

CS480 – INTRODUCTION TO ARTIFICIAL INTELLIGENCE

TOPIC: MAKING SIMPLE
DECISIONS
CHAPTER: 16



Mustafa Bilgic



<http://www.cs.iit.edu/~mbilgic>



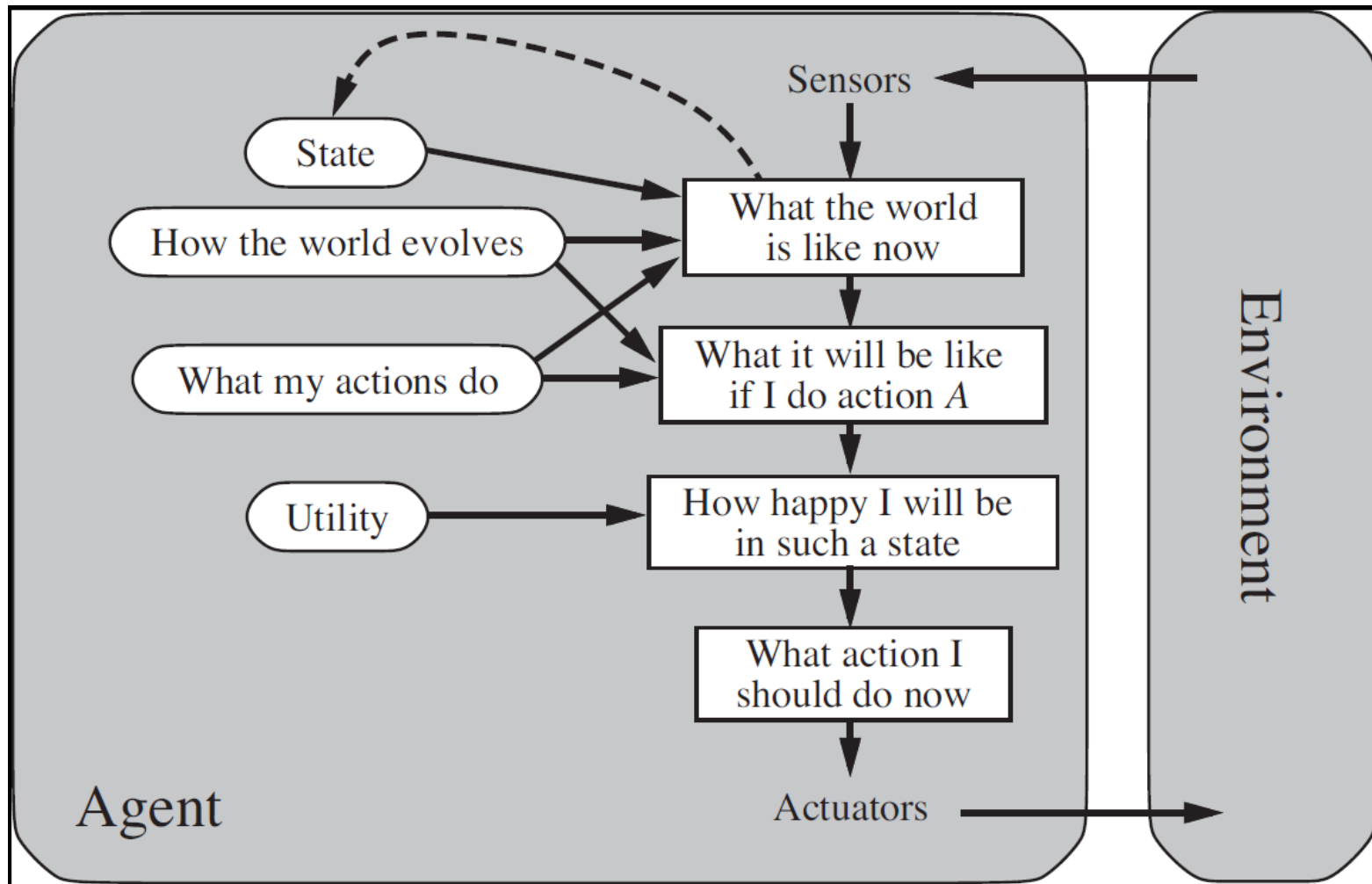
<https://twitter.com/bilgicm>

MOTIVATION

- Goal-based agent of Chapter 3
 - Fully observable & deterministic
- Now – Chapter 16
 - The world might be partially observable
 - The actions might be non-deterministic

We discuss “how an agent should make decisions so that it gets what it wants— on average, at least.”

UTILITY-BASED AGENT



UTILITY

- $P(RESULT(a) = s' \mid a, e)$
 - The probability of ending up in state s' after taking action a , given that we have so far observed e
- The agent's preferences are captured by a utility function $U(s)$
- Expected utility of an action a given evidence e , $EU(a \mid e)$, is the average utility of the possible outcomes weighted by their probabilities

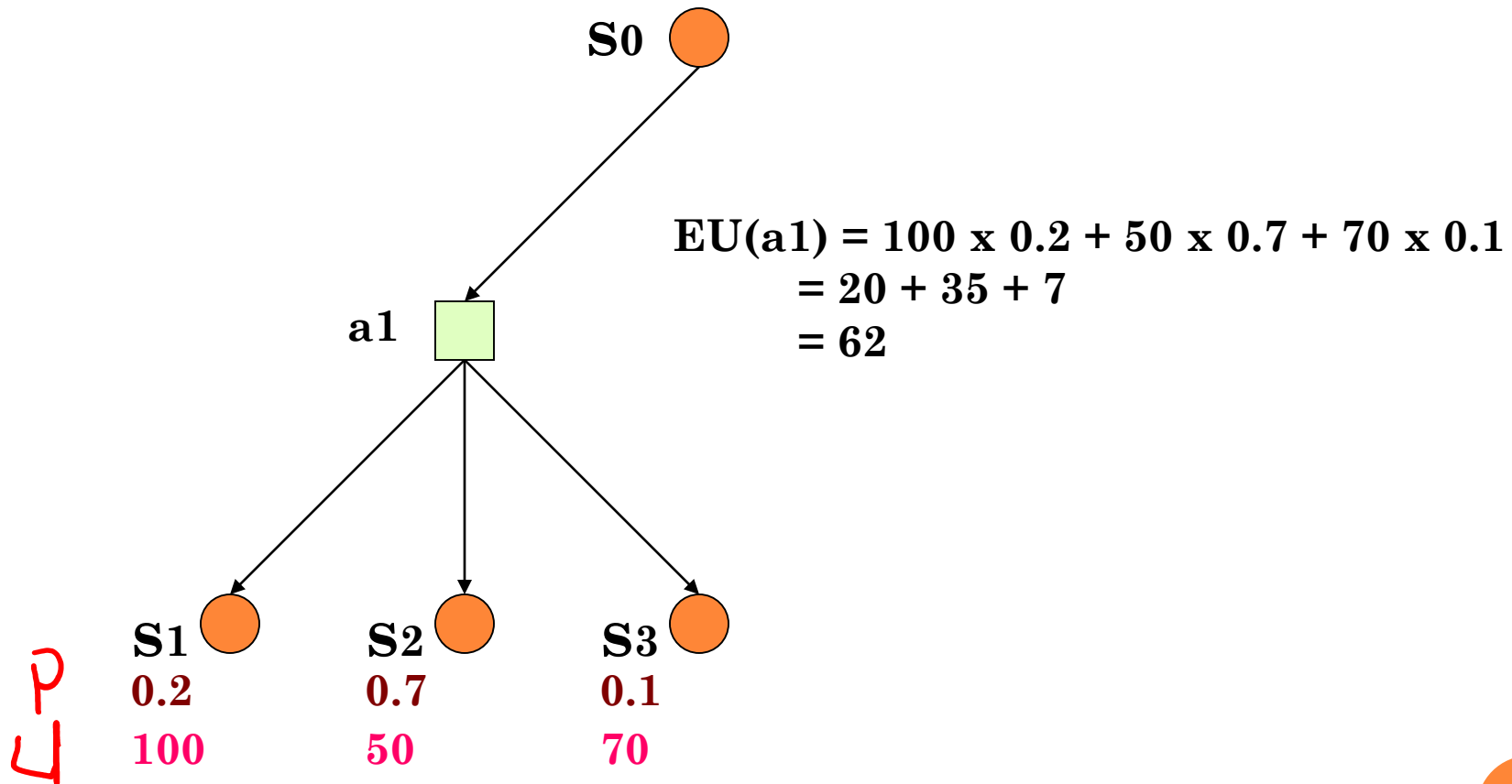
$$EU(a \mid e) = \sum_{s'} P(RESULT(a) = s' \mid a, e) \times U(s')$$

MAXIMUM EXPECTED UTILITY PRINCIPLE (MEU)

- Choose action that maximizes the expected utility

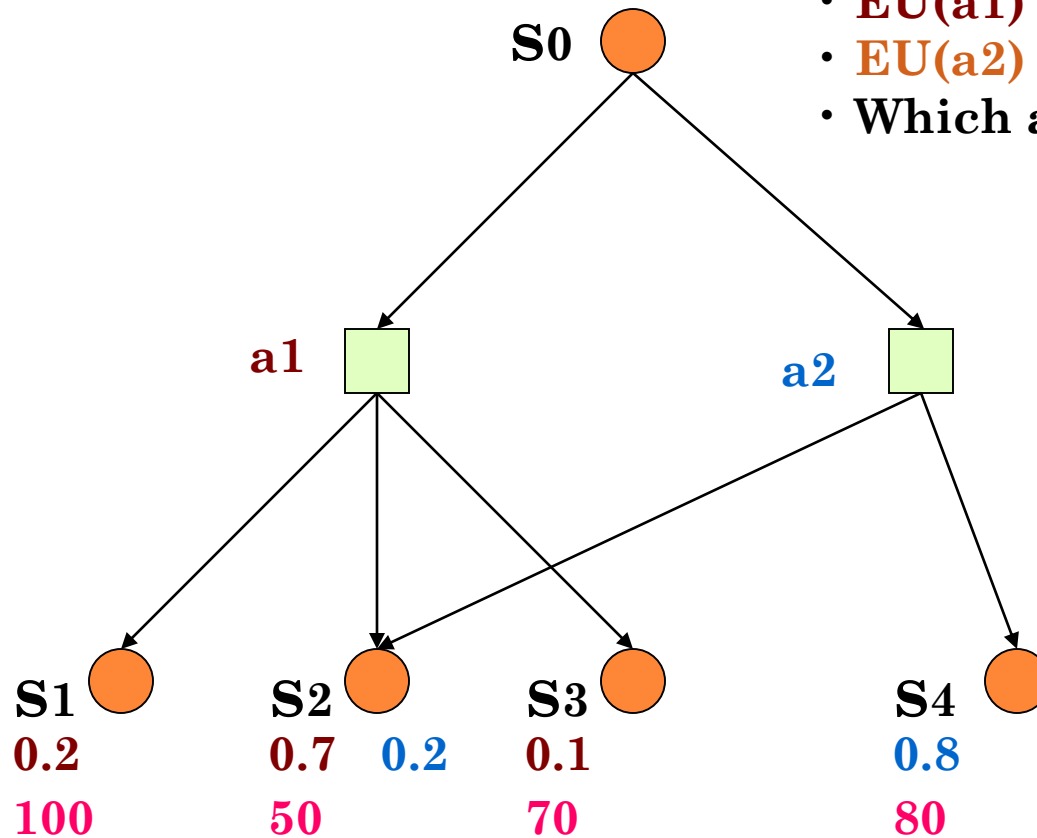
$$action = \arg \max_a EU(a | e)$$

ONE ACTION EXAMPLE



TWO ACTIONS EXAMPLE

- **EU(a1) = 62**
- **EU(a2) = 0.2*50+0.8*80 = 74**
- Which action to take?



UTILITY THEORY – RATIONAL PREFERENCES

○ Notation

- $A > B$: the agent prefers A over B
- $A \sim B$: the agent is indifferent between A and B
- $A \geq B$: the agent prefers A over B or is indifferent between them

○ Lottery: n possible outcomes with probabilities

- $[p_1, S_1; p_2, S_2; \dots p_n, S_n]$
- Each S_i can be an atomic state or another lottery

AXIOMS OF UTILITY THEORY

1. Orderability

- $A > B$, $B > A$, or $A \sim B$

2. Transitivity

- $A > B$ and $B > C \Rightarrow A > C$

3. Continuity

- $A > B > C \Rightarrow \exists p [p, A; (1-p), C] \sim B$

AXIOMS OF UTILITY THEORY

4. Substitutability

- $A \sim B \Rightarrow [p, A; (1-p), C] \sim [p, B; (1-p), C]$
- $A > B \Rightarrow [p, A; (1-p), C] > [p, B; (1-p), C]$

5. Monotonicity

- $A > B \Rightarrow (p > q \Leftrightarrow [p, A; (1-p), B] > [q, A; (1-q), B])$

6. Decomposability

- $[p, A; (1-p), [q, B; (1-q), C]] \sim [p, A; (1-p)q, B; (1-p)(1-q), C]$

PREFERENCES LEAD TO UTILITY

○ Existence of utility

- If an agent's preferences obey the axioms of utility, then there exists a function such that
 - $U(A) > U(B) \Leftrightarrow A > B$, and
 - $U(A) = U(B) \Leftrightarrow A \sim B$.

○ Expected utility of a lottery

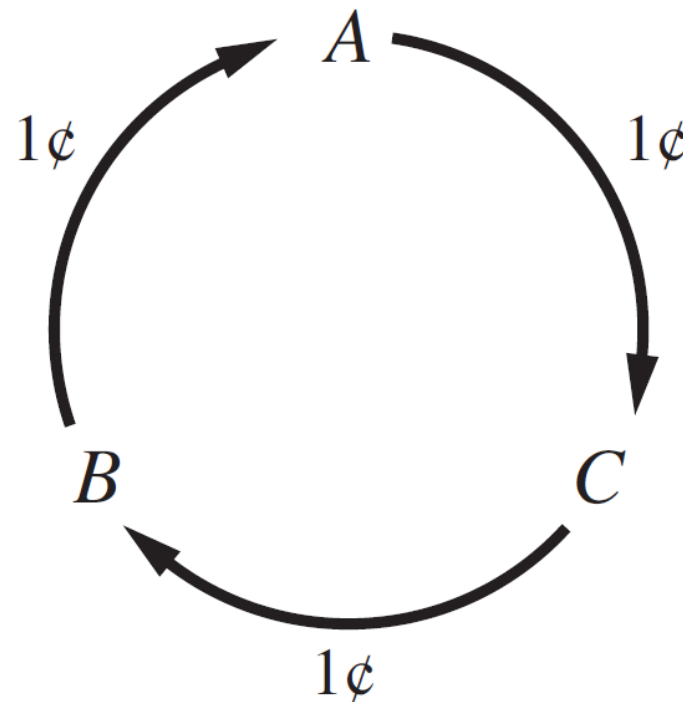
- $U([p_1, S_1; p_2, S_2; \dots p_n, S_n]) = p_1 U(S_1) + p_2 U(S_2) + \dots + p_n U(S_n)$

RATIONALITY

- If an agent's preferences do not obey the axioms of utility theory, then that agent can be made to behave irrationally
- For e.g., if an agent's preferences do not obey transitivity for three or more products, then the agent can be tricked to pay money in a cyclic manner indefinitely (or till the agent runs out of money)

EXAMPLE: VIOLATING TRANSITIVITY

- $A > B$
- $B > C$
- Transitivity requires $A > C$, but instead assume the agent prefers C over A , i.e., $C > A$
- Then the agent can be stripped of all of its money through cyclic transactions



RATIONALITY

- An agent is rational if its preferences obey the axioms of utility theory, not matter how odd its preferences are
- An agent might have completely different preferences from another agent and both can still be rational, if and only if, their individual preferences obey the axioms of utility theory

UTILITY \neq MONEY

- Most agents prefer more money to less money,
 - Thus it obeys the monotonicity constraint,
 - But this does not mean money behaves as a utility function
- For example, which lottery would you prefer
 - L_1 : [1, \$1 Million]
 - L_2 : [0.5, \$0; 0.5, \$2.5 Million]
- If money served as a utility function, then you'd prefer L_2 no matter what, but the answer *often* depends on how much money you currently have
 - The utility of money depends on what you prefer
 - If you are short on cash, a little more *certain* money can help
 - If you are already billionaire, you might take the risk
 - Or if you are swimming in debt, you might like to gamble

UTILITY \neq MONEY

- Let's say you currently have \$k and let S_k represent the state of having \$k
- $EU(L_1) = U(S_{k+1M})$
- $EU(L_2) = 0.5 * U(S_k) + 0.5 * U(S_{k+2.5M})$
- The rational choice depends on your preferences for S_k , S_{k+1M} , and $S_{k+2.5M}$
 - i.e, it depends on the values of $U(S_k)$, $U(S_{k+1M})$, and $U(S_{k+2.5M})$
- $U(S_i)$ does not have to be a linear function of i, and for people it often is not
 - However, $U(.)$ has to obey the six axioms

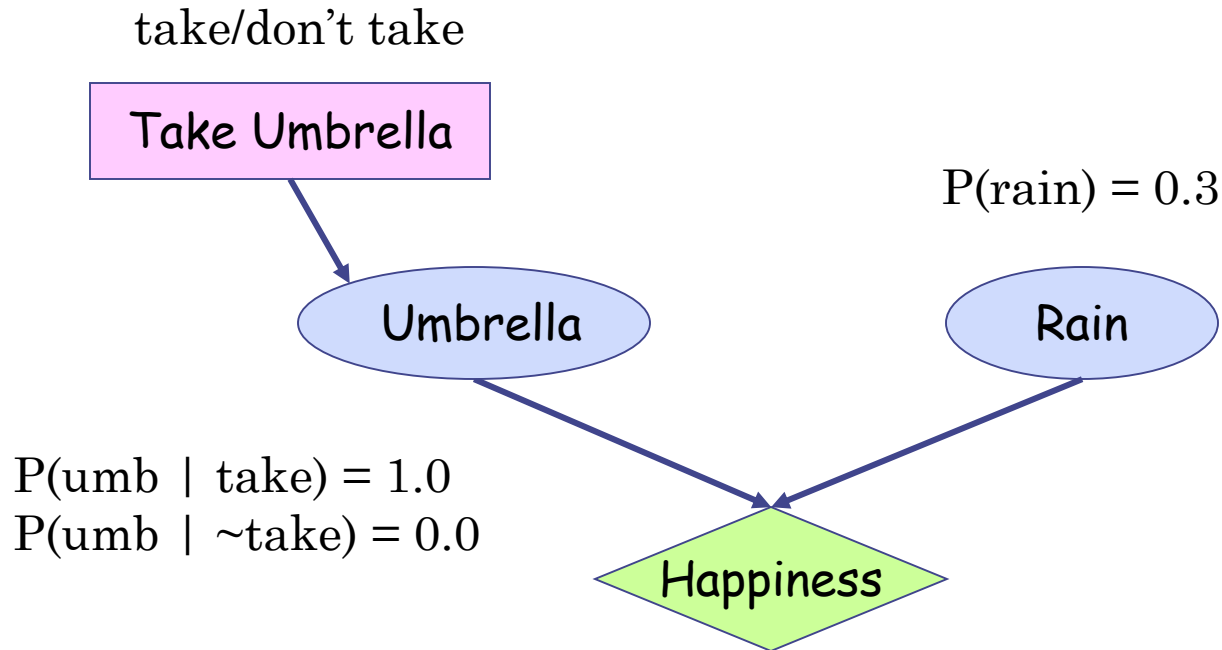
HUMAN JUDGMENT

- Read 16.3.2 – 16.3.4

DECISION NETWORKS

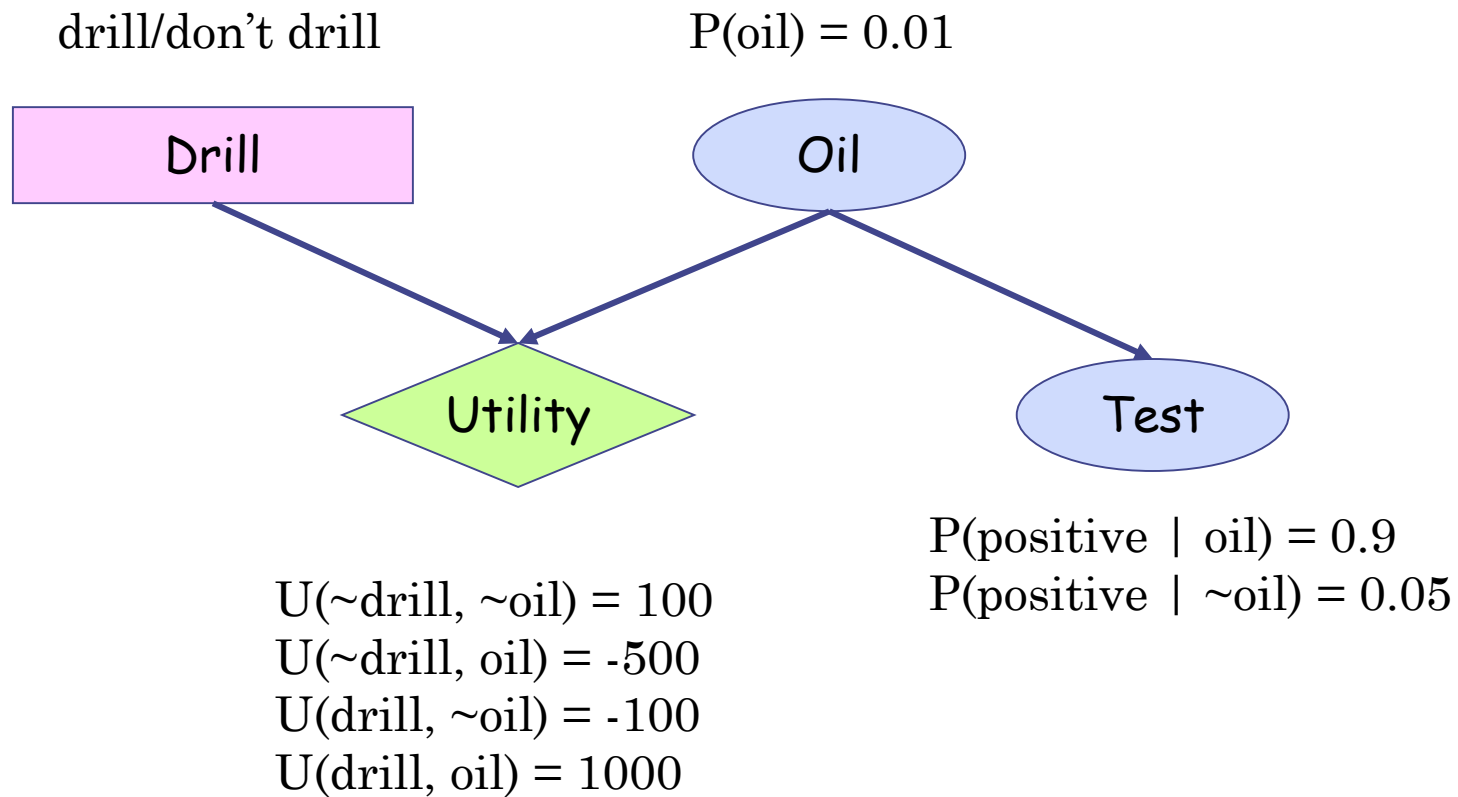
- Builds on Bayesian networks
- In addition to the chance nodes (ovals), decision networks have
 - Decision nodes – square
 - Represents actions
 - Utility nodes – diamond
 - Represents utilities for possible states and actions

UMBRELLA EXAMPLE



$$\begin{aligned}U(\sim\text{umb}, \sim\text{rain}) &= 100 \\U(\sim\text{umb}, \text{rain}) &= 0 \\U(\text{umb}, \sim\text{rain}) &= 20 \\U(\text{umb}, \text{rain}) &= 70\end{aligned}$$

DRILL



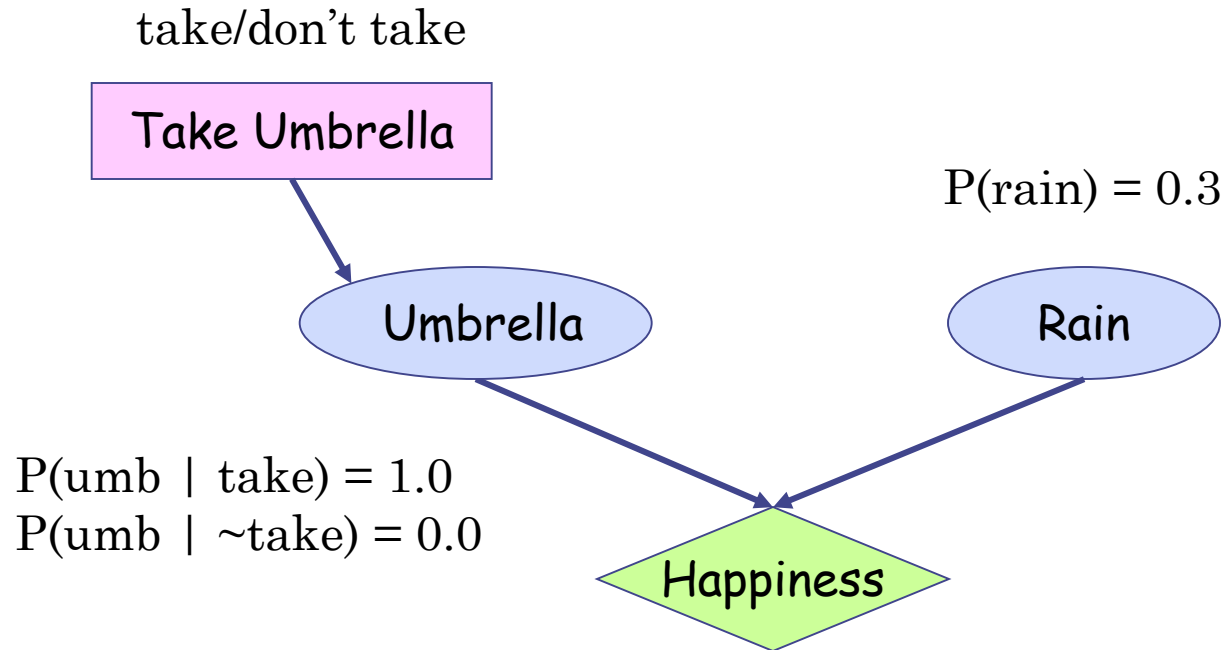
DECISION NETWORKS - APPLICATIONS

- Used for
 - What action to take
 - What information to gather
 - How much to pay for a piece of information
- For example:
 - Medical diagnosis: which test to perform, which treatment to prescribe, ...
 - Marketing: which project to invest in, how much to spend on marketing, how much to spend on user surveys, ...

EVALUATING DECISION NETWORKS

- Set evidence nodes **E** to their values **e**
- For each choice **a** of action **A**
 - Set **A=a**
 - Compute the posterior probability of the parent chance nodes of the utility node; i.e., compute $P(\text{Pa}(\text{Utility}) \mid \mathbf{e}, \mathbf{a})$
 - Compute expected utility using the utility node and the probability distribution $P(\text{Pa}(\text{Utility}) \mid \mathbf{e}, \mathbf{a})$
- Choose action **a** with the maximum expected utility

UMBRELLA EXAMPLE

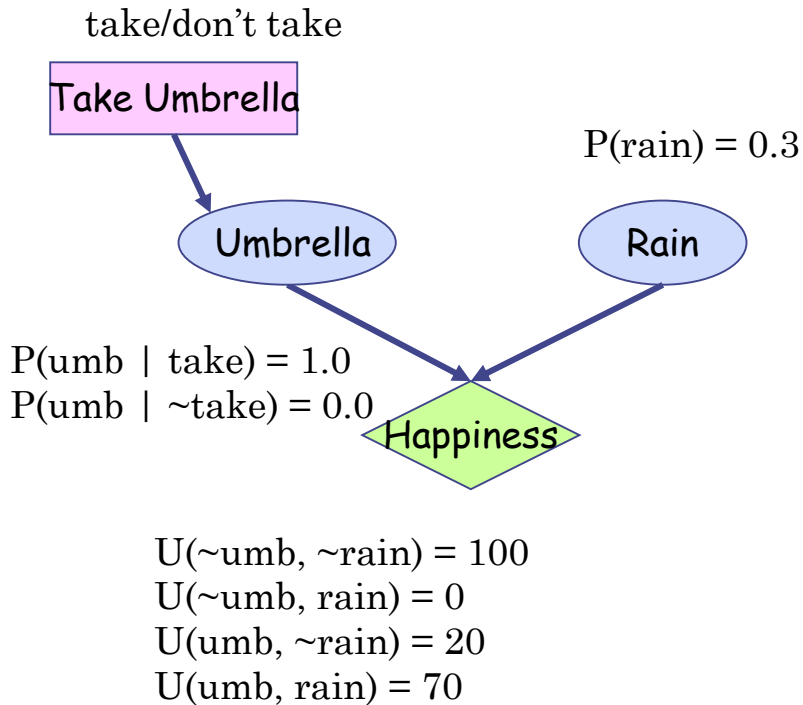


$$\begin{aligned}U(\sim\text{umb}, \sim\text{rain}) &= 100 \\U(\sim\text{umb}, \text{rain}) &= 0 \\U(\text{umb}, \sim\text{rain}) &= 20 \\U(\text{umb}, \text{rain}) &= 70\end{aligned}$$

UMBRELLA EXAMPLE

- Take Umbrella = take
 - Compute $P(\text{Umbrella}, \text{Rain} \mid \text{take})$
 - Compute expected utility
- Take umbrella = $\sim\text{take}$
 - Compute $P(\text{Umbrella}, \text{Rain} \mid \sim\text{take})$
 - Compute expected utility
- MEU principle: choose the action with the highest expected utility

UMBRELLA EXAMPLE



Take Umbrella = take

Umb	Rain	$P(\text{Umb}, \text{Rain} \mid \text{take})$
$\sim\text{umb}$	$\sim\text{rain}$	$0 \times 0.7 = 0$
$\sim\text{umb}$	rain	$0 \times 0.3 = 0$
umb	$\sim\text{rain}$	$1 \times 0.7 = 0.7$
umb	rain	$1 \times 0.3 = 0.3$

Expected Utility = $0 \times 100 + 0 \times 0 + 0.7 \times 20 + 0.3 \times 70 = 35$

Take Umbrella = $\sim\text{take}$

Umb	Rain	$P(\text{Umb}, \text{Rain} \mid \sim\text{take})$
$\sim\text{umb}$	$\sim\text{rain}$	$1 \times 0.7 = 0.7$
$\sim\text{umb}$	rain	$1 \times 0.3 = 0.3$
umb	$\sim\text{rain}$	$0 \times 0.7 = 0$
umb	rain	$0 \times 0.3 = 0$

Expected Utility = $0.7 \times 100 + 0.3 \times 0 + 0.0 \times 20 + 0.0 \times 70 = 70$

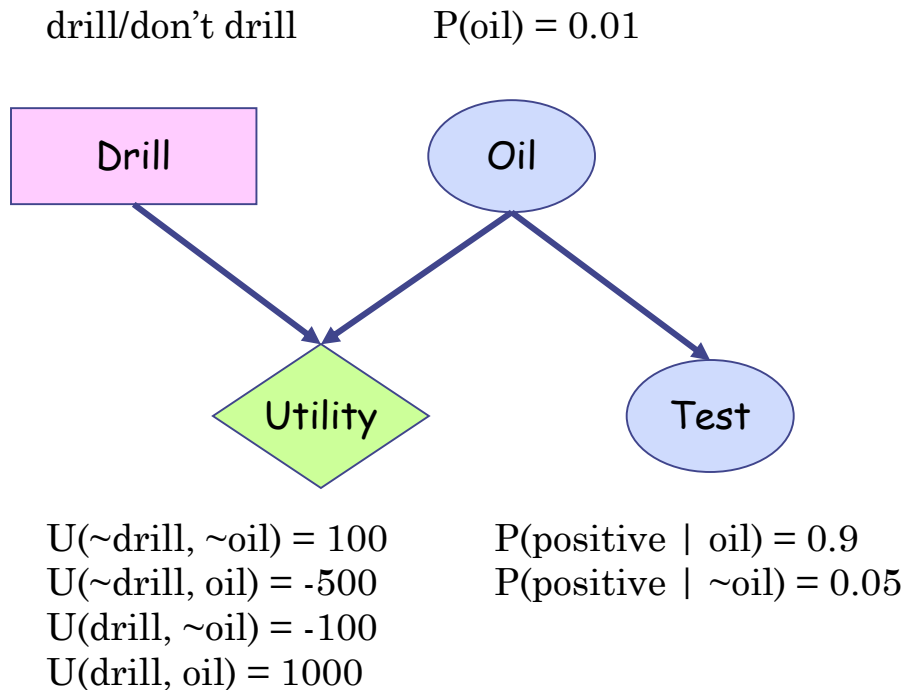
MEU Principle: Don't take it.

VALUE OF INFORMATION

- If I am allowed to observe the value of a chance node, how much valuable is that information to me?
- Value of information
 - Expected utility after the information is acquired
 - Minus
 - Expected utility before the information is acquired
- There is one catch: we do not know the content of the information before we acquire it
 - Solution: take an expectation over the possible outcomes

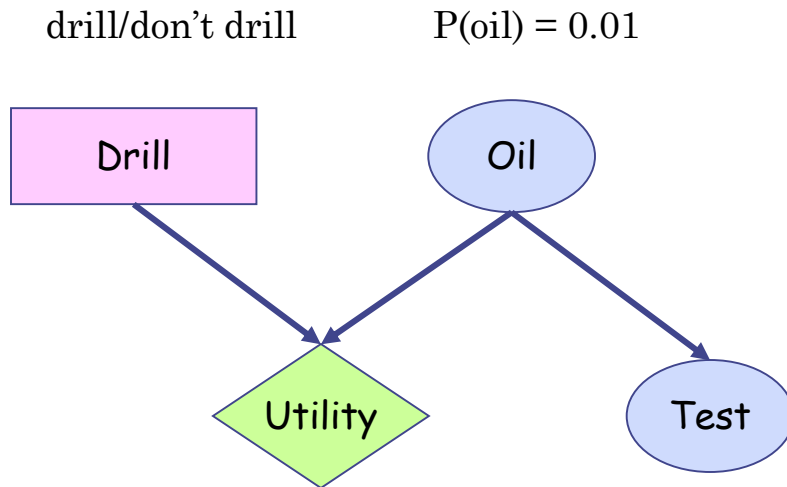
How much is the Test worth?

DRILL



1. Compute MEU before Test
2. Compute MEU
 - a. Assuming Test = positive
 - b. Assuming Test = negative
3. $\text{VOI}(\text{Test}) =$
 $P(\text{Test} = \text{pos}) * (\text{MEU} \mid \text{Test} = \text{pos}) +$
 $P(\text{Test} = \text{neg}) * (\text{MEU} \mid \text{Test} = \text{neg}) -$
MEU before Test

DRILL



$U(\sim\text{drill}, \sim\text{oil}) = 100$
 $U(\sim\text{drill}, \text{oil}) = -500$
 $U(\text{drill}, \sim\text{oil}) = -100$
 $U(\text{drill}, \text{oil}) = 1000$

$P(\text{positive} \mid \text{oil}) = 0.9$
 $P(\text{positive} \mid \sim\text{oil}) = 0.05$

MEU before Test

Drill = drill \Rightarrow

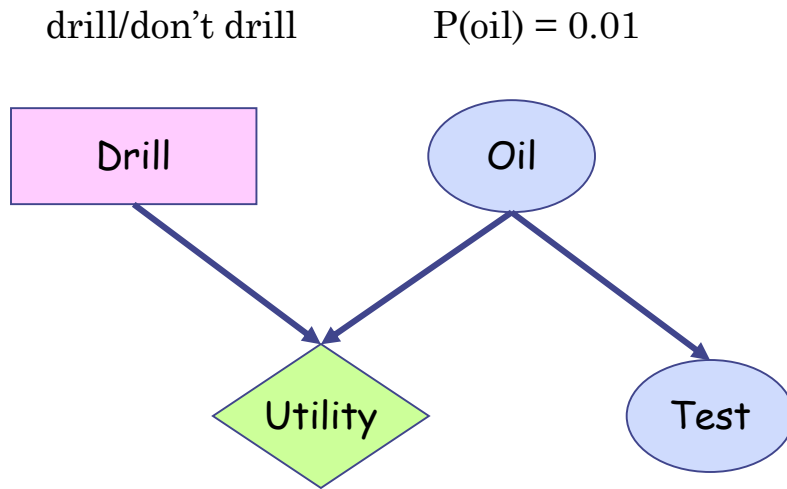
$$\begin{aligned} EU &= P(o \mid d) * U(d, o) + P(\sim o \mid d) * U(d, \sim o) \\ &= 0.01 * 1000 + 0.99 * -100 \\ &= 10 - 99 \\ &= -89 \end{aligned}$$

Drill = \sim drill \Rightarrow

$$\begin{aligned} EU &= P(o \mid \sim d) * U(\sim d, o) + P(\sim o \mid \sim d) * U(\sim d, \sim o) \\ &= 0.01 * -500 + 0.99 * 100 \\ &= -5 + 99 \\ &= 94 \end{aligned}$$

MEU before Test = 94

DRILL



$U(\sim\text{drill}, \sim\text{oil}) = 100$
 $U(\sim\text{drill}, \text{oil}) = -500$
 $U(\text{drill}, \sim\text{oil}) = -100$
 $U(\text{drill}, \text{oil}) = 1000$

$P(\text{positive} \mid \text{oil}) = 0.9$
 $P(\text{positive} \mid \sim\text{oil}) = 0.05$

MEU if Test = pos

Drill = drill \Rightarrow

$$EU = P(o \mid p, d) * U(d, o) + P(\sim o \mid p, d) * U(d, \sim o)$$

$$= ? * 1000 + ? * -100$$

$$= ?$$

$$= ?$$

Drill = \sim drill \Rightarrow

$$EU = P(o \mid p, \sim d) * U(\sim d, o) + P(\sim o \mid p, \sim d) * U(\sim d, \sim o)$$

$$= ? * -500 + ? * 100$$

$$= ?$$

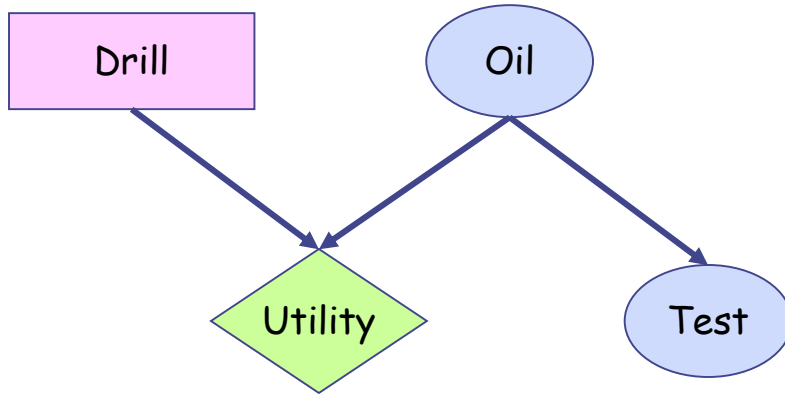
$$= ?$$

MEU | pos = ?

DRILL

drill/don't drill

$P(\text{oil}) = 0.01$



$U(\sim\text{drill}, \sim\text{oil}) = 100$

$U(\sim\text{drill}, \text{oil}) = -500$

$U(\text{drill}, \sim\text{oil}) = -100$

$U(\text{drill}, \text{oil}) = 1000$

$P(\text{positive} \mid \text{oil}) = 0.9$

$P(\text{positive} \mid \sim\text{oil}) = 0.05$

MEU if Test = neg

Drill = drill \Rightarrow

$EU = P(o \mid n, d) * U(d, o) + P(\sim o \mid n, d) * U(d, \sim o)$

$= ? * 1000 + ? * -100$

$= ?$

$= ?$

Drill = \sim drill \Rightarrow

$EU = P(o \mid n, \sim d) * U(\sim d, o) + P(\sim o \mid n, \sim d) * U(\sim d, \sim o)$

$= ? * -500 + ? * 100$

$= ?$

$= ?$

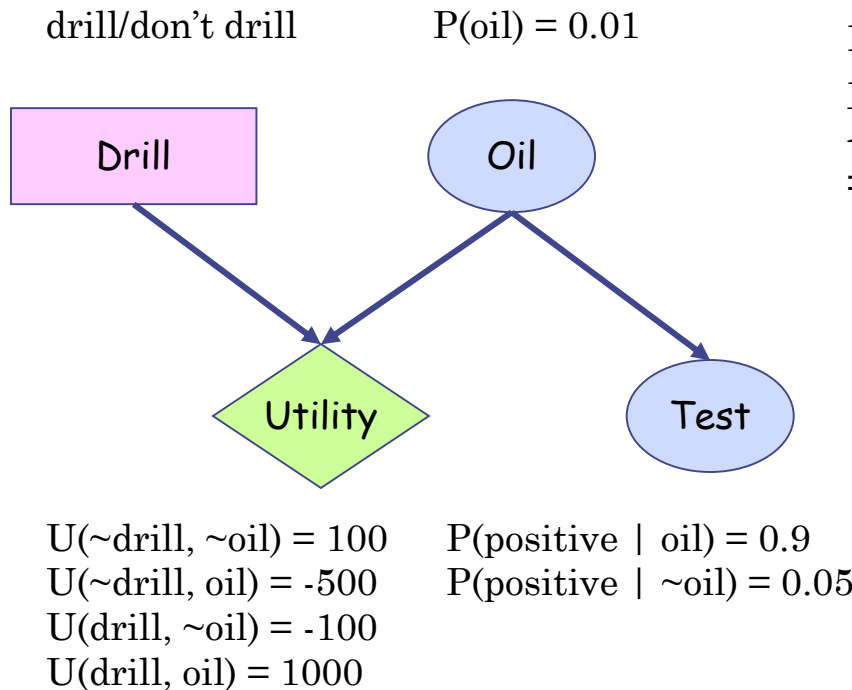
MEU | neg = ?

DRILL

VOI(Test)

VOI(Test) =

$$\begin{aligned} & P(\text{Test} = \text{pos}) \times (\text{MEU} \mid \text{pos}) + \\ & P(\text{Test} = \text{neg}) \times (\text{MEU} \mid \text{neg}) - \\ & \text{MEU before Test} \\ & = ? * ? + ? * ? - 94 \end{aligned}$$



NEXT

- Chapter 17 – making complex decisions