

Q2. a)

$$P(x) = x^4 + x + 1 = 10011 = 5 \text{ bits}$$

$$D(x) = 10010011011 = x^0 + x^7 + x^4 + x^3 + x + 1$$

EXTRA bit to be added =  $5 - 1 = 4$

$$D' = 100100110110000$$

$$D'(x) = x^{14} + x^{11} + x^8 + x^7 + x^5 + x^4$$

To Encode

$$x^{14} + x^{11} + x^8 + x^7 + x^5 + x^4 \leftarrow Q(x)$$

$$\begin{array}{r} x^{14} + x^{11} + x^8 + x^7 + x^5 + x^4 \\ x^{14} + x^{11} + x^{10} + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 \\ \hline x^{10} + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 \\ x^{10} + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 \\ \hline x^8 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 \\ x^8 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 \\ \hline x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 \\ \hline x^3 + x^2 \end{array}$$

Transmitted  $(x) = x^{14} + x^{11} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2$

Encoded bits = 100100110111100

Q2. b)

Error Pattern = 1000100000000000

Transmitted bits = 100100110111100

(To be filtered from Q)

Filtered Data = 000110110111100

(Flip 1 to 0 / 0 to 1)

Received Data

$P(x) = x^4 + x + 1 = 10011$  = Divided by

To detect error

$$D(x) = 00011011011100 = x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2$$

$$\begin{array}{r} x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2 \\ x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2 \\ \hline \end{array}$$

$$\begin{array}{r}
 x^7 + x \\
 x^4 + x + 1 \overline{) x^{11} + x^{10} + x^8 + x^7} \\
 \underline{x^{11} + x^{10} + x^8 + x^7} \\
 x^{10} + x^7 + x^6 \\
 \underline{x^{10} + x^7 + x^6} \\
 x^5 + x^4 + x^3 + x^2 \\
 \underline{x^5 + x^4 + x^3 + x^2} \\
 x^7 + x^6 + x^5 + x^4 + x^3 \\
 \underline{x^7 + x^6 + x^5 + x^4 + x^3} \\
 x^6 + x^5 + x^2 \\
 \underline{x^6 + x^5} \\
 x^3 + x^2 \\
 \underline{x^3 + x^2} \\
 x^5 + x^3 \\
 \underline{x^5 + x^3} \\
 x^2 + x \\
 \underline{x^2 + x} \\
 x^3 + x^2 + x
 \end{array}$$

Dividend (Polynomial) =  $x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2$

Divided by =  $x^4 + x + 1$

Quotient =  $x^7 + x^6 + x^3 + x^2 + x$

Remainder =  $x^3 + x^2 + x = 1110$

As we see, the Remainder after division is  $x^3 + x^2 + x (1110)$ , which is non zero, The errors are detected.

Q.2)

ERROR Pattern =  $\underline{1} \ 0 \ 0 \ \underline{1} \ \underline{1} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$

To be Filtered Data =  $\underline{1} \ 0 \ 0 \ \underline{1} \ \underline{0} \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0$

Filtered Data =  $0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0$   
(Flip at Position 1, 4, 5)

Therefore, Received Data =  $\overset{14}{0} \ \overset{13}{0} \ \overset{12}{0} \ \overset{11}{0} \ \overset{10}{1} \ \overset{9}{0} \ \overset{8}{1} \ \overset{7}{1} \ \overset{6}{0} \ \overset{5}{1} \ \overset{4}{1} \ \overset{3}{1} \ \overset{2}{1} \ \overset{1}{1} \ \overset{0}{0}$

$D(x) = x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2$

$P(x) = x^4 + x + 1$

To detect error,

$$\begin{array}{r}
 x^6 + x^4 + x^2 + 1 \\
 \hline
 \end{array}$$

To detect

$$\begin{array}{r}
 x^6 + x^4 + x^2 + 1 \\
 x^4 + x + 1 \overline{) x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2} \\
 \underline{x^{10} + x^8 + x^7 + x^6} \phantom{+ x^5 + x^4 + x^3 + x^2} \\
 x^8 + x^6 + x^5 + x^4 \phantom{+ x^3 + x^2} \\
 \underline{x^8 + x^6 + x^5 + x^4} \phantom{+ x^3 + x^2} \\
 x^6 + x^3 + x^2 \\
 \underline{x^6 + x^3 + x^2} \\
 0 \quad 0 \quad 0
 \end{array}$$

Remainder = 0000

Conclusion:

The string 000010110111100 is received, corresponding to  $x^{10} + x^8 + x^7 + x^5 + x^4 + x^3 + x^2$ . The Remainder after division by  $x^4 + x + 1$  is 0000, which is zero,  
 Hence, ERROR not detected.