Final exam, Artificial Intelligence (EPS – UAM)
2022-05-31

Grades published on 2022-06-03

Last (family) name(s):
First (given) name: David

INSTRUCTIONS:

1. **Write your full name on every sheet of paper you use.**
2. Give concise and clear answers. Include all the information asked in an ordered manner.
3. Your answers should be reasoned in order to get points: A correct answer without any justification may not receive full credit.
4. No books, mobile phones or external aids can be used.
5. Calculators are allowed
6. **Turn in each exercise in a different set of sheets.**

1. **Logistic regression and neural networks [3.3 points].**
Consider a neuron with sigmoidal activation function:

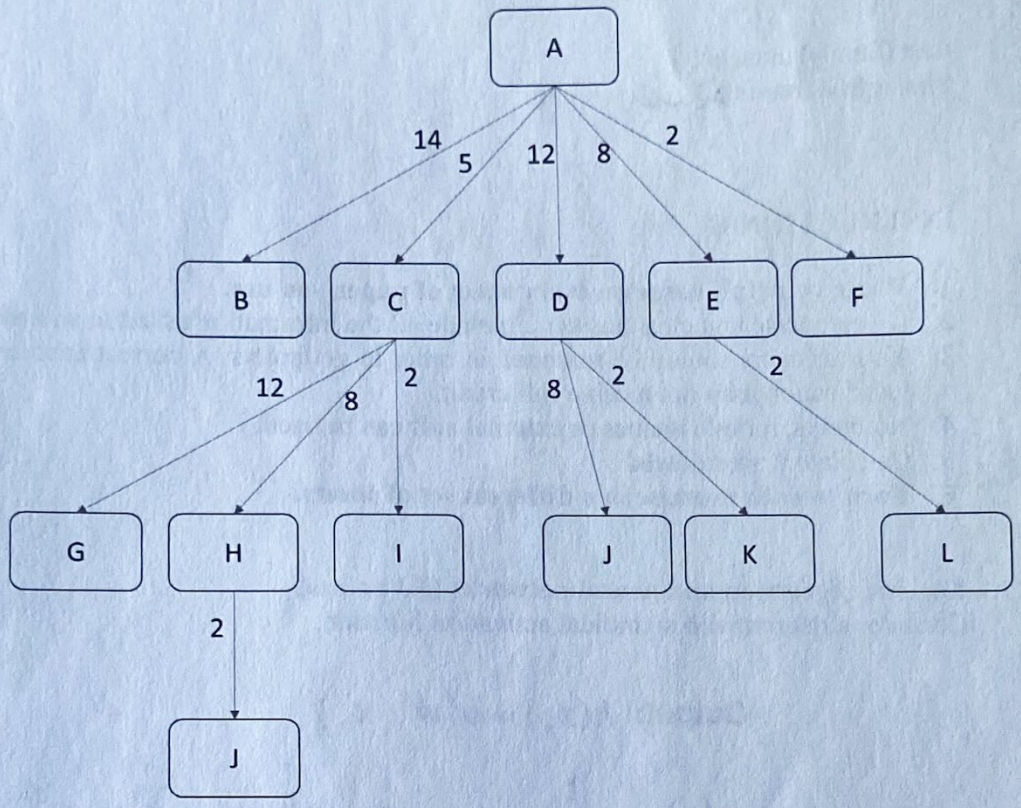
$$\textbf{Output: } h(\mathbf{x}_n) = \sigma\left(\mathbf{w}^T \cdot \mathbf{x}_n\right)$$

$$\sigma(z) = \frac{1}{1+e^{-z}} \qquad \mathbf{z} = \mathbf{w}^T \cdot \mathbf{x}_n = \sum_{d=0}^{D} w_d x_n^{(d)}$$

Prove that the neuron implements a linear separator in its input space (x^1, x^2, … x^D). For this you can use mathematical arguments, examples, or graphs.

2. **Informed Search and A* (3.4 points).**
Given this graph and this table:

Node	Heuristic
A	7
B	2
C	10
D	2
E	2
F	13
G	2
H	2
I	8
J	0
K	5
L	5



Answer the following questions (You can do them on the exam sheet):

a. Draw the tree that A* would generate with elimination of repeated states.
   i. Indicate the expansion order and generation order (between siblings from left to right)
   ii. Indicate at each step the value on which you base yourself to establish said order.
   iii. Indicate the path to the solution
b. With this heuristic and elimination of repeated states, does A* guarantees to find the optimal solution? Explain the reason.
c. Does A* with this heuristic and without elimination of repeated states guarantees to find the optimal solution? Explain the reason.
d. Is the heuristic indicated in the graph monotonic? why?
e. Give an example and justify what change or changes you should make so that it stops being monotonous or not monotonous.

3. **[3.3 points]** Consider the following ontology in the domain "family":

	Name	Arity	Meaning
Predicates	M	2	M(x,y): x is the mother of y.
	C	2	C(x,y): x is the child of y.
	S	2	S(x,y): x is the sibling of y.
Functions	ma	1	ma(x): reference to the mother of x.

Use other functions and predicates (including the equality predicate) only if necessary.

a. Complete the missing information in the following knowledge base:

WFF in predicate logic	**Meaning**
[1] $\forall x\ M(ma(x), x)$	
[2] $\forall x\ C(x, ma(x))$	
[3]	Siblings are people who have the same mother.
[4]	If a person is the mother of another one, the second is the child of the first one.

b. Transform the knowledge base into conjunctive normal form indicating at each step the rule used for the transformation.

c. Use refutation with resolution among clauses to prove that every person has exactly one mother. To answer this question, you may need to include additional well-formed formulas in the original knowledge base.