

COMP 540 Assignment #1

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0 Background refresher(30 points)

- Plot the categorical distribution.

Figure 1: Categorical distribution

- Plot the Univariate normal distribution with mean of and standard deviation of 1.

Figure 2: Univariate Normal Distribution

- Produce a scatter plot of the samples for a 2-D Gaussian.

Figure 3: Univariate Normal Distribution

- Test mixture sampling code Code can be seen in sampler.py. Mixture Gaussian plot is shown below

Figure 4: Univariate Normal Distribution

- Prove that the sum of two independent Poisson random variables is also a Poisson random variable. Suppose $X \sim \mathcal{P}(\lambda)$ and $Y \sim \mathcal{P}(\mu)$. Now Prove that $X + Y \sim \mathcal{P}(\lambda + \mu)$.

$$\begin{aligned}
 P(X + Y = k) &= \sum_{i=0}^k P(X + Y = k, X = i) \\
 &= \sum_{i=0}^k P(Y = k - i, X = i) \\
 &= \sum_{i=0}^k P(Y = k - i)P(X = i) \\
 &= \sum_{i=0}^k e^{-\mu} \frac{\mu^{k-i}}{(k-i)!} e^{-\lambda} \frac{\lambda^i}{i!} \\
 &= e^{-(\mu+\lambda)} \frac{1}{k!} \sum_{i=0}^k \frac{k!}{i!(k-i)!} \mu^{k-i} \lambda^i \\
 &= e^{-(\mu+\lambda)} \frac{1}{k!} \sum_{i=0}^k \binom{k}{i} \mu^{k-i} \lambda^i \\
 &= \frac{(\mu + \lambda)^k}{k!} \cdot e^{-(\mu+\lambda)}
 \end{aligned}$$

So $X + Y \sim \mathcal{P}(\lambda + \mu)$.

- Find α, μ_1 and σ_1 .
We have X_0 and X_1 be continuous random variables. If

$$\begin{aligned}
 p(X_0 = x_0) &= \alpha_0 e^{-\frac{(x_0 - \mu_0)^2}{2\sigma_0^2}} \\
 P(X_1 = x_1 | X_0 = x_0) &= \alpha_1 e^{-\frac{(x_1 - x_0)^2}{2\sigma^2}}
 \end{aligned}$$

$$\begin{aligned}
p(X_1 = x_1) &= \int P(X_1 = x_1 | X_0 = x_0) \cdot p(X_0 = x_0) dx_0 \\
&= \alpha_0 \alpha_1 \int e^{-\frac{\sigma^2(x_0 - \mu_0)^2 + \sigma_0^2(x_1 - x_0)^2}{2\sigma_0^2\sigma^2}} dx_0 \\
&= \alpha_0 \alpha_1 \int e^{-\frac{(\sigma^2 + \sigma_0^2)x_0^2 - 2(\sigma^2\mu_0 + \sigma_0^2x_1)x_0 + \sigma^2\mu_0^2 + \sigma_0^2x_1^2}{2\sigma_0^2\sigma^2}} dx_0 \\
&= \alpha_0 \alpha_1 \int e^{-\frac{1}{2\sigma_0^2\sigma^2}[(\sqrt{\sigma^2 + \sigma_0^2}x_0 - \frac{\sigma^2\mu_0 + \sigma_0^2x_1}{\sqrt{\sigma^2 + \sigma_0^2}})^2 + \sigma^2\mu_0^2 + \sigma_0^2x_1^2 - \frac{\sigma^2\mu_0^2 + \sigma_0^2x_1^2}{\sigma^2 + \sigma_0^2}]} dx_0
\end{aligned}$$

Since

$$\int \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 1$$

$$\begin{aligned}
p(X_1 = x_1) &= \frac{\alpha\alpha_0\sqrt{2\pi}\sigma\sigma_0}{\sqrt{\sigma^2 + \sigma_0^2}} e^{-\frac{1}{2\sigma_0^2\sigma^2}(\sigma^2\mu_0^2 + \sigma_0^2x_1^2 - \frac{\sigma^2\mu_0^2 + \sigma_0^2x_1^2}{\sigma^2 + \sigma_0^2})} \\
&= \alpha e^{-\frac{(x_1 - \mu_0)^2}{2(\sigma^2 + \sigma_0^2)}}
\end{aligned}$$

Thus we can solve that:

$$\begin{aligned}
\alpha &= \frac{\alpha\alpha_0\sqrt{2\pi}\sigma\sigma_0}{\sqrt{\sigma^2 + \sigma_0^2}} \\
\mu_1 &= \mu_0 \\
\sigma_1 &= \sqrt{\sigma^2 + \sigma_0^2}
\end{aligned}$$

- Show that if $P(A|B, C) > P(A|B)$ then $P(A|B, C^C) < P(A|B)$

$$P(A|B, C) = \frac{P(A \cap B \cap C)}{P(B \cap C)}$$

$$\frac{P(A \cap B \cap C)}{P(B \cap C)} > \frac{P(A \cap B)}{P(B)}$$

$$\frac{P(C|A \cap B)}{P(C|B)} > 1$$

$$P(C|A \cap B) > P(C|B)$$

$$1 - P(C|A \cap B) < 1 - P(C|B)$$

$$P(C^C|A \cap B) < P(C^C|B)$$

$$\frac{P(C^C|A \cap B)}{P(C^C|B)} < 1$$

$$\begin{aligned}
P(A|B, C^C) &= \frac{P(A \cap B \cap C^C)}{P(B \cap C^C)} \\
&= \frac{P(C^C|A \cap B)P(A \cap B)}{P(C^C|B)P(B)} \\
&= \frac{P(C^C|A \cap B)}{P(C^C|B)P(B)} P(A|B) \\
&< P(A|B)
\end{aligned}$$

Thus, if $P(A|B, C) > P(A|B)$ then $P(A|B, C^C) < P(A|B)$.

- Consider the vectors $\mathbf{u} = \begin{bmatrix} 1 & 2 \end{bmatrix}^T$ and $\mathbf{v} = \begin{bmatrix} 2 & 3 \end{bmatrix}^T$. Define the matrix $\mathbf{M} = \mathbf{u}\mathbf{v}^T$. Compute the eigenvalues and eigenvectors of \mathbf{M} .

$$\begin{aligned}
\mathbf{M} &= \mathbf{u}\mathbf{v}^T \\
&= \begin{bmatrix} 1 & 2 \end{bmatrix}^T \cdot \begin{bmatrix} 2 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}
\end{aligned}$$

$$|\lambda \mathbf{I} - \mathbf{M}| = \begin{vmatrix} \lambda - 2 & -3 \\ -4 & \lambda - 6 \end{vmatrix} = 0$$

Then we can solve it as

$$\begin{aligned}
(\lambda - 2)(\lambda - 6) - 12 &= 0 \\
\lambda^2 - 8\lambda &= 0 \\
\lambda(\lambda - 8) &= 0 \\
\lambda_1 = 0, \lambda_2 = 8
\end{aligned}$$

Let $\lambda = 0$:

$$\begin{aligned}
(\lambda \mathbf{I} - \mathbf{M}) \cdot \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T &= 0 \\
\begin{bmatrix} -2 & -3 \\ -4 & -6 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
-2x_1 - 3x_2 &= 0
\end{aligned}$$

Let $x_1 = 3$, Then $x_2 = -2$. So eigenvector is $\begin{bmatrix} 3 & -2 \end{bmatrix}^T$.

Let $\lambda = 8$:

$$\begin{aligned}
(\lambda \mathbf{I} - \mathbf{M}) \cdot \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T &= 0 \\
\begin{bmatrix} 6 & -3 \\ -4 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
6x_1 - 3x_2 &= 0 \\
-4x_1 + 2x_2 &= 0
\end{aligned}$$

Let $x_1 = 1$, Then $x_2 = 2$. So eigenvector is $\begin{bmatrix} 1 & 2 \end{bmatrix}^T$.

Thus, when eigenvalue $\lambda = 0$, eigenvector is $\begin{bmatrix} 3 & -2 \end{bmatrix}^T$, when eigenvalue $\lambda = 8$, eigenvector is $\begin{bmatrix} 1 & 2 \end{bmatrix}^T$.

- Show that if A is positive semi-definite, then all eigenvalues of A are non-negative.
Definition of an eigenvalue and eigenvector is:

$$A\mathbf{v} = \lambda\mathbf{v}$$

If A is positive semi-definite, $\mathbf{x}^T A \mathbf{x} \geq 0$ for all \mathbf{x} .

$$\mathbf{v}^T A \mathbf{v} = \mathbf{v}^T \mathbf{v} \lambda$$

Since $\mathbf{v}^T \mathbf{v}$ is necessarily a positive number, in order for $\mathbf{v}^T A \mathbf{v}$ to be greater than or equal to 0, λ must be $\lambda \geq 0$.

- Provide one example for each of the following cases.

As for $(A + B)^2 \neq A^2 + 2AB + B^2$. Suppose $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Since $A \cdot B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and

$$B \cdot A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

So $(A + B)^2 = A^2 + AB + BA + B^2 \neq A^2 + 2AB + B^2$.

As for $AB = 0, A \neq 0, B \neq 0$, Suppose $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

- Show that \mathbf{A} is orthogonal.
Given $\mathbf{u}^T \mathbf{u} = 1$ and $\mathbf{A} = \mathbf{I} - 2\mathbf{u}\mathbf{u}^T$.

$$\begin{aligned} \mathbf{A}^T \mathbf{A} &= (\mathbf{I} - 2\mathbf{u}\mathbf{u}^T)^T (\mathbf{I} - 2\mathbf{u}\mathbf{u}^T) \\ &= (\mathbf{I} - 2\mathbf{u}\mathbf{u}^T) (\mathbf{I} - 2\mathbf{u}\mathbf{u}^T) \\ &= \mathbf{I} - 2\mathbf{u}\mathbf{u}^T - 2\mathbf{u}\mathbf{u}^T + 4\mathbf{u}\mathbf{u}^T \mathbf{u}\mathbf{u}^T \\ &= \mathbf{I} - 2\mathbf{u}\mathbf{u}^T - 2\mathbf{u}\mathbf{u}^T + 4\mathbf{u}\mathbf{u}^T \\ &= \mathbf{I} \end{aligned}$$

So \mathbf{A} is orthogonal.

- Prove the following assertions.
-As for $f(x) = x^3$ for $x \geq 0$.

$$\begin{aligned} f(x) &= x^3 \\ f''(x) &= 6x \end{aligned}$$

Since $x \geq 0$, $f''(x) = 6x \geq 0$. Thus $f(x) = x^3$ is convex for $x \geq 0$.

-As for $f(x_1, x_2) = \max(x_1, x_2)$. Let $\mathbf{x} = (x_1, x_2)$ and $\mathbf{y} = (y_1, y_2)$ and $\lambda \in [0, 1]$.

$$\begin{aligned} f(\lambda\mathbf{x} + (1-\lambda)\mathbf{y}) &= f(\lambda x_1 + (1-\lambda)y_1, \lambda x_2 + (1-\lambda)y_2) \\ &= \max(\lambda x_1 + (1-\lambda)y_1, \lambda x_2 + (1-\lambda)y_2) \\ &\leq \max(\lambda x_1, \lambda x_2) + \max((1-\lambda)y_1, (1-\lambda)y_2) \\ &= \lambda \max(x_1, x_2) + (1-\lambda) \max(y_1, y_2) \\ &= \lambda f(\mathbf{x}) + (1-\lambda)f(\mathbf{y}) \end{aligned}$$

So $f(x_1, x_2) = \max(x_1, x_2)$ is convex on R^2 .

–As for function $f + g$. If univariate functions f and g are convex on S , then $f''(x) \geq 0$ and $g''(x) \geq 0$.

So $(f + g)''(x) = f''(x) + g''(x) \geq 0$. Thus if univariate functions f and g are convex on S , then $f + g$ is convex on S .

–As for fg . If univariate functions f and g are convex and non-negative on S .

$$\begin{aligned}(fg)''(x) &= (f'g + fg')'(x) \\ &= (f''g + f'g' + f'g' + fg'')(x) \\ &= (f''g + 2f'g' + fg'')(x)\end{aligned}$$

Since f and g have their minimum within S at the same point. Before the minimum point, both f and g are decreasing. After the minimum point, both f and g are increasing. So $f'g' \geq 0$, $f'' \geq 0$, $g'' \geq 0$, $f \geq 0$ and $g \geq 0$. Thus $(fg)''(x) = (f''g + 2f'g' + fg'')(x) \geq 0$. Then fg is convex on S .

- Find the highest entropy of categorical distribution.

The entropy of a categorical distribution on K values is defined as

$$H(p) = - \sum_{i=1}^K p_i \log(p_i)$$

The probability and constraints are defined below:

$$P(X = x_i) = p_i \quad \text{for } i = 1, 2, \dots, K$$

$$s.p. \begin{cases} \sum_{i=1}^K p_i = 1 \\ p_i \geq 0 \quad \text{for } i = 1, 2, \dots, K \end{cases}$$

Constrain function is

$$\varphi(p_i) = \sum_{i=1}^K p_i - 1 = 0$$

Define Lagrange multipliers:

$$\begin{aligned}\mathcal{L} &= H(p) + \lambda \varphi(p_i) \\ &= - \sum_{i=1}^K p_i \log(p_i) + \lambda \left(\sum_{i=1}^K p_i - 1 \right)\end{aligned}$$

To find the highest entropy, we should find the point where derivative is 0.

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial p_i} &= -\log(p_i) - 1 + \lambda = 0 \\ p^* &= e^{\lambda-1}\end{aligned}$$

Thus when all $p_i, i = 1, 2, \dots, K$ are equally equal to $e^{\lambda-1}$, the categorical distribution has the highest entropy.

1 Locally weighted linear regression(20 points)

- Find an appropriate diagonal matrix W .

$$J(\theta) = (X\theta - y)^T W (X\theta - y)$$

Let W be

$$W = \begin{bmatrix} \frac{1}{2}w^{(1)} & & & & 0 \\ & \frac{1}{2}w^{(2)} & & & \\ & 0 & & \ddots & \\ & & & & \frac{1}{2}w^{(i)} \end{bmatrix}$$

X is the $m \times d$ input matrix and y is a $m \times 1$ vector.

$$X = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & \cdots & x_d^{(1)} \\ x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & \cdots & x_d^{(2)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ x_1^{(m)} & x_2^{(m)} & x_3^{(m)} & \cdots & x_d^{(m)} \end{bmatrix}$$

$$y = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(m)} \end{bmatrix}$$

$$\theta = \begin{bmatrix} \theta_{(1)} \\ \theta_{(2)} \\ \vdots \\ \theta_{(d)} \end{bmatrix}$$

$$X\theta - y = \begin{bmatrix} (\theta_1 x_1^{(1)} + \theta_2 x_2^{(1)} + \cdots + \theta_d x_d^{(1)}) - y^{(1)} \\ (\theta_1 x_1^{(2)} + \theta_2 x_2^{(2)} + \cdots + \theta_d x_d^{(2)}) - y^{(2)} \\ \vdots \\ (\theta_1 x_1^{(m)} + \theta_2 x_2^{(m)} + \cdots + \theta_d x_d^{(m)}) - y^{(m)} \end{bmatrix}$$

$$W(X\theta - y) = \begin{bmatrix} \frac{1}{2}w^{(1)} \times ((\theta_1 x_1^{(1)} + \theta_2 x_2^{(1)} + \cdots + \theta_d x_d^{(1)}) - y^{(1)}) \\ \frac{1}{2}w^{(2)} \times ((\theta_1 x_1^{(2)} + \theta_2 x_2^{(2)} + \cdots + \theta_d x_d^{(2)}) - y^{(2)}) \\ \vdots \\ \frac{1}{2}w^{(m)} \times ((\theta_1 x_1^{(m)} + \theta_2 x_2^{(m)} + \cdots + \theta_d x_d^{(m)}) - y^{(m)}) \end{bmatrix}$$

$$\begin{aligned} (X\theta - y)^T W (X\theta - y) &= \frac{1}{2}w^{(1)} \times ((\theta_1 x_1^{(1)} + \theta_2 x_2^{(1)} + \cdots + \theta_d x_d^{(1)}) - y^{(1)})^2 \\ &+ \frac{1}{2}w^{(2)} \times ((\theta_1 x_1^{(2)} + \theta_2 x_2^{(2)} + \cdots + \theta_d x_d^{(2)}) - y^{(2)})^2 + \cdots \\ &+ \frac{1}{2}w^{(m)} \times ((\theta_1 x_1^{(m)} + \theta_2 x_2^{(m)} + \cdots + \theta_d x_d^{(m)}) - y^{(m)})^2 \end{aligned}$$

So $J(\theta) = (X\theta - y)^T W (X\theta - y)$ can be written in the form $J(\theta) = (X\theta - y)^T W (X\theta - y)$ when choosing W as above.

- Solve for θ

$$\begin{aligned}
J(\theta) &= (X\theta - y)^T W (X\theta - y) \\
&= ((X\theta)^T - y^T) W (X\theta - y) \\
&= ((X\theta)^T W - y^T W) (X\theta - y) \\
&= \theta^T X^T W X \theta - 2\theta^T X^T W y + y^T W y
\end{aligned}$$

$$\begin{aligned}
\nabla_{\theta} J(\theta) &= 2X^T W X \theta - 2X^T W y = 0 \\
\theta &= (X^T W X)^{-1} X^T W y \\
&= (X^T X)^{-1} X^T y
\end{aligned}$$

Thus $\theta = (X^T X)^{-1} X^T y$.

- Write down an algorithm for calculating θ by batch gradient descent for locally weighted linear regression. We know that $\theta = (X^T X)^{-1} X^T y$. Algorithm is

Algorithm 1: Calculating θ by batch gradient descent

Input: x

Output: θ

for $i < 10000$ **do**

$\theta_j \leftarrow \theta_j - \alpha \sum_i w^{(i)} (\sum_k (\theta_k x_x^{(i)}) - y^{(i)}), j = 0$
 $\theta_j \leftarrow \theta_j - \alpha \sum_i w^{(i)} (\sum_k (\theta_k x_x^{(i)}) - y^{(i)}) x_j^{(i)}, j \neq 0$

Locally weighted linear regression is a non-parametric method.

2 Properties of the linear regression estimator(10 points)

- Show that $E[\theta] = \theta^*$

$$\begin{aligned}
\theta &= (X^T X)^{-1} X^T y \\
&= (X^T X)^{-1} X^T (X\theta^* + \epsilon) \\
&= (X^T X)^{-1} X^T X \theta^* + (X^T X)^{-1} X^T \epsilon
\end{aligned}$$

$$\begin{aligned}
E(\theta) &= E((X^T X)^{-1} X^T X \theta^* + (X^T X)^{-1} X^T \epsilon) \\
&= E((X^T X)^{-1} X^T X \theta^*) + E((X^T X)^{-1} X^T \epsilon) \\
&= \theta^* + (X^T X)^{-1} X^T E(\epsilon) \\
&= \theta^*
\end{aligned}$$

- Show the variance of the least squares estimator is $Var(\theta) = (X^T X)^{-1} \sigma^2$. Suppose \mathbf{b} is an estimator of θ

$$\begin{aligned}
Var(\theta) &= E[(\mathbf{b} - \theta)(\mathbf{b} - \theta)^T | X] \\
&= E[(X^T X)^{-1} X^T \epsilon \epsilon^T X (X^T X)^{-1} | X] \\
&= (X^T X)^{-1} X^T E[\epsilon \epsilon^T | X] X (X^T X)^{-1} \\
&= (X^T X)^{-1} \sigma^2
\end{aligned}$$

3 Implementing linear regression and regularized linear regression(90 points)

3.1 Implementing linear regression(45 points)

- A1: Computing the cost function $J(\theta)$
See the implementation for the *loss* function in the file *linear_regressor.py*.
- A2: Implementing gradient descent
See the implementation for the *train* function in the file *linear_regressor.py*.

Figure 5: Fitting a linear model

Figure 6: Convergence of gradient descent

- A3: Predicting on unseen data
For lower status percentage = 5, we predict a median home value of 298034.49
For lower status percentage = 50, we predict a median home value of -129482.13
- B1: Feature normalization
See the implementation for the *feature_normalize* function in the file *utils.py*.
- B2: Loss function and gradient descent
See the implementation for the *train* and *loss* function in the file *linear_regressor_multi.py*.

Figure 7: Convergence of gradient descent with multiple variables

- B3: Making predictions on unseen data
For average home in Boston suburbs, we predict a median home value of 230467.11
- B4: Normal equations
See the implementation for the *normal_eqn* function in the file *linear_regressor_multi.py*.
Theta computed by direct solution is: $[3.64594884e + 01 - 1.08011358e - 014.64204584e - 022.05586264e - 022.68673382e + 00 - 1.77666112e + 013.80986521e + 006.92224640e - 04 - 1.47556685e + 003.06049479e - 01 - 1.23345939e - 02 - 9.52747232e - 019.31168327e - 03 - 5.24758378e - 01]$
For average home in Boston suburbs, we predict a median home value of 230406.54.

- B5: Exploring convergence of gradient descent

((a)) learning rate=0.01

((b)) learning rate=0.03

((c)) learning rate=0.1

((d)) learning rate=0.3

Figure 8: Convergence of gradient descent with multiple variables according to different learning rates

According to the above figures, when learning $rate = 0.01$ is a good choice.

When learning rate = 0.01, choosing different iteration times.

((a)) iteration time = 50

((b)) iteration time = 500

((c)) iteration time = 5000

((d)) iteration time = 50000

Figure 9: Convergence of gradient descent with multiple variables according to different iteration times

According to the above figures, iteration time is 500 is a good choice.

3.2 Implementing regularized linear regression(45 points)

- A1: Regularized linear regression cost function
See the implementation for the *loss* function in the class *Reg_LinearRegression_SquaredLoss* in the file *reg_linear_regressor_multi.py*.
- A2: Gradient of the Regularized linear regression cost function
See the implementation for the *grad_loss* function in the class *Reg_LinearRegression_SquaredLoss* in the file *reg_linear_regressor_multi.py*.
- A3: Learning curves
- A4: Adjusting the regularization parameter

((a)) Polynomial Regression fit

((b)) Learning curve for linear regression

Figure 10: $\lambda = 0.0$

((a)) Polynomial Regression fit

((b)) Learning curve for linear regression

Figure 11: $\lambda = 1.0$

((a)) Polynomial Regression fit

((b)) Learning curve for linear regression

Figure 12: $\lambda = 10.0$

((a)) Polynomial Regression fit

((b)) Learning curve for linear regression

Figure 13: $\lambda = 100.0$

- A5: Selecting λ using a validation set According to the figure above, the best λ is 3.

Figure 14: Fitting a linear model

- A6: Computing test set error
Test error is 5.944297830865587.
- A7: Plotting learning curves with randomly selected examples
- A8: Comparing ridge regression and lasso regression models
Lasso regression converges faster than Ridge regression.

Figure 15: Fitting a linear model

4 Extra Credits

- The lowest achievable error on the test set with $\lambda = 0$
Test error is 12.930601133114878 when $\lambda = 0$.
- Select the best value for λ and report the test set error with the best λ .
The best $\lambda = 30.0$. Test error at $\lambda = 30.0$ is 13.256266000933937.
- What is the test set error with quadratic features with the best λ chosen with the validation set?
The best $\lambda = 30.0$. Test error at $\lambda = 30.0$ is 4.747514558496794.

Figure 16: Fitting a linear model

Figure 17: Fitting a linear model

- What is the test set error with cubic features with the best λ chosen with the validation set?
The best $\lambda = 30.0$. Test error at $\lambda = 30.0$ is 4.767539493628556.

Figure 18: Fitting a linear model