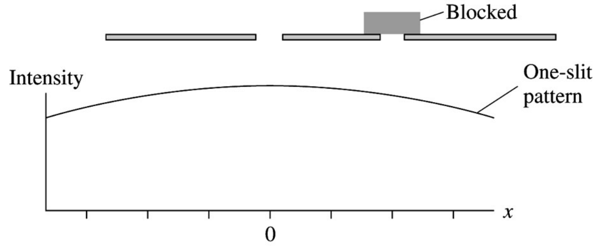
33

WAVE OPTICS

Conceptual Questions

33.1.



The initial light pattern is a double-slit interference pattern. It is centered behind the midpoint of the slits. The slight decrease in intensity going outward from the middle indicates that the light from each of the individual slits is not uniform but slowly decreases toward the edges of the screen. If the right slit is covered, light comes through only the left slit. Without a second slit, there is no interference. Instead, we get simply the spread-out pattern of light diffracting through a single slit, such as in the center of the photograph of Figure 33.2. The intensity is a maximum directly behind the left slit, and—as we discerned from the intensities in the double-slit pattern—the single-slit intensity fades gradually toward the edges of the screen.

33.2. Because $y_m = \dfrac{m\lambda L}{d}$ increasing λ and L increases the fringe spacing. Increasing d decreases the fringe spacing. Submerging the experiment in water decreases λ and decreases the fringe spacing. So the answers are (a) and (c).

33.3. The fringe separation for the light intensity pattern of a double-slit is determined by $\Delta y = \lambda L/d$. (a) If λ increases, Δy will decrease. (b) If d decreases, Δy will increase. (c) If L decreases, Δy will decrease. (d) Since the dot is in the $m = 2$ bright fringe, the path length difference from the two slits is $2\lambda = 1000$ nm.

33.4. **(a)** The equation for gratings does not contain the number of slits, so increasing the number of slits can't affect the angles at which the bright fringes appear as long as d is the same. So the number of fringes on the screen stays the same. **(b)** The number of slits does not appear in the equation for the fringe spacing, so the spacing stays the same. **(c)** Decreases; the fringes become narrower. **(d)** The equation for intensity does contain the number of slits, so each fringe becomes brighter: $I_{\max} = N^2 I_1$.

33.5. $\lambda < a$. Several secondary maxima appear. For $a\sin\theta_p = p\lambda,$ the first minima from the central maximum require $a\sin\theta_1 = \lambda,$ which must be less than 1.

33.6. **(a)** The width increases because $\theta_1 = \dfrac{1.22\lambda}{D}$. **(b)** The width decreases because $\theta_1 = \dfrac{1.22\lambda}{D}$. **(c)** Almost uniformly gray with no minima.

33.7. When the experiment is immersed in water the frequency of the light stays the same. But the speed is slower, so the wavelength is smaller. When the wavelength is smaller the fringes get closer together because $\Delta y = \dfrac{\lambda L}{d}$.

33.8. **(a)** Because the hole is so large we do not use the wave model of light. The ray model says we treat the light as if it travels in straight lines, so increasing the hole diameter will increase the size of the spot on the screen. **(b)** When the hole is smaller than 1 mm we need to consider diffraction effects; making the hole 20% larger will decrease the diameter of the spot on the screen.

33.9. Moving a mirror by 200 nm increases or decreases the path length by 400 nm. Since one mirror is moved in and the other out each increases the path length difference by 400 nm. The total is a path length difference of 800 nm, so the spot will still be bright.

33.10. If the wavelength is changed to $\lambda/2$ there will still be crests everywhere there were crests before (plus new crests halfway between, where there used to be troughs). If the interferometer was set up to display constructive interference originally, then crests were arriving from each arm of the interferometer together at the detector. That will still be true when the wavelength is halved. So the central spot remains bright.

Exercises and Problems

Exercises

Section 33.2 The Interference of Light

33.1. **Model:** Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference pattern looks like the photograph of Figure 33.4(b).
Solve: The bright fringes are located at positions given by Equation 33.4, $d\sin\theta_m = m\lambda$. For the m = 3 bright orange fringe, the interference condition is $d\sin\theta_3 = 3(600\times10^{-9}\text{ m})$. For the m = 4 bright fringe the condition is $d\sin\theta_4 = 4\lambda$. Because the position of the fringes is the same,

$$d\sin\theta_3 = d\sin\theta_4 = 4\lambda = 3(600\times10^{-9}\text{ m}) \Rightarrow \lambda = \tfrac{3}{4}(600\times10^{-9}\text{ m}) = 450\text{ nm}$$

33.2. **Model:** Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference pattern looks like the photograph of Figure 33.4(b). It is symmetrical with the $m=3$ fringes on both sides of and equally distant from the central maximum.
Solve: The bright fringes occur at angles θ_m such that

$$d\sin\theta_m = m\lambda \quad m = 0,\ 1,\ 2,\ 3,\ \ldots$$

$$\Rightarrow \sin\theta_3 = \frac{3(600\times10^{-9}\text{ m})}{(80\times10^{-6}\text{ m})} = 0.0225 \Rightarrow \theta_3 = 0.023\text{ rad} = 0.023\text{ rad}\times\frac{180°}{\pi\text{ rad}} = 1.3°$$

33.3. **Model:** Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference pattern looks like the photograph of Figure 33.4(b).
Solve: The formula for fringe spacing is

$$\Delta y = \frac{\lambda L}{d} \Rightarrow 1.8\times10^{-3}\text{ m} = (600\times10^{-9}\text{ m})\frac{L}{d} \Rightarrow \frac{L}{d} = 3000$$

The wavelength is now changed to 400 nm, and L/d, being a part of the experimental setup, stays the same. Applying the above equation once again,

$$\Delta y = \frac{\lambda L}{d} = (400\times 10^{-9}\ \text{m})(3000) = 1.2\ \text{mm}$$

33.4. Model: Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference pattern looks like the photograph of Figure 33.4(b). It is symmetrical, with the $m=2$ fringes on both sides of and equally distant from the central maximum.
Solve: The two paths from the two slits to the $m=2$ bright fringe differ by $\Delta r = r_2 - r_1$, where

$$\Delta r = m\lambda = 2\lambda = 2(500\ \text{nm}) = 1000\ \text{nm}$$

Thus, the position of the $m=2$ bright fringe is 1000 nm farther away from the more distant slit than from the nearer slit.

33.5. Visualize: The fringe spacing for a double-slit pattern is $\Delta y = \dfrac{\lambda L}{d}.$ We are given d = 0.25 mm and $\lambda = 630$ nm. We also see from the figure that $\Delta y = \frac{1}{3}\text{cm}.$
Solve: Solve the equation for L.

$$L = \frac{d\Delta y}{\lambda} = \frac{(0.25\ \text{mm})\left(\frac{1}{3}\times 10^{-2}\ \text{m}\right)}{(630\times 10^{-9}\ \text{m})} = 1.3\ \text{m}$$

Assess: This is a typical screen distance.

33.6. Model: Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference fringes are equally spaced on both sides of the central maximum. The interference pattern looks like Figure 33.4(b).
Solve: In the small-angle approximation

$$\Delta\theta = \theta_{m+1} - \theta_m = (m+1)\frac{\lambda}{d} - m\frac{\lambda}{d} = \frac{\lambda}{d}$$

Since $d = 200\lambda,$ we have

$$\Delta\theta = \frac{\lambda}{d} = \frac{1}{200}\ \text{rad} = 0.286^\circ$$

33.7. Model: Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference pattern looks like the photograph of Figure 33.4(b).
Solve: The fringe spacing is

$$\Delta y = \frac{\lambda L}{d} \Rightarrow d = \frac{\lambda L}{\Delta y} = \frac{(589\times 10^{-9}\ \text{m})(150\times 10^{-2}\ \text{m})}{4.0\times 10^{-3}\ \text{m}} = 0.22\ \text{mm}$$

33.8. Model: Two closely spaced slits produce a double-slit interference pattern.
Visualize: The interference pattern looks like the photograph of Figure 33.4(b).
Solve: The dark fringes are located at positions given by Equation 33.9:

$$y'_m = \left(m + \tfrac{1}{2}\right)\frac{\lambda L}{d} \qquad m = 0,\ 1,\ 2,\ 3,\ \ldots$$

$$\Rightarrow y'_5 - y'_1 = \left(5+\tfrac{1}{2}\right)\frac{\lambda L}{d} - \left(1+\tfrac{1}{2}\right)\frac{\lambda L}{d} \Rightarrow 6.0\times 10^{-3}\ \text{m} = \frac{4\lambda(60\times 10^{-2}\ \text{m})}{0.20\times 10^{-3}\ \text{m}} \Rightarrow \lambda = 500\ \text{nm}$$

Section 33.3 The Diffraction Grating

33.9. Model: A diffraction grating produces an interference pattern.
Visualize: The interference pattern looks like the diagram in Figure 33.9.
Solve: The bright constructive-interference fringes are given by Equation 33.15:

$$d\sin\theta_m = m\lambda \qquad m = 0,\ 1,\ 2,\ \ldots$$

$$\Rightarrow \sin\theta_1 = \frac{(1)(550\times10^{-9}\ \text{m})}{(4.0\times10^{-2}\ \text{m})/2000} = 0.0275 \Rightarrow \theta_1 = 1.6°$$

Likewise, $\sin\theta_2 = 0.055$ and $\theta_2 = 3.2°$.

33.10. Model: A diffraction grating produces an interference pattern.
Visualize: The interference pattern looks like the diagram in Figure 33.9.
Solve: The bright constructive-interference fringes are given by Equation 33.15:

$$d\sin\theta_m = m\lambda \Rightarrow d = \frac{m\lambda}{\sin\theta_m} = \frac{(2)(600\times10^{-9}\ \text{m})}{\sin(39.5°)} = 1.89\times10^{-6}\ \text{m}$$

The number of lines in per millimeter is $(1\times10^{-3}\ \text{m})/(1.89\times10^{-6}\ \text{m}) = 530$.

33.11. Model: A diffraction grating produces a series of constructive-interference fringes at values of θ_m determined by Equation 33.15.
Solve: We have

$$d\sin\theta_m = m\lambda \qquad m = 0,\ 1,\ 2,\ 3,\ \ldots \Rightarrow d\sin 20.0° = 1\lambda \text{ and } d\sin\theta_2 = 2\lambda$$

Dividing these two equations,

$$\sin\theta_2 = 2\sin 20.0° = 0.6840 \Rightarrow \theta_2 = 43.2°$$

33.12. Model: A diffraction grating produces a series of constructive-interference fringes at values of θ_m that are determined by Equation 33.15.
Solve: For the m = 3 maximum of the red light and the m = 5 maximum of the unknown wavelength, Equation 33.15 gives

$$d\sin\theta_3 = 3(660\times10^{-9}\ \text{m}) d\sin\theta_5 = 5\lambda_{\text{unknown}}$$

The $m = 5$ fringe and the $m = 3$ fringe have the same angular positions. This means $\theta_5 = \theta_3$. Dividing the two equations,

$$5\lambda_{\text{unknown}} = 3(660\times10^{-9}\ \text{m}) \Rightarrow \lambda_{\text{unknown}} = 396\ \text{nm}$$

33.13. Model: A diffraction grating produces an interference pattern.
Visualize: The interference pattern looks like the diagram of Figure 33.9.
Solve: The bright interference fringes are given by

$$d\sin\theta_m = m\lambda \quad \text{m} = 0,\ 1,\ 2,\ 3,\ \ldots$$

The slit spacing is $d = 1\ \text{mm}/500 = 2.00\times10^{-6}\ \text{m}$ and $m = 1$. For the red and blue light,

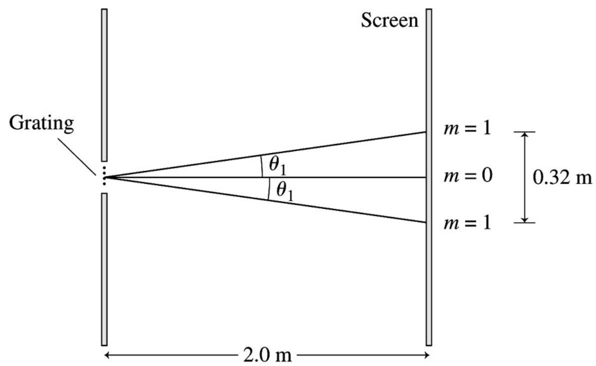
$$\theta_{1\ \text{red}} = \sin^{-1}\left(\frac{656\times10^{-9}\ \text{m}}{2.00\times10^{-6}\ \text{m}}\right) = 19.15 \qquad \theta_{1\ \text{blue}} = \sin^{-1}\left(\frac{486\times10^{-9}\ \text{m}}{2.00\times10^{-6}\ \text{m}}\right) = 14.06°$$

The distance between the fringes, then, is $\Delta y = y_{1\text{ red}} - y_{1\text{ blue}}$ where

$$y_{1\text{ red}} = (1.5\text{ m})\tan 19.15° = 0.521\text{ m}$$
$$y_{1\text{ blue}} = (1.5\text{ m})\tan 14.06° = 0.376\text{ m}$$

So, $\Delta y = 0.145\text{ m} = 14.5\text{ cm}.$

33.14. Model: A diffraction grating produces an interference pattern.
Visualize: The interference pattern looks like the diagram in Figure 33.9.



Solve: The bright fringes are given by Equation 33.15:

$$d\sin\theta_m = m\lambda \qquad m = 0, 1, 2, 3, \ldots \Rightarrow d\sin\theta_1 = (1)\lambda \Rightarrow d = \lambda/\sin\theta_1$$

The angle θ_1 can be obtained from geometry as follows:

$$\tan\theta_1 = \frac{(0.32\text{ m})/2}{2.0\text{ m}} = 0.080 \Rightarrow \theta_1 = \tan^{-1}(0.080)\ 4.57°$$

Using $\sin\theta_1 = \sin 4.57° = 0.07968,$

$$d = \frac{633\times 10^{-9}\text{ m}}{0.07968} = 7.9\ \mu\text{m}$$

Section 33.4 Single-Slit Diffraction

33.15. Model: A narrow slit produces a single-slit diffraction pattern.
Visualize: The intensity pattern for single-slit diffraction will look like Figure 33.15.
Solve: The width of the central maximum for a slit of width $a = 200\lambda$ is

$$w = \frac{2\lambda L}{a} = \frac{2\lambda(2.0\text{ m})}{200\lambda} = 20\text{ mm}$$

33.16. Model: A narrow single slit produces a single-slit diffraction pattern.
Visualize: The intensity pattern for single-slit diffraction will look like Figure 33.15.
Solve: The minima occur at positions

$$y_p = p\frac{L\lambda}{a}$$

$$\text{So } \Delta y = y_2 - y_1 = \frac{2\lambda L}{a} - \frac{1\lambda L}{a} = \frac{\lambda L}{a} \Rightarrow a = \frac{\lambda L}{\Delta y} = \frac{(633\times 10^{-9}\text{ m})(1.5\text{ m})}{0.00475\text{ m}} = 2.0\times 10^{-4}\text{ m} = 0.20\text{ mm}$$

33.17. Visualize: Use $w = \dfrac{2\lambda L}{a}$. We are given $\lambda = 630$ nm and a = 0.15 mm. We see from the figure that $w = 1.0$ cm.
Solve: Solve the equation for L.

$$L = \frac{wa}{2\lambda} = \frac{(0.010\text{ m})(0.15\text{ mm})}{2\left(630\times10^{-9}\text{ m}\right)} = 1.2\text{ m}$$

Assess: This is a typical screen distance.

33.18. Model: A narrow slit produces a single-slit diffraction pattern.
Visualize: The intensity pattern for single-slit diffraction will look like Figure 33.15.
Solve: The width of the central maximum for a slit of width $a = 200\lambda$ is

$$w = \frac{2\lambda L}{a} = \frac{2(500\times10^{-9}\text{ m})(2.0\text{ m})}{0.0005\text{ m}} = 0.0040\text{ m} = 4.0\text{ mm}$$

33.19. Model: The spacing between the two buildings is like a single slit and will cause the radio waves to be diffracted.
Solve: Radio waves are electromagnetic waves that travel with the speed of light. The wavelength of the 800 MHz waves is

$$\lambda = \frac{3\times10^{8}\text{ m/s}}{800\times10^{6}\text{ Hz}} = 0.375\text{ m}$$

To investigate the diffraction of these waves through the spacing between the two buildings, we can use the general condition for complete destructive interference: $a\sin\theta_p = p\lambda$ ($p = 1, 2, 3, \ldots$) where a is the spacing between the buildings. Because the width of the central maximum is defined as the distance between the two $p = 1$ minima on either side of the central maximum, we will use $p = 1$ and obtain the angular width $\Delta\theta = 2\theta_1$ from

$$a\sin\theta_1 = \lambda \Rightarrow \theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{0.375\text{ m}}{15\text{ m}}\right) = 1.43°$$

Thus, the angular width of the wave after it emerges from between the buildings is $\Delta\theta = 2(1.43°) = 2.86° \approx 2.9°$.

33.20. Model: A narrow slit produces a single-slit diffraction pattern.
Visualize: The diffraction-intensity pattern from a single slit will look like Figure 33.15.
Solve: These are not small angles, so we can't use equations based on the small-angle approximation. As given by Equation 33.19, the dark fringes in the pattern are located at $a\sin\theta_p = p\lambda$, where $p = 1, 2, 3, \ldots$ For the first minimum of the pattern, $p = 1$. Thus,

$$\frac{a}{\lambda} = \frac{p}{\sin\theta_p} = \frac{1}{\sin\theta_1}$$

For the three given angles the slit width to wavelength ratios are

$$\textbf{(a)}: \left(\frac{a}{\lambda}\right)_{30°} = \frac{1}{\sin 30°} = 2, \quad \textbf{(b)}: \left(\frac{a}{\lambda}\right)_{60°} = \frac{1}{\sin 60°} = 1.15, \quad \textbf{(c)}: \left(\frac{a}{\lambda}\right)_{90°} = \frac{1}{\sin 90°} = 1$$

Assess: It is clear that the smaller the a/λ ratio, the wider the diffraction pattern. This is a conclusion that is contrary to what one might expect.

33.21. Model: A narrow slit produces a single-slit diffraction pattern.
Visualize: The diffraction-intensity pattern from a single slit will look like Figure 33.15.

Solve: As given by Equation 33.19, the dark fringes in the pattern are located at $a\sin\theta_p = p\lambda,$ where $p = 1, 2, 3, \ldots$ For the diffraction pattern to have no minima, the first minimum must be located at least at $\theta_1 = 90°$. From the constructive-interference condition $a\sin\theta_p = p\lambda,$ we have

$$a = \frac{p\lambda}{\sin\theta_p} \Rightarrow a = \frac{\lambda}{\sin 90°} = \lambda = 633 \text{ nm}$$

Section 33.5 A Closer Look at Diffraction

33.22. Model: Assume L is large so we can use the small angle approximation.
Visualize: We are given $y_{\text{max 2}} = 26$ mm.
Solve: The numerical approximation for the first secondary maximum is

$$y_{\text{max 1}} = 1.43\frac{\lambda L}{a} \Rightarrow \frac{\lambda L}{a} = \frac{y_{\text{max 1}}}{1.43} = \frac{26 \text{ mm}}{1.43} = 18 \text{ mm}$$

We do not know a, L, or λ individually, but we do not need to; we only need the combination. Now we use the small angle approximation for the minima.

$$y_p = p\frac{\lambda L}{a} \Rightarrow y_1 = (1)(18 \text{ mm}) = 18 \text{ mm}$$

Assess: This is a bit smaller than the first secondary maximum, as expected.

33.23. Model: Use the small angle approximation.
Visualize: We want to know the order m for the two-slit interference maximum that occurs at the first one-slit ($p = 1$) diffraction minimum (this is where the "missing orders" occur). We won't see the maximum for that m, but we should for the ms less than that (on both sides) plus the central one for $m = 0$. Thus the number of bright interference fringes inside the central diffraction maximum should be $2(m-1)+1$.
Solve: The first minimum for the one-slit diffraction is at

$$y_p = \frac{p\lambda L}{a} \Rightarrow y_1 = \frac{\lambda L}{a}$$

Now we solve the two-slit interference equation for m and insert the value above for y_m.

$$y_m = \frac{m\lambda L}{d} \Rightarrow m = \frac{d(y_m)}{\lambda L} = \frac{d(y_{p=1})}{\lambda L} = \frac{d(\lambda L/a)}{\lambda L} = \frac{d}{a} = \frac{0.25 \text{ mm}}{50\ \mu\text{m}} = 5$$

So $m = 5$ is the missing interference order on each side. Therefore, within the central one-slit diffraction maximum we will see $2(m-1)+1 = 2(5-1)+1 = 9$ bright interference fringes.

Assess: This seems to be in the right ballpark given the photographs and diagrams we've seen. Notice that λ and L canceled, so the answer does not depend on them, only a and d.

33.24. Model: Use the small angle approximation.
Visualize: First find the y for the point halfway between the center and the first minimum. The minimums for the two-slit interference pattern are given by

$$y'_m = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} \Rightarrow y'_0 = \left(\frac{1}{2}\right)\frac{\lambda L}{d}$$

The y we put into the intensity formula is half of the answer above: $y = \lambda L/4d$.

Solve:

$$\frac{I_{\text{double}}}{I_0}=\left[\frac{\sin(\pi ay/\lambda L)}{\pi ay/\lambda L}\right]^2\cos^2\left(\frac{\pi dy}{\lambda L}\right)=\left[\frac{\sin(\pi a(\lambda L/4d)/\lambda L)}{\pi a(\lambda L/4d)/\lambda L}\right]^2\cos^2\left(\frac{\pi d(\lambda L/4d)}{\lambda L}\right)$$

$$=\left[\frac{\sin(\pi a/4d)}{\pi a/4d}\right]^2\cos^2\left(\frac{\pi}{4}\right)$$

Do a side calculation:

$$\frac{\pi a}{4d}=\frac{\pi(0.12\text{ mm})}{4(0.30\text{ mm})}=0.314$$

Insert this back into the intensity equation.

$$\frac{I_{\text{double}}}{I_0}=\left[\frac{\sin(\pi a/4d)}{\pi a/4d}\right]^2\cos^2\left(\frac{\pi}{4}\right)=\left[\frac{\sin(0.314)}{0.314}\right]^2\cos^2\left(\frac{\pi}{4}\right)=0.48$$

Assess: This seems to be in the right ballpark given the photographs and diagrams we've seen. Notice that λ and L canceled, so the answer does not depend on them, only a and d.

Section 33.6 Circular-Aperture Diffraction

33.25. Model: Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.
Solve: The width of the central maximum for a circular aperture of diameter D is

$$w=\frac{2.44\lambda L}{D}=\frac{(2.44)(500\times10^{-9}\text{ m})(2.0\text{ m})}{0.50\times10^{-3}\text{ m}}=4.9\text{ mm}$$

33.26. Model: Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.
Visualize: The intensity pattern will look like Figure 33.21.
Solve: According to Equation 33.30, the angle that locates the first minimum in intensity is

$$\theta_1=\frac{1.22\lambda}{D}=\frac{(1.22)(2.5\times10^{-6}\text{ m})}{0.20\times10^{-3}\text{ m}}=0.01525\text{ rad}=0.874°$$

These should be rounded to $0.015\text{ rad}=0.87°$.

33.27. Model: Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.
Visualize: The intensity pattern will look like Figure 33.21.
Solve: From Equation 33.31,

$$L=\frac{Dw}{2.44\lambda}=\frac{(0.12\times10^{-3}\text{ m})(1.0\times10^{-2}\text{ m})}{2.44(633\times10^{-9}\text{ m})}=78\text{ cm}$$

33.28. Model: Use the small angle approximation. Assume the screen is the same distance behind both apertures, that is, $L_{\text{red}}=L_{\text{violet}}=L$.
Visualize: We are given the width of the central maximum from a circular aperture with diameter D:

$$w=\frac{2.44\lambda L}{D}$$

The problem states that we want $w_{\text{red}}=w_{\text{violet}}=w$.

Solve: Solve the above equation for D.

$$\frac{D_{\text{red}}}{D_{\text{violet}}}=\frac{\dfrac{2.44\lambda_{\text{red}}L}{w}}{\dfrac{2.44\lambda_{\text{violet}}L}{w}}=\frac{\lambda_{\text{red}}}{\lambda_{\text{violet}}}=\frac{670\text{ nm}}{410\text{ nm}}=1.6$$

Assess: This is a reasonable ratio of diameters.

33.29. Model: Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.
Visualize: The intensity pattern will look like Figure 33.21.
Solve: From Equation 33.31, the diameter of the circular aperture is

$$D=\frac{2.44\lambda L}{w}=\frac{2.44(633\times10^{-9}\text{ m})(4.0\text{ m})}{2.5\times10^{-2}\text{ m}}=0.25\text{ mm}$$

Section 33.8 Interferometers

33.30. Model: An interferometer produces a new maximum each time L_2 increases by $\frac{1}{2}\lambda$ causing the path-length difference Δr to increase by λ.
Visualize: Please refer to the interferometer in Figure 33.24.
Solve: From Equation 33.30, the number of fringe shifts is

$$\Delta m=\frac{2\Delta L_2}{\lambda}=\frac{2(1.00\times10^{-2}\text{ m})}{656.45\times10^{-9}\text{ m}}=30{,}467$$

33.31. Model: An interferometer produces a new maximum each time L_2 increases by $\frac{1}{2}\lambda$ causing the path-length difference Δr to increase by λ.
Visualize: Please refer to the interferometer in Figure 33.24.
Solve: From Equation 33.30, the wavelength is

$$\lambda=\frac{2\Delta L_2}{\Delta m}=\frac{2(100\times10^{-6}\text{ m})}{500}=4.0\times10^{-7}\text{ m}=400\text{ nm}$$

33.32. Model: An interferometer produces a new maximum each time L_2 increases by $\frac{1}{2}\lambda$ causing the path-length difference Δr to increase by λ.
Visualize: Please refer to the interferometer in Figure 33.24.
Solve: For sodium light of the longer wavelength (λ_1) and of the shorter wavelength (λ_2),

$$\Delta L=m\frac{\lambda_1}{2}\qquad \Delta L=(m+1)\frac{\lambda_2}{2}$$

We want the same path difference $2(L_2-L_1)$ to correspond to one extra wavelength for the sodium light of shorter wavelength (λ_2). Thus, we combine the two equations to obtain:

$$m\frac{\lambda_1}{2}=(m+1)\frac{\lambda_2}{2}\Rightarrow m(\lambda_1-\lambda_2)=\lambda_2\Rightarrow m=\frac{\lambda_2}{\lambda_1-\lambda_2}=\frac{589.0\text{ nm}}{589.6\text{ nm}-589.0\text{ nm}}=981.67\cong982$$

Thus, the distance by which M_2 is to be moved is

$$\Delta L=m\frac{\lambda_1}{2}=982\left(\frac{589.6\text{ nm}}{2}\right)=0.2895\text{ mm}$$

Problems

33.33. Model: Two closely spaced slits produce a double-slit interference pattern with the intensity graph looking like Figure 33.4(b). The intensity pattern due to a single-slit diffraction looks like Figure 33.15. Both the spectra consist of a central maximum flanked by a series of secondary maxima and dark fringes.

Solve: (a) The light intensity shown in Figure P33.33 corresponds to a double-slit aperture. This is because the fringes are equally spaced and the decrease in intensity with increasing fringe order occurs slowly.

(b) From Figure P33.33, the fringe spacing is $\Delta y = 1.0 \text{ cm} = 1.0\times10^{-2} \text{ m}$. Therefore,

$$\Delta y = \frac{\lambda L}{d} \Rightarrow d = \frac{\lambda L}{\Delta y} = \frac{(6.00\times10^{-9} \text{ m})(2.5 \text{ m})}{0.010 \text{ m}} = 0.15 \text{ mm}$$

33.34. Model: Two closely spaced slits produce a double-slit interference pattern with the intensity graph looking like Figure 33.4(b). The intensity pattern due to a single-slit diffraction looks like Figure 33.15. Both the spectra consist of a central maximum flanked by a series of secondary maxima and dark fringes.

Solve: (a) The light intensity shown in Figure P33.34 corresponds to a single-slit aperture. This is because the central maximum is twice the width and much brighter than the secondary maximum.

(b) From Figure P33.34, the separation between the central maximum and the first minimum is $y_1 = 1.0 \text{ cm} = 1.0\times10^{-2}$ m. Therefore, using the small-angle approximation, Equation 33.21 gives the condition for the dark minimum:

$$y_p = \frac{pL\lambda}{d} \Rightarrow a = \frac{L\lambda}{y_1} = \frac{(2.5 \text{ m})(600\times10^{-9} \text{ m})}{1.0\times10^{-2} \text{ m}} = 0.15 \text{ mm}$$

33.35. Model: Two closely spaced slits produce a double-slit interference pattern.

Visualize: The interference pattern looks like the photograph of Figure 33.4(b).

Solve: In a span of 12 fringes, there are 11 gaps between them. The formula for the fringe spacing is

$$\Delta y = \frac{\lambda L}{d} \Rightarrow \left(\frac{52\times10^{-3} \text{ m}}{11}\right) = \frac{(633\times10^{-9} \text{ m})(3.0 \text{ m})}{d} \Rightarrow \text{d} = 0.40 \text{ mm}$$

Assess: This is a reasonable distance between the slits, ensuring $d/L = 1.34\times10^{-4} \ll 1$.

33.36. Solve: According to Equation 33.7, the fringe spacing between the m fringe and the $m + 1$ fringe is $\Delta y = \lambda L/d$. Δy can be obtained from Figure P33.36. The separation between the $m = 2$ fringes is 2.0 cm, implying that the separation between the two consecutive fringes is $\frac{1}{4}(2.0 \text{ cm}) = 0.50 \text{ cm}$. Thus,

$$\Delta y = 0.50\times10^{-2} \text{ m} = \frac{\lambda L}{d} \Rightarrow L = \frac{d\Delta y}{\lambda} = \frac{(0.20\times10^{-3} \text{ m})(0.50\times10^{-2} \text{ m})}{600\times10^{-9} \text{ m}} = 167 \text{ cm}$$

Assess: A distance of 167 cm from the slits to the screen is reasonable.

33.37. Solve: According to Equation 33.7, the fringe spacing between the m fringe and the $m + 1$ fringe is $\Delta y = \lambda L/d$. Δy can be obtained from Figure P33.36. Because the separation between the $m = 2$ fringes is 2.0 cm, two consecutive fringes are $\Delta y = \frac{1}{4}(2.0 \text{ cm}) = 0.50 \text{ cm}$ apart. Thus,

$$\Delta y = 0.50\times10^{-2} \text{ m} = \frac{\lambda L}{d} \Rightarrow \lambda = \frac{d\Delta y}{L} = \frac{(0.20\times10^{-3} \text{ m})(0.50\times10^{-2} \text{ m})}{2.0 \text{ m}} = 500 \text{ nm}$$

33.38. Solve: The intensity of light of a double-slit interference pattern at a position y on the screen is

$$I_{\text{double}} = 4I_1 \cos^2\left(\frac{\pi d}{\lambda L} y\right)$$

where I_1 is the intensity of the light from each slit alone. At the center of the screen, that is, at $y = 0$ m, $I_1 = \frac{1}{4} I_{\text{double}}$. From Figure P33.36, I_{double} at the central maximum is $12\ \text{mW/m}^2$. So, the intensity due to a single slit is $I_1 = 3\ \text{mW/m}^2$.

33.39. Model: A diffraction grating produces an interference pattern.
Visualize: The interference pattern looks like the diagram in Figure 33.9.
Solve: 500 lines per mm on the diffraction grating gives a spacing between the two lines of $d = 1\ \text{mm}/500 = (1\times10^{-3}\ \text{m})/500 = 2.0\times10^{-6}\ \text{m}$. The wavelength diffracted at angle $\theta_m = 30°$ in order *m* is

$$\lambda = \frac{d\sin\theta_m}{m} = \frac{(2.0\times10^{-6}\ \text{m})\sin 30°}{m} = \frac{1000\ \text{nm}}{m}$$

We're told it is *visible* light that is diffracted at $30°$, and the wavelength range for visible light is 400–700 nm. Only $m = 2$ gives a visible light wavelength, so $\lambda = 500$ nm.

33.40. Model: Each wavelength of light is diffracted at a different angle by a diffraction grating.
Solve: Light with a wavelength of 501.5 nm creates a first-order fringe at *y* = 21.90 cm. This light is diffracted at angle

$$\theta_1 = \tan^{-1}\left(\frac{21.90\ \text{cm}}{50.00\ \text{cm}}\right) = 23.65°$$

We can then use the diffraction equation $d\sin\theta_m = m\lambda$, with $m = 1$, to find the slit spacing:

$$d = \frac{\lambda}{\sin\theta_1} = \frac{501.5\ \text{nm}}{\sin(23.65°)} = 1250\ \text{nm}$$

The unknown wavelength creates a first-order fringe at $y = 31.60$ cm, or at angle

$$\theta_1 = \tan^{-1}\left(\frac{31.60\ \text{cm}}{50.00\ \text{cm}}\right) = 32.29°$$

With the split spacing now known, we find that the wavelength is

$$\lambda = d\sin\theta_1 = (1250\ \text{nm})\sin(32.29°) = 667.8\ \text{nm}$$

Assess: The distances to the fringes and the first wavelength were given to 4 significant figures. Consequently, we can determine the unknown wavelength to 4 significant figures.

33.41. Model: Assume the incident light is coherent and monochromatic.
Solve:
(a) Where the path length difference is λ between adjacent slits there will be constructive interference from all three slits, and $I_{\max} = N^2 I_1$ applies.

$$I_{\max} = 3^2 I_1 = 9I_1$$

(b) For the case where the path length difference is $\lambda/2$ adjacent slits will produce destructive interference, so the third slit will just give a total intensity of I_1.

33.42. Model: We will assume that the listeners did not hear any other loud spots between the center and 1.4 m on each side, $m = 1$. We'll use the diffraction grating equations. First solve Equation 33.16 for θ_1 and insert it into Equation 33.15.
We need the wavelength of the sound waves, and we'll use the fundamental relationship for periodic waves to get it.

$$\lambda = \frac{v}{f} = \frac{340\ \text{m/s}}{10000\ \text{Hz}} = 3.4\ \text{cm} = 0.034\ \text{m}$$

We are also given $L = 10$ m and $y_1 = 1.4$ m.

Solve:

$$\theta_1 = \tan^{-1}\left(\frac{y_1}{L}\right) = \tan^{-1}\left(\frac{1.4\text{ m}}{10\text{ m}}\right) = 0.14\text{ rad}$$

(This may be small enough to use the small-angle approximation, but we are almost finished with the problem without it, so maybe we can save the approximation and try it in the assess step.)

$$d = \frac{(1)\lambda}{\sin\theta_1} = \frac{0.034\text{ m}}{\sin(0.14\text{ rad})} = 0.25\text{ m} = 25\text{ cm}$$

Assess: These numbers seem reasonable given the size (wavelength) of sound waves.
With the small-angle approximation, $\theta \approx \sin\theta \approx \tan\theta$,

$$d = \frac{m\lambda}{y_m/L} = \frac{1(0.034\text{ m})}{1.4\text{ m}/10\text{ m}} = 24\text{ cm}$$

This is almost the same, but rounded down to two significant figures rather than up. The θ_1 we computed seemed almost small enough to use the approximation, and it would probably be OK for many applications when the angle is this small.

33.43. Model: A diffraction grating produces an interference pattern.
Visualize: The interference pattern looks like the diagram of Figure 33.9.
Solve: (a) A grating diffracts light at angles $\sin\theta_m = m\lambda/d$. The distance between adjacent slits is $d = 1\text{ mm}/600 = 1.667\times10^{-6}\text{ m} = 1667\text{ nm}$. The angle of the $m = 1$ fringe is

$$\theta_1 = \sin^{-1}\left(\frac{500\text{ nm}}{1667\text{ nm}}\right) = 17.46°$$

The distance from the central maximum to the $m = 1$ bright fringe on a screen at distance L is

$$y_1 = L\tan\theta_1 = (2\text{ m})\tan 17.46^\circ = 0.629\text{ m}$$

(Note that the small angle approximation is *not* valid for the maxima of diffraction gratings, which almost always have angles $> 10°$.) There are two $m = 1$ bright fringes, one on either side of the central maximum. The distance between them is $\Delta y = 2y_1 = 1.258\text{ m} \approx 1.3\text{ m}$.

(b) The maximum number of fringes is determined by the maximum value of m for which $\sin\theta_m$ does not exceed 1 because there are no physical angles for which $\sin\theta > 1$. In this case,

$$\sin\theta_m = \frac{m\lambda}{d} = \frac{m(500\text{ nm})}{1667\text{ nm}}$$

We can see by inspection that $m = 1$, $m = 2$, and $m = 3$ are acceptable, but $m = 4$ would require a physically impossible $\sin\theta_4 > 1$. Thus, there are three bright fringes on either side of the central maximum plus the central maximum itself for a total of seven bright fringes.

33.44. Model: Assume the screen is centered behind the slit. We actually want to solve for *m*, but given the other data, it is unlikely that we will get an integer from the equations for the edge of the screen, so we will have to truncate our answer to get the largest order fringe on the screen.
Visualize: Refer to Figure 33.8. Use Equation 33.15: $d\sin\theta_m = m\lambda$, and Equation 33.16: $y_m = L\tan\theta_m$. We are given $\lambda = 510$ nm, $L = 2.0$ m, and $d = \frac{1}{500}$ mm. As mentioned above, we are not guaranteed that a bright fringe will occur exactly at the edge of the screen, but we will kind of assume that one does and set $y_m = 1.0$ m; if we do not get an integer for *m* then the fringe was not quite at the edge of the screen and we will truncate our answer to get an integer *m*.

Solve: Solve Equation 33.16 for θ_m and insert it in Equation 33.15.

$$\theta_m = \tan^{-1}\frac{y_m}{L}$$

Solve Equation 33.15 for m.

$$m = \frac{d}{\lambda}\sin\theta_m = \frac{d}{\lambda}\sin\left(\tan^{-1}\frac{y_m}{L}\right) = \frac{\frac{1}{500}\text{ mm}}{510\text{ nm}}\sin\left(\tan^{-1}\frac{1.0\text{ m}}{2.0\text{ m}}\right) = 1.8$$

Indeed, we did not get an integer, so truncate 1.8 to get $m = 1$. This means we will see three fringes, one for $m = 0$, and one on each side for $m = \pm 1$.

Assess: Our answer fits with the statement in the text: "Practical gratings, with very small values for d, display only a few orders."

33.45. Model: A diffraction grating produces an interference pattern that is determined by both the slit spacing and the wavelength used. The visible spectrum spans the wavelengths 400 nm to 700 nm.

Solve: According to Equation 33.16, the distance y_m from the center to the *m*th maximum is $y_m = L\tan\theta_m$. The angle of diffraction is determined by the constructive-interference condition $d\sin\theta_m = m\lambda$, where $m = 0, 1, 2, 3, \ldots$ The width of the rainbow for a given fringe order is thus $w = y_{\text{red}} - y_{\text{violet}}$. The slit spacing is

$$d = \frac{1\text{ mm}}{600} = \frac{1.0\times10^{-3}\text{ m}}{600} = 1.6667\times10^{-6}\text{ m}$$

For the red wavelength and for the $m = 1$ order,

$$d\sin\theta_1 = (1)\lambda \Rightarrow \theta_1 = \sin^{-1}\frac{\lambda}{d} = \sin^{-1}\frac{700\times10^{-9}\text{ m}}{1.6667\times10^{-6}\text{ m}} = 24.83°$$

From the equation for the distance of the fringe,

$$y_{\text{red}} = L\tan\theta_1 = (2.0\text{ m})\tan(24.83°) = 92.56\text{ cm}$$

Likewise for the violet wavelength,

$$\theta_1 = \sin^{-1}\left(\frac{400\times10^{-9}\text{ m}}{1.6667\times10^{-6}\text{ m}}\right) = 13.88° \Rightarrow y_{\text{violet}} = (2.0\text{ m})\tan(13.88°) = 49.42\text{ cm}$$

The width of the rainbow is thus $92.56\text{ cm} - 49.42\text{ cm} = 43.14\text{ cm} \approx 43\text{ cm}$.

33.46. Model: Assume that we are looking at the first-order spectral line in each case: $m = 1$. We do not assume the small angle approximation for the diffraction grating.

Visualize: We are given $L = 15.0$ cm, $y_{\text{known}} = 9.95$ cm, $y_{\text{A}} = 8.55$ cm, $y_{\text{B}} = 12.15$ cm, and $\lambda_{\text{known}} = 461$ nm.

Solve: We are given the position of a bright fringe from a diffraction grating for first order where $m = 1$:

$$y = L\tan\left[\sin^{-1}\frac{\lambda}{d}\right]$$

We do not know d, one of the specs on the grating.

Solve the above equation for λ/d.

$$\frac{\lambda}{d} = \sin\left[\tan^{-1}\frac{y}{L}\right]$$

Now use ratios and d will cancel out.

$$\frac{\lambda_A/d}{\lambda_{known}/d} = \frac{\sin\left[\tan^{-1}\frac{y_A}{L}\right]}{\sin\left[\tan^{-1}\frac{y_{known}}{L}\right]} \Rightarrow$$

$$\lambda_A = \lambda_{known}\frac{\sin\left[\tan^{-1}\frac{y_A}{L}\right]}{\sin\left[\tan^{-1}\frac{y_{known}}{L}\right]} = (461\text{ nm})\frac{\sin\left[\tan^{-1}\frac{8.55\text{ cm}}{15.0\text{ cm}}\right]}{\sin\left[\tan^{-1}\frac{9.95\text{ cm}}{15.0\text{ cm}}\right]} = 413\text{ nm}$$

Repeat the calculation with $y_B = 12.15\text{ cm}$ and get $\lambda_B = 525\text{ nm}$.

Assess: These answers are in the right ballpark. We expected $\lambda_A < \lambda_{known}$ and $\lambda_B > \lambda_{known}$.

33.47. Model: Assume that we are looking at the first-order spectral line case: $m = 1$. Assume the small angle approximation: $\tan\theta \approx \sin\theta \approx \theta$.

Visualize: We are given $L = 12.0\text{ cm}$, $d = (333\text{ line/mm})^{-1} = 0.00300\text{ mm}$, $y_B = 12.15\text{ cm}$, and $\lambda = 550\text{ nm}$.

Solve: (a) With $\tan\theta \approx \sin\theta \approx \theta$ then $d\sin\theta = \lambda$ becomes $d\theta = \lambda$ and $y = L\tan\theta$ becomes $y = L\theta$. So the position of the first bright fringe is

$$y = L\theta = \frac{L\lambda}{d}$$

(b) Both L and d are constants, so given $y = L\lambda/d$

$$\Delta y = \frac{L}{d}\Delta\lambda$$

(c) Solve for the resolution $\Delta\lambda_{min}$ with $\Delta y = (100\text{ pixels/mm})^{-1} = 0.0100\text{ mm}$.

$$\Delta\lambda_{min} = \frac{d}{L}\Delta y = \frac{0.00300\text{ mm}}{12.0\text{ cm}}0.0100\text{ mm} = 0.250\text{ nm}$$

Assess: These answers are in the right ballpark. We expected $\lambda_A < \lambda_{known}$ and $\lambda_B > \lambda_{known}$.

33.48. Model: A diffraction grating produces an interference pattern that is determined by both the slit spacing and the wavelength used.

Solve: (a) If blue light (the shortest wavelengths) is diffracted at angle θ, then red light (the longest wavelengths) is diffracted at angle $\theta + 30°$. In the first order, the equations for the blue and red wavelengths are

$$\sin\theta = \frac{\lambda_b}{d} \qquad d\sin(\theta + 30°) = \lambda_r$$

Combining the two equations we get for the red wavelength,

$$\lambda_r = d(\sin\theta\cos 30° + \cos\theta\sin 30°) = d(0.8660\sin\theta + 0.50\cos\theta) = d\left(\frac{\lambda_b}{d}\right)0.8660 + d(0.50)\sqrt{\left(1 - \frac{\lambda_b^2}{d^2}\right)}$$

$$\Rightarrow (0.50)d\sqrt{1 - \frac{\lambda_b^2}{d^2}} = \lambda_r - 0.8660\lambda_b \Rightarrow (0.50)^2(d^2 - \lambda_b^2) = (\lambda_r - 0.8660\lambda_b)^2$$

$$\Rightarrow d = \sqrt{\left(\frac{\lambda_r - 0.8660\lambda_b}{0.50}\right)^2 + \lambda_b^2}$$

Using $\lambda_{\text{b}} = 400\times10^{-9}$ m and $\lambda_{\text{r}} = 700\times10^{-9}$ m, we get $d = 8.125\times10^{-7}$ m. This value of *d* corresponds to

$$\frac{1\text{ mm}}{d} = \frac{1.0\times10^{-3}\text{ m}}{8.125\times10^{-7}\text{ m}} = 1230\text{ lines/mm}$$

(b) Using the value of *d* from part (a) and $\lambda = 589\times10^{-9}$ m, we can calculate the angle of diffraction as follows:

$$d\sin\theta_1 = (1)\lambda \Rightarrow (8.125\times10^{-7}\text{ m})\sin\theta_1 = 589\times10^{-9}\text{ m} \Rightarrow \theta_1 = 46.5°$$

33.49. Model: A diffraction grating produces an interference pattern that is determined by both the slit spacing and the wavelength used.

Solve: An 800 line/mm diffraction grating has a slit spacing $d = (1.0\times10^{-3}\text{ m})/800 = 1.25\times10^{-6}$ m. Referring to Figure P33.49, the angle of diffraction is given by

$$\tan\theta_1 = \frac{y_1}{L} = \frac{0.436\text{ m}}{1.0\text{ m}} = 0.436 \Rightarrow \theta_1 = 23.557° \Rightarrow \sin\theta_1 = 0.400$$

Using the constructive-interference condition $d\sin\theta_m = m\lambda$,

$$\lambda = \frac{d\sin\theta_1}{1} = (1.25\times10^{-6}\text{ m})(0.400) = 500\text{ nm}$$

We can obtain the same value of λ by using the second-order interference fringe. We first obtain θ_2:

$$\tan\theta_2 = \frac{y_2}{L} = \frac{0.436\text{ m} + 0.897\text{ m}}{1.0\text{ m}} = 1.333 \Rightarrow \theta_2 = 53.12° \Rightarrow \sin\theta_2 = 0.800$$

Using the constructive-interference condition,

$$\lambda = \frac{d\sin\theta_2}{2} = \frac{(1.25\times10^{-6}\text{ m})(0.800)}{2} = 500\text{ nm}$$

Assess: Calculations with the first-order and second-order fringes of the interference pattern give the same value for the wavelength.

33.50. Model: A diffraction grating produces an interference pattern that is determined by both the slit spacing and the wavelength used.

Solve: From Figure P33.49,

$$\tan\theta_1 = \frac{0.436\text{ m}}{1.0\text{ m}} = 0.436 \Rightarrow \theta_1 = 23.557° \Rightarrow \sin\theta_1 = 0.400$$

Using the constructive-interference condition $d\sin\theta_m = m\lambda$,

$$d\sin 23.557° = (1)(600\times10^{-9}\text{ m}) \Rightarrow d = \frac{600\times10^{-9}\text{ m}}{\sin(23.557°)} = 1.50\times10^{-6}\text{ m}$$

Thus, the number of lines per millimeter is

$$\frac{1.0\times10^{-3}\text{ m}}{1.50\times10^{-6}\text{ m}} = 670\text{ lines/mm}$$

Assess: The same answer is obtained if we perform calculations using information about the second-order bright constructive-interference fringe.

33.51. Model: A narrow slit produces a single-slit diffraction pattern.

Visualize: The diffraction-intensity pattern from a single slit will look like Figure 33.15.

Solve: The dark fringes are located at

$$y_p = \frac{p\lambda L}{a} \quad p = 0, 1, 2, 3, \ldots$$

The locations of the first and third dark fringes are

$$y_1 = \frac{\lambda L}{a} \qquad y_3 = \frac{3\lambda L}{a}$$

Subtracting the two equations,

$$(y_3 - y_1) = \frac{2\lambda L}{a} \Rightarrow a = \frac{2\lambda L}{y_3 - y_1} = \frac{2(589\times10^{-9}\ \text{m})(0.75\ \text{m})}{7.5\times10^{-3}\ \text{m}} = 0.12\ \text{mm}$$

33.52. Visualize: The relationship between the diffraction grating spacing d, the angle at which a particular order of constructive interference occurs θ_m, the wavelength of the light, and the order of the constructive interference m is $d\sin\theta_m = m\lambda$. Also note $N = 1/d$.

Solve: The first-order diffraction angle for green light is

$$\theta_1 = \sin^{-1}(\lambda/d) = \sin^{-1}(5.5\times10^{-7}\ \text{m}/2.0\times10^{-6}\ \text{m}) = \sin^{-1}(0.275) = 0.278\ \text{rad} = 16°$$

Assess: This is a reasonable angle for a first-order maximum.

33.53. Model: The peacock feather is acting as a reflection grating, so we may use Equation 33.15: $d\sin\theta_m = m\lambda$, with $m = 1$ (we are told it is first-order diffraction), $\lambda = 470$ nm, and $\theta_1 = 15° = 0.262$ rad.

Solve: Solve Equation 33.15 for d.

$$d = \frac{m\lambda}{\sin\theta_m} = \frac{(1)(470\times10^{-9}\ \text{m})}{\sin(0.262\ \text{rad})} = 1.8\ \mu\text{m}$$

Assess: The answer is small, but plausible for the bands on the barbules.

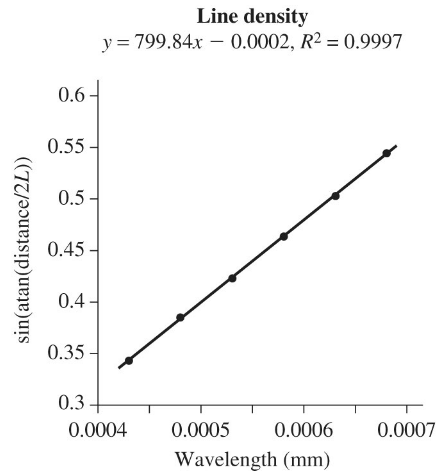
33.54. Model: Assume the incident light is coherent and monochromatic.
Visualize: The distances given in the table are $2y_1$ since the measurement is between the two first-order fringes. Also note that $m = 1$ in this problem.
Solve: The equation for diffraction gratings is $d\sin\theta_m = m\lambda$ where d is the distance (in mm) between slits; we seek $\frac{1}{d}$, the number of lines per mm.

$$\sin\theta_1 = \frac{1}{d}\lambda$$

From $y_m = L\tan\theta_m$ and $y_1 = \frac{\text{distance}}{2}$, where "distance" is the distance between first-order fringes as given in the table, we arrive at

$$\sin\left(\tan^{-1}\left(\frac{\text{distance}}{2L}\right)\right) = \frac{1}{d}\lambda$$

This leads us to believe that a graph of $\sin\left(\tan^{-1}\left(\frac{\text{distance}}{2L}\right)\right)$ vs. λ will produce a straight line whose slope is $\frac{1}{d}$ and whose intercept is zero.

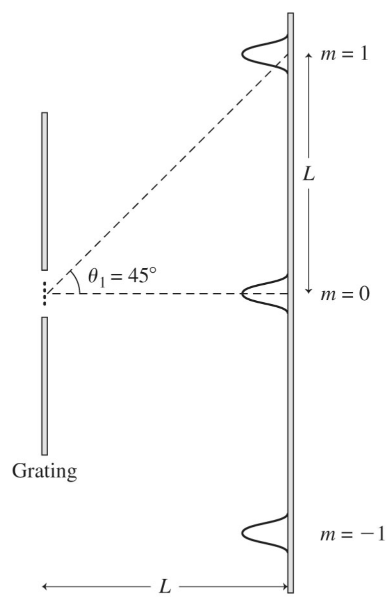


We see from the spreadsheet that the linear fit is excellent and that the slope is $799.84\ \text{mm}^{-1}$ and the intercept is very small. Because the number of lines per mm is always reported as an integer we round this answer to 800 lines/mm.

Assess: This number of lines per mm is a typical number for a decent grating.

33.55. Model: Small angle approximations would not apply here.

Visualize: We see from the figure that if $y_1 = L$ then $\theta_1 = 45°$.



Solve: From $d\sin\theta_m = m\lambda$ we get

$$\sin\theta_1 = \frac{\lambda}{d}$$

but $\theta_1 = 45°$, and $\sin 45° = \frac{\sqrt{2}}{2}$.

$$\lambda = \frac{\sqrt{2}}{2}d$$

Assess: This large diffraction angle corresponds to a wavelength that is about the same size as the slit spacing.

33.56. Model: A narrow slit produces a single-slit diffraction pattern.
Visualize: The dark fringes in this diffraction pattern are given by Equation 33.21:

$$y_p = \frac{p\lambda L}{a} \qquad p = 1, 2, 3, \ldots$$

We note that the first minimum in the figure is 0.50 cm away from the central maximum. We are given a = 0.02 nm and $L = 1.5$ m.
Solve: Solve the above equation for λ.

$$\lambda = \frac{y_p a}{pL} = \frac{(0.50\times10^{-2}\ \text{m})(0.20\times10^{-3}\ \text{m})}{(1)(1.5\ \text{m})} = 670\ \text{nm}$$

Assess: 670 nm is in the visible range.

33.57. Model: A narrow slit produces a single-slit diffraction pattern.
Solve: The dark fringes in this diffraction pattern are given by Equation 33.21:

$$y_p = \frac{p\lambda L}{a} \qquad p = 1, 2, 3, \ldots$$

We note from Figure P33.56 that the first minimum is 0.50 cm away from the central maximum. Thus,

$$L = \frac{ay_p}{p\lambda} = \frac{(0.15\times10^{-3}\ \text{m})(0.50\times10^{-2}\ \text{m})}{(1)(600\times10^{-9}\ \text{m})} = 1.3\ \text{nm}$$

Assess: This is a typical slit to screen separation.

33.58. Model: Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.
Solve: Within the small angle approximation, which is almost always valid for the diffraction of light, the width of the central maximum is

$$w = 2y_1 = 2.44\frac{\lambda L}{D}$$

From Figure P33.56, $w = 1.0$ cm, so

$$D = \frac{2.44\lambda L}{w} = \frac{2.44(500\times10^{-9}\ \text{m})(1.0\ \text{m})}{(1.0\times10^{-2}\ \text{m})} = 0.12\ \text{mm}$$

Assess: This is a typical size for an aperture to show diffraction.

33.59. Model: Assume that we are looking at the first-order spectral line case: m = 1. Assume the small angle approximation: $\tan\theta \approx \sin\theta \approx \theta$.
Visualize: We are given $L = 1.5$ m, $y = 1.6$ cm, and $m = 5$.
Solve: (a) The fifth order is missing likely because it falls on the first minimum of the single-slit diffraction pattern due to the width of the slits.
(b) The first minimum for the single-slit diffraction occurs at

$$y = \frac{\lambda L}{a}$$

This value of y is where the fifth maximum for the two-slit interference pattern would have been.

$$y_5 = \frac{5\lambda L}{d} \Rightarrow \frac{d}{\lambda} = \frac{5L}{y}$$

Set the two ys equal to each other and solve for a.

$$a = \frac{\lambda L}{y} = \frac{\lambda L}{(5\lambda L/d)} = \frac{d}{\lambda}\frac{\lambda L}{5L} = \left(\frac{5L}{y}\right)\frac{\lambda L}{5L} = \frac{\lambda L}{y} = \frac{(530\ \text{nm})(1.5\ \text{m})}{0.016\ \text{m}} = 50\ \mu\text{m}$$

Assess: This is a reasonable slit width.

33.60. Model: The slit produces a single-slit diffraction pattern. The angles are small enough to justify the small-angle approximation.
Visualize: We are given $L = 0.70$ m, $a = 35$ μm, and $y = 7.2$ mm.
Solve: The intensity at position y is

$$I = I_0\left[\frac{\sin(\pi ay/\lambda L)}{\pi ay/\lambda L}\right]^2$$

The intensity will have fallen to 25% of maximum I_0 when

$$\frac{\sin(\pi ay/\lambda L)}{\pi ay/\lambda L} = \frac{1}{2}$$

Use the trial and error method as shown in the example in the text. Let $x = \pi ay/\lambda L.$ Then the equation we want to solve is $\sin x/x = 1/2 = 0.50$ where x is in radians. The first minimum, $y_1 = \lambda L/a,$ has $x = \pi$ rad, and we know that the solution is less than this. We make our first guess bigger than the one in the example because the intensity has fallen off more in this problem.

First try: x = 2.0 rad gives $\sin x/x = 0.455.$
Second try: x = 1.9 rad gives $\sin x/x = 0.498.$
Third try: x = 1.89 rad gives $\sin x/x = 0.502.$

So $1.89 < x < 1.90$; since the one is about as high as the other is low, we take the average and say $x = 1.895$ which gives $\sin x / x = 0.5002.$ This is close enough. We now know x; so we solve for $\lambda.$

$$x = \frac{\pi ay}{\lambda L} \Rightarrow \lambda = \frac{\pi ay}{xL} = \frac{\pi(35\ \mu\text{m})(7.2\ \text{mm})}{1.895(0.70\ \text{m})} = 600\ \text{nm}$$

to two significant figures.
Assess: The function $\sin x / x$ is called the sinc function. It is best to find x to more than two significant figures for the final answer to be good to two significant figures. The trial and error method above will not work with the calculator in degree mode. 600 nm is a typical wavelength.

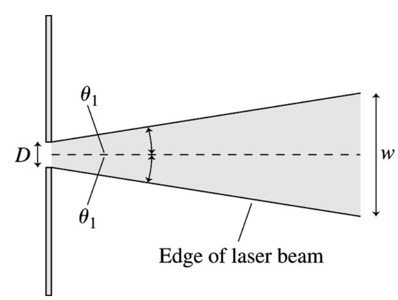
33.61. Model: Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.
Solve: Within the small-angle approximation, the width of the central maximum is

$$w = 2.44\frac{\lambda L}{D}$$

Because $w = D,$ we have

$$D = 2.44\frac{\lambda L}{D} \Rightarrow D = \sqrt{2.44\lambda L} \Rightarrow D = \sqrt{(2.44)(633\times10^{-9}\ \text{m})(0.50\ \text{m})} = 0.88\ \text{mm}$$

33.62. Model: The laser beam is diffracted through a circular aperture.
Visualize:



Solve: (a) No. The laser light emerges through a circular aperture at the end of the laser. This aperture causes diffraction, hence the laser beam must gradually spread out. The diffraction angle is small enough that the laser beam *appears* to be parallel over short distances. But if you observe the laser beam at a large distance it is easy to see that the diameter of the beam is slowly increasing.
(b) The position of the first minimum in the diffraction pattern is more or less the "edge" of the laser beam. For diffraction through a circular aperture, the first minimum is at an angle

$$\theta_1 = \frac{1.22\lambda}{D} = \frac{1.22(633\times10^{-9}\text{ m})}{0.0010\text{ m}} = 7.72\times10^{-4}\text{ rad} = 0.044^\circ$$

(c) The diameter of the laser beam is the width of the diffraction pattern:

$$w = \frac{2.44\lambda L}{D} = \frac{2.44(633\times10^{-9}\text{ m})(3.0\text{ m})}{0.0010\text{ m}} = 4.6\text{ mm}$$

(d) At $L = 1\text{ km} = 1000\text{ m}$, the diameter is

$$w = \frac{2.44\lambda L}{D} = \frac{2.44(633\times10^{-9}\text{ m})(1000\text{ m})}{0.0010\text{ m}} = 1.5\text{ m}$$

33.63. Model: Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.
Solve: (a) Because the visible spectrum spans wavelengths from 400 nm to 700 nm, we take the average wavelength of sunlight to be 550 nm.
(b) Within the small-angle approximation, the width of the central maximum is

$$w = 2.44\frac{\lambda L}{D} \Rightarrow (1\times10^{-2}\text{ m}) = (2.44)\frac{(550\times10^{-9}\text{ m})(3\text{ m})}{D} \Rightarrow D = 4.03\times10^{-4}\text{ m} = 0.40\text{ mm}$$

33.64. Model: The antenna is a circular aperture through which the microwaves diffract.
Solve: (a) Within the small-angle approximation, the width of the central maximum of the diffraction pattern is $w = 2.44\lambda L/D$. The wavelength of the radiation is

$$\lambda = \frac{c}{f} = \frac{3\times10^8\text{ m/s}}{12\times10^9\text{ Hz}} = 0.025\text{ m} \Rightarrow w = \frac{2.44(0.025\text{ m})(30\times10^3\text{ m})}{2.0\text{ m}} = 920\text{ m}$$

That is, the diameter of the beam has increased from 2.0 m to 915 m, a factor of 458.
(b) The average microwave intensity is

$$I = \frac{P}{\text{area}} = \frac{100\times10^3\text{ W}}{\pi\left[\frac{1}{2}(915\text{ m})\right]^2} = 0.15\text{ W/m}^2$$

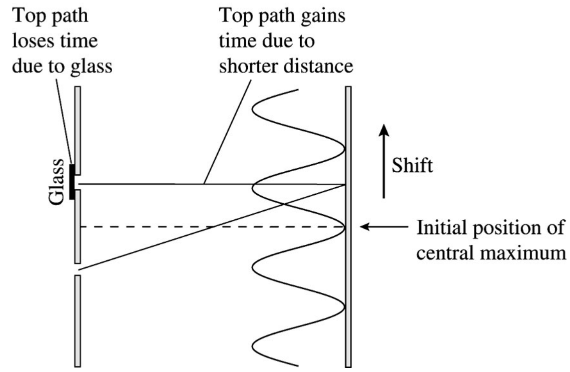
33.65. Model: The laser light is diffracted by the circular opening of the laser from which the beam emerges.
Solve: The diameter of the laser beam is the width of the central maximum. We have

$$w = \frac{2.44\lambda L}{D} \Rightarrow D = \frac{2.44\lambda L}{w} = \frac{2.44(532\times10^{-9}\text{ m})(3.84\times10^8\text{ m})}{1000\text{ m}} = 0.50\text{ m}$$

In other words, the laser beam must emerge from a laser of diameter 50 cm.

33.66. Model: Two closely spaced slits produce a double-slit interference pattern.

Visualize:



Solve: (a) The $m=1$ bright fringes are separated from the $m=0$ central maximum by

$$\Delta y=\frac{\lambda L}{d}=\frac{(600\times10^{-9}\text{ m})(1.0\text{ m})}{0.0002\text{ m}}=0.0030\text{ m}=3.0\text{ mm}$$

(b) The light's frequency is $f=c/\lambda=5.00\times10^{14}$ Hz. Thus, the period is $T=1/f=2.00\times10^{-15}$ s. A delay of $5.0\times10^{-16}\text{ s}=0.50\times10^{-15}$ s is $\frac{1}{4}T$.

(c) The wave passing through the glass is delayed by $\frac{1}{4}$ of a cycle. Consequently, the two waves are not in phase as they emerge from the slits. The slits are the sources of the waves, so there is now a phase difference $\Delta\phi_0$ between the two sources. A delay of a full cycle $(\Delta t=T)$ would have no effect at all on the interference because it corresponds to a phase difference $\Delta\phi_0=2\pi$ rad. Thus a delay of $\frac{1}{4}$ of a cycle introduces a phase difference $\Delta\phi_0=\frac{1}{4}(2\pi)=\frac{1}{2}\pi$ rad.

(d) The text's analysis of the double-slit interference experiment assumed that the waves emerging from the two slits were in phase, with $\Delta\phi_0=0$ rad. Thus, there is a central maximum at a point on the screen exactly halfway between the two slits, where $\Delta\phi=0$ rad and $\Delta r=0$ m. Now that there is a phase difference between the sources, the central maximum—still defined as the point of constructive interference where $\Delta\phi=0$ rad—will shift to one side. The wave leaving the slit with the glass was delayed by $\frac{1}{4}$ of a period. If it travels a *shorter* distance to the screen, taking $\frac{1}{4}$ of a period *less* than the wave coming from the other slit, it will make up for the previous delay and the two waves will arrive in phase for constructive interference. Thus, the central maximum will shift *toward* the slit with the glass. How far? A phase difference $\Delta\phi_0=2\pi$ would shift the fringe pattern by $\Delta y=3.0$ mm, making the central maximum fall exactly where the $m=1$ bright fringe had been previously. This is the point where $\Delta r=(1)\lambda$, exactly compensating for a phase shift of 2π at the slits. Thus, a phase shift of $\Delta\phi_0=\frac{1}{2}\pi=\frac{1}{4}(2\pi)$ will shift the fringe pattern by $\frac{1}{4}(3\text{ mm})=0.75$ mm. The net effect of placing the glass in the slit is that the central maximum (and the entire fringe pattern) will shift 0.75 mm toward the slit with the glass.

33.67. Model: A diffraction grating produces an interference pattern, which looks like the diagram of Figure 33.9.
Solve: (a) Nothing has changed while the aquarium is empty. The order of a bright (constructive interference) fringe is related to the diffraction angle θ_m by $d\sin\theta_m=m\lambda$, where $m=0, 1, 2, 3, \ldots$ The space between the slits is

$$d=\frac{1.0\text{ mm}}{600}=1.6667\times10^{-6}\text{ m}$$

For $m = 1$,

$$\sin\theta_1 = \frac{\lambda}{d} \Rightarrow \theta_1 = \sin^{-1}\left(\frac{633\times10^{-9}\text{ m}}{1.6667\times10^{-6}\text{ m}}\right) = 22.3°$$

(b) The path-difference between the waves that leads to constructive interference is an integral multiple of the wavelength in the medium in which the waves are traveling, that is, water. Thus,

$$\lambda = \frac{633\text{ nm}}{n_{\text{water}}} = \frac{633\text{ nm}}{1.33} = 4.759\times10^{-7}\text{ m} \Rightarrow \sin\theta_1 = \frac{\lambda}{d} = \frac{4.759\times10^{-7}\text{ m}}{1.6667\times10^{-6}\text{ m}} = 0.2855 \Rightarrow \theta_1 = 16.6°$$

33.68. Model: The gas increases the number of wavelengths in one arm of the interferometer. Each additional wavelength causes one bright-dark-bright fringe shift.
Solve: From the equation in Challenge Example 33.8, the number of fringe shifts is

$$\Delta m = m_2 - m_1 = (n-1)\frac{2d}{\lambda_{\text{vac}}} = (1.00028-1)\frac{(2)(2.00\times10^{-2}\text{ m})}{600\times10^{-9}\text{ m}} = 19$$

33.69. Model: The arms of the interferometer are of equal length, so without the crystal the output would be bright.
Visualize: We need to consider how many more wavelengths fit in the electro-optic crystal than would have occupied that space $(6.70\ \mu\text{m})$ without the crystal; if it is an integer then the interferometer will produce a bright output; if it is a half-integer then the interferometer will produce a dark output. But the wavelength we need to consider is the wavelength inside the crystal, not the wavelength in air.

$$\lambda_n = \frac{\lambda}{n}$$

We are told the initial *n* with no applied voltage is 1.522, and the wavelength in air is $\lambda = 1.000\ \mu\text{m}$.
Solve: The number of wavelengths that would have been in that space without the crystal is

$$\frac{6.70\ \mu\text{m}}{1.000\ \mu\text{m}} = 6.70$$

(a) With the crystal in place $(\text{and } n = 1.522)$ the number of wavelengths in the crystal is

$$\frac{6.70\ \mu\text{m}}{1.000\ \mu\text{m}/1.522} = 10.20$$
$$10.20 - 6.70 = 3.50$$

which shows there are a half-integer number more wavelengths with the crystal in place than if it weren't there. Consequently the output is dark with the crystal in place but no applied voltage.
(b) Since the output was dark in the previous part, we want it to be bright in the new case with the voltage on. That means we want to have just one-half more extra wavelengths in the crystal (than if it weren't there) than we did in the previous part. That is, we want 4.00 extra wavelengths in the crystal instead of 3.5, so we want $6.70 + 4.00 = 10.70$ wavelengths in the crystal.

$$\frac{6.70\ \mu\text{m}}{1.000\ \mu\text{m}/n} = 10.70 \qquad \Rightarrow \qquad n = \frac{10.70(1.000\ \mu\text{m})}{6.70\ \mu\text{m}} = 1.597$$

Assess: It seems reasonable to be able to change the index of refraction of a crystal from 1.522 to 1.597 by applying a voltage.

33.70. Model: A diffraction grating produces an interference pattern like the one shown in Figure 33.9. We also assume that the small-angle approximation is valid for this grating.
Solve: (a) The general condition for constructive-interference fringes is

$$d\sin\theta_m = m\lambda \qquad m = 0,\ 1,\ 2,\ 3,\ \ldots$$

When this happens, we say that the light is diffracted at an angle θ_m. Since it is usually easier to measure distances rather than angles, we will consider the distance y_m from the center to the mth maximum. This distance is $y_m = L\tan\theta_m$. In the small-angle approximation, $\sin\theta_m \approx \tan\theta_m$, so we can write the condition for constructive interference as

$$d\frac{y_m}{L} = m\lambda \Rightarrow y_m = \frac{m\lambda L}{d}$$

The fringe separation is

$$y_{m+1} - y_m = \Delta y = \frac{\lambda L}{d}$$

(b) Now the laser light falls on a film that has a series of "slits" (*i.e.*, bright and dark stripes), with spacing

$$d' = \frac{\lambda L}{d}$$

Applying once again the condition for constructive interference:

$$d'\sin\theta_m = m\lambda \Rightarrow d'\frac{y'_m}{L} = m\lambda \Rightarrow y'_m = \frac{m\lambda L}{d'} = \frac{m\lambda L}{\lambda L/d} = md$$

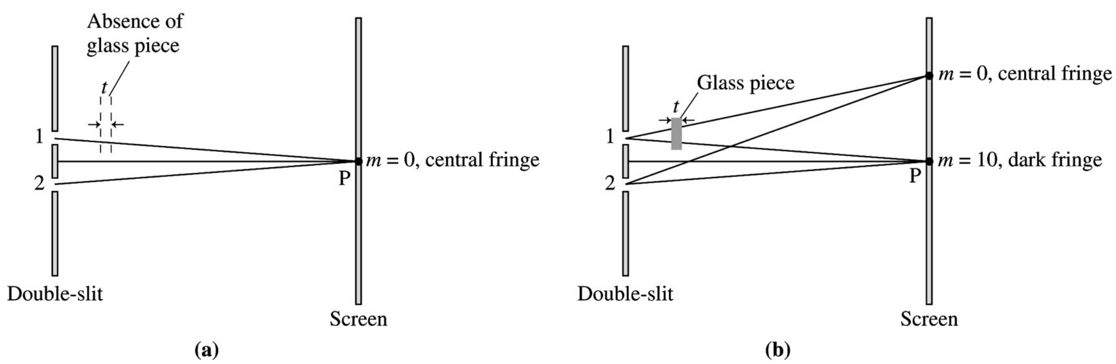
The fringe separation is $y'_{m+1} - y'_m = \Delta y' = d$.

That is, using the film as a diffraction grating produces a diffraction pattern whose fringe spacing is d, the spacing of the original slits.

Challenge Problems

33.71. Model: Two closely spaced slits produce a double-slit interference pattern. The interference pattern is symmetrical on both sides of the central maximum.

Visualize:



In figure (a), the interference of laser light from the two slits forms a constructive-interference central ($m = 0$) fringe at P. The paths 1P and 2P are equal. When a glass piece of thickness t is inserted in the path 1P, the interference between the two waves yields the 10^{th} dark fringe at P. Note that the glass piece is not present in figure (a).

Solve: The number of wavelengths in the air-segment of thickness t is

$$m_1 = \frac{t}{\lambda}$$

The number of wavelengths in the glass piece of thickness t is

$$m_2 = \frac{t}{\lambda_{\text{glass}}} = \frac{t}{\lambda/n} = \frac{nt}{\lambda}$$

The path length has thus *increased* by Δm wavelengths, where

$$\Delta m = m_2 - m_1 = (n-1)\frac{t}{\lambda}$$

From the 10^{th} dark fringe to the 1^{st} dark fringe is 9 fringes and from the 1^{st} dark fringe to the 0^{th} bright fringe is one-half of a fringe. Hence,

$$\Delta m = 9 + \frac{1}{2} = \frac{19}{2} \Rightarrow \frac{19}{2} = (n-1)\frac{t}{\lambda} \Rightarrow t = \frac{19}{2}\frac{\lambda}{(n-1)} = \frac{19}{2}\frac{(633\times10^{-9}\text{ m})}{1.5-1.0} = 12.0\ \mu\text{m}$$

33.72. Visualize: To find the location y where the intensity is I_1 use Equation 33.14: $I_{\text{double}} = 4I_1\cos^2\left(\frac{\pi d}{\lambda L}y\right)$. Then divide by the distance to the first minimum $y_0 = \frac{1}{2}\frac{\lambda L}{d}$ to get the fraction desired.

Solve: First set $I_{\text{double}} = I_1$.

$$I_{\text{double}} = 4I_1\cos^2\left(\frac{\pi d}{\lambda L}y\right)$$
$$I_1 = 4I_1\cos^2\left(\frac{\pi d}{\lambda L}y\right)$$
$$\frac{1}{4} = \cos^2\left(\frac{\pi d}{\lambda L}y\right)$$
$$\frac{1}{2} = \cos\left(\frac{\pi d}{\lambda L}y\right)$$
$$y = \frac{\lambda L}{\pi d}\cos^{-1}\left(\frac{1}{2}\right)$$

Now set up the ratio that will give the desired fraction.

$$\frac{y}{y_0'} = \frac{\frac{\lambda L}{\pi d}\cos^{-1}\left(\frac{1}{2}\right)}{\frac{1}{2}\frac{\lambda L}{d}} = \frac{2}{\pi}\cos^{-1}\left(\frac{1}{2}\right) = \frac{2}{\pi}\frac{\pi}{3} = \frac{2}{3}$$

Assess: The fraction must be less than 1, and $\frac{2}{3}$ seems reasonable.

33.73. Model: The intensity in a double-slit interference pattern is determined by diffraction effects from the slits.
Solve: (a) For the two wavelengths λ and $\lambda + \Delta\lambda$ passing simultaneously through the grating, their first-order peaks are at

$$y_1 = \frac{\lambda L}{d} \qquad y_1' = \frac{(\lambda + \Delta\lambda)L}{d}$$

Subtracting the two equations gives an expression for the separation of the peaks:

$$\Delta y = y_1' - y_1 = \frac{\Delta\lambda L}{d}$$

(b) For a double-slit, the intensity pattern is

$$I_{\text{double}} = 4I_1\cos^2\left(\frac{\pi d}{\lambda L}y\right)$$

The intensity oscillates between zero and $4I_1$, so the maximum intensity is $4I_1$. The width is measured at the point where the intensity is half of its maximum value. For the intensity to be $\frac{1}{2}I_{\max} = 2I_1$ for the $m=1$ peak:

$$2I_1 = 4I_1\cos^2\left(\frac{\pi d}{\lambda L}y_{\text{half}}\right) \Rightarrow \cos^2\left(\frac{\pi d}{\lambda L}y_{\text{half}}\right) = \frac{1}{2} \Rightarrow \frac{\pi d}{\lambda L}y_{\text{half}} = \frac{\pi}{4} \Rightarrow y_{\text{half}} = \frac{\lambda L}{4d}$$

The width of the fringe is twice y_{half}. This means

$$w = 2y_{\text{half}} = \frac{\lambda L}{2d}$$

But the location of the $m=1$ peak is $y_1 = \lambda L/d$, so we get $w = \frac{1}{2}y_1$.

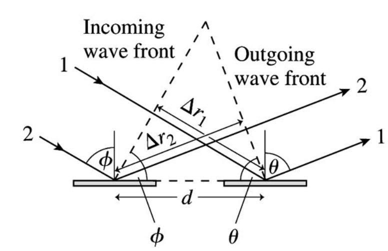
(c) We can extend the result obtained in (b) for two slits to $w = y_1/N$. The condition for barely resolving two diffraction fringes or peaks is $w = \Delta y_{\min}$. From part (a) we have an expression for the separation of the first-order peaks and from part (b) we have an expression for the width. Thus, combining these two pieces of information,

$$\frac{y_1}{N} = \Delta y_{\min} = \frac{\Delta\lambda_{\min}L}{d} \Rightarrow \Delta\lambda_{\min} = \frac{y_1 d}{LN} = \left(\frac{\lambda L}{d}\right)\frac{d}{NL} = \frac{\lambda}{N}$$

(d) Using the result of part (c),

$$N = \frac{\lambda}{\Delta\lambda_{\min}} = \frac{656.27\times10^{-9}\ \text{m}}{0.18\times10^{-9}\ \text{m}} = 3646\ \text{lines}$$

33.74. Solve: (a)



We have two incoming and two diffracted light rays at angles ϕ and θ and two wave fronts perpendicular to the rays. We can see from the figure that the wave 1 travels an extra distance $\Delta r = d\sin\phi$ to reach the reflection spot. Wave 2 travels an extra distance $\Delta r = d\sin\theta$ from the reflection spot to the outgoing wave front. The path *difference* between the two waves is

$$\Delta r = \Delta r_1 - \Delta r_2 = d(\sin\theta - \sin\phi)$$

(b) The condition for diffraction, with all the waves in phase, is still $\Delta r = m\lambda$. Using the results from part (a), the diffraction condition is

$$d\sin\theta_m = m\lambda + d\sin\phi \qquad m = \ldots -2, -1, 0, 1, 2, \ldots$$

Negative values of *m* will give a different diffraction angle than the corresponding positive values.

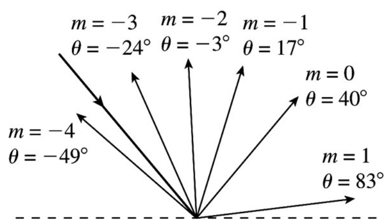
(c) The "zeroth order" diffraction from the reflection grating is $m = 0$. From the diffraction condition of part (b), this implies $d\sin\theta_0 = d\sin\phi$ and hence $\theta_0 = \phi$. That is, the zeroth order diffraction obeys the *law of reflection*—the angle of reflection equals the angle of incidence.

(d) A 700 lines per millimeter grating has spacing $d = \frac{1}{700}\ \text{mm} = 1.429\times10^{-6}\ \text{m} = 1429\ \text{nm}$. The diffraction angles are given by

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d} + \sin\phi\right) = \sin^{-1}\left(\frac{m(500\ \text{nm})}{1429\ \text{nm}} + \sin 40°\right)$$

M	θ_m
≤ -5	not defined
-4	$-49.2°$
-3	$-24.0°$
-2	$-3.3°$
-1	$17.0°$
0	$40.0°$
1	$83.1°$
≥ 2	not defined

(e)



33.75. Model: Diffraction patterns from two objects can just barely be resolved if the central maximum of one image falls on the first dark fringe of the other image.

Solve: (a) Using Equation 33.31 with the width equal to the aperture diameter,

$$w = D = \frac{2.44\lambda L}{D} \Rightarrow D = \sqrt{2.44\lambda L} = \sqrt{(2.44)(550\times10^{-7}\text{ m})(0.20\text{ m})} = 0.52\text{ mm}$$

(b) We can now use Equation 33.30 to find the angle between two distant sources that can be resolved. The angle

$$\alpha = 1.22\frac{\lambda}{D} = \frac{1.22(550\times10^{-9}\text{ m})}{0.52\times10^{-3}\text{ m}} = 1.29\times10^{-3}\text{ rad} = 0.074°$$

(c) The distance that can be resolved is

$$(1000\text{ m})\alpha = (1000\text{ m})(1.29\times10^{-3}\text{ rad}) = 1.3\text{ m}$$