

# Theory of Automata CS-3005

Course Instructor(s):  
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Alam, Ms. Tajwar Mehmood, and  
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Sections A,B,C,D,E,H,G,J,K

## Sessional-II Exam

Total Time (Hrs): 1  
Total Marks: 60  
Total Questions: 3

Date: Nov 5, 2024

Instructions : Attempt all questions on answer sheet and properly mention the question number, otherwise it will not be marked.  
Do not write below this line.

Attempt all the questions.

LO 1 : Identify formal language classes and prove language membership properties]

Question 1: Show that  $L = \{a^n b^k c^{n+k} : n \geq 0, k \geq 0\}$  is not regular, by using the pumping lemma

prem.

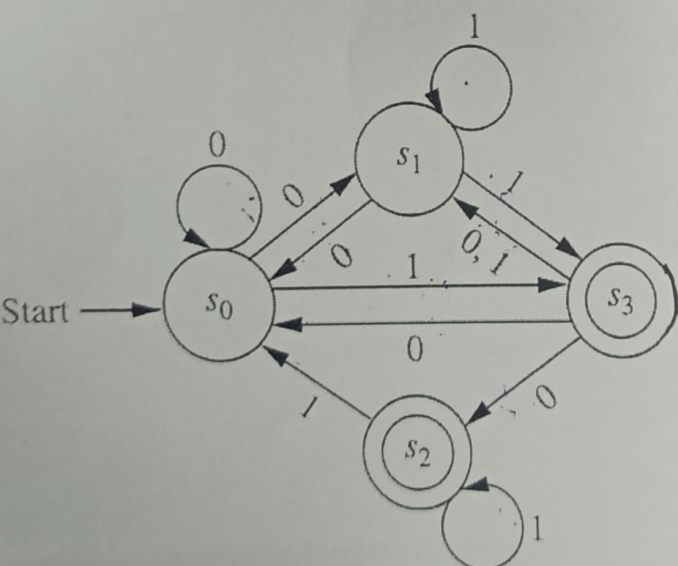
[10 Marks]

LO 2: Differentiate and manipulate formal descriptions of languages, automata and grammars with focus on non-regular, regular, context-free languages using automata (DFA, NFA, PDA) and Turing Machines.]

Question 2:

[30 Marks]

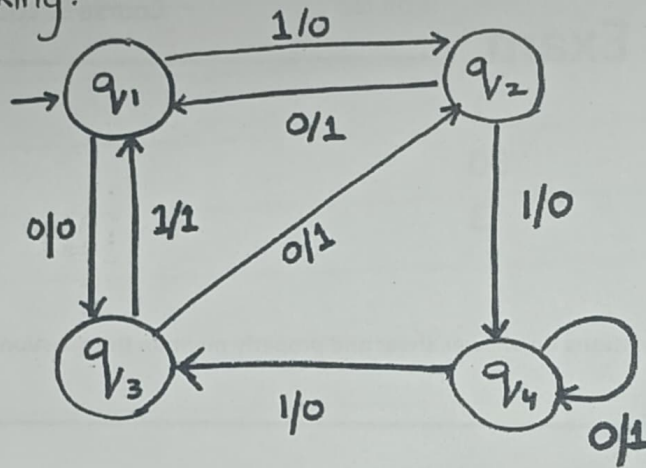
- [10 Marks] Design cfg for the language  $L = \{a^n b^n a^m b^t a^t b^m \mid n, m \geq 0, t > 0\}$ ,
- [10 Marks] Design cfg for equal number of a's and b's in string.
- [10 Marks] Convert the following NFA to DFA? Show the complete working.



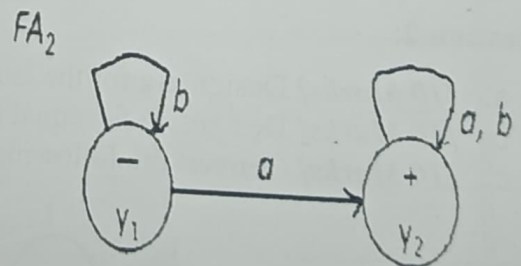
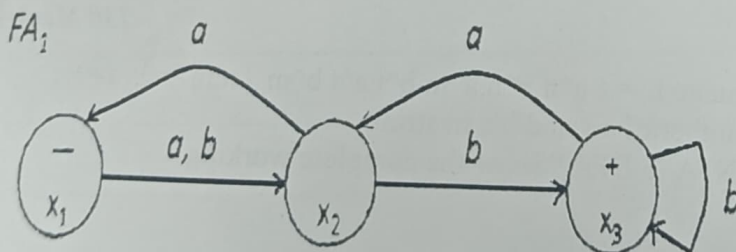
[CLO 3: Prove and disprove theorems establishing key properties of formal languages and automata]

Question 3:

- a. [10 Marks] Convert the following Mealy Machine into Moore Machine. [20 Marks] Show the complete working.



- b. [10 Marks] Let  $r_1$  and  $r_2$  be regular expressions and FA1 and FA2 be finite automata that accept exactly the languages defined by  $r_1$  and  $r_2$  respectively. By applying Kleene's theorem, build another FA that accepts all the words of the language defined by  $r_1 + r_2$ . Show the complete working.



### Question #01

$$L = \{a^n b^k c^{n+k} : n \geq 0, k \geq 0\}$$

To prove its irregularity,

Let  $n = 2$  and  $k = 2$ , (can be any number)

The string will be =  $aabccccc$

Let's divide it into 3 parts:

$aa, bbc, ccc$

$\Rightarrow$   $\underbrace{aa}_x \underbrace{bbc}_y \underbrace{ccc}_z$

Let's pump  $y^i$  with  $i = 2$

$\Rightarrow aabbcbbccccc$

As the string formed  $\notin L$  then  
the string is not regular.



Question # 02

(a)

$$L = \{a^n b^n a^m b^t a^t b^m \mid n, m \geq 0, t > 0\}$$

CFG:

$$S \rightarrow XY$$

$$X \rightarrow aXb \mid \lambda$$

$$Y \rightarrow aYb \mid Z$$

$$Z \rightarrow bZt \mid bt$$

10

(b)

Equal number of a's and b's

$$L = \{a^n b^n \mid n \geq 0\}$$

CFG =

$$S \rightarrow SabS \mid SbSaS \mid \lambda$$

$$S \rightarrow SaaSbS \mid SbSSaS \mid \lambda$$

✓

10

(C)

## NFA to DFA

Transition table of given NFA:

States	Input 0	Input 1
$\rightarrow S_0$	$S_0 S_1$	$S_3$
$S_1$	$S_0$	$S_1 S_3$
$+ S_2$	—	$S_2 S_0$
$+ S_3$	$S_0 S_1 S_2$	$S_1$

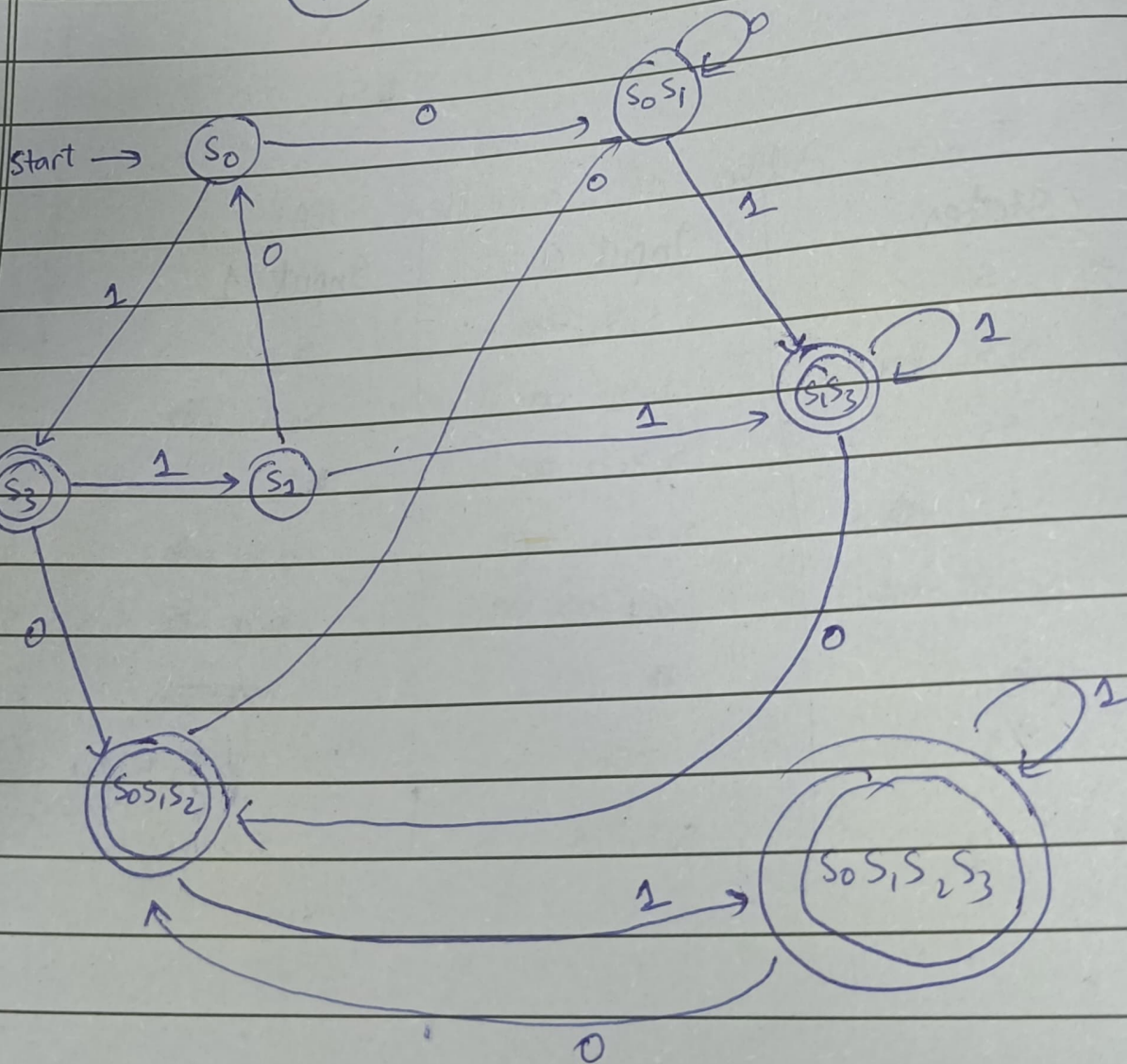
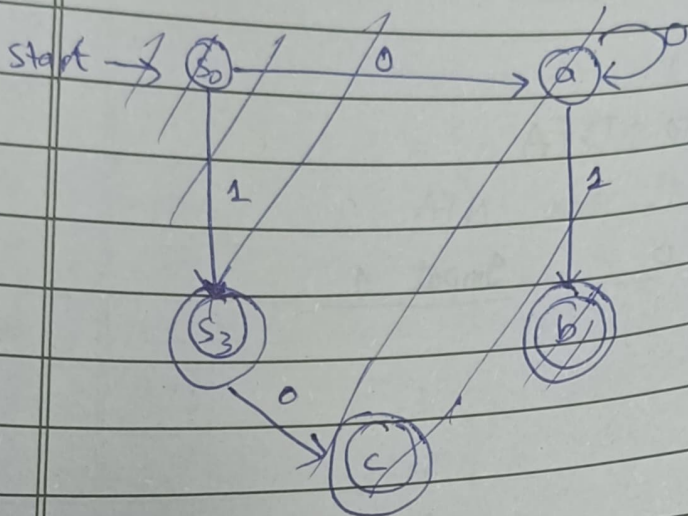
Transition table of converted DFA:

States	Input 0	Input 1
$\rightarrow S_0$	$S_0 S_1$	$S_3$
$S_0 S_1$	$S_0 S_1$	$S_1 S_3$
$+ S_3$	$S_0 S_1 S_2$	$S_1$
$+ S_1 S_3$	$S_0 S_1 S_2$	$S_1 S_3$
$+ S_0 S_1 S_2$	$S_0 S_1$	$S_0 S_1 S_2 S_3$
$S_1$	$S_0$	$S_1 S_3$
$+ S_0 S_1 S_2 S_3$	$S_0 S_1 S_2$	$S_0 S_1 S_2 S_3$

10/10



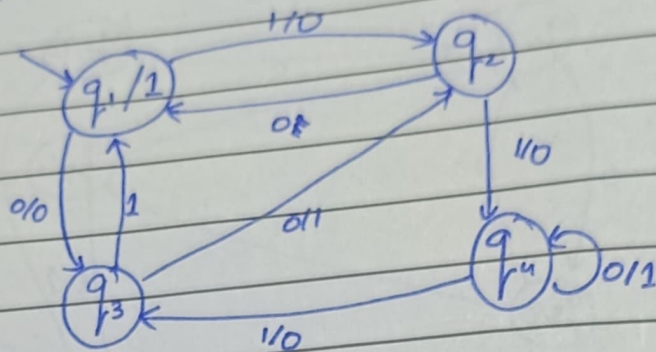
# Converted DFA



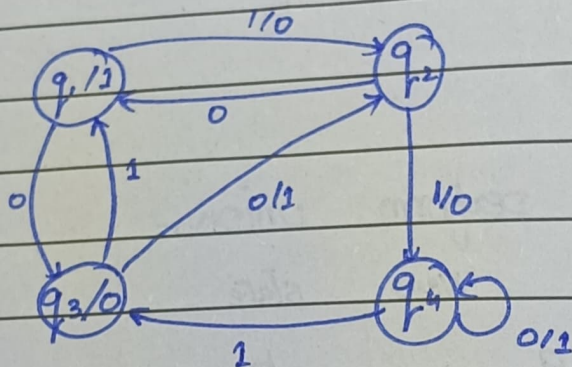
# Question # 03

Part A:-

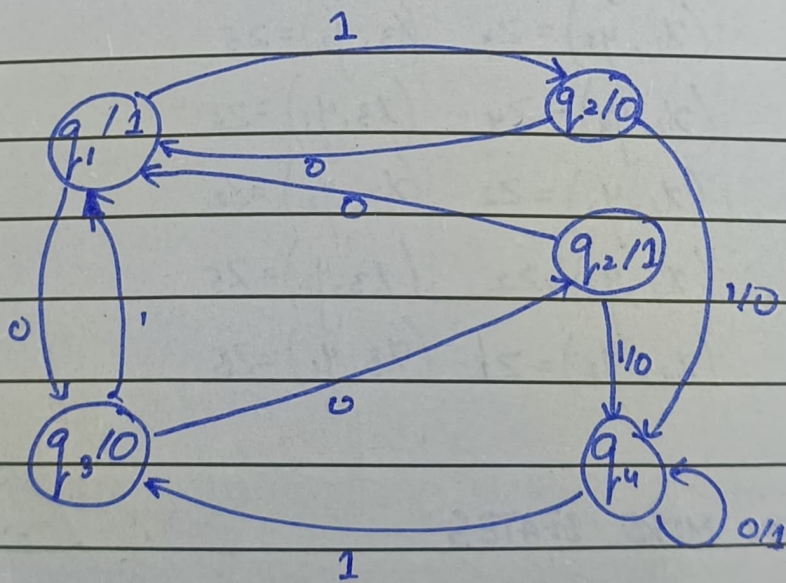
Step 1:-



Step 2:-

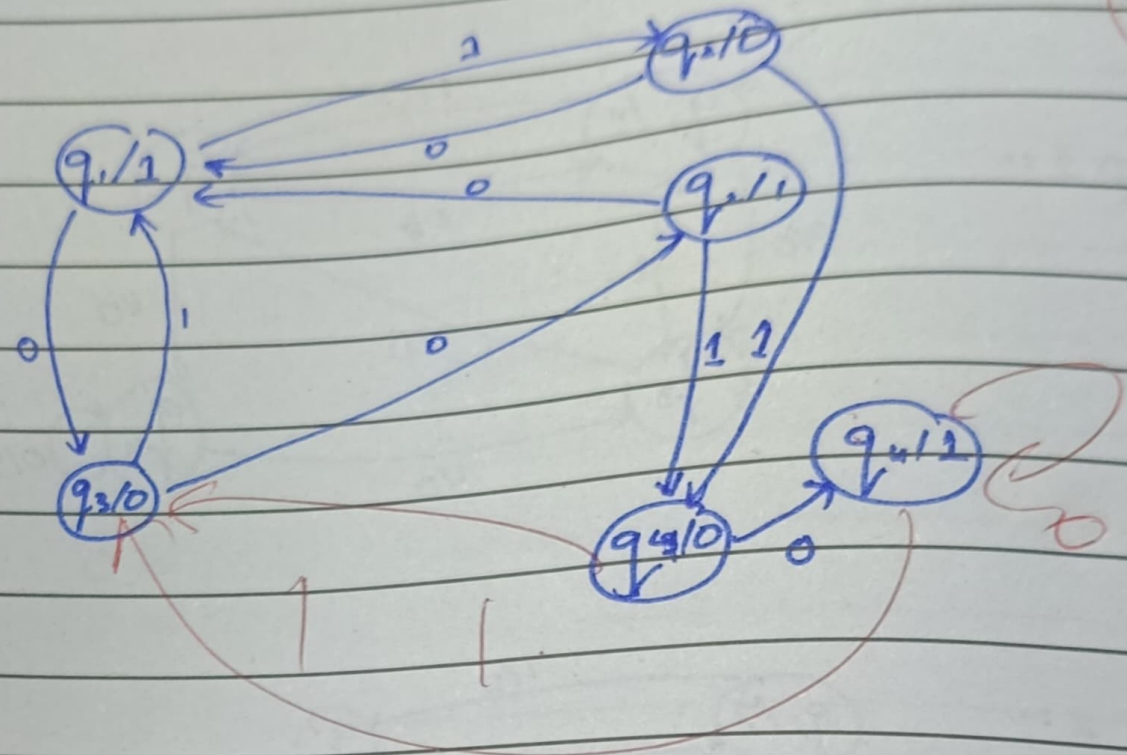


Step 3:-



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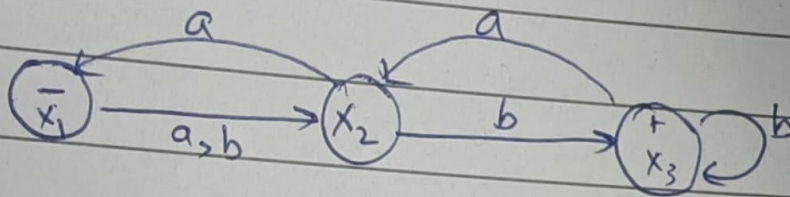
Step 4:



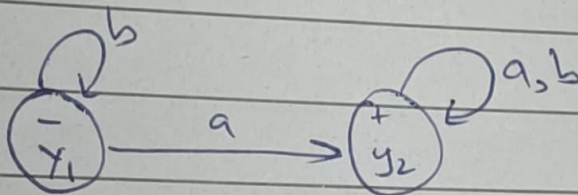


(b)  
Union of FA's

FA<sub>1</sub>



FA<sub>2</sub>



10 | 10

for FA<sub>3</sub>, transition table:

<u>states</u>	<u>"a"</u>	<u>"b"</u>
→ x <sub>1</sub> y <sub>1</sub> = z <sub>1</sub>	x <sub>2</sub> y <sub>2</sub> = z <sub>2</sub>	x <sub>2</sub> y <sub>1</sub> = z <sub>3</sub>
+ x <sub>2</sub> y <sub>2</sub> = z <sub>2</sub>	x <sub>1</sub> y <sub>2</sub> = z <sub>4</sub>	x <sub>3</sub> y <sub>2</sub> = z <sub>5</sub>
x <sub>2</sub> y <sub>1</sub> = z <sub>3</sub>	x <sub>1</sub> y <sub>2</sub> = z <sub>4</sub>	x <sub>3</sub> y <sub>1</sub> = z <sub>6</sub>
+ x <sub>1</sub> y <sub>2</sub> = z <sub>4</sub>	x <sub>2</sub> y <sub>2</sub> = z <sub>2</sub>	x <sub>2</sub> y <sub>2</sub> = z <sub>2</sub>
+ x <sub>3</sub> y <sub>2</sub> = z <sub>5</sub>	x <sub>2</sub> y <sub>2</sub> = z <sub>2</sub>	x <sub>3</sub> y <sub>2</sub> = z <sub>5</sub>
+ x <sub>3</sub> y <sub>1</sub> = z <sub>6</sub>	x <sub>2</sub> y <sub>2</sub> = z <sub>2</sub>	x <sub>3</sub> y <sub>1</sub> = z <sub>6</sub>

Q/Part No.

 $FA_1 + FA_2 = FA_3 :$ 