CS973: Machine Learning for Cyber Security

Practice Questions

Q1. Suppose $A, B \in \mathbb{R}^{3\times3}$ are two square symmetric matrices. Then which of the following statements is true (more than one statement may be true)?

a. If both A, B are invertible, then $P = A + B$ must be invertible.
b. If both A, B are invertible, then $Q = A \cdot B$ must be invertible.
c. If both A, B are invertible, then $R = A - B$ must be invertible.
d. If both A, B are invertible, then $S = A \cdot B \cdot B$ must be invertible.

Q2. Let X be a real-valued random variable such that $\mathbb{E}[X] = \mathbb{E}[-X]$. Suppose it is known that $\mathbb{E}[X^2] = 1$. Calculate the variance of X.

Q3. Melbo has a coin that lands heads with probability $p > 0$. Melbo tosses the coin repeatedly and defines a random variable N to denote the number of tosses before the first heads appears. Calculate $\mathbb{E}[N]$.

Q4. Let A be a real symmetric $n \times n$ matrix whose least eigenvalue is 1/3. Let $\mathbf{x}$ be an n-dimensional vector with real entries. Which option is correct?

a. $\mathbf{x}^\top A\mathbf{x}$ always evaluates to 0
b. $\mathbf{x}^\top A\mathbf{x}$ always takes a non-negative value
c. $\mathbf{x}^\top A\mathbf{x}$ always takes a non-positive value
d. $\mathbf{x}^\top A\mathbf{x}$ can take positive, negative and 0 value depending on $\mathbf{x}$

Q5. Consider 3 random variables x, y, z with distributions $p(x|z), p(y|z), p(z)$ i.e., the distributions of x, y depend on z but distribution of z does not depend on the other variables. Then the conditional distribution $p(x, y|z)$ is equal to

a. $p(x) \cdot p(y)$
b. $p(x|z) \cdot p(y)$
c. $p(x|y) \cdot p(y|z)$
d. $p(x|z) \cdot p(y|z)$

Q6. A is a 3×3 positive definite matrix with trace 12 and eigenvalues being three consecutive integers. Find the determinant of A.