

A MACROSCOPIC DESCRIPTION OF MATTER

Conceptual Questions

18.1. Convert them to the absolute Kelvin scale to compare.

$$T_1 = 0 \text{ K}$$

$$T_2 = 0^\circ\text{C} = 273 \text{ K because } T_K = T_C + 273$$

$$T_3 = 0^\circ\text{F} = 255 \text{ K because } T_C = \frac{5}{9}(T_F - 32) = -17.8^\circ\text{C} \Rightarrow T_K = 255 \text{ K}$$

So $T_1 < T_3 < T_2$.

18.2. We must use the absolute temperature scale when we double the temperature.

$$10^\circ\text{C} = 283 \text{ K} \quad 2 \cdot 283 \text{ K} = 566 \text{ K} = 293^\circ\text{C}$$

18.3. (a) Yes. Because the water-ice phase boundary has a negative slope down to the triple point and the solid-gas phase boundary has a positive slope up to the triple point, ice does not exist for temperatures greater than the triple point.

(b) No. The positive slope of the solid-vapor boundary begins at the point (0 K, 0 atm). This means that as long as the pressure is low enough, water can always exist in the vapor phase.

18.4. The pressure must be the same on both sides of the piston; were that not the case then there would be a greater force on one side and the net force would cause the piston to accelerate.

18.5. (a) The ideal-gas law says

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow T_2 = \left(\frac{p_2 V_2}{p_1 V_1} \right) T_1 = \left(\frac{3 p_1}{p_1} \right) \left(\frac{2 V_1}{V_1} \right) T_1 = 6 T_1$$

$$\text{(b)} \quad T_2 = \left(\frac{p_2 V_2}{p_1 V_1} \right) T_1 = \left(\frac{3 p_1}{p_1} \right) \left(\frac{\frac{1}{2} V_1}{V_1} \right) T_1 = \frac{3}{2} T_1.$$

18.6. (a) Because the container is sealed the number of moles in the container remains constant.

$$\text{(b)} \quad \frac{n_1}{V_1} = \frac{p_1}{RT_1} \frac{n_2}{V_2} = \frac{p_2}{RT_2} = \frac{3 p_1}{R(2T_1)} \frac{n_2}{V_2} = \frac{3}{2} \frac{n_1}{V_1}.$$

18.7. It is likely that the air pressure in the underwater apartment is higher than normal atmospheric air pressure, so the freezing point of water would be lower than the value at the surface, and the boiling point would be higher than the value at the surface. This is because the solid-liquid transition line in the phase diagram has a negative slope, but the liquid-gas transition line has a positive slope.

18.8. (a) As the piston squeezes the sample the pressure increases. To see what happens refer to the phase diagram in Figure 18.6. As the pressure on the gas increases from that initial point on the phase diagram it passes from the gas region to the solid region and eventually to the liquid region.

(b) When the initial temperature is warmer ($+0.02^\circ\text{C}$) the sample never passes through the solid state, but goes directly from the gas state to the liquid state.

18.9. (a) The ideal-gas law says

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow p_2 = \left(\frac{V_1}{V_2} \right) \left(\frac{T_2}{T_1} \right) p_1 = \left(\frac{V_1}{2V_1} \right) \left(\frac{3T_1}{T_1} \right) p_1 = \frac{3}{2} p_1$$

$$\text{(b)} \quad p_2 = \left(\frac{V_1}{\frac{1}{2}V_1} \right) \left(\frac{3T_1}{T_1} \right) p_1 = 6p_1$$

18.10. We want T_2/T_1 .

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow \frac{T_2}{T_1} = \frac{p_2 V_2}{p_1 V_1} \quad \text{but } p_2 = p_1, \text{ and from the figure } V_2 = 3V_1, \text{ so}$$

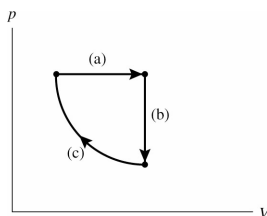
$$\frac{T_2}{T_1} = \frac{3V_1}{V_1} = 3$$

18.11. The figure shows that $V_2 = V_1$. We also know $p_1 = 2 \text{ atm}$, $T_1 = 300 \text{ K}$, and $T_2 = 1200 \text{ K}$.

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow p_2 = \left(\frac{V_1}{V_2} \right) \left(\frac{T_2}{T_1} \right) p_1 = \left(\frac{V_1}{V_1} \right) \left(\frac{1200 \text{ K}}{300 \text{ K}} \right) (2 \text{ atm}) = 8 \text{ atm}$$

18.12. Part **(a)** of the figure represents a constant pressure, or isobaric, expansion of the gas. Part **(b)** represents a constant volume reduction of pressure of the gas. During part **(b)**, the temperature also decreases. Part **(c)** represents a decrease in volume along with an increase in pressure. However, part **(c)** is not isothermal since the graph is a straight line. Isothermal processes are hyperbolas on pV diagrams.

A correct diagram would look like the following figure.



Exercises and Problems

Exercises

Section 18.1 Solids, Liquids, and Gases

18.1. Model: Recall the density of water is 1000 kg/m^3 .

Solve: The mass of gold $m_{\text{Au}} = \rho_{\text{Au}} V_{\text{Au}} = (19,300 \text{ kg/m}^3)(100 \times 10^{-6} \text{ m}^3) = 1.93 \text{ kg}$. For water to have the same mass its volume must be

$$V_{\text{water}} = \frac{m_{\text{water}}}{\rho_{\text{water}}} = \frac{1.93 \text{ kg}}{1000 \text{ kg/m}^3} = 0.00193 \text{ m}^3 \approx 1900 \text{ cm}^3$$

Assess: Since the lead is 19.3 times as dense we expect the water to take 19.3 times the volume.

18.2. Model: Assume the nucleus is spherical.

Solve: The volume of the uranium nucleus is

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(7.5 \times 10^{-15} \text{ m})^3 = 1.767 \times 10^{-42} \text{ m}^3$$

The density of the uranium nucleus is

$$\rho_{\text{nucleus}} = \frac{m_{\text{nucleus}}}{V_{\text{nucleus}}} = \frac{4.0 \times 10^{-25} \text{ kg}}{1.767 \times 10^{-42} \text{ m}^3} = 2.3 \times 10^{17} \text{ kg/m}^3$$

Assess: This density is extremely large compared to the typical density of materials.

18.3. Solve: The volume of the aluminum cube is 10^{-3} m^3 and its mass is

$$m_{\text{Al}} = \rho_{\text{Al}} V_{\text{Al}} = (2700 \text{ kg/m}^3)(1.0 \times 10^{-3} \text{ m}^3) = 2.7 \text{ kg}$$

The volume of the copper sphere with this mass is

$$V_{\text{Cu}} = \frac{4\pi}{3}(r_{\text{Cu}})^3 = \frac{m_{\text{Cu}}}{\rho_{\text{Cu}}} = \frac{2.7 \text{ kg}}{8920 \text{ kg/m}^3} = 3.027 \times 10^{-4} \text{ m}^3$$

$$\Rightarrow r_{\text{Cu}} = \left[\frac{3(3.027 \times 10^{-4} \text{ m}^3)}{4\pi} \right]^{1/3} = 0.042 \text{ m}$$

The diameter of the copper sphere is $0.0833 \text{ m} = 8.3 \text{ cm}$.

Assess: The diameter of the sphere is a little less than the length of the cube, and this is reasonable considering the density of copper is greater than the density of aluminum.

18.4. Model: The volume of a hollow sphere is

$$V = \frac{4\pi}{3}(r_{\text{out}}^3 - r_{\text{in}}^3)$$

Solve: We are given $m = 0.690 \text{ kg}$, $r_{\text{out}} = 0.050 \text{ m}$, and we know that for aluminum $\rho = 2700 \text{ kg/m}^3$.

Solve the above equation for r_{in} .

$$\begin{aligned} r_{\text{in}} &= \sqrt[3]{r_{\text{out}}^3 - \frac{V}{\frac{4\pi}{3}}} \\ &= \sqrt[3]{r_{\text{out}}^3 - \frac{m/\rho}{\frac{4\pi}{3}}} \\ &= \sqrt[3]{0.050^3 \text{ m}^3 - \frac{0.690 \text{ kg}/2700 \text{ kg/m}^3}{\frac{4\pi}{3}}} \\ &= 0.040 \text{ m} \end{aligned}$$

So the inner diameter is 8.0 cm .

Assess: We are happy that the inner diameter is less than the outer diameter, and in a reasonable range.

Section 18.2 Atoms and Moles

18.5. Solve: The volume of the copper cube is $8.0 \times 10^{-6} \text{ m}^3$ and its mass is

$$M = \rho V = (8920 \text{ kg/m}^3)(8.0 \times 10^{-6} \text{ m}^3) = 0.07136 \text{ kg} = 71.36 \text{ g}$$

Because the atomic mass number of Cu is 64, one mole of Cu has a mass of 64 g. The number of moles in the cube is

$$n = \left(\frac{1 \text{ mol}}{64 \text{ g}} \right) (71.36 \text{ g}) = 1.1 \text{ mol}$$

Assess: This answer is in the same ballpark as the next exercise.

18.6. Solve: The volume of the aluminum cube $V = 8.0 \times 10^{-6} \text{ m}^3$ and its mass is

$$M = \rho V = (2700 \text{ kg/m}^3)(8.0 \times 10^{-6} \text{ m}^3) = 0.0216 \text{ kg} = 21.6 \text{ g}$$

One mole of aluminum (^{27}Al) has a mass of 27 g. The number of atoms is

$$N = \left(\frac{6.02 \times 10^{23} \text{ atoms}}{1 \text{ mol}} \right) \left(\frac{1 \text{ mol}}{27 \text{ g}} \right) (21.6 \text{ g}) = 4.8 \times 10^{23} \text{ atoms}$$

Assess: This is almost one mole of atoms, which is a reasonable value.

18.7. Solve: (a) The number density is defined as N/V , where N is the number of particles occupying a volume V .

Because Al has a mass density of 2700 kg/m^3 , a volume of 1 m^3 has a mass of 2700 kg. We also know that the molar mass of Al is 27 g/mol or 0.027 kg/mol. So, the number of moles in a mass of 2700 kg is

$$n = (2700 \text{ kg}) \left(\frac{1 \text{ mol}}{0.027 \text{ kg}} \right) = 1.00 \times 10^5 \text{ mol}$$

The number of Al atoms in 1.00×10^5 mols is

$$N = nN_A = (1.00 \times 10^5 \text{ mol})(6.02 \times 10^{23} \text{ atoms/mol}) = 6.02 \times 10^{28} \text{ atoms}$$

Thus, the number density is

$$\frac{N}{V} = \frac{6.02 \times 10^{28} \text{ atoms}}{1 \text{ m}^3} = 6.02 \times 10^{28} \text{ atoms/m}^3$$

(b) Pb has a mass of 11,300 kg in a volume of 1 m^3 . Since the atomic mass number of Pb is 207, the number of moles in 11,300 kg is

$$n = (11,300 \text{ kg}) \left(\frac{1 \text{ mole}}{0.207 \text{ kg}} \right)$$

The number of Pb atoms is thus $N = nN_A$, and hence the number density is

$$\frac{N}{V} = \frac{nN_A}{V} = \left(\frac{11,300 \text{ kg}}{0.207 \text{ kg}} \right) \left(6.02 \times 10^{23} \frac{\text{atoms}}{\text{mol}} \right) \frac{(1 \text{ mol})}{1 \text{ m}^3} = 3.28 \times 10^{28} \frac{\text{atoms}}{\text{m}^3}$$

Assess: We expected to get very large numbers like this.

18.8. Solve: The mass density is $\rho = M/V$. The mass M of the sample is related to the number of atoms N and the mass m of each atom by $M = Nm$. Combining these, the atomic mass is

$$m = \frac{M}{N} = \frac{\rho V}{N} = \frac{\rho}{N/V} = \frac{1750 \text{ kg/m}^3}{4.39 \times 10^{28} \text{ atoms/m}^3} = 3.986 \times 10^{-26} \text{ kg/atom}$$

the atomic mass in $m = A$ u, where A is the atomic mass number. Thus

$$A = \frac{m}{1 \text{ u}} = \frac{3.986 \times 10^{-26} \text{ kg}}{1.661 \times 10^{-27} \text{ kg}} = 24$$

The element's atomic mass number is 24.

Assess: This is a reasonable answer for an isotope of neon (although neon is a gas at normal temperatures), sodium, or magnesium.

18.9. Model: Assume the gold is shaped into a solid sphere of volume $V = \frac{4}{3}\pi r^3$.

Visualize: We want to know $D = 2r$. Because the atomic mass number of gold is 197, 1.0 mol of gold has a mass of 197 g or 0.197 kg. Table 18.1 gives $\rho = 19,300 \text{ kg/m}^3$.

Solve:

$$V = \frac{4}{3}\pi r^3 = M/\rho \Rightarrow r = \sqrt[3]{\frac{3}{4\pi} \frac{M}{\rho}} = \sqrt[3]{\frac{3}{4\pi} \frac{0.197 \text{ kg}}{19,300 \text{ kg/m}^3}} = 0.0135 \text{ m} = 1.35 \text{ cm}$$

$$D = 2r = 2(1.35 \text{ cm}) = 2.7 \text{ cm}.$$

Assess: This is about the right size for a chunk that contains one mole of material.

18.10. Solve: The mass of mercury is

$$M = \rho V = (13,600 \text{ kg/m}^3)(10 \text{ cm}^3) \left(\frac{10^{-6} \text{ m}^3}{1 \text{ cm}^3} \right) = 0.136 \text{ kg} = 136 \text{ g}$$

and the number of moles is

$$n = \frac{M}{M_{\text{mol}}} = \frac{0.136 \text{ g}}{201 \text{ g/mol}} = 0.6766 \text{ mol}$$

The mass of aluminum with 0.6766 mol of Al is

$$M = (0.6766 \text{ mol})M_{\text{mol}} = (0.6766 \text{ mol}) \left(\frac{27 \text{ g}}{\text{mol}} \right) = 18.27 \text{ g} = 0.01827 \text{ kg}$$

This mass M of aluminum corresponds to a volume of

$$V = \frac{M}{\rho} = \frac{0.01827 \text{ kg}}{2700 \text{ kg/m}^3} = 6.8 \times 10^{-6} \text{ m}^3 = 6.8 \text{ cm}^3$$

Assess: We expected an answer in the same order of magnitude. The size of atoms doesn't vary as much as the density of atoms from element to element.

Section 18.3 Temperature

18.11. Solve: The lowest temperature is

$$T_F = \frac{9}{5}T_C + 32^\circ \Rightarrow -129^\circ\text{F} = \frac{9}{5}T_C + 32^\circ \Rightarrow T_C = -89.4^\circ\text{C} \Rightarrow T_K = (-89.4 + 273) = 184 \text{ K}$$

In the same way, the highest temperature is

$$134^\circ\text{F} = \frac{9}{5}T_C + 32^\circ \Rightarrow T_C = 56.7^\circ\text{C} = 330 \text{ K}$$

Assess: On the absolute scale the highest recorded temperature is not quite twice the lowest.

18.12. Solve: Let $T_F = T_C = T$:

$$T_F = \frac{9}{5}T_C + 32^\circ \Rightarrow T = \frac{9}{5}T + 32^\circ \Rightarrow T = -40^\circ$$

That is, the Fahrenheit and the Celsius scales give the same numerical value at -40° .

Assess: It is usually unnecessary to specify the scale when the temperature is reported as -40° .

18.13. Model: A temperature scale is a linear scale.

Solve: (a) We need a conversion formula for $^{\circ}\text{C}$ to $^{\circ}\text{Z}$, analogous to the conversion of $^{\circ}\text{C}$ to $^{\circ}\text{F}$. Since temperature scales are linear, $T_{\text{C}} = aT_{\text{Z}} + b$, where a and b are constants to be determined. We know the boiling point of liquid nitrogen is 0°Z and -196°C . Similarly, the melting point of iron is 1000°Z and 1538°C . Thus

$$\begin{aligned}-196 &= 0a + b \\ 1538 &= 1000a + b\end{aligned}$$

From the first, $b = -196^{\circ}$. Then from the second, $a = (1538 + 196)/1000 = 1734/1000$. Thus the conversion is $T_{\text{C}} = (1734/1000)T_{\text{Z}} - 196^{\circ}$. Since the boiling point of water is $T_{\text{C}} = 100^{\circ}\text{C}$, its temperature in $^{\circ}\text{Z}$ is

$$T_{\text{Z}} = \left(\frac{1000}{1734}\right)(100^{\circ} + 196^{\circ}) = 171^{\circ}\text{Z}$$

(b) A temperature $T_{\text{Z}} = 500^{\circ}\text{Z}$ is

$$T_{\text{C}} = \left(\frac{1734}{1000}\right)500^{\circ} - 196^{\circ} = 671^{\circ}\text{C} = 944\text{ K}$$

Section 18.4 Thermal Expansion

Section 18.5 Phase Changes

18.14. Model: Assume the concrete expands linearly.

Visualize: We are given the coefficient of linear expansion for concrete: $\alpha = 1.2 \times 10^{-5} (^{\circ}\text{C})^{-1}$. We must convert the temperatures to degrees Celsius. $115^{\circ}\text{F} = 46.1^{\circ}\text{C}$, $0^{\circ}\text{F} = -1.77^{\circ}\text{C}$, so $\Delta T = 63.9^{\circ}\text{C}$.

Solve:

$$\frac{\Delta L}{L} = \alpha \Delta T \Rightarrow \Delta L = \alpha L \Delta T = (1.2 \times 10^{-5} (^{\circ}\text{C})^{-1})(3.25\text{ m})(63.9^{\circ}\text{C}) = 2.5\text{ mm}$$

Assess: This is a small gap because concrete has a small coefficient of expansion.

18.15. Model: Assume the distance between stakes does not contract in the cold; only the measuring tape does.

Visualize: We look up the coefficient of linear expansion for steel: $\alpha = 1.1 \times 10^{-5} (^{\circ}\text{C})^{-1}$. $\Delta T = -17^{\circ}\text{C}$.

Solve: The tape contracts which means it will read a higher number than the true number, so she needs to subtract a correction factor. The amount she will need to subtract is the 3°C reading, L' , minus $L_0 = 65.175\text{ m}$.

$$\frac{\Delta L}{L} = \alpha \Delta T \Rightarrow \Delta L = \alpha L \Delta T = (1.1 \times 10^{-5} (^{\circ}\text{C})^{-1})(65.175\text{ m})(17^{\circ}\text{C}) = 12\text{ mm}$$

Assess: 12 mm is not insignificant. Thermal expansion must be taken into account in surveying.

18.16. Model: Assume the rods expand linearly with temperature.

Visualize: We look up the coefficients of linear expansion: $\alpha_{\text{br}} = 1.9 \times 10^{-5} (^{\circ}\text{C})^{-1}$ and $\alpha_{\text{invt}} = 0.09 \times 10^{-5} (^{\circ}\text{C})^{-1}$. $\Delta T = 5.0^{\circ}\text{C}$.

Solve:

$$\frac{\Delta L_{\text{br}}}{L} = \alpha_{\text{br}} \Delta T \Rightarrow \Delta L_{\text{br}} = \alpha_{\text{br}} L \Delta T = (1.9 \times 10^{-5} (^{\circ}\text{C})^{-1})(0.655\text{ m})(5.0^{\circ}\text{C}) = 0.062\text{ mm}$$

$$\frac{\Delta L_{\text{invt}}}{L} = \alpha_{\text{invt}} \Delta T \Rightarrow \Delta L_{\text{invt}} = \alpha_{\text{invt}} L \Delta T = (0.09 \times 10^{-5} (^{\circ}\text{C})^{-1})(0.655\text{ m})(5.0^{\circ}\text{C}) = 0.0029\text{ mm}$$

The difference is $0.062\text{ mm} - 0.0029\text{ mm} = 0.059\text{ mm}$.

Assess: The brass rod expanded more because its coefficient of linear expansion is greater.

18.17. Model: Assume the gasoline expands linearly with temperature.

Visualize: We look up the coefficient of volume expansion for gasoline: $\beta_{\text{gas}} = 9.6 \times 10^{-4} (^{\circ}\text{C})^{-1}$. $\Delta T = 30^{\circ}\text{C}$. We look for the original volume V_0 which expands to 60.00 L.

Solve: Use fractional change in volume formula: $\frac{\Delta V}{V} = \beta \Delta T$.

$$V_f = V_0 + \Delta V = V_0 + V_0 \beta \Delta T = V_0 (1 + \beta \Delta T) \Rightarrow$$

$$V_0 = \frac{V_f}{1 + \beta \Delta T} = \frac{60.00 \text{ L}}{1 + (9.6 \times 10^{-4} (\text{°C})^{-1})(30 \text{°C})} = 58.32 \text{ L}$$

The volume needed to be left empty is $V_f - V_0 = 60.00 \text{ L} - 58.32 \text{ L} = 1.68 \text{ L}$

Assess: We have assumed that the air at the top of the tank is very compressible.

18.18. Solve: (a) The triple point of water is $T = 0.01 \text{°C}$ and $p = 0.006 \text{ atm}$. Thus

$$T_F = \frac{9}{5} T_C + 32 \text{°} = \frac{9}{5} (0.01) + 32 = 32.02 \text{°F}$$

$$p = 0.006 \text{ atm} \times \frac{1.013 \times 10^5 \text{ Pa}}{1 \text{ atm}} = 608 \text{ Pa}$$

(b) The triple point of carbon dioxide is $T = -56 \text{°C}$ and $p = 5 \text{ atm}$. Thus

$$T_F = \frac{9}{5} T_C + 32 \text{°} = \frac{9}{5} (-56 \text{°}) + 32 \text{°} = -68.8 \text{°F}$$

$$p = 5.0 \text{ atm} = (5.0 \text{ atm}) \left(\frac{1.013 \times 10^5 \text{ Pa}}{1 \text{ atm}} \right) = 5.06 \times 10^5 \text{ Pa}$$

Section 18.6 Ideal Gases

18.19. Model: Treat the nitrogen gas in the closed cylinder as an ideal gas.

Solve: (a) The density before and after the compression are $\rho_{\text{before}} = m_1/V_1$ and $\rho_{\text{after}} = m_2/V_2$. Noting that $m_1 = m_2$ and $V_2 = \frac{1}{2} V_1$,

$$\frac{\rho_{\text{after}}}{\rho_{\text{before}}} = \frac{m}{V_2} \frac{V_1}{m} = 2 \Rightarrow \rho_{\text{after}} = 2\rho_{\text{before}}$$

The mass density has changed by a factor of 2.

(b) The number of atoms in the gas is unchanged, implying that the number of moles in the gas remains the same; hence the number density is unchanged.

18.20. Model: Treat the gas in the container as an ideal gas.

Solve: From the ideal-gas law $pV = nRT$, the pressure of the gas is

$$p = \frac{nRT}{V} = \frac{(3.0 \text{ mol})(8.31 \text{ J/mol K})[(273 - 120) \text{ K}]}{(2.0 \times 10^{-3} \text{ m}^3)} = 1.9 \times 10^6 \text{ Pa} = 19 \text{ atm}$$

Assess: 19 atm is a high pressure, but not unreasonable.

18.21. Model: Treat the gas in the sealed container as an ideal gas.

Solve: (a) From the ideal gas law equation $pV = nRT$, the volume V of the container is

$$V = \frac{nRT}{p} = \frac{(2.0 \text{ mol})(8.31 \text{ J/mol K})[(273 + 30) \text{ K}]}{1.013 \times 10^5 \text{ Pa}} = 0.050 \text{ m}^3$$

Note that pressure *must* be in Pa in the ideal-gas law.

(b) The before-and-after relationship of an ideal gas in a sealed container (constant volume) is

$$\frac{p_1 V}{T_1} = \frac{p_2 V}{T_2} \Rightarrow p_2 = p_1 \frac{T_2}{T_1} = (1.0 \text{ atm}) \frac{(273 + 130) \text{ K}}{(273 + 30) \text{ K}} = 1.3 \text{ atm}$$

Note that gas-law calculations *must* use T in kelvins.

18.22. Model: Treat the gas as an ideal gas in the sealed container.

Solve: (a) For $p_1 = p_0$, the before-and-after relationship of an ideal gas in a sealed container is

$$\frac{V_1}{T_1} = \frac{V_0}{T_0} \Rightarrow V_1 = V_0 \frac{T_1}{T_0} = V_0 \frac{473 \text{ K}}{373 \text{ K}} = 1.27V_0$$

where $T_0 = (273 + 100) \text{ K}$ and $T_1 = (273 + 200) \text{ K}$.

(b) When the Kelvin temperature is doubled, $T_1 = 2T_0 = 2(373 \text{ K}) = 746 \text{ K}$ and the above equation becomes

$$V_1 = V_0 \frac{T_1}{T_0} = V_0 \frac{746 \text{ K}}{373 \text{ K}} = 2V_0$$

Assess: When we use the Kelvin scale we expect the volume to double when the temperature doubles (if the pressure is kept constant).

18.23. Model: We'll assume that air is an ideal gas so we can use the ideal gas law, $pV = nRT$.

Visualize: We are given $V = 5.0 \text{ L} = 0.0050 \text{ m}^3$, $p = 1 \text{ atm} = 101.3 \text{ kPa}$, and $T = 37^\circ\text{C} = 310 \text{ K}$.

Also recall that $R = 8.31 \text{ J/(mol} \cdot \text{K)}$ and oxygen makes up 20% of the air.

Solve: Solve Equation 12.12 for n , the number of moles of air.

$$n = \frac{pV}{RT} = \frac{(101.3 \text{ kPa})(0.0050 \text{ m}^3)}{(8.31 \text{ J/(mol} \cdot \text{K)})(310 \text{ K})} = 0.20 \text{ mol}$$

Multiply the number of moles of air by 20% to get the number of moles of oxygen: $(0.20 \text{ mol})(0.20) = (0.040 \text{ mol})$
 $(6.02 \times 10^{23} \text{ molecules/mol}) = 2.4 \times 10^{22} \text{ molecules of oxygen}$.

Assess: The answer is a small number of moles of oxygen, but a large number of molecules of oxygen.

18.24. Model: Treat the oxygen gas in the cylinder as an ideal gas.

Solve: (a) The number of moles of oxygen is

$$n = \frac{M}{M_{\text{mol}}} = \frac{50 \text{ g}}{32 \text{ g/mol}} = 1.563 \text{ mol} \approx 1.6 \text{ mol}$$

(b) The number of molecules is

$$N = nN_A = (1.563 \text{ mol})(6.02 \times 10^{23} \text{ mol}^{-1}) = 9.41 \times 10^{23} \approx 9.4 \times 10^{23}$$

(c) The volume of the cylinder $V = \pi r^2 L = \pi (0.10 \text{ m})^2 (0.40 \text{ m}) = 1.257 \times 10^{-2} \text{ m}^3$. Thus,

$$\frac{N}{V} = \frac{9.41 \times 10^{23}}{1.257 \times 10^{-2} \text{ m}^3} = 7.5 \times 10^{25} \text{ m}^{-3}$$

(d) From the ideal-gas law $pV = nRT$ we can calculate the absolute pressure to be

$$p = \frac{nRT}{V} = \frac{(1.563 \text{ mol})(8.31 \text{ J/mol K})(293 \text{ K})}{1.257 \times 10^{-2} \text{ m}^3} = 303 \text{ kPa}$$

where we used $T = 20^\circ\text{C} = 293 \text{ K}$. But a pressure gauge reads *gauge pressure*:

$$p_g = p - 1 \text{ atm} = 303 \text{ kPa} - 101 \text{ kPa} = 202 \text{ kPa} \approx 200 \text{ kPa}$$

18.25. Model: Assume the gas in the solar corona is an ideal gas.

Solve: The number density of particles in the solar corona is N/V . Using the ideal-gas equation,

$$pV = Nk_B T \Rightarrow \frac{N}{V} = \frac{p}{k_B T}$$

$$\frac{N}{V} = \frac{(0.03 \text{ Pa})}{(1.38 \times 10^{-23} \text{ J/K})(2 \times 10^6 \text{ K})} = 1.1 \times 10^{15} \text{ particles/m}^3$$

Assess: The density of the solar corona is much smaller than the density of earth's atmosphere at sea level.

18.26. Model: Assume the gas is an ideal gas.

Solve: The before-and-after relationship of an ideal gas is

$$\frac{p_0 V_0}{T_0} = \frac{p_2 V_2}{T_2} \Rightarrow T_2 = T_0 \frac{p_2}{p_0} \frac{V_2}{V_0} = T_0 \frac{\frac{1}{2} p_0}{p_0} \frac{3V_0}{V_0} = \frac{3}{2} T_0$$

Section 18.7 Ideal-Gas Processes

18.27. Model: The gas is assumed to be ideal and it expands isothermally.

Solve: (a) Isothermal expansion means the temperature stays unchanged. That is $T_2 = T_1$.

(b) The before-and-after relationship of an ideal gas under isothermal conditions is

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_1} \Rightarrow p_2 = p_1 \frac{V_1}{V_2} = p_1 \left(\frac{V_1}{2V_1} \right) = \frac{p_1}{2}$$

18.28. Model: The gas is assumed to be ideal.

Solve: (a) Isochoric means the volume stays unchanged. That is $V_2 = V_1$.

(b) The before-and-after relationship of an ideal gas under isochoric conditions is

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_1} \Rightarrow T_2 = T_1 \frac{p_2}{p_1} = T_1 \left(\frac{\frac{1}{3} p_1}{p_1} \right) = \frac{T_1}{3}$$

18.29. Model: The rigid sphere's volume does not change, so this is an isochoric process. The air is assumed to be an ideal gas.

Solve: (a) When the valve is closed, the air inside is at $p_1 = 1$ atm and $T_1 = 100^\circ\text{C}$. The before-and-after relationship of an ideal gas in the closed sphere (constant volume) is

$$\frac{p_1 V}{T_1} = \frac{p_2 V}{T_2} \Rightarrow p_2 = p_1 \left(\frac{T_2}{T_1} \right) = (1.0 \text{ atm}) \frac{(273 + 0) \text{ K}}{(273 + 100) \text{ K}} = 0.73 \text{ atm}$$

(b) Dry ice is CO_2 . From Figure 18.6, we can see that the solid-gas transition line gives a temperature of -78°C when $p = 1$ atm. Cooling the sphere to -78°C gives

$$p_2 = p_1 \left(\frac{T_2}{T_1} \right) = (1.0 \text{ atm}) \frac{(273 - 78) \text{ K}}{373 \text{ K}} = 0.52 \text{ atm}$$

18.30. Model: The gas is assumed to be ideal, and since the container is rigid $V_2 = V_1$.

Solve: Convert both temperatures to the Kelvin scale.

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_1} \Rightarrow p_2 = p_1 \frac{T_2}{T_1} = 3 \text{ atm} \left(\frac{253 \text{ K}}{293 \text{ K}} \right) = 2.6 \text{ atm}$$

Assess: On the absolute scale the temperature went up only a little bit, so we expect the pressure to rise a little bit.

18.31. Model: Assume the gas is an ideal gas.

Visualize: The pressure in the gas exerts exactly enough upward force to balance the weight of the piston plus the pressure of the air above the piston. The initial temperature is $T_1 = 303^\circ\text{C} + 273 = 576 \text{ K}$.

Solve: (a) The pressure of the gas in the cylinder is

$$p = 1 \text{ atm} + \frac{F}{A} = 101300 \text{ Pa} + \frac{mg}{\pi r^2} = 101300 \text{ Pa} + \frac{(20 \text{ kg})(9.80 \text{ m/s}^2)}{\pi(0.12 \text{ m})^2} = 105600 \text{ Pa}$$

(b) The pressure is determined only by the weight of the piston. It does not change with temperature. The temperature change is an isobaric process, and the pressure remains 105600 Pa.

For an isobaric process,

$$\frac{V_i}{T_i} = \frac{V_f}{T_f} \Rightarrow \frac{h_i A}{T_i} = \frac{h_f A}{T_f} \Rightarrow \frac{h_i}{T_i} = \frac{h_f}{T_f}$$

where $V = hA$ is the volume of a cylinder of cross section area A . The final piston height is

$$h_f = h_i \frac{T_f}{T_i} = 82 \text{ cm} \times \frac{(15 + 273) \text{ K}}{576 \text{ K}} = 42 \text{ cm}$$

Assess: The temperature decreased dramatically, so a large decrease in piston height is to be expected.

18.32. Model: In an isochoric process, the volume of the container stays unchanged. Argon gas in the container is assumed to be an ideal gas.

Solve: (a) The container has only argon inside with $n = 0.1 \text{ mol}$, $V_1 = 50 \text{ cm}^3 = 50 \times 10^{-6} \text{ m}^3$, and $T_1 = 20^\circ\text{C} = 293 \text{ K}$. This produces a pressure

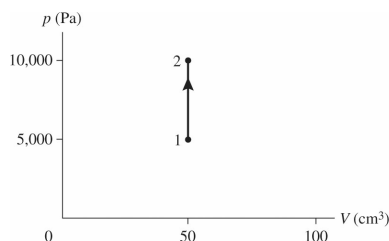
$$p_1 = \frac{nRT}{V_1} = \frac{(0.1 \text{ mol})(8.31 \text{ J/mol K})(293 \text{ K})}{50 \times 10^{-6} \text{ m}^3} = 4.87 \times 10^6 \text{ Pa} = 4870 \text{ kPa} \approx 4900 \text{ kPa}$$

An ideal gas process has $p_2 V_2 / T_2 = p_1 V_1 / T_1$. Isochoric heating to a final temperature $T_2 = 300^\circ\text{C} = 573 \text{ K}$ has $V_2 = V_1$, so the final pressure is

$$p_2 = \frac{V_1 T_2}{V_2 T_1} p_1 = 1 \times \frac{573}{293} \times 4870 \text{ kPa} = 9520 \text{ kPa} \approx 9500 \text{ kPa}$$

Note that it is essential to express temperatures in kelvins.

(b)



Assess: All isochoric processes will be a straight vertical line on a pV diagram.

18.33. Model: The isobaric heating means that the pressure of the argon gas stays unchanged.

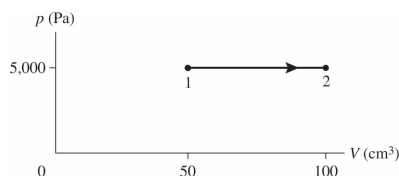
Solve: (a) The container has only argon inside with $n = 0.1 \text{ mol}$, $V_1 = 50 \text{ cm}^3 = 50 \times 10^{-6} \text{ m}^3$, and $T_1 = 20^\circ\text{C} = 293 \text{ K}$. This produces a pressure

$$p_1 = \frac{nRT_1}{V_1} = \frac{(0.1 \text{ mol})(8.31 \text{ J/mol K})(293 \text{ K})}{50 \times 10^{-6} \text{ m}^3} = 4.87 \times 10^6 \text{ Pa} = 4870 \text{ kPa} \approx 4900 \text{ kPa}$$

An ideal gas process has $p_2 V_2 / T_2 = p_1 V_1 / T_1$. Isobaric heating to a final temperature $T_2 = 300^\circ\text{C} = 573 \text{ K}$ has $p_2 = p_1$, so the final volume is

$$V_2 = \frac{p_1 T_2}{p_2 T_1} V_1 = 1 \times \frac{573}{293} \times 50 \text{ cm}^3 = 97.8 \text{ cm}^3 \approx 98 \text{ cm}^3$$

(b)



Assess: All isobaric processes will be a straight horizontal line on a pV diagram.

18.34. Model: In an isothermal expansion, the temperature stays the same. The argon gas in the container is assumed to be an ideal gas.

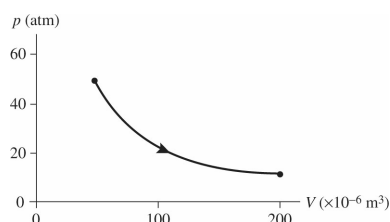
Solve: (a) The container has only argon inside with $n = 0.1$ mol, $V_1 = 50 \text{ cm}^3 = 50 \times 10^{-6} \text{ m}^3$, and $T_1 = 20^\circ\text{C} = 293 \text{ K}$. This produces a pressure

$$p_1 = \frac{nRT_1}{V_1} = \frac{(0.1 \text{ mol})(8.31 \text{ J/mol K})(293 \text{ K})}{50 \times 10^{-6} \text{ Pa}} = 4.87 \times 10^6 \text{ Pa} = 12.02 \text{ atm} \approx 12 \text{ atm}$$

An ideal-gas process obeys $p_2V_2/T_2 = p_1V_1/T_1$. Isothermal expansion to $V_2 = 200 \text{ cm}^3$ gives a final pressure

$$p_2 = \frac{T_2}{T_1} \frac{V_1}{V_2} p_1 = 1 \times \frac{200}{50} \times 12.02 \text{ atm} = 48 \text{ atm}$$

(b)



18.35. Model: Assume the gas to be an ideal gas.

Solve: (a) Because the volume stays unchanged, the process is isochoric.

(b) The ideal-gas law $p_1V_1 = nRT_1$ gives

$$T_1 = \frac{p_1V_1}{nR} = \frac{(3 \times 1.013 \times 10^5 \text{ Pa})(200 \times 10^{-6} \text{ m}^3)}{(0.0040 \text{ mol})(8.31 \text{ J/mol K})} = 1829 \text{ K} = 1556^\circ\text{C}$$

The final temperature T_2 is calculated as follows for an isochoric process:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow T_2 = T_1 \frac{p_2}{p_1} = (1829 \text{ K}) \left(\frac{1 \text{ atm}}{3 \text{ atm}} \right) = 609.5 \text{ K} = 337^\circ\text{C}$$

Assess: All straight vertical lines on a pV diagram represent isochoric processes.

18.36. Model: Assume that the gas is ideal.

Solve: (a) Because the process is at a constant pressure, it is isobaric.

(b) For an ideal gas at constant pressure,

$$\frac{V_2}{T_2} = \frac{V_1}{T_1} \Rightarrow T_2 = T_1 \frac{V_2}{V_1} = [(273 + 900) \text{ K}] \frac{100 \text{ cm}^3}{300 \text{ cm}^3} = 391 \text{ K} = 118^\circ\text{C}$$

(c) Using the ideal-gas law $p_2V_2 = nRT_2$,

$$n = \frac{p_2V_2}{RT_2} = \frac{(2 \times 1.013 \times 10^5 \text{ Pa})(100 \times 10^{-6} \text{ m}^3)}{(8.31 \text{ J/mol K})(391 \text{ K})} = 6.24 \times 10^{-3} \text{ mol}$$

Assess: All straight horizontal lines on a pV diagram represent isobaric processes.

18.37. Model: Assume that the gas is an ideal gas.

Solve: (a) The graph shows that the pressure is inversely proportional to the volume. The process is isothermal.

(b) From the ideal-gas law,

$$T_1 = \frac{p_1V_1}{nR} = \frac{(1 \times 1.013 \times 10^5 \text{ Pa})(400 \times 10^{-6} \text{ m}^3)}{(0.020 \text{ mol})(8.31 \text{ J/mol K})} = 244 \text{ K} = -29^\circ\text{C}$$

T_2 is also -29°C , because the process is isothermal.

(c) The before-and-after relationship of an ideal gas under isothermal conditions is

$$p_1 V_1 = p_2 V_2 \Rightarrow V_2 = V_1 \frac{p_1}{p_2} = (400 \text{ cm}^3) \left(\frac{1 \text{ atm}}{3 \text{ atm}} \right) = 133 \text{ cm}^3$$

18.38. Model: Assume the gas is an ideal gas.

Solve: (a) Using the ideal-gas law,

$$= (1 \text{ atm}) \left(\frac{2926 \text{ K}}{244 \text{ K}} \right) \left(\frac{100 \text{ cm}^3}{300 \text{ cm}^3} \right) = 4 \quad T_1 = \frac{p_1 V_1}{nR} = \frac{(1.013 \times 10^5 \text{ Pa})(100 \times 10^{-6} \text{ m}^3)}{(5.0 \times 10^{-3} \text{ mol})(8.31 \text{ J/mol K})} = 244 \text{ K} = -29^\circ\text{C}$$

(b) Using the before and after relationship of an ideal gas,

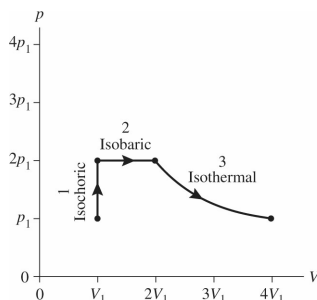
$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow p_2 = p_1 \frac{T_2}{T_1} \frac{V_1}{V_2} = (1 \text{ atm}) \left(\frac{2926 \text{ K}}{244 \text{ K}} \right) \left(\frac{100 \text{ cm}^3}{300 \text{ cm}^3} \right) = 4.0 \text{ atm}$$

(c) Using the before and after relationship of an ideal gas,

$$\frac{p_3 V_3}{T_3} = \frac{p_2 V_2}{T_2} \Rightarrow V_3 = \frac{p_2}{p_3} \frac{T_3}{T_2} V_2 = \left(\frac{4 \text{ atm}}{2 \text{ atm}} \right) \left(\frac{2438 \text{ K}}{2926 \text{ K}} \right) (300 \text{ cm}^3) = 500 \text{ cm}^3$$

18.39. Model: Assume that the gas is an ideal gas.

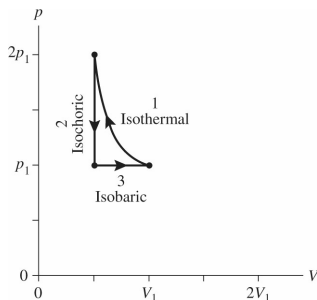
Solve:



Assess: For the isothermal process, the pressure must be halved as the volume doubles. This is because p_1 is proportional to $1/V_1$ for isothermal processes.

18.40. Model: Assume that the gas is an ideal gas.

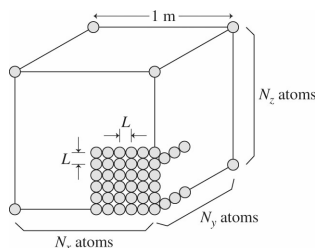
Solve:



Assess: For the isothermal process, the pressure must double as the volume is halved. This is because p is proportional to $1/V$ for isothermal processes.

Problems

18.41. Visualize:



Solve: Suppose we have a $1\text{ m} \times 1\text{ m} \times 1\text{ m}$ block of copper of mass M containing N atoms. The atoms are spaced a distance L apart along all three axes of the cube. There are N_x atoms along the x -edge of the cube, N_y atoms along the y -edge, and N_z atoms along the z -edge. The total number of atoms is $N = N_x N_y N_z$. If L is expressed in meters, then the number of atoms along the x -edge is $N_x = (1\text{ m})/L$. Thus,

$$N = \frac{1\text{ m}}{L} \times \frac{1\text{ m}}{L} \times \frac{1\text{ m}}{L} = \frac{1\text{ m}^3}{L^3} \Rightarrow L = \left(\frac{1\text{ m}^3}{N} \right)^{1/3}$$

This relates the spacing between atoms to the number of atoms in a 1-meter cube. The mass of the large cube of copper is

$$M = \rho_{\text{Cu}} V = (8920\text{ kg/m}^3)(1\text{ m}^3) = 8920\text{ kg}$$

But $M = mN$, where $m = 64\text{ u} = 64 \times (1.661 \times 10^{-27}\text{ kg})$ is the mass of an individual copper atom. Thus,

$$N = \frac{M}{m} = \frac{8920\text{ kg}}{64 \times (1.661 \times 10^{-27}\text{ kg})} = 8.39 \times 10^{28}\text{ atoms}$$

$$\Rightarrow L = \left(\frac{1\text{ m}^3}{8.39 \times 10^{28}} \right)^{1/3} = 2.28 \times 10^{-10}\text{ m} = 0.228\text{ nm}$$

Assess: This is a reasonable interatomic spacing in a crystal lattice.

18.42. Model: Assume the density of the liquid water is $\frac{1.0\text{ g}}{1.0\text{ cm}^3}$.

Visualize: There are 10 protons in each water molecule. 1 mol of water molecules has a mass of 18 g.

Solve:

$$1.0\text{ L} = 1000\text{ cm}^3 \left(\frac{1.0\text{ g}}{1.0\text{ cm}^3} \right) \left(\frac{1\text{ mol}}{18\text{ g}} \right) \left(\frac{6.02 \times 10^{23}\text{ molecules}}{1\text{ mol}} \right) \left(\frac{10\text{ protons}}{1\text{ molecule}} \right) = 3.3 \times 10^{26}\text{ protons}$$

Assess: This is an unimaginably large number, but reasonable considering the data.

18.43. Model: Assume the brass expands linearly in each direction with temperature. We will consider this a one-dimensional problem along a diameter, heating both objects.

Visualize: We look up the coefficient of linear expansion for brass: $\alpha_{\text{br}} = 1.9 \times 10^{-5} (\text{°C})^{-1}$ and the coefficient of linear expansion for steel $\alpha_{\text{st}} = 1.1 \times 10^{-5} (\text{°C})^{-1}$. The hole will expand the same amount as if there were no hole and it were solid brass.

Solve: The linear dimension across the diameter will be the same at the requisite hot temperature for both the ring and the rod: $L_{\text{br}} = L_{\text{st}} = L$:

$$L_{\text{br}} = L_{\text{st}} \Rightarrow L_{\text{br}0}(1 + \alpha_{\text{br}}\Delta T) = L_{\text{st}0}(1 + \alpha_{\text{st}}\Delta T)$$

Solve for ΔT and assume that the new length is not much different from the original length L_0 :

$$\Delta T = \frac{L_{\text{br } 0} - L_{\text{st } 0}}{L_{\text{st}} \alpha_{\text{st}} - L_{\text{br}} \alpha_{\text{br}}} = \frac{L_{\text{br } 0} - L_{\text{st } 0}}{L(\alpha_{\text{st}} - \alpha_{\text{br}})} \approx \frac{\Delta L_0}{L_0(\alpha_{\text{st}} - \alpha_{\text{br}})}$$

$$\Delta T \approx \frac{\Delta L_0}{L_0(\alpha_{\text{st}} - \alpha_{\text{br}})} = \frac{50 \mu\text{m}}{(0.0200 \text{ m})(1.9 \times 10^{-5} (\text{°C})^{-1} - 1. \times 10^{-5} (\text{°C})^{-1})} = 312.5^\circ\text{C}$$

$$T = T_0 + \Delta T = 20^\circ\text{C} + 312.5^\circ\text{C} = 333^\circ\text{C}$$

The ring needs to be raised by 312.5°C which means it must be heated to a temperature of $20^\circ\text{C} + 312.5^\circ\text{C} = 333^\circ\text{C}$.

Assess: The temperature must be raised quite a bit. We did not need to know the outer diameter of the ring.

18.44. Model: Assume the aluminum

Visualize: The bar will try to expand but will be held by the rigid walls, so it will exert a force on those walls according to the section on elasticity. We are given $\Delta T = 10.0^\circ\text{C}$. Look up the coefficient of thermal expansion for aluminum: $\alpha = 2.3 \times 10^{-5} (\text{°C})^{-1}$. We also need Young's modulus for aluminum: $Y = 7 \times 10^7 \text{ N/m}^2$.

Solve: We remember what we learned from the chapter on elasticity.

$$\frac{F}{A} = Y \frac{\Delta L}{L} \Rightarrow F = AY \frac{\Delta L}{L}$$

where Y is Young's modulus. But we also have $\Delta L/L = \alpha \Delta T$ from this chapter, so

$$F = AY \frac{\Delta L}{L} = AY \alpha \Delta T = \pi (1.0 \text{ cm})^2 (7 \times 10^{10} \text{ N/m}^2) (2.3 \times 10^{-5} \text{°C}) (10.0^\circ\text{C}) = 5.06 \text{ kN}$$

The bar will exert this much force on each wall. We report this as 5 kN to one significant figure.

Assess: 5.2 kN is quite a bit of force, but reasonable in this situation.

18.45. Model: Assume that the gas in the vacuum chamber is an ideal gas.

Solve: (a) The fraction is

$$\frac{p_{\text{vacuum chamber}}}{p_{\text{atmosphere}}} = \frac{1.0 \times 10^{-10} \text{ mm of Hg}}{760 \text{ mm of Hg}} = 1.3 \times 10^{-13}$$

(b) The volume of the chamber $V = \pi (0.20 \text{ m})^2 (0.30 \text{ m}) = 0.03770 \text{ m}^3$. From the ideal-gas equation $pV = Nk_B T$, the number of molecules of gas in the chamber is

$$N = \frac{pV}{k_B T} = \frac{(1.32 \times 10^{-13})(1.013 \times 10^5 \text{ Pa})(0.03770 \text{ m}^3)}{(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})} = 1.2 \times 10^{11} \text{ molecules}$$

18.46. Model: Model the oxygen molecules as an ideal gas.

Visualize: Do some preliminary calculations. $n = 100 \text{ mg O}_2 \left(\frac{1 \text{ mol}}{32 \text{ g}} \right) = 0.003125 \text{ mol}$. The area of the cap is

$A = \pi r^2 = \pi (0.030 \text{ m})^2 = 0.002827 \text{ m}^2$. The volume of the cylinder is $V = \pi r^2 L = \pi (0.030 \text{ m})^2 (0.10 \text{ m}) = 0.0002827 \text{ m}^3$.

The pressure inside the cylinder will be the outside pressure minus the force needed to remove the cap divided by the

area of the cap. $p = 100 \text{ kPa} - \frac{184 \text{ N}}{0.002827 \text{ m}^2} = 34.9 \times 10^3 \text{ Pa}$.

Solve: Solve the ideal gas law for temperature.

$$T = \frac{pV}{nR} = \frac{(34.9 \times 10^3 \text{ Pa})(0.0002827 \text{ m}^3)}{(0.003125 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})} = 380 \text{ K} = 107^\circ\text{C}$$

Assess: The answer is much hotter than room temperature, but the pressure was pretty low.

18.47. Model: Assume the nebula gas is ideal.

Visualize: Use $pV = Nk_B T$. We are given $N/V = 100 \text{ atoms/cm}^3 = 1 \times 10^8 \text{ atoms/m}^3$.

Solve:

$$p = \frac{Nk_B T}{V} = (1.0 \times 10^8 \text{ atoms/m}^3)(1.38 \times 10^{-23} \text{ J/K})(7500 \text{ K}) = 1.0 \times 10^{-11} \text{ Pa} = 1.0 \times 10^{-16} \text{ atm}$$

Assess: This is much lower pressure than the best vacuum we can achieve in a laboratory, but it is a higher pressure than in non-nebula space.

18.48. Model: The carbon dioxide in the cube is an ideal gas.

Solve: Using the ideal gas equation and $n = M/M_{\text{mol}}$,

$$pV = nRT \Rightarrow V = \frac{nRT}{p} = \frac{MRT}{pM_{\text{mol}}}$$

The molar mass of CO_2 is 44 g/mol or 0.044 kg/mol. Thus,

$$V = \frac{(10,000 \text{ kg})(8.31 \text{ J/mol K})(273 \text{ K})}{(1.013 \times 10^5 \text{ Pa})(0.044 \text{ kg/mol})} = 5090 \text{ m}^3$$

The length of the cube is $L = (V)^{1/3} = 17.2 \text{ m}$.

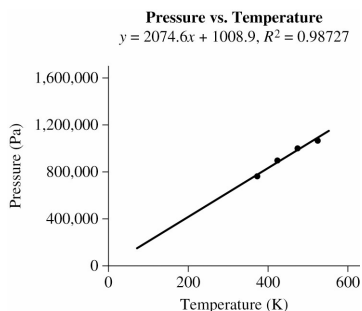
Assess: This is sobering when you multiply it by the number of people in the industrialized world. Good thing plants take up CO_2 in large quantities.

18.49. Model: Model the neon as an ideal gas.

Visualize: Because the cylinder is rigid, V doesn't change. The ideal-gas law requires that everything be in SI units: $2.0 \text{ L} = 0.0020 \text{ m}^3$

Solve: From the ideal-gas law $pV = nRT$, we solve for p as a function of T : $p = \left(\frac{nR}{V}\right)T$. This leads us to believe that a graph of pressure vs. temperature would be a straight line whose slope is $\frac{nR}{V}$. Make sure the given data is restated in SI units. Given values of pressure p in atm, we must re-write as absolute pressures in Pa: $(p + 1) \times 101,300 \text{ Pa}$.

Given temperatures T in degrees Celsius, we must convert to kelvin: $T + 273 \text{ K}$.



From the graph we see that the fit is reasonably good and that the slope is 2074.6 Pa/K.

$$\text{slope} = \frac{nR}{V} \Rightarrow n = \frac{\text{slope} \times V}{R} = \frac{(2074.6 \text{ Pa/K})(0.0020 \text{ m}^3)}{8.31 \text{ J/mol} \cdot \text{K}} = 0.50 \text{ mol}$$

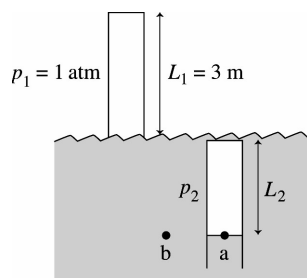
For neon, 1 mole has a mass of 20 g.

$$0.50 \text{ mol} \left(\frac{20 \text{ g}}{1 \text{ mol}} \right) = 10 \text{ g}$$

Assess: The units check out. We are just a bit bothered that the intercept of the line isn't as close to zero as we would like; there appears to be a bit of systematic error in all of the measurements, but that doesn't affect the slope.

18.50. Model: Assume the trapped air to be an ideal gas.

Visualize:



Initially, as the pipe is touched to the water surface and the gas inside is thus closed off from the air, the pressure $p_1 = p_{\text{atmos}} = 1 \text{ atm}$ and the volume is $V_1 = L_1 A$, where A is the cross-sectional area of the pipe. By pushing the pipe in *slowly*, the gas temperature in the pipe remains the same as the water temperature. Thus, this is an isothermal compression of the gas with $T_2 = T_1$.

Solve: From the ideal-gas law,

$$p_2 V_2 = p_1 V_1 \Rightarrow p_2 L_2 A = p_1 L_1 A \Rightarrow p_2 L_2 = p_{\text{atmos}} L_1 \Rightarrow p_2 = p_{\text{atmos}} (L_1 / L_2)$$

As the pipe is pushed down, the increasing water pressure pushes water up into the pipe, compressing the air. In equilibrium, the pressure at points a and b, along a horizontal line, must be equal. (This is like the barometer. If the pressures at a and b weren't equal, the pressure difference would cause the liquid level in the pipe to move up or down.) The pressure at point a is just the gas pressure inside the pipe: $p_a = p_2$. The pressure at point b is the pressure at depth L_2 in water: $p_b = p_{\text{atmos}} + \rho g L_2$. Equating these gives

$$p_2 = p_{\text{atmos}} = \rho g L_2$$

Substituting the expression for p_2 from the ideal-gas equation above, the pressure equation becomes

$$\frac{p_{\text{atmos}} L_1}{L_2} = p_{\text{atmos}} + \rho g L_2 \Rightarrow \rho g L_2^2 + p_{\text{atmos}} L_2 - p_{\text{atmos}} L_1 = 0$$

This is a quadratic equation for L_2 with solutions

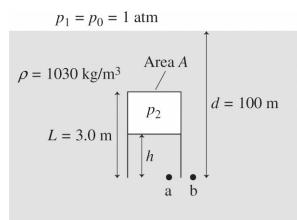
$$L_2 = \frac{-p_{\text{atmos}} \pm \sqrt{(p_{\text{atmos}})^2 + 4\rho g p_{\text{atmos}} L_1}}{2\rho g}$$

Length has to be a positive quantity, so the one physically acceptable solution is

$$L_2 = \frac{-101,300 \text{ Pa} + \sqrt{(101,300 \text{ Pa})^2 + 4(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(101,300 \text{ Pa})(3.0 \text{ m})}}{2(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 2.4 \text{ m}$$

18.51. Model: The air in the diving bell is an ideal gas.

Visualize:



Solve: (a) Initially $p_1 = p_0$ (atmospheric pressure), $V_1 = AL$, and $T_1 = 293$ K. When the diving bell is submerged to $d = 100$ m at the bottom edge, the water comes up height h inside. The volume is $V_2 = A(L - h)$ and the temperature is $T_2 = 283$ K. Like a barometer, the pressure at points a and b must be the same. Thus $p_2 + \rho gh = p_0 + \rho gd$, or $p_2 = p_0 + \rho g(d - h)$. Using the before and after relationship of an ideal gas,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow \frac{p_0 AL}{293 \text{ K}} = \frac{[p_0 + \rho g(d - h)]A(L - h)}{283 \text{ K}}$$

Multiplying this out gives the following quadratic equation for h :

$$\rho gh^2 - [p_0 + \rho g(d + L)]h + \left(1 - \frac{283}{293}\right)p_0 L + \rho gLd = 0$$

Inserting the known values (using $\rho = 1030 \text{ kg/m}^3$ for seawater) and dividing by ρg gives

$$h^2 - 113.04h + 301.03 = 0 \Rightarrow h = 110 \text{ m or } 2.7 \text{ m}$$

The first solution is not physically meaningful, so the water rises to height $h = 2.7$ m.

(b) To expel all the water, the air pressure inside the bell must be increased to match the water pressure at the bottom edge of the bell, $d = 100$ m. The necessary pressure is

$$p = p_0 + \rho gd = 101,300 \text{ Pa} + (1030 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(100 \text{ m}) = 1111 \text{ kPa} = 10.96 \text{ atm} \approx 11 \text{ atm}$$

18.52. Model: Assume that the steam is an ideal gas.

Solve: (a) The volume of water is

$$V = \frac{M}{\rho} = \frac{nM_{\text{mol}}}{\rho} = \frac{pV}{RT} \frac{M_{\text{mol}}}{\rho} = \frac{50(1.013 \times 10^5 \text{ Pa})(5.0 \text{ m}^3)(0.018 \text{ kg/mol})}{(8.31 \text{ J/mol K})(673 \text{ K})(1000 \text{ kg/m}^3)} = 0.0815 \text{ m}^3 = 81.5 \text{ L} \approx 82 \text{ L}$$

(b) Using the before-and-after relationship of an ideal gas,

$$\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1} \Rightarrow V_2 = \frac{T_2}{T_1} \frac{p_1}{p_2} V_1 = \left(\frac{273 + 150}{673 \text{ K}}\right) \left(\frac{50 \text{ atm}}{2.0 \text{ atm}}\right) (5.0 \text{ m}^3) = 78.6 \text{ m}^3 \approx 79 \text{ m}^3$$

18.53. Model: We assume that the volume of the tire and that of the air in the tire is constant.

Solve: A gauge pressure of 30 psi corresponds to an absolute pressure of $(30 \text{ psi}) + (14.7 \text{ psi}) = 44.7 \text{ psi}$. Using the before-and-after relationship of an ideal gas for an isochoric (constant volume) process,

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow p_2 = \frac{T_2}{T_1} p_1 = \left(\frac{273 + 45}{273 + 15}\right) (44.7 \text{ psi}) = 49.4 \text{ psi}$$

Your tire gauge will read a gauge pressure $p_g = 49.4 \text{ psi} - 14.7 \text{ psi} = 34.7 \text{ psi} \approx 35 \text{ psi}$.

18.54. Model: The air is assumed to be an ideal gas.

Solve: At 20°C and 1 atm pressure, the number of moles in the container is

$$n_1 = \frac{p_1 V_1}{RT_1} = \frac{(1.013 \times 10^5 \text{ Pa})(10^{-3} \text{ m}^3)}{(8.31 \text{ J/mol K})(293 \text{ K})} = 0.0416 \text{ mol}$$

At 100°C and 1 atm pressure, the number of moles is

$$n_2 = \frac{p_2 V_2}{RT_2} = \frac{(1.013 \times 10^5 \text{ Pa})(10^{-3} \text{ m}^3)}{(8.31 \text{ J/mol K})(373 \text{ K})} = 0.0327 \text{ mol}$$

When heated, the pressure will rise as the number of moles remains n_1 . When opened, the pressure drops to 1 atm as gas escapes. Thus, the number of moles of air that escape as the container is opened is $n_1 - n_2 = 0.0416 \text{ mol} - 0.0327 \text{ mol} = 0.0089 \text{ mol}$.

18.55. Model: Assume that the steam (as water vapor) is an ideal gas.

Solve: The volume of the liquid water is

$$V = \frac{m}{\rho} = \frac{nM_{\text{mol}}}{\rho} = \left(\frac{pV}{RT} \right) \frac{M_{\text{mol}}}{\rho} = \frac{20(1.013 \times 10^5 \text{ Pa})(10,000 \times 10^{-6} \text{ m}^3)(0.018 \text{ kg/mol})}{(8.31 \text{ J/mol K})(473 \text{ K})(1000 \text{ kg/m}^3)}$$

$$= 9.28 \times 10^{-5} \text{ m}^3 = 92.8 \text{ cm}^3 \approx 93 \text{ cm}^3$$

Assess: The liquid takes a lot smaller volume than the same number of atoms as a gas.

18.56. Model: Assume the gas in the manometer is an ideal gas.

Solve: In the ice-water mixture the pressure is

$$p_1 = p_{\text{atmos}} + \rho_{\text{Hg}}g(0.120 \text{ m})$$

$$= 1.013 \times 10^5 \text{ Pa} + (13,600 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.120 \text{ m}) = 1.173 \times 10^5 \text{ Pa}$$

In the freezer the pressure is

$$p_2 = p_{\text{atm}} + \rho_{\text{Hg}}g(0.030 \text{ m})$$

$$= 1.013 \times 10^5 \text{ Pa} + (13,600 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.030 \text{ m}) = 1.053 \times 10^5 \text{ Pa}$$

Assume that a drop in length of 90 mm produces a very small change in gas volume compared with the total volume of the gas cell. This means the volume of the chamber can be considered constant. Hence,

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow T_2 = T_1 \left(\frac{p_2}{p_1} \right) = (273 \text{ K}) \frac{1.053 \times 10^5 \text{ Pa}}{1.173 \times 10^5 \text{ Pa}} = 245 \text{ K} = -28^\circ\text{C}$$

Assess: This is a reasonable temperature for an industrial freezer.

18.57. Model: The air in the closed section of the U-tube is an ideal gas.

Visualize: The length of the tube is $l = 1.0 \text{ m}$ and its cross-sectional area is A .

Solve: Initially, the pressure of the air in the tube is $p_1 = p_{\text{atmos}}$ and its volume is $V_1 = Al$. After the mercury is poured in, compressing the air, the air-pressure force supports the weight of the mercury. Thus the compressed pressure equals the pressure at the bottom of the column: $p_2 = p_{\text{atmos}} + \rho gL$. The volume of the compressed air is $V_2 = A(l - L)$. Because the mercury is poured in slowly, we will assume that the gas remains in thermal equilibrium with the surrounding air, so $T_2 = T_1$. In an isothermal process, pressure and volume are related by

$$p_1V_1 = p_{\text{atmos}}Al = p_2V_2 = (p_{\text{atmos}} + \rho gL)A(l - L)$$

Canceling the A , multiplying through, and solving for L gives

$$L = l - \frac{p_{\text{atmos}}}{\rho g} = 1.00 \text{ m} - \frac{101,300 \text{ Pa}}{(13,600 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 0.24 \text{ m} = 24 \text{ cm}$$

18.58. Model: Assume that the compressed air is an ideal gas.

Solve: (a) Because the piston is floating in equilibrium,

$$F_{\text{net}} = (p_1 - p_{\text{atmos}})A - w = 0 \text{ N}$$

where the piston's cross-sectional area $A = \pi(r^2) = \pi(0.050 \text{ m})^2 = 7.854 \times 10^{-3} \text{ m}^2$ and the piston's weight $w = (50 \text{ kg})(9.8) = 490 \text{ N}$. Thus,

$$p_1 = \frac{w}{A} + p_{\text{atmos}} = \frac{490 \text{ N}}{7.854 \times 10^{-3} \text{ m}^2} + 1.013 \times 10^5 \text{ Pa} = 1.637 \times 10^5 \text{ Pa}$$

Using the ideal-gas equation $p_1V_1 = nRT_1$,

$$(1.637 \times 10^5 \text{ Pa})Ah_1 = (0.12 \text{ mol})(8.31 \text{ J/mol K})[(273 + 30) \text{ K}]$$

With the value of A given above, this equation yields $h_1 = 0.227 \text{ m} \approx 23 \text{ cm}$.

(b) When the temperature is increased from $T_1 = 303 \text{ K}$ to $T_2 = (303 + 100) \text{ K} = 403 \text{ K}$, the volume changes from $V_1 = Ah_1$ to $V_2 = Ah_2$ at a constant pressure $p_2 = p_1$. From the before-and-after relationship of the ideal gas:

$$\frac{p_1(Ah_1)}{T_1} = \frac{p_2(Ah_2)}{T_2}$$

$$\Rightarrow h_2 = \frac{p_1}{p_2} \frac{T_2}{T_1} h_1 = (1) \left(\frac{403 \text{ K}}{303 \text{ K}} \right) (0.227 \text{ m}) = 0.302 \text{ m} \approx 30 \text{ cm}$$

Thus, the piston moves $h_2 - h_1 = 7.5 \text{ cm}$.

18.59. Model: Assume that the air bubble is always in thermal equilibrium with the surrounding water, and the air in the bubble is an ideal gas.

Solve: The pressure inside the bubble matches the pressure of the surrounding water. At 50 m deep, the pressure is

$$p_1 = p_0 + \rho_{\text{water}}gd = 1.013 \times 10^5 \text{ Pa} + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(50 \text{ m}) = 5.913 \times 10^5 \text{ Pa}$$

At the lake's surface, $p_2 = p_0 = 1.013 \times 10^5 \text{ Pa}$. Using the before-and-after relationship of an ideal gas,

$$\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1} \Rightarrow V_2 = V_1 \frac{p_1}{p_2} \frac{T_2}{T_1}$$

$$\Rightarrow \frac{4\pi}{3} r_2^3 = \frac{4\pi}{3} (0.005 \text{ m})^3 \left(\frac{5.913 \times 10^5 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}} \right) \left(\frac{293 \text{ K}}{283 \text{ K}} \right) \Rightarrow r_2 = 0.0091 \text{ m}$$

The diameter of the bubble is $2r_2 = 0.0182 \text{ m} \approx 1.8 \text{ cm}$.

Assess: The bubble's diameter just about doubled; this seems reasonable.

18.60. Model: Assume that the helium gas is an ideal gas.

Visualize: Process $1 \rightarrow 2$ is isochoric, process $2 \rightarrow 3$ is isothermal, and process $3 \rightarrow 1$ is isobaric.

Solve: The number of moles of helium is

$$n = \frac{M}{M_{\text{mol}}} = \frac{8.0 \text{ g}}{4 \text{ g/mol}} = 2.0 \text{ mol}$$

Using the ideal-gas equation,

$$V_1 = \frac{nRT_1}{p_1} = \frac{(2.0 \text{ mol})(8.31 \text{ J/mol K})[(273 + 37) \text{ K}]}{2(1.013 \times 10^5 \text{ Pa})} = 0.0254 \text{ m}^3 \approx 0.025 \text{ m}^3$$

For the isochoric process $V_2 = V_1$, and

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow p_2 = p_1 \frac{T_2}{T_1} = (2 \text{ atm}) \left(\frac{657 + 273}{37 + 273} \right) = 6.0 \text{ atm}$$

For the isothermal process, the equation $p_3 V_3 = p_2 V_2$ is

$$V_3 = V_2 \frac{p_2}{p_3} = (0.0254 \text{ m}^3) \left(\frac{6 \text{ atm}}{2 \text{ atm}} \right) = 0.0762 \text{ m}^3 \approx 0.076 \text{ m}^3$$

For the isothermal process, $T_3 = T_2 = 657^\circ\text{C}$.

18.61. Model: Assume the nitrogen gas is an ideal gas.

Solve: (a) The number of moles of nitrogen is

$$n = \frac{M}{M_{\text{mol}}} = \frac{1 \text{ g}}{28 \text{ g/mol}} = \left(\frac{1}{28} \right) \text{ mol}$$

Using the ideal-gas equation,

$$p_1 = \frac{nRT_1}{V_1} = \frac{(1/28 \text{ mol})(8.31 \text{ J/mol K})(298 \text{ K})}{(100 \times 10^{-6} \text{ m}^3)} = 8.84 \times 10^5 \text{ Pa} = 884 \text{ kPa} \approx 880 \text{ kPa}$$

(b) For the process from state 1 to state 3:

$$\frac{p_1 V_1}{T_1} = \frac{p_3 V_3}{T_3} \Rightarrow T_3 = T_1 \frac{p_3}{p_1} \frac{V_3}{V_1} = (298 \text{ K}) \left(\frac{1.5 p_1}{p_1} \right) \left(\frac{50 \text{ cm}^3}{100 \text{ cm}^3} \right) = 223.5 \text{ K} \approx -49^\circ\text{C}$$

For the process from state 3 to state 2:

$$\frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3} \Rightarrow T_2 = T_3 \left(\frac{p_2}{p_3} \right) \left(\frac{V_2}{V_3} \right) = (223.5 \text{ K}) \left(\frac{2.0 p_1}{1.5 p_1} \right) \left(\frac{100 \text{ cm}^3}{50 \text{ cm}^3} \right) = 596 \text{ K} = 323^\circ\text{C}$$

For the process from state 1 to state 4:

$$\frac{p_4 V_4}{T_4} = \frac{p_1 V_1}{T_1} \Rightarrow T_4 = T_1 \frac{p_4}{p_1} \frac{V_4}{V_1} = (298 \text{ K}) \left(\frac{1.5 p_1}{p_1} \right) \left(\frac{150 \text{ cm}^3}{100 \text{ cm}^3} \right) = 670.5 \text{ K} \approx 398^\circ\text{C}$$

18.62. Model: The gas is an ideal gas.

Solve: (a) Using the ideal-gas equation,

$$T_1 = \frac{p_1 V_1}{nR} = \frac{(1.0 \times 10^5 \text{ Pa})(2.0 \text{ m}^3)}{(80 \text{ mol})(8.31 \text{ J/mol K})} = 301 \text{ K}$$

Because points 1 and 2 lie on the isotherm, $T_2 = T_1 = 301 \text{ K}$. The temperature of the isothermal process is 301 K.

(b) The straight-line process $1 \rightarrow 2$ can be represented by the equation

$$p = (3 - V) \times 10^5$$

where V is in m^3 and p is in Pa. We can use the ideal gas law to find that the temperature along the line varies as

$$T = \frac{pV}{nR} = (3V - V^2) \times \frac{10^5}{nR}$$

We can maximize T by setting the derivative dT/dV to zero:

$$\frac{dT}{dV} = (3 - 2V_{\max}) \times \frac{10^5}{nR} = 0 \Rightarrow V_{\max} = \frac{3}{2} \text{ m}^3 = 1.50 \text{ m}^3$$

At this volume, the pressure is $p_{\max} = 1.5 \times 10^5 \text{ Pa}$ and the temperature is

$$T_{\max} = \frac{p_{\max} V_{\max}}{nR} = \frac{(1.50 \times 10^5 \text{ Pa})(1.5 \text{ m}^3)}{(80 \text{ mol})(8.31 \text{ J/mol K})} = 338 \text{ K}$$

18.63. Model: We assume the oxygen gas is ideal.

Visualize: From the figure we glean $p_2 = p_1/3$ and $V_2 = V_1/3$. We are given $T_1 = 820^\circ\text{C} + 273 = 1093 \text{ K}$.

Solve: Use the before-and-after version of the ideal-gas law.

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$T_2 = \frac{p_2 V_2}{p_1 V_1} T_1 = \frac{(p_1/3)(V_1/3)}{p_1 V_1} T_1 = T_1/9 = (1093 \text{ K})/9 = 121 \text{ K} = -152^\circ\text{C}$$

Assess: This is a cold temperature—colder than the condensation point of oxygen.

18.64. Model: Assume CO_2 gas is an ideal gas.

Solve: (a) The molar mass for CO_2 is $M_{\text{mol}} = 44 \text{ g/mol}$, so a 10 g piece of dry ice is 0.2273 mol. This becomes 0.227 mol of gas at 0°C . With $V_1 = 10,000 \text{ cm}^3 = 0.010 \text{ m}^3$ and $T_1 = 0^\circ\text{C} = 273 \text{ K}$, the pressure is

$$p_1 = \frac{nRT_1}{V_1} = \frac{(0.2273 \text{ mol})(8.31 \text{ J/mol K})(273 \text{ K})}{0.010 \text{ m}^3} = 5.156 \times 10^4 \text{ Pa} = 0.509 \text{ atm} \approx 0.50 \text{ atm}$$

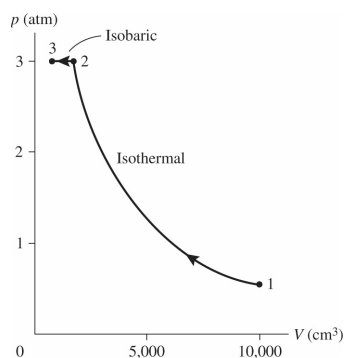
(b) From the isothermal compression,

$$p_2 V_2 = p_1 V_1 \Rightarrow V_2 = V_1 \frac{p_1}{p_2} = (0.010 \text{ m}^3) \left(\frac{0.509 \text{ atm}}{3.0 \text{ atm}} \right) = 1.70 \times 10^{-3} \text{ m}^3 = 1700 \text{ cm}^3$$

From the isobaric compression,

$$T_3 = T_2 \frac{V_3}{V_2} = (273 \text{ K}) \left(\frac{1000 \text{ cm}^3}{1700 \text{ cm}^3} \right) = 161 \text{ K} = -112^\circ\text{C}$$

(c)



18.65. Model: The gas in the container is assumed to be an ideal gas.

Solve: (a) The gas starts at pressure $p_1 = 2.0 \text{ atm}$, temperature $T_1 = 127^\circ\text{C} = (127 + 273) \text{ K} = 400 \text{ K}$ and volume V_1 . It is first compressed at a constant temperature $T_2 = T_1$ until $V_2 = \frac{1}{2}V_1$ and the pressure is p_2 . It is then further compressed at constant pressure $p_3 = p_2$ until $V_3 = \frac{1}{2}V_2$. From the ideal-gas law,

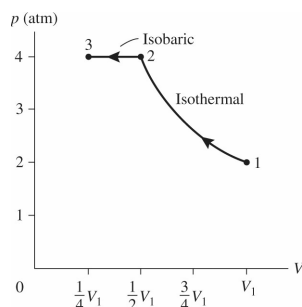
$$\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1} \Rightarrow p_2 = p_1 \frac{V_1}{V_2} \frac{T_2}{T_1} = (2.0 \text{ atm}) \frac{V_1}{\frac{1}{2}V_1} \times 1 = 4.0 \text{ atm}$$

Note that $T_2 = T_1 = 400 \text{ K}$. Using the ideal-gas law once again,

$$\frac{p_3 V_3}{T_3} = \frac{p_2 V_2}{T_2} \Rightarrow T_3 = T_2 \frac{V_3}{V_2} \frac{p_3}{p_2} = (400 \text{ K}) \frac{\frac{1}{2}V_2}{V_2} \times 1 = 200 \text{ K} = -73^\circ\text{C}$$

The final pressure and temperature are 4.0 atm and -73°C .

(b)



18.66. Model: Assume that the nitrogen gas is an ideal gas.

Solve: (a) The molar mass of N_2 gas is 28 g/mol. The number of moles is $n = (5 \text{ g})/(28 \text{ g/mol}) = 0.1786 \text{ mol}$. The initial conditions are $p_1 = 3.0 \text{ atm}$ and $T_1 = 293 \text{ K}$. We use the ideal gas law to find the initial volume as follows:

$$V_1 = \frac{nRT_1}{p_1} = \frac{(0.1786 \text{ mol})(8.31 \text{ J/mol K})(293 \text{ K})}{3.0 \text{ atm} \times 101,300 \text{ Pa/atm}} = 1.430 \times 10^{-3} \text{ m}^3 = 1430 \text{ cm}^3 \approx 1400 \text{ cm}^3$$

An isobaric expansion until the volume triples results in $V_2 = 3V_1 = 4290 \text{ cm}^3$.

(b) After the expansion,

$$\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1} \Rightarrow T_2 = \frac{p_2 V_2}{p_1 V_1} T_1 = 1 \times 3 \times T_1 = 3T_1 = 879 \text{ K} = 606^\circ\text{C}$$

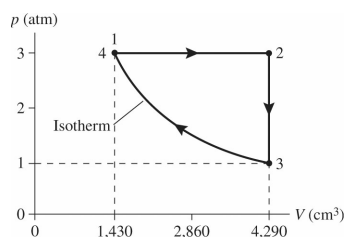
(c) A constant volume decrease at $V_3 = V_2 = 4290 \text{ cm}^3$ back to $T_3 = T_1 = \frac{1}{3}T_2$ results in the following:

$$\frac{p_3 V_3}{T_3} = \frac{p_2 V_2}{T_2} \Rightarrow p_3 = \frac{T_3 V_2}{T_2 V_3} p_2 = \frac{1}{3} \times 1 \times p_2 = \frac{1}{3} \times 3.0 \text{ atm} = 1.0 \text{ atm}$$

(d) An isothermal compression at $T_4 = T_3$ back to the initial volume $V_4 = V_1 = \frac{1}{3}V_3$ results in the following:

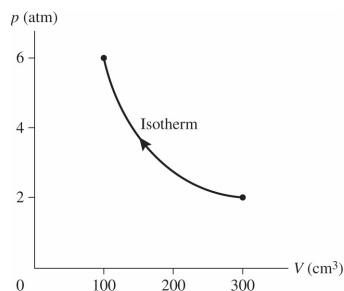
$$\frac{p_4 V_4}{T_4} = \frac{p_3 V_3}{T_3} \Rightarrow p_4 = \frac{T_4 V_3}{T_3 V_4} p_3 = 1 \times \frac{1}{\frac{1}{3}} \times p_3 = 3 \times 1.0 \text{ atm} = 3.0 \text{ atm}$$

(e)



18.67. Solve: (a) A gas is compressed isothermally from a volume 300 cm^3 at 2 atm to a volume of 100 cm^3 . What is the final pressure?

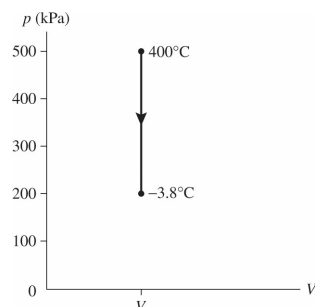
(b)



(c) The final pressure is $p_2 = 6 \text{ atm}$.

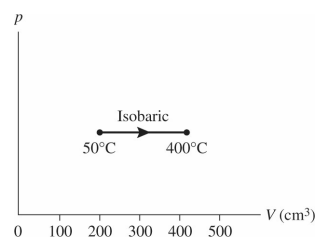
18.68. Solve: (a) A gas at 400°C and 500 kPa is cooled at a constant volume to a pressure of 200 kPa . What is the final temperature in $^\circ\text{C}$?

(b)

(c) The final temperature is $T_2 = -3.8^\circ\text{C}$.

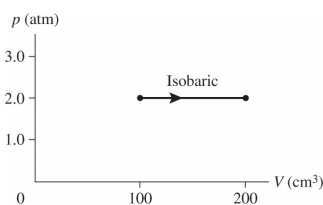
18.69. Solve: (a) A gas expands at constant pressure from 200 cm^3 at 50°C until the temperature is 400°C . What is the final volume?

(b)

(c) The final volume is $V_2 = 417\text{ cm}^3$.

18.70. Solve: (a) 0.12 g of neon gas at 2.0 atm and 100 cm^3 expands isobarically to twice its initial volume. What is the final temperature of the gas?

(b)



(c) The number of moles of neon is

$$n = \frac{M}{M_{\text{mol}}} = \frac{0.12\text{ g}}{20\text{ g}} = 0.006\text{ mol}$$

The initial temperature is

$$T_1 = \frac{p_1 V_1}{nR} = \frac{2(1.013 \times 10^5\text{ Pa})(1.0 \times 10^{-4}\text{ m}^3)}{(0.006\text{ mol})(8.31\text{ J/mol K})} = 406\text{ K}$$

The final temperature is

$$T_2 = \frac{p_2}{p_1} \frac{V_2}{V_1} T_1 = \frac{p}{p} \frac{2V_1}{V_1} (406 \text{ K}) = 812 \text{ K}$$

Challenge Problems

18.71. Model: Assume the air is pure N_2 , with a molar mass of $M_{\text{mol}} = 28 \text{ g/mol}$.

Visualize: We will use $pV = nRT$ both before and after. Our intermediate goal is $n_1 - n_2$. We are given $T_2 = T_1 = T = 20^\circ\text{C} + 273 = 293 \text{ K}$ and $V_2 = V_1 = V = \pi r^2 C = \pi(0.011 \text{ m}^2)(2.0 \text{ m}) = 7.60 \times 10^{-4} \text{ m}^3$.

Solve: We convert the gauge pressures to absolute pressures with $p = p_g + 1 \text{ atm} = p_g + 14.7 \text{ psi}$. $p_1 = 110 \text{ psi} + 14.7 \text{ psi} = 124.7 \text{ psi} = 860 \text{ kPa}$. $p_2 = 80 \text{ psi} + 14.7 \text{ psi} = 94.7 \text{ psi} = 653 \text{ kPa}$.

$$\begin{aligned} n_1 - n_2 &= \frac{p_1 V_1}{RT_1} - \frac{p_2 V_2}{RT_2} \\ &= \frac{V}{RT} (p_1 - p_2) \\ &= \frac{7.60 \times 10^{-4} \text{ m}^3}{(8.31 \text{ J/mol} \cdot \text{K})(293 \text{ K})} (860 \text{ kPa} - 653 \text{ kPa}) \\ &= 0.0646 \text{ mol} \end{aligned}$$

Thus 0.0646 mol of N_2 was lost; this is

$$0.0646 \text{ mol} \left(\frac{28 \text{ g}}{1 \text{ mol}} \right) = 1.8 \text{ g}$$

Assess: The result seems to be a reasonable number.

18.72. Model: The gas in the cylinder is assumed to be an ideal gas.

Solve: The gas at $T_1 = 20^\circ\text{C} = 293 \text{ K}$ has a pressure $p_1 = 1 \text{ atm}$ and a volume $V_1 = AL_0$. (The pressure has to be 1 atm to balance the external force of the air on the piston.) At a temperature $T_2 = 100^\circ\text{C} = 373 \text{ K}$, its volume becomes $V_2 = A(L_0 + \Delta x)$ and its pressure increases to

$$p_2 = p_1 + \frac{F}{A} = p_1 + \frac{k\Delta x}{A}$$

where $F = k\Delta x$ is the spring force on the piston. Using the before-and-after relationship of an ideal-gas equation,

$$\begin{aligned} \frac{p_1 V_1}{T_1} &= \frac{p_2 V_2}{T_2} \Rightarrow \frac{p_1 L_0 A}{T_1} = \frac{(p_1 + k\Delta x/A)(L_0 + \Delta x)A}{T_2} \\ \Rightarrow \frac{T_2}{T_1} &= \frac{(p_1 + k\Delta x/A)(L_0 + \Delta x)}{p_1 L_0} = \left(1 + \frac{k\Delta x}{p_1 A} \right) \left(1 + \frac{\Delta x}{L_0} \right) \end{aligned}$$

L_0 can be obtained from the ideal-gas law as follows:

$$L_0 = \frac{V_1}{A} = \left(\frac{nRT_1}{p_1} \right) \frac{1}{A} = \frac{(0.004 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(293 \text{ K})}{(1.013 \times 10^5 \text{ Pa})(10 \times 10^{-4} \text{ m}^2)} = 0.0961 \text{ m} = 9.61 \text{ cm}$$

Substituting this value of L_0 and the values for p_1 , T_1 , T_2 , and k , the above equation can be simplified to

$$154.02\Delta x^2 + 25.210\Delta x - 0.2730 = 0 \Rightarrow \Delta x = 0.0102 \text{ m} = 1.02 \text{ cm}$$

The spring is compressed by 1.0 cm.

18.73. Model: The gas in containers A and B is assumed to be an ideal gas.

Solve: (a) The number of moles of gas in containers A and B can be expressed as follows:

$$n_A = \frac{p_A V_A}{RT_A} = \frac{(1.0 \times 10^5 \text{ Pa}) V_A}{R(300 \text{ K})} = 333.3 \frac{V_A}{R} \quad n_B = \frac{p_B V_B}{RT_B} = \frac{(5.0 \times 10^5 \text{ Pa})(4V_A)}{R(400 \text{ K})} = 5000 \frac{V_A}{R}$$

where n_A and n_B are expressed in moles. Let n'_A and n'_B be the number of moles after the valve is opened. Since the total number of moles is the same,

$$n_A + n_B = 5333.3 \frac{V_A}{R} = n'_A + n'_B$$

The pressure is now the same in both containers:

$$p'_A = \frac{n'_A RT_A}{V_A} = p'_B = \frac{n'_B RT_B}{V_B}$$

$$\Rightarrow n'_A = n'_B \frac{T_B}{T_A} \frac{V_A}{V_B} = n'_B \left(\frac{400 \text{ K}}{300 \text{ K}} \right) \frac{V_A}{4V_A} \Rightarrow 3n'_A = n'_B$$

Solving the above equations,

$$5333.3 \frac{V_A}{R} = n'_A + 3n'_A = 4n'_A \Rightarrow n'_A = 1333.3 \frac{V_A}{R} \Rightarrow n'_B = 4000 \frac{V_A}{R}$$

The new pressure is

$$p'_A = \frac{n'_A RT_A}{V_A} = \frac{1333.3 \left(\frac{V_A}{R} \right) RT_A}{V_A} = 4.0 \times 10^5 \text{ Pa}$$

$$p'_B = \frac{n'_B RT_B}{V_B} = \frac{4000 \left(\frac{V_A}{R} \right) RT_B}{4V_A} = 4.0 \times 10^5 \text{ Pa}$$

Gas flows from B to A until the pressure is $4.0 \times 10^5 \text{ Pa}$.

(b) The change is irreversible because opening the valve is like breaking a membrane. It is not a quasi-static process.

18.74 Model: Assume the gas in the left chamber is ideal.

Visualize: For the piston to be in equilibrium $(F_x)_{\text{net}} = 0$. The initial volume is $V_0 = L_0 A$.

Solve:

(a) The force to the right on the piston by the gas in the left chamber is simply $p_0 A$. The magnitude of the force to the left on the piston by the spring is $k(\Delta L)$.

$$(F_x)_{\text{net}} = p_0 A - k(\Delta L) \Rightarrow \Delta L = \frac{p_0 A}{k}$$

(b) When the piston is moved x to the right the spring is compressed a total of $\Delta L + x$ and the pressure in the left chamber changes to p_1 according to the ideal gas law. When the temperature doesn't change the ideal gas equation reduces to $p_0 V_0 = p_1 V_1$, where $V_0 = L_0 A$ and $V_1 = (L_0 + x) A$. Therefore

$$p_1 = \frac{p_0 L_0}{(L_0 + x)}$$

Now, similar to part (a) above,

$$(F_x)_{\text{net}} = p_1 A - k(\Delta L + x)$$

Substitute in ΔL from part (a) and p_1 from above.

$$(F_x)_{\text{net}} = \frac{p_0 L_0}{(L_0 + x)} A - k \left(\frac{p_0 A}{k} + x \right)$$

Now it is time to apply the binomial approximation: $(L_0 + x)^{-1} \approx 1/L_0 (1 - x/L_0)$ for $x \ll L_0$.

$$\begin{aligned}(F_x)_{\text{net}} &= (p_0 L_0)(1/L_0(1 - x/L_0))A - p_0 A - kx \\ &= p_0 A - p_0 A x/L_0 - p_0 A - kx \\ &= -(p_0 A/L_0 + k)x\end{aligned}$$

This is clearly a linear restoring force of the form of Hooke's law where the usual k is replaced with $(p_0 A/L_0 + k)$. That is, the gas in the left chamber is increasing the k of the system (by $p_0 A/L_0$) and making it act like a stiffer spring.

(c) Since we have a Hooke's law for which we know the modified k , we simply replace k in the period equation for harmonic oscillators with the modified constant.

$$T = 2\pi \sqrt{\frac{M}{p_0 A/L_0 + k}}$$

Assess: This is simple harmonic motion when the approximation $x \ll L_0$ is valid.