CS972: Assignment 1

April, 2023

Time: 3 days **Maximum Marks**: 50

Question (Full marks = 50) Consider the set of solutions S of a linear equation:

$$S = \{(x_1, x_2, \dots, x_n) \in \mathbb{Q}^n \mid a_1x_1 + a_2x_2 + \cdots + a_nx_n = 0\}.$$

Here the $\mathbb{Q}$ is the set of rational numbers and $a_1, a_2, \dots, a_n \in \mathbb{Q}$.

1. Define a linear function $f : \mathbb{Q}^n \mapsto \mathbb{Q}$ such that S is precisely the null space of f. Show that the dimension of S is $n-1$ if not all a_i's are 0. [5+10]

Now consider a collection of m linear equations:

$$\begin{aligned} a_{1,1}x_1 + a_{1,2}x_2 + \cdots + a_{1,n}x_n &= 0 \\ a_{2,1}x_1 + a_{2,2}x_2 + \cdots + a_{2,n}x_n &= 0 \\ \vdots \quad &\quad \vdots \\ a_{m,1}x_1 + a_{m,2}x_2 + \cdots + a_{m,n}x_n &= 0 \end{aligned}$$

with $a_{i,j} \in \mathbb{Q}$. Let $S \subseteq \mathbb{Q}^n$ be the set of solutions of these equations.

2. Define a linear function $f : \mathbb{Q}^n \mapsto \mathbb{Q}^m$ such that S is precisely the null space of f. Obtain a matrix representation F of f. [5+5]

3. Let F' be the matrix obtained by doing Gaussian elimination on the columns of F. Show that the null space of F' is also S. [10]

4. Computation of F' allows us to easily find solutions of the collection given. Show how to use F' to find a basis for vector space S. [15]