From equation (1) to equation (7)

$$\frac{\partial I_\nu(\hat{\gamma})}{\partial \tau} = -I_\nu(\hat{\gamma}) + \frac{3}{16\pi}\int d^2\hat{\gamma}' \cdot I_\nu(\hat{\gamma}')(1+\cos^2\theta_{sc}) \qquad (1)$$

Let $\hat{\gamma} = (\theta, \phi)$, $\hat{\gamma}' = (\theta', \phi')$, then $\cos\theta_{sc} = \hat{\gamma}\cdot\hat{\gamma}'$

$$1+\cos^2\theta_{sc} = P_0 + \frac{1}{3}(P_0 + 2P_2) = \frac{4}{3}P_0(\hat{\gamma}\cdot\hat{\gamma}') + \frac{2}{3}P_2(\hat{\gamma}\cdot\hat{\gamma}')$$

By the addition theorem: $P_0(\hat{\gamma}\cdot\hat{\gamma}') = 4\pi Y_{00}(\hat{\gamma}')Y_{00}^*(\hat{\gamma})$

$$P_2(\hat{\gamma}\cdot\hat{\gamma}') = \frac{4\pi}{5}\sum_{m=-2}^{2} Y_{2m}(\hat{\gamma}')Y_{2m}^*(\hat{\gamma})$$

So $1+\cos^2\theta_{sc} = \left[\frac{16\pi}{3} Y_{00}(\hat{\gamma}')Y_{00}^*(\hat{\gamma}) + \frac{8\pi}{15}\sum_{m=-2}^{2} Y_{2m}(\hat{\gamma}')Y_{2m}^*(\hat{\gamma})\right]$

$$\frac{3}{16\pi}\int d^2\hat{\gamma}' \cdot I_\nu(\hat{\gamma}')(1+\cos^2\theta_{sc})$$

$$= \frac{3}{16\pi}\cdot\int d^2\hat{\gamma}' \sum_{l=0}^{\infty}\sum_m^l a_{lm}^I(\nu)Y_{lm}(\hat{\gamma}')\left[\frac{16\pi}{3} Y_{00}(\hat{\gamma}')Y_{00}^*(\hat{\gamma}) + \frac{8\pi}{15}\sum_{m=-2}^{2} Y_{2m}(\hat{\gamma}')Y_{2m}^*(\hat{\gamma})\right]$$

$$= \frac{1}{2}\sum_{l=0}^{\infty}\int d^2\hat{\gamma}' \sum_{m=-l}^{l} a_{lm}^I(\nu)Y_{lm}(\hat{\gamma}')\left[2Y_{00}(\hat{\gamma}')Y_{00}^*(\hat{\gamma}) + \frac{1}{5}\sum_{m=-2}^{2} Y_{2m}(\hat{\gamma}')Y_{2m}^*(\hat{\gamma})\right]$$

$$= \frac{1}{2}\left\{\sum_{l=0}^{\infty}\int d^2\hat{\gamma}' \sum_{m=-l}^{l} a_{lm}^I(\nu)Y_{lm}(\hat{\gamma}')\, 2Y_{00}(\hat{\gamma}')Y_{00}^*(\hat{\gamma}) + \sum_{l=0}^{\infty}\int d^2\hat{\gamma}' \sum_{m=-l}^{l} a_{lm}^I(\nu)Y_{lm}(\hat{\gamma}')\,\frac{1}{5}\sum_{m=-2}^{2} Y_{2m}(\hat{\gamma}')Y_{2m}^*(\hat{\gamma})\right\}$$

$$= \frac{1}{2}\left\{\int d^2\hat{\gamma}'\, a_{00}^I(\nu)Y_{00}(\hat{\gamma}')Y_{00}^*(\hat{\gamma}')\cdot 2Y_{00}(\hat{\gamma}) + \int d^2\hat{\gamma}' \sum_{m=-2}^{2} a_{2m}^I(\nu)Y_{2m}(\hat{\gamma}')\sum_{m=-2}^{2} Y_{2m}^*(\hat{\gamma}')\cdot\frac{1}{5}Y_{2m}(\hat{\gamma})\right\}$$

$$= Y_{00}(\hat{\gamma})a_{00}^I(\nu) + \sum_{m=-2}^{2} a_{2m}^I(\nu)\cdot\frac{1}{10}Y_{2m}(\hat{\gamma})$$

$$\delta I_\nu^{(l)}(\hat{\gamma}) = \sum_m^l a_{lm}^I(\nu)Y_{lm}(\hat{\gamma})$$

this cancels out $\delta I_\nu^{(0)}(\hat{\gamma})$

this reduces the coefficient of $\delta I_\nu^{(2)}(\hat{\gamma})$ from 1 to $\frac{9}{10}$

$$I_\nu(\hat{\gamma}) = \sum_{l=0}^{\infty}\sum_m^l a_{lm}^I(\nu)Y_{lm}(\hat{\gamma}) = \sum_{l=0}^{\infty}\delta I_\nu^{(l)}(\hat{\gamma})$$

From equation (2) to (8)

generalized addition thm: ${}_sY_{ls}(\theta_{sc},0)e^{-is\phi_{sc}} = (-1)^s\sqrt{\frac{4\pi}{2l+1}}\sum_m^l Y^*_{lm}(\hat{\gamma}')\cdot {}_sY_{lm}(\hat{\gamma})$

let $s=2$, $\sin^2\theta_{sc}e^{-2i\phi_{sc}} = \sqrt{\frac{32\pi}{15}}\underbrace{{}_2Y_{22}(\theta_{sc},0)}_{}e^{-2i\phi_{sc}}$

$\sum_{m=-2}^{2} Y^*_{2m}(\hat{\gamma})\,{}_{\pm2}Y_{2m}(\hat{\gamma}')$

${}_2Y_{22} = \frac{1}{8}\sqrt{\frac{5}{\pi}}(1-\cos\theta)^2e^{i2\phi}$

${}_2Y_{22} = \sqrt{\frac{5}{4\pi}}\sin^4(\frac{\theta}{2})\times\binom{0}{0}\binom{4}{0}e^{i2\phi}$

$\binom{l-s}{r} = \frac{(l-s)!}{r!(l-s-r)!}$

$\sin^4(\frac{\theta}{2}) = \frac{(1-\cos\theta)^2}{4}$

${}_2Y_{22} = \sqrt{\frac{5}{32\pi}}[1+\cos^2\theta-2\cos\theta]$

$\sin^2\beta = \sqrt{\frac{8}{3}}D^2_{0\pm2}(\gamma_1,\beta,-\gamma_2)e^{\mp2i\gamma_2} = \sqrt{\frac{8}{3}}\cdot\frac{4\pi}{5}\sum_{m=-2}^{2}Y^*_{2m}(\hat{e}_1)\,{}_{\pm2}Y_{2m}(\hat{e}_2)e^{\mp2i\gamma_2}$

$D^2_{0\pm2}(\gamma_1,\beta,-\gamma_2) = \sum_m\frac{4\pi}{5}\,{}_0Y^*_{2m}(\hat{e}_1)\,{}_{\pm2}Y_{2m}(\hat{e}_2)$

$D^2_{0\pm2} = \sqrt{\frac{4\pi}{5}}\,{}_{\pm2}Y_{20}(\hat{e})$

$\sin^2\beta = \sqrt{\frac{32\pi}{15}}\,{}_{\pm2}Y_{20}(\hat{e})e^{\mp2i\gamma_2}$