16

TRAVELING WAVES

Conceptual Questions

16.1. $v_a = v_b = v_c$. Wave speed is independent of wave amplitude.

16.2. **(a)** $v = \sqrt{\dfrac{T}{\mu}}$ $\quad v' = \sqrt{\dfrac{2T}{\mu}} = \sqrt{2}v$ $\quad v' = 280$ cm/s **(b)** $v' = \sqrt{\dfrac{T}{4\mu}} = \dfrac{v}{2}$ $\quad v' = 100$ cm/s

(c) $v' = \sqrt{\dfrac{T}{\mu/4}} = 2v$ $\quad v' = 400$ cm/s **(d)** $\mu' = \dfrac{4m}{4L} = \mu$ so the speed is unchanged: $v' = 200$ cm/s.

16.3. The constant 1 mm displacement interval appears at later times. Therefore, the displacement at this point reached 2 mm *before* it settled at 1 mm.

16.4. **(a)** The wave is traveling to the right. It reaches the 2 cm point at later times, 7 starting from the left. **(b)** 200 cm/s. The leading edge reaches the 2 cm point at $t = 0.03$ s. At $t = 0.01$ s it was 4 cm to the left, at -2 cm.

$$v = \frac{\Delta x}{\Delta t} = \frac{4 \text{ cm}}{(0.03 \text{ s} - 0.01 \text{ s})} = 200 \text{ cm/s}$$

16.5. $\lambda_a > \lambda_b > \lambda_c$ because $v = \lambda f$ = constant for these frequencies, so large f implies small λ.

16.6. $\lambda_1 = 2\lambda_0$ $\ f_1 = f_0$. The frequency is unchanged because that is the frequency of the driving force that moves successive oscillators. But $v = \lambda f$, so if the speed is doubled, so is the wavelength.

16.7. The amplitude of the wave is the maximum displacement, which is 4.0 cm. The wavelength is the distance between two consecutive peaks, which gives $\lambda = 14 \text{ m} - 2 \text{ m} = 12$ m. The frequency of the wave is

$$f = \frac{v}{\lambda} = \frac{24 \text{ m/s}}{12 \text{ m}} = 2.0 \text{ Hz}$$

We solve for ϕ_0 from the initial conditions at $x = 0$ m and $t = 0$ s: $2 = 4\sin(0 + \phi_0) \Rightarrow \dfrac{1}{2} = \sin(\phi_0) \Rightarrow \phi_0 = \dfrac{\pi}{6}$.

16.8. The amplitude of the wave is the maximum displacement, which is 1.0 cm. The wavelength is the distance between two consecutive peaks, which gives $\lambda = 2.0$ m. The frequency of the wave is

$$f = \frac{v}{\lambda} = \frac{1.0 \text{ m/s}}{2.0 \text{ m}} = 0.50 \text{ Hz}$$

We solve for ϕ_0 from the initial conditions at $x = 0$ m and $t = 1$ s: $1 = 1\sin\left(2\pi\left(0 - \frac{1}{2}\right) + \phi_0\right) \Rightarrow \frac{\pi}{2} = \frac{-2\pi}{2} + \phi_0 \Rightarrow$

$\phi_0 = -\frac{\pi}{2}$.

16.9. **(a)** 0; they are on the same wave front. **(b)** 4π rad; because they are two wave crests apart. **(c)** π rad; because F is on a crest and E on an adjacent trough.

16.10. $P_C = P_B > P_A$. Use $P = \frac{E}{\Delta t}$ in each case.

$$P_A = \frac{2 \text{ J}}{2 \text{ s}} = 1 \text{ W} \qquad P_B = \frac{10 \text{ J}}{5 \text{ s}} = 2 \text{ W} \qquad P_C = \frac{2 \text{ J}}{1 \text{ s}} = 2 \text{ W}$$

16.11. For one professor $I = (1.0\times10^{-12} \text{ W/m}^2)\times10^{5.2} = 1.5849\times10^{-7} \text{ W/m}^2$; for 100 professors $I = 1.5849\times 10^{-5} \text{ W/m}^2$. The new sound intensity level is

$$\beta = (10 \text{ dB})\log_{10}\left(\frac{1.5849\times10^{-5} \text{ W/m}^2}{1.0\times10^{-12} \text{ W/m}^2}\right) = 72 \text{ dB}$$

Notice that for 10 times the intensity, the sound intensity level goes up by 10 dB; for 100 times the intensity, the sound intensity level goes up by 20 dB.

16.12. The correct answer is D because the initial shift is to a lower frequency, which means the source is initially moving away from you until $t = 2$ s. Then the shift is to a higher frequency, which means the source is moving toward you.

Exercises and Problems

Exercises

Section 16.1 An Introduction to Waves

16.1. Model: The wave is a traveling wave on a stretched string.

Solve: The wave speed on a stretched string with linear density μ is $v_{\text{string}} = \sqrt{T_S/\mu}$. The wave speed if the tension is doubled will be

$$v'_{\text{string}} = \sqrt{\frac{T_S}{2\mu}} = \frac{1}{\sqrt{2}}v_{\text{string}} = \frac{1}{\sqrt{2}}(200 \text{ m/s}) = 141 \text{ m/s}.$$

16.2. Model: The wave is a traveling wave on a stretched string.

Solve: The wave speed on a stretched string with linear density μ is

$$v_{\text{string}} = \sqrt{\frac{T_S}{\mu}} \Rightarrow 150 \text{ m/s} = \sqrt{\frac{75 \text{ N}}{\mu}} \Rightarrow \mu = 3.333\times10^{-3} \text{ kg/m}$$

For a wave speed of 180 m/s, the required tension will be

$$T_S = \mu v_{\text{string}}^2 = (3.333\times10^{-3} \text{ kg/m})(180 \text{ m/s})^2 = 110 \text{ N}$$

16.3. Solve:

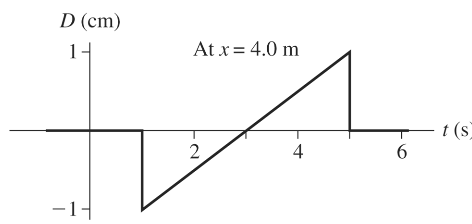
$$L = v\Delta t = \sqrt{\frac{T_S}{\mu}}\,\Delta t = \sqrt{\frac{T_S}{m/L}}\,\Delta t = \sqrt{\frac{T_S L}{m}}\,\Delta t \Rightarrow \sqrt{L} = \sqrt{\frac{T_S}{m}}\,\Delta t \Rightarrow$$

$$L = \frac{T_S}{m}(\Delta t)^2 = \frac{20\text{ N}}{0.025\text{ kg}}(50\text{ ms})^2 = 2.0\text{ m}$$

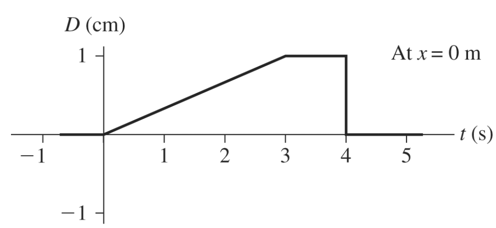
Assess: 2.0 m seems like a reasonable length for a string.

Section 16.2 One-Dimensional Waves

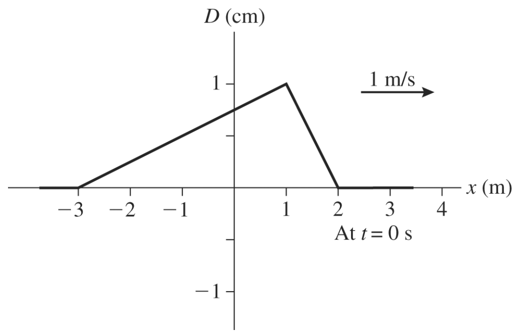
16.4. Model: This is a wave traveling at constant speed. The pulse moves 1 m to the right every second.
Visualize: The snapshot graph shows the wave at all points on the x-axis at $t = 2$ s. We can directly read from the snapshot graph that at $t = 2$ s the displacement at $x = 4.0$ m is –1/2. The leading edge of the wave—moving right—reached $x = 4.0$ m 1 s earlier, at $t = 1$ s. The first part of the wave causes a sudden upward displacement at $t = 1$ s. The falling slope of the wave is 4 m wide, so it will take 4 s for the displacement at $x = 4.0$ m to increase from -1 cm to $+1$ cm. The trailing edge of the pulse arrives at $x = 4.0$ m at $t = 5$ s, which is 3 s after the figure given in the problem. The displacement now becomes zero and stays zero for all later times.
Solve:



16.5. Model: This is a wave traveling at constant speed. The pulse moves 1 m to the left every second.
Visualize: The snapshot graph shows the wave at all points on the x-axis at $t = 0$ s. The leading edge of the wave reaches $x = 0$ m right at 0 s as shown in the snapshot graph. The first part of the wave causes a sudden upward displacement at $t = 1$ s. The flat top of the wave is 3 m wide, so it will take 3 s to pass the $x = 5.0$ m point, keeping this point high until $t = 4$ s. The falling slope is 1 m wide, so it will take 1 s for the displacement at $x = 5.0$ m to drop back to zero. The trailing edge of the pulse arrives at $x = 5.0$ m at $t = 1$ s, which is 5 s after the figure given in the problem. The displacement now becomes zero and stays zero for all later times

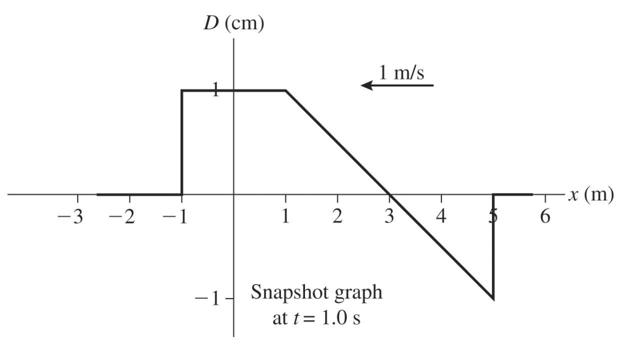


16.6. Model: This is a wave traveling to the right at a constant speed of 1 m/s.
Visualize: This is the history graph of a wave at $x = 2$ m. At $t = 0$ s, the time for which we're to draw the snapshot graph, the displacement at $x = 2$ m is 0 cm but is just beginning to rise. That is, the leading edge of a right-moving pulse is just reaching $x = 2$ m at $t = 0$ s. The trailing edge reaches $x = 2$ m at $t = 4$ s. That's 4 s after our snapshot graph, so at $t = 0$ s the trailing edge must still be 4 m left of $x = 2$ m. Thus at $t = 0$ s the wave will stretch from $x = -2$ m to $x = 2$ m.

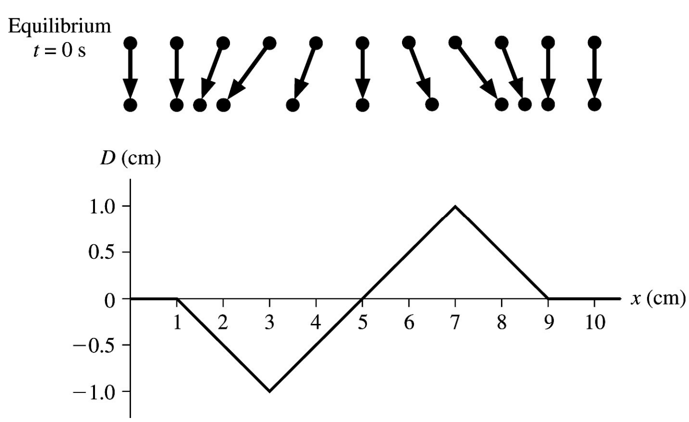


16.7. Model: This is a wave traveling to the left at a constant speed of 1 m/s.

Visualize: This is the history graph of a wave at $x = 0$ m. At $t = 1$ s, the time for which we're to draw the snapshot graph, the displacement at $x = 0$ m is $+1$ cm. The graph shows that the $x = 0$ m point first "went high" at $t = 0$ s, and this leading edge will have moved 1 m to the left by the time of our snapshot graph at $t = 1$ s. The trailing edge reaches $x = 0$ m at $t = 6$ s. That's 5 s after our snapshot graph, so at $t = 1$ s the trailing edge must still be 5 m right of $x = 0$ m. Thus at $t = 1$ s the wave will stretch from $x = -1$ to $x = +5$ m.



16.8. Visualize:



We first draw the particles of the medium in the equilibrium positions, with an inter-particle spacing of 1.0 cm. Just underneath, the positions of the particles as a longitudinal wave is passing through are shown at time $t = 0$ s. It is clear that relative to the equilibrium the particle positions are displaced negatively on the left side and positively on the right side. For example, the particles at $x = 0$ cm and $x = 1$ cm are at equilibrium, the particle at $x = 2$ cm is displaced left by 0.5 cm, the particle at $x = 3$ cm is displaced left by 1.0 cm, the particle at $x = 4$ cm is displaced left by 0.5 cm, and the particle at . $x = 5$ cm is undisplaced. The behavior of particles for $x > 5$ cm is opposite of that for $x < 5$ cm.

16.9. Visualize:

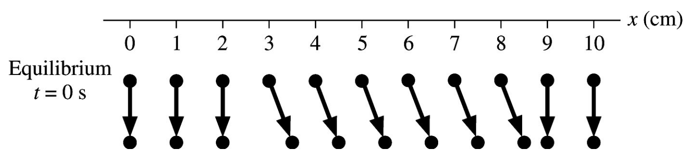


Figure EX16.9 shows a snapshot graph at $t = 0$ s of a longitudinal wave. This diagram shows a row of particles with an inter-particle separation of 1.0 cm at equilibrium. Because the longitudinal wave has a positive amplitude of 0.5cm between $x = 3$ cm and $x = 8$ cm, the particles at $x = 3, 4, 5, 6, 7,$ and 8 cm are displaced to the right by 0.5 cm.

Section 16.3 Sinusoidal Waves

16.10. Solve: (a) The wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2.0 \text{ m}} = 3.1 \text{ rad/m}$$

(b) The wave speed is

16.11. Solve: (a) The wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.5 \text{ rad/m}} = 4.2 \text{ m}$$

(b) The frequency is

$$f = \frac{v}{\lambda} = \frac{200 \text{ m/s}}{4.19 \text{ m}} = 48 \text{ Hz}$$

16.12. Model: The wave is a traveling wave.

Solve: (a) A comparison of the wave equation with Equation 16.14 yields: $A = 5.2$ cm, $k = 5.5$ rad/m, $\omega =$ 72 rad/s, and $\phi_0 = 0$ rad. The frequency is

$$f = \frac{\omega}{2\pi} = \frac{72 \text{ rad/s}}{2\pi} = 11.5 \text{ Hz} \approx 11 \text{ Hz}$$

(b) The wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{5.5 \text{ rad/m}} = 1.14 \text{ m} \approx 1.1 \text{ m}$$

(c) The wave speed $v = \lambda f = 13$ m/s.

16.13. Model: The wave is a traveling wave.

Solve: (a) A comparison of the wave equation with Equation 16.14 yields: $A = 3.5$ cm, $k = 2.7$ rad/m, $\omega = 124$ rad/s, and $\phi_0 = 0$ rad. The frequency is

$$f = \frac{\omega}{2\pi} = \frac{124 \text{ rad/s}}{2\pi} = 19.7 \text{ Hz} \approx 20 \text{ Hz}$$

(b) The wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2.7 \text{ rad/m}} = 2.33 \text{ m} \approx 2.3 \text{ m}$$

(c) The wave speed $v = \lambda f = 46$ m/s.

16.14. Solve: The amplitude of the wave is the maximum displacement, which is 6.0 cm. The period of the wave is 0.60 s, so the frequency $f = 1/T = 1/0.60 \text{ s} = 1.67$ Hz. The wavelength is

$$\lambda = \frac{v}{f} = \frac{2 \text{ m/s}}{1.667 \text{ Hz}} = 1.2 \text{ m}$$

Section 16.4 The Wave Equation on a String

16.15. Solve: Take the appropriate partial derivatives of $D(x,t) = cx^2 + dt^2$.

$$\frac{\partial D}{\partial x} = 2cx \qquad \frac{\partial D}{\partial t} = 2dt$$
$$\frac{\partial^2 D}{\partial x^2} = 2c \qquad \frac{\partial^2 D}{\partial t^2} = 2d$$

Plug the derivatives into the wave equation: $\dfrac{\partial^2 D}{\partial t^2} = v^2 \dfrac{\partial^2 D}{\partial x^2}$

$$2d = v^2 2c$$

This is satisfied if $v = \sqrt{d/c}$.

16.16. Solve: Take the appropriate partial derivatives of $D(x,t) = \ln(ax + bt)$.

$$\frac{\partial D}{\partial x} = a(ax+bt)^{-1} \qquad \frac{\partial D}{\partial t} = b(ax+bt)^{-1}$$
$$\frac{\partial^2 D}{\partial x^2} = -a^2(ax+bt)^{-2} \qquad \frac{\partial^2 D}{\partial t^2} = -b^2(ax+bt)^{-2}$$

Plug the derivatives into the wave equation: $\dfrac{\partial^2 D}{\partial t^2} = v^2 \dfrac{\partial^2 D}{\partial x^2}$

$$-b^2(ax+bt)^{-2} = v^2[-a^2(ax+bt)^{-2}]$$

Cancel the quantity in parentheses and the minus sign and solve for v.

$$b^2 = v^2 a^2 \Rightarrow v = b/a.$$

Section 16.5 Sound and Light

16.17. Solve: (a) In aluminum, the speed of sound is 6420 m/s. The wavelength is thus equal to

$$\lambda = \frac{v}{f} = \frac{6420 \text{ m/s}}{2.0 \times 10^6 \text{ Hz}} = 3.21 \times 10^{-3} \text{ m} = 3.21 \text{ mm} \approx 3.2 \text{ mm}$$

(b) The speed of an electromagnetic wave is c. The frequency would be

$$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{3.21 \times 10^{-3} \text{ m}} = 9.3 \times 10^{10} \text{ Hz}$$

16.18. Solve: (a) The frequency is

$$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{0.20 \text{ m}} = 1.5 \times 10^9 \text{ Hz} = 1.5 \text{ GHz}$$

(b) The speed of a sound wave in water is $v_{\text{water}} = 1480$ m/s. The wavelength of the sound wave would be

$$\lambda = \frac{v_{\text{water}}}{f} = \frac{1480 \text{ m/s}}{1.50 \times 10^9 \text{ Hz}} = 9.87 \times 10^{-7} \text{ m} \approx 990 \text{ nm}$$

16.19. Model: Light is an electromagnetic wave that travels with a speed of 3×10^8 m/s.

Solve: (a) The frequency of the blue light is

$$f_{\text{blue}} = \frac{c}{\lambda} = \frac{3.0\times10^8\ \text{m/s}}{450\times10^{-9}\ \text{m}} = 6.67\times10^{14}\ \text{Hz}$$

(b) The frequency of the red light is

$$f_{\text{red}} = \frac{3.0\times10^8\ \text{m/s}}{650\times10^{-9}\ \text{m}} = 4.62\times10^{14}\ \text{Hz}$$

(c) Calculate the index of refraction,

$$\lambda_{\text{material}} = \frac{\lambda_{\text{vacuum}}}{n} \Rightarrow n = \frac{\lambda_{\text{vacuum}}}{\lambda_{\text{material}}} = \frac{650\ \text{nm}}{450\ \text{nm}} = 1.44$$

16.20. Model: Radio waves are electromagnetic waves that travel with speed *c*.
Solve: (a) The wavelength is

$$\lambda = \frac{c}{f} = \frac{3.0\times10^8\ \text{m/s}}{101.3\ \text{MHz}} = 2.96\ \text{m}$$

(b) The speed of sound in air at $20°\text{C}$ is 343 m/s. The frequency is

$$f = \frac{v_{\text{sound}}}{\lambda} = \frac{343\ \text{m/s}}{2.96\ \text{m}} = 116\ \text{Hz}$$

16.21. Model: Microwaves are electromagnetic waves that travel with a speed of 3×10^8 m/s.

Solve: (a) The frequency of the microwave is

$$f_{\text{microwaves}} = \frac{c}{\lambda} = \frac{3.0\times10^8\ \text{m/s}}{3.0\times10^{-2}\ \text{m}} = 1.0\times10^{10}\ \text{Hz} = 10\ \text{GHz}$$

(b) The refractive index of air is 1.0003, so the speed of microwaves in air is $v_{\text{air}} = c/1.00 \approx \text{c}$. The time for the microwave signal to travel is

$$t = \frac{50\ \text{km}}{v_{\text{air}}} = \frac{50\times10^3\ \text{m}}{(3.0\times10^8\ \text{m}/1.00)} = 0.167\ \text{ms} \approx 0.17\ \text{ms}$$

Assess: A small time of 0.17 ms for the microwaves to cover a distance of 50 km shows that the electromagnetic waves travel very fast.

16.22. Solve: Two pulses of sound are detected because one pulse travels through the metal to the microphone while the other travels through the air to the microphone. The time interval for the sound pulse traveling through the air is

$$\Delta t_{\text{air}} = \frac{\Delta x}{v_{\text{air}}} = \frac{4.00\ \text{m}}{343\ \text{m/s}} = 0.01166\ \text{s} = 11.66\ \text{ms}$$

Sound travels *faster* through solids than gases, so the pulse traveling through the metal will reach the microphone *before* the pulse traveling through the air. Because the pulses are separated in time by 9.00 ms, the pulse traveling through the metal takes $\Delta t_{\text{metal}} = 2.66$ ms to travel the 4.00 m to the microphone. Thus, the speed of sound in the metal is

$$v_{\text{metal}} = \frac{\Delta x}{\Delta t_{\text{metal}}} = \frac{4.00\ \text{m}}{0.00266\ \text{s}} = 1504\ \text{m/s} \approx 1500\ \text{m/s}$$

16.23. Model: Assume that the glass has index of refraction $n = 1.5$. This means that $v_{\text{glass}} = c/n = 2\times10^8$ m/s.

Visualize: We apply $v = \lambda f$ twice, once in air and then in the glass. The frequency will be the same in both cases.

Solve: **(a)** In the air

$$f_{\text{air}} = \frac{v_{\text{air}}}{\lambda_{\text{air}}} = \frac{3.0\times10^8\ \text{m/s}}{0.35\ \text{m}} = 8.57\times10^8\ \text{Hz} \approx 8.6\times10^8\ \text{Hz}$$

The frequency is the same in both media, so $f_{\text{glass}} = 8.6\times10^8$ Hz.

(b) Now that we know f_{glass} and v_{glass}, we can find λ_{glass}.

$$\lambda_{\text{glass}} = \frac{v_{\text{glass}}}{f_{\text{glass}}} = \frac{2.0\times10^8\ \text{m/s}}{8.57\times10^8\ \text{Hz}} = 23\ \text{cm}$$

Assess: We get the same answer from $\lambda_{\text{glass}} = \lambda_{\text{air}}/n_{\text{glass}} = 35\ \text{cm}/1.5 = 23\ \text{cm}$.

16.24. Model: Light is an electromagnetic wave.
Solve: **(a)** The time light takes is

$$t = \frac{3.0\ \text{mm}}{v_{\text{glass}}} = \frac{3.0\times10^{-3}\ \text{m}}{c/n} = \frac{3.0\times10^{-3}\ \text{m}}{(3.0\times10^8\ \text{m/s})/1.50} = 1.5\times10^{-11}\ \text{s}$$

(b) The thickness of water is

$$d = v_{\text{water}}t = \frac{c}{n_{\text{water}}}t = \frac{3.0\times10^8\ \text{m/s}}{1.33}(1.5\times10^{-11}\ \text{s}) = 3.4\ \text{mm}$$

16.25. Solve: **(a)** The speed of light in a material is given by Equation 16.37:

$$n = \frac{c}{v_{\text{mat}}} \Rightarrow v_{\text{mat}} = \frac{c}{n}$$

The refractive index is

$$n = \frac{\lambda_{\text{vac}}}{\lambda_{\text{mat}}} \Rightarrow v_{\text{solid}} = c\frac{\lambda_{\text{solid}}}{\lambda_{\text{vac}}} = (3.0\times10^8\ \text{m/s})\frac{420\ \text{nm}}{670\ \text{nm}} = 1.88\times10^8\ \text{m/s}$$

(b) The frequency is

$$f = \frac{v_{\text{solid}}}{\lambda_{\text{solid}}} = \frac{1.88\times10^8\ \text{m/s}}{420\ \text{nm}} = 4.48\times10^{14}\ \text{Hz}$$

16.26. Model: $v = \lambda f$ applies.

Solve: **(a)** The frequency must remain the same since the harmonic oscillators in one medium excite the oscillators in the second medium. So $f_{\text{water}} = 440$ Hz.

(b) The table in the chapter gives $v_{\text{water}} = 1480$ m/s.

$$\lambda = \frac{v}{f} = \frac{1480\ \text{m/s}}{440\ \text{Hz}} = 3.4\ \text{m}$$

Assess: This is a reasonable wavelength for a sound wave.

Section 16.6 The Wave Equation in a Fluid

16.27. Model: We can use the general expression for the speed of sound in air.
Visualize: Convert the Fahrenheit temperatures to Celsius: $-25°\text{F} = -31.7°\text{C}$, $125°\text{F} = 51.7°\text{C}$.
Solve: **(a)**

$$v_{\text{sound in air}} = 331\ \text{m/s} \times \sqrt{\frac{T(°\text{C}) + 273}{273}} = 331\ \text{m/s} \times \sqrt{\frac{-32.7°\text{C} + 273}{273}} = 311\ \text{m/s}$$

(b)

$$v_{\text{sound in air}} = 331 \text{ m/s} \times \sqrt{\frac{T(°\text{C}) + 273}{273}} = 331 \text{ m/s} \times \sqrt{\frac{51.7°\text{C} + 273}{273}} = 361 \text{ m/s}$$

16.28. Model: We can use the general expression for the speed of sound in a fluid.

Visualize: Look up the bulk modulus of mercury at $20°\text{C}$: $B_{\text{Hg}} = 2.85\times10^{10}$ Pa.

Solve:

$$v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.85\times10^{10} \text{ Pa}}{13{,}600 \text{ kg/m}^3}} = 1450 \text{ m/s}$$

Assess: This speed is greater than the speed of sound in air, but we expected it to be because of the large *B*.

Section 16.7 Waves in Two and Three Dimensions

16.29. Solve: According to Equation 16.56, the phase difference between two points on a wave is

$$\Delta\phi = \phi_2 - \phi_1 = 2\pi\frac{\Delta r}{\lambda} = \frac{2\pi}{\lambda}(r_2 - r_1) \Rightarrow (3\pi \text{ rad} - 0 \text{ rad}) = \frac{2\pi}{\lambda}(80 \text{ cm} - 20 \text{ cm}) \Rightarrow \lambda = 40 \text{ cm}$$

16.30. Solve: According to Equation 16.56, the phase difference between two points on a wave is

$$\Delta\phi = \phi_2 - \phi_1 = \frac{2\pi}{\lambda}(r_2 - r_1)$$

If $\phi_1 = \pi$ rad at $r_1 = 4.0$ m, we can determine ϕ_2 at any *r* value at the same instant using this equation. At $r_2 = 3.5$ m,

$$\phi_2 = \phi_1 + \frac{2\pi}{\lambda}(r_2 - r_1) = \pi \text{ rad} + \frac{2\pi}{2.0 \text{ m}}(3.5 \text{ m} - 4.0 \text{ m}) = \frac{\pi}{2} \text{ rad}$$

At $r_2 = 4.5$ m, $\phi = \frac{3}{2}\pi$ rad.

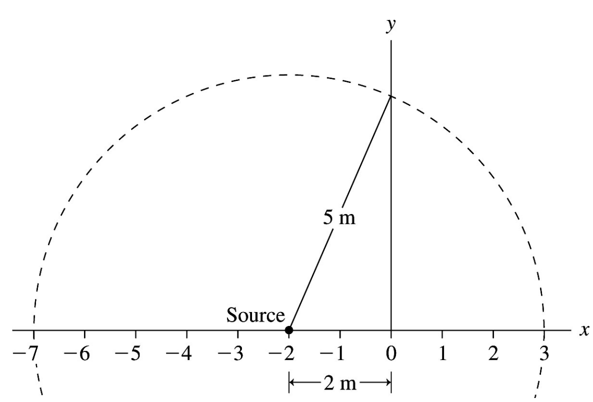
16.31. Visualize: A phase difference of 2π rad corresponds to a distance of λ. Set up a ratio.

Solve:

$$\frac{x}{\lambda} = \frac{5.5 \text{ rad}}{2\pi \text{ rad}} \Rightarrow x = \left(\frac{5.5 \text{ rad}}{2\pi \text{ rad}}\right)\lambda = \left(\frac{5.5 \text{ rad}}{2\pi \text{ rad}}\right)\frac{v}{f} = \left(\frac{5.5 \text{ rad}}{2\pi \text{ rad}}\right)\frac{340 \text{ m/s}}{120 \text{ Hz}} = 2.5 \text{ m}$$

Assess: 2.5 m seems like a reasonable distance.

16.32. Visualize:



Solve: **(a)** Because the same wavefront simultaneously reaches listeners at $x = -7.0$ m and $x = +3.0$ m,

$$\Delta\phi = 0 \text{ rad} = \frac{2\pi}{\lambda}(r_2 - r_1) \Rightarrow r_2 = r_1$$

Thus, the source is at $x = -2.0$ m, so that it is equidistant from the two listeners.

(b) The third person is also 5.0 m away from the source. Her y-coordinate is thus $y = \sqrt{(5 \text{ m})^2 - (2 \text{ m})^2} = 4.6$ m.

Section 16.8 Power, Intensity, and Decibels

16.33. Solve: The energy delivered to the eardrum in time t is $E = Pt$, where P is the power of the wave. The intensity of the wave is $I = P/a$ where a is the area of the ear drum. Putting the above information together, we have

$$E = Pt = (Ia)t = I\pi r^2 t = (2.0\times10^{-3} \text{ W/m}^2)\pi(3.0\times10^{-3} \text{ m})^2(60 \text{ s}) = 3.4\times10^{-6} \text{ J}$$

16.34. Solve: The energy delivered to an area a in time t is $E = Pt$, where the power P is related to the intensity I as $I = P/a$. Thus, the energy received by your back is

$$E = Pt = Iat = (0.80)(1400 \text{ W/m}^2)(0.30\times50 \text{ m}^2)(3600 \text{ s}) = 6.0\times10^5 \text{ J}$$

16.35. Solve: **(a)** The intensity of a uniform spherical source of power P_{source} a distance r away is $I = P_{\text{source}}/4\pi r^2$. Thus, the intensity at the position of the microphone is

$$I_{50 \text{ m}} = \frac{35 \text{ W}}{4\pi(50 \text{ m})^2} = 1.1\times10^{-3} \text{ W/m}^2$$

(b) The sound energy impinging on the microphone per second is

$$P = Ia = (1.1\times10^{-3} \text{ W/m}^2)(1.0\times10^{-4} \text{ m}^2) = 1.1\times10^{-7} \text{ W} = 1.1\times10^{-7} \text{ J/s}$$

$$\Rightarrow \text{Energy impinging on the microphone in 1 second} = 1.1\times10^{-7} \text{ J}$$

16.36. Model: Assume the intensity scales inversely with the square of the distance.

Visualize: $\beta = (10 \text{ dB})\log_{10}\left(\frac{I}{I_0}\right)$.

Solve: The intensity at 1.0 km is $I' = \left(\frac{30 \text{ m}}{1000 \text{ m}}\right)^2 I$.

$$\beta = (10 \text{ dB})\log_{10}\left(\frac{I'}{I_0}\right) = (10 \text{ dB})\log_{10}\left(\frac{\left(\frac{30 \text{ m}}{1000 \text{ m}}\right)^2 I}{I_0}\right)$$

$$= (10 \text{ dB})\log_{10}\left(\frac{30 \text{ m}}{1000 \text{ m}}\right)^2 + (10 \text{ dB})\log_{10}\left(\frac{I}{I_0}\right) = -30 \text{ db} + 140 \text{ dB} = 110 \text{ dB}$$

Assess: 110 dB is still loud, but not as damaging.

16.37. Solve: Because the sun radiates waves uniformly in all directions, the intensity I of the sun's rays when they impinge upon the earth is

$$I = \frac{P_{\text{sun}}}{4\pi r^2} \Rightarrow I_{\text{earth}} = \frac{P_{\text{sun}}}{4\pi r_{\text{earth}}^2} = \frac{4\times10^{26} \text{ W}}{4\pi(1.496\times10^{11} \text{ m})^2} = 1400 \text{ W/m}^2$$

With $r_{\text{sun-Venus}} = 1.082\times10^{11}$ m and $r_{\text{sun-Mars}} = 2.279\times10^{11}$ m, the intensities of electromagnetic waves at these planets are $I_{\text{venus}} = 2700 \text{ W/m}^2$ and $I_{\text{Mars}} = 610 \text{ W/m}^2$.

16.38. Visualize: Equation 16.61 gives the sound intensity level as

$$\beta = (10\text{ dB})\log_{10}\left(\frac{I}{I_0}\right)$$

where $I_0 = 1.0\times10^{-12}\ \text{W/m}^2$.

Solve:

(a)

$$\beta = (10\text{ dB})\log_{10}\left(\frac{I}{I_0}\right) = (10\text{ dB})\log_{10}\left(\frac{3.0\times10^{-6}\ \text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2}\right) = 65\text{ dB}$$

(b)

$$\beta = (10\text{ dB})\log_{10}\left(\frac{I}{I_0}\right) = (10\text{ dB})\log_{10}\left(\frac{3.0\times10^{-2}\ \text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2}\right) = 105\text{ dB}$$

Assess: As mentioned in the chapter, each factor of 10 in intensity changes the sound intensity level by 10 dB; between the first and second parts of this problem the intensity changed by a factor of 10^4, so we expect the sound intensity level to change by 40 dB.

16.39. Model: Assume the pole is tall enough that we don't have to worry about the ground absorbing or reflecting sound.

Visualize: The area of a sphere of radius R is $A = 4\pi R^2$. Also, $I = P/A$. We seek P when $R = 20$ m.

Solve:

$$P = IA = (I_0\times10^{\beta/10\text{ dB}})(4\pi R^2) = (I_0\times10^{90\text{ dB}/10\text{ dB}})(4\pi(20\text{ m})^2) = 5.0\text{ W}$$

Assess: 5.0 W is a reasonable power output for a speaker.

16.40. Model: Assume the saw is far enough off the ground that we don't have to worry about reflected sound.

Visualize: First note that $\beta_1 - \beta_2 = 20\text{dB} \Rightarrow I_1/I_2 = 10\cdot10 = 100$ (a change of 10 dB corresponds to a change in intensity by a factor of 10). Then use $I_1A_1 = P$ and then $P = I_2A_2 \Rightarrow A_2 = P/I_2$, and finally solve for $R_2 = \sqrt{A_2/4\pi}$.

Solve: Put all of the above together.

$$R_2 = \sqrt{\frac{A_2}{4\pi}} = \sqrt{\frac{\frac{P}{I_2}}{4\pi}} = \sqrt{\frac{\frac{I_1A_1}{I_2}}{4\pi}} = \sqrt{\frac{\frac{I_1}{I_2}(4\pi R_1^2)}{4\pi}} = R_1\sqrt{\frac{I_1}{I_2}} = R_1\sqrt{100} = (5.0\text{ m})(10) = 50\text{ m}$$

Assess: The scaling laws help and the answer is reasonable.

Section 16.9 The Doppler Effect

16.41. Model: Your friend's frequency is altered by the Doppler effect. The frequency of your friend's note increases as he races toward you (moving source and a stationary observer). The frequency of your note for your approaching friend is also higher (stationary source and a moving observer).

Solve: (a) The frequency of your friend's note as heard by you is

$$f_+ = \frac{f_0}{1-\frac{v_S}{v}} = \frac{400\text{ Hz}}{1-\frac{25.0\text{ m/s}}{340\text{ m/s}}} = 432\text{ Hz}$$

(b) The frequency heard by your friend of your note is

$$f_+ = f_0\left(1+\frac{v_0}{v}\right) = (400\text{ Hz})\left(1+\frac{25.0\text{ m/s}}{340\text{ m/s}}\right) = 429\text{ Hz}$$

16.42. Model: The frequency of the opera singer's note is altered by the Doppler effect.
Solve: (a) Using $90\ \text{km/h} = 25\ \text{m/s}$, the frequency as her convertible approaches the stationary person is

$$f_+ = \frac{f_0}{1 - v_S/v} = \frac{600\ \text{Hz}}{1 - \dfrac{25\ \text{m/s}}{343\ \text{m/s}}} = 650\ \text{Hz}$$

(b) The frequency as her convertible recedes from the stationary person is

$$f_- = \frac{f_0}{1 + v_S/v} = \frac{600\ \text{Hz}}{1 + \dfrac{25\ \text{m/s}}{343\ \text{m/s}}} = 560\ \text{Hz}$$

16.43. Model: The bat's chirping frequency is altered by the Doppler effect. The frequency is increased as the bat approaches and it decreases as the bat recedes away.
Solve: The bat must fly away from you, so that the chirp frequency observed by you is less than 25 kHz. From Equation 16.34,

$$f_- = \frac{f_0}{1 + v_S/v} \Rightarrow 20{,}000\ \text{Hz} = \frac{25{,}000\ \text{Hz}}{1 + \left(\dfrac{v_S}{343\ \text{m/s}}\right)} \Rightarrow v_S = 85.8\ \text{m/s} \approx 86\ \text{m/s}$$

Assess: This is a rather large speed: $85.8\ \text{m/s} \approx 180\ \text{mph}$. This is not possible for a bat.

16.44. Model: The mother hawk's frequency is altered by the Doppler effect.
Solve: The frequency is f_+ as the hawk approaches you is

$$f_+ = \frac{f_0}{1 - v_S/v} \Rightarrow 900\ \text{Hz} = \frac{800\ \text{Hz}}{1 - \dfrac{v_S}{343\ \text{m/s}}} \Rightarrow v_S = 38.1\ \text{m/s}$$

Assess: The mother hawk's speed of $38.1\ \text{m/s} \approx 80\ \text{mph}$ is reasonable.
If x is a point just to the right of the origin and is very small, the angle $(2\pi x/\lambda + \phi_0)$ is just slightly bigger than the angle ϕ_0. Now $\sin 31° > \sin 30°$, but $\sin 151° < \sin 150°$, so the value $\phi_0 = \frac{1}{6}\pi$ rad is the phase constant for which the displacement increases as x increases.
(c) The equation for a sinusoidal traveling wave can be written as

$$D(x, t) = A\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t + \phi_0\right) = A\sin\left[2\pi\left(\frac{x}{\lambda} - ft\right) + \phi_0\right]$$

Substituting in the values found above,

$$D(x, t) = (1.0\ \text{mm})\sin\left[2\pi\left(\frac{x}{2.0\ \text{m}} - (5.0\ \text{s}^{-1})t\right) + \frac{\pi}{6}\right]$$

Problems

16.45. Solve: (a) We see from the history graph that the period $T = 0.20$ s and the wave speed $v = 4.0$ m/s. Thus, the wavelength is

$$\lambda = \frac{v}{f} = vT = (4.0\ \text{m/s})(0.20\ \text{s}) = 0.80\ \text{m}$$

(b) The phase constant ϕ_0 is obtained as follows:

$$D(0\ \text{m}, 0\ \text{s}) = A\sin\phi_0 \Rightarrow -2\ \text{mm} = (2\ \text{mm})\sin\phi_0 \Rightarrow \sin\phi_0 = -1 \Rightarrow \phi_0 = -\tfrac{1}{2}\pi\ \text{rad}$$

(c) The displacement equation for the wave is

$$D(x,t) = A\sin\left(\frac{2\pi x}{\lambda} - 2\pi ft + \phi_0\right) = (2.0\text{ mm})\sin\left(\frac{2\pi x}{0.80\text{ m}} - \frac{2\pi t}{0.20\text{ s}} - \frac{\pi}{2}\right) = (2.0\text{ mm})\sin(2.5\pi x - 10\pi t - \tfrac{1}{2}\pi)$$

where x and t are in m and s, respectively.

16.46. Solve: (a) We can see from the graph that the wavelength is $\lambda = 2.0$ m. We are given that the wave's frequency is $f = 5.0$ Hz. Thus, the wave speed is $v = \lambda f = 10$ m/s.
(b) The snapshot graph was made at $t = 0$ s. Reading the graph at $x = 0$ m, we see that the displacement is

$$D(x = 0\text{ m}, t = 0\text{ s}) = D(0\text{ m}, 0\text{ s}) = 0.5\text{ mm} = \tfrac{1}{2}A$$

Thus

$$D(0\text{ m, }0\text{ s}) = \tfrac{1}{2}A = A\sin\phi_0 \Rightarrow \phi_0 = \sin^{-1}\left(\tfrac{1}{2}\right) = \frac{\pi}{6}\text{rad or }\frac{5\pi}{6}\text{rad}$$

Note that the value of D(0 m, 0 s) alone gives two possible values of the phase constant. One of the values will cause the displacement to start at 0.5 mm and increase with distance—as the graph shows—while the other will cause the displacement to start at 0.5 mm but *decrease* with distance. Which is which? The wave equation for $t = 0$ s is

$$D(x, t = 0) = A\sin\left(\frac{2\pi x}{\lambda} + \phi_0\right)$$

16.47. Model: The wave pulse is a traveling wave on a stretched string.
Solve: While the tension T_S is the same in both the strings, the wave speeds in the two strings are not. We have

$$v_1 = \sqrt{\frac{T_S}{\mu_1}} \qquad \text{and} \qquad v_2 = \sqrt{\frac{T_S}{\mu_2}} \Rightarrow v_1^2\mu_1 = v_2^2\mu_2 = T_S$$

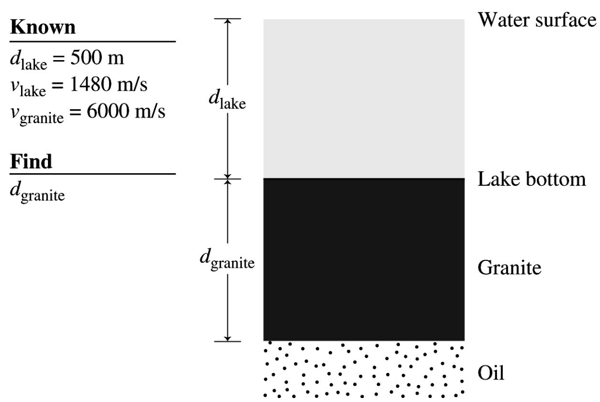
Because $v_1 = L_1/t_1$ and $v_2 = L_2/t_2$, and because the pulses are to reach the ends of the string simultaneously, the above equation can be simplified to

$$\frac{L_1^2\mu_1}{t^2} = \frac{L_2^2\mu_2}{t^2} \Rightarrow \frac{L_1}{L_2} = \sqrt{\frac{\mu_2}{\mu_1}} = \sqrt{\frac{4.0\text{ g/m}}{2.0\text{ g/m}}} = \sqrt{2} \Rightarrow L_1 = \sqrt{2}L_2$$

Since $L_1 + L_2 = 4$ m,

$$\sqrt{2}L_2 + L_2 = 4\text{ m} \Rightarrow L_2 = 1.66\text{ m} \approx 1.7\text{ m} \qquad \text{and} \qquad L_1 = \sqrt{2}(1.66\text{ m}) = 2.34\text{ m} \approx 2.3\text{ m}$$

16.48. Visualize:

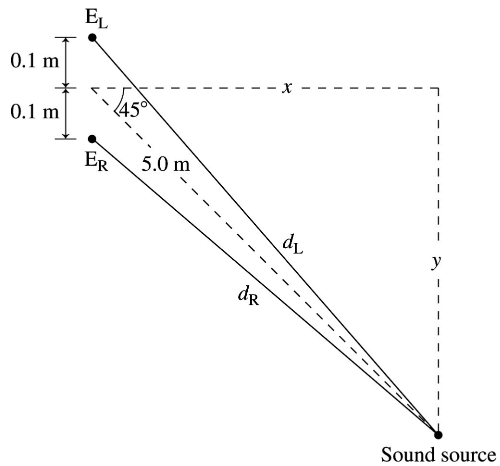


Solve: The explosive's sound travels down the lake and into the granite, and then it is reflected by the oil surface. The echo time is thus equal to

$$t_{\text{echo}} = t_{\text{water down}} + t_{\text{granite down}} + t_{\text{granite up}} + t_{\text{water up}}$$

$$0.94 \text{ s} = \frac{500 \text{ m}}{1480 \text{ m/s}} + \frac{d_{\text{granite}}}{6000 \text{ m/s}} + \frac{d_{\text{granite}}}{6000 \text{ m/s}} + \frac{500 \text{ m}}{1480 \text{ m/s}} \Rightarrow d_{\text{granite}} = 790 \text{ m}$$

16.49. Model: Assume a room temperature of 20°C.
Visualize:



Solve: The distance between the source and the left ear (E_L) is

$$d_\text{L} = \sqrt{x^2 + (y + 0.1 \text{ m})^2} = \sqrt{[(5.0 \text{ m})\cos 45°]^2 + [(5.0 \text{ m})\sin 45° + 0.1 \text{ m}]^2} = 5.0712 \text{ m}$$

Similarly $d_\text{R} = 4.9298$ m. Thus,

$$d_\text{L} - d_\text{R} = \Delta d = 0.1414 \text{ m}$$

For the sound wave with a speed of 343 m/s, the difference in arrival times at your left and right ears is

$$\Delta t = \frac{\Delta d}{343 \text{ m/s}} = \frac{0.1414 \text{ m}}{343 \text{ m/s}} = 410 \ \mu\text{s}$$

16.50. Model: The laser beam is an electromagnetic wave that travels with the speed of light.
Solve: The speed of light in the liquid is

$$v_{\text{liquid}} = \frac{30 \times 10^{-2} \text{ m}}{1.38 \times 10^{-9} \text{ s}} = 2.174 \times 10^8 \text{ m/s}$$

The liquid's index of refraction is

$$n = \frac{c}{v_{\text{liquid}}} = \frac{3.0 \times 10^8}{2.174 \times 10^8} = 1.38$$

Thus the wavelength of the laser beam in the liquid is

$$\lambda_{\text{liquid}} = \frac{\lambda_{\text{vac}}}{n} = \frac{633 \text{ nm}}{1.38} = 459 \text{ nm}$$

16.51. Solve: The difference in the arrival times for the P and S waves is

$$\Delta t = t_\text{S} - t_\text{P} = \frac{d}{v_\text{S}} - \frac{d}{v_\text{P}} \Rightarrow 120 \text{ s} = d\left(\frac{1}{4500 \text{ m/s}} - \frac{1}{8000 \text{ m/s}}\right) \Rightarrow d = 1.23 \times 10^6 \text{ m} = 1230 \text{ km}$$

Assess: d is approximately one-fifth of the radius of the earth and is reasonable.

16.52. **Model:** We can use the general expression for the speed of sound in a fluid.

Visualize: Look up the bulk modulus of helim at $20°\text{C}$ and 1 atm: $B_{\text{He}} = 1.688\times10^5$ Pa.

Solve:

$$v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{B_0}{\rho_0}\frac{T(°\text{C})+273}{273}} = \sqrt{\frac{1.688\times10^5\ \text{Pa}}{0.18\ \text{kg/m}^3}\left(\frac{-268+273}{273}\right)} = 130\ \text{m/s}$$

Assess: The speed of sound in helium at room temperature and pressure is almost 3x the speed of sound in air (which is why people sound like Donald Duck when the talk after breathing helium), but it is much slower at this cold temperature.

16.53. **Model:** We can use the general expression for the speed of sound in a fluid.

Visualize: We can't look up the bulk modulus of the unknown liquid, so we compute it from $B = pV/\Delta V$ **.**

Solve:

$$v_{\text{sound}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\Delta pV}{\rho\Delta V}} = \sqrt{\frac{\Delta pV}{(m/V)\Delta V}} = \sqrt{\frac{\Delta pV^2}{m\Delta V}} = \sqrt{\frac{\Delta p(\pi r^2 L)^2}{m(\pi r^2 L)}} = \sqrt{\frac{\Delta p\pi r^2 L^2}{m\Delta L}}$$

$$= \sqrt{\frac{(41.0\ \text{atm})(101.3\ \text{kPa/1 atm})\pi(5.00\ \text{cm})^2(0.200\ \text{m})^2}{(1.34\ \text{kg})(1.00\ \text{mm})}} = 987\ \text{m/s}$$

Assess: The speed of sound in this liquid is in the ballpark of other sound speeds in liquids.

16.54. **Model:** This is a sinusoidal wave.

Solve: **(a)** The equation is of the form $D(y,t) = A\sin(ky+\omega t+\phi_0)$, so the wave is traveling along the y-axis. Because it is $+\omega t$ rather than $-\omega t$ the wave is traveling in the *negative* y-direction.

(b) Sound is a longitudinal wave, meaning that the medium is displaced *parallel* to the direction of travel. So the air molecules are oscillating back and forth along the y-axis.

(c) The wave number is $k = 8.96\ \text{m}^{-1}$, so the wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{8.96\ \text{m}^{-1}} = 0.701\ \text{m}$$

The angular frequency is $\omega = 3140\ \text{s}^{-1}$, so the wave's frequency is

$$f = \frac{\omega}{2\pi} = \frac{3140\ \text{s}^{-1}}{2\pi} = 500\ \text{Hz}$$

Thus, the wave speed $v = \lambda f$ = (0.70 m)(500 Hz) = 350 m/s. The period $T = 1/f$ = 0.00200 s = 2.00 ms.

Assess: The wave is a sound wave with speed v = 350 m/s. This is greater than the room-temperature speed of 343 m/s, so the air temperature must be greater than $20°$.

16.55. **Model:** This is a sinusoidal wave.

Solve: **(a)** The displacement of a wave traveling in the positive x-direction with wave speed v must be of the form $D(x,t) = D(x-vt)$. Since the variables x and t in the given wave equation appear together as $x + vt$, the wave is traveling toward the left, that is, in the $-x$ direction.

(b) The speed of the wave is

$$v = \frac{\omega}{k} = \frac{2\pi/0.20\ \text{s}}{2\pi\ \text{rad}/2.4\ \text{m}} = 12\ \text{m/s}$$

The frequency is

$$f = \frac{\omega}{2\pi} = \frac{2\pi\ \text{rad}/0.20\ \text{s}}{2\pi} = 5.0\ \text{Hz}$$

The wave number is

$$k = \frac{2\pi \text{ rad}}{2.4 \text{ m}} = 2.6 \text{ rad/m}$$

(c) The displacement is

$$D(0.20 \text{ m}, 0.50 \text{ s}) = (3.0 \text{ cm})\sin\left[2\pi\left(\frac{0.20 \text{ m}}{2.4 \text{ m}} + \frac{0.50 \text{ s}}{0.20 \text{ s}} + 1\right)\right] = -1.5 \text{ cm}$$

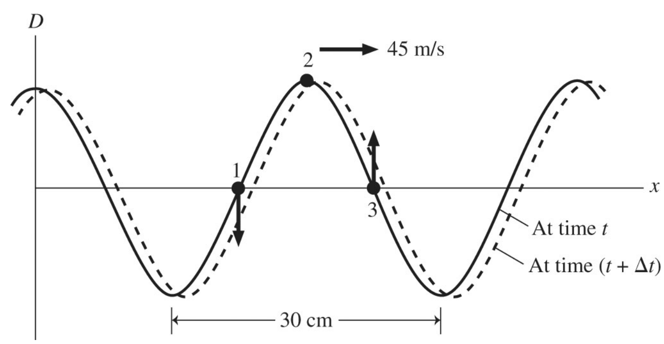
16.56. Model: This is a sinusoidal wave traveling on a stretched string in the $+x$ direction.
Solve: **(a)** From the displacement equation of the wave, $A = 2.0$ cm, $k = 12.57$ rad/m, and $\omega = 638$ rad/s. Using the equation for the wave speed in a stretched string,

$$v_{\text{string}} = \sqrt{\frac{T_S}{\mu}} \Rightarrow T_S = \mu v_{\text{string}}^2 = \mu\left(\frac{\omega}{k}\right)^2 = (5.00 \times 10^{-3} \text{ kg/m}^3)\left(\frac{638 \text{ rad/s}}{12.57 \text{ rad/m}}\right)^2 = 12.6 \text{ N}$$

(b) The maximum displacement is the amplitude $D_{\text{max}}(x, t) = 2.00$ cm.
(c) From Equation 16.17,

$$v_{y \text{ max}} = \omega A = (638 \text{ rad/s})(2.0 \times 10^{-2} \text{ m}) = 12.8 \text{ m/s}$$

16.57. Model: We have a wave traveling to the right on a string.
Visualize:

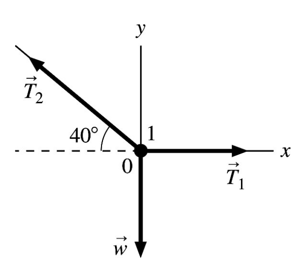


Solve: The snapshot of the wave as it travels to the right for an infinitesimally small time Δt shows that the velocity at point 1 is downward, at point 3 is upward, and at point 2 is zero. Furthermore, the speed at points 1 and 3 is the maximum speed given by Equation 16.17: $v_1 = v_3 = \omega A.$ The frequency of the wave is

$$\omega = 2\pi f = 2\pi\frac{v}{\lambda} = \frac{2\pi(45 \text{ m/s})}{0.30 \text{ m}} = 300\pi \text{ rad/s} \Rightarrow \omega A = (300\pi \text{ rad/s})(2.0 \times 10^{-2} \text{ m}) = 19 \text{ m/s}$$

Thus, $v_1 = -19$ m/s, $v_2 = 0$ m/s, and $v_3 = +19$ m/s.

16.58. Model: The wave pulse is a traveling wave on a stretched string. The two masses hanging from the steel wire are in static equilibrium.
Visualize:



Solve: The wave speed along the wire is

$$v_{\text{wire}} = \frac{4.0 \text{ m}}{0.024 \text{ s}} = 166.7 \text{ m/s}$$

Using Equation 16.2,

$$v_{\text{wire}} = 166.7 \text{ m/s} = \sqrt{\frac{T_1}{\mu}} = \sqrt{\frac{T_1}{(0.060 \text{ kg}/8.0 \text{ m})}} \Rightarrow T_1 = 208.4 \text{ N}$$

Because point 1 is in static equilibrium, with $\vec{F}_{\text{net}} = \vec{0}$,

$$(F_{\text{net}})_x = T_1 - T_2\cos 40° \Rightarrow T_2 = \frac{T_1}{\cos 40°} = 2721 \text{ N}$$

$$(F_{\text{net}})_y = T_2\sin 40° - w = 0 \text{ N} \Rightarrow w = mg = T_2\sin 40° \Rightarrow m = \frac{(272.1 \text{ N})\sin 40°}{9.8 \text{ m/s}^2} = 17.8 \text{ kg}$$

16.59. Solve: The wave speeds along the two metal wires are

$$v_1 = \sqrt{\frac{T}{\mu_1}} = \sqrt{\frac{2250 \text{ N}}{0.009 \text{ kg/m}}} = 500 \text{ m/s} \qquad v_2 = \sqrt{\frac{T}{\mu_2}} = \sqrt{\frac{2250 \text{ N}}{0.025 \text{ kg/m}}} = 300 \text{ m/s}$$

The wavelengths along the two wires are

$$\lambda_1 = \frac{v_1}{f} = \frac{500 \text{ m/s}}{1500 \text{ Hz}} = \frac{1}{3} \text{ m} \qquad \lambda_2 = \frac{v_2}{f} = \frac{300 \text{ m/s}}{1500 \text{ Hz}} = \frac{1}{5} \text{ m}$$

Thus, the number of wavelengths over two sections of the wire are

$$\frac{1.0 \text{ m}}{\lambda_1} = \frac{1.0 \text{ m}}{\left(\frac{1}{3} \text{ m}\right)} = 3 \qquad \frac{1.0 \text{ m}}{\lambda_2} = \frac{1.0 \text{ m}}{\left(\frac{1}{5} \text{ m}\right)} = 5$$

The number of complete cycles of the wave in the 2.00-m-long wire is 8.

16.60. Model: The object is in static equilibrium. μ is the linear density of the string, *not* a coefficient of friction.

Visualize: Use tilted axes with the x-direction along the string. The tension in the string is T_S.

Solve: From a free-body diagram we see that

$$\sum F_x = T_S - Mg\sin\theta = 0 \Rightarrow T_S = Mg\sin\theta$$

$$v = \sqrt{\frac{T_S}{\mu}} = \sqrt{\frac{Mg\sin\theta}{\mu}}$$

16.61. Model: The wave is traveling on a stretched string.
Solve: The wave speed on the string is

$$v = \sqrt{\frac{T_S}{\mu}} = \sqrt{\frac{50 \text{ N}}{0.005 \text{ kg/m}}} = 100 \text{ m/s}$$

The speed of the particle on the string, however, is given by Equation 16.17. The maximum speed is calculated as follows:

$$v_y = -\omega A\cos(kx - \omega t + \phi_0) \Rightarrow v_{y\text{ max}} = \omega A = 2\pi fA = 2\pi\frac{v}{\lambda}A = 2\pi\left(\frac{100 \text{ m/s}}{2.0 \text{ m}}\right)(0.030 \text{ m}) = 9.4 \text{ m/s}$$

16.62. Model: As the guitar string is stretched its linear density will decrease, but only slightly, so we will assume that $\mu = 1.3$ g/m will apply through the problem.

Visualize: The fact that we have a length L, a tension force, and are looking for a change in length suggests this is a Young's modulus problem.

Solve: First solve for the tension force from $v=\sqrt{\frac{T_S}{\mu}}$. This tension force becomes the F in the Young's modulus equation. From the chapter on elasticity we have

$$\Delta L=\frac{LF}{YA}$$

where $Y=20\times10^{10}\ \text{N/m}^2$ is Young's modulus for steel from Table 14.3, $F=T_S=v^2\mu$, and $A=\pi R^2$ is the cross-sectional area of the string.

$$\Delta L=\frac{LF}{YA}=\frac{L(v^2\mu)}{Y(\pi R^2)}=\frac{(0.75\ \text{m})(250\ \text{m/s})^2(0.0013\ \text{kg/m})}{(20\times10^{10}\ \text{N/m}^2)\pi(0.00023\ \text{m})^2}=1.8\ \text{mm}$$

Assess: That $f=196\ \text{Hz}$ when the string is properly tuned is correct but irrelevant information. That frequency, however, illustrates that only 65 cm of the string is really vibrating, the other 10 cm are wrapped around the tuning screw.

16.63. Model: A sinusoidal wave is traveling along a stretched string.

Solve: Equation 16.17 gives $v_{\max}=\omega A$. The derivative of Equation 16.17 gives $a=\frac{dv}{dt}=-\omega^2 A\sin(kx-\omega t+\phi_0)\Rightarrow a_{\max}=\omega^2 A$. These two equations can be combined to give

$$\omega=\frac{a_{\max}}{v_{\max}}=\frac{200\ \text{m/s}^2}{2.0\ \text{m/s}}=100\ \text{rad/s}\ \Rightarrow f=\frac{\omega}{2\pi}=15.9\ \text{Hz}\approx16\ \text{Hz}\qquad A=\frac{v_{\max}}{\omega}=\frac{2.0\ \text{m/s}}{100\ \text{rad/s}}=2.0\ \text{cm}$$

Assess: This frequency and amplitude are typical for a wave on a string.

16.64. Solve: Take the appropriate partial derivatives of $D(x,t)=(0.10-0.10x^2+xt-2.5t^2)\ \text{m}=\frac{1}{10}(1-(x-5t)^2)\ \text{m}$.

$$\frac{\partial D}{\partial x}=-\tfrac{2}{10}(x-5t)(1)\qquad \frac{\partial D}{\partial t}=-\tfrac{2}{10}(x-5t)(-5)\ \text{m/s}=(x-5t)\ \text{m/s}$$

$$\frac{\partial^2 D}{\partial x^2}=-\tfrac{2}{10}\ 1/\text{m}\qquad \frac{\partial^2 D}{\partial t^2}=-5\ \text{m/s}^2$$

Plug the derivatives into the wave equation: $\frac{\partial^2 D}{\partial t^2}=v^2\frac{\partial^2 D}{\partial x^2}$

$$-5.0\ \text{m/s}^2=v^2\left(-\tfrac{2}{10}\ 1/\text{m}\right)\Rightarrow v^2=25\ \text{m}^2/\text{s}^2\Rightarrow v=5.0\ \text{m/s}$$

So, yes, $D(x,t)=(0.10-0.10x^2+xt-2.5t^2)\ \text{m}$ is a possible traveling wave with a speed of 5.0 m/s.

Assess: Any function of $(kx-\omega t)$ is a possible traveling wave, in that it satisfies the wave equation. The trick is to see (by factoring the trinomial) that $D(x,t)=(0.10-0.10x^2+xt-2.5t^2)\ \text{m}$ is of this form.

16.65. Solve: Take the appropriate partial derivatives of $D(x,t)=(3.0\times10^{-3}\ \text{m})\ e^{i(2.0x+8.0t+5.0)}$.

$$\frac{\partial D}{\partial x}=(3.0\times10^{-3})(2.0i)\ e^{i(2.0x+8.0t+5.0)}\qquad \frac{\partial D}{\partial t}=(3.0\times10^{-3}\ \text{m/s})(8.0i)\ e^{i(2.0x+8.0t+5.0)}$$

$$\frac{\partial^2 D}{\partial x^2}=(3.0\times10^{-3}\ 1/\text{m})(2.0i)^2\ e^{i(2.0x+8.0t+5.0)}\qquad \frac{\partial^2 D}{\partial t^2}=(3.0\times10^{-3}\ \text{m/s}^2)(8.0i)^2\ e^{i(2.0x+8.0t+5.0)}$$

Plug the derivatives into the wave equation: $\frac{\partial^2 D}{\partial t^2}=v^2\frac{\partial^2 D}{\partial x^2}$

$$(3.0\times10^{-3}\ \text{m/s}^2)(8.0i)^2\ e^{i(2.0x+8.0t+5.0)}=v^2(3.0\times10^{-3}\ 1/\text{m})(2.0i)^2\ e^{i(2.0x+8.0t+5.0)}$$

$$\Rightarrow -64\ \text{m/s}^2=v^2(-4.0)\ 1/\text{m}\Rightarrow v=4.0\ \text{m/s}$$

So, yes, $D(x,t) = (3.0\times10^{-3}\text{ m})\,e^{i(2.0x+8.0t+5.0)}$ is a possible traveling wave with a speed of 4.0 m/s.
Assess: Any function of $(kx-\omega t)$ is a possible traveling wave, in that it satisfies the wave equation.

16.66. Model: The radio wave is an electromagnetic wave.
Solve: At a distance *r*, the 25 kW power station spreads out waves to cover the surface of a sphere of radius *r*. The surface area of a sphere is $4\pi r^2$. Thus, the intensity of the radio waves is

$$I = \frac{P_{\text{source}}}{4\pi r^2} = \frac{25\times10^3\text{ W}}{4\pi(10\times10^3\text{ m})^2} = 2.0\times10^{-5}\text{ W/m}^2$$

16.67. Visualize: To find the power of a laser pulse, we need the energy it contains, U, and the time duration of the pulse, Δt. Then to find the intensity, we need the area of the pulse. Its radius is 0.50 mm.
Solve: **(a)** Using $P = U/\Delta t$, we find the following:

$$P = (1.0\times10^{-3}\text{ J})/(15\times10^{-9}\text{ s}) = 6.67\times10^4\text{ W}$$

(b) Then from $I = P/a$, we obtain

$$I = \frac{(6.67\times10^4\text{ W})}{\pi(5.0\times10^{-4}\text{ m})^2} = 8.5\times10^{10}\text{ W/m}^2$$

16.68. Model: We have a traveling wave radiated by the tornado siren.
Solve: **(a)** The power of the source is calculated as follows:

$$I_{50\text{ m}} = 0.10\text{ W/m}^2 = \frac{P_{\text{source}}}{4\pi r^2} = \frac{P_{\text{source}}}{4\pi(50\text{ m})^2} \Rightarrow P_{\text{source}} = (0.10\text{ W/m}^2)4\pi(50\text{ m})^2 = (1000\pi)\text{ W}$$

The intensity at 1000 m is

$$I_{1000\text{ m}} = \frac{P_{\text{source}}}{4\pi(1000\text{ m})^2} = \frac{(1000\pi)\text{ W}}{4\pi(1000\text{ m})^2} = 250\ \mu\text{W/m}^2$$

(b) The maximum distance is calculated as follows:

$$I = \frac{P_{\text{source}}}{4\pi r^2} \Rightarrow 1.0\times10^{-6}\text{ W/m}^2 = \frac{(1000\pi)\text{ W}}{4\pi r^2} \Rightarrow r = 16\text{ km}$$

16.69. Model: Assume the energy is radiated spherically symmetrically.
Solve: (a)

$$I = \frac{P}{a} = \frac{P}{4\pi(6.0R_{\text{sun}})^2} = \frac{6.8\times10^{29}\text{ W}}{4\pi(6.0\times6.96\times10^8\text{ m})^2} = 3.10\times10^9\text{ W/m}^2$$

(b) The energy from the star will be radiated out into space in all directions equally. The ratio of the power output by the star to the power falling on the planet's surface is the same as the ratio of the surface area of a sphere with radius equal to the planet's orbital radius to the cross-sectional area of the planet (a disk).

$$\frac{P_{\text{star}}}{P_{\text{received}}} = \frac{4\pi R_{\text{orbit}}^2}{\pi R_{\text{planet}}^2} \Rightarrow R_{\text{orbit}} = \sqrt{\frac{R_{\text{planet}}^2 P_{\text{star}}}{4P_{\text{received}}}} = \frac{R_{\text{planet}}}{2}\sqrt{\frac{P_{\text{star}}}{P_{\text{received}}}} = R_{\text{earth}}\sqrt{\frac{P_{\text{star}}}{P_{\text{received}}}}$$

$$= 6.37\times10^6\text{ m}\sqrt{\frac{6.8\times10^{29}\text{ W}}{(9.4\times10^{22}\text{ J})/((8.0\text{ h})(3600\text{ s/h}))}} = 2.9\times10^{12}\text{ m} = 19\text{ au}$$

Assess: The planet's distance from the star is about the distance of Uranus from the sun.

16.70. Model: We would expect the intensity to be proportional to $1/r^3$ in a 4D universe.
Visualize: The energy cancels in all of the following ratios.

Solve: (a)

$$\left(\frac{I_{1\text{ m}}}{I_{5\text{ m}}}\right)_{2D} = \frac{(1.0\text{ m})}{(5.0\text{ m})} = \frac{1}{5.0} = 0.20$$

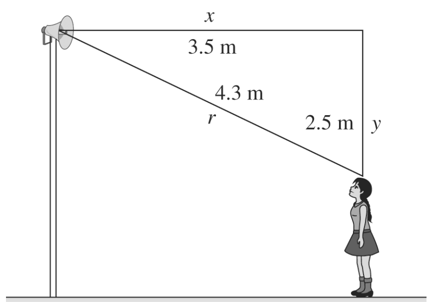
(b)

$$\left(\frac{I_{1\text{ m}}}{I_{5\text{ m}}}\right)_{3D} = \frac{(1.0\text{ m})^2}{(5.0\text{ m})^2} = \frac{1}{5.0^2} = 0.040$$

(c)

$$\left(\frac{I_{1\text{ m}}}{I_{5\text{ m}}}\right)_{4D} = \frac{(1.0\text{ m})^3}{(5.0\text{ m})^3} = \frac{1}{5.0^3} = 0.0080$$

16.71. Model: Assume the ¾ of the energy directed forward is uniformly distributed in the front hemisphere.
Visualize:



Solve: Solve for the intensity toward the front in terms of the sound level in decibels.

$$\beta_{\text{front}} = 85\text{ dB} = (10\text{ dB})\log_{10}\left(\frac{I_{\text{front}}}{I_0}\right) \Rightarrow I_{\text{front}} = I_0(10^{\beta_{\text{front}}/10\text{ dB}}) = (10^{-12}\text{ W/m}^2)(10^{85\text{ dB}/10\text{ dB}})$$

Now use the power-to-area ratio $I = P/a$ and solve for P. Note that ¾ of the power goes into half the area.

$$I_{\text{front}} = \frac{\frac{3}{4}P}{2\pi r^2} \Rightarrow P = \frac{8}{3}\pi r^2 I_{\text{front}} = \frac{8}{3}\pi(x^2+y^2)I_0(10^{\beta_{\text{front}}/10\text{ dB}})$$

$$= \frac{8}{3}\pi((3.5\text{ m})^2 + (2.5\text{ m})^2)(10^{-12}\text{ W/m}^2)(10^{85\text{ dB}/10\text{ dB}}) = 49\text{ mW}$$

Assess: The speed of sound in this liquid is in the ballpark of other sound speeds.

16.72. Solve: If we solve the equation for I, we have:

$$I = I_0 \times 10^{(\beta/10\text{ dB})}$$

Now plugging in 60 dB for β, we get $I = 10^{-6}\text{ W/m}^2$ and plugging in 61 dB for β, we get $I = 1.3\times10^{-6}\text{ W/m}^2$. The ratio of the latter to the former is 1.3.

16.73. Visualize: Take the $\log_{10}$ of both sides of $I = cP_{\text{source}}r^{-x}$.

$$\log I = \log(cP_{\text{source}}) + \log(r^{-x}) = \log(cP_{\text{source}}) - x\log r$$

Solve: Now use the equation for sound level intensity.

$$\beta = (10\text{dB})\log\left(\frac{I}{I_0}\right) = (10\text{dB})[\log I - \log I_0]$$

Insert the expression for log I from above.

$$\beta = (10\text{dB})[\log(cP_{\text{source}}) - x\log r - \log I_0]$$

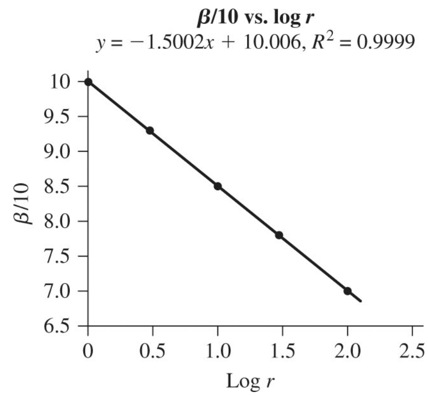
Divide both sides by 10 dB and note $\log 10^{-12} = -12$.

$$\frac{\beta}{10\text{ dB}} = \log(cP_{\text{source}}) - x\log r - (-12)$$

Group constant terms.

$$\frac{\beta}{10\text{ dB}} = -x\log r + (\log(cP_{\text{source}}) + 12)$$

This equation leads us to believe that a graph of $\frac{\beta}{10\text{ dB}}$ vs. log r would produce a straight line whose slope is $-x$ and whose intercept is $\log(cP_{\text{source}}) + 12$.



The spreadsheet shows that the linear fit is excellent and that the slope is -1.5. So we conclude that $x = 1.5$.
Assess: We expected a number less than 2. Also, the intercept of 10 tells us that $\log(cP_{\text{source}}) = -2$.

16.74. Model: The sound generator's frequency is altered by the Doppler effect. The frequency increases as the generator approaches the student, and it decreases as the generator recedes from the student.
Solve: The generator's speed is

$$v_S = r\omega = r(2\pi f) = (1.0\text{ m})2\pi\left(\frac{100}{60}\text{ rev/s}\right) = 10.47\text{ m/s}$$

The frequency of the approaching generator is

$$f_+ = \frac{f_0}{1 - v_S/v} = \frac{600\text{ Hz}}{1 - \dfrac{10.47\text{ m/s}}{343\text{ m/s}}} = 619\text{ Hz} \approx 620\text{ Hz}$$

Doppler effect for the receding generator, on the other hand, is

$$f_- = \frac{f_0}{1 + v_S/v} = \frac{600\text{ Hz}}{1 + \dfrac{10.47\text{ m/s}}{343\text{ m/s}}} = 582\text{ Hz} \approx 580\text{ Hz}$$

Thus, the highest and the lowest frequencies heard by the student are 620 Hz and 580 Hz.

16.75. Model: Assume the air is at $20°\text{C}$.
Solve: Use the Doppler formula for a moving source approaching the observer(s).

$$\frac{f_+}{f_0} = \frac{1}{1 - v_s/v} = 2^{1/12} \Rightarrow \frac{1}{2^{1/12}} = 1 - \frac{v_s}{v} \Rightarrow v_s = v(1 - 2^{-1/12}) = (343\text{ m/s})(1 - 2^{-1/12}) = 19\text{ m/s}$$

Assess: The is a possible speed for a bat.

16.76 Model: Assume you are somehow walking directly toward the speaker, even though it is not on the ground.
Solve: (a)

$$\beta = (10\text{ db})\log_{10}\left(\frac{I}{I_0}\right) = (10\text{ db})\log_{10}\left(\frac{P/4\pi r^2}{I_0}\right) = (10\text{ db})\log_{10}\left(\frac{(100\text{ W})/4\pi(20\text{ m})^2}{1\times10^{-12}\text{ W/m}^2}\right) = 100\text{ dB}$$

(b) To get the rate of change we must take the time derivative of the expression for β and evaluate at $r = 20$ m. Use the hint $\log_{10} x = \ln x/\ln 10$, the chain rule, and the fact that $dr/dt = v$.

$$\frac{d\beta}{dt} = \frac{d}{dt}(10\text{ db})\log_{10}\left(\frac{I}{I_0}\right) = \frac{d}{dt}\left(\frac{10\text{ db}}{\ln 10}\right)\ln\left(\frac{P/4\pi r^2}{I_0}\right) = \frac{d}{dt}\left(\frac{10\text{ db}}{\ln 10}\right)\ln\left(\frac{P}{I_0 4\pi}r^{-2}\right)$$

$$= \left(\frac{10\text{ db}}{\ln 10}\right)\left(\frac{P}{I_0 4\pi}r^{-2}\right)^{-1}\left(\frac{P}{I_0 4\pi}(-2)r^{-3}\right)\frac{dr}{dt} = \left(\frac{10\text{ db}}{\ln 10}\right)(r^2)(-2r^{-3})v = \left(\frac{10\text{ db}}{\ln 10}\right)\left(\frac{-2}{r}\right)v$$

$$= \left(\frac{10\text{ db}}{\ln 10}\right)\left(\frac{-2}{20\text{ m}}\right)(-0.80\text{ m/s}) = 0.35\text{ dB/s}$$

Assess: The answer to **(a)** is loud, but realistic for such a power output from the speaker. The answer to **(b)** seems like it could be in the right ballpark.

16.77. Solve: We will closely follow the details of Section 16.9 in the textbook. Figure 16.30 shows that the wave crests are stretched out behind the source. The wavelength detected by Pablo is $\lambda_- = \frac{1}{3}d$, where d is the distance the wave has moved plus the distance the source has moved at time $t = 3T$. These distances are $\Delta x_{\text{wave}} = vt = 3vT$ and $\Delta x_{\text{source}} = v_S t = 3v_S T$. The wavelength of the wave emitted by a receding source is thus

$$\lambda_- = \frac{d}{3} = \frac{\Delta x_{\text{wave}} + \Delta x_{\text{source}}}{3} = \frac{3vT + 3v_S T}{3} = (v + v_S)T$$

The frequency detected in Pablo's direction is thus

$$f_- = \frac{v}{\lambda_-} = \frac{v}{(v + v_s)T} = \frac{f_0}{1 + v_S/v}$$

16.78. Model: We are looking at the Doppler effect for the light of an approaching source.
Solve: (a) The time is

$$t = \frac{54\times10^6\text{ km}}{3\times10^5\text{ km/s}} = 180\text{ s} = 3.0\text{ min}$$

(b) Using Equation 20.40, the observed wavelength is

$$\lambda = \sqrt{\frac{1 - v_s/c}{1 + v_s/c}}\lambda_0 = \sqrt{\frac{1 - 0.1c/c}{1 + 0.1c/c}}(540\text{ nm}) = (0.9045)(540\text{ nm}) = 488\text{ nm} \approx 490\text{ nm}$$

Assess: 490 nm is slightly blue shifted from green.

16.79. Model: The Doppler effect for light of a receding source yields an increased wavelength.
Solve: Because the measured wavelengths are 0.5% longer, that is, $\lambda = 1.005\lambda_0$, the distant galaxy is receding away from the earth. Using Equation 16.66,

$$\lambda = 1.005\lambda_0 = \sqrt{\frac{1 + v_s/c}{1 - v_s/c}}\lambda_0 \Rightarrow (1.005)^2 = \frac{1 + v_s/c}{1 - v_s/c} \Rightarrow v_s = 0.0050\ c = 1.5\times10^6\text{ m/s}$$

16.80. Model: The Doppler effect for light of an approaching source leads to a decreased wavelength.
Solve: The red wavelength $(\lambda_0 = 650\text{ nm})$ is Doppler shifted to green $(\lambda = 540\text{ nm})$ due to the approaching light source. In relativity theory, the distinction between the motion of the source and the motion of the observer

disappears. What matters is the relative approaching or receding motion between the source and the observer. Thus, we can use Equation 16.66 as follows:

$$\lambda = \lambda_0\sqrt{\frac{1-v_s/c}{1+v_s/c}} \Rightarrow 540\text{ nm} = (650\text{ nm})\sqrt{\frac{1-v_s/c}{1+v_s/c}}$$

$$\Rightarrow v_s = 5.5\times10^4\text{ km/s} = 2.0\times10^8\text{ km/h}$$

The fine will be

$$(2.0\times10^8\text{ km/h} - 50\text{ km/h})\left(\frac{1\ \$}{1\text{ km/h}}\right) = \$200\text{ million}$$

Assess: The police officer knew his physics.

Challenge Problems

16.81. Solve: The time for the wave to travel from California to the South Pacific is

$$t = \frac{d}{v} = \frac{8.00\times10^6\text{ m}}{1480\text{ m/s}} = 5405.4\text{ s}$$

A time decrease to 5404.4 s implies the speed has changed to $v = \frac{d}{t} = 1480.28$ m/s.

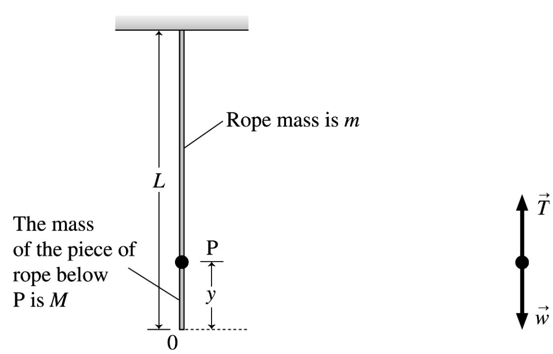
Since the 4.0 m/s increase in velocity is due to an increase of 1°C, an increase of 0.28 m/s occurs due to a temperature increase of

$$\left(\frac{1°\text{C}}{4.0\text{ m/s}}\right)(0.28\text{ m/s}) = 0.07\ °\text{C}$$

Thus, a temperature increase of approximately 0.07 °C can be detected by the researchers.

16.82. Model: The wave pulse is a traveling wave on a stretched wire.
Visualize:

Pictorial representation



Solve: **(a)** At a distance y above the lower end of the rope, the point P is in static equilibrium. The upward tension in the rope must balance the weight of the rope that hangs below this point. Thus, at this point

$$T = w = Mg = (\mu y)g$$

where $\mu = m/L$ is the linear density of the entire rope. Using Equation 16.2, we get

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\mu y g}{\mu}} = \sqrt{gy}$$

(b) The time to travel a distance dy at y, where the wave speed is $v = \sqrt{gy}$, is

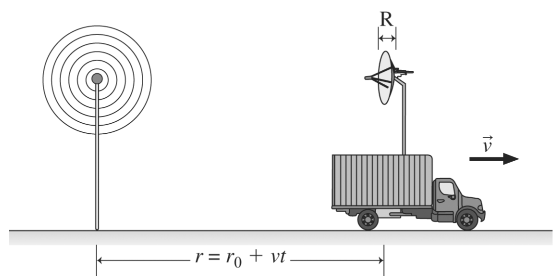
$$dt = \frac{dy}{v} = \frac{dy}{\sqrt{gy}}$$

Finding the time for a pulse to travel the length of the rope requires integrating from one end of the rope to the other:

$$\Delta t = \int_0^T dt = \int_0^L \frac{dy}{\sqrt{gy}} = \frac{1}{\sqrt{g}}\left(2\sqrt{y}\Big|_0^L\right) = \frac{2}{\sqrt{g}}\sqrt{L} \Rightarrow \Delta t = 2\sqrt{\frac{L}{g}}$$

16.83 Model: Assume the radiation that hits the ground is absorbed so the energy emitted goes into $4\pi r^2$ worth of surface area.

Visualize: P_{dish} is the power received by the dish antenna. $a_{\text{sphere}} = 4\pi r^2$ is the area the base station power is radiated into.



Known
$P_{\text{base}} = 2.5$ kW
$R_{\text{dish}} = R = 22$ cm
$v = 50$ km/h $= 13.80$ m/s
$r_0 = 10$ km
$t_f = 1$ h $= 3600$ s

Find
E

Solve:

$$E = \int dE = \int P_{\text{dish}} dt = \int I_{\text{dish}} a_{\text{dish}} dt$$

a_{dish} is constant, but $I_{\text{dish}} = P_{\text{base}}/a_{\text{sphere}} = P_{\text{base}}/4\pi r^2$ is not.

$$E = \int I_{\text{dish}} a_{\text{dish}} = P_{\text{base}} a_{\text{dish}} \int \frac{dt}{a_{\text{hemisphere}}} = P_{\text{base}}(\pi R^2)\int \frac{dt}{4\pi r^2} = \frac{P_{\text{base}} R^2}{4}\int_0^{t_f} \frac{dt}{(r_0 + vt)^2}$$

$$= \frac{P_{\text{base}} R^2}{4}\left[\frac{-1}{v}(r_0 + vt)^{-1}\right]_0^{t_f} = \frac{P_{\text{base}} R^2}{4}\left[\frac{-1}{v(r_0 + vt)}\right]_0^{t_f} = \frac{P_{\text{base}} R^2}{4}\left[\frac{-1}{v(r_0 + vt)} - \frac{-1}{vr_0}\right]$$

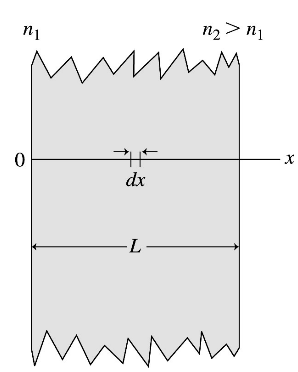
$$= \frac{P_{\text{base}} R^2}{4}\left[\frac{1}{vr_0} - \frac{1}{v(r_0 + vt)}\right] = \frac{P_{\text{base}} R^2}{4v}\left[\frac{1}{r_0} - \frac{1}{(r_0 + vt)}\right]$$

$$= \frac{(2.5\text{ kW})(22\text{ cm})^2}{4(13.89\text{ m/s})}\left[\frac{1}{(10\text{ km})} - \frac{1}{\left(10\text{ km} + (13.89\text{ m/s})(3600\text{ s})\right)}\right]$$

$$= 0.18\text{ mJ}$$

Assess: The energy received by radio antennas is notoriously small.

16.84. Visualize:



Solve: **(a)** Using the graph, the refractive index n as a function of distance x can be mathematically expressed as

$$n = n_1 + \frac{n_2 - n_1}{L}x$$

At position x, the light speed is $v = c/n$. The time for the light to travel a distance dx at x is

$$dt = \frac{dx}{v} = \frac{n}{c}dx = \frac{1}{c}\left(n_1 + \frac{n_2 - n_1}{L}x\right)dx$$

To find the total time for the light to cover a thickness L of a glass we integrate as follows:

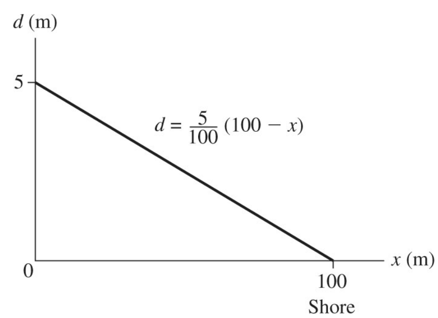
$$T = \int_0^T dt = \frac{1}{c}\int_0^L\left(n_1 + \frac{n_2 - n_1}{L}x\right)dx = \frac{n_1}{c}\int_0^L dx + \frac{(n_2 - n_1)}{cL}\int_0^L x\,dx = \frac{n_1}{c}L + \left(\frac{n_2 - n_1}{cL}\right)\frac{L^2}{2} = \left(\frac{n_1 + n_2}{2c}\right)L$$

(b) Substituting the given values into this equation,

$$T = \frac{(1.50 + 1.60)}{2(3.0\times10^8\text{ m/s})}\times 0.010\text{ m} = 5.17\times10^{-11}\text{ s}$$

16.85. **Model:** Use the shallow wave equation $v = \sqrt{gd}$.

Visualize: Say the wave is moving to the right from an initial depth of 5.0 m at 100 m from shore to a depth of 0.0 m at the shore.



Solve: We want $T = \int dt$ where $dt = dx/v = dx/\sqrt{gd}$.

$$T = \int dt = \int_0^{100}\frac{dx}{\sqrt{gd}} = \int_0^{100}\frac{dx}{\sqrt{g\left(\frac{5}{100}(100 - x)\right)}} = \frac{10}{\sqrt{5g}}\int_0^{100}\frac{dx}{\sqrt{100 - x}}$$

$$= \frac{10}{\sqrt{5g}}\left[-2\sqrt{100 - x}\right]_0^{100} = \frac{-20}{\sqrt{5g}}\left[0 - 10\right] = 29\text{ s}$$

Assess: 29 s seems like a reasonable time for a shallow wave to travel 100 m.