

CS480 – INTRODUCTION TO ARTIFICIAL INTELLIGENCE

TOPIC: UNCERTAINTY
CHAPTER: 13



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MEDICAL DIAGNOSIS - TOOTHACHE

- Toothache \Rightarrow Cavity
 - What's wrong?
 - Not all toothaches are due to cavity
- Toothache \Rightarrow Cavity \vee GumProblem \vee Abscess \vee ...
 - What's wrong?
 - We have to add almost an unlimited number of possible problems
- Cavity \Rightarrow Toothache
 - What's wrong?
 - Not all cavities cause toothache

FOL

○ Fails for three main reasons

1. Laziness

- Too much work to list all premises and conclusions
- Too hard to use such rules

2. Theoretical ignorance

- No complete theory for the domain

3. Practical ignorance

- Even if we knew all the rules, some information might be missing; e.g., lab tests for diagnosis

SOME SOURCES OF UNCERTAINTY

○ **Uncertainty in knowledge**

- E.g., We do not know all the causes of all the diseases

○ **Uncertainty in actions;** we cannot list all the pre-conditions of actions

- E.g., To be able to fly a plane from SFO to JFK, it must not be broken, the weather conditions have to be appropriate, the pilot must not be sick, you need to have enough fuel, ...

○ **Uncertainty in sensors**

- E.g., lightning conditions for a camera might not be enough

TASKS

1. Representation

- What is the formal and appropriate language to represent uncertainty?

2. Inference

- How can we infer uncertainty before or after we gather more information?

3. Decision making

- How can a rational agent act in an uncertain world?

PROBABILITY MODEL

- It's all about the state the world is in, i.e., a possible world
- A possible world is an assignment of truth values to the predicates
 - Logical assertions rule out some of the possible worlds
 - E.g., $\text{cavity} \Rightarrow \text{toothache}$ rules out the worlds where $\text{cavity}=\text{true} \wedge \text{toothache}=\text{false}$
 - Probabilistic reasoning determines how probable the various worlds are
 - E.g., a world where $\text{cavity}=\text{true} \wedge \text{toothache}=\text{true}$ is more probable than the world where $\text{cavity}=\text{true} \wedge \text{toothache}=\text{false}$
- The set of all possible worlds is called the sample space
- The possible worlds are *mutually exclusive* and *exhaustive*
 - *Mutually exclusive*: the world can be only in one state
 - *Exhaustive*: the world has to be in one of the states

PROBABILITY MODEL

- A **probability model** associates a numerical probability $P(w)$ with each possible world w
 - $P(w)$ sums to 1 over all possible worlds
- An **event** is the set of possible worlds where a given predicate is true
 - Roll two dice; the possible worlds are (1,1), (1,2), ..., (6,6); 36 possible worlds
 - Predicate = two dice sum to 10
 - Event = {(4,6), (5,5), (6,4)}

AXIOMS OF PROBABILITY

1. The probability $P(a)$ of a proposition a is a real number between 0 and 1
2. $P(\text{true}) = 1$, $P(\text{false}) = 0$
3. $P(a \vee b) = P(a) + P(b) - P(a \wedge b)$

$P(\neg a)$

- $P(a \vee \neg a) = P(a) + P(\neg a) - P(a \wedge \neg a)$
- $P(\text{true}) = P(a) + P(\neg a) - P(\text{false})$
- $1 = P(a) + P(\neg a) - 0$
- $P(\neg a) = 1 - P(a)$
- Intuitive explanation:
 - The probability of all possible worlds is 1
 - Either a or $\neg a$ holds in one world
 - The worlds that a holds and the worlds that $\neg a$ holds are mutually exclusive and exhaustive

RANDOM VARIABLE

- Like CSP
 - A factored representation of the world; random variables
 - Each variable has a domain
 - Probabilities over the domain values of a variable sum to 1
 - The possible worlds where a random variable takes a certain value are mutually exclusive and exhaustive (from the viewpoint of that variable)
- E.g.
 - $D_1: \{1, 2, 3, 4, 5, 6\}$

JOINT DISTRIBUTION

- We have n random variables, V_1, V_2, \dots, V_n
- We are interested in the probability of a possible world, where
 - $V_1=v_1, V_2=v_2, \dots, V_n = v_n$
- $P(V_1, V_2, \dots, V_n)$ associates a probability for each possible world \equiv the **joint distribution**
 - How many entries are there, if we assume the variables are all binary?
 - How is this related to the truth tables in logic?

2^n

TOOTHACHE EXAMPLE

Toothache	Cavity	P(T,C)
toothache	cavity	0.15
toothache	\neg cavity	0.10
\neg toothache	cavity	0.05
\neg toothache	\neg cavity	0.70

These probabilities are different from what is given in the textbook

NOTATION

- An upper-case letter A_1 represents a variable and its all possible values
- A lower-case letter a_1 represents a particular value of the variable A_1
- $P(A_1)$ represents a table/function specifying a probability for each possible value of A_1
- $P(A_1=a_1)$ represents a scalar value specifying the probability of $A_1=a_1$
- We often abbreviate $P(A_1=a_1)$ as $P(a_1)$

PRIOR AND POSTERIOR

- Prior probability
 - Probability of a proposition in the absence of any other information
 - E.g., $P(V_1, V_3, V_5)$
- Conditional/posterior probability
 - Probability of a proposition given another piece of information
 - E.g., $P(V_2, V_3 \mid V_5 = T, V_7 = F)$
 - $P(A \mid B) = P(A \wedge B) / P(B)$

NUMBER OF PARAMETERS

- Assuming everything is binary
- $P(V_1)$ requires
 - 1 independent parameter
- $P(V_1, V_2, \dots, V_n)$ requires
 - $2^n - 1$ independent parameters
- $P(V_1 | V_2)$ requires
 - 2 independent parameters
- $P(V_1, V_2, \dots, V_n | V_{n+1}, V_{n+2}, \dots, V_{n+m})$ requires
 - $2^m \times (2^n - 1)$ independent parameters

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MARGINALIZATION

- Given $P(V_1, V_2, \dots, V_n \mid \underline{V_{n+1}, V_{n+2}, \dots, V_{n+m}})$, where $n > 0$ and $m \geq 0$, we can find, for example
 - $P(V_i, V_j, V_k \mid V_{n+1}, V_{n+2}, \dots, V_{n+m})$ where $i, j, k < n$ by summing out all the irrelevant variables
- Examples

$$P(A, B, C \mid D)$$
$$P(B \mid D) = \sum_A \sum_C P(A, B, C \mid D)$$

LET'S ANSWER A FEW QUERIES

Toothache	Cavity	P(T,C)
toothache <i>t</i>	cavity <i>t</i>	0.15
toothache <i>t</i>	¬cavity <i>f</i>	0.10
¬toothache <i>f</i>	cavity <i>t</i>	0.05
¬toothache <i>f</i>	¬cavity <i>f</i>	0.70

- $P(\text{cavity}) = ?$ $P(C=t) = 0.15 + 0.05 = 0.2$
- $P(\neg\text{cavity}) = ?$ $P(C=f) = 1 - P(C=t) = 1 - 0.2 = 0.8$
- $P(\text{toothache}) = ?$ $P(T=t) = 0.15 + 0.10 = 0.25$
- $P(\neg\text{toothache}) = ?$ $P(T=f) = 1 - 0.25 = 0.75$

LET'S ANSWER A FEW QUERIES

Toothache	Cavity	P(T,C)
toothache	cavity	0.15
toothache	¬cavity	0.10
¬toothache	cavity	0.05
¬toothache	¬cavity	0.70

- $P(\overset{t}{\text{cavity}} \mid \overset{t}{\text{toothache}}) = ? \rightarrow \frac{P(C=t, T=t)}{P(T=t)} = \frac{0.15}{0.15+0.10} = \frac{0.15}{0.25} = 0.6$
- $P(\overset{f}{\text{cavity}} \mid \overset{f}{\neg\text{toothache}}) = ?$
- $P(\overset{f}{\neg\text{cavity}} \mid \overset{t}{\text{toothache}}) = ?$
- $P(\overset{f}{\neg\text{cavity}} \mid \overset{f}{\neg\text{toothache}}) = ?$
- $P(\text{toothache} \mid \text{cavity}) = ? \rightarrow \frac{0.15}{0.15+0.05} = \frac{0.15}{0.20} = 0.75$
- $P(\neg\text{toothache} \mid \text{cavity}) = ?$
- $P(\text{toothache} \mid \neg\text{cavity}) = ?$
- $P(\neg\text{toothache} \mid \neg\text{cavity}) = ?$

BAYES' RULE

- $P(B | A) = P(A | B) * P(B) / P(A)$
- Example use
 - $P(\text{cause} | \text{effect}) = P(\text{effect} | \text{cause}) * P(\text{cause}) / P(\text{effect})$
- Why is this useful?
 - Because in practice it is easier to get probabilities for $P(\text{effect} | \text{cause})$ and $P(\text{cause})$ than for $P(\text{cause} | \text{effect})$
 - E.g., $P(\text{disease} | \text{symptoms}) = P(\text{symptoms} | \text{disease}) * P(\text{disease}) / P(\text{symptoms})$
 - It is easier to know what symptoms diseases cause. It is harder to diagnose a disease given symptoms

TUBERCULOSIS TEST

○ Tuberculosis test

- The test is 90% accurate
 - If you have TB, the test is positive with 90% probability
 - If you don't have TB, the test is negative with 90% probability
- John takes the test and the result is positive
- What is the probability that John has TB?

○ Formally

- $P(+ | TB) = 0.9$
- $P(- | \neg TB) = 0.9$
- $P(TB | +) = ?$

TASKS

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DECISION MAKING

- Instead of T/F, we have probabilities
- Preferences for certain outcomes (world states)
 - Utility theory
- Maximize expected outcome
 - Decision theory = probability theory + utility theory
- **Maximum Expected Utility** principle
 - An agent is rational if and only if it chooses the action that yields the highest expected utility, averaged over all the possible outcomes of the action