National University of Computer and Emerging Sciences

FAST School of Computing **Spring-2022** **Islamabad Campus**

MT-2005: Probability and Statistics

Serial No:

Sessional Exam-II

Total Time: 1 Hour

Total Marks: 40

Wednesday, 11th May, 2022

Course Instructors

Dr. Shahzad Saleem

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Signature of Invigilator

Student Name Roll No. Section Signature

DO NOT OPEN THE QUESTION BOOK OR START UNTIL INSTRUCTED.

Instructions:

1. Attempt on question paper. Attempt all of them. Read the question carefully, understand the question, and then attempt it.
2. No additional sheet will be provided for rough work. Use the back of the last page for rough work.
3. If you need more space, write on the back side of the paper and clearly mark question and part number etc.
4. After asked to commence the exam, please verify that you have **seven (7)** different printed pages including this title page. There is a total of **3** questions.
5. Calculator sharing is strictly prohibited.
6. Use permanent ink pens only. Any part done using soft pencil will not be marked and cannot be claimed for rechecking.

	Q-1	Q-2	Q-3	Total
Marks Obtained				
Total Marks	15	15	10	40

Question 1 [15 Marks]

a) Suppose there are six persons represented by A, B, C, D, E, F. [2+3]

i. In how many ways these six can line up?

Six people can line up in $6! = 720$ ways

ii. Suppose two persons A and B don't want to sit next to each other. Considering AB as a single unit, find the probability of the event X defined as

$$X = \{\text{A and B don't want to sit next to each other}\}.$$

Considering AB as a unit. Now the number of ways in which AB together is 5! & BA is 5! So, the total number of ways in which AB are together is $2 \times 5!$

Let X be the event that a particular pair (i.e., A & B) don't sit next to each other.

$$P[X'] = P[A \,\&\, B \text{ sit next to each other}] = \frac{2 \times 5!}{6!}$$

$$P[X] = 1 - P[X'] = 1 - \frac{2 \times 5!}{6!} = \frac{2}{3}$$

b) Components from three different manufacturers have been procured: 100 from Manufacturer A, of which 10% are defective; 300 from Manufacturer B, of which 5% are defective; and 500 from Manufacturer C, of which 20% are defective. We randomly select the shipment from one of the manufacturers and then randomly pick a component from it.

i. What is the probability that the selected component is defective? [5]

$$\begin{aligned} P(D) &= P(A)P(D/A) + P(B)P(D/B) + P(C)P(D/C) \\ &= \frac{100}{900} \times 0.1 + \frac{300}{900} \times 0.05 + \frac{500}{900} \times 0.2 = 0.139 \end{aligned}$$

ii. If the selected component is found to be defective, what is the probability that it came from Manufacturer A? [5]

$$P(A/D) = \frac{P(A)P(D/A)}{P(D)} = \frac{\frac{1}{9} \times 0.1}{0.139} = 0.08.$$

Question 2 [15 Marks]

a) Write down the properties of cumulative distribution function. [2]

i) $F(+\infty) = 1,\ F(-\infty) = 0.$

ii) It is a non-decreasing function of x, i.e., $x_1 < x_2$, then $F(x_1) \leq F(x_2)$

iii) It is discontinuous for a discrete random variable and continuous for a continuous random variable.

iv) $P(x_1 < X \leq x_2) = F(x_2) - F(x_1)$

b) The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-8x}, & x \geq 0 \end{cases}$$

Find the probability of waiting less than 12 minutes between successive speeders

i. using the cumulative distribution function of *X*. [4]

ii. using the probability density function of *X*. [4]

iii. Verify that area under the curve is unity. [5]

i) $P\left(X < \frac{12}{60} = 0.2\right) = F(0.2) = 1 - e^{-1.6} = 0.7981$

ii) $f(x) = \frac{dF(x)}{dx} = 8e^{-8x}.$

Therefore, $P(X < 0.2) = 8\int_0^{0.2} e^{-8x}dx = -e^{-8x}|_0^{0.2} = 0.7981.$

iii) $\int_0^{\infty} f(x)dx = \int_0^{\infty} 8e^{-8x}dx = -e^{-8x}\Big|_0^{\infty} = -[e^{-\infty} - e^0] = 1$

Hence, area under the curve is unity.

Question 3 [10 Marks]

In a set of independent trials, a certain discrete random variable K takes on the values:

$$5, 7, 2, 8, 2, 7, 2, 7, 7, 8, 7, 2, 7, 2, 5, 7.$$

a) Find the probability mass function (PMF) of K. [4]

k	2	5	7	8
$P(K = k)$	$\frac{5}{16}$	$\frac{2}{16}$	$\frac{7}{16}$	$\frac{2}{16}$

b) Find the cumulative distribution function of K. [3]

$$F(2) = P(K \leq 2) = f(2) = \frac{5}{16}$$

$$F(5) = P(K \leq 5) = f(2) + f(5) = \frac{5}{16} + \frac{2}{16} = \frac{7}{16}$$

$$F(7) = P(K \leq 7) = f(2) + f(5) + f(7) = \frac{5}{16} + \frac{2}{16} + \frac{7}{16} = \frac{7}{8}$$

$$F(8) = P(K \leq 8) = f(2) + f(5) + f(7) + f(8) = \frac{5}{16} + \frac{2}{16} + \frac{7}{16} + \frac{2}{16} = 1$$

$$F(k) = \begin{cases} 0, & k < 2 \\ \frac{5}{16}, & 2 \leq k < 5 \\ \frac{7}{16}, & 5 \leq k < 7 \\ \frac{7}{8}, & 7 \leq k < 8 \\ 1, & k \geq 8 \end{cases}$$

c) Using the PMF obtained in (a), determine the mean of K. [3]

$$\mu = E[K] = \sum_k k\, f(k) = 2 \times \frac{5}{16} + 5 \times \frac{1}{8} + 7 \times \frac{7}{16} + 8 \times \frac{1}{8} = 5.3125.$$