Scientific Programming with Python

Projectile Motion (with and without air resistance) description

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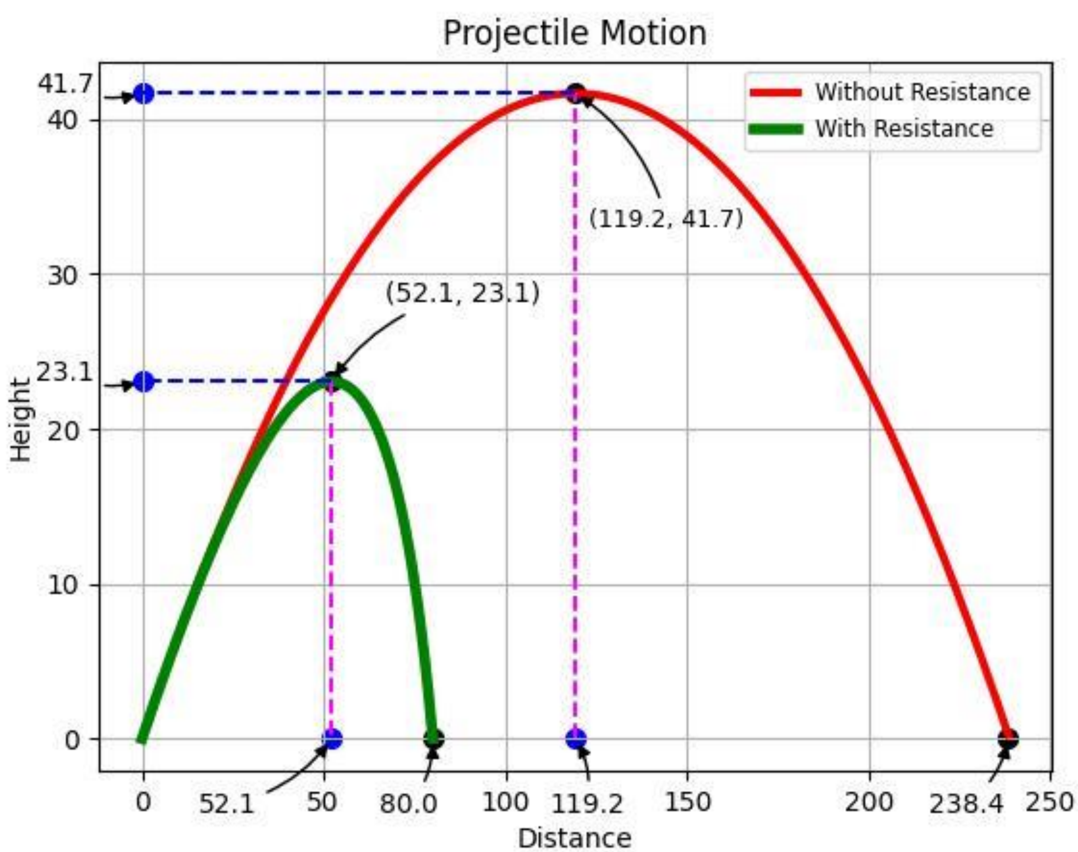
This is a basic and simple model for the motion of an object in air (**Projectile Motion**). One can analyze the **difference** between the motion of the object **with air resistance** and **without air resistance** with some same fixed initial values**.**

There are three main steps in the **(Python) code**.

1) **Setting some initial values**
2) **Defining a function for the motion without air resistance**
3) **Defining another function for the motion with air resistance**

The plot (graph) of the motion is given by:



The **formulas** used in this model are given at the end of the description. One can check the detail of the model by visiting the python code file.

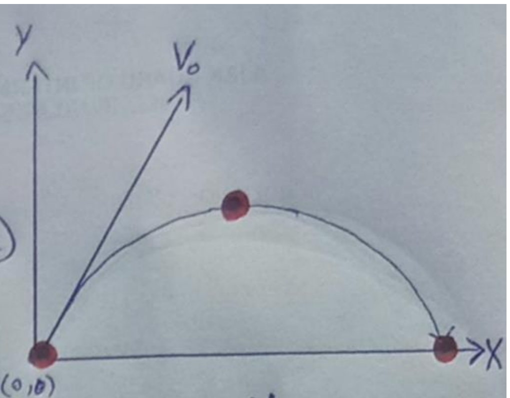
7 Formulas:

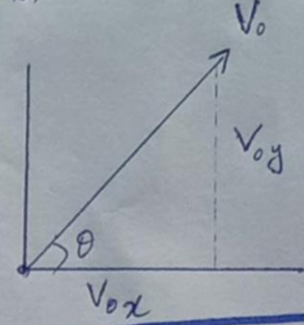
A): Without Resistance:

As we know that (without resistance)

$$\boxed{\begin{aligned} a_x &= 0 \\ a_y &= -g \end{aligned}} \text{ — (2)}$$

∵ velocity is not changing in x-direction.





$$\Rightarrow \boxed{\begin{aligned} V_{0x} &= V_0 \cos\theta \\ V_{0y} &= V_0 \sin\theta \end{aligned}} \text{ — (1)}$$

Also we have

$V = u + at$

$\Rightarrow V_x = V_{0x} + 0 = V_{0x}$

$$\boxed{V_y = V_{0y} - gt} \text{ — (3)}$$

To get the x & y distances, we have

$S = ut + \frac{1}{2}at^2$

$$\Rightarrow \boxed{\begin{aligned} x &= V_{0x}t + 0 = V_0 \cos\theta\, t \\ y &= V_{0y}t - \frac{1}{2}gt^2 = V_0 \sin\theta\, t - \frac{1}{2}gt^2 \end{aligned}} \text{ — (4)}$$

B): With Resistance:

The net force acting on the Object, which is moving in the presence of resistance is given by;

$\vec{F} = -mg\hat{j} - mb|\vec{V}|\cdot\vec{V}$

where "b" is drag coefficient

and $= \frac{\text{object constant}}{\text{mass}} = \frac{D}{m}$

$\Rightarrow m\cdot\bar{a} = -mg\cdot\hat{j} - mbV(V_x\hat{i} + V_y\hat{j})$

where $V = \sqrt{V_x^2 + V_y^2}$

$\Rightarrow a_x\hat{i} + a_y\hat{j} = -g\cdot\hat{j} - bV(V_x\hat{i} + V_y\hat{j})$

$$\Rightarrow \boxed{\begin{aligned} a_x &= -bVV_x \\ a_y &= -g - bVV_y \end{aligned}} \text{ — (5)}$$

Also we have

$x = V_{0x}t + \frac{1}{2}(-bVV_x)t^2$

or

$$\boxed{x = V_0\cos\theta\, t - \frac{1}{2}bVV_x t^2}$$

&

$y = V_{0y}t + \frac{1}{2}(-g - bVV_y)t^2$

$$\Rightarrow \boxed{y = V_0\sin\theta\, t - \frac{1}{2}(g + bVV_y)t^2} \text{ — (6)}$$

$V = u + at$

&

$$\Rightarrow \boxed{\begin{aligned} V_x &= V_{0x} + a_x t \\ V_y &= V_{0y} + a_y t \end{aligned}} \text{ — (7)}$$