CS480 – Introduction to Artificial Intelligence

Topic: Complex Decisions
Chapter: 17

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Comparison with ch16

- Similar
  - The outcomes of each action are stochastic
  - The agent is trying to maximize expected utility
- Different
  - Instead of an episodic decision (chapter 16), the agent's utility depends on a sequence of actions (chapter 17)

We

- Will cover
  - 17.1 – Intro to sequential decision making
  - 17.2 – Value iteration
  - 17.3 – Policy iteration
- Will not cover
  - 17.4 – Partially-observable environments
  - 17.5 – Multiple agents
  - 17.6 – Mechanism design

EXAMPLE

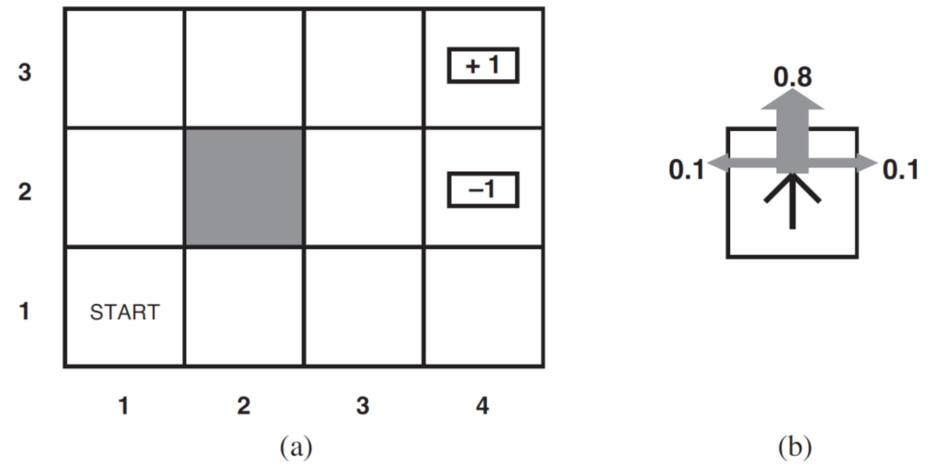


Figure 17.1 FILES: figures/sequential-decision-world.eps (Tue Nov 3 16:23:43 2009). (a) A simple 4×3 environment that presents the agent with a sequential decision problem. (b) Illustration of the transition model of the environment: the "intended" outcome occurs with probability 0.8, but with probability 0.2 the agent moves at right angles to the intended direction. A collision with a wall results in no movement. The two terminal states have reward +1 and –1, respectively, and all other states have a reward of –0.04.

Representation

- **Fully observable**: the agent knows where it is
- **Stochastic**: the agent's actions have probabilistic outcomes
  - $P(s'|\, s, a)$ Probability of arriving at state s' given we are at state s and take action a
- Transition model is **Markovian**
  - $P(s'|\, s, a)$ depends on only a and s and not the entire history of actions and states
- **Sequential**: the utility depends on the sequence of states
- Utility is **additive**: the utility is the sum (with a potential discounting factor) of the reward, R(s), received at each state s that the agent visits

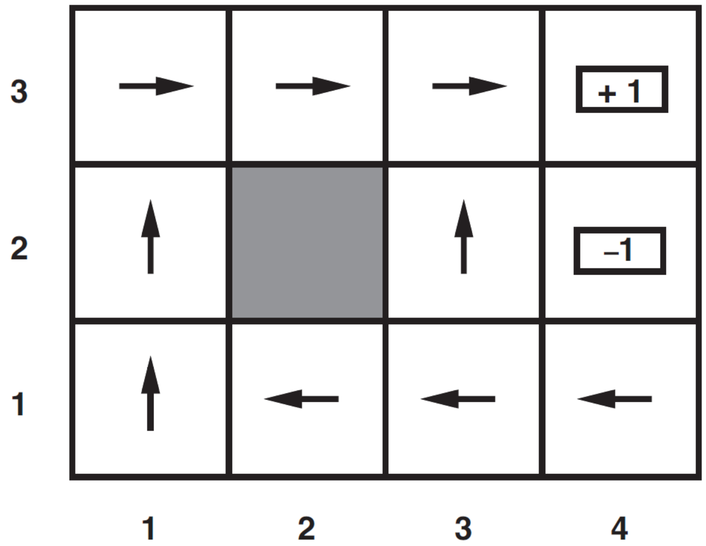
Markov Decision Process

- “*A sequential decision problem for a fully-observable stochastic environment with a Markov transition model and additive rewards is called a **Markov Decision Process (MDP)***” – Textbook page 647

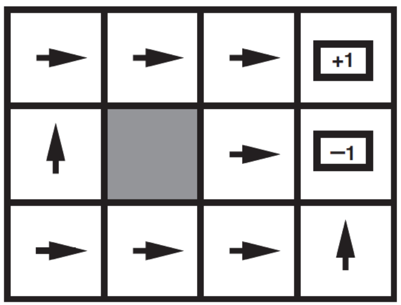
SOLUTION?

- A fixed action sequence is not the answer due to stochasticity
  - For example, [Up, Up, Right, Right, Right] is not a solution
  - It would be a solution if the environment was deterministic
- A solution must specify the agent should do in any state that the agent might reach
  - This is called a **policy**
- Policy notation: $\boldsymbol{\pi}$
  - $\pi(s)$ specifies what action the agent should take at state s
- An **optimal policy** is the one that maximizes the expected utility
  - $\boldsymbol{\pi^*}$

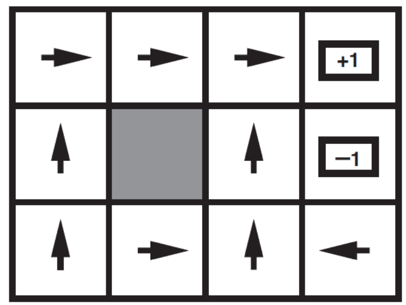
Solution to Earlier Example



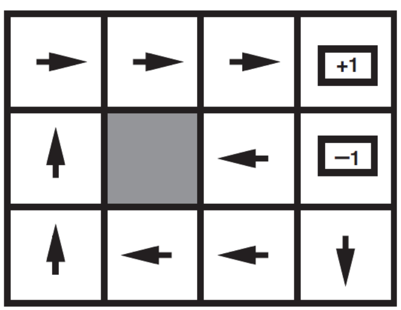
Solutions for different R(s)



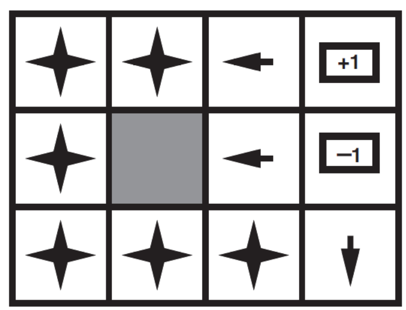
$R(s) < -1.6284$



$-0.4278 < R(s) < -0.0850$



$-0.0221 < R(s) < 0$



$R(s) > 0$

Utilities over Time

- How should we calculate $U([s_0, s_1, s_2, \dots])$?
- Additive rewards
  - $U([s_0, s_1, s_2, \dots]) = R(s_0) + R(s_1) + R(s_2) + \cdots$
- Discounted rewards
  - $U([s_0, s_1, s_2, \dots]) = R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) + \cdots$
  - The discounting factor γ is a number between 0 and 1
  - The agent prefers current rewards to future rewards
  - When γ is close to 0, distant future is insignificant
  - When $\gamma = 1$, it is equivalent to additive rewards

Utility of States

- The agent receives a reward at each state
- Utility of a state s given a policy π is the expected reward that the agent will get starting from state s and following policy π
- Let S_t denote the state that the agent reaches at time t
- $U^{\pi}(s) = E[\sum_{t=0}^{\infty} \gamma^t R(S_t)]$
- The expectation is with respect to the transition probabilities

The optimal Policy

- The optimal policy is the one that maximizes the expected utility
  - $\pi_s^* = \underset{\pi}{\operatorname{argmax}}\, U^{\pi}(s)$
- Remember that π_s^* is a policy; that is, it recommends an action for each state, regardless of whether it is the starting state or not
- It is optimal when the starting state is s
- When the rewards are discounted, the optimal policy is independent of the start state
  - Wait, what?
- The optimality of the policy does not depend on the starting state but of course the action sequence depends on the starting state
- True utility of each state is defined as $U^{\pi^*}(s)$ -- the expected rewards the agent will receive if it executes the optimal policy starting at s

U(S) VS R(S)

- $R(s)$ is the short-term immediate reward at s
- $U(s)$ is the long-term cumulative reward from s and onward

U(S) FOR THE EXAMPLE

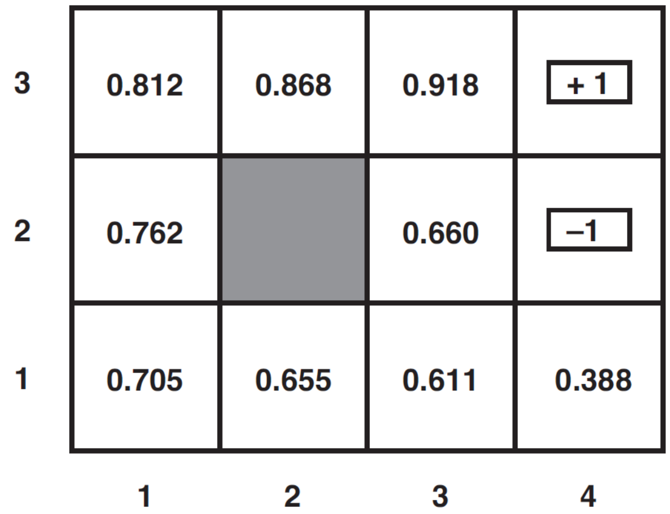


Figure 17.3 FILES: figures/sequential-decision-values.eps (Tue Nov 3 16:23:42 2009). The utilities of the states in the 4×3 world, calculated with $\gamma = 1$ and $R(s) = -0.04$ for nonterminal states.

If We Were Given U(s)

- $\pi^*(s) = \underset{a \in A(s)}{\operatorname{argmax}} \sum_{s'} P(s' \mid s, a) U(s')$
- However, we are not given $U(s)$
- Two algorithms for finding optimal policies

1. Value iteration
2. Policy iteration

Value Iteration

- $U(s) = R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' | s, a) U(s')$
- This is called the **Bellman equation**
- Exercise: write the Bellman equation for U(1,1) for our example
- n possible states, n Bellman equations, one for each state
- However, these are non-linear equations, due to the max operator
- One approach: iterative
  - Start with an initial guess (could be random)
  - Iterate until convergence

VALUE ITERATION FUNCTION

function VALUE-ITERATION(mdp, ϵ) **returns** a utility function
inputs: mdp, an MDP with states S, actions $A(s)$, transition model $P(s' \mid s, a)$,
rewards $R(s)$, discount γ
ϵ, the maximum error allowed in the utility of any state
local variables: U, U', vectors of utilities for states in S, initially zero
δ, the maximum change in the utility of any state in an iteration

repeat
$U \leftarrow U'$; $\delta \leftarrow 0$
for each state s **in** S **do**
$$U'[s] \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' \mid s, a)\, U[s']$$
if $|U'[s] - U[s]| > \delta$ **then** $\delta \leftarrow |U'[s] - U[s]|$
until $\delta < \epsilon(1-\gamma)/\gamma$
return U

Optimal Policy

- Once the value iteration algorithm terminates, use $\pi^*(s) = \underset{a \in A(s)}{\operatorname{argmax}} \sum_{s'} P(s' \mid s, a) U(s')$ to determine the optimal policy, i.e., the optimal action for each s

SKIPPED DETAILS

- Is the value iteration algorithm guaranteed to converge to the true values, and under what conditions
- How long does it take to converge?
- Do we need the algorithm fully converge to find the optimal policy or is an approximate computation of the utilities often enough?

Policy Iteration

- Start with an initial policy π_0
- Alternate between

1. Policy iteration: given policy π_i, calculate U^{π_i}
2. Policy improvement: Calculate a new MEU policy π_{i+1}, using the utilities calculated in the previous step

- Stop when utilities no longer change

Summary – Important Equations

- $U([s_0, s_1, s_2, \dots]) = R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) + \cdots$
  - Discounted rewards

- $U(s) = R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' \mid s, a) U(s')$
  - Bellman equation; utility under optimal policy

- $\pi^*(s) = \underset{a \in A(s)}{\operatorname{argmax}} \sum_{s'} P(s' \mid s, a) U(s')$
  - Optimal policy when utilities are given

- $U^{\pi_i}(s) = R(s) + \gamma \sum_{s'} P(s' \mid s, \pi_i(s)) U^{\pi_i}(s')$
  - Utility under fixed policy π_i

EXERCISE

- Define a simple MDP
- Solve it manually
- Solve it using Python
  - See examples at https://github.com/aimacode/aima-python/blob/master/mdp.ipynb

NEXT

- Part V – Learning