Q # 1

Rate of change of Salt in the tank is equal to the rate at which Salt is flowing in minus the rate at which it is flowing out.

$$\frac{dM}{dt} = \text{rate in} - \text{rate out}$$ 02

$$\text{rate in} = \tfrac{1}{3}P$$

$$\text{rate out} = \frac{PM}{140}$$

$$\frac{dM}{dt} = \frac{P}{3} - \frac{PM}{140}$$

$$\frac{dM}{dt} + \frac{P}{140}M = P/3 \longrightarrow (1)$$ 03

$M(0) = M_0 \rightarrow (2)$, Equation (1) is linear nonhomogeneous Differential Eq so

$$I.F = e^{\int P/140\, dt} = e^{P/140\, t} \rightarrow (3)$$

$$e^{P/140\, t}\frac{dM}{dt} + \frac{P}{140}M\, e^{P/140\, t} = \frac{P}{3}e^{Pt/140}$$

(2)

$$\int \frac{d}{dt}\left(M(t)\, e^{pt/140}\right) = \int \frac{p}{3}\, e^{pt/140}$$

$$M(t)\, e^{pt/140} = \frac{p}{3}\, \frac{e^{\frac{pt}{140}}}{p/140}$$

$$m(t)\, e^{pt/140} = \frac{\cancel{p}}{3} \times \frac{140}{\cancel{p}}\, e^{\frac{pt}{140}} + c$$

03

$$\boxed{m(t) = \frac{140}{3} + c\, e^{-pt/140}} \longrightarrow (4)$$

consequently we might expect that ultimate amount of salt in the tank is very close to $\frac{140}{3}$. so $M_L = \frac{140}{3}$ $\longrightarrow$ (3)

where c is any arbitrary constant.

In order to satisfy condition (2), $c = M_0 - \frac{140}{3}$.

$$\text{so } M(t) = \frac{140}{3} + \left(M_0 - \frac{140}{3}\right) e^{-pt/140} \longrightarrow (5^*)$$

$$= \frac{140}{3}\left(1 - e^{-pt/140}\right) + M_0\, e^{-pt/140} \longrightarrow (6)$$

02

③

From eq (6) we can see that for large t means $t \to \infty$.

$$M(t) = \frac{140}{3}.$$

Now if $p = 5$, $M_0 = 3M_L$

$$M_0 = \cancel{3} \times \frac{140}{\cancel{3}} \quad \text{so from eq(5)}^*$$

$$M(t) = \frac{140}{3} + \left(140 - \frac{140}{3}\right) e^{-pt/140}$$

$$M(t) = \frac{140}{3} + \left(\frac{420 - 140}{3}\right) e^{-0.05t/140}$$

03

Now 3% of $M_L = (0.03)\left(\frac{140}{3}\right) = 1.4$

So Total $M(t) = 48.066$

$$48.066 = 46.666 + 93.333\, e^{-0.05t/140}$$

$$\ln(0.0150) = -\frac{0.05}{140} t\,(\ln e)$$

$$t = -\frac{140}{0.05} \ln(0.0150) = 11{,}759 \text{ min.}$$

02

Q#3

(4)

$$f(x,z) = \frac{xz^2}{x^2+z^4}$$

$\lim_{(x,z)\to(0,0)} f(x,z)$ exist? Find the limt if it exist.

Solution:- $(x,z)\to(0,0)$ along any nonvertical line through the origin. Then $z = mx$, where m is the slope 02

$$f(x,z) = f(x,mx) = \frac{x(mx)^2}{x^2+m^4x^4} = \frac{m^2x^3}{\cancel{x^2}(1+m^4x^2)}$$

So $f(x,z)\to 0$ as $(x,z)\to(0,0)$ along $z = mx$.

Thus f has same limiting value along every non vertical line through the origin. But that does not show the given limit is 0. for if 03
we let $(x,z)\to(0,0)$ along the parabola $x = z^2$ we have.

$$f(x,z) = f(z^2,z) = \frac{z^2\,z^2}{z^4+z^4} = \frac{\cancel{z^4}}{2\cancel{z^4}} = 1/2$$

$f(x,z)\to\frac{1}{2}$ as $(x,z)\to(0,0)$ along $x = z^2$.

Since different path leads to different limit values. 03

The given limit does not exist. (8)

02

Q#4

$(x^2-5x+6)y''-5y'-2y=0$, $y(0)=2$

$y'(0)=3$

$y=a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+a_5x^5+a_6x^6+\cdots$

$y'=a_1+2a_2x+3a_3x^2+4a_4x^3+5a_5x^4+6a_6x^5$

$y''=2a_2+6a_3x+12a_4x^2+20a_5x^3+30a_6x^4$

03

$y(0)=2=a_0+a_1(0)+a_2(0)^2+\cdots$

$\boxed{a_0=2}$

01

$y'(0)=3=a_1+2a_2(0)+3a_3(0)+\cdots$

$\boxed{a_1=3}$

01

$(x^2-5x+6)(2a_2+6a_3x+12a_4x^2+20a_5x^3+30a_6x^4+\cdots)-5(a_1+2a_2x$

$+3a_3x^2+4a_4x^3+5a_5x^4+\cdots)-2(a_0+a_1x+a_2x^2$

$+a_3x^3+a_4x^4+a_5x^5\cdots)=0$

02

Now comparing coefficents of x^2, x^3, x^4 we can write

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$$y(x) = 2 + 3x + \frac{19}{12}x^2 + \frac{26}{27}x^3 + \frac{65}{108}x^4 + \cdots$$ 02

Q#5 :-

$$\left(\frac{1}{r} + \frac{1}{r^2} + \frac{\theta}{r^2+\theta^2}\right)dr + \left(\theta e^{\theta} + \frac{r}{r^2+\theta^2}\right)d\theta = 0$$

$$M(r,\theta)\,dr + N(r,\theta)\,d\theta = 0$$ 02

For exact :- $M_\theta = N_r$

$$M_\theta = \frac{\partial M}{\partial \theta} = 0 + 0 + \frac{(r^2+\theta^2)(1) - \theta(2\theta)}{(r^2+\theta^2)^2}$$

$$\boxed{M_\theta = \frac{r^2 - \theta^2}{(r^2+\theta^2)^2}}$$ 04

$$N_r = \frac{\partial N}{\partial r} = 0 + \frac{(r^2+\theta^2)(1) - r(2r)}{(r^2+\theta^2)^2}$$

$$= \frac{r^2+\theta^2-2r^2}{(r^2+\theta^2)^2} = \frac{\theta^2-r^2}{(r^2+\theta^2)^2}$$

Non Exact DE.

04

Integrating Factors.

Now

$$\frac{M_\theta - N_r}{N} = \frac{\dfrac{r^2 - \theta^2 - \theta^2 + r^2}{(r^2+\theta^2)(r^2+\theta^2)}}{\left(\theta e^\theta + \dfrac{r}{r^2+\theta^2}\right)}$$

$$= \frac{2(r^2-\theta^2)}{(r^2+\theta^2)\cancel{(r^2+\theta^2)}} \times \frac{\cancel{(r^2+\theta^2)}}{\left(\theta e^\theta (r^2+\theta^2) + r\right)}$$

can not be use as an I.F.

Also

$$\frac{N_r - M_\theta}{M} = \frac{\dfrac{\theta^2 - r^2 - r^2 + \theta^2}{(r^2+\theta^2)^2}}{\left(\dfrac{1}{r} + \dfrac{1}{r^2} + \dfrac{\theta}{r^2+\theta^2}\right)}$$

$$= \frac{2\theta^2 - 2r^2}{(r^2+\theta^2)^2} \Big/ \left(\frac{1}{r} + \frac{1}{r^2} + \frac{\theta}{r^2+\theta^2}\right)$$

can not be use as an I.F.

Solution can not be possible.
Because Non Exact Equation can not become exact. Neither we not given any $u(r, \theta)$ to make it Exact.

01

(16)

Q #6 $t^2 s''' - 3ts'' + 6s' - \frac{6}{t}s = 3 + \ln t^3$ → (B)

For complementary solution. Equation will be homogeneous.

$$t^2 s''' - 3ts'' + 6s' - \frac{6}{t}s = 0$$

$$t^3 s''' - 3t^2 s'' + 6ts' - 6s = 0$$

let $t = e^x \Rightarrow x = \ln t$

$$dx = \frac{1}{t}dt$$

$$\frac{dx}{dt} = \frac{1}{t} \qquad \text{Also } t^3 = e^{3x}$$

$$t^3 \frac{d^3s}{dt^3} - 3t^2 \frac{d^2s}{dt^2} + 6t\frac{ds}{dt} - 6s = 0. \rightarrow (A)$$

$$\frac{ds}{dt} = \frac{ds}{dx} \times \frac{dx}{dt} = \frac{1}{t}\frac{ds}{dx} \rightarrow (1)$$

$$\frac{d^2s}{dt^2} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d}{dt}\left(\frac{1}{t}\frac{ds}{dx}\right)$$

$$= -\frac{1}{t^2}\frac{ds}{dx} + \frac{1}{t}\frac{d}{dt}\left(\frac{ds}{dx}\right)$$

$$= -\frac{1}{t^2}\frac{ds}{dx} + \frac{1}{t}\frac{d}{dx}\left(\frac{ds}{dt}\right)$$

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$$= -\frac{1}{t^2}\frac{ds}{dx} + \frac{1}{t}\frac{d}{dx}\left(\frac{1}{t}\frac{ds}{dx}\right)$$

$$= \frac{1}{t^2}\left(\frac{d^2s}{dx^2} - \frac{ds}{dx}\right) \longrightarrow (2)$$

Similarly $$\frac{d^3s}{dt^3} = \frac{1}{t^3}\left(\frac{d^3s}{dx^3} - 3\frac{d^2s}{dx^2} + 2\frac{ds}{dx}\right) \longrightarrow (3)$$

Now substituting values from eq (1) (2) and (3) to eq (A)

$$\cancel{t^3}\frac{1}{\cancel{t^3}}\left(\frac{d^3s}{dx^3} - 3\frac{d^2s}{dx^2} + 2\frac{ds}{dx}\right) - 3\cancel{t^2}\times\frac{1}{\cancel{t^2}}\left(\frac{d^2s}{dx^2} - \frac{ds}{dx}\right) + 6\cancel{t}\,\frac{1}{\cancel{t}}\frac{ds}{dx} - 6s = 0$$

$$\Rightarrow \frac{d^3s}{dx^3} - 6\frac{d^2s}{dx^2} + 11\frac{ds}{dx} - 6s = 0$$

Now by assuming

$$s = e^{mx}$$

$$s' = me^{mx}$$

$$s'' = m^2e^{mx}$$

$$s''' = m^3e^{mx}$$

Now substituting values

$$e^{mx}(m^3-6m^2+11m-6)=0$$

$$e^{mx}\neq 0 \quad \text{so} \quad (m^3-6m^2+11m-6)=0.$$

Roots :- $(m-1)(m-2)(m-3)=0$

$$m_1=1, \quad m_2=2, \quad m_3=3$$

Now

$$\boxed{y_c=c_1e^{x}+c_2e^{2x}+c_3e^{3x}}$$

Now for $y_p=?$

As Right Hand Side of eq (B). is

$$3+\ln t^3 = 3+\ln e^{3x} = 3+3x\ln e$$

$$=3+3x.$$

Now for particular solution, we choose particular Solution

$$y_p = Ax+B$$

$$y_p' = A, \quad y_p'' = 0$$

$$y_p''' = 0$$

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$0 - 0 + 11(A) - 6(Ax + B) = 3 + 3x$

<u>NOW</u> $-6A = 3$

$\boxed{A = -\frac{1}{2}}$

$11A - 6B = 3$

$6B = 11A - 3 = 11\left(-\frac{1}{2}\right) - 3$

$6B = \frac{-11 - 6}{2} = \frac{-17}{2}$

$\boxed{B = -17/12}$

$y_p = -\frac{1}{2}x - \frac{17}{12}$

$y = y_c + y_p = c_1 e^{x} + c_2 e^{2x} + c_3 e^{3x} - \frac{x}{2} - \frac{17}{12}$

$y = c_1 t + c_2 t^2 + c_3 t^3 - \frac{1}{2} \ln t - \frac{17}{12}$.

Q#2

$$f(s,q) = \sqrt{s^2+q^2-1} + \ln(4-s^2-q^2)$$

$s^2+q^2-1 \geq 0$ and $4-s^2-q^2 > 0$

$s^2+q^2 \geq 1$ and $4 > s^2+q^2$

so $1 \leq s^2+q^2 < 4$

Q#7

the Soln be

$u(x,y) = X(x)\,Y(y)$

eq ① =>

$X''\cancel{Y} = -c X Y''$

$\frac{X''}{X} = -c \frac{\cancel{Y''}}{Y} = -\lambda$

$X'' + \lambda X = 0$ — ⑥

$Y'' - \frac{\lambda}{c} Y = 0$ — ⑦

Q#7 ①

Conditions

$Y(0) = 0$

$Y(\pi) = 0$

$X'(0) = \frac{X(0)}{S}$

Case i $\lambda = 0$

$\cancel{X}'' = 0$

$\cancel{Y} = c_1 + c_2 y$

$Y(0) = 0$

$c_1 = 0$

$Y(\pi) = 0 = 0 + c_2\pi$, $\pi \neq 0$

$c_2 = 0$

$Y = 0$

$u = XY = 0$ Trivial

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$\lambda = -\alpha^2$

eq (7) $\Rightarrow$

$$X'' - \alpha^2 X = 0 \quad - (8)$$

$$Y'' + \frac{\alpha^2}{c} Y = 0 - (9)$$

eq (8) $\Rightarrow$ $X'' - \alpha^2 X = 0$

$m^2 = \alpha^2$, $m = \pm\alpha$

$$X = c_3 \cosh \alpha x + c_4 \sinh \alpha x$$

$Y'' = -\frac{\alpha^2}{c} Y$ Let $\frac{\alpha^2}{c} = \alpha_1^2$

$m^2 = -\frac{\alpha^2}{c} = -\alpha_1^2$

$m = \pm i\alpha_1$

$$Y = c_5 \cos \alpha_1 y + c_6 \sin \alpha_1 y$$

$Y(0) = 0 = c_5$

$Y(\pi) = 0 = 0 + c_6 \sin \alpha_1 (\pi)$, $c_6 \neq 0$

$\alpha_1 \pi = n\pi$ $\alpha_1 = n$ $n = 1, 2, 3 \ldots$

$X'(0) = \frac{X(0)}{s}$

$c_3 \alpha \sinh(0) + c_4 \alpha \cosh(0) = \frac{c_3}{s} \cosh(0) + \frac{c_4}{s} \sinh(0)$

$\alpha c_4 = \frac{c_3}{s}$, $\alpha = n$, $n = 1, 2, \ldots$

$c_4 = \frac{c_3}{ns}$

$u = XY$ Q#7 (3)

$n = 1, 2, \ldots$

$u = \sum_{n=1}^{\infty} \quad X = C_3 \cosh n x + \frac{c_3}{n5} \sinh n x$

$Y = c_6 \sin \alpha_1 y = c_6 \sin \frac{\alpha^2}{c} y = c_6 \sin\left(\frac{n^2}{\sqrt{c}} y\right)$

$$u = \sum_{n=1}^{\infty} \left\{ A_n \left[\cosh n x + \frac{1}{5n} \sinh n x \right] \sin\left(\frac{n^2 y}{\sqrt{c}}\right) \right. \quad (10)$$

$$u(2, y) = 5 = \sum_{n=1}^{\infty} A_n \left[\cosh 2n + \frac{\sinh 2n}{5n} \right] \sin\left(\frac{n^2 y}{\sqrt{c}}\right)$$

This is half range sine fourier series

$$A_n \left[\cosh 2n + \frac{\sinh 2n}{5n} \right] = \int_0^{\pi} 5 \sin\left(\frac{n^2 y}{\sqrt{c}}\right) dy$$

$$= -5 \cos\left(\frac{n^2 y}{\sqrt{c}}\right) \left(\frac{\sqrt{c}}{n^2}\right) \Big|_0^{\pi}$$

$$= -\frac{5\sqrt{c}}{n^2} \left[\cos n\pi - 1 \right]$$

$$A_n = \frac{-5\sqrt{c} \left[(-1)^n - 1 \right]}{n \left[\cosh 2n + \frac{\sinh 2n}{5n} \right]}$$

put A_n in eq (10)

$$u_t = k u_{xx} \quad \text{—— (1)}$$

$$u(-L, t) = u(L, t) \quad \text{— (2)}$$

$$u_x(-L, t) = u_x(L, t) \quad \text{— (3)}$$

$$u(x, 0) = \sin x \quad \text{—— (4)}$$

Soln

Let the soln be

$$u(x, t) = X(x)\, T(t)$$

(1) =>

$$X T' = k X'' T$$

$$\frac{T'}{kT} = \frac{X''}{X} = -\lambda$$

$$T' + k\lambda T = 0 \quad \text{— (5)}$$

$$X'' + \lambda X = 0 \quad \text{— (6)}$$

For conditions (2)

$$u(-L, t) = u(L, t)$$

$$X(-L)\, T(t) = X(L)\, T(t)$$

$$X(-L) = X(L)$$

condition (3) is $u_x(-L, t) = u_x(L, t)$

$$X'(-L) = X'(L)$$

Q # 8 (2)

$\underline{\lambda = 0}$

eq (6) $\Rightarrow$

$$X'' = 0 \qquad m^2 = 0$$

$$X = C_1 + C_2 x$$

$$X(-L) = X(L)$$

$$C_1 + C_2(-L) = C_1 + C_2 L$$

$$C_2(2L) = 0$$

$$\Rightarrow C_2 = 0$$

$$X'(-L) = X'(L)$$

$$\Rightarrow \quad C_2 = C_2 \checkmark$$

i.e $\quad X = C_1$

eq (5) $\Rightarrow$

$$T' = 0$$

$$m = 0$$

$$T(t) = C_3$$

$$u = XT = C_1 C_3 = A_0$$

$\underline{\text{Case II}}$ $\quad \lambda < 0, \quad \lambda = -\alpha^2$

$$T' - k\alpha^2 T = 0$$

$$X'' - \alpha^2 X = 0$$

For X

$$m^2 = \alpha^2, \quad m = \pm\alpha$$

Q#8 ③

$X = c_4 \cosh \alpha x + c_5 \sinh \alpha x$

$X(-L) = X(L)$

$\Rightarrow c_4 \cosh(-\alpha L) + c_5 \sinh(-\alpha L) = c_4 \cosh(\alpha L) + c_5 \sinh(\alpha L)$

$\Rightarrow$ as $\begin{cases} \cosh(-x) = \cosh x \\ \sinh(-x) = -\sinh x \end{cases}$

$c_4 \cancel{\cosh \alpha L} - c_5 \sinh(\alpha L) = c_4 \cancel{\cosh(\alpha L)} + c_5 \sinh(\alpha L)$

$2 c_5 \sinh(\alpha L) = 0$ $\quad \sinh(\alpha L) \neq 0$

i.e $c_5 = 0$

Now $X = c_4 \cosh \alpha x$

$X'(-L) = X'(L)$

$c_4 \alpha \sinh(-\alpha L) = c_4 \alpha \sinh(\alpha L)$

$-c_4 \alpha \sinh(\alpha L) = c_4 \alpha \sinh(\alpha L)$

$2 c_4 \alpha \sinh(\alpha L) = 0$

$c_4 = 0$

$X = 0$

$u = XY = 0$

Trivial soln

$\lambda > 0$

Q#8 (4)

$\lambda = \alpha^2$

eq (6) $\Rightarrow$

$$X'' + \alpha^2 X = 0$$

$$m^2 = -\alpha^2$$

$$m = \pm i\alpha$$

$$X = c_6 \cos\alpha x + c_7 \sin\alpha x \,, \quad X' = -\alpha c_6 \sin\alpha x + \alpha c_7 \cos\alpha x$$

$$X(-L) = X(L)$$

$$c_6 \cos(-\alpha L) + c_7 \sin(-\alpha L) = c_6 \cos(\alpha L) + c_7 \sin(\alpha L)$$

$$\cancel{c_6 \cos(\alpha L)} - c_7 \sin(\alpha L) = \cancel{c_6 \cos(\alpha L)} + c_7 \sin(\alpha L)$$

$$2 c_7 \sin(\alpha L) = 0$$

$$\sin(\alpha L) = 0 = \sin(n\pi) \qquad n = 1, 2, \ldots$$

$$\alpha L = n\pi \qquad , \quad \alpha = \frac{n\pi}{L}$$

$$X'(-L) = X'(L)$$

$$-\alpha c_6 \sin(-\alpha L) + \alpha c_7 \cos(-\alpha L) = -\alpha c_6 \sin\alpha L + \alpha c_7 \cos\alpha L$$

$$+\alpha c_6 \sin(+\alpha L) + \cancel{\alpha c_7 \cos(\alpha L)} = -c_6 \alpha \sin(\alpha L) + \cancel{c_7 \alpha \cos(\alpha L)}$$

$$2\alpha c_6 \sin(\alpha L) = 0 \qquad \text{where } \alpha = \frac{n\pi}{L}$$

$$2\alpha c_6 \sin(n\pi) = 0 \qquad , \quad \cancel{\cos(n\pi) = (-1)^n}$$

$$\sin(n\pi) = 0$$

So $c_6 \neq 0$

$$X = c_6 \cos\alpha x + c_7 \sin\alpha x$$

eq (5) $\Rightarrow$

Q#8 (8)

$$T' + K\alpha^2 T = 0$$

$$m = -K\alpha^2$$ $n = 1, 2, \ldots$

$$T = c_1 e^{-K\alpha^2 t}$$ $\alpha = \frac{n\pi}{L}$

$$u = \sum_{n=1}^{\infty} X T + u_0$$

$$= \sum_{n=1}^{\infty} \left[c_6 \cos\alpha x + c_7 \sin\alpha x \right] c_1 e^{-K\alpha^2 t} + A_0$$

$$u(x,0) = \sin x = A_0 + \sum_{n=1}^{\infty} \left[c_6 \cos\alpha x + c_7 \sin\alpha x \right] c_1$$

$$\sin x = A_0 + \sum_{n=1}^{\infty} A_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{L}$$

where $c_1 c_6 = A_n$, $c_1 c_7 = B_n$

#9

Q#9 ①

$$\left(\frac{d}{dx}+3\right)v + 5s = \sin 2x$$

$$\left(\frac{d}{dx}+2\right)s + \left(\frac{d}{dx}+1\right)v = \cos 2x$$

Check it

1984

A_2

Soln

663 Q_3

X

$$(D+3)v + 5s = \sin 2x \quad ①$$

$$(D+2)s + (D+1)v = \cos 2x \quad ②$$

Multiply eq ① by $(D+2)$ and -5 by eq ②

$$(D+2)(D+3)v + 5\cancel{(D+2)s} = (D+2)\sin 2x$$

$$-5\cancel{(D+2)s} + (-5)(D+1)v = -5\cos 2x$$

$$(D+2)(D+3)v - 5(D+1)v = (D+2)\sin 2x - 5\cos 2x$$

$$(D^2+3D+2D+6)v - 5Dv - 5v = 2\cos 2x + 2\sin 2x - 5\cos 2x$$

$$D^2v + 5Dv + 6v - 5Dv - 5v = 2\sin 2x - 3\cos 2x$$

$$D^2v + v = 2\sin 2x - 3\cos 2x$$

$$(D^2+1)v = 2\sin 2x - 3\cos 2x \quad ③$$

corresponding homogeneous eq is

$$(D^2+1)v = 0$$

$$m^2 = -1, \quad m = \pm i$$

$$v = c_1 e^{ix} + c_2 e^{-ix}$$

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Q#7 (2)

$$V_p = A\sin 2x + B\cos 2x$$

$$DV_p = 2A\cos 2x - 2B\sin 2x$$

$$D^2V_p = -4A\sin 2x - 4B\cos 2x$$

put it in eq (3)

$$-4A\sin 2x - 4B\cos 2x + A\sin 2x + B\cos 2x = 2\sin 2x - 3\cos 2x$$

$$-3A\sin 2x - 3B\cos 2x = 2\sin 2x - 3\cos 2x$$

$$-3A = 2 \Rightarrow A = -2/3$$

$$-3B = -3 \Rightarrow B = 1$$

$$V_p = -\frac{2}{3}\sin 2x + \cos 2x.$$

$$V = V_c + V_p$$

$$V = c_1 e^{it} + c_2 e^{-it} + \left(-\frac{2}{3}\right)\sin 2x + \cos 2x$$

Reconsider eq (1) & (2) ×ing

$-(D+1)$ by eq (1) & $(D+3)$ by eq (2)

$$-(D+1)(D+3)V - 5(D+1)S = -(D+1)\sin 2x$$

$$(D+2)(D+3)S + (D+3)(D+1)V = (D+3)\cos 2x$$

$$(D+2)(D+3)S - 5(D+1)S = -D\sin 2x - \sin 2x + D\cos 2x + 3\cos 2x$$

$(D^2+2D+3D+6)S-5DS-5S=-2\cos 2x-\sin 2x$ Q#9 (3)

$\div 2\sin 2x+3\cos 2x$

$(D^2S+5DS+6S)-5DS-5S=\cos 2x-3\sin 2x$

$D^2S+S=\cos 2x-3\sin 2x \quad -(4)$

corresponding homogeneous eq is

$D^2S+S=0$

$m^2+1=0 \quad , \quad m^2=-1 \quad , \quad m=\pm i$

$S_c=c_3\cos x+c_4\sin x$

Now let perticular sol as

$S_p=C\cos 2x+D\sin 2x$

$DS_p=-2C\sin 2x+2D\cos 2x$

$D^2S_p=-4C\cos 2x-4D\sin 2x$

put in eq (4)

$-4C\cos 2x-4D\sin 2x+C\cos 2x+D\sin 2x=\cos 2x-3\sin 2x$

$-3C\cos 2x-3D\sin 2x=\cos 2x-3\sin 2x$

$C=-1/3 \quad , \quad D=1$

$S_p=-\frac{1}{3}\cos 2x+\sin 2x$

$S=S_c+S_p=C_3\cos x+C_4\sin x-\frac{1}{3}\cos 2x+\sin 2x$

Now put the values of Q#9 ④

S and v in eq ①

$$(D+3)\,c_1 e^{it} + c_2 e^{-it} - \frac{2}{3}\sin 2x + \cos 2x + 5 c_3 \cos x$$
$$+ 5c_4 \sin x + \frac{5}{3}\cos 2x + 5\sin 2x = \sin 2x$$

$$c_1 i e^{it} + 3c_1 e^{it} + c_2 e^{-it} + \left(-\frac{2}{3} + 5\right)\sin 2x$$
$$+ \left(1 + \frac{5}{3}\right)\cos 2x + 5c_3 \cos x + 5c_4 \sin x = \sin x$$

$$c_1 i(\cos x + \sin x) + 3c_1(\cos x + \sin x) + c_2(\cos x - \sin x)$$
$$+ \frac{13}{3}\sin 2x + \frac{8}{3}\cos 2x + 5c_3 \cos 2x + 5c_4 \sin x = \sin x$$