National University of Computer and Emerging Sciences

Multivariable Calculus

(MT1008)

Date: February 27th 2024

Course Instructor(s)

Dr. Hamda Khan, Dr. Imran Shahzad,

Mr. Arif Hussain

Sessional-I Exam

Total Time: 1 Hour Total Marks: 45 Total Questions: 04

Semester: SP-2024 Campus: Islamabad

Dept: Computer Science

Instructions:

- 1. Use permanent ink pens only. Any part done using soft pencil will not be marked and cannot be claimed for rechecking.
- 2. Show full steps for scoring full credit.
- 3. Solve the questions using techniques learnt in this course. Using a method other than the required method will result in deduction or zero marks.
- 4. Start each question on new page.

Q1: You are in charge of installing a radio telescope on a newly discovered planet. To minimize interference, you want to place it where the magnetic field of the planet is weakest. The planet is spherical, with a radius of 6 units. Based on a coordinate system whose origin is at the center of the planet, the strength of the magnetic field is given by $M(x,y,z) = 6x - y^2 + xz + 60$. Where should you locate the radio telescope? Solve using the method of Lagrange multipliers. [15 marks]

Q2: The temperature T in a metal ball is inversely proportional to the distance from the center of the ball, which we take to be the origin. The temperature at the point (1,2,2) is 1200. [10 marks]

- a) Find the function for temperature T.
- b) Find the rate of change of T at (1,2,2) in the direction toward the point (2,1,3). Hint: First calculate the vector between the two points before calculating the rate of change.

Page 1 of 2



National University of Computer and Emerging Sciences

Q3A: Check if the limit exists $\lim_{(x,y)\to(0,2)} (\sin(xy^2))/(xy)$. [04 marks]

Q3B: Find the absolute maximum and minimum values of f on the set D. $f(x,y) = xy^2 + 5$. D =

$$\{(x,y)|x \ge 0, y \ge 0, x^2 + y^2 \le 3\}$$
 [08 marks]

Q4: Find and sketch/shade the given functions' domain. Determine if the domain is open or closed or neither. Also decide if it is bounded or unbounded. [08 marks]

i)
$$f(x,y) = \sqrt{y+1} + \ln(x^2 - y)$$

ii)
$$f(x,y) = \frac{\left(\ln(y+1) + \sin\left(\frac{y}{x}\right)\right)}{\sqrt{x^2 + y^2 - 4}}$$