MT-2005: Probability & Statistics

BS(CS) All Sections

Thursday, 25th May, 2023

Serial No:

Final Exam
Total Time: 3 Hours
Total Marks: 125

Course Instructors:

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Signature of Invigilator

Student Name | Roll No. | Course Section | Student Signature

DO NOT OPEN THE QUESTION BOOK OR START UNTIL INSTRUCTED.

Instructions:

1. Attempt on question paper. Attempt all of them. Read the question carefully, understand the question, and then attempt it.
2. No additional sheet will be provided for rough work. Use the back of the last page for rough work.
3. If you need more space write on the back side of the paper and clearly mark question and part number etc.
4. After asked to commence the exam, please verify that you have **Seventeen (17)** different printed pages including this title page. There are a total of **Nine (09) questions.**
5. Calculator sharing is strictly prohibited.
6. Use permanent ink pens only. Any part done using soft pencil will not be marked and cannot be claimed for rechecking.
7. Pages 15 to 17 contain the statistical tables. Use these statistical tables wherever required.

	Q-1	Q-2	Q-3	Q-4	Q-5	Q-6	Q-7	Q-8	Q-9	Total
Marks Obtained										
Total Marks	15	10	15	10	15	10	25	10	15	125

Question#1 [05+03+07=15 Marks]

Part (a):

A brokerage survey reports that 30 % of individual investors have used a discount broker i.e. one which does not charge the full commission. In a random sample of 9 individuals, what is the probability that:

I. At least three of the sampled individuals have used a discount broker.

$p = 0.30 \quad q = 0.70 \quad n = 9$

$$P(X \geq 3) = 1 - P(X \leq 2)$$
$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$
$$= 1 - \left[\binom{9}{0}(0.3)^0(0.7)^9 + \binom{9}{1}(0.3)^1(0.7)^8 + \binom{9}{2}(0.3)^2(0.7)^7\right]$$
$$P(X \geq 3) = 0.5372$$

II. Exactly two of the sampled individuals have not used a discount broker.

$p = 0.7 \quad q = 0.3 \quad n = 9$

$$\binom{9}{2}(0.7)^2(0.3)^7$$
$$= 0.00385$$

Part (b):

A new automated production process has an average of 2 breakdowns per day. Because of the cost associated with a breakdown, management is concerned about the possibility of having three or more breakdowns during a day. Assume that breakdowns occur randomly that the probability of breakdown is the same for any two time intervals of equal length, and that breakdowns in one period are independent of breakdowns in the other periods. What is the probability of having three or more breakdowns during a day?

$\lambda = 2 \qquad P(X \geq 3) = 1 - [P(X \leq 2)]$

$f(x) = e^{-\lambda} \cdot \lambda^x / x!$

$P(X \geq 3) =$

$$1 - \left[\frac{e^{-2} \cdot 2^0}{0!} + \frac{e^{-2} \cdot 2^1}{1!} + \frac{e^{-2} \cdot 2^2}{2!}\right]$$

$$P(X \geq 2) = 0.3233$$

Question#2 [04+03+03=10 Marks]

A market research agency that conducts interviews by telephone has found from the past experience that there are 40% chances that a call made between 2:30 PM and 5:30 PM will be answered:

I: What is the probability that an interviewer receives 10th answer on his 20th call?

$P = 0.40$ $k = 20$ $r = 10$

$$\binom{k-1}{r-1} p^r \cdot q^{k-r}$$

$$\binom{19}{9} (0.4)^9 (.6)$$

$$= 0.058$$

II: What is the probability that an interviewer receives the first answer on his 3rd call?

$$q^{k-1} \cdot p$$

$$(0.6)^2 \cdot (0.4) = 0.144$$

III: Find the expected number of calls to obtain seven answers?

$$E(x) = \frac{k}{p}$$

$$= 7/0.4 = 17.5 \approxeq 18 \text{ calls}$$

Question#3 [07+08= 15 Marks]
Part (a):

In a certain federal prison, it is known that 2/3 of the inmates are under 25 years of age. It is also known that 3/5 of the inmates are male, furthermore 5/8 of the inmates are female or 25 years of age or older. What is the probability that a prisoner selected at random from this prison is female and at least 25 years old?

$P(\text{Under } 25) = 2/3$, $P(\text{Male}) = 3/5$

$P(\bar{M} \cup \bar{U}) = 5/8$

$P(\bar{M} \cap \bar{U}) = P(\bar{M}) + P(\bar{U}) - P(\bar{M} \cup \bar{U})$

$= 2/5 + 1/3 - \frac{5}{8} = 0.4 + 0.333 - 0.625$

$= 0.108$.

Part (b):

Police plan to enforce speed limits by using radar traps at four di□erent locations within the city limits. The radar traps at each of the locations: L1, L2, L3, and L4 will be operated 40%, 30%, 20%, and 30% of the time. If a person who is speeding on his way to work has probabilities of 0.2, 0.1, 0.5, and 0.2 respectively, of passing through these locations, then what is the probability that he will receive a speeding ticket?

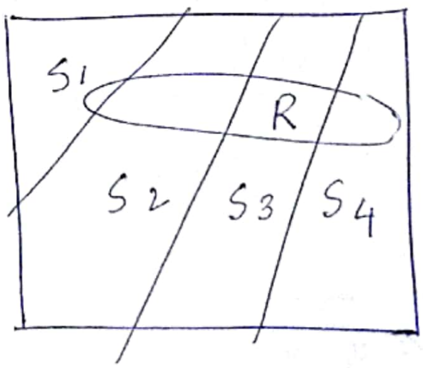
$P(S_1) = 0.2$, $P(S_2) = 0.1$, $P(S_3) = 0.5$

$P(S_4) = 0.2$

$P(\text{speeding ticket}) = ?$

$= P(S_1 \cap R) + P(S_2 \cap R)$

$+ P(S_3 \cap R) + P(S_4 \cap R)$



$= P(S_1)P(R/S_1) + P(S_2)P(R/S_2) + P(S_3)P(R/S_3)$

$+ P(S_4)\, P(R/S_4)$

$$= (0.20)(0.40) + (0.1)(0.30) + (0.5)(0.20) + (0.20)(0.30) = 0.27$$

Question#4 [10 Marks]

In a post office, three clerks are assigned to process incoming mail. The first clerk A, processes 40 percent; the second clerk B, processes 35 percent; and the third clerk C, processes 25 percent of the mail. The first clerk has an error rate of 0.04, the second clerk has an error rate of 0.06 and the third clerk has an error rate of 0.03. A mail selected at random from a day's output is found to have an error. The postmaster wishes to know the probability that it was processed by clerk C.

$P(A) = 0.40, \quad P(B) = 0.35, \quad P(C) = 0.25$

E = error rate

$P(E/A) = 0.04, \quad P(E/B) = 0.06$

$P(E/C) = 0.03$

$$P(C/E) = \frac{P(C \cap E)}{P(E)} = \frac{P(C)P(E/C)}{P(A)P(E/A) + P(B)P(E/B) + P(C)P(E/C)}$$

$$= \frac{(0.25)(0.03)}{(0.40)(0.04) + (0.35)(0.06) + (0.25)(0.03)}$$

$$= \frac{0.0075}{0.0445} = 0.1685393$$

Question#5 [04+04+07= 15 Marks]

Part (a):

The average number of acres burned by forest and range fires in a large New Mexico county is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal.

I. What is the probability that between 2,500 and 4,200 acres will be burned in any given year?

$P(2500 < x < 4200) = P(z < -0.133) - P(z < -2.4)$

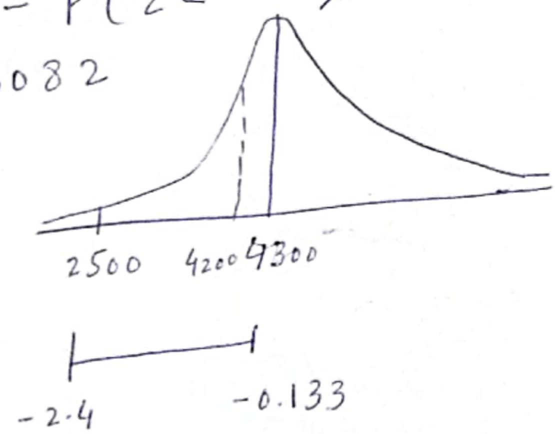
$= 0.4483 - .0082$

$= 0.4401$

$z = \frac{x - \mu}{\delta} =$

$z_1 = \frac{4200 - 4300}{750} = -0.133$

$z_2 = \frac{2500 - 4300}{750} = -2.4$



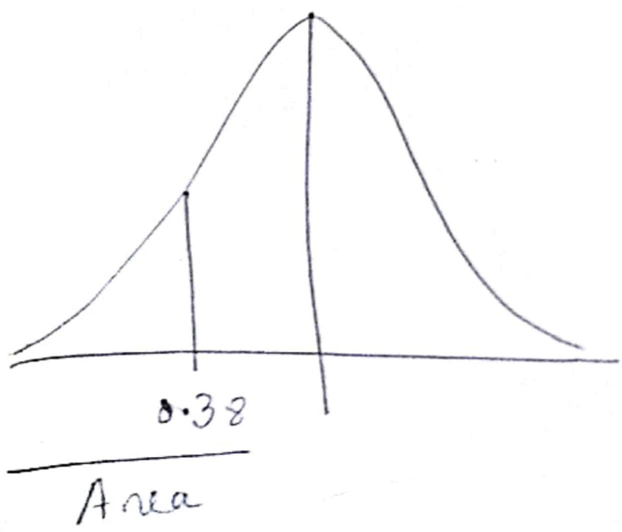
II. What number of burnt acres corresponds to the 38th percentile?

$Z = \frac{x - \mu}{\delta}$

$\Rightarrow x = \mu + \delta(Z)$

$x = 4300 + 750(-0.31)$

$x = 4067.5$

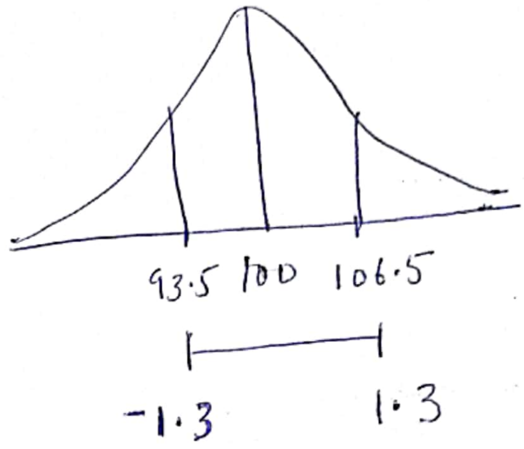


Part (b):

Chocolate bars are produced in a factory by a process that cannot guarantee the same mass for each bar. The mass of a (randomly chosen) chocolate bar is normally distributed with mean 100 g and standard deviation 5 g. A chocolate bar is randomly selected. Chocolate bars are accepted if they are within 1.3 times the standard deviation from the mean. What is the probability that the chocolates are rejected?

$$Z = \frac{x - \mu}{\sigma}$$

$$Z_1 = \frac{93.5 - 100}{5} = -1.3$$



$$Z_2 = \frac{106.5 - 100}{5} = 1.3$$

$$P(-1.3 < Z < 1.3) = P(Z < 1.3) - P(Z < -1.3)$$

$$= 0.9032 - 0.0968$$

$$= 0.8064 \rightarrow \text{Acceptance}$$

$$P(\text{rejection}) = 1 - 0.8064 = 0.1936$$

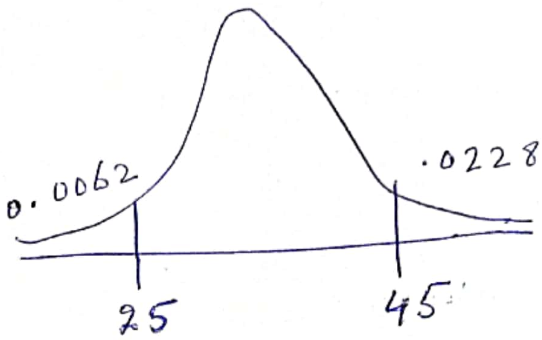
Question#6 [10 Marks]

Let random variable X be normally distributed, $X \sim N(\mu, \sigma^2)$. It is given that $P(X > 45) = 0.0228$ and $P(X < 25) = 0.0062$. Find μ and σ.

$$P(X > 45) = 0.0228$$

$$P(X < 25) = 0.0062$$

$$Z = \frac{X - \mu}{\delta}$$



$$X = \mu + \delta Z$$

$$\Rightarrow 25 = \mu + \delta(-2.5) \longrightarrow ①$$

$$\Rightarrow 45 = \mu + \delta(2) \longrightarrow ②$$

$$\Rightarrow \begin{array}{rcl} 45 & = & \cancel{\mu} + 2\delta \\ 25 & = & \cancel{\mu} - 2.5\delta \\ - & & - \quad + \\ \hline 20 & = & 4.5\delta \end{array}$$

$$\boxed{\delta = 4.444}$$

$$\mu = 45 - 2(4.444)$$

$$\mu = 45 - 8.888$$

$$\boxed{\mu = 36.112}$$

Question#7 [8+8+3+2+2+2=25Marks]

A probability density function is given below. Find:

$$f(x) = \begin{cases} \frac{1}{3}x & , \quad 0 \le x < 2 \\ 2 - \frac{2}{3}x, & \quad 2 \le x < 3 \\ 0 & , \quad \text{otherwise} \end{cases}$$

I. Cumulative distribution function (CDF).

$x \le 0 \qquad F_x(x) = 0$

$0 \le x < 2 \quad = \quad F_x(x) \int_{-\infty}^{x} f(t)\,dt$

$F_x(x) = \int_0^x \frac{1}{3} t\,dt = \left| \frac{1}{6} t^2 \right|_0^x = \frac{x^2}{6}$

$2 \le x < 3 \qquad F_x(x) = \int_2^x \left(2 - \frac{2}{3} t\right) dt$

$\left| 2t \right|_2^x - \frac{2}{3} \left. \frac{t^2}{2} \right|_2^x$ by putting values

$2 \le x < 3 \quad = \quad -\frac{x^2}{3} + 2x - \frac{8}{3}$

$$F_x(x) \begin{cases} 0 & x < 0 \\ x^2/6 & 0 \le x < 2 \\ \frac{-x^2}{3} + 2x - 8/3 & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

II. Coefficient of variation. $\text{C.V.} = \frac{\text{SD}}{\text{Mean}} \times 100$

$$\text{mean} = E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx$$

$$= \int_0^2 x\left(\tfrac{1}{3}x\right)dx + \int_2^3 x\left(2-\tfrac{2}{3}x\right)dx$$

$$\frac{1}{3}\left|\frac{x^3}{3}\right|_0^2 + 2\left|\frac{x^2}{2}\right|_2^3 - \frac{2}{3}\left|\frac{x^2}{2}\right|_2^3$$

by putting ~~total~~ limits we get

$$\boxed{E(X) = 5/3}$$

$$E(x^2) = \int_0^2 x^2\left(\tfrac{1}{3}x\right)dx + \int_2^3 x^2\left(2-\tfrac{2}{3}x\right)dx$$

$$\frac{1}{3}\left|\frac{x^4}{4}\right|_0^2 + 2\left|\frac{x^3}{3}\right|_2^3 - \frac{2}{3}\left|\frac{x^4}{4}\right|_2^3$$

by putting limits we get $E(x^2) = \frac{43}{6}$

$$V(X) = E(x^2) - [E(X)]^2 = \frac{43}{6} - \left(\frac{5}{3}\right)^2 = 4.388$$

$$S.D(X) = 2.095$$

$$C.V = \frac{2.095}{1.67} \times 100 = 125.45\%$$

III. $P(1.5 < X \leq 4)$ $\int_{1.5}^{2} \frac{1}{3}x\,dx + \int_{2}^{3} (2 - \frac{2}{3}x)\,dx$

$\frac{1}{3}\left|\frac{x^2}{2}\right|_{1.5}^{2} + \left|2x\right|_{2}^{3} - \frac{2}{3}\left|\frac{x^2}{2}\right|_{2}^{3}$

by solving we get

$P(1.5 \leq X \leq 4) = 5/8$

IV. $P(X \leq 1)$ $= \int_{0}^{1} \frac{1}{3}x\,dx = \frac{1}{3}\left|\frac{x^2}{2}\right|_{0}^{1}$

$= \frac{1}{3}\left(\frac{1}{2} - 0\right) = \frac{1}{6}$

$P(X \leq 1) = 1/6$

V. $P(X = 1.5)$ $= 0$

VI. Which of the following quantities can exceed one in magnitude? Encircle them.

Expected Value ✓	Probability Density Function ✓	Probability Mass Function	Cumulative Distribution Function

Question#8 [04+06 = 10 Marks]
Part (a):

The coefficient of quartile deviation is 10.34% and the difference of first and third quartile is 15, then find the sum of upper and lower quartile?

$$C.Q.D = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

by Putting given information.

$$0.1034 = \frac{15}{Q_3 + Q_1}$$

$$Q_3 + Q_1 = 145.067$$

Part (b):

An instructor taught two sections (W and T) of probability and Statistics in the last semester. The number of students in both section were 50. Section W had 20 students while section T had 30 students. A term project was given. After all the projects had been turned in, the instructor randomly selects 15 projects. Find the mean and standard deviation for the number of projects that are from the section W?

$N = 50 \quad n = 15 \quad K = 20 \quad N-K = 30$

$$\text{mean} = n \cdot \frac{K}{N} = (15)\left(\frac{20}{50}\right) = 6$$

$$S.D = \sqrt{\left(n\frac{K}{N}\right)\left(1-\frac{K}{N}\right)\left(\frac{N-n}{N-1}\right)}$$

$$= \sqrt{(15)\left(\frac{20}{50}\right)\left(\frac{30}{50}\right)\left(\frac{35}{49}\right)} = 1.603$$

Question#9 [05+05+05 =15 Marks]
Part (a):

Suppose X has the following probability distribution:

$$f(x) = K(x^2+4) \quad \textit{for } x = 0,1,2,3.$$

Where "K" is any constant, determine "K" and find the probability distribution of X.

x	$f(x)$
0	4K
1	5K
2	8K
3	13K

Pd of X

x	$f(x)$
0	$4/30$
1	$5/30$
2	$8/30$
3	$13/30$

As $\sum f(x) = 1$

$30K = 1$

$K = 1/30$

Part (b):

Let X be a continuous random variable with function:

$$f(x) = \begin{cases} 4x(1-x) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Check whether the given function is a valid pdf or not? If not, then under what condition it will become a valid pdf.

for a valid Pdf we have

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$= \int_0^1 4x(1-x)\,dx = 1$$

$$4\int_0^1 (x-x^2)\,dx = 1$$

$$4\left[\int_0^1 x\,dx - \int_0^1 x^2\,dx\right] = 1$$

$$= 4\left[\frac{x^2}{2} - \frac{x^3}{3}\right] = 1$$

$$4\left[\frac{1}{2} - \frac{1}{3}\right] = 1$$

$$\frac{4}{6} \neq 1$$

So the given functn is Not a valid Pdf. if we multiply the given $\frac{6}{4}$ we will a valid Pdf as

$f(x) = 6x(1-x) \quad 0 \le x \le 1$

Part (c):

It is often desired to inspect a portion of the shipment and judge the quality of the entire shipment on the basis of the observed quality of the sample. Suppose a buyer has received a shipment of 10 video cameras, 4 of which are defective. Three video cameras are randomly selected among the 10 and tested. Let *X* be the number of defective cameras, find the probability distribution of *X*.

$N = 10$, $K = 4$ $N - K = 6$

$n = 3$

x	
0	$\binom{4}{0}\binom{6}{3}/\binom{10}{3} = 20/120$
1	$\binom{4}{1}\binom{6}{2}/\binom{10}{3} = 60/120$
2	$\binom{4}{2}\binom{6}{1}/\binom{10}{3} = 36/120$
3	$\binom{4}{3}\binom{6}{0}/\binom{10}{3} = 4/120$