TOSHKENT AMALIY FANLAR UNIVERSITETI

11 – Amaliy mashg'ulot

Funksiya differensiali va yuqori tartibli hosilalar.

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Misol 1. Quyidagi funksiyalarning ko‘rsatilgan tartibdagi hosilalarni toping.

a) $y = x^3 + 2x^2 - x - 3$ $\quad y''' = ?$

b) $s = \ln t$ $\quad s''' = ?$

v) $s = t^3 - t - 3$ $\quad s''(0) = ?$

g) $f(x) = \sin 2x$ $\quad f'''\left(\frac{\pi}{2}\right) = ?$

Yechimi: a) Birinchi tartibli hosilani olamiz:

$$y' = 3x^2 + 2 \cdot 2x - 1$$

undan yana hosila olamiz:

$$y'' = 6x + 4$$

yana bir marta hosila olsak, uchinchi tartibli hosila kelib chiqadi, demak,

$$y''' = 6$$

b) Berilgan funksiyadan ketma – ket to‘rt marta hosila olamiz:

$$s' = (\ln t)' = \frac{1}{t};$$

$$s'' = (s')' = \left(\frac{1}{t}\right)' = -\frac{1}{t^2};$$

$$s''' = (s'')' = \left(-\frac{1}{t^2}\right)' = \frac{1}{t^3};$$

$$s'''' = (s''')' = \left(\frac{1}{t^3}\right)' = (2t^{-3}) = 2\cdot(-3)t^{-4} = -\frac{6}{t^4}$$

demak, $s'''' = -\frac{6}{t^4}$.

v) $s = t^3 - t + 3$

$s' = \left(t^3 - t + 3\right)' = 3t^2 - 1$

$s'' = \left(s'\right)' = \left(3t^2 - 1\right)' = 6t$. $t = 0$ da

$s''(0) = 6 \cdot 0 = 0$

demak,

$$s''(0) = 0$$

g) $f(x) = \sin 2x$

$f'(x) = 2\cos 2x$

$f''(x) = \left(f'(x)\right)' = \left(2\cos 2x\right)' = -4\sin 2x$

$f'''(x) = \left(f''(x)\right)' = \left(-4\sin 2x\right)' = -8\cos 2x$

$f''''(x) = \left(f'''(x)\right)' = \left(-8\cos 2x\right)' = 16\sin 2x$

$f'''''(x) = \left(f''''(x)\right)' = \left(16\sin 2x\right)' = 32\sin 2x$

$$f''''\left(\frac{\pi}{2}\right)=32\cos\left(2\frac{\pi}{2}\right)=32\cos\pi=-32$$

Demak, $f''''\left(\frac{\pi}{2}\right)=-32$

Misol 2. $y=\cos 2x$ funksiyaning $y''+4y=0$ tenglamani qanoatlantirishini isbotlang.

Yechimi:

$$y'=-2\sin 2x;\quad y''=\left(-2\sin 2x\right)'=-4\cos 2x$$

o‘rniga qo‘ysak

$$-4\cos 2x+4\cdot\cos 2x=0;\quad 0=0$$

Mashqlar

Quyidagi funksiyalarni ko‘rsatilgan tartibdagi hosilalarini toping:
(Javobi qavs ichida berilgan)

1. $y = x^3 + 4x^2 - 7x + 1;$ $y''' = ?$ (0)
2. $f(x) = x^8;$ $f'''(1) = ?$ (336)
3. $y = x^5 + 4x^3 - x;$ $y'''' = ?$ (120)
4. $y = \cos x;$ $y''' = ?$ $(\cos x)$

6.1.12. Berilgan hosilalar uchun y''' ni toping:

$1)\ y=(x^2-1)^3;\quad 2)\ y=e^{2x}\cos x;\quad 3)\ y=(1+x^2)arctgx;\quad 4)\ y=x^2(\ln x-1).$

6.1.12. $1)\ y'''=24x(5x^2-3);\ \ 2)\ y'''=e^{2x}(2\cos x-11\sin x);\ \ 3)\ y'''=\dfrac{4}{(1+x^2)^2};\ \ 4)\ y'''=\dfrac{2}{x}$

6.1.14. Berilgan funksiyalar uchun $\dfrac{d^2y}{dx^2}$ ni toping:

$1)\ \begin{cases} x=t^2+1, \\ y=t^3-1; \end{cases}$

$2)\ \begin{cases} x=a\cos t, \\ y=a\sin t; \end{cases}$

$3)\ \begin{cases} x=\ln(1+t^2), \\ y=t-arctg\,t; \end{cases}$

$4)\ \begin{cases} x=\arcsin t, \\ y=\sqrt{1-t^2}. \end{cases}$

6.1.14. $1)\ \dfrac{3}{4t};\ \ 2)\ -\dfrac{1}{a\sin^3 t};\ \ 3)\ \dfrac{1+t^2}{4t};\ \ 4)\ -\sqrt{1-t^2}.$

6.1.18. Material nuqta $s = s(t)$ qonun bilan to‘g‘ri chiziqli harakat qilmoqda. Qaysi vaqtda material nuqtaning tezlanishi $a(m/c^2)$ ga teng bo‘ladi?

$1)\ s(t) = 2t^3 - \frac{5}{2}t^2 + 3t + 1(m),\ a = 19;$ $2)\ s(t) = t^3 + \frac{3}{2}t^2 - 4t + 3(m),\ a = 9.$

6.1.18. $1)\ t = 2c;\ 2)\ t = 1c.$

E’tiboringiz uchun rahmat!