Material complementario análisis de algoritmos

• basic arithmetic progression

$$S_N = \sum_1^N K \qquad S_N = \frac{N(N+1)}{2} = \frac{N^2}{2} + O(N)$$

• basic geometric progression

Serie geométrica

$$S_N = \sum_1^N x^K \text{ for an } x>0,\ x \neq 1 \qquad \sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

$$S_N = \frac{x^{N+1} - x}{x-1}$$

Hay un truco para recordar está fórmula general

$$\sum_M^N R^K = \frac{UR-P}{R-1}$$

→ where $U = R^N$ is the last term

→ $P = R^M$ is the

Sums of Powers

• Consider $S_N^K = \sum_1^N i^K = \sum_1^N f(i)$ with $f(x) = x^K$

• Furthermore Es

$$\int_0^N x^K dx \leq S_N^K \leq \int_1^{N+1} x^K dx$$

• Therefore

$$\left[\frac{x^{K+1}}{K+1}\right]_0^N = \frac{N^{K+1}}{K+1} \leq S_N^K \leq \frac{(N+1)^{K+1}}{K+1} - \frac{1}{K+1}$$

dividimos los términos entre $\frac{N^{K+1}}{K+1}$

$$1 \leq \frac{S_N^K}{\frac{N^{K+1}}{K+1}} \leq \frac{\frac{(N+1)^{K+1}}{\cancel{K+1}}}{\frac{N^{K+1}}{\cancel{K+1}}} - \frac{\frac{1}{\cancel{K+1}}}{\frac{N^{K+1}}{\cancel{K+1}}} = \frac{(N+1)^{K+1}}{N^{K+1}} - \frac{1}{N^{K+1}}$$

$$\lim_{N\to\infty} \frac{(N+1)^{K+1}}{N^{K+1}} = \lim_{n\to\infty} \left(\frac{N+1}{N}\right)^{K+1} \to 1$$

$$1 \leq \frac{S_N^K}{\frac{N^{K+1}}{K+1}} \leq 1$$

Por ende $S_N^K \sim \frac{N^{K+1}}{K+1}$

Aproximación $\log N!$

- Consider $S_N = \log N! = \sum_1^N \log i$
- $\log x$ is also increasing and we then have

$$\int_0^N \log x\,dx \le S_N \le \int_1^{N+1} \log x\,dx$$

$$\int \log x\,dx = \int 1\cdot\log x\,dx = x\log x - \int \frac{x}{x}\,dx = x\log x - x.$$

$u = \log x \quad du = 1/x$

$dv = 1 \quad v = x$

como el $\ln 0$ es una indeterminación

$$\int_1^N \log x\,dx \le S_N \le \int_2^{N+1} \log x\,dx \le \int_1^{N+1} \log x\,dx$$

$$N\log N - N + 1 \le S_N \le (N+1)\log(N+1) - N$$

dividir todo entre $N\log N$ y obtener

$$1 - \frac{1}{\log N} + \frac{1}{N\log N} \le \frac{S_N}{N\log N} \le \frac{N+1}{N}\,\frac{\log(N+1)}{\log N} - \frac{1}{\log N}$$

$$\lim_{x\to\infty} \frac{\log(x+1)}{\log(x)} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{\frac{1}{x+1}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{x}{x+1} = 1$$

$$1 \le \frac{S_N}{N\log N} \le 1$$

$$S_N \sim N\log N$$

The Harmonic Number

- Consider $H_N := \sum_1^N \frac{1}{i}$
- $\frac{1}{x}$ decreases

$$\int_0^N \frac{dx}{x} \ge H_N \ge \int_1^{N+1} \frac{dx}{x} \quad \text{modificamos } H_N = 1 + \sum_2^N \frac{1}{i}$$

$$H_N \sim \log N$$

$$\boxed{\int f^n \cdot f' = \frac{f^{n+1}}{n+1}}$$