E/P **25** x satisfies the differential equation

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 16x = \cos 4t, \; t \geqslant 0$$

a Find the general solution of the differential equation. **(8 marks)**

b Find the particular solution of this differential equation for which, at $t = 0$, $x = \frac{1}{2}$ and $\frac{dx}{dt} = 0$. **(5 marks)**

c Describe the behaviour of the function for large values of t. **(2 marks)**

Auxilary Equation:

$m^2 + 8m + 16 = 0$ $\quad \times 16$ $\quad +8$

1	16
2	8
4	4

$(m+4)(m+4) = 0$

$m = -4$

1 solution $\Rightarrow$ CF form:

$(A+Bt)e^{\alpha t}$

$(A+Bt)e^{-4t}$

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$f(t) = \cos 4t \Rightarrow$

PI form:

$x = \lambda \sin 4t + \mu \cos 4t$

$\frac{dx}{dt} = 4\lambda \cos 4t - 4\mu \sin 4t$

$\frac{d^2x}{dt^2} = -16\lambda \sin 4t + 16\mu \cos 4t$

Substituting

$-16\lambda \sin 4t + 16\mu \cos 4t + 32\lambda \cos 4t - 32\mu \sin 4t + 16\lambda \sin 4t - 16\mu \cos 4t = \cos 4t$

$32\lambda \cos 4t - 32\mu \sin 4t = \cos 4t$

$\therefore \lambda = \frac{1}{32}, \mu = 0$

$\therefore$ PI : $\frac{\sin 4t}{32}$

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PI: $\frac{\sin 4t}{32}$

CF: $(A+Bx)e^{-4x}$

GS = PI + CF

GS: $x = (A+Bt)e^{-4t} + \frac{\sin 4t}{32}$

$t = 0, \; x = \frac{1}{2}$

$\frac{1}{2} = (A+0)(1) + (0)$

$\frac{1}{2} = A$

PS: $x = (\frac{1}{2} + Bt)e^{-4t} + \frac{\sin 4t}{32}$

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$$x = \left(\frac{1}{2} + Bt\right)e^{-4t} + \frac{\sin 4t}{32}$$

$$\frac{dx}{dt} = 0 = Be^{-4t} - 4e^{-4t}\left(\frac{1}{2} + Bt\right) + \frac{\cos 4t}{8}$$

$(t = 0)$

$$0 = B - 4\left(\frac{1}{2}\right) + \frac{1}{8}$$

$$B = \frac{15}{8}$$

The ps at $t = 0, x = \frac{1}{2}, \frac{dx}{dt} = 0$ is:

$$x = \left(\frac{1}{2} + \frac{15t}{8}\right)e^{-4t} + \frac{\sin 4t}{32}$$

$$x = \underbrace{\frac{\left(\frac{1}{2} + \frac{15t}{8}\right)}{e^{4t}}}_{\text{Approaches 0}} + \underbrace{\frac{\sin 4t}{32}}_{\text{Oscilates with period pi/2 around x=+/-(1/32)}}$$

As t approaches infinity: