35

Optical Instruments

Conceptual Questions

35.1. The picture will be underexposed if the shutter speed remains the same. The problem stated that D is not changed. $I \propto \dfrac{D^2}{f^2}$. If f is increased then I is decreased so the picture is underexposed.

35.2. The answer is C. Rays from all parts of the object hit all parts of the lens, so the whole image still appears, but because the area of the aperture is smaller then the image will be less bright. Contrary to choice a, the image actually gets sharper because the off-axis rays contribute to blurriness and a smaller aperture eliminates many of those rays.

35.3. If you try to see underwater with no face mask there is little refraction at the water-cornea boundary because the indices of refraction are so similar. To make up for this loss of refraction (as compared to seeing in air) it would be helpful to have glasses with converging lenses.

35.4. The card is red because it reflects red light and absorbs the other colors. When it is illuminated by red light the red light reflects off the card into your eyes and you see the red card as red. If the card is illuminated with blue light the light is all absorbed. No light is reflected, so the card looks black. If you illuminate the card with white light and look at it through a blue filter it will again look black because the red light reflected by the card is not passed by the blue filter.

35.5. Because scattering increases as wavelength decreases, it is better to use infrared light to reduce the impact of scattering by hydrogen gas.

35.6. The formula for the resolution of a microscope does not contain the magnification. It depends on the diameter of the objective lens (by way of NA and α) but your friend's eyepiece has the same diameter as yours. So his claim is not valid.

35.7. Equation 35.12 gives the width of the spot as $w_{\text{min}} \approx \dfrac{2.44\lambda f}{D}$. The minimum spot width is achieved for incoming parallel rays when the lens-to-screen distance is f. **(a)** Decreasing λ decreases w. **(b)** Decreasing D increases w. **(c)** Decreasing f decreases w. **(d)** Decreasing the lens-to-screen distance means the screen is no longer in the focal plane and the spot is larger than its minimum in the focal plane.

35.8. You would use a lens with a small f-number. The minimum width is given by $w_{\text{min}} \approx \dfrac{2.44\lambda f}{D}$ but since $f\text{-number} = f/D$, then $w_{\text{min}} \approx 2.44\lambda\ (f\text{-number})$.

35.9. Equation 35.13 gives the angular resolution of a telescope as $\theta_{\text{min}} = \dfrac{1.22\lambda}{D}$. Two objects are marginally resolvable if $\alpha = \theta_{\text{min}}$ (our original condition) and are resolvable if $\alpha > \theta_{\text{min}}$. **(a)** Yes, using a shorter wavelength decreases θ_{min} which makes $\alpha > \theta_{\text{min}}$, so the resolution is improved. **(b)** Changing the focal length does not affect the resolution because *f* is not in the formula for resolution. **(c)** Yes, using a larger lens increases the resolution; increasing *D* decreases θ_{min} which makes $\alpha > \theta_{\text{min}}$. **(d)** Changing magnification does not affect the resolution.

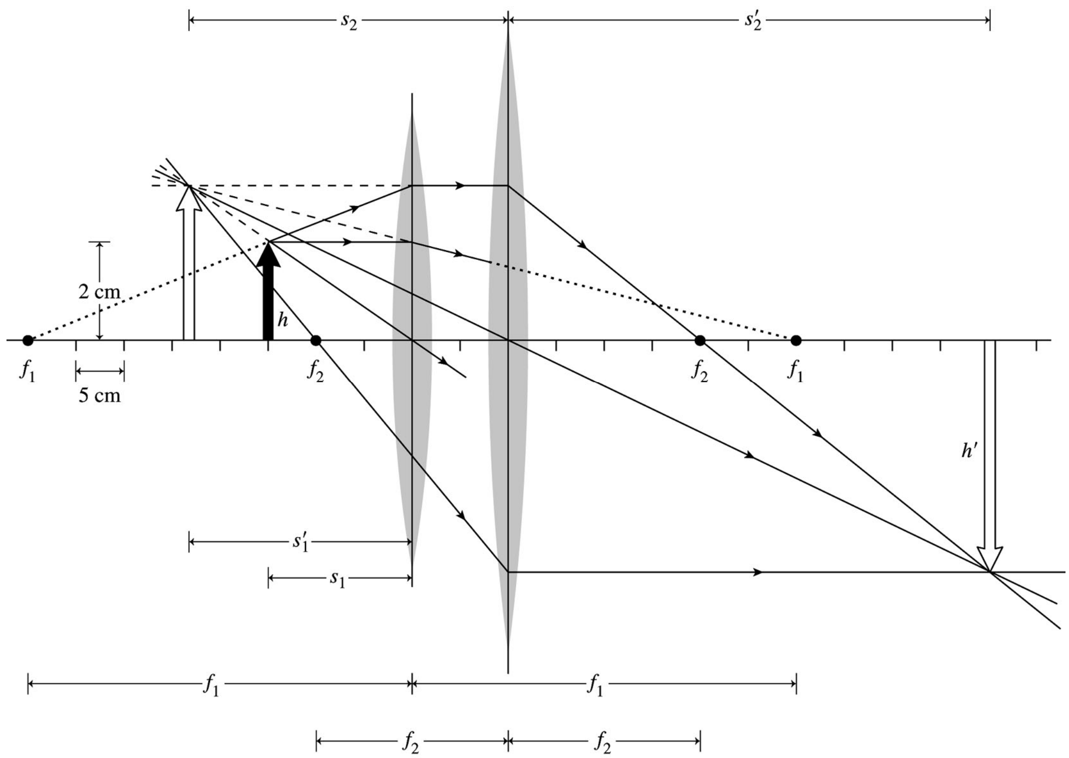
Exercises and Problems

Exercises

Section 35.1 Lenses in Combination

35.1. Model: Each lens is a thin lens. The image of the first lens is the object for the second lens.

Visualize:



The figure shows the two lenses and a ray-tracing diagram. The ray-tracing shows that the lens combination will produce a real, inverted image behind the second lens.

Solve: (a) From the ray-tracing diagram, we find that the image is ≈50 cm from the second lens and the height of the final image is 4.5 cm.

(b) $s_1 = 15\text{ cm}$ is the object distance of the first lens. Its image, which is a virtual image, is found from the thin-lens equation:

$$\frac{1}{s_1'} = \frac{1}{f_1} - \frac{1}{s_1} = \frac{1}{40\text{ cm}} - \frac{1}{15\text{ cm}} = -\frac{5}{120\text{ cm}} \Rightarrow s_1' = -24\text{ cm}$$

The magnification of the first lens is

$$m_1 = -\frac{s_1'}{s_1} = -\frac{(-24\text{ cm})}{15\text{ cm}} = 1.6$$

The image of the first lens is now the object for the second lens. The object distance is $s_2 = 24\text{ cm} + 10\text{ cm} = 34\text{ cm}$. A second application of the thin-lens equation yields:

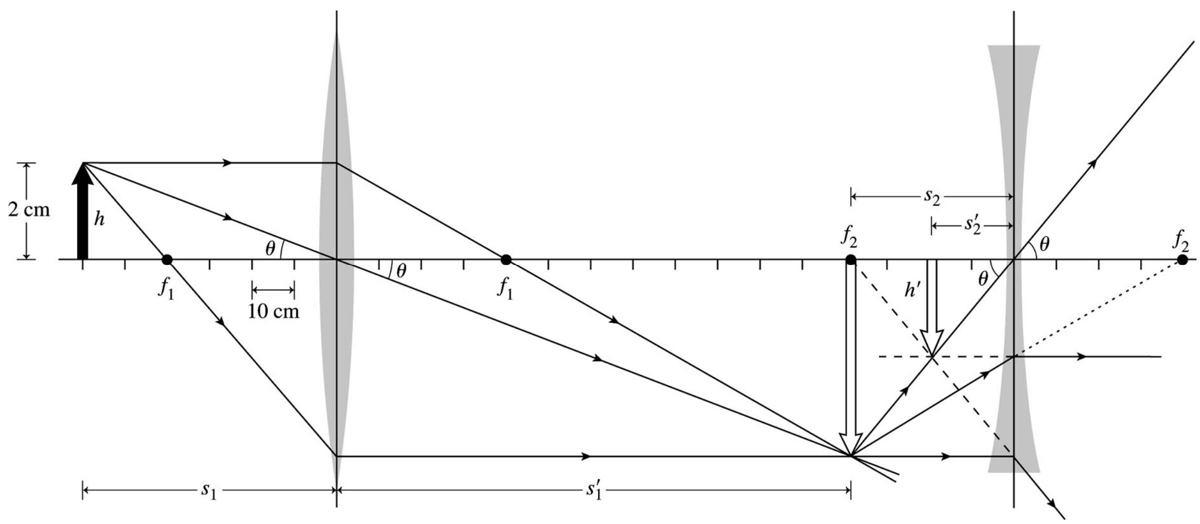
$$\frac{1}{s_2'} = \frac{1}{f_2} - \frac{1}{s_2} = \frac{1}{20\text{ cm}} - \frac{1}{34\text{ cm}} \Rightarrow s_2' = \frac{680\text{ cm}}{14} = 48.6\text{ cm} \approx 49\text{ cm}$$

The magnification of the second lens is

$$m_2 = -\frac{s_2'}{s_2} = -\frac{48.6\text{ cm}}{34\text{ cm}} = -1.429$$

The combined magnification is $m = m_1 m_2 = (1.6)(-1.429) = -2.286$. The height of the final image is (2.286)(2.0 cm) = 4.6 cm. These calculated values are in agreement with those found in part **(a)**.

35.2. Model: Each lens is a thin lens. The image of the first lens is the object for the second lens.
Visualize:



The figure shows the two lenses and a ray-tracing diagram. The ray tracing shows that the lens combination will produce a virtual, inverted image in front of the second lens.
Solve: (a) From the ray-tracing diagram, we find that the image is 20 cm in front of the second lens and the height of the final image is 2.0 cm.
(b) $s_1 = 60\text{ cm}$ is the object distance of the first lens. Its image, which is a real image, is found from the thin-lens equation:

$$\frac{1}{s_1'} = \frac{1}{f_s} - \frac{1}{s_1} = \frac{1}{40\text{ cm}} - \frac{1}{60\text{ cm}} = \frac{1}{120\text{ cm}} \Rightarrow s_1' = 120\text{ cm.}$$

The magnification of the first lens is

$$m_1 = -\frac{s_1'}{s_1} = -\frac{120\text{ cm}}{60\text{ cm}} = -2$$

The image of the first lens is now the object for the second lens. The object distance is $s_2 = 160\text{ cm} - 120\text{ cm} = 40\text{ cm}$. A second application of the thin-lens equation yields:

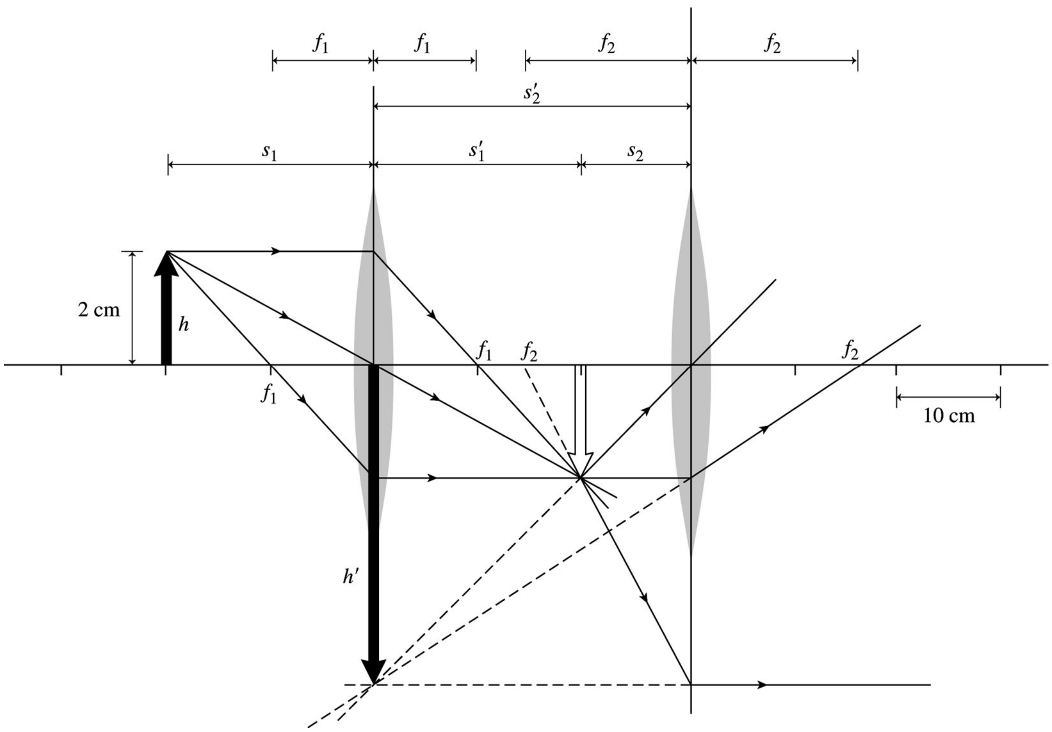
$$\frac{1}{s_2'} = -\frac{1}{s_2} + \frac{1}{f_2} = \frac{-1}{+40\text{ cm}} + \frac{1}{-40\text{ cm}} \Rightarrow s_2' = -20\text{ cm}$$

The magnification of the second lens is

$$m_2 = -\frac{s_2'}{s_2} = -\frac{-20\text{ cm}}{40\text{ cm}} = +0.5$$

The overall magnification is $m = m_1 m_2 = (-2)(0.5) = -1.0$. The height of the final image is $(+1.0)(2.0\text{ cm}) = 2.0\text{ cm}$. The image is inverted because m has a negative sign. These calculated values are in agreement with those found in part **(a)**.

35.3. Model: Each lens is a thin lens. The image of the first lens is the object for the second lens.
Visualize:



The figure shows the two lenses and a ray-tracing diagram. The ray tracing shows that the lens combination will produce a virtual, inverted image at the first lens.
Solve: (a) From the ray-tracing diagram, we find that the image is 30 cm in front of the second lens and the height of the final image is 6 cm.
(b) $s_1 = 20\text{ cm}$ is the object distance of the first lens. Its image, which is real and inverted, is found from the thin lens equation:

$$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1} \Rightarrow s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(10\text{ cm})(20\text{ cm})}{20\text{ cm} - 10\text{ cm}} = 20\text{ cm}$$

The magnification of the first lens is

$$m_1 = -\frac{s_1'}{s_1} = -\frac{20\text{ cm}}{20\text{ cm}} = -1$$

The image of the first lens is now the object for the second lens. The object distance is $s_2 = 30\ \text{cm} - 20\ \text{cm} = 10\ \text{cm}$. A second application of the thin lens equation yields

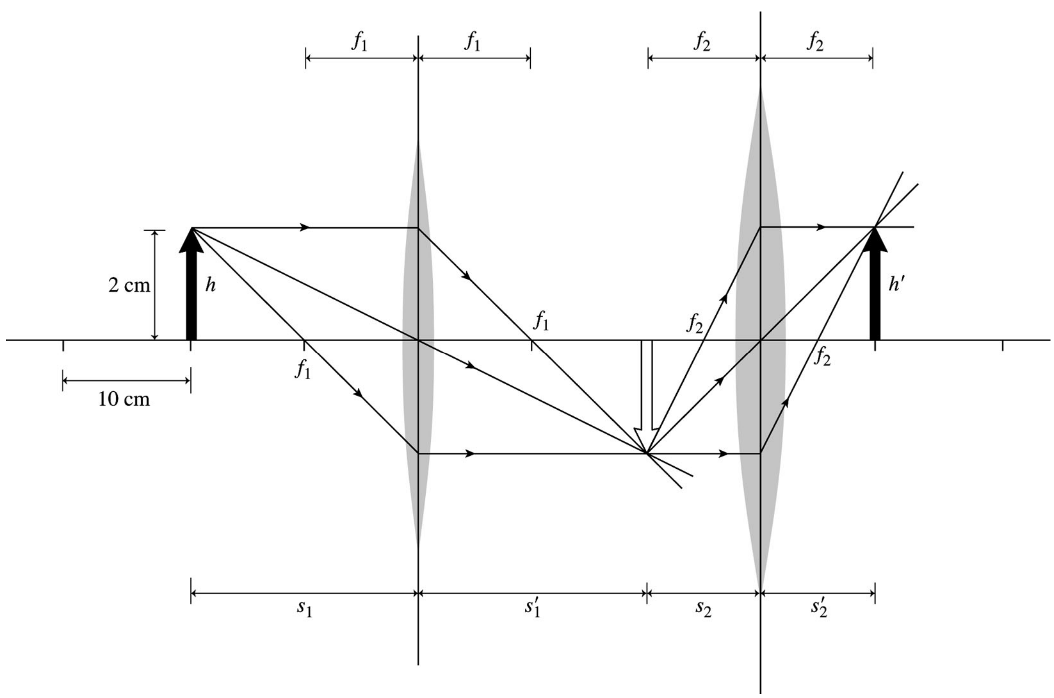
$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2} \Rightarrow s_2' = \frac{f_2 s_2}{s_2 - f_2} = \frac{(15\ \text{cm})(10\ \text{cm})}{10\ \text{cm} - 15\ \text{cm}} = -30\ \text{cm}$$

The magnification of the second lens is

$$m_2 = -\frac{s_2'}{s_2} = -\frac{(-30\ \text{cm})}{10\ \text{cm}} = 3$$

The combined magnification is $m = m_1 m_2 = (-1)(3) = -3$. The height of the final image is $(3)(2.0\ \text{cm}) = 6.0\ \text{cm}$. The image is inverted because m has a negative sign. These calculated values are in agreement with those found in part **(a)**.
Assess: The thin lens equation agrees with the ray tracing.

35.4. Model: Each lens is a thin lens. The image of the first lens is the object for the second lens.
Visualize:



The figure shows the two lenses and a ray-tracing diagram. The ray tracing shows that the lens combination will produce a real, upright image behind the second lens.
Solve: (a) From the ray-tracing diagram, we find that the image is 10 cm behind the second lens and the height of the final image is 2 cm.
(b) $s_1 = 20\ \text{cm}$ is the object distance of the first lens. Its image, which is real and inverted, is found from the thin lens equation:

$$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1} \Rightarrow s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(10\ \text{cm})(20\ \text{cm})}{20\ \text{cm} - 10\ \text{cm}} = 20\ \text{cm}$$

The magnification of the first lens is

$$m_1 = -\frac{s_1'}{s_1} = -\frac{20\ \text{cm}}{20\ \text{cm}} = -1$$

The image of the first lens is now the object for the second lens. The object distance is $s_2 = 30\text{ cm} - 20\text{ cm} = 10\text{ cm}$. A second application of the thin-lens equation yields

$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2}\Rightarrow s_2'=\frac{f_2 s_2}{s_2-f_2}=\frac{(5\text{ cm})(10\text{ cm})}{10\text{ cm}-5\text{ cm}}=10\text{ cm}$$

The magnification of the second lens is

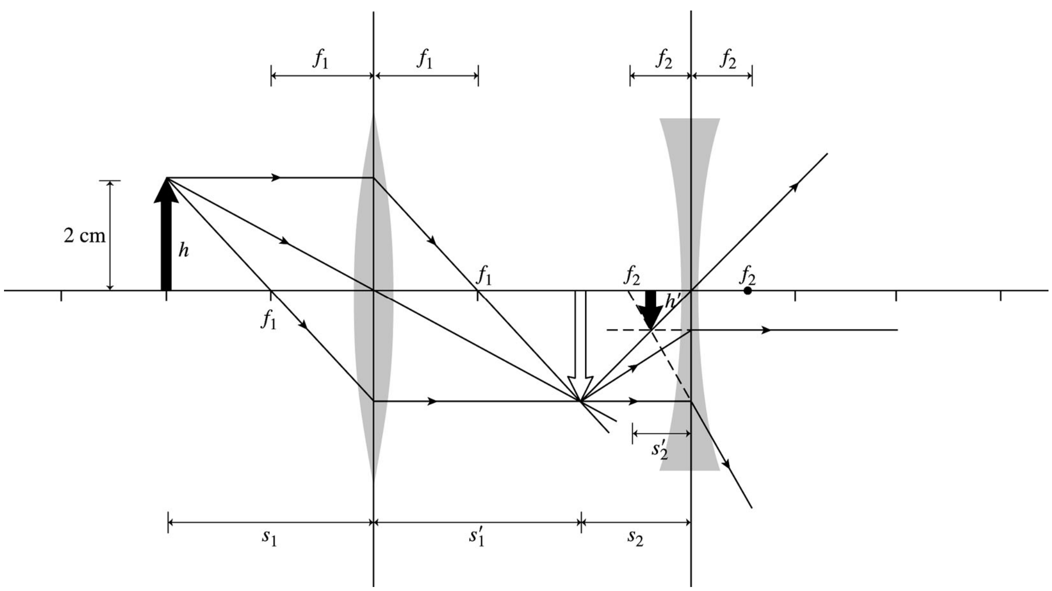
$$m_2=-\frac{s_2'}{s_2}=-\frac{10\text{ cm}}{10\text{ cm}}=-1$$

The combined magnification is $m = m_1 m_2 = (-1)(-1) = 1$. The height of the final image is $(1)(2.0\text{ cm}) = 2.0\text{ cm}$. These calculated values are in agreement with those found in part **(a)**.

Assess: The thin-lens equation agrees with the ray tracing.

35.5. Model: Each lens is a thin lens. The image of the first lens is the object for the second lens.

Visualize:



The figure shows the two lenses and a ray-tracing diagram. The ray tracing shows that the lens combination will produce a virtual, inverted image in front of the second lens.

Solve: (a) From the ray-tracing diagram, we find that the image is 3.3 cm behind the second lens and the height of the final image is ≈ 0.7 cm.

(b) $s_1 = 20\text{ cm}$ is the object distance of the first lens. Its image, which is real and inverted, is found from the thin-lens equation:

$$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}\Rightarrow s_1'=\frac{f_1 s_1}{s_1-f_1}=\frac{(10\text{ cm})(20\text{ cm})}{20\text{ cm}-10\text{ cm}}=20\text{ cm}$$

The magnification of the first lens is

$$m_1=-\frac{s_1'}{s_1}=-\frac{20\text{ cm}}{20\text{ cm}}=-1$$

The image of the first lens is now the object for the second lens. The object distance is $s_2 = 30\text{ cm} - 20\text{ cm} = 10\text{ cm}$. A second application of the thin-lens equation yields

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2} \Rightarrow s_2' = \frac{f_2 s_2}{s_2 - f_2} = \frac{(-5\text{ cm})(10\text{ cm})}{10\text{ cm} + 5\text{ cm}} = -3.33\text{ cm}$$

The magnification of the second lens is

$$m_2 = -\frac{s_2'}{s_2} = -\frac{(-3.3\text{ cm})}{10\text{ cm}} = 0.33$$

The combined magnification is $m = m_1 m_2 = (-1)(0.33) = -0.33$. The height of the final image is $(0.33)(2.0\text{ cm}) = 0.66\text{ cm}$. The image is inverted because m has a negative sign. These calculated values are in agreement with those found in part **(a)**.

Assess: The thin-lens equation agrees with the ray tracing.

Section 35.2 The Camera

35.6. Model: $s \gg f$ so we can use Equation 35.1: $m = -f/s$.

Solve:

$$h' = mh = -\frac{f}{s}h = -\frac{15\text{ mm}}{10\text{ m}}(2.0\text{ m}) = -3.0\text{ mm}$$

The height of the image on the detector is 3.0 mm.

Assess: This seems reasonable given typical focal lengths and detector sizes.

35.7. Visualize: Equation 35.2 gives f-number $= f/D$.

Solve:

$$f\text{-number} = \frac{f}{D} = \frac{35\text{ mm}}{7.0\text{ mm}} = 5.0$$

Assess: This is in the range of f-numbers for typical camera lenses.

35.8. Visualize: Solve Equation 35.2 for D.

Solve:

$$D = \frac{f}{f\text{-number}} = \frac{12\text{ mm}}{4.0} = 3.0\text{ mm}$$

Assess: This is in the same ballpark as the example after Equation 35.2.

35.9. Visualize: We want the same exposure in both cases. The exposure depends on $I\Delta t_{\text{shutter}}$. We'll also use Equation 35.3.

Solve:

$$\text{exposure} = I\Delta t \propto \frac{1}{(f\text{-number})^2}\Delta t$$

$$\frac{1}{(f\text{-number})^2}\Delta t = \frac{1}{(f\text{-number})'^2}\Delta t'$$

$$\Delta t' = \frac{(f\text{-number})'^2}{(f\text{-number})^2}\Delta t = \frac{(4.0)^2}{(5.6)^2}\left(\frac{1}{125}\text{ s}\right) = \frac{1}{245}\text{ s} \approx \frac{1}{250}\text{ s}$$

Assess: An alternate approach without a lot of calculation is that since we changed the lens (opened) by one *f* stop, that doubles the intensity so we need half the time interval to achieve the same exposure.

35.10. Visualize: We want the same exposure in both cases. The exposure depends on $I\Delta t_{\text{shutter}}$. We'll also use Equation 35.3. The lens is the same lens in both cases, so $f = f'$.

Solve:

$$\text{exposure} = I\Delta t \propto \frac{D^2}{f^2}\Delta t$$

$$\frac{D^2}{f^2}\Delta t = \frac{D'^2}{f'^2}\Delta t'$$

Solve for D'; then simplify.

$$D' = \sqrt{D^2\left(\frac{f'}{f}\right)^2\left(\frac{\Delta t}{\Delta t'}\right)} = D\sqrt{\frac{\Delta t}{\Delta t'}} = (3.0\text{ mm})\sqrt{\frac{1/125\text{ s}}{1/500\text{ s}}} = (3.0\text{ mm})\sqrt{4} = 6.0\text{ mm}$$

Assess: Since we decreased the shutter speed by a factor of 4 we need to increase the aperture area by a factor of 4, and this means increase the diameter by a factor of 2.

Section 35.3 Vision

35.11. Model: Ignore the small space between the lens and the eye.

Visualize: Refer to Example 35.4, but we want to solve for s', the near point.

Solve:

(a) The power of the lens is positive which means the focal length is positive, so Ramon wears converging lenses. This is the remedy for hyperopia.

(b) We want to know where the image should be for an object $s = 25\text{ cm}$ given $1/f = 2.0\text{ m}^{-1}$.

$$f = \frac{1}{P} = 0.50\text{ m}$$

$$s' = \frac{fs}{s-f} = \frac{(0.50\text{ m})(0.25\text{ m})}{0.25\text{ m} + 0.50\text{ m}} = -0.50\text{ m}$$

So the near point is 50 cm.

Assess: The negative sign on s' is expected because we need the image to be virtual.

35.12. Model: Ignore the small space between the lens and the eye.

Visualize: Refer to Example 35.5, but we want to solve for s', the far point.

Solve:

(a) The power of the lens is negative which means the focal length is negative, so Ellen wears diverging lenses. This is the remedy for myopia.

(b) We want to know where the image should be for an object $s = \infty\text{ m}$ given $1/f = -1.0\text{ m}^{-1}$.

$$f = \frac{1}{P} = -1.0\text{ m}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

When $s = \infty\text{ m}$,

$$\frac{1}{\infty\text{ m}} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = f = -1.0\text{ m}$$

So the far point is 100 cm.

Assess: The negative sign on s' is expected because we need the image to be virtual.

35.13. Model: With normal vision the farthest distance at which a relaxed eye can focus (the far point) is infinity. Rays coming from infinity are nearly parallel, so the focal length of the lens would be 24 mm, the same as the length of the eye. However, the far point is less than infinity for many people, and most sources quote the focal length of the eye as 17 mm–22 mm. However, for simplicity of calculation, assume the vision is normal and the focal length of the lens/cornea combination is 24 mm

Visualize: Equation 35.2 gives f-number $= f/D$.

Solve: For the fully dilated pupil (dark-adapted eye):

$$f\text{-number} = \frac{f}{D} = \frac{24\text{ mm}}{8.0\text{ mm}} = 3.0$$

Assess: These answers correspond to the values given in the text.

Section 35.4 Optical Systems That Magnify

35.14. Model: Assume the thin lens equation is valid.

Visualize: We are given $s' = 25\text{ cm}$ and $M = 5\times$. We also have $M = \dfrac{25\text{ cm}}{f}$.

Solve: $M = 5\times = \dfrac{25\text{ cm}}{f} i \quad \Rightarrow \quad f = 5\text{ cm}$. Now use the thin lens equation.

$$\frac{1}{s} = \frac{1}{f} - \frac{1}{s'} = \frac{1}{5\text{ cm}} - \frac{1}{25\text{ cm}} = \frac{4}{25\text{ cm}} \quad \Rightarrow \quad s = \frac{25}{4}\text{cm} \approx 6.3\text{ cm}$$

Assess: 6.3 cm seems like a reasonable object distance for a magnifying glass.

35.15. Visualize: Equation 35.10 relates the variables in question:

$$M = -\frac{L}{f_{\text{obj}}}\frac{25\text{ cm}}{f_{\text{eye}}}$$

We are given $M = 500\times$, $L = 20\text{ cm}$, and $f_{\text{eye}} = 5.0\text{ cm}$

Solve: Solve for f_{obj}.

$$f_{\text{obj}} = -\frac{L}{M}\frac{25\text{ cm}}{f_{\text{eye}}} = -\frac{20\text{ cm}}{-500}\frac{25\text{ cm}}{5.0\text{ cm}} = 0.20\text{ cm} = 2.0\text{ mm}$$

Assess: This is in the same ball park as the case in the book.

35.16. Visualize: Equation 35.10 relates the variables in question:

$$M = -\frac{L}{f_{\text{obj}}}\frac{25\text{ cm}}{f_{\text{eye}}}$$

We are given $M = 100\times$, $L = 160\text{ mm}$, and $f_{\text{obj}} = 8.0\text{ cm}$.

Solve: Solve for f_{eye}.

$$f_{\text{eye}} = -\frac{L}{f_{\text{obj}}}\frac{25\text{ cm}}{M} = -\frac{160\text{ mm}}{8.0\text{ mm}}\frac{25\text{ cm}}{(-100)} = 5.0\text{ cm}$$

Assess: This is the same f_{eye} as in the previous exercise.

35.17. Model: Assume the thin-lens equation is valid.

Visualize: We are given $f_{\text{obj}} = 9.0\text{ mm}$ and

$$m = -\frac{s'}{s} = -40 \quad \Rightarrow \quad s' = 40s$$

Solve: Use the thin-lens equation.

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f_{\text{obj}}}$$

$$\frac{1}{s}+\frac{1}{40s}=\frac{1}{f_{\text{obj}}}$$

$$\frac{41}{40s}=\frac{1}{f_{\text{obj}}}$$

$$s=\frac{41}{40}f_{\text{obj}}=\frac{41}{40}(9.0\text{ mm})=9.2\text{ mm}$$

Assess: This value of the object distance is typical for a simple microscope.

35.18. Visualize: Figure 35.15 shows from similar triangles that for the eyepiece lens to collect all the light

$$\frac{D_{\text{obj}}}{f_{\text{obj}}}=\frac{D_{\text{eye}}}{f_{\text{eye}}}$$

We also see from Equation 35.11 that $M=-f_{\text{obj}}/f_{\text{eye}}$. We are given $M=-20$ and $D_{\text{obj}}=12\text{ cm}$.

Solve:

$$D_{\text{eye}}=D_{\text{obj}}\frac{f_{\text{eye}}}{f_{\text{obj}}}=\frac{D_{\text{obj}}}{-M}=\frac{12\text{ cm}}{20}=0.60\text{ cm}=6.0\text{ mm}$$

Assess: The answer is almost as wide as a dark-adapted eye.

35.19. Model: Assume the eyepiece is a simple magnifier with $M_{\text{eye}}=25\text{ cm}/f_{\text{eye}}$.

Visualize: $f_{\text{eye}}=25\text{ cm}/10=2.5\text{ cm}$.

Solve:
(a) The magnification of a telescope is

$$M=-\frac{f_{\text{obj}}}{f_{\text{eye}}}=\frac{100\text{ cm}}{2.5\text{ cm}}=40$$

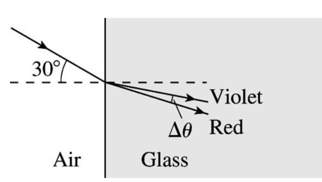
(b)

$$f\text{-number}=\frac{f}{D}=\frac{1.00\text{ m}}{0.20\text{ m}}=5.0$$

Assess: These results are in reasonable ranges for magnification and *f*-number.

Section 35.5 Color and Dispersion

35.20. Model: Use the ray model of light.
Visualize:



Solve: Using Snell's law,

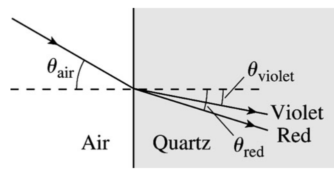
$$n_{\text{air}}\sin 30° = n_{\text{red}}\sin\theta_{\text{red}} \quad\Rightarrow\quad \theta_{\text{red}} = \sin^{-1}\left(\frac{\sin 30°}{1.52}\right) = 19.2°$$

$$n_{\text{air}}\sin 30° = n_{\text{violet}}\sin\theta_{\text{violet}} \quad\Rightarrow\quad \theta_{\text{violet}} = \sin^{-1}\left(\frac{\sin 30°}{1.55}\right) = 18.8°$$

Thus the angular spread is

$$\Delta\theta = \theta_{\text{red}} - \theta_{\text{violet}} = 19.2° - 18.8° = 0.4°$$

35.21. Model: Use the ray model of light and the phenomenon of dispersion.
Visualize:



Solve: Using Snell's law for the red light,

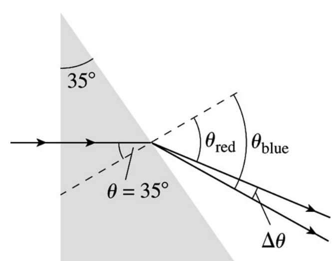
$$n_{\text{air}}\sin\theta_{\text{air}} = n_{\text{red}}\sin\theta_{\text{red}} \quad\Rightarrow\quad (1.00)\sin\theta_{\text{air}} = 1.45\sin(26.3°) \quad\Rightarrow\quad \theta_{\text{air}} = \frac{\sin^{-1}(1.45\sin 26.3°)}{1.00} = 40.0°$$

Now using Snell's law for the violet light,

$$n_{\text{air}}\sin\theta_{\text{air}} = n_{\text{violet}}\sin\theta_{\text{violet}} \quad\Rightarrow\quad (1.00)\sin 40.0° = n_{\text{violet}}\sin 25.7° \quad\Rightarrow\quad n_{\text{violet}} = 1.48$$

Assess: As expected, n_{violet} is slightly larger than n_{red}.

35.22. Model: Use the ray model of light and the phenomenon of dispersion.
Visualize:



Solve: **(a)** From the graph in Figure 35.18, we estimate the index of refraction for the red light (656 nm) to be $n_{\text{red}} = 1.572$ and for the blue light (456 nm) to be $n_{\text{blue}} = 1.587$.
(b) The angle of incidence onto the rear of the prism is $35°$. Using these values for the refractive index and Snell's law,

$$n_{\text{red}}\sin 35° = n_{\text{air}}\sin\theta_{\text{red}} \quad\Rightarrow\quad \theta_{\text{red}} = \sin^{-1}\left(\frac{(1.572)\sin 35°}{1.00}\right) = 64.4°$$

$$n_{\text{blue}}\sin 35° = n_{\text{air}}\sin\theta_{\text{blue}} \quad\Rightarrow\quad \theta_{\text{blue}} = \sin^{-1}\left(\frac{(1.587)\sin 35°}{1.00}\right) = 65.5° \Rightarrow$$

$$\Delta\theta = \theta_{\text{blue}} - \theta_{\text{red}} = 1.1°$$

35.23. Model: The intensity of scattered light is inversely proportional to the fourth power of the wavelength.

Solve: We want to find the wavelength of infrared light such that $I_{\text{IR}} = 0.01 I_{500}$. Because $I_{500} \propto (500\text{ nm})^{-4}$ and $I_{\text{IR}} \propto \lambda^{-4}$, we have

$$\frac{I_{500}}{I_{\text{IR}}} = \left(\frac{\lambda}{500\text{ nm}}\right)^4 = 100 \quad \Rightarrow \quad \lambda = 1580\text{ nm} \approx 1600\text{ nm}$$

Section 35.6 The Resolution of Optical Instruments

35.24. Model: Diffraction prevents focusing light to an arbitrarily small point. Model the lens of diameter D as an aperture in front of an ideal lens with an 8.0 cm focal length.

Solve: (a) Assuming that the incoming laser beam is parallel, the focal length of the lens should be 8.0 cm.

(b) From Equation 35.12, the minimum spot size in the focal plane of this lens is

$$w = \frac{2.44\lambda f}{D} \Rightarrow 10\times10^{-6}\text{ m} = \frac{2.44(633\times10^{-9}\text{ m})(8.0\times10^{-2}\text{ m})}{D} \Rightarrow D = 0.012\text{ m} = 1.2\text{ cm}$$

35.25. Model: Two objects are marginally resolvable if the angular separation between the objects, as seen from the lens, is $\alpha = 1.22\lambda/D$.

Solve: Let Δy be the separation between the two light bulbs, and let L be their distance from a telescope. Thus,

$$\alpha = \frac{\Delta y}{L} = \frac{1.22\lambda}{D} \Rightarrow L = \frac{\Delta y\, D}{1.22\lambda} = \frac{(1.0\text{ m})(4.0\times10^{-2}\text{ m})}{1.22(600\times10^{-9}\text{ m})} = 55\text{ km}$$

Problems

35.26. Visualize: Parallel rays coming into the lens will focus at the focal point of the lens, then continue on and hit the mirror and diverge back to the left from the mirror.

Solve: The first location is at the focal point 2.0 cm right of the lens.

The first focus location becomes a source a distance $s = 3.0$ cm to the left of the mirror, so we compute the image distance for the mirror.

$$s' = \frac{sf}{s-f} = \frac{(3.0\text{ cm})(-1.5\text{ cm})}{3.0\text{ cm} - (-1.5\text{ cm})} = -1.0\text{ cm}$$

Therefore, the second location is 1.0 cm to the right of the mirror, or 6.0 cm to the right of the lens.

Assess: Both answers are reasonable.

35.27. Visualize: Physically, the light rays can either go directly through the lens or they can reflect from the mirror and then go through the lens. We can consider the image from the lens alone and then consider the image from mirror becoming the object for the lens.

Solve:

(a) First case: the lens gets the subscript 1's and the mirror the 2's. The location of the image from the lens is

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(10\text{ cm})(5\text{ cm})}{5\text{ cm} - 10\text{ cm}} = -10\text{ cm}$$

The image is right at the mirror plane and a calculation for a mirror shows that when $s_2 = 0$ then $s_2' = 0$, too. So the final image is at the mirror, 10 cm to the left of the lens.

$$m = -\frac{s_1'}{s_1} = -\frac{-10\text{ cm}}{5\text{ cm}} = 2.0$$

so $h' = hm = (1.0\text{ cm})(2.0) = 2.0\text{ cm}$.

Second case: the mirror gets the subscript 1's and the lens the 2's. The location of the image from the mirror is

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(10\text{ cm})(5\text{ cm})}{5\text{ cm} - 10\text{ cm}} = -10\text{ cm}$$

or 10 cm behind (to the left of) the mirror. This image now becomes the object for the lens and $s_2 = 20$ cm.

$$s_2' = \frac{f_2 s_2}{s_2 - f_2} = \frac{(10\text{ cm})(20\text{ cm})}{20\text{ cm} - 10\text{ cm}} = 20\text{ cm}$$

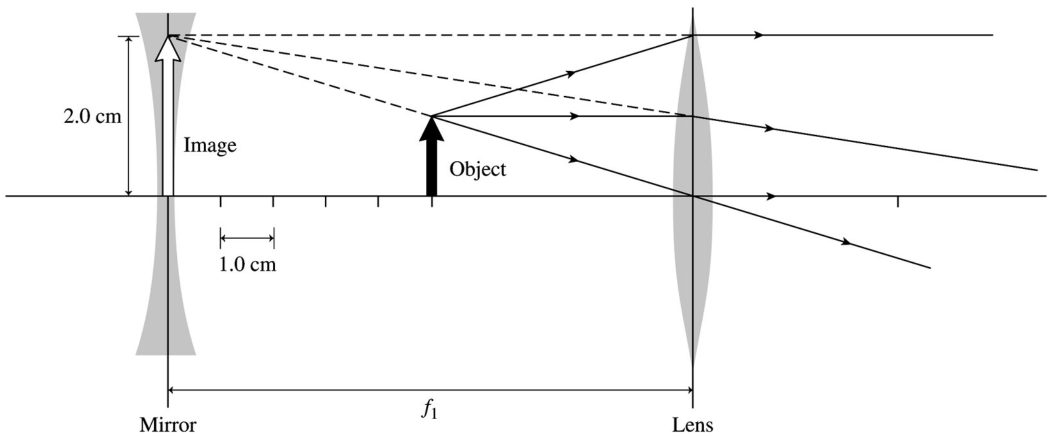
So the image is 20 cm to the right of the lens.

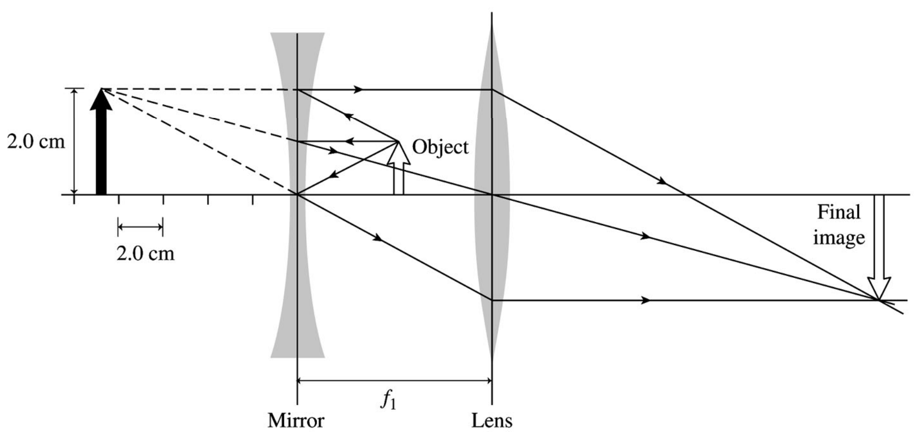
$$m = \left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right) = \left(-\frac{-10\text{ cm}}{5\text{ cm}}\right)\left(-\frac{20\text{ cm}}{20\text{ cm}}\right) = -2.0$$

so $h' = hm = (1.0\text{ cm})(-2.0) = -2.0$ cm, where the negative sign indicates the image is inverted.

In summary, both images are 2.0 cm tall; one is upright 10 cm left of the lens, the other is inverted 20 cm to the right of the lens.

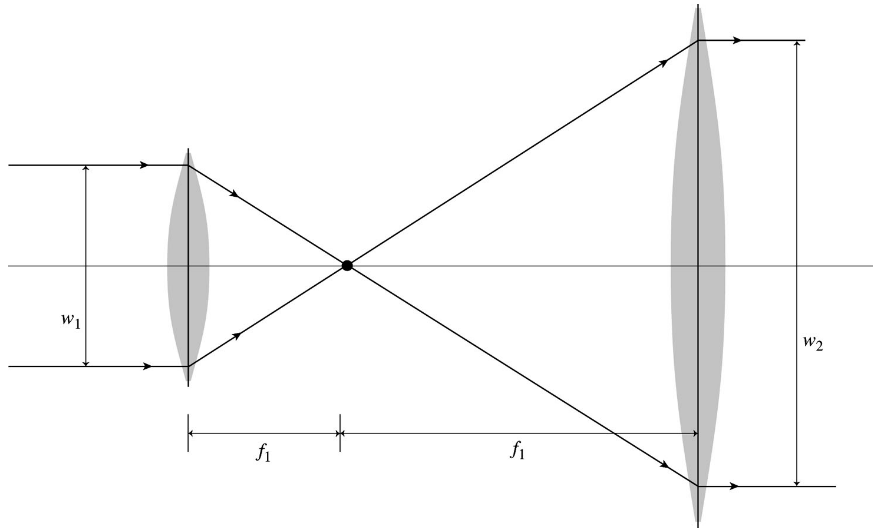
(b)





Assess: The ray tracing verifies the calculations.

35.28. Visualize: Parallel rays coming into the first lens will focus at the focal point of the first lens. If that position is also the focal point of the second lens then the rays will also leave the second lens parallel.



Solve: (a) This is similar to a telescope.

$$d = f_1 + f_2$$

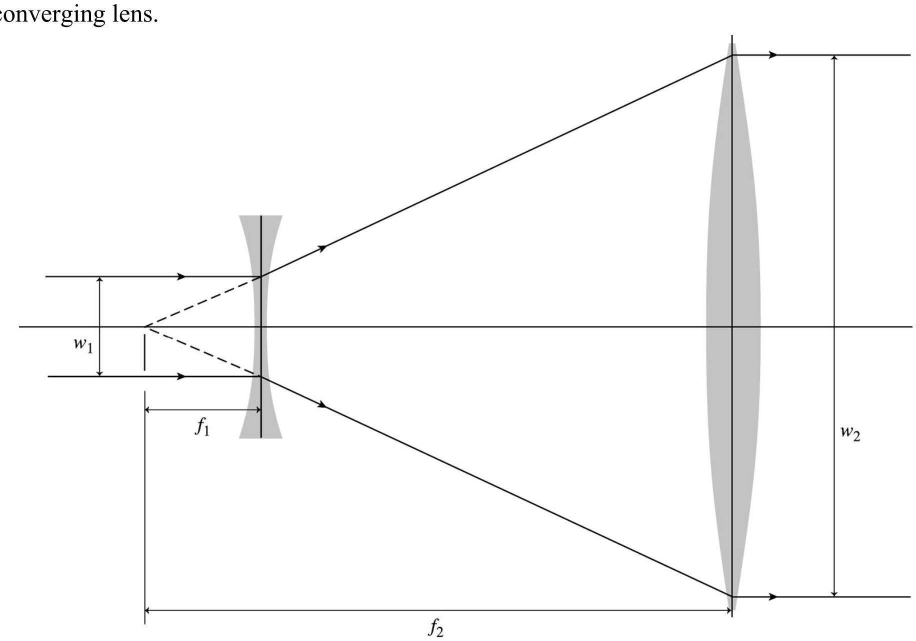
(b) Looking at the similar triangles in the diagram shows that

$$\frac{w_1}{f_1} = \frac{w_2}{f_2}$$

$$w_2 = \frac{f_2}{f_1} w_1$$

Assess: Figure P35.28 says $f_2 > f_1$ and our answer then shows that $w_2 > w_1$ which is the goal of a beam expander.

35.29. Visualize: Hard thought shows that if the left focal points for both lenses coincide then the parallel rays before and after the beam splitter are reproduced. The first lens diverges the rays as if they had come from the focal point of the converging lens.



Solve: (a)

$$d = f_2 - |f_1|$$

But since we are given $f_1 < 0$, this is equivalent to

$$d = f_2 + f_1$$

(b) Looking at the similar triangles in the diagram shows that

$$\frac{w_1}{|f_1|} = \frac{w_2}{f_2}$$

$$w_2 = \frac{f_2}{|f_1|} w_1$$

Assess: Figure P35.29 says $f_2 > |f_1|$ and our answer then shows that $w_2 > w_1$ which is the goal of a beam expander.

35.30. Visualize: The object is within the focal length of the converging lens, so we expect the image to be upright, virtual, and to the left of the lens. The image of the lens becomes the object for the mirror, and we expect the second image to be upright and virtual behind (to the right of) the mirror. That becomes the object for the lens as the light goes back through the lens a second time.
Solve:

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(10\text{ cm})(5\text{ cm})}{5\text{ cm} - 10\text{ cm}} = -10\text{ cm}$$

The image of the lens becomes the object for the mirror $\Rightarrow s_2 = 15$ cm, and we expect the second image to be upright and virtual behind (to the right of) the mirror.

$$s_2' = \frac{f_2 s_2}{s_2 - f_2} = \frac{(-30\text{ cm})(15\text{ cm})}{15\text{ cm} - (-30\text{ cm})} = -10\text{ cm}$$

The image from the mirror becomes the object for the lens as the light goes back through the lens a second time. $s_3 = 15$ cm. This is the same lens as before, so $f_3 = f_1 = 10$ cm.

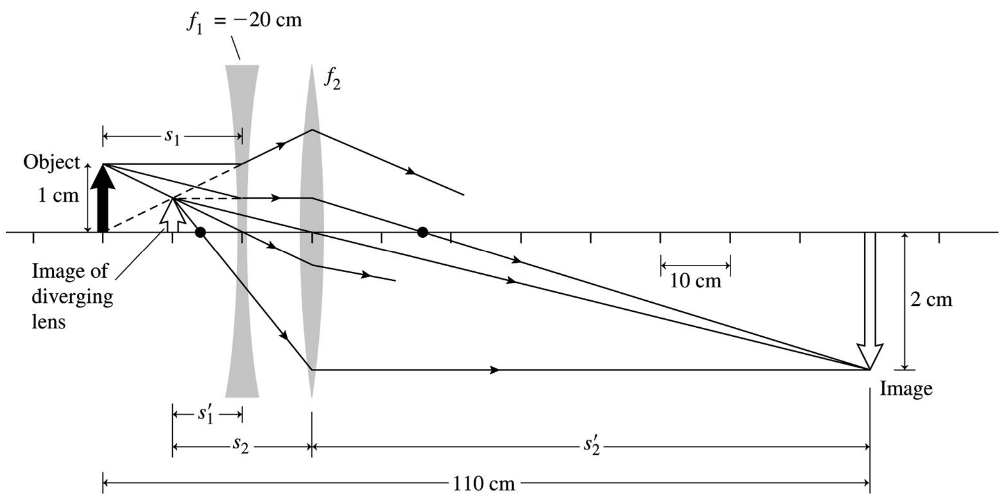
$$s_3' = \frac{f_3 s_3}{s_3 - f_3} = \frac{(10\text{ cm})(15\text{ cm})}{15\text{ cm} - (10\text{ cm})} = 30\text{ cm}$$

This is left of the lens since the object this time is on the right of the lens.

$$h' = h\frac{s_1'}{s_1}\frac{s_2'}{s_2}\frac{s_3'}{s_3} = (1.0\text{ cm})\left(\frac{-10\text{ cm}}{5.0\text{ cm}}\right)\left(\frac{-10\text{ cm}}{15\text{ cm}}\right)\left(\frac{30\text{ cm}}{15\text{ cm}}\right) = 2.7\text{ cm}$$

The final image is 30 cm to the left of the lens. It is upright with a height of 2.7 cm.
Assess: Ray tracing will verify the answer.

35.31. Model: Use the ray model of light. Assume both the lenses are thin lenses.
Visualize:



Solve: Begin by finding the image of the diverging lens,

$$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1} \Rightarrow \frac{1}{20\text{ cm}}+\frac{1}{s_1'}=\frac{1}{-20\text{ cm}} \Rightarrow s_1' = -10\text{ cm}$$

This image is the object for the second lens. Its distance from the screen is $s_2 + s_2' = 110\text{ cm} - 10\text{ cm} = 100\text{ cm}$. The overall magnification is

$$M = m_1 m_2 = -\frac{h'}{h} = -2$$

The magnification of the diverging lens is

$$m_1 = -\frac{s_1'}{s_1} = -\frac{(-10\text{ cm})}{20\text{ cm}} = \frac{1}{2}$$

Thus the magnification of the converging lens needs to be

$$m_2 = -\frac{s_2'}{s_2} = -4 \Rightarrow s_2' = 4s_2$$

Substituting this result into $s_2 + s_2' = 100\text{ cm}$, we have $s_2 + 4s_2 = 100\text{ cm}$, which means $s_2 = 20\text{ cm}$ and $s_2' = 80\text{ cm}$. We can find the focal length by using the thin-lens equation for the converging lens:

$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2} \Rightarrow \frac{1}{f_2}=\frac{1}{20\text{ cm}}+\frac{1}{80\text{ cm}} \Rightarrow f_2 = 16\text{ cm}$$

Hence, the second lens is a converging lens of focal length 16 cm. It must be placed 10 cm in front of the diverging lens, toward the screen, or 80 cm from the screen.

35.32. Visualize: We simply need to work backwards. We are given $f_1 = 7.0\text{ cm}$ and $f_2 = 15\text{ cm}$. We are also given $s_2' = -10\text{ cm}$. We use this to find s_2.
Solve: (a)

$$s_2 = \frac{f_2 s_2'}{s_2' - f_2} = \frac{(15\text{ cm})(-10\text{ cm})}{-10\text{ cm} - 15\text{ cm}} = 6.0\text{ cm}$$

So the final image is 6.0 cm to the left of the second lens, or 14 cm to the right of the first lens. That is, the object for the second lens is the image from the first lens, so $s_1' = 20\text{ cm} = -6.0\text{ cm} = 14\text{ cm}$.

$$s_1 = \frac{f_1 s_1'}{s_1' - f_1} = \frac{(7.0\text{ cm})(14\text{ cm})}{14\text{ cm} - 7.0\text{ cm}} = 14\text{ cm}$$

Thus, $L = 14\text{ cm}$.

(b) To find the height and orientation we need to look at the magnification.

$$m = m_1 m_2 = \left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right) = \left(-\frac{14\text{ cm}}{14\text{ cm}}\right)\left(-\frac{-10\text{ cm}}{6.0\text{ cm}}\right) = -1.7$$

$$h' = hm = (1.0\text{ cm})(-1.7) = -1.7\text{ cm}$$

The negative sign indicates that the image is inverted.

Assess: Ray tracing would verify the answers.

35.33. Model: Yang has myopia. Normal vision will allow Yang to focus on a very distant object. In measuring distances, we'll ignore the small space between the lens and her eye.

Solve: Because Yang can see objects at 150 cm with a fully relaxed eye, we want a lens that creates a virtual image at $s' = -150\text{ cm}$ (negative because it's a virtual image) of an object at $s = \infty\text{ cm}$. From the thin-lens equation,

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty\text{ m}} + \frac{1}{-1.5\text{ m}} = -0.67\text{ D}$$

So Yang gets a prescription for a -0.67 D lens that has $f = -150\text{ cm}$.

Since Yang can accommodate to see things as close as 20 cm we need to create a virtual image at 20 cm of objects that are at $s = \text{new near point}$. That is, we want to solve the thin-lens equation for s when $s' = -20\text{ cm}$ and $f = -150\text{ cm}$.

$$s = \frac{fs'}{s' - f} = \frac{(-150\text{ cm})(-20\text{ cm})}{-20\text{ cm} - (-150\text{ cm})} = 23\text{ cm}$$

Assess: Diverging lenses are always used to correct myopia.

35.34. Visualize: Use Equation 34.20:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

where $n_1 = 1.00$ for air and $n_2 = 1.34$ for aqueous humor. If we think of incoming parallel rays coming to a focus in the humor then we have $s = \infty$ and $s' = f$.

Solve:

$$\frac{1.0}{\infty} + \frac{n_2}{f} = \frac{n_2 - n_1}{R}$$

Solve for *R*.

$$R = f\frac{n_2 - n_1}{n_2} = (3.0\text{ cm})\frac{1.34 - 1.00}{1.34} = 0.76\text{ cm}$$

Assess: If you think about the dimensions of an eye, this answer seems physically possible.

35.35. Visualize: Use Equation 34.26, the lens makers' equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For a symmetric lens $R_1 = R_2$ and

$$f = \frac{R}{2(n-1)} \text{ and } R = 2(n-1)f$$

Also needed will be the magnification of a telescope: $M = -f_{\text{obj}}/f_{\text{eye}} \Rightarrow f_{\text{eye}} = -f_{\text{obj}}/M$ (but we will drop the negative sign).
We are given $R_{\text{obj}} = 100 \text{ cm}$ and $M = 20$.
Solve:

$$R_{\text{eye}} = 2(n-1)f_{\text{eye}} = 2(n-1)\frac{f_{\text{obj}}}{M} = 2(n-1)\frac{\frac{R_{\text{obj}}}{2(n-1)}}{M} = \frac{R_{\text{obj}}}{M} = \frac{100 \text{ cm}}{20} = 5.0 \text{ cm}$$

Assess: We expect a short focal length and small radius of curvature for telescope eyepieces.

35.36. Model: In the small angle approximation the angle subtended by Mars without the telescope is $\theta_{\text{obj}} = D/d$ where D is the diameter and d is the distance from the earth.
Visualize: We are given $f_{\text{eye}} = 2.5 \text{ cm}$.
Solve:

$$M = \frac{\theta_{\text{eye}}}{\theta_{\text{obj}}} = \frac{0.50°}{D/d} = \frac{0.50°}{6800\times10^3 \text{ m}/1.1\times10^{11} \text{ m}}\left(\frac{\pi \text{ rad}}{180°}\right) = 141$$

Ignoring the negative sign,

$$M = \frac{f_{\text{obj}}}{f_{\text{eye}}}$$

$$f_{\text{obj}} = Mf_{\text{eye}} = 141(2.5 \text{ cm}) = 353 \text{ cm}$$

The length of the telescope is

$$L = f_{\text{obj}} + f_{\text{eye}} = 352.5 \text{ cm} + 2.5 \text{ cm} = 355 \text{ cm} = 3.55 \text{ m} \approx 3.5 \text{ m}$$

Assess: This is longer than most amateur telescopes.

35.37. Model: Assume that each lens is a simple magnifier with $M = 25 \text{ cm}/f$.
Visualize:

$$M_{\text{obj}} = \frac{25 \text{ cm}}{f_{\text{obj}}} \Rightarrow f_{\text{obj}} = \frac{25 \text{ cm}}{M_{\text{obj}}}$$

$$M_{\text{eye}} = \frac{25 \text{ cm}}{f_{eye}} \Rightarrow f_{\text{eye}} = \frac{25 \text{ cm}}{M_{\text{eye}}}$$

Solve: (a) The magnification of a telescope is

$$M = -\frac{f_{\text{obj}}}{f_{\text{eye}}} = -\frac{\frac{25 \text{ cm}}{M_{\text{obj}}}}{\frac{25 \text{ cm}}{M_{\text{eye}}}} = -\frac{M_{\text{eye}}}{M_{\text{obj}}}$$

The way to maximize the magnitude of this is to have $M_{\text{eye}} > M_{\text{obj}}$.

$$M = -\frac{5.0}{2.0} = -2.5$$

The magnification is usually given without the negative sign, so it is $2.5\times$.

(b) To achieve this we used the $2.0\times$ lens as the objective, which coincides with the text which says the objective should have a long focal length and the eyepiece a short focal length.
(c)

$$L = f_{\text{obj}} + f_{\text{eye}} = \frac{25\text{ cm}}{M_{\text{obj}}} + \frac{25\text{ cm}}{M_{\text{eye}}} = \frac{25\text{ cm}}{2.0} + \frac{25\text{ cm}}{5.0} = 17.5\text{ cm}$$

Assess: This is not a very powerful telescope.

35.38. Model: To make a telescope you need an objective with a long focal length and an eyepiece with a short focal length.
Visualize:

$$f = \frac{1}{P}$$

Solve:
(a) The lens with the smaller refractive power has the longer focal length and should be used as the object—that's the lens with $P = +3.0\text{ D}$. The $+4.5\text{ D}$ lens should be used as the eyepiece.
(b)

$$M = -\frac{f_{\text{obj}}}{f_{\text{eye}}} = -\frac{P_{\text{eye}}}{P_{\text{obj}}} = \frac{4.5\text{ D}}{3.0\text{ D}} = -1.5$$

(c) In a telescope the lenses should be a distance apart equal to the sum of their focal lengths.

$$d = f_{\text{obj}} + f_{\text{eye}} = \frac{1}{P_{\text{obj}}} + \frac{1}{P_{\text{eye}}} = \frac{1}{3.0\text{ D}} + \frac{1}{4.5\text{ D}} = 0.56\text{ m}$$

Assess: These numbers are reasonable, although a $+4.5\text{ D}$ lens is fairly strong. You really could make yourself a telescope by holding the lenses a half-meter apart and get a little $(1.5\times)$ magnification.
This cannot be done with the glasses of nearsighted people since they wear diverging lenses.

35.39. Model: While $s \approx f_{\text{obj}}$ we will not assume they are equal.
Visualize: Equation 35.9 says $m_{\text{obj}} \approx -L/f_{\text{obj}}$. We are given $L = 180\text{ mm}$ and $m_{\text{obj}} = -40$, where the negative sign means the image is inverted.
Solve: Solve for f_{obj}.

$$f_{\text{obj}} = -\frac{L}{m_{\text{obj}}} = \frac{180\text{ mm}}{40} = 4.5\text{ mm}$$

From Equation 35.8, $M_{\text{eye}} = (25\text{ cm})/f_{\text{eye}} \Rightarrow f_{\text{eye}} = 25\text{ cm}/20 = 1.25\text{ cm}$. For relaxed eye viewing the image of the objective must be 1.25 cm = 12.5 mm from the eyepiece, so $s' = 180\text{ mm} - 12.5\text{ mm} = 167.5\text{ mm}$. Thus the sample distance is

$$s = \left(\frac{1}{4.5\text{ mm}} - \frac{1}{167.5\text{ mm}}\right)^{-1} = 4.6\text{ mm}$$

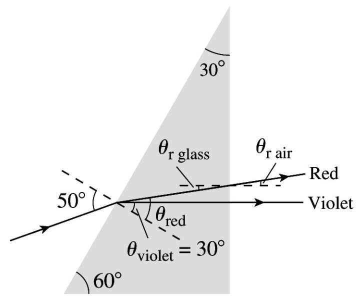
Assess: You need a short focal length to achieve $800\times$ magnification. We can also verify that $s \approx f_{\text{obj}}$.

35.40. Visualize: The magnification of a microscope is the magnification of the objective multiplied by the magnification of the eyepiece (or, in this case, the photo-ocular).
Solve: (a) The magnification is $M = m_{\text{obj}}M_{\text{ocular}} = (40\times)(2.5\times) = 100\times$, so the width on the stage will be 100 times less than the width of the sensor, or $22.5\text{ mm}/100 = 0.225\text{ mm}$.
(b) First find the width of the photo of the cell in µm.

$$(120\text{ pixels})(4.0\ \mu\text{m/pixel}) = 480\ \mu\text{m}$$

The cell's actual diameter is 100 times smaller than this, or 4.8 μm.
Assess: This is about right for the size of a cell.

35.41. Model: Use the ray model of light and the phenomena of refraction and dispersion.
Visualize:



Solve: Since violet light is perpendicular to the second surface, it must reflect at $\theta_{\text{violet}} = 30°$ at the first surface. Using Snell's law at the air-glass boundary where the ray is incident, we have

$$n_{\text{air}} \sin\theta_{\text{air}} = n_{\text{violet}} \sin\theta_{\text{violet}} \quad \Rightarrow \quad n_{\text{violet}} = \frac{n_{\text{air}} \sin\theta_{\text{air}}}{\sin\theta_{\text{violet}}} = \frac{(1.0)\sin(50°)}{\sin(30°)} = 1.5321$$

Since $n_{\text{violet}} = 1.020\ n_{\text{red}}$, $n_{\text{red}} = 1.5021$. Using Snell's law for the red light at the first surface gives

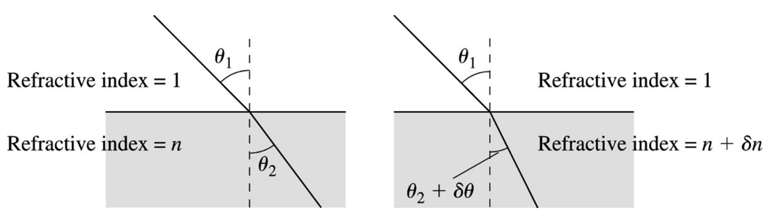
$$n_{\text{red}} \sin\theta_{\text{red}} = n_{\text{air}} \sin\theta_{\text{air}} \quad \Rightarrow \quad \theta_{\text{red}} = \sin^{-1}\left(\frac{(1.0)\sin(50°)}{1.5021}\right) = 30.664°$$

The angle of incidence on the rear face of the prism is thus $\theta_{\text{r glass}} = 30.664° - 30° = 0.664°$. Using Snell's law once again for the rear face and for the red wavelength,

$$n_{\text{red}} \sin\theta_{\text{r glass}} = n_{\text{air}} \sin\theta_{\text{r air}} \quad \Rightarrow \quad \theta_{\text{r air}} = \sin^{-1}\left(\frac{n_{\text{red}} \sin\theta_{\text{r glass}}}{n_{\text{air}}}\right) \quad \Rightarrow \quad = \sin^{-1}\left(\frac{(1.5021)\sin(0.664°)}{1.0}\right) = 0.997°$$

Because $\theta_{\text{v air}} = 0°$ and $\theta_{\text{r air}} = 0.997°$, $\phi = \theta_{\text{r air}} - \theta_{\text{v air}} - 0.997° \approx 1.0°$.

35.42. Model: Use the ray model of light.
Visualize:



The angle of refraction is $\theta_2 + \delta\theta$ for those wavelengths that have a refractive index of $n + \delta n$.
Solve: (a) Applying Snell's law to the diagrams,

$$(1)\sin\theta_1 = n\sin\theta_2 \qquad (1)\sin\theta_1 = (n + \delta n)\sin(\theta_2 + \delta\theta)$$

Equating the right-hand sides of the above two equations and using the formula for the sine of a sum gives

$$n\sin\theta_2 = (n + \delta n)(\sin\theta_2 \cos\delta\theta + \cos\theta_2 \sin\delta\theta) = (n + \delta n)(\sin\theta_2 + \cos\theta_2 \delta\theta)$$

where we have assumed that $\delta\theta \ll \theta$. Multiplying the expressions gives

$$n\sin\theta_2 = n\sin\theta_2 + n\cos\theta_2\delta\theta + \delta n\sin\theta_2 + \delta n\delta\theta\cos\theta_2$$

We can ignore the last term on the right-hand side because it is the product of two small terms. The equation becomes

$$n\cos\theta_2\delta\theta = -\delta n\sin\theta_2 \quad\Rightarrow\quad \delta\theta = -\left(\frac{\delta n}{n}\right)\tan\theta_2$$

Note that $\delta\theta$ has to be in radians.
We can obtain the same result in the following way as well. From Snell's law,

$$\sin\theta_2 = \frac{\sin\theta_1}{n}$$

Differentiating with respect to n

$$\delta\left(\frac{\sin\theta_2}{\delta n}\right) = \cos\theta_2\frac{\delta\theta}{\delta n} = (\sin\theta_1)\left(-\frac{1}{n^2}\right) = -\frac{n\sin\theta_2}{n^2} = \frac{-\sin\theta_2}{n} \quad\Rightarrow\quad \delta\theta = -\left(\frac{\delta n}{n}\right)\tan\theta_2$$

(b) We have $\theta_1 = 30°$ and $n_{\text{red}} = 1.552$. Because the red wavelength is longer than the violet wavelength, $n_{\text{red}} < n_{\text{violet}}$. Also, if the refraction angle for the red light is θ_2, the refraction angle for the violet is less than θ_2. Thus, $\delta\theta = -0.28°$. From the formula obtained in part **(a)**,

$$\delta\theta = -\left(\frac{\delta n}{n}\right)\tan\theta_2 \quad\Rightarrow\quad \delta n = -\frac{n}{\tan\theta_2}\delta\theta$$

To determine $\tan\theta_2$, we note that

$$n_{\text{red}}\sin\theta_2 = n_{\text{air}}\sin 30° \quad\Rightarrow\quad \theta_2 = \sin^{-1}\left(\frac{\sin 30°}{1.552}\right) = 18.794° \quad\Rightarrow\quad \tan\theta_2 = 0.3403$$

Thus, the expression for the change in the index of refraction is

$$\delta n = -\frac{1.552}{0.3403}(-0.28°)\left(\frac{\pi\text{ rad}}{180°}\right) = 0.0223 \quad\Rightarrow\quad n_{\text{violet}} = n_{\text{red}} + \delta n = 1.552 + 0.022 = 1.574$$

35.43. Model: The width of the central maximum that accounts for a significant amount of diffracted light intensity is inversely proportional to the size of the aperture. The lens is an aperture that focuses light.
Solve: To focus a laser beam, which consists of parallel rays from $s = \infty$, the focal length needs to match the distance to the target: $f = L = 5.0\text{ cm}$. The minimum spot size to which a lens can focus is

$$w = \frac{2.44\lambda f}{D} \Rightarrow 5.0\times10^{-6}\text{ m} = \frac{2.44(1.06\times10^{-6}\text{ m})(5.0\times10^{-2}\text{ m})}{D} \Rightarrow D = 2.6\text{ cm}.$$

35.44. Model: Two objects are marginally resolved if the angular separation between the objects, as seen from your eye lens, is $\alpha = 1.22\lambda/D$, but the λ we want to use is the λ in the eye: $\lambda = \lambda_{\text{air}}/n$. Let Δx be the separation between the two headlights of the oncoming car and let L be the distance of these lights from your eyes. For small angles, $\Delta x = \alpha L$.
We are given $D = 7\times10^{-3}\text{ m}$, $\Delta x = 1.2\text{ m}$, and $\lambda = \lambda_{\text{air}}/n = 600\text{ nm}/1.33 = 450\text{ nm}$.
Solve: Let Δy be the separation between the two headlights of the incoming car and let L be the distance of these lights from your eyes. Then,

$$\alpha = \frac{\Delta x}{L} = \frac{1.20\text{ m}}{L} = \frac{1.22\lambda}{D} = \frac{1.22(450\text{ nm})}{(7.0\times10^{-3}\text{ m})} \Rightarrow L = \frac{(1.20\text{ m})(7.0\times10^{-3}\text{ m})}{(1.22)(450\times10^{-9}\text{ m})} = 15\text{ km}$$

Assess: The two headlights are not resolvable if L > 15 km, marginally resolvable at 15 km, and resolvable at L < 15 km.

35.45. Model: The object is so far away that the rays are practically parallel when they get to the lens, so $s' = f$.

Solve: (a) In the diffraction-less case the two point sources of light must fall on different sensor pixels 2.5 μm apart. The magnitude of the lateral magnification is

$$m = \frac{f}{s} = \frac{20 \text{ mm}}{100 \text{ m}} = 0.00020$$

Now find the separation h of the two point sources that produce images 2.5 μm apart on the sensor.

$$h = \frac{h'}{m} = \frac{2.5\ \mu\text{m}}{0.00020} = 1.3 \text{ cm}$$

(b) Recall the equation for the diffraction from a circular aperture. $w = 2.44\lambda L/D$. We will say that w is the width of one pixel in the camera sensor and that $L = f$. Solve the equation for D.

$$D = \frac{2.44\lambda L}{w} = \frac{2.44(600 \text{ nm})(20 \text{ mm})}{2.5\ \mu\text{m}} = 1.2 \text{ cm}$$

(c)

$$f\text{-number} = \frac{f}{D} = \frac{20 \text{ mm}}{1.2} = f/1.7$$

Assess: The *f*-number is close to the range for the camera mentioned in the text. The width of the lens on smartphones or point-and-shoot cameras is about 1.2 cm.

35.46. Visualize: For telescopes the angular resolution is

$$\theta = \frac{1.22\lambda}{D}$$

And for small angles, $s = \theta r$. We want to know s. We are given

$\lambda = 650$ nm, $D = 2.4$ m, and $r = 33{,}000 \text{ ly} = 3.1\times10^{17}$ km.

Solve: Combine the two equations.

(a)

$$s = \theta r = \frac{1.22\lambda}{D} r = \frac{(1.22)(650\times10^{-9} \text{ m})}{2.4 \text{ m}} 3.1\times10^{17} \text{ km} = 1.0\times10^{11} \text{ km}$$

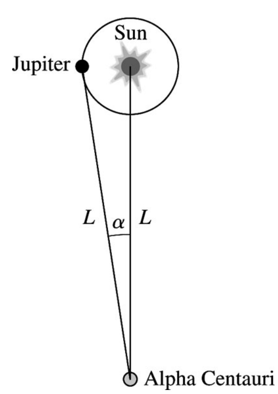
(b) The distance from the sun to Jupiter is 7.8×10^{11} m. So we divide the answer from part **(a)** by this number.

$$\frac{1.0\times10^{11} \text{ km}}{7.8\times10^{11} \text{ m}} = 130$$

Assess: The HST is good but not good enough to resolve two objects as close together as the sun and Jupiter from a distance of 33,000 ly.

35.47. Model: Two objects are marginally resolved if the angular separation between the objects is $\alpha = 1.22\lambda/D$.

Visualize:



Solve: (a) The angular separation between the sun and Jupiter is

$$\alpha = \frac{780\times10^9\text{ m}}{4.3\text{ light years}} = \frac{780\times10^9\text{ m}}{4.3\times(3.0\times10^8)\times(365\times24\times3600)\text{ m}} = 1.92\times10^{-5}\text{ rad}$$

$$\alpha = \frac{1.22\lambda}{D} = \frac{1.22(600\times10^{-9}\text{ m})}{D} \Rightarrow D = 0.038\text{ m} = 3.8\text{ cm}$$

(b) The sun is vastly brighter than Jupiter, which is much smaller and seen only dimly by reflected light. In theory it may be possible to resolve Jupiter and the sun, but in practice the extremely bright light from the sun will overwhelm the very dim light from Jupiter.

Challenge Problems

35.48. Visualize: We'll use the thin-lens equation and also Equation 35.10:

$$M = -\frac{L}{f_{\text{obj}}}\frac{25\text{ cm}}{f_{\text{eye}}}$$

Also recall that $P = 1/f$.

Solve: The power of each lens is

$$P_1 = \frac{1}{f_1} = \frac{1}{0.020\text{ m}} = 50\text{ D} \qquad P_2 = \frac{1}{f_1} = \frac{2}{0.010\text{ m}} = 100\text{ D}$$

Since we want to use the more powerful lens as the objective, the lens labeled P_2 will be the objective. This means the focal lengths of the objective and eyepiece are $f_{\text{obj}} = 1.0\text{ cm}$ and $f_{\text{eye}} = 2.0\text{ cm}$.

(a) We want the eyepiece to be $L = 16\text{ cm}$ from the objective, so $s' = 16\text{ cm} - f_{\text{eye}} = 16\text{ cm} - 2.0\text{ cm} = 14\text{ cm}$. The object distance for the objective is

$$s = \frac{fs'}{s'-f} = \frac{(1.0\text{ cm})(14\text{ cm})}{14\text{ cm} - 1.0\text{ cm}} = 1.1\text{ cm}$$

(b) $M = m_{\text{obj}}M_{\text{eye}} = \frac{-s'}{s}\frac{25\text{ cm}}{f_{\text{eye}}} = -\frac{(14\text{ cm})}{(1.08\text{ cm})}\frac{25\text{ cm}}{(2.0\text{ cm})} = -160$

Assess: As expected, s is just beyond the focal point. We can use approximation in Equation 35.9 to get a similar answer:

$$M = \frac{L}{f_{\text{obj}}}\frac{25\text{ cm}}{f_{\text{eye}}} = -\frac{(16\text{ cm})}{(1.0\text{ cm})}\frac{25\text{ cm}}{(2.0\text{ cm})} = -200$$

but the approximation isn't very good for this microscope.

35.49. Visualize: The plane left face will not refract any of the rays (which are parallel to each other and perpendicular to the face), so nothing happens until the rays hit the first curved surface between lens 1 and lens 2. We'll need to twice (once for each curved surface) apply Equation 34.20:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

Solve: (a) For the first curved surface we say $s = \infty$ because the incoming rays are parallel.

$$\frac{n_1}{\infty} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow s' = R\frac{n_2}{n_2 - n_1}$$

For the second curved surface (where the rays exit into the air) n_2 is on the left and $n_{\text{air}} = 1.0$ is on the right (so those will take the place of n_1 and n_2, respectively). Since the distance between the lenses is zero, s' from the previous result will be plugged in for s for the second case. The final thing to note is that the magnitude of the new

$s' = f$ because the doublet brings parallel rays to a focus at *f*, but s' is negative due to the sign convention in Table 23.3.

$$\frac{n_2}{R\frac{n_2}{n_2 - n_1}} + \frac{n_{\text{air}}}{-f} = \frac{n_{\text{air}} - n_2}{R}$$

Now solve for *f*.

$$\frac{n_2 - n_1}{R} + \frac{1}{-f} = \frac{1 - n_2}{R}$$

$$\frac{1}{-f} = \frac{1 - n_2 - n_2 + n_1}{R}$$

$$f = \frac{R}{2n_2 - n_1 - 1}$$

(b)

$$f_{\text{blue}} = \frac{R}{2(n_2)_{\text{blue}} - (n_1)_{\text{blue}} - 1} \quad f_{\text{red}} = \frac{R}{2(n_2)_{\text{red}} - (n_1)_{\text{red}} - 1}$$

In the condition we desire $f_{\text{blue}} = f_{\text{red}}$, so the two denominators must be equal.

$$2(n_2)_{\text{blue}} - (n_1)_{\text{blue}} - 1 = 2(n_2)_{\text{red}} - (n_1)_{\text{red}} - 1$$

$$2[(n_2)_{\text{blue}} - (n_1)_{\text{red}}] = (n_1)_{\text{blue}} - (n_1)_{\text{red}}$$

$$\Delta n_2 = \frac{1}{2}\Delta n_1$$

(c) Simply find the Δn for each type of glass and hope one is twice the other.

$$\Delta n_{\text{crown}} = 1.525 - 1.517 = 0.008$$

$$\Delta n_{\text{flint}} = 1.632 - 1.616 = 0.016$$

Since $\Delta n_{crown} = \frac{1}{2}\Delta n_{\text{flint}}$ then crown glass must be the second material, or the converging lens, while flint glass must be the first material, or the diverging lens.

(d) Solve the original focal length expression for *R*.

$$R = f(2n_2 - n_1 - 1)$$

Since $f_{\text{blue}} = f_{\text{red}}$, it doesn't matter which color we choose for the *n*'s (as long as we are consistent). Say we pick blue, so $n_1 = 1.632$ and $n_2 = 1.525$. We are given $f = 10.0$ cm.

$$R = (10.0\text{ cm})[2(1.525) - 1.632 - 1] = 4.18\text{ cm}$$

Assess: The answers to the various parts fit together and the final result is reasonable.

35.50. Visualize: The effective focal length is defined as the distance from the midpoint between the two lenses to the point that initially parallel rays come to a focus. Because $d < f$ the image from the first lens is to the right of the second lens, so we will use a negative object distance when we analyze the second lens.

Solve: (a) If the original object distance is very large $(s_1 \approx \infty)$ then $s_1' = f$. This image is to the right of the right lens by an amount $f - d$, but since we are treating this as a negative object distance we will put in $s_2 = -(f - d) = d - f$. The thin-lens equation for the second lens (the diverging one, with a negative focal length) becomes:

$$\frac{1}{d - f} + \frac{1}{s_2'} = \frac{1}{-f}$$

Solve for s_2'.

$$s_2' = \frac{-f(d-f)}{(d-f)-(-f)} = \frac{f_2 - fd}{d}$$

This is the distance of the focus to the right of the second lens, however, we want the distance from the midpoint between the lenses, so we add $\frac{1}{2}d$ to the answer.

$$f_{\text{eff}} = s_2' + \frac{1}{2}d = \frac{f^2 - fd}{d} + \frac{1}{2}d = \frac{f_2 - fd}{d} + \frac{\frac{1}{2}d^2}{d} = \frac{f^2 - fd + \frac{1}{2}d^2}{d}$$

(b) We'll plug $d = \frac{1}{2}f$ and $d = \frac{1}{4}f$ in turn into the previous result. Then Equation 35.1 shows that if we take the ratios of the resulting f_{eff} 's we'll have the zoom.

$$\text{zoom} = \frac{(f_{\text{eff}})_{d=1/4f}}{(f_{\text{eff}})_{d=1/2f}} = \frac{\left(\dfrac{f^2 - f\left(\frac{1}{4}f\right) + \frac{1}{2}\left(\frac{1}{4}f\right)^2}{\frac{1}{4}f}\right)}{\left(\dfrac{f^2 - f\left(\frac{1}{2}f\right) + \frac{1}{2}\left(\frac{1}{2}f\right)^2}{\frac{1}{2}f}\right)} = \left(\frac{\frac{1}{2}}{\frac{1}{4}}\right)\frac{f^2 - \frac{1}{4}f^2 + \frac{1}{32}f^2}{f^2 - \frac{1}{2}f^2 + \frac{1}{8}f^2}$$

Cancel f^2 from each term.

$$\left(\frac{4}{2}\right)\frac{1 - \frac{1}{4} + \frac{1}{32}}{1 - \frac{1}{2} + \frac{1}{8}} = (2)\frac{\frac{25}{32}}{\frac{5}{8}} - (2)\frac{5}{4} = \frac{5}{2} = 2.5$$

So the lens is a $2.5\times$ zoom lens.

Assess: This is a reasonable amount of zoom. The magnification spans a factor of 2.5.