

National University of Computer and Emerging Sciences

FAST School of Computing

Fall-2023

Islamabad Campus

Question 1 [3+4=7 Marks]

- (a) Four married couples have bought 8 seats in the same row for a concert. In how many different ways can they be seated
- with no restrictions?
 - If each couple is to sit together?
 - If all the men sit together to the right of all the women?
- (b) A certain carton of eggs has 3 bad eggs and 9 good eggs. An omelet is made of 3 eggs randomly chosen from the carton. What is the probability that omelet contains
- at least one bad egg.
 - at most one bad egg.

a) a) $8! = 40320$ ✓

b) $4! \times 2! \times 2! \times 2! \times 2! = 384$ ✓

c) $2! \times 4! \times 4!$
 $= 1152$ right of all women

02.5

$$\begin{array}{cccc} \overline{2} & \overline{2} & \overline{2} & \overline{2} \\ 2 & 2 & 2 & 2 \\ & 2 & \times & 1 \\ \hline \overline{4!} & & \overline{2!} & \overline{4!} \\ 4! & 2! & 4! & \end{array}$$

b) $n(s) = {}^{12}C_3$

a) $P(\text{at least 1 bad egg}) =$
 $= 1 - P(\text{No bad egg})$

$$= 1 - \left[\frac{{}^3C_0 \cdot {}^9C_3}{{}^{12}C_3} \right] = 1 - 0.3818$$

$$= 0.6181$$

b) $P(\text{at most 1 bad egg}) = P(X=0) + P(\text{1 bad egg})$

$$P(1 \text{ bad egg}) = \frac{{}^3C_1 \cdot {}^9C_2}{{}^{12}C_3} = 0.4909$$

$$P(\text{at most 1 bad egg}) = 0.3818 + 0.4909$$

$$= 0.8727$$

40 / 50
0.5

they are under 25g

it that inmate is a male.

$P(M) = 0.625$

$$P(T/M) = \frac{P(T \cap M)}{P(M)}$$

$$= \frac{P(M/T) \cdot P(T)}{P(M)} \quad \text{--- ①}$$

$$P(M) = P(T) \cdot P(M/T) + P(T') \cdot P(M/T')$$

$$= (0.5)(0.9) + (0.5)(0.65)$$

$$P(M) = 0.775$$

put in ①.

$$P(T/M) = \frac{P(M/T) \cdot P(T)}{P(M)} = \frac{0.9 \times 0.5}{0.775}$$

$$= 0.5806$$

08

let w be the event that Hyekha walks
and c be the event that she cycles

$$P(w) = 0.3 \quad P(c) = 0.65$$

let L be the event that she is late.

$$P(L/w) = 0.15 \quad P(L/c) = 0.1$$

let B be the event that she takes bus.

$$\begin{aligned} P(B) &= P(w' \cap c') = P(w \cup c)' \\ &= 1 - P(w \cup c) = 1 - ((0.65) + 0.3) \\ &= 0.05 \end{aligned}$$

$$P(L/B) = 0.6 \quad P(C/L) = ?$$

$$P(C/L) = \frac{P(C \cap L)}{P(L)} = \frac{P(L/c) \cdot P(c)}{P(L)} \quad \text{--- ①}$$

$$\begin{aligned} P(L) &= P(w) \cdot P(L/w) + P(c) \cdot P(L/c) + P(B) \cdot P(L/B) \\ &= (0.3)(0.15) + (0.65)(0.1) + (0.05)(0.6) \\ &= 0.14 \end{aligned}$$

put in ①

$$P(C/L) = \frac{(0.1)(0.65)}{0.14} = 0.4642$$

✓
0.4642

Question 2 [3+4=7 Marks]

(a) In a certain federal prison, it is known that $\frac{2}{3}$ of the inmates are under 25 years of age. It is also known that $\frac{3}{5}$ of the inmates are male and that $\frac{5}{8}$ of the inmates are female or 25 years of age or older. What is the probability that a prisoner selected at random from this prison is female and at least 25 years old?

(b) When Ayesha goes to school, the probability that she walks is 0.3 and the probability that she cycles is 0.65. If she does not walk or cycle she takes the bus. When Ayesha walks the probability that she is late is 0.15. when she cycles the probability that she is late is 0.1 and when she takes bus the probability that she is late is 0.6. Given that Ayesha is late, find the probability that she cycles.

a) Let U be the event that they are under 25 yrs of age.

$$P(U) = 0.666$$

Let M be the event that inmate is a male.

$$P(M) = 0.6$$

$$P(M' \cap U') = 0.625$$

$$P(M' \cap U') = ?$$

$$P(U') = 1 - 0.666 = 0.334 \quad P(M') = 1 - 0.6 = 0.4$$

$$P(M' \cup U') = P(M') + P(U') - P(M' \cap U')$$

$$P(M' \cap U') = P(M') + P(U') - P(M' \cup U')$$

$$= 0.4 + 0.334 - 0.625$$

$$= 0.109$$

Question 3 [4+4=8 Marks]

- ✓ (a) There are 90 applicants for a job with the news department of a television station. Some of them are college graduates and some are not; some of them have at least three years' experience and some have not, with the exact breakdown being

	G College graduates	G' Not college graduates	
T At least three years' experience	18	9	= 27
T' Less than three years' experience	36	27	= 63
	= 54	= 36	

If the order in which the applicants are interviewed by the station manager is random, G is the event that the first applicant interviewed is a college graduate, and T is the event that the first applicant interviewed has at least three years' experience, determine each of the following probabilities directly from the entries and the row and column totals of the table:

(a) $P(T|G)$

(b) $P(G^c|T^c)$

- ✓ (b) A firm is accustomed to training operators who do certain tasks on a production line. Those operators who attend the training course are known to be able to meet their production quotas 90% of the time. New operators who do not take the training course only meet their quotas 65% of the time. Fifty percent of new operators attend the course. Given that a new operator meets her production quota, what is the probability that she attended the program?

a) $P(T|G) = \frac{P(T \cap G)}{P(G)} = \frac{18}{54} = 0.333$ ✓

b) $P(G'|T) = \frac{P(G' \cap T)}{P(T)} = \frac{9}{27} = 0.333$ ✓

- b) Let M be the event that they meet the quotas and T be the event that they attend training

$P(M|T) = 0.9$ $P(M|T') = 0.65$

$P(T) = 0.5$ $P(T') = 0.5$

$P(T|M) = ?$