National University of Computer and Emerging Sciences

FAST School of Computing **Spring-2023** **Islamabad Campus**

MT-2005: Probability & Statistics

BS(CS) All Sections

Saturday, 8th April, 2023

Serial No:

Sessional - II Exam

Total Time: 1 Hour

Total Marks: 50

Course Instructors:

Dr. Khalil Ullah Awan, Dr. Muhammad Usman Ashraf,
Mr. Waqas Munir, Engr. Moomal Bukhari Engr. Sana Saleh

Signature of Invigilator

Student Name	Roll No.	Course Section	Student Signature

DO NOT OPEN THE QUESTION BOOK OR START UNTIL INSTRUCTED.

Instructions:

1. Attempt on question paper. Attempt all of them. Read the question carefully, understand the question, and then attempt it.
2. No additional sheet will be provided for rough work. Use the back of the last page for rough work.
3. If you need more space write on the back side of the paper and clearly mark question and part number etc.
4. After asked to commence the exam, please verify that you have **Seven (07)** different printed pages including this title page. There are a total of **Five (05) questions**.
5. Calculator sharing is strictly prohibited.
6. Use permanent ink pens only. Any part done using soft pencil will not be marked and cannot be claimed for rechecking.

	Q-1	Q-2	Q-3	Q-4	Q-5	Total
Marks Obtained						
Total Marks	**10**	**10**	**10**	**10**	**10**	**50**

Question#1 [02+02+03+03=10 Marks]

Consider the permutations of the letters of the word "FACETIOUS".

(a) How many permutations are possible if there is no restriction?

$9! = 362880$

(b) In how many possible ways all the vowels are together?

$5! \times 5! = 14400$

(c) In how many possible ways any two vowels are next to one another?

$^5P_2 \times 8! = 806400$

(d) If five alphabets are selected at random from the word "FACETIOUS" then in how many possible ways three vowels and two consonants can be selected?

$\binom{5}{3}\binom{4}{2} = 60$

Question#2 [10 Marks]

The probability that trainee will remain with a company is 0.60. The probability that an employee earns more than Rs. 10,000 per month is 0.50. The probability that an employee who is trainee remained with the company or who earns more than Rs.10,000 per month is 0.70. What is the probability that an employee earns more than Rs. 10,000 per month given that he is a trainee who stayed with the company?

Let T be the event that trainee remains with company

$$P(T) = 0.60$$

Let E be the event that employee earns more than Rs 10,000

$$P(E) = 0.50$$

$$P(T \cup E) = 0.70$$

$$P(E/T) = \frac{P(T \cap E)}{P(T)} \quad \text{---- (i)}$$

$$P(T \cup E) = P(T) + P(E) - P(T \cap E)$$

$$P(T \cap E) = P(T) + P(E) - P(T \cup E)$$

$$= 0.6 + 0.5 - 0.7$$

$$= 0.40$$

Solving eq (i)

$$P(E/T) = \frac{0.40}{0.60}$$

$$= 0.67$$

Question#3 [03+07=10 Marks]

Let **M1** be the event that first stage of machining operation meets specification and **M2** be the event that the second stage of machining operation meets specification. The probability that the first stage of numerically controlled machining operation for high-rpm pistons meets specifications is 0.90. Failures are due to metal variations, fixture alignment, cutting blade condition, vibration and ambient environment condition. Given that the first stage meets the specification, the probability that the second stage machining meets the specification is 0.95. The probability that the second stage machine didn't meet the specification, given that first stage didn't meet the specification is 0.03.

(a): What is the probability that both stages meet the specifications?

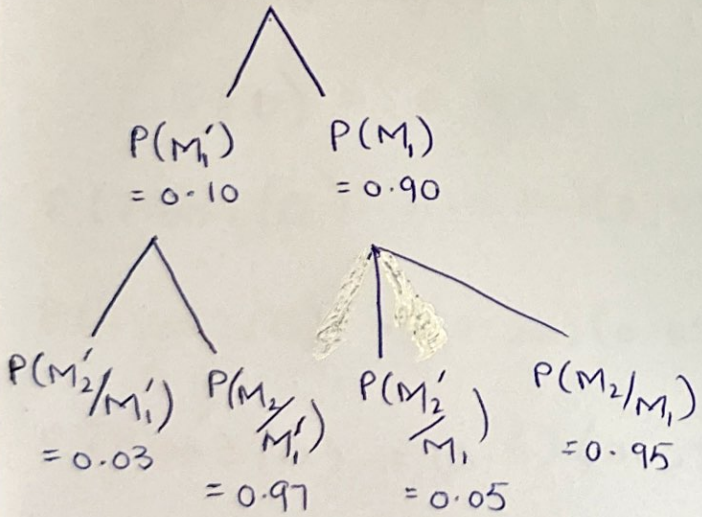
$P(M_1) = 0.90$ $\quad P(M_2/M_1) = 0.95$

$P(M_1 \cap M_2) = P(M_1) \cdot P(M_2/M_1)$

$= (0.90)(0.95)$

$= 0.855$

(b): Are the two events **M1** and **M2** independent? Show your working.



$P(M_2) = P(M_1 \cap M_2) + P(M_1' \cap M_2)$

$= P(M_1) \cdot P(M_2/M_1) + P(M_1') \cdot P(M_2/M_1')$

$= (0.90)(0.95) + (0.10)(0.97) = 0.952$

For independence

$P(M_1 \cap M_2) = P(M_1) \cdot P(M_2)$

$= (0.90)(0.952)$

$= 0.8568$

Since $P(M_1 \cap M_2) \neq P(M_1) \cdot P(M_2)$

So the two events are dependent

Question#4 [10 Marks]

A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 30%, 20%, and 50% of the products, respectively. The non-defective rates for the three plans are: 99%, 95%, and 98% respectively. If a product is selected at random and found to be defective, then according to your opinion which plan is most likely used and thus responsible?

$P(Plan1) = 0.30 \quad P(Plan2) = 0.20 \quad P(Plan3) = 0.50$

$P(D/Plan1) = 0.01 \quad P(D/Plan2) = 0.05 \quad P(D/Plan3) = 0.02$

Let D be the event that product is defective

$$P(D) = P(D \cap Plan1) + P(Plan2 \cap D) + P(Plan3 \cap D)$$

$$= P(Plan1) \cdot P(D/Plan1) + P(Plan2) \cdot P(D/Plan2) + P(Plan3) \cdot P(D/Plan3)$$

$$= (0.30)(0.01) + (0.20)(0.05) + (0.5)(0.02)$$

$$P(D) = 0.023$$

$$P(Plan1/D) = (0.30)(0.01)/0.023 = 0.13043$$

$$P(Plan2/D) = (0.20)(0.05)/0.023 = 0.434782$$

$$P(Plan3/D) = (0.5)(0.02)/0.023 = 0.4347826$$

Question#5 [07+03=10 Marks]

A certain type of competitive exam is conducted in four different departments of the FAST School of Computing: Computer Science (CS), Software Engineering (SE), Artificial Intelligence (AI) and Data Science (DS). The number of candidates participated from each department, the average scores and standard deviations are given in the following table:

	Departments			
	CS	**SE**	**DS**	**AI**
Number of Candidates	45	30	40	35
Average scores	70	65	50	53
Standard Deviation	15	13	10	11

(a): Calculate the combined coefficient of variation for the scores of all four departments

$$\bar{X}_c = \frac{n_1\bar{X}_{CS} + n_2\bar{X}_{SE} + n_3\bar{X}_{DS} + n_4\bar{X}_{AI}}{n_1 + n_2 + n_3 + n_4}$$

$$\bar{X}_c = 59.7$$

Combine variance

$$S_c^2 = \frac{\sum n_i\left[S_i^2 + (\bar{x}_i - \bar{X}_c)^2\right]}{\sum n_i}$$

$$= \frac{n_1\left[S_1^2 + (\bar{x}_1 - \bar{X}_c)^2\right] + n_2\left[S_2^2 + (\bar{x}_2 - \bar{X}_c)^2\right] + n_3\left[S_3^2 + (\bar{x}_3 - \bar{X}_c)^2\right] + n_4\left[S_4^2 + (\bar{x}_4 - \bar{X}_c)^2\right]}{n_1 + n_2 + n_3 + n_4}$$

$$S_c^2 = 229.21$$

$S_c = 15 \cdot 14$

Combine $C.V = \frac{S_c}{\bar{X}_c} \times 100$

$= \frac{15 \cdot 14}{59.7} \times 100$

$= 25.36\%$

(b): The profit (P) for a new product is measured through the equation P = 3X +7Y − 2Z +5, where X, Y and Z are the independent variables with SD(X) = 8, SD(Y) = 3 and SD(Z)= 4. Find the variance of the profit.

$V(P) = V\{3X + 7Y - 2Z + 5\}$

$= 9V(X) + 49V(Y) + 4V(Z)$

$= 9(64) + 49(9) + 4(16)$

$= 1081$

Good Luck