COMP 477 N

Project Report

Elastic Deformation of Rods in the Context of Sports

Alexis Bolduc 40126092

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Concordia University, Montreal, Qc, Canada

1. Introduction

I came up with the idea of simulating a net in the context of sports, because I wanted to conceive something original and that would relate to my other interests. I am a hockey player, and I would be proud to complete a project that combines both my main interests: sports and computer sciences. The domains of computer games and animation are the ones that intrigue me the most and I think this project is the perfect way to showcase my passions and abilities.

I wanted to accomplish something I would be proud of due to the difficulty of the challenge. I think it is fair to say that I succeeded in doing so.

1.1 Initial Beliefs

I first approached Prof. Tiberiu Popa with the intent of simulating a hockey puck hitting the inside of a net. I was thinking it would be nice if I could even make the netting tear apart if the shot was too strong. I was quickly talked out of this plan due to the complexity of simulating destruction with physics equations. It was also highly suggested to avoid dealing with unregular shapes such as a hockey puck for net simulation, since a ball could yield some nice and easier results.

1.2 Objectives

The goal of the project was to create a system of rods that could properly behave to a collision with balls, similarly to scenario in sports game with balls. Starting with a model of a single rod, I was planning on setting up a flat net, similar to the ones used in soccer, and introduce a collision scenario with a sphere to simulate the physics in the scoring of a goal. Using the net created for the soccer goal, I finally wanted to create another scenario in another sport. The choice for the last scenario was basketball.

2 . Research and Development of Model

This project's starting point is the paper "Discrete Elastic Rods" by Bergou et al.[1]. It describes a precise model for the twisting and bending energies of a rod. According to this paper, a single rod can be represented by a set of edges and points, and the energy of twisting and bending could be computed by comparing the pre-set coordinates of the points and edges with the current ones.

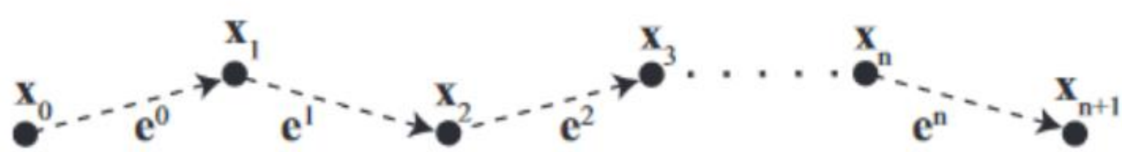

Image Representing the model of a single rod from Bergou et al.[1]

2.1 Energy models

This paper advised against the usage of stretching energy and to keep both twisting and bending energies, but based on my discussion with Prof. Popa, and for the scope of my project, the rod model I should recreate should only include bending and should also take stretching energy into consideration.

For the bending energy, I used the equation in Bergou et al.(1) for the discrete curvature binormal, to then compute the sum of bending energy at each point.

$$(\kappa \mathbf{b})_i = \frac{2\mathbf{e}^{i-1} \times \mathbf{e}^i}{|\bar{\mathbf{e}}^{i-1}||\bar{\mathbf{e}}^i| + \mathbf{e}^{i-1} \cdot \mathbf{e}^i} \ .$$

Curvature at point **i** using the edge before and after the point. (Bergou et al.(1))

$$\sum_{i=1}^{n} \frac{\alpha(\kappa \mathbf{b}_i)^2}{\bar{l}_i} \ .$$

Sum of all bending energy for rod, where l_i is the sum of the length of e^{i-1} and e^i. (Bergou et al.(1))

It is worth noting that I have made some research in order to find other ways to model an elastic rod. I have investigated Bergou's following paper called "Discrete viscous threads"(2), but, while interesting, the suggested equations were too complex for the simplified version of a rod. The presentation of Khalid Jawed on "Mechanics of elastic rods" was also interesting, but I did not end up using any specific implementations of their presented equations, since I have consolidated a bending energy model from Bergou et al. (1) and would only need to use Hooke's law for the stretching forces.

My rod now had a bending energy model and a vague attempt at a stretching energy model.

2.2 Forces

In order to properly simulate a rod, it is important to convert the energies into forces. I started on the basis that the force is the derivative of the energy. I tried to come up with a simple force formula for bending.

Where x_i is the vector of coordinate of the point on the screen and X_i, Y_i, and Z_i are the coordinates of x_i

$$F_1 = \frac{\left(E(x_i) - E(x_{i-1})\right)}{|x_i - x_{i-1}|} \langle X_i - X_{i-1} | Y_i - Y_{i-1} | Z_i - Z_{i-1} \rangle$$

$$F_2 = \frac{\left(E(x_{i+1}) - E(x_i)\right)}{|x_{i+1} - x_i|} \langle X_{i+1} - X_i | Y_{i+1} - Y_i | Z_{i+1} - Z_i \rangle$$

Then simply adding F_1 and F_2 together would give us the force for the bending.

Additionally to being mathematically and physically suspicious, this attempt of converting energy to force was inconclusive. Prof. Popa helped me by suggesting to approximate my bending energy derivative with a small change in the x component of the position of a point on my rod, then to calculate the difference over the small change. Repeating this with y and z would then give me the force at a point. For the stretching force at a point, I implemented a basic calculation of Hooke's law for springs.

2.3 Time integration

With a functioning model for stretching and bending forces, the last step was to implement a time integration. Given that my only known variables at all times are the position of each point and their velocity, the Explicit Euler integration seemed like the best choice.

3 Nets, collision and render

Using the model of the rod, I created a not model, which is the link between two to four rods. Using a combination of rods and nots I created a soccer net and a basketball net. Nots that would not be in contact with a ball were clamped in order to simulate them being attached to a ring or posts.

The collision between the nets and a sphere was done by evaluating the position of the sphere and its radius compared to the position of some key unclamped points of the nets. I used the equation of the sphere in order to find the new accurate position of each rod points or nots that are being touched.

For rendering, I decided to render several points instead of a net. This choice was made in order to focus on the accuracy of the physics, rather than to try to master OpenGL.

4 Difficulties

The main difficulties I had in this project were threefold. Firstly, I had a hard time to figure out what equations to use or not in order to build an accurate model of a single rod. Thankfully, Prof. Popa helped me during key moments of my project. Secondly, it was hard to accurately contain the nets and the single rods. For numerous days, my models would completely diverge until I set an initial offset of bending force. The problem was due to the fact that bending forces were computed using approximative derivative, which assumes that the model is not at rest. Similarly, the basketball net would not go back to its rest state after a collision. A small workaround was to add an additional invisible level of clamping points in order to keep the apparent last ones in place. Thirdly, this project made me realized how little I knew about OpenGL. I did not even think I would be able to render a sphere.

5 Conclusion

To sum up, I have succeeded in modelling a single rod, and using this single rod in order to represent two different types of sports nets. I used some specific equations for bending energy, and more traditional physics equations for the forces and collision. One important observation I am now able to make is that the fact I knew little about computer animation when starting this project pushed me into underestimating the effort needed into some of the tasks. Overall, I am pleased and proud with the results, it has been an intense but deeply nourishing learning experience.

Alexis Bolduc 40126092

References

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(2) Miklós Bergou, Basile Audoly, Etienne Vouga, Max Wardetzky, and Eitan Grinspun. 2010. Discrete viscous threads. In *ACM SIGGRAPH 2010 papers* (*SIGGRAPH '10*). Association for Computing Machinery, New York, NY, USA, Article 116, 1–10. DOI:https://doi.org/10.1145/1833349.1778853

(3) Khalid Jawed. "Mechanics of elastic rods", Massachusetts Institute of Technology, 2016. https://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/16311/www/s17/syllabus/ppp/Rods_Mechanics_MK.pdf