}}{6.02\times10^{23}} = 5.32\times10^{-26}\text{ kg}$

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7. Two containers (A and B) initially hold N balls. Once a second, one of the balls is chosen at random and switched to the other container. After a long time has passed, you record the number of balls in each container every second. In 100,000 s, you find 49 times when all the balls were in container A. How many balls are there?

- ☐ 8
- ☐ 9
- ☐ 10
- ■ 11
- ☐ 12

$$\text{prob}\begin{pmatrix}\text{all balls}\\ \text{in container } A\end{pmatrix} = \frac{1}{2^N}$$

in 100,000 tries: $\frac{100000}{2^N} = 49$

$$\Rightarrow N = \log_2\left(\frac{100000}{49}\right) \approx 10.99$$