

Software tools for collider data analyses within the Electroweak Chiral Lagrangian framework

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The Electroweak Effective Theory is a model capable of testing the Standard Model. Beyond the Standard Model physics could be hidden inside the couplings of the theory, which can be put to test at colliders like the LHC. The large number of operators that appear in the theory makes it too troublesome to work with by hand. Thus, implementing the theory model into a computer is essential to get the needed results to compare with the data. In this paper, we review the existing tools capable of performing such computations, which are included in MATHEMATICA-based packages. In particular, we will be working with the packages FEYNRULES, FEYNARTS and FEYNCALC. With the help of this packages we will be implementing the Electroweak Chiral Lagrangian in the software, getting the vertices of our theory and calculating some example amplitudes for different Feynman diagrams, thus showing the capabilities of these tools.

I. INTRODUCTION

After the successful runs at the LHC confirming the validity of the Standard Model (SM), different new theories have been developed trying to find physics beyond the Standard Model (BSM).

The approach these theories take is the construction of some effective field theory (EFT) [1] where the information about the ultraviolet (UV) and the infrared (IR) is separated.

To build an EFT, one must select an energy scale at which the new physics should take place. Then, one proceeds to write the most general Lagrangian preserving the underlying gauge symmetries of the SM in terms of the light fields present at that energy scale. The dynamics of the UV will then be accounted for within the couplings of the effective Lagrangian.

In this paper, we will study one particular EFT known as the Electroweak Effective Theory (EWET), Higgs Effective Field Theory (HEFT) or Electroweak Chiral Lagrangian (EWChL) [2, 3]. This theory is a non-linear generalization of the SM Effective Field Theory (SMEFT).

The motivation for such theory is the lack of new particle states observed at the LHC implying the existence of a mass gap between the electroweak and new physics scales. Therefore, one could expect the new low-energy signals to be suppressed by higher powers of the energy scale parameterized with higher order operators.

The formal development of the theory to next to leading order is well-known [2–8], but due to the complexity and large number of the operators that can appear for a given process not many vertices have been obtained. It is not common in the literature to find the vertices used for a study and even those that can be found [9, 10] may not have considered a given term of the Lagrangian (because their contribution was not taken into account for

a variety of reasons). Because of this, a program capable of automatize the calculation of the vertices is a very needed and useful tool.

Our goal is to create a code that automatizes this calculations and can be easily modified to get new terms of the Lagrangian expansion. The language on which we will be working is MATHEMATICA. This program has some very useful packages built for working with a Lagrangian. In particular, we will be using the packages FEYNRULES[11], FEYNARTS[12] and FEYNCALC[13]. The FEYNRULES web database [14] offers some version of this Lagrangian [2, 3] but they are not meant to be extended. Thus, it is not possible to obtain extra terms when needed using the existing codes. This is what we try to prevent with our code.

In this work we will automatize the calculations of the boson sector of the theory for any reader to extend. Furthermore, we will provide a file with every possible vertex up to 6 particles within the Lagrangian up to next to leading order and another, smaller, with the up to 4 particles ones (the reader could use the code to ask for vertices with any given number of particles).

II. ELECTROWEAK CHIRAL LAGRANGIAN

The Effective Chiral Lagrangian is based on the electroweak chiral symmetry breaking (EWSB) of the scalar sector of the SM, $\mathcal{G} \equiv SU(2)_L \otimes SU(2)_R \rightarrow \mathcal{H} \equiv SU(2)_{L+R}$.

The particle content of the model will be taken from the bosonic fields SM: the Higgs boson (h), the three EW Goldstone bosons (π^a) and the four EW gauge bosons (W^\pm, Z, A). We'll not be including the fermionic fields nor the color sector (these are included in Ref. [2]). The Higgs will be considered a scalar singlet with $m_h = 125$ GeV (not part of a doublet together with the three Goldstones).

We will be implementing the model following Pich *et al.* [3], where the reader can find a further study of the theory construction.

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The Lagrangian will be organized as a low-energy expansion in powers of generalized momenta (derivatives):

$$\mathcal{L}_{\text{EWET}} = \sum_{\hat{d} \geq 2} \mathcal{L}^{(\hat{d})}, \quad \mathcal{L}^{(\hat{d})} = \mathcal{O}(p^{\hat{d}}), \quad (1)$$

where the chiral dimension \hat{d} reflects the infrared behaviour at low momenta (notice that the operators are not ordered according to their canonical dimension). The scale characterizing the EWSB will be defined as the vacuum expectation value ($v = (\sqrt{2}G_F)^{-1/2} = 246$ GeV at leading order (p^2)).

This counting classifies the terms of the Lagrangian depending how they contribute to the dimension of the amplitude \mathcal{M} of a process. The order $\mathcal{O}(p^{\hat{d}})$ of a diagram is given by

$$\frac{2(L+1) + \sum_{\hat{d}} N_{\hat{d}}(\hat{d}-2)}{p}, \quad (2)$$

where L is the number of loops and $N_{\hat{d}}$ the number of vertices of dimension \hat{d} . We consider leading order (LO) the diagrams of order $\mathcal{O}(p^2)$. The diagrams that can contribute to LO must contain only dimension 2 vertices (from the LO term of the Lagrangian $\mathcal{L}^{(2)}$) and no loops. We consider next to leading order (NLO) the diagrams of order $\mathcal{O}(p^4)$. The diagrams that can contribute to NLO must have one loop with only vertices of dimension 2 or no loop with exactly one vertex of dimension 4 (from the NLO term of the Lagrangian $\mathcal{L}^{(4)}$) and any vertex of dimension 2.

The chiral dimension of the objects used to build our theory (as reviewed in [3]) is as follows:

$$v, \frac{h}{v}, \frac{\pi^a}{v}, \frac{W^{a,\mu}}{v}, \frac{B^\mu}{v} \sim \mathcal{O}(p^0), \quad (3)$$

$$\partial_\mu, m_h, m_W, m_Z, g_W, g_1 \sim \mathcal{O}(p),$$

where the gauge coupling constants g_W and g_1 are usually known in the literature as g and g' respectively. With the dimension of these objects, the reader can obtain the dimension of any other operator built with them.

We will split the terms of the expansion into two parts, one denoted as $\mathcal{L}_{\text{Scalar}}^{(\hat{d})}$ including only the operators u_μ, h and \mathcal{T} (later defined in Sec. II A) and one $\mathcal{L}_{\text{FS}}^{(\hat{d})}$ including the field strength operators along with the previous ones. This choice is made so the code is easier to read. We have then,

$$\mathcal{L}^{(\hat{d})} = \mathcal{L}_{\text{Scalar}}^{(\hat{d})} + \mathcal{L}_{\text{FS}}^{(\hat{d})} (+ \text{fermionic} / \text{color terms}). \quad (4)$$

At this point, we could take into account the consequences of the quantization of the theory. This will would make appear two extra contributions to our Lagrangian, namely the gauge-fixing Lagrangian and the Faddeev-Popov ghost Lagrangian. There is a family of renormalizability gauges [15], known as the R_ξ , parameterized with the real gauge parameter ξ (this parameter is related to

the mass of the Goldstones of the theory and the form of the propagator). In the following study, we will be using the Landau gauge $\xi = 0$ (taking into account that the physical properties of the theory are gauge independent) in which the masses of the Goldstones become zero and the ghost fields decouple from the Goldstones (not so with the gauge bosons) [16]. Thus, because not writing these extra terms has no impact in the Goldstones diagrams, they will be omitted for simplicity (remembering that the gauge bosons may have further corrections with ghosts).

A. The fields

Firstly, we have the scalar Higgs field h (dependence on the local coordinates x being omitted for the fields), which is a singlet under the \mathcal{G} group transformations.

Then, we introduce the Goldstones fields

$$\pi^0 = \pi^3, \quad \pi^\pm = \frac{\pi^1 \mp i\pi^2}{\sqrt{2}}, \quad (5)$$

collected in the 2×2 matrix

$$\mathcal{M}(\pi) = \sigma^a \pi^a = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}, \quad (6)$$

where the Einstein summation notation is used for repeated indices and σ^a are the Pauli matrices ($a = 1, 2, 3$).

Using the exponential representation we define the 2×2 matrix $U(\pi)$ which transforms under $g \equiv (g_L, g_R) \in \mathcal{G}$ as

$$U(\pi) = \exp\{i\mathcal{M}(\pi)/v\} \longrightarrow g_L U(\pi) g_R^\dagger. \quad (7)$$

Furthermore, given the coset \mathcal{G}/\mathcal{H} with coordinates ($u_L(\pi), u_R(\pi)$), we can adopt the canonical choice of coset representative $u(\pi) \equiv u_L(\pi) = u_R^\dagger(\pi)$ related to the matrix $U(\pi)$ as

$$u(\pi) = [U(\pi)]^{1/2} \longrightarrow g_L u(\pi) g_h^\dagger = g_h u(\pi) g_R^\dagger, \quad (8)$$

where $g_h \equiv g_h(\pi, g)$ is a compensating transformation to preserve the chosen coset representative, which depends on the Goldstones and the group element.

Next, we introduce the $SU(2)_L$ and $SU(2)_R$ matrix fields, \hat{W}_μ and \hat{B}_μ respectively, transforming under \mathcal{G} as

$$\begin{aligned} \hat{W}^\mu &\longrightarrow g_L \hat{W}^\mu g_L^\dagger + ig_L \partial^\mu g_L^\dagger, \\ \hat{B}^\mu &\longrightarrow g_R \hat{B}^\mu g_R^\dagger + ig_R \partial^\mu g_R^\dagger, \end{aligned} \quad (9)$$

which we can identify as

$$\hat{W}^\mu = -g_W \frac{\sigma^a}{2} W^{a,\mu}, \quad \hat{B}^\mu = -g_1 \frac{\sigma^3}{2} B^\mu, \quad (10)$$

where $W^{a,\mu}$ and B^μ are the gauge fields of SM and will end up related to the physical gauge bosons.

We also introduce the covariant derivative

$$D_\mu U = \partial_\mu U - i\hat{W}_\mu U + iU\hat{B}_\mu \longrightarrow g_L D_\mu U g_R^\dagger, \quad (11)$$

and the corresponding field-strength tensors

$$\begin{aligned} \hat{W}_{\mu\nu} &= \partial_\mu \hat{W}_\nu - \partial_\nu \hat{W}_\mu - i[\hat{W}_\mu, \hat{W}_\nu] \longrightarrow g_L \hat{W}_{\mu\nu} g_L^\dagger, \\ \hat{B}_{\mu\nu} &= \partial_\mu \hat{B}_\nu - \partial_\nu \hat{B}_\mu - i[\hat{B}_\mu, \hat{B}_\nu] \longrightarrow g_R \hat{B}_{\mu\nu} g_R^\dagger. \end{aligned} \quad (12)$$

Lastly, we introduce some useful quantities that appear when building the Lagrangian

$$\begin{aligned} u_\mu &= iu(D_\mu U)^\dagger u = -iu^\dagger D_\mu U u^\dagger = u_\mu^\dagger, \\ f_\pm^{\mu\nu} &= u^\dagger \hat{W}^{\mu\nu} u \pm u \hat{B}^{\mu\nu} u^\dagger, \\ \hat{X}^{\mu\nu} &= \partial_\mu \hat{X}_\nu - \partial_\nu \hat{X}_\mu, \quad \hat{X}_\mu = -g_1 B_\mu, \\ \mathcal{T} &= u\mathcal{T}_R u^\dagger, \quad \mathcal{T}_R = -g_1 \frac{\sigma^3}{2}. \end{aligned} \quad (13)$$

These ingredients are all we need to proceed writing the EWChL.

B. Leading order Lagrangian. $\mathcal{L}^{(2)}$.

The FS leading order (LO) Lagrangian corresponds to the Yang-Mills (YM) Lagrangian for the EW gauge bosons:

$$\mathcal{L}_{\text{FS}}^{(2)} = -\frac{1}{2g_W^2} \langle \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \rangle - \frac{1}{2g_1^2} \langle \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \rangle, \quad (14)$$

where $\langle \cdot \rangle$ denotes the trace over the $SU(2)$ space.

The scalar leading order Lagrangian can be written in terms of the invariant operator $\langle u_\mu u^\mu \rangle$. We can multiply our operator with an arbitrary polynomial of h (compensated with the corresponding power of the scale v so that the chiral dimension do not increase), since it is a singlet under \mathcal{G} . We write then the bosonic Lagrangian:

$$\begin{aligned} \mathcal{L}_{\text{Scalar}}^{(2)} &= \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{2} m_h^2 h^2 - V(h/v) + \\ &+ \frac{v^2}{4} \mathcal{F}_u(h/v) \langle u_\mu u^\mu \rangle, \end{aligned} \quad (15)$$

with

$$\begin{aligned} V(h/v) &= \frac{1}{2} m_h^2 v^2 \left[\sum_{n \geq 3} b_n \left(\frac{h}{v} \right)^n \right], \\ \mathcal{F}_u(h/v) &= 1 + \sum_{n=1} c_n^{(u)} \left(\frac{h}{v} \right)^n. \end{aligned} \quad (16)$$

where we will rename $c_1^{(u)} \equiv 2a$ and $c_2^{(u)} \equiv b$.

The SM Lagrangian is recovered for $b_3 = 1$, $b_4 = 1/4$, $b_{n>4} = 0$, $a = b = 1$ and $c_{n>2}^{(u)} = 0$.

Also, inside the last term of the scalar Lagrangian, we can identify the usual SM gauge bosons

$$\begin{aligned} W^{\pm, \mu} &= \frac{W^{1, \mu} \mp iW^{2, \mu}}{\sqrt{2}}, \\ Z^\mu &= c_W W^{3, \mu} - s_W B^\mu, \\ A^\mu &= s_W W^{3, \mu} + c_W B^\mu, \end{aligned} \quad (17)$$

where $c_W = \cos(\theta_W)$ and $s_W = \sin(\theta_W)$ with θ_W the Weinberg angle. We have $\tan(\theta_W) = g_1/g_W$, $g_W s_W = e$ (with e the electron charge) $m_Z^2 = (g_W^2 + g_1^2)v^2/4$, $m_W^2 = g_W^2 v^2/4$ and $m_A^2 = 0$ at LO.

C. Next to leading order Lagrangian. $\mathcal{L}^{(4)}$.

At next to leading order (NLO), the $\mathcal{O}(p^4)$ operators must be considered alongside the one-loop corrections of the LO Lagrangian (they are of the same order).

Following Ref. [3], the NLO FS Lagrangian can be written as

$$\begin{aligned} \mathcal{L}_{\text{FS}}^{(4)} &= \sum_{i=1}^3 \left[\mathcal{F}_i(h/v) \mathcal{O}_i + \tilde{\mathcal{F}}_i(h/v) \tilde{\mathcal{O}}_i \right] + \\ &+ \mathcal{F}_9(h/v) \mathcal{O}_9 + \mathcal{F}_{11}(h/v) \mathcal{O}_{11}, \end{aligned} \quad (18)$$

and the NLO scalar Lagrangian as

$$\mathcal{L}_{\text{Scalar}}^{(4)} = \sum_{i=4}^8 \mathcal{F}_i(h/v) \mathcal{O}_i + \mathcal{F}_{10}(h/v) \mathcal{O}_{10}, \quad (19)$$

where the coefficients $\mathcal{F}_i(h/v)$ and $\tilde{\mathcal{F}}_i(h/v)$ are polynomials of h/v of the form

$$\begin{aligned} \mathcal{F}_i(h/v) &= \sum_{n=0} \mathcal{F}_{i,n} \left(\frac{h}{v} \right)^n, \\ \tilde{\mathcal{F}}_i(h/v) &= \sum_{n=0} \tilde{\mathcal{F}}_{i,n} \left(\frac{h}{v} \right)^n. \end{aligned} \quad (20)$$

The operator \mathcal{O}_i used to build these terms are collected in Table I.

III. THE SOFTWARE TOOLS

For implementing the software we have made use of the computation program MATHEMATICA which offers multiple packages for implementing our theory and getting some results.

The packages we have used are FEYNRULES[11], FEYNARTS[12] and FEYNCALC[13].

i	\mathcal{O}_i	$\tilde{\mathcal{O}}_i$
1	$\frac{1}{4}\langle f_+^{\mu\nu} f_{+\mu\nu} - f_-^{\mu\nu} f_{-\mu\nu} \rangle$	$\frac{i}{2}\langle f_-^{\mu\nu} [u_\mu, u_\nu] \rangle$
2	$\frac{1}{2}\langle f_+^{\mu\nu} f_{+\mu\nu} + f_-^{\mu\nu} f_{-\mu\nu} \rangle$	$\langle f_+^{\mu\nu} f_{-\mu\nu} \rangle$
3	$\frac{i}{2}\langle f_+^{\mu\nu} [u_\mu, u_\nu] \rangle$	$\frac{(\partial_\mu h)}{v}\langle f_+^{\mu\nu} u_\nu \rangle$
4	$\langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle$	—
5	$\langle u_\mu u^\mu \rangle^2$	—
6	$\frac{(\partial_\mu h)(\partial^\mu h)}{v^2} \langle u_\nu u^\nu \rangle$	—
7	$\frac{(\partial_\mu h)(\partial_\nu h)}{v^2} \langle u^\mu u^\nu \rangle$	—
8	$\frac{(\partial_\mu h)(\partial^\mu h)(\partial_\nu h)(\partial^\nu h)}{v^4}$	—
9	$\frac{(\partial_\mu h)}{v}\langle f_-^{\mu\nu} u_\nu \rangle$	—
10	$\langle \mathcal{T} u_\mu \rangle^2$	—
11	$\hat{X}_{\mu\nu} \hat{X}^{\mu\nu}$	—

TABLE I. CP -invariant operators of the $\mathcal{O}(p^4)$ EWET Lagrangian. P -even (P -odd) operators are shown in the left (right) column. Ref. [3]

A. The packages

1. FEYNRULES

FEYNRULES[11] is a MATHEMATICA-based package capable of computing the Feynman rules in the momentum space of a given theory.

FEYNRULES needs to load a model-file (*.fr*) in which one must write the content of the theory: the gauge groups, the fields, the parameters... This model file may contain the Lagrangian but we will leave it outside. This way, the model file doesn't have to be edited if changes to the Lagrangian are needed and we get a better visualization of its individual terms.

With the model loaded, FEYNRULES works with the Lagrangian to get all the possible vertices (we can also, for example, check the hermiticity) and exports them to a language readable by multiple other programs. For our purposes, we need to generate a group of files with extensions *.mod*, *.gen* and *.pars* which contain the main information of the theory (including the vertices) and can be read by FEYNARTS and FEYNCALC.

2. FEYNARTS

FEYNARTS[12] is a MATHEMATICA package for the generation and visualization of Feynman diagrams and amplitudes. We use this package as an add-on of FEYN-CALC[13].

With FEYNARTS one can create the topology for a given process with multiple options settings: number of external particles, number of loops, one-particle irreducible topologies...

FEYNARTS works with the files generated by FEYNRULES to insert the model particles and vertices in the previously generated topologies, resulting in the generation of all the Feynman diagrams for the selected processes (with the given options).

EWET software flowchart

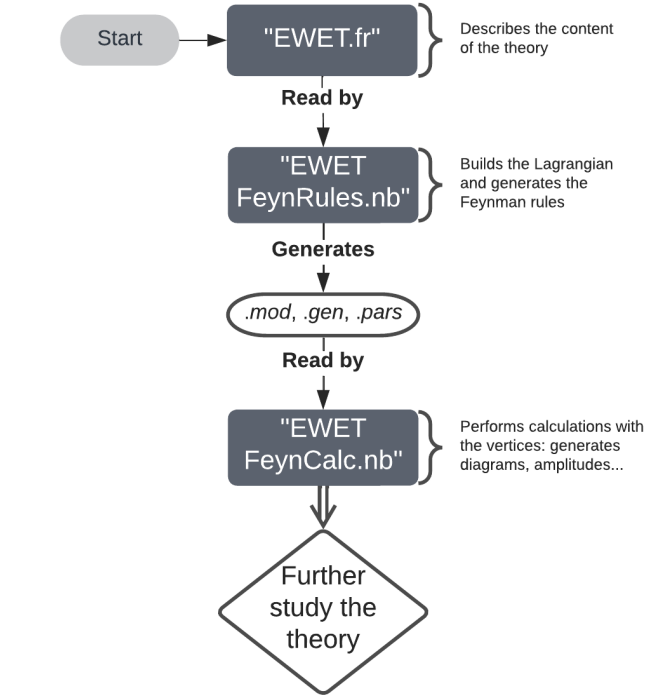


FIG. 1. Overview of the model implementation

Ultimately, FEYNARTS calculates the amplitudes of the diagrams it wrote, which is then translated to a FEYNCALC compatible language.

3. FEYNCALC

FEYNCALC[13] is a MATHEMATICA package for symbolic evaluation of Feynman diagrams.

With FEYNCALC we can perform the needed calculations to simplify the amplitudes obtained with FEYNARTS: substitute the Mandelstam variables, change the values of the parameters, perform the needed Feynman integral for loops...

Later on, this amplitude could be used to calculate cross-sections, compare our processes with the experimental data and further study the theory.

B. Implementing the EWET Lagrangian

The files we talk about in this paper are available in GitHub [17]. The flowchart in Fig. 1 provides an overview of the process of implementing the theory.

First, we have built the model file of the theory in "EWET.fr"[17]. This file contains the fields used to build the different operators: the physical gauge fields A^μ , Z^μ

and $W^{\pm,\mu}$; the unphysical gauge fields B^μ and $W^{a,\mu}$; and the scalar fields h , π^0 and π^\pm . The $SU(2)_L$ interaction of the weak bosons and the $U(1)_Y$ of the photon are also declared.

All the parameters needed for the construction have been established. The declared names are intuitively related to the quantity they represent, for example the coupling $c_3^{(u)}$ is declared as `c3u` and the coupling $\mathcal{F}_{i,n}$ with $i = 1$ and $n = 2$ as `F1n2` (the $\tilde{\mathcal{F}}_{i,n}$ are represented by double `F` at the beginning of its name, for example `FF1n2` stands for $\tilde{\mathcal{F}}_{1,2}$). A definition of any particular parameter can be found within the model file.

Due to the complexity of the Lagrangian, we have opted to write a MATHEMATICA notebook "EWET FeynRules.nb"[17] where we describe the operators and the Lagrangian and get the Feynman rules of the theory. We have prepared the program to get every operator up to 6 fields, so that we get every vertex needed for studying LO (with tree level $\mathcal{L}^{(2)}$) and NLO (with 1 loop $\mathcal{L}^{(2)}$ and tree level $\mathcal{L}^{(4)}$) diagrams with up to 4 external legs. To avoid computing terms with a higher number of fields, we have declared a variable `epsf` which is attached to every field so that every term we compute is expanded up to `epsf`⁶. Once we get the full Lagrangian up to 6 fields, we eliminate `epsf` replacing it by 1. Here the reader could extend the theory and get the operators up to any number of fields needed.

To implement the matrices in (7) and (8) we have expanded them in a power series up to order 5, taking into account that they always appear multiplied by at least one other field. The reader could expand the power series further to get vertices with more particles. With this construction, the rest of operators are easily declared and we build the Lagrangian.

The computation of the vertices takes several minutes and then, returns 384 different vertices up to 6 particles for the Lagrangian. This information is stored in the `.mod`, `.gen` and `.pars` files under the name "SixPartVertsLagr"[17]. The hermiticity of the Lagrangian has also been checked.

In addition, we compute other `.mod`, `.gen` and `.pars` files under the name "FourPartVertsLagr"[17] with every vertex up to 4 particles, which are simpler than the previous, in case the reader does not want to study loop diagrams. This files can be used for tree-level computations of four particle legs diagrams (including the NLO ones from $\mathcal{L}^{(4)}$). These vertices account for 84 of the vertices.

Also, we have extracted the `.mod`, `.gen` and `.pars` files with only the $\mathcal{L}^{(2)}$ rules under the name "LOLagr". This is useful when calculating diagrams with one loop if we want to avoid loops with $\mathcal{L}^{(4)}$ vertices (which contribute at least as $\mathcal{O}(p^6)$) to keep the amplitude terms contributing until $\mathcal{O}(p^4)$. There are 169 vertices of $\mathcal{L}^{(2)}$.

The last part of the notebook contains every vertex up to 6 particles, obtained from the $\mathcal{L}^{(2)}$ and $\mathcal{L}^{(4)}$ Lagrangians studied, computed and printed on screen for the reader. This results are ordered by the number of

particles of the vertex. There are 23 three-particles vertices, 61 four-particles vertices, 110 five-particles vertices and 190 six-particles vertices.

Once we have the files `.mod`, `.gen` and `.pars` generated thanks to the "EWET FeynRules.nb"[17] notebook, we do not need to execute that notebook again unless we need to change something from the theory (add fermions, $\mathcal{O}(p^6)$ operators...). The reader needs to take into account that executing that notebooks takes some computation time and using the provided `.mod`, `.gen` and `.pars` the fastest way to study the theory like presented in this paper.

Given that MATHEMATICA needs to kill the kernel before executing a new package and that we do not want to keep using the prior notebook, we have created a new notebook "EWET FeynCalc.nb"[17] which runs FEYN-CALC (with a FEYNARTS add-on). With this notebook we can draw the diagrams for a given process, get the amplitudes and simplify them just by specifying the location of the model `.mod` and the generic model `.gen`.

The option to set up the Mandelstam variables in FEYN-CALC needs to be carefully selected for every diagram. When calculating the different amplitudes, we have considered the different definition of the Mandelstam sum considering the particles mass (remember that in our selected gauge the Goldstones are massless).

The "EWET FeynCalc.nb"[17] notebook only contains some examples of vertices and amplitudes calculations (which are easy to edit to change the process to a particular one the reader may be looking for), precisely those mentioned later in the paper. Nonetheless, we remind the reader that all the computed vertices can be found as outputs in the notebook "EWET FeynRules.nb"[17].

IV. CHECKING THE MODEL IMPLEMENTATION

Now that we have finished building the model to the computer and we can start getting the Feynman amplitudes for a process, we need to check that the theory works as expected. This can be achieved by comparing some of the existing results the literature with those we get as an output of FEYN-CALC. We will check both local vertices and amplitudes.

Given that FEYN-CALC computes the amplitude of a given process, to check the vertices we have extracted them by hand from the amplitudes. Some of the changes made are adding the global imaginary unit i factor which FEYN-CALC takes out of the amplitudes and extracting the polarization vectors that appear already contracted in the amplitude. This could be avoided just by searching for the vertex in the "EWET FeynCalc.nb"[17] notebook, but those lists are not really user-friendly to read considering they contain 383 different vertices in its own chosen particles order.

The convention used is that p_i (k_i) stands for the momentum of an incoming (outgoing) particle (the time of

the diagrams is taken from left to right). The number i corresponds to the position of an incoming/outgoing particle among the other incoming/outgoing particles counting from the top of the diagram. The Greek indices will stand for the polarization of the gauge bosons.

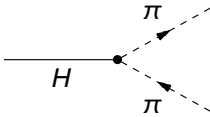
Each diagram has underneath it the corresponding process to easily identify momentum and polarization of a particle.

We will make a distinction between the terms found in Refs. [9, 10] (black colored), with which we have checked our results, and the new ones (blue colored), which therefore cannot be checked but show nonetheless the potential of having an automatized code capable of performing the calculation with all the operators. Please notice that Ref. [9] contains some of the blue results in the limit $m_h = 0$.

For the checks one needs to relate our coupling constants to some different ones, relations which can be found in [9, 18] and are:

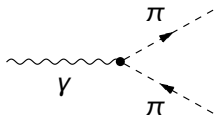
$$\begin{aligned}
 a_i &= \mathcal{F}_{i,0} \quad \text{for } i = 1, 4, 5, \\
 a_2 &= \frac{\mathcal{F}_{3,0} - \tilde{\mathcal{F}}_{1,0}}{2}, \\
 a_3 &= -\frac{\mathcal{F}_{3,0} + \tilde{\mathcal{F}}_{1,0}}{2}, \\
 \lambda_3 &= b_3 \frac{m_h^2}{2v}, \\
 \lambda_4 &= b_4 \frac{m_h^2}{2v^2}.
 \end{aligned} \tag{21}$$

With these considerations, some of the extracted local vertices we get from our model are:



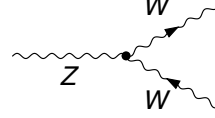
$$= -i \frac{2a}{v} (k_1 k_2)$$

FIG. 2. $H \rightarrow \pi^+(k_1) \pi^-(k_2)$ vertex.



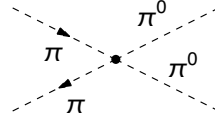
$$= ie(k_2 - k_1)_\mu + i \frac{4e\mathcal{F}_{3,0}}{v^2} [(p_1 k_2) k_{1\mu} - (p_1 k_1) k_{2\mu}]$$

FIG. 3. $A_\mu(p_1) \rightarrow \pi^+(k_1) \pi^-(k_2)$ vertex.



$$\begin{aligned}
 &= -ig_W c_W [g_{\mu\nu}(p_1 + k_1)_\rho + \\
 &\quad -g_{\rho\nu}(p_1 + k_2)_\mu - g_{\mu\rho}(k_1 - k_2)_\nu] + \\
 &\quad + i \frac{g_W^3}{c_W} \left\{ -\frac{\mathcal{F}_{3,0} + \tilde{\mathcal{F}}_{1,0}}{2} [g_{\mu\rho}(k_2 - k_1)_\nu + \right. \\
 &\quad + g_{\mu\nu}(c_W^2 p_1 + k_1)_\rho + g_{\nu\rho}(-k_2 - c_W^2 p_1)_\mu] + \\
 &\quad + s_W^2 \left[\mathcal{F}_{1,0} - \frac{\mathcal{F}_{3,0} - \tilde{\mathcal{F}}_{1,0}}{2} \right] (g_{\nu\rho} p_{1\mu} - g_{\mu\nu} p_{1\rho}) + \\
 &\quad + 2c_W^2 (\mathcal{F}_{2,0} - \tilde{\mathcal{F}}_{2,0}) [g_{\mu\rho}(k_2 - k_1)_\nu \\
 &\quad + g_{\mu\nu}(p_1 + k_1)_\rho - g_{\nu\rho}(p_1 + k_2)_\mu] \}
 \end{aligned}$$

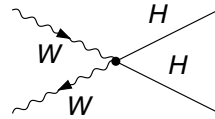
FIG. 4. $Z_\nu(p_1) \rightarrow W_\mu^+(k_1) W_\rho^-(k_2)$ vertex.



$$\begin{aligned}
 &= \frac{i}{3v^2} \{ 2[(k_1 k_2) + (p_1 p_2)] + (p_1 + p_2)^2 \} + \\
 &\quad + i \frac{16}{v^2} \{ \mathcal{F}_{4,0} [(k_1 p_2)(k_2 p_1) + (k_1 p_1)(k_2 p_2)] + \\
 &\quad + 2\mathcal{F}_{5,0}(k_1 k_2)(p_1 p_2) \} + \\
 &\quad + i \frac{4e^2 \mathcal{F}_{10,0}}{3v^4} [4(k_1 k_2) + (p_1 + p_2)^2]
 \end{aligned}$$

FIG. 5. $\pi^+(p_1) \pi^-(p_2) \rightarrow \pi^0(k_1) \pi^0(k_2)$ vertex.

$$\begin{aligned}
 &= i \frac{bg_W^2}{2} g_{\mu\nu} + \\
 &\quad - i \frac{2g_W^2 \mathcal{F}_{6,0}}{v^2} (k_1 k_2) g_{\mu\nu} +
 \end{aligned}$$



$$\begin{aligned}
 &- i \frac{g_W^2 \mathcal{F}_{7,0}}{v^2} (k_{1\nu} k_{2\mu} + k_{1\mu} k_{2\nu}) + \\
 &\quad + i \frac{g_W^2 (\mathcal{F}_{9,1} + \tilde{\mathcal{F}}_{3,1})}{2v^2} [p_{1\nu}(k_1 + k_2)_\mu + \\
 &\quad + p_{2\mu}(k_1 + k_2)_\nu - (p_1 + p_2)^2 g_{\mu\nu}] + \\
 &\quad + i \frac{4g_W^2 (\mathcal{F}_{2,2} + \tilde{\mathcal{F}}_{2,2})}{v^2} [p_{1\nu} p_{2\mu} - (p_1 p_2) g_{\mu\nu}]
 \end{aligned}$$

FIG. 6. $W_\mu^+(p_1) W_\nu^-(p_2) \rightarrow H(k_1) H(k_2)$ vertex.

$$\begin{aligned}
&= i \frac{2be}{v^2} (p_2 - p_1)_\mu + \\
&+ i \frac{8e\mathcal{F}_{6,0}}{v^4} (k_2 k_3) (p_1 - p_2)_\mu + \\
&+ i \frac{4e\mathcal{F}_{7,0}}{v^4} [k_3 (p_1 - p_2) k_{2\mu} + \\
&+ k_2 (p_1 - p_2) k_{3\mu}] + \\
&+ i \frac{2e\mathcal{F}_{9,1}}{v^4} [k_1 (p_1 - p_2) (k_2 + k_3)_\mu + \\
&- k_1 (k_2 + k_3) (p_1 - p_2)_\mu] + \\
&+ i \frac{8e\mathcal{F}_{3,2}}{v^4} [(k_1 p_2) p_{1\mu} - (k_1 p_1) p_{2\mu}]
\end{aligned}$$

FIG. 7. $\pi^+(p_1) \pi^-(p_2) \rightarrow A_\mu(k_1) H(k_2) H(k_3)$ vertex.

The vertices in Figs. 2 and 5 have been checked with Ref. [9] and are exactly the same. Taking into account that Ref. [9] defines the photon coupling at LO with a global sign difference in relation to ours, the results of our vertices in Figs. 3 and 7 are also checked. The last two vertices in Figs. 4 and 6 are checked with Ref. [10], where the diagrams are considered with all particles incoming, and they also check right.

The same way we have done with the vertices, we have computed the following amplitudes:

$$\begin{aligned}
\mathcal{M}(\pi^+ \pi^- \rightarrow \pi^0 \pi^0) &= \frac{s}{v^2} \left(1 + \frac{a^2 s}{m_h^2 - s} \right) + \\
&+ \frac{g_W^2}{4} \left(\frac{s-u}{m_W^2 - t} + \frac{s-t}{m_W^2 - u} \right) + \frac{4\mathcal{F}_{4,0}}{v^4} (t^2 + u^2) + \\
&+ \frac{8\mathcal{F}_{5,0}}{v^4} s^2 + \frac{g_W^2 (\mathcal{F}_{3,0} + \tilde{\mathcal{F}}_{1,0})}{2v^2} \left(\frac{s^2 - u^2}{m_W^2 - t} + \frac{s^2 - t^2}{m_W^2 - u} \right) + \\
&+ \frac{g_W^2 (\mathcal{F}_{3,0} + \tilde{\mathcal{F}}_{1,0})^2}{v^4} \left[\frac{t(u^2 - s^2)}{m_W^2 - t} + \frac{u(t^2 - s^2)}{m_W^2 - u} \right] + \\
&+ \frac{e^2 \mathcal{F}_{10,0}}{c_W^2 v^2} \left[g_W^2 \left(\frac{s-u}{m_W^2 - t} + \frac{s-t}{m_W^2 - u} \right) + \frac{4s}{v^2} \right] + \\
&+ \frac{a \mathcal{F}_{10,1} e^2 s^2}{c_W^2 v^4 (m_h^2 - s)},
\end{aligned} \tag{22}$$

$$\begin{aligned}
\mathcal{M}(\pi^+ \pi^- \rightarrow HH) &= -\frac{1}{v^2} \left[bs + a^2 \frac{(m_h^2 - t)^2}{t} + \right. \\
&+ a^2 \frac{(m_h^2 - u)^2}{u} - \frac{3ab_3 s m_h^2}{m_h^2 - s} \left. \right] + \\
&+ \frac{a^2 g_W^2 s}{2} \left(\frac{1}{m_W^2 - t} + \frac{1}{m_W^2 - u} \right) + \\
&+ \frac{2s\mathcal{F}_{6,0}}{v^2} (s - 2m_h^2) + \frac{\mathcal{F}_{7,0}}{v^2} [2m_h^2 (s - m_h^2) + t^2 + u^2] + \\
&+ \frac{a g_W^2 (\mathcal{F}_{9,0} + \tilde{\mathcal{F}}_{3,0})}{4v^2} \left[m_h^2 \left(\frac{u-t-2s}{m_W^2 - t} + \frac{t-u-2s}{m_W^2 - u} \right) + \right. \\
&+ \frac{s^2 + tu - u^2}{m_W^2 - t} + \frac{s^2 + tu - t^2}{m_W^2 - u} \left. \right] + \\
&+ \frac{g_W^2 (\mathcal{F}_{9,0} + \tilde{\mathcal{F}}_{3,0})^2}{8v^4} \left\{ \frac{1}{m_W^2 - t} [t(u^2 - s^2) + \right. \\
&+ (u^2 - s^2 + 2st - 2tu)m_h^2 + (2s - 2u + t)m_h^4 + m_h^6] + \\
&+ \frac{1}{m_W^2 - u} [u(t^2 - s^2) + (t^2 - s^2 + 2su - 2tu)m_h^2 \\
&+ (2s - 2t + u)m_h^4 + m_h^6] \left. \right\},
\end{aligned} \tag{23}$$

$$\begin{aligned}
\mathcal{M}(HH \rightarrow HH) &= -\frac{12b_4 m_h^2 v^2}{v^4} + \\
&+ \frac{9b_3^2 m_h^4}{v^2} \left(\frac{1}{m_h^2 - t} + \frac{1}{m_h^2 - u} + \frac{1}{m_h^2 - s} \right) + \\
&+ \frac{2\mathcal{F}_{8,0}}{v^4} [(2m_h^2 - s)^2 + (2m_h^2 - t)^2 + (2m_h^2 - u)^2].
\end{aligned} \tag{24}$$

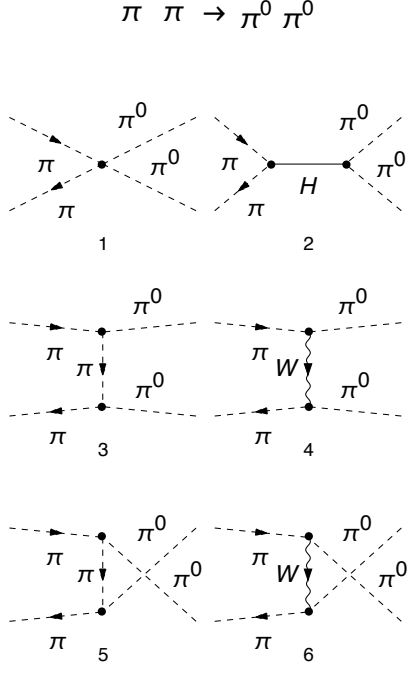
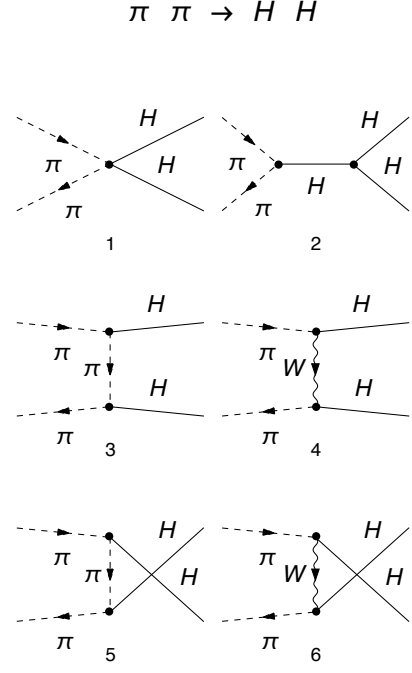
The amplitudes in (22), (23) and (24) are obtained from the diagrams in Fig. 8, Fig. 9 and Fig. 11 respectively. Again, the terms in the literature (black colored) have been checked, in particular with Ref. [9].

The reader can check any extra amplitude with existing ones, for example Ref. [9] shows some amplitudes including photons and Ref. [19] some with gauge bosons.

V. THE CODE CAPABILITY: AN EXAMPLE

To this point, we hope to have proved the reader of the power the code carries. After the checks, we assume the code is working smoothly and we can ask for some new physics results.

We remind the reader that the code is capable of to perform amplitudes up to NLO as stated previously. To show the capability of the code, we will get all the possible contributions to one loop diagrams including those with $\mathcal{L}^{(4)}$ vertices. Even though those terms contribute to a higher order in the amplitude and we will be missing other same higher order corrections ($\mathcal{O}(p^6)$ could also have contributions from two loops diagrams with $\mathcal{L}^{(2)}$ vertices and tree level with one $\mathcal{L}^{(6)}$ vertex).

FIG. 8. $\pi^+ \pi^- \rightarrow \pi^0 \pi^0$ amplitude.FIG. 9. $\pi^+ \pi^- \rightarrow H H$ amplitude.

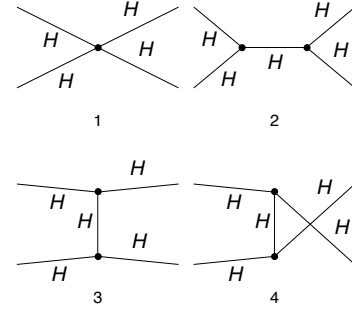
We have asked our code for the process $H(p_1) \rightarrow A^\mu(k_1) \pi^0(k_2)$. The tree level contribution is given by the diagram in Fig. 11 which returns the amplitude:

$$\mathcal{M}_0 = \frac{e\tilde{\mathcal{F}}_{3,0}}{v^2} [(s - m_h^2)(p_1 \varepsilon_{k_1}^*) + (t - m_h^2)(k_2 \varepsilon_{k_1}^*)] \quad (25)$$

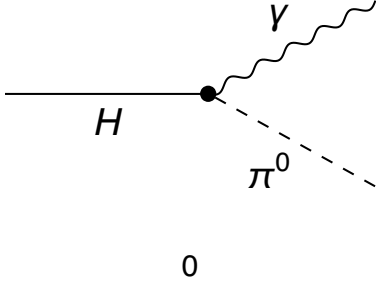
To go one step further, we have also asked for the one-loop contributions to the process. The code returns then 179 different diagrams and the amplitude. This result is absurdly huge and there will be no point in presenting it in this paper, but the reader has it available in the "EWET FeynCalc.nb"[17] notebook. If actually looking for the amplitude up to $\mathcal{O}(p^4)$, with only the one loop diagrams with $\mathcal{L}^{(2)}$ vertices, there would only be 22 possible diagrams.

As an example we have included the 50 diagrams (Fig. 12) contributing to the amplitude excluding those with tadpoles or wavefunction corrections. Of those diagrams we have selected some of their amplitudes to write here: one three-point (1), two two-points (30,47) and one one-point (50). The amplitudes are numerated with the number of the diagram they correspond to. The internal momenta of the loop is denoted by q . The polarization of the outgoing photon is denoted by $\varepsilon_{k_1}^*$.

$H H \rightarrow H H$

FIG. 10. $H H \rightarrow H H$ amplitude.

$$\mathcal{M}_1 = \int \frac{d^d q}{(2\pi)^d} \frac{3e\tilde{\mathcal{F}}_{3,0} (ac_W^2 v^2 + e^2 \mathcal{F}_{10,1}) (b_3 v^2 m_H^2 - 2\mathcal{F}_{10,3})}{4\pi^4 c_W^2 v^8 (q^2 - m_H^2)} \cdot \left[\frac{-(k_1 \varepsilon_{k_1}^*) (k_1 q)}{-2(k_1 q) + q^2} \right] \cdot \left[\frac{(k_1 k_2) - (k_2 q)}{2(k_1 k_2) - 2(k_1 q) - 2(k_2 q) + q^2 - m_H^2} \right], \quad (26)$$

FIG. 11. $H H \rightarrow H H$ amplitude.

$$\mathcal{M}_{30} = \int \frac{d^d q}{(2\pi)^d} \frac{a e (k_1 q + k_2 q - q^2)}{2\pi^4 c_W^2 v^4 q^2 [2(k_1 k_2) - 2(k_1 q) - 2(k_2 q) + q^2]} \cdot [e^2 \mathcal{F}_{10,0}(k_2 \varepsilon_{k_1}^*) + c_W^2 \tilde{\mathcal{F}}_{1,0}(k_1 \varepsilon_{k_1}^*)(k_1 k_2)] , \quad (27)$$

$$\mathcal{M}_{47} = - \int \frac{d^d q}{(2\pi)^d} \frac{e g_W^4 (\mathcal{F}_{2,1} + \tilde{\mathcal{F}}_{2,1})(\mathcal{F}_{3,0} + \tilde{\mathcal{F}}_{1,0})}{8\pi^4 v^2 (q^2 - m_W^2) [2(k_2 q) + q^2 - m_W^2]} \cdot \{2q^2 (k_2 \varepsilon_{k_1}^*) + (k_2 q) [7(k_2 \varepsilon_{k_1}^*) + 10(q \varepsilon_{k_1}^*)]\} , \quad (28)$$

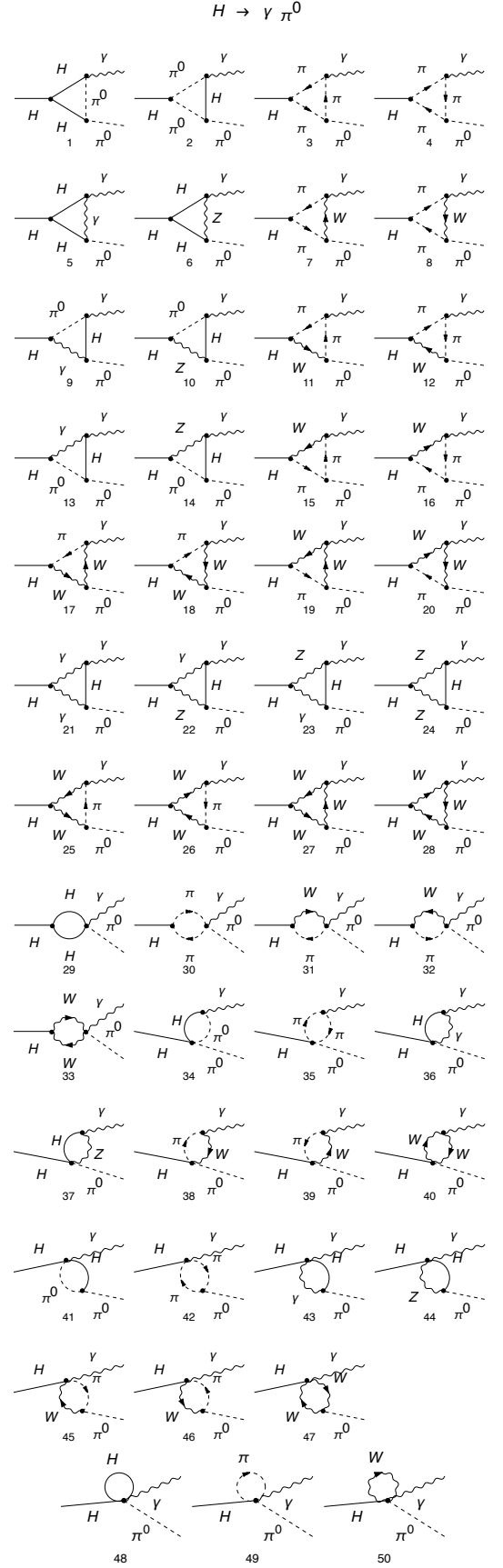
$$\mathcal{M}_{50} = - \int \frac{d^d q}{(2\pi)^d} \frac{3 e g_W^2 (\mathcal{F}_{3,1} + \tilde{\mathcal{F}}_{1,1})(k_2 \varepsilon_{k_1}^*)}{8\pi^4 v^2 (q^2 - m_W^2)} \quad (29)$$

Even though we have showed some isolated amplitudes, the code actually computes the full amplitude with every diagram. These results can be used for further calculations, like getting the cross section of an event to compare with the experimental data.

VI. CONCLUSION

With this work we have been able to create an automatized code [17] capable of calculating any term of the Lagrangians $\mathcal{L}^{(2)}$ and $\mathcal{L}^{(4)}$. This allows us to calculate amplitudes with all the contributions up to $\mathcal{O}(p^4)$. We have build the part of the theory containing only the Higgs, the Goldstones and the electroweak bosons. The next step would be including the fermions, the ghosts and the gluons. This should be easier now that we have the other terms to guide us.

This is a helpful tool as we have seen that the literature only accounted for some of the possible vertices which are even scattered in different articles. Nevertheless, some of

FIG. 12. $H(p_1) \rightarrow A_\mu(k_1) \pi^0(k_2)$ amplitude.

those articles [9, 10] have been essential to check that the code has been built correctly.

It is important to remark that thanks to the code, hours of handmade calculations can be avoided. With the `.mod`, `.gen` and `.pars` files one can start working directly with the vertices, saving the time of extracting them from the Lagrangian. These files can be read by FEYN CALC to get amplitudes, integrate loops and much more.

Thanks to this code, extra contributions to the vertices in the literature have been found (the blue ones). Furthermore, we have found many more vertices that can be used by the scientific community to improve our knowledge of the theory.

We have showed that the software [11–13] is an incredible tool developed by scientists. Thanks to them, we have been able to produce all diagrams at one loop for a process and get their amplitudes. With our implementation, this can be done for any diagram with four external legs (or even more if one gets terms with more fields from the Lagrangian, which is done automated).

We hope this code can help scientists everywhere when further studying the theory. We are also looking forward to giving a more complete version with the extra sectors of the Lagrangian: fermions, gluons and ghosts.

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