Starting with equation (2) in the E.Bunn paper

$$a_{2m}(\vec r) = \int d^3k\, \Delta_2(k;r)\, \delta_\Phi(\vec k)\, e^{i\vec k\cdot\vec r}\, Y^*_{2m}(\hat k') \qquad (2)$$

using the plane wave expansion:

$$e^{i\vec k\cdot\vec r} = 4\pi \sum_{l=0}^{\infty}\sum_{m=-l}^{l} i^l j_l(kr)\, Y_{lm}(\hat k)\, Y^*_{lm}(\hat r)$$

equation (2) becomes

$$a_{2m}(\vec r) = \int k^2 dk \sin\theta\, d\theta\, d\varphi\;\; \Delta_2(k;r)\, \delta_\Phi(k;r)\, 4\pi \sum_{l=0}^{\infty}\sum_{m=-l}^{l} i^l j_l(kr)\, Y_{lm}(\hat k)\, Y^*_{lm}(\hat r)\, Y^*_{2m}(\hat k')$$

$$\int dk\, Y_{lm}(\hat k)\, Y^*_{2m}(\hat k') = \delta_{l2}\ ?$$

$$= 4\pi \sum_{l=0}^{\infty}\sum_{m=-l}^{l} i^l\, Y^*_{lm}(\hat r) \int d^3k\, \Delta_2(k;r)\, \delta_\Phi(k;r)\, Y_{lm}(\hat k)\, Y^*_{2m}(\hat k)$$

$$= \int k^2 dk\, \Delta_2(k;r)\, \delta_\Phi(k;r) \cdot \int d\Omega\, Y_{lm}(\hat k)\, Y^*_{2m}(\hat k)$$

$$= \frac{4\pi}{(2l+1)} \cdot \delta_{l2}$$

$$= -\frac{16\pi^2}{5}\, Y^*_{2m}(\hat r) \cdot \int k^2 dk\, \Delta_2(k;r)\, \delta_\Phi(k;r)$$