0076

National University of Computer and Emerging Sciences

FAST School of Computing | Spring-2024 | Islamabad Campus

MT-1008: Multivariable Calculus (CS)

Sessional-II Exam

Total Time: 1 Hour
Total Marks: 55

Date: 06 April, 2024

Course Instructor(s)

Dr. Hamda Khan, Dr. Imran Shahzad, Mr. Arif Hussain

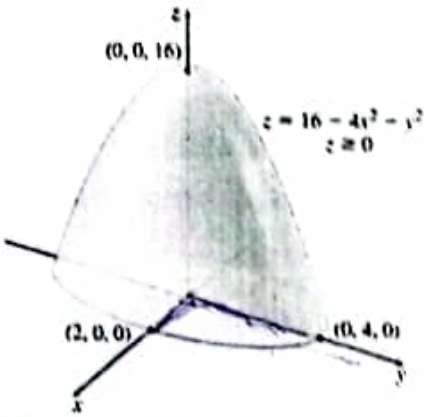
Muhammad Isam	231-0735	CS-B	Isam
Student Name	Roll No.	Course Section	Student Signature

Do not write anything on the question paper except the information required above.

Instructions:

1. Read the question carefully, understand the question, and then attempt your answers in the provided answer booklet.
2. Verify that you have **Two (02)** printed page of the question paper including this page. There are **Six (06)** questions.
3. Calculator sharing is strictly prohibited.
4. Use permanent ink pens only. Any part done using soft pencil will not be marked and cannot be claimed for rechecking.
5. Show full steps for scoring full credit.
6. Solve the questions using techniques learnt in this course. Using a method other than the required method will result in deduction or zero marks.
7. Start each question on new page.

Q1: Setup an iterated integral (triple integral) giving the volume V of the solid E in the first octant that is enclosed by the paraboloid $z = 16 - 4x^2 - y^2$ and the $xy-plane$. Sketch the projection of E onto $xy-plane$. Also, give the integrands and bounds, but do not evaluate. **[5 marks]**



Q2: An agricultural sprinkler distributes water in a circular pattern of radius 100 feet. It supplies water to a depth of e^{-r} feet per hour at a distance of r feet from the sprinkler.

a) If $0 < R \leq 100$, what is the total amount of water supplied per hour to the region inside the circle of radius R centered at the sprinkler?

b) Determine an expression for the average amount of water per hour per square foot supplied to the region inside the circle of radius R.

[10 marks]

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Q3: In neural networks and machine learning, deriving efficiently is vital. Traditional methods are computationally heavy, but automatic differentiation offers a swift alternative. As a computer science student, your task is to apply automatic differentiation to compute the partial derivatives of $z = \sin(x_1 + x_2) + x_2^2 x_3$. **[15 marks]**

i- Create the computational graph.

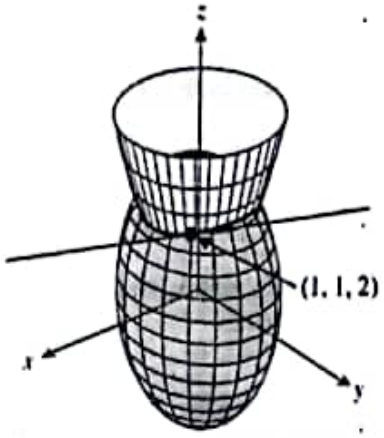
ii- Find $\frac{\partial z}{\partial x_1}$, $\frac{\partial z}{\partial x_2}$, and $\frac{\partial z}{\partial x_3}$ using reverse mode automatic differentiation.

iii- Directly calculate $\frac{\partial z}{\partial x_1}$, $\frac{\partial z}{\partial x_2}$, and $\frac{\partial z}{\partial x_3}$ and compare your answers.

Q4: Use Gradient Descent method (3 iterations) to find a point which minimize the quadratic surface $f(x,y) = x^2 + y^2 - 4x - 6y + 13$ by taking the initial point (1,1). **[9 marks]**

Q5: Evaluate $\iint_R 2x - y^2 dA$ over the region R enclosed between the lines $y = x + 1$, $y = -x + 1, and\ y = 3$. Also sketch the region. **[6 marks]**

Q6: Find parametric equation of the tangent line to the curve of the intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $3x^2 + 2y^2 + z^2 = 9$ at the point (1,1,2). **[10 marks]**



End of Question Paper