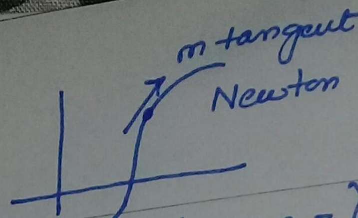
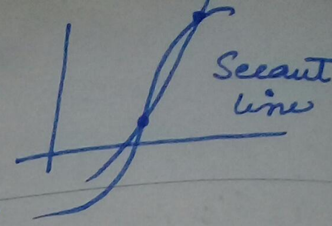
b) $P_4(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2) + a_4(x-x_0)(x-x_1)(x-x_2)(x-x_3)$

$$P_4(x) = 2 + 21(x-1) - 21.904766(x-1)(x-1.2) + 22.85511261(x-1)(x-1.2)(x-1.9) - 7.020797006(x-1)(x-1.2)(x-1.9)(x-3.2)$$

Put $(3.1) = 61.1025042$

4DD
$\frac{0.388562192 - 22.85511261}{4.2-1} =$
$-7.020797006 \; a_4$





DERIVATIVE:

{AVERAGE CHANGE}

→ Rate of change

1- $t_1 = 1990$ $\quad n_1 = 1000$

$t_2 = 2000$ $\quad n_2 = 5000$

$$\frac{\Delta n}{\Delta t} = \frac{n_2 - n_1}{t_2 - t_1} = \frac{5000-1000}{2000-1990} = \frac{400\cancel{0}}{1\cancel{0}} = 400 \text{ ply}$$

2- $t_1 = 0hr$ $\quad d_1 = 0Km$

$t_2 = 3hr$ $\quad d_2 = 180 Km$

$$\frac{\Delta d}{\Delta t} = \frac{d_2 - d_1}{t_2 - t_1} = \frac{180-0}{3-0} = \frac{180}{3} = 60 \text{ Km/Hr}$$

3- $P_1 = 10$ $\quad q_1 = 5000$

$P_2 = 20$ $\quad q_2 = 2000$

$$\frac{\Delta q}{\Delta p} = \frac{q_2 - q_1}{P_2 - P_1} = \frac{2000-5000}{20-10} = -\frac{300\cancel{0}}{1\cancel{0}} = -300 \text{ Ui/Rs}$$

INSTANTANIOUS CHANGE:

Labneiz theory ایک لمحہ کیلئے جو تبدیلی آئے

Secant line convert into Tangent

Instantanious rate of change Price or Cost ki bhi Hoti Hai*

Formula of areas:

stepsize = 0.25

$f(x) = x^{2x}$

f	f(x)
1.5	

1) Trapezoidal Method

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + f(b)\right]$$

$$h = b - a$$

2) Simpson's 1/3

$$\int_a^b f(x)\,dx = h/3\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

$$h = \frac{b-a}{2}$$

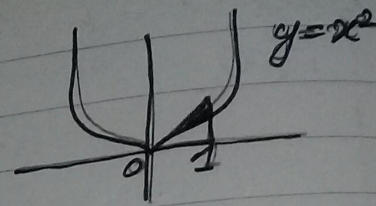
3) Simpson's 3/8

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right]$$

$$h = \frac{b-a}{3}$$

$n=1$

Find $\int_0^1 x^2\,dx$



$h = b-a = 1-0 = 1$

$f(x) = x^2$

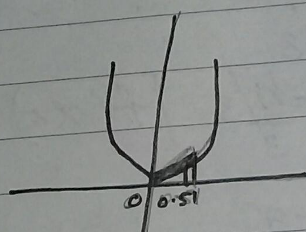
$f(0) = 0^2$

$f(1) = 1^2$

$$\int_0^1 x^2\,dx = \frac{h}{2}\left[f(0)+f(1)\right]$$

$$= \frac{1}{2}\left[0+1\right] = \frac{1}{2} = 0.5$$

Find $\int_0^1 x^2\,dx$ $\quad n=2$



$h = \dfrac{b-a}{n}$

$h = \dfrac{1-0}{2} = 0.5$

$x_0 = 0$	$f(0) = 0^2 = 0$
$x_1 = 0.5$	$f(0.5) = 0.5^2 = 0.25$
$x_2 = 1$	$f(1) = 1^2 = 1$

Q2- $$\int_0^1 x^2\,dx = \frac{h}{2}\left[f(0)+f(0.5)\right] + \frac{h}{2}\left[f(0.5)+f(1)\right]$$

$$= \frac{0.5}{2}\left[0+0.25\right] + \frac{0.5}{2}\left[0.25+1\right]$$

$= 0.375$

Q#03 $\int_0^1 x^2\,dx$ $\quad n=4$

$h = \dfrac{b-a}{n} = \dfrac{1-0}{4} = 0.25$

x	$f(x)$
0	0
0.25	0.0625
0.5	0.25
0.75	0.5625

$$\int_0^1 f(x)\,dx = \frac{h}{2}\left[f(0)+f(0.25)\right]+\frac{h}{2}\left[f(0.25)+f(0.5)\right]+$$

$$\frac{h}{2}\left[f(0.5)+f(0.75)\right]+\frac{h}{2}\left[f(0.75)+f(1)\right]$$

$$=\frac{0.25}{2}\left[0+0.0625\right]+\frac{0.25}{2}(0.0625+0.25)+$$

$$\frac{0.25}{2}(0.25+0.5625)+\frac{0.25}{2}(0.5625+1)$$

$$=0.34375$$

Q - Find $\int_0^1 x^2\,dx$ by simson $1/3$

$$\int_a^b f(x)\,dx = \frac{h}{3}\left(f(x_0)+4f(x_1)+f(x_2)\right]$$

$$h=\frac{b-a}{2}$$

$$h=\frac{1-0}{2}=0.5$$

X	f(x)
0	0
0.5	0.25
1	1

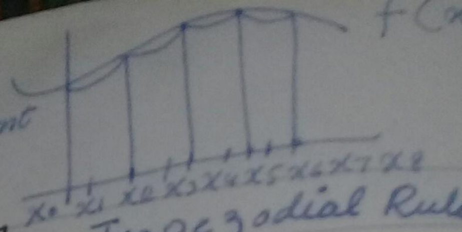
$$\int_0^1 x^2\,dx = \frac{h}{3}\left[f(0)+4f(0.5)+f(1)\right]$$

$$=\frac{0.5}{3}\left[0+4(0.25)+1\right]$$

$$=0.3333$$

جب 3 پوائنٹ ملتے ہیں تو پیرابولا بنتا ہے۔

co-efficient میں ویلیو انٹر کریں۔



INTEGRATION:

1) $\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a)+f(b)\right]$ Trapezodial Rule

$h = b-a/n$

2) $\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0)+4f(x_1)+f(x_2)\right]$ Simpson's $\frac{1}{3}$ Rule

$h = \frac{b-a}{2n}$

3) $\int_a^b f(x)\,dx = \frac{3h}{8}\left[f(x_0)+3f(x_1)+3f(x_2)+f(x_3)\right]$ Simpson's $\frac{3}{8}$ Rule

$h = \frac{b-a}{3n}$

Composite Form,

Q#01- Find $\int_{0.0001}^{1} x^x + x\cos x\,dx$ by simpson's Rule $\frac{1}{3}$

with $n=4$

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0)+4f(x_1)+f(x_2)\right]+\frac{h}{3}\left[f(x_2)+4f(x_3)+f(x_4)\right]+\frac{h}{3}\left[f(x_4)+4f(x_5)+f(x_6)\right]+\frac{h}{3}\left[f(x_6)+4f(x_7)+f(x_8)\right]$$

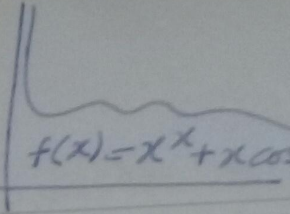
$$= \frac{h}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+4f(x_5)+2f(x_6)+4f(x_7)+f(x_8)\right]$$

$h = \frac{b-a}{(2n)}$

جتنی n کی ویلیو انکریز کریں گے اتنی h کی ویلیو ڈکریز ہوگی۔

$a=0,\ b=1,\ n=4,\ h = \frac{1-0.001}{2(4)} = \frac{0.\overline{1}}{8} = 0.124875$

$\frac{\infty}{\infty}, \frac{0}{0}, 0^0, \infty^0$, 1 $\quad x_1 = x_0 + h = h^2$



$x_{i+1} = x_i + h$

0.001 x	$f(x)$	
0.125875 x_0	0.99917939	$f(x_0)$
0.25075 x_1	0.8951265169	$f(x_1)$
0.375625 x_2	0.9492398248	$f(x_2)$
0.5005 x_3	1.04096645	$f(x_3)$
0.625375 x_4	1.14555613	$f(x_4)$
0.75025 x_5	1.251871861	$f(x_5)$
0.875125 x_6	1.354177561	$f(x_6)$
1 x_7	1.45002594	$f(x_7)$
1.124875 x_8	1.539673267	$f(x_8)$

$$\int_{0.0001}^{1} x^x + x\cos x \, dx = \frac{h}{3}\left[f(0.0001) + 4f(0.124975) + 2f(0.24985) + \cdots\cdots 4f(0.875) + f(\quad) \right]$$

$$= \frac{h}{3}\left[f_0 + 4f_1 + 2f_2 + 4f_3 + \cdots\cdots + 4f_7 + f_8 \right]$$

$$= \frac{0.124875}{3}\left[0.99917939 + 4(0.89512 55169) + 2 \right.$$

$$= 1.166$$

Q#02 Use three simpson 3/8 to integral $(n=3)$

$$\int_0^{\pi/2} \frac{x\cos x + e^x}{x^2+5x+3}\,dx$$

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[f(x_0)+3f(x_1)+3f(x_2)+f(x_3)\right] + \frac{3h}{8}\left[f(x_3)+3f(x_4)+3f(x_5)+f(x_6)\right] + \frac{3h}{8}\left[f(x_6)+3f(x_7)+3f(x_8)+f(x_9)\right]$$

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[f(x_0)+3f(x_1)+3f(x_2)+2f(x_3)+3f(x_4) + 3f(x_5)+2f(x_6)+3f(x_7)+3f(x_8)+ f(x_9)\right]$$

$$\therefore h = \frac{b-a}{3(n)} \Rightarrow \frac{b-a}{9}$$

$a=0$, $b=\pi/2$ $n=3$, $h=0.1745432$

$$h = \frac{\pi/2 - 0}{9}$$

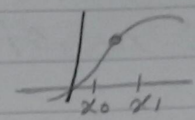
	x	f(x)
x_0	0	0.333333
x_1	0.1745432	0.349098145
x_2	0.3490864	0.358680487
x_3	0.5236184	0.363457082
x_4	0.6981616	0.364685692
x_5	0.8727048	0.363601554
x_6	1.047248	0.361447267
x_7	1.2217912	0.359476036
x_8	1.3963344	0.358945967
x_9	1.5708776	
x_{10}	1.7454208	

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

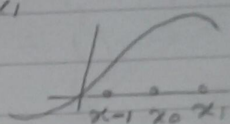
0.5633673 (Computation)
0.56337 (Calculation)

FORMULAS FOR FIRST DERIVATIVE:

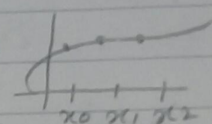
1- $f'(x_0) = \dfrac{f_1 - f_0}{h}$



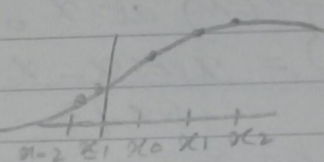
2- $f'(x_0) = \dfrac{\overset{x_0+h}{f_1} - f_{-1}\ {\scriptstyle x_0-h}}{2h}$



3- $f'(x_0) = \dfrac{-f_2 - 4f_1 - 3f_0}{2h}$



4- $f'(x_0) = \dfrac{-f_2 + 8f_1 - 8f_{-1} + f_{-2}}{2h}$



Formula For Second derivative

$f''(x_0) = \dfrac{f_2 - 2f_1 + f_0}{h^2}$

$f''(x_0) = \dfrac{f_1 - 2f_0 + f_{-1}}{h^2}$

$f''(x_0) = \dfrac{-f_3 + 4f_2 - 5f_1 + 2f_0}{h^2}$

Find $f'(1) = x^x$

$h = 0.01$

$f'(x_0) = \dfrac{f_1 - f_0}{h}$

$x_0 = 1$

$x_1 = x_0 + h = 1 + 0.01 = 1.01$

$f(x_0) = f_0 = 1^1 = 1$

$f(x_1) = f_1 = 1.01^{1.01} = 1.010100503$

$f'(1) = \dfrac{f(1.01) - f(1)}{h}$

$$f'(1) = \frac{1.010100503 - 1}{0.01}$$

$$f'(1) = 1.010050334$$

$$f(x) = (x)^x$$

$f'(1) = ?$

$h = 0.00001$

$f(1) = 1^1 = 1$

$f(1.00001) = 1.00001 \qquad 1.00001 = 1.00001$

$$f'(1) = \frac{1.00001 - 1}{0.0001} = \frac{0.0001}{0.001} = 1.00001$$

Find $f''(x)$ at $x = 1.5$

$$f(x) = \frac{x\cos x - x}{2x + 3}$$

$h = 0.1$

$$f''(x) = \frac{f_1 - 2f_0 + f_{-1}}{h^2}$$

$x_0 = 1.5$

$x_1 = x_0 + h = 1.5 + 0.1 = 1.6$

$x_{-1} = x_0 - h = 1.5 - 0.1 = 1.4$

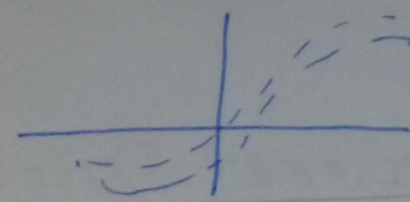
$x_{-1} = 1.4 \qquad f_{-1} = f(x_{-1}) = \dfrac{1.4\cos(1.4) - 1.4}{2(1.4) + 3} = -$

$x_0 = 1.5 \quad f_0 = \quad -0.232315699$

$x_1 = 1.6 \quad f_1 =$

$$f''(1.5) = \frac{(\qquad) - 2(\qquad) + (\qquad)}{(0.1)^2}$$

Face Potrait

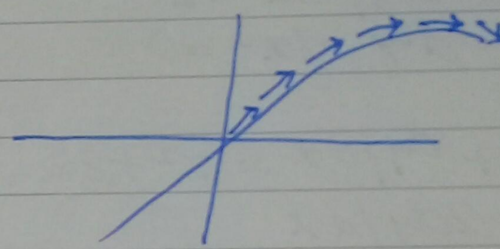


$y = x^3$

$\frac{dy}{dx} = 3x^2 \quad \frac{d^2y}{dx^2} = 6x \quad \frac{d^3y}{dx^3} = 6 \quad \frac{d^4y}{dx^4} = 0$

$\frac{dy}{dx} + y = 3x^2 + x^3$

$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{d^4y}{dx^4}\right)^1 = (6x)^3 + (0) = 6^3x^3$

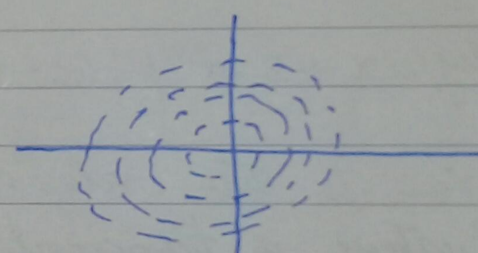


$dy = 3x^2 dx$

$\int dy = \int 3x^2 dx$

$y = \frac{\cancel{3}x^3}{\cancel{3}} + c$

$y = x^3 + c$



ORDINARY DIFFERENTIAL EQUATIONS

Two types of ordinary differential Equation D.E

→ Partial $f(x, y, \frac{dy}{dx}) = 0$ $\frac{dy}{dx} = f(x, y)$

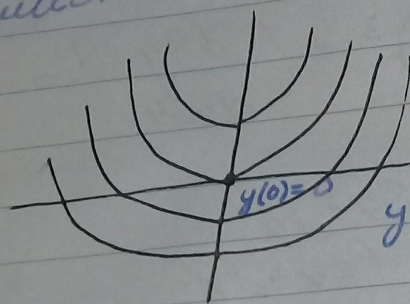
→ Ordinary

$\boxed{y = x^2}$ → Integral Solution

$dy/dx = 2x$

$d^2y/dx^2 = 2$

$d^3y/dx^3 = 0$



$y = x^2 + 1$

$y = x^2$

$y = x^2 - 1$

$y = x^2 - 2$

$y + dy/dx = x^2 + 2x$

$2dy/dx$

$2\, dy/dx + 3\, d^3y/dx^3 = 2(2x) + 0 = 4x$

→ Degree is the order of Power

→ Derivative is tangent

$ax^2 + bx + c$

$dy/dx = 2x$

$dy = 2x\, dx$

$\int dy = \int 2x\, dx$

$y = \frac{2x^2}{2} + c$

$\boxed{y = x^2 + c}$

$y(0) = 1$ $y(2) = ?$

$1 = 0^2 + c$

$c = 1$

$y = x^2 + 1$

$y = 2^2 + 1 = 5$

① $\frac{dy}{dx} = 2x$ $\quad y(1) = 1 \quad y(2) = ?$

$x \quad y$

$y' = 2x$

EULER METHOD:

$y_{i+1} = y_i + hy'_i$

$a = 1 ; \quad b = 2 \quad , h = \frac{b-a}{n} \quad , n = 4$

$h = \frac{2-1}{4} = \frac{1}{4} = 0.25$

x_i	y_i	y_i'	hy_i'	$y_{i+1} = y_i + hy'_i$
1	1	2	0.5	1.5
1.25	1.5	2.5	0.625	2.125
1.5	2.125	3	0.75	2.875
1.75	2.875	3.5	0.875	3.75
2	3.75			

$\frac{dy}{dx} = 2x \quad y(1) = 1 , y(2) = ? \quad h = 0.1$

x_i	y_i	y_i'	hy_i'	$y_{i+1} = y_i + hy'_i$
1	1	2	0.2	1.2
1.1	1.2	2.2	0.22	1.42
1.2	1.42	2.4	0.24	1.66
1.3	1.66	2.6	0.26	1.92
1.4	1.92	2.8	0.28	2.2
1.5	2.2	3	0.3	2.5
1.6	2.5	3.2	0.32	2.82
1.7	2.82	3.4	0.34	3.16
1.8	3.16	3.6	0.36	3.52
1.9	3.5	3.8	0.38	3.9
2.0	3.9	4	0.4	4.3
	4.3			

(Predictor Corrector Method)

MODIFIED EULER METHOD: $y_i' = 2x$
$y(1) = 1$
$h = 0.1$

$y_p{}_{i+1} = y_i + hy_i'$
$y_{i+1} = y_i + \frac{h}{2}\left[y_i' + y_p'{}_{i+1}\right]$

x_i	y_i	y_i'	hy_i'	y_{i+1}	y'_{i+1}	$y_i' + y'_{i+1}/2$	$h[y_i' + y'_{i+1}/2]$	$y_c{}_{i+1}$
1	1	2	0.2	1.2	2.2	2.1	0.21	1.21
1.1	1.21	2.2	0.22	1.43	2.4	2.3	0.23	1.44
1.2	1.44	2.4	0.24	2.6	2.6	2.6	1.3	2.74
1.3								
1.4								
1								

RK - 4

RANGE KUTTA METHOD: Euler Modified

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_i, y_i)$$

$$k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$$

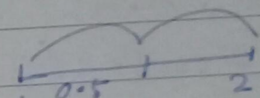
$$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_i + h, y_i + k_3)$$

Question # 01

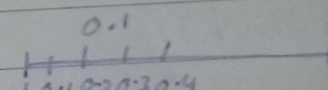
Solve $\frac{dy}{dx} = x^2 + y^2 + 3$ (H should be moderate)

$f(x,y) =$



$y(1) = 0.5$

$y(2) = ?$



$h = \frac{b-a}{n}$ $a = 1, b = 2, n = 4$

$h = \frac{2-1}{4} = \frac{1}{4} = 0.25$

	x	y	k_1	k_2	k_3	k_4	y_{i+1}
x_0	1	0.5	1.0625	1.332275	1.406375	2.0491941	1.931499
x_1	1.25	1.931499					
x_2	1.5						
x_3	1.75						
x_4	2						

FOR K1

$K1 = hf(x_i, y_i)$

$K1 = 0.25 f(1, 0.5)$

$K1 = 0.25 \, [1^2 + 0.5^2 + 3]$

$K1 = 1.0625$

FOR K2:

$K2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$

$K2 = 0.25 f\left(1 + \frac{0.25}{2}, 0.5 + \frac{1.0625}{2}\right)$

$K2 = 0.25 f(1.125, 1.03125)$

$K2 = 0.25(1.125^2 + 1.03125^2 + 3)$

$K2 = 1.332275$

FOR K3:

$K3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$

$K3 = 0.25 f\left(1 + \frac{0.25}{2}, 0.5 + \frac{1.332275}{2}\right)$

$K3 = 0.25 f(1.125, 1.1661375)$

$K3 = 0.25(1.125^2 + 1.1661375^2 + 3$

$K3 = 1.4063754$

FOR K4:

$K4 = hf(x_i + h, y_i + K3)$

$K4 = 0.25 f(1 + 0.25, 0.5 + 1.406375)$

$K4 = 0.25 f(1.25, 1.906375)$

$K4 = 0.25(1.25^2 + 1.906375^2 + 3)$

$K4 = 2.04919141$

$y_1 = y_0 + \frac{1}{6} k_1 + 2k_2 + 2k_3 + k_4)$

$y_1 = 0.5 + \frac{1}{6} (1.0625 + 2(1.332275) + 2(1.406375) + 2.0491941)$

$y_1 = 1.931499$

FOR K_1:

$K_1 = hf(x_i, y_i)$

$K_1 = 0.25 f(1.25, 1.931499)$

$K_1 = 0.25 (1.25^2 + 1.931499^2 + 3)$

$K_1 = 2.073297$

FOR K_2:

$K_2 = hf(x_i + \frac{h}{2}, y_i + \frac{K_1}{2})$

$K_2 = 0.25 f(1.25 + \frac{0.25}{2}, \frac{1.931499 + 2.073297}{2}$

$K_2 = 0.25 f(1.375, 2.9681475)$

$K_2 = 0.25 [1.375^2 + 2.9681475^2 + 3)$

$K_2 = 3.425126$

FOR K_3:

$K_3 = hf(x_i + \frac{h}{2}, y_i + \frac{K_2}{2})$

$K_3 = 0.25 f(1.25 + \frac{0.25}{2}, 1.931499 + \frac{3.425126}{2})$

$K_3 = 0.25 f(1.375, 3.644062$

$K_3 = 0.25 (1.375^2 + 3.644062^2 + 3)$

$K_3 = 4.5424532$

FOR K_4:

$K_4 = hf(x_i + h, y_i + K_3)$

$K_4 = 0.25 f(1.25 + 0.25, 1.931499 + 4.5424532)$

$K_4 = 0.25 f(1.5, 6.4739522)$

$K_4 = 0.25 (1.5^2 + 6.4739522^2 + 3)$

$K_4 = 11.790523$

$y_2 = y_1 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$y_2 = 1.931499 + \frac{1}{6}(21.8313992)$

$y_2 = 6.897997$

1- Root
2- System of Linear Equation
3- Interpolation
4- Derivation
5- Integration
6- D.E
Euler, Modified, RK Method

$y_p'{}_{i+1} = 0.1 + 1.1 - (0.1)(1.1) = 1.09$

$y_p'{}_{i+1} = 0.2 + 1.213905 - (0.2)(1.213905)$

Question #01

Use Euler Method to get Solution to:

$$\frac{dy}{dx} = x + y - xy \qquad y(0) = 1 \Rightarrow y(x_0) = y_0$$

at $x = 0.1$ & $x = 0.5$

Let $h = 0.1$

$y_{i+1} = y_i + hy'_i$

$y_0' = 0 + 1 - 0(1) = 1$

x_i	y_i	y_i'	hy'_i	y_{i+1}
0	1	1	0.1	1.1
0.1	1.1	1.09	0.109	1.199
0.2	1.199	1.1592	0.11592	1.27512
0.3	1.27512	1.192584	0.1192584	1.3118424
0.4	1.3118424	1.1871054	0.1187105	1.30581594
0.5	1.30581594	1.15290797	0.1152907	

Question #02:

Repeat Question #2 but Now use Modified Euler Method & Compare the result

1- $y_{p\,i+1} = y_i + hy'_i$

2- $y_{c\,i+1} = y_i + \frac{h}{2}[y_i' + y_p'{}_{i+1}] \Rightarrow$ Alternate $y_{i+1} = y_i + h\left[\frac{y_i' + y_p'{}_{i+1}}{2}\right]$

x_i	y_i	y_i'	hy_i'	$y_{p\,i+1}$	$y_p'{}_{i+1}$	$\frac{y_i' + y_p'{}_{i+1}}{2}$
0	1	1	0.1	1.1	1.09	1.045
0.1	1.1045	1.09405	0.109405	1.213905	1.1711605	1.32605
0.2	1.21776	1.174208	0.1174208	1.335180	1.234626	1.204417
0.3	1.338201	1.2367407	0.12367407	1.461875	1.277125	1.256431
0.4	1.463894	1.2783364	0.12783364	1.5917274	1.295863	1.287095
0.5	1.592603					

$h\frac{(y_i' + y_p' i+1)}{2}$	$y_c i+1$
	1.1045
0.1045	1.21776
0.1132605	1.338201
0.1204417	1.463894
0.125693	1.592603
0.128709	

Question #03
Repeat Q# 01 by RK Method $h = 0.1$
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Q#35: Use the data in the table to find the Integral b/w $x = 1.0$ & $x = 1.8$ using the trapezoidal rule
a) with $h = 0.1$
b) with $h = 0.2$

x	$f(x)$
1.0	1.543
1.1	1.669
1.2	1.811
1.3	1.971
1.4	2.151
1.5	2.352
1.6	2.577
1.7	2.828
1.8	3.107

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_0)+f(x_1)\right]$$

$$\int_1^{1.8} f(x)\,dx = \frac{0.1}{2}\left[f(1.0)+f(1.1)\right]+\frac{0.1}{2}\left[f(1.1)+f(1.2)\right]+$$

$$\cdots\ \frac{0.1}{2}\left[f(1.7)+f(1.8)\right]$$

$$= \frac{0.1}{2}\left[f(1.0)+2f(1.1)+2f(1.2)+2f(1.3)\right.$$
$$\left.+2f(1.4)+2f(1.5)+2f(1.6)+2f(1.7)+f(1.8)\right]$$

$$= \frac{0.1}{2}\left[1.543+2(1.669)+2(1.811)+2(1.971)\right.$$
$$+2(2.151)+2(2.352)+2(2.577)+2$$
$$\left.(2.828)+(3.107)\right]$$

$$= \frac{0.1}{2}\ (\text{Sum of above})$$

$$= 1.7684$$

(b) $$\int_1^{1.8} f(x)\,dx = \frac{0.2}{2}\left[f(1.0)+f(1.2)\right]+\frac{0.2}{2}\left[f(1.2)+\right.$$
$$\left.f(1.4)\right]+\frac{0.2}{2}\left[f(1.4)+f(1.6)\right]+\frac{0.2}{2}\left[f(1.6)+\right.$$
$$\left.f(1.8)\right]$$

$$\int_1^{1.8} fx\,dx = \frac{0.2}{2}\left[f(1.0)+2f(1.2)+2f(1.4)+2f(1.6)+f(1.8)\right]$$

$$\int_1^{1.8} fx\,dx = \frac{0.2}{2}\left[(1.543)+2(1.811)+2(2.151)+2(2.577)\right.$$
$$\left.+\,3.107\right]$$

$$\int_1^{1.8} fx\,dx = 1.7728$$