2. Predicate Logic: Translations and Interpretations (10 Marks) [5+5]

In a seaside town, regulations state that every boat must have a life jacket for each passenger to ensure safety. One day, a boat is borrowed without the owner's permission by someone who is not aware that the boat is short on life jackets. When the coast guard stops the boat, they find that there are not enough life jackets for all passengers.

1 Express the following of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. [5]

(a) Boat users should ensure they have a life jacket for each passenger. [2.5]

B(x) : x is a boat user. ~~E(x): x should ens~~

P(y) : y is a passenger. J(x): x has a li

$\forall x \exists y B(x) \rightarrow J(x) \wedge P(y)$

$\neg(\forall x \exists y B(x) \rightarrow J(x) \wedge P(y))$

$\exists x \forall y \neg(B(x) \rightarrow J(x) \wedge P(y))$

(b) No boat user should be penalized for lacking life jackets if they were unaware of the shortage. [2.5]

U(x): x is unaware

P(x): x should be penalized

L(x): x has lack of jackets

1

ational University of Computer and Emerging Sciences

FAST School of Computing Islamabad Campus 00

3. Predicate Logic: Transformational Proof (10 Marks) [10]

DY

ow transformational proof of $\neg\exists x\exists y \bullet Q(x,y) \leftrightarrow Q(y,x) \Longleftrightarrow \forall x\forall y \bullet (Q(x,y) \vee Q(y,x)) \wedge \forall y\forall x \bullet$ $\neg Q(y,x) \vee \neg Q(x,y))$

.)

$\neg \exists x \exists y\, Q(x,y) \longleftrightarrow Q(y,x)$

$\forall x \forall y \neg (Q(x,y) \longleftrightarrow Q(y,x))$ De-Morgan

$\forall x \forall y \neg ((Q(x,y) \rightarrow Q(y,x)) \wedge (Q(y,x) \rightarrow Q(x,y)))$ Equivalente

$\forall x \forall y \neg ((\neg Q(x,y) \vee Q(y,x)) \wedge (\neg Q(y,x) \vee Q(x,y)))$ Implication 2

$\forall x \forall y (\neg(\neg Q(x,y) \vee Q(y,x)) \vee \neg(\neg Q(y,x) \vee Q(x,y)))$ De-Morg

$\forall x \forall y ((\neg\neg Q(x,y) \wedge \neg Q(y,x)) \vee (\neg\neg Q(y,x) \wedge \neg Q(x,y)))$ De Morg

$\forall x \forall y ((Q(x,y) \wedge \neg Q(y,x)) \vee (Q(y,x) \wedge \neg Q(x,y)))$ Double negat

Premises is not equivalent to conclusion. Thu the formula is invalid.

Q4. Predicate Logic: Natural Deduction Proof (10 Marks) [10]

Proof the following by using natural deduction. you are only allowed to use inference rules. $\forall x \bullet P(x) \rightarrow Q(x) \wedge S(x)$, $\forall x \bullet P(x) \wedge R(x) \vdash_N \forall x \bullet R(x) \wedge S(x)$

10

1- $\forall x . P(x) \rightarrow Q(x) \wedge S(x)$ premise

2- $\forall x . P(x) \wedge R(x)$ premise

3- x_g Assumption

4- $P(x_g) \wedge R(x_g)$ $\forall$-E.2

5- $P(x_g)$ $\wedge$-E-4

6- $R(x_g)$ $\wedge$-E-4

7- $P(x_g) \rightarrow Q(x_g) \wedge S(x_g)$ $\forall$-E-1

8- $Q(x_g) \wedge S(x_g)$ $\rightarrow$E-7

9- $S(x_g)$ $\wedge$-E_6

10- $R(x_g) \wedge S(x_g)$ $\wedge$-I-6,9

11- $\forall x\ R(x) \wedge S(x)$ $\forall$-I_10

Proved

Q1. Propositional Logic: Semantic Tableaux (10 Marks) [10]

Determine whether $A \to B$, $C \to D$, $B \vee D \to E$, $\neg E$ logically follows $\neg A \wedge \neg C$ or not.

1- $A \to B$
2- $C \to D$
3- $B \vee D \to E$
4- $\neg E$
5- $\neg(\neg A \wedge \neg C)$

↓ same 4

6- $\neg E$

implies 3

7- $\neg(B \vee D)$ 8- E
Closed 6,8

Not And 7

9- $\neg B$
10- $\neg D$

implies 2

11- $\neg C$ 12- D
Closed 10,12

implies 1

13- $\neg A$ 14- B
Closed 9,14

15- $\neg\neg A$ 16- $\neg\neg C$

Not Not 15 Not Not 16

17 A 18- C
Closed 13,17 Closed 11,18

10

All branches closed, therefore the given premises are not consistent with $\neg(\neg A \wedge \neg C)$. However they are consistent with $\neg A \wedge \neg C$.

National University of Computer and Emerging Sciences

FAST School of Computing Islamabad Campus

2 Let domain Dx=Dy={-2, -1, 0, 1, 2}. Explain whether the following statements are true or not. If they are true then define the truth set.

(a) $\forall x, \exists y \bullet x + y = 0$ [2.5]

Sol. This statement is true for each $x = -y$

$x = \{-2, -1, 0, 1, 2\}$ and $y = \{-x\}$

2.5

2.4

(b) $\exists x, \forall y \bullet x + y = y$ [2.5]

This statement is true iff

$x = \{0\}$ and $y = \{-2, -1, 0, 1, 2\}$.

2