MT-224: Differential Equations (Cal II)

Serial No:

Remote Final Exam

Attempt Time: 3 Hours
Submission (through email only) Time: 15 minutes

Monday, 22nd June, 2020

Total Marks: 140

Course Instructors

Dr. Farah Jabeen Awan, Mr. Muhammad Usman Ashraf

Instructions:

1) The final exam will be attempted offline in the student's own handwriting (in readable way).
2) You are required to use A4 size blank white sheets to attempt the exam (portrait format unless a diagram or table requires landscape). Each sheet of the A4 size paper **MUST** have the Roll Number, Name, the course code, name of the course, and Signature of the student at the top of **EACH** sheet.
3) You can use cam-scanner, MS lens, or an equivalent application to scan and convert your hand-written answer sheets into a **SINGLE** pdf file (keeping the correct order of pages and question numbers), which you will email to the course instructor in a response email to the instructor (through which the question paper was sent). You will be given 15 minutes for this purpose.
4) You should carry a clean scanning, free from any marks/stains etc.
5) You must use the standard file name format (Full course code - Roll number, section e.g. MT-224-section X- 19i-abcd).
6) You must CC the email to yourself for verification of correct submission of the exam.
7) For proven cheating/ plagiarism, student will get an F grade even if the student had opted for S/U grade, and the case will be referred to DDC (Department's Disciplinary Committee). Instructors will conduct vivas of randomly selected students or in case of doubt (significantly different attempt as compared to past performance in the course or matching attempt with other students). Plagiarism includes sharing an attempt to other students (copy providing). Students who are not able to satisfactorily answer instructor's questions (based on the exam as well as slightly lateral but related concepts) during viva will also be considered as plagiarism cases.
8) Bonus two points will be given to all those students who solve complete paper in ascending order.
9) Understanding of the questions is also a part of the exam.

Q. No	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Total**
Total marks	**15**	**05**	**10**	**15**	**15**	**15**	**25**	**20**	**20**	**140**
Obtained marks										

Q#1

At time $t = 0$ a tank contains M_0 lb of salt dissolved in *140* gal of water. Assume that water containing *1/3* lb of salt/gal is entering the tank at a rate of p gal/min and that the well-stirred mixture is draining from the tank at the same rate. Set up the initial value problem that describes this flow process. Find the amount of salt $M(t)$ in the tank at any time, and also find the limiting amount M_L that is present after a very long time. If $p = 5$ and $M_0 = 3M_L$, find the time T after which the salt level is within 3% of M_L.Also find the flow rate that is required if the value of T is not to exceed 40 min.

Q#2

Find the domain of the functions

$$f(s,q) = \sqrt{s^2 + q^2 - 1} + \ln(4 - s^2 - q^2)$$

Q#3

If

$$f(x,z) = \frac{xz^2}{x^2 + z^4}$$

Does $\lim_{(x,z)\to(0,0)} f(x,z)\ \ exist$? Find the limit if it exist.

Q#4

Using power series find the solution up to x^4.

$$(x^2 - 5x + 6)y'' - 5y' - 2y = 0,\ \ where \quad y(0) = 2 \quad , y'(0) = 3$$

Q#5

Solve the given differential equation.

$$\left(\frac{1}{r} + \frac{1}{r^2} + \frac{\theta}{r^2 + \theta^2}\right) dr = -\left(\theta e^{\theta} + \frac{r}{r^2 + \theta^2}\right) d\theta$$

Q#6

Find the general solution of the differential equation.

$$t^2 s''' - 3ts'' + 6s' - \frac{6}{t}s = 3 + lnt^3.$$

Q#7

Show each step to find the series solution of

$$u_{xx} = -cu_{yy} \qquad (0 \le x \le 2), (0 \le y \le \pi)$$
$$u(x,0) = 0$$
$$u(x,\pi) = 0, \qquad t > 0$$
$$u_x(0,y) = \frac{u(0,y)}{u(2,y)}, \qquad u(2,y) = s \quad (0 \le x \le 2)$$

where c and s are constants.

Q#8

The purpose of this problem is to solve the heat equation on a one-dimensional ring-shaped domain of total length $2L$. The idea is to solve the heat equation on the domain $-L \leq x \leq L$, where $x = -L \; and \; x = L$ correspond to the same physical location. To simulate this, we set up periodic boundary conditions as follows: (Show each step)

$$u_t = \kappa \, u_{xx} \qquad (-L \leq x \leq L)$$
$$u(-L, t) = u(L, t)$$
$$u_x(-L, t) = u_x(L, t)$$
$$u(x, 0) = sinx$$

Q#9

Show each step to solve the following system using systematic elimination

$$\left(\frac{d}{dx} + 3\right) v + 5s = sin2x$$
$$\left(\frac{d}{dx} + 2\right) s + \left(\frac{d}{dx} + 1\right) v = cos2x.$$

Good Luck