NC (LECTURE #01)

Algebra:

Solution of Non-linear Equation

1- Find roots of $x+5=9 \Rightarrow x=-5+9 \Rightarrow x=4$

2- $\underbrace{x^2+3x+2}=0$

Quadratic Equation (can be solved by break mid term) or by applying Quadratic Equation

$x^2+2x+x+2=0 \Rightarrow x(x+2)+1(x+2)=0$

$(x+1)(x+2) \rightarrow \boxed{x=-1}$ or $\boxed{x=-2}$

3- $x^3+x^2=0 \Rightarrow$ we can solve it by Synthetic division

$x^2(x+1)=0$

$\boxed{x=0}$, $x+1=0 \Rightarrow$ or $\boxed{x=-1}$

4- $e^x+\sin x-2=0$

5- $\ln x+e^x+\sin x+\sinh x-e^{2x^2}-3=0$

We use the system of Non-linear Method

SYSTEM OF LINEAIR EQUATION:

1- $x+y=2$ & $x-y=0$

$\boxed{x=1, y=1}$

2- $x+y+z=3$ & $x-y+z=1$

$2x+2y-z=3$

3- $x_1+x_2+x_3+\cdots\cdots+x_{100}=105$ &

$x_1+3x_2+\cdots\cdots+x_{100}=101$

→ In Computer Truncation Error Occured

→ Computer maintains the accuracy of 256 decimal pla

$=2169\cdot219(216\cdot2996+21\cdot6213)+316\cdot921(129\cdot621+2617\cdot32$

$=21619\cdot21(216\cdot29+21\cdot62)+316\cdot92(129\cdot62+2617\cdot32)$

$=21619\cdot21(237\cdot91)+316\cdot92(2746\cdot94)$

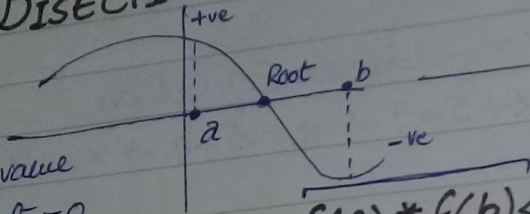
$=5143426\cdot25+870560\cdot22$

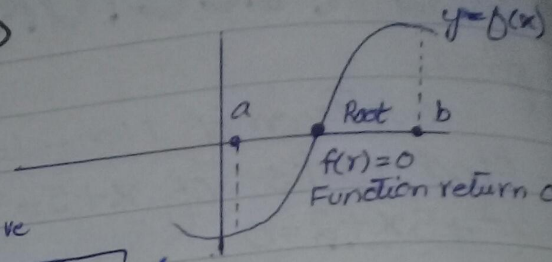
$=6013986\cdot47$

$=6014226\cdot934-6013986\cdot47$

$=240\cdot46$

BISECTION METHOD





→ Intermediate value theorem product =0

$$f(a) * f(b) < 0$$
$$r \in [a, b]$$

→ Find the root of $f(x) = x^2 + 3x - 4.5$ with the tollerance of 10^{-2} by Bisection Method.

$f(x) = x^2 + 3x - 4.5$

$f(1) = (1)^2 + 3(1) - 4.5 = -0.5$ (One Negative Value)

$f(2) = (2)^2 + 3(2) - 4.5 = 5.5$ (One Positive Value)

$f(1) * f(2) = -0.5 * 5.5 < 0$

$= -2.75 < 0$

-ve x_1	+ve x_2	$x = \frac{x_1 + x_2}{2}$	$f(x)$	Equation
1	2	1.5	2.25	$= (1.5)^2 + 3(1.5) - 4.5$
1	1.5	1.25	0.8125	$= (1.25)^2 + 3(1.25) - 4.5$
1	1.25	1.125	0.140625	=
1	1.125	1.0625	-0.18359375	=
1.0625	1.125	1.09375	-0.022460937	=
1.09375	1.125	1.109375	0.0588378[illegible]	=
1.09375	1.109375	1.1015625	0.018127441	=
1.09375	1.1015625	1.09765625	-2.182006×10^{-3}	=

→ Error is always Positive

$E = |x_7 - x_6|$

$E = |1.1015625 - 1.109375|$

$E = |-0.0078125|$

$E = 0.0078125 < 10^{-2}$

error < tollerance

* Bisection Method ka Convergence Rate Slow Hota Hai
* Bisection Method may root zarur milta Hae

2- Find the root of $f(x) = x^x - e^x - 2$ by Bisection Method with tollerance of 10^{-2}

$f(0) = 0^0 - e^0 - 2 = -1$

$f(1) = 1^1 - e^1 - 2 = -3.718281828$

$f(2) = 2^2 - e^2 - 2 = -5.389056099$

$f(3) = 3^3 - e^3 - 2 = 4.914463077$

$f(2) \cdot f(3) = (-5.389056) * (4.914463) < 0$

$= -26.484316 < 0$

-ve x_1	+ve x_2	$x = \frac{x_1 + x_2}{2}$	$f(x)$
2	3	2.5	-4.300376273
2.5	3	2.75	-1.492915194
2.75	3	2.875	1.099518618
2.75	2.875	2.8125	-0.325281379
2.8125	2.875	2.84375	0.35224435
2.8125	2.84375	2.828125	-10.91542684
2.828125	2.84375	2.8359375	-11.00383228
2.8359375	2.84375	2.83984375	-11.04837888

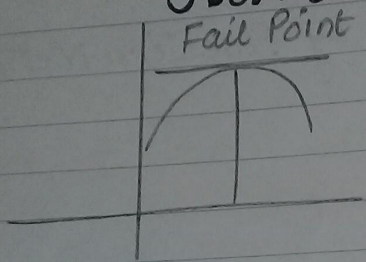
$E = |x_7 - x_6|$

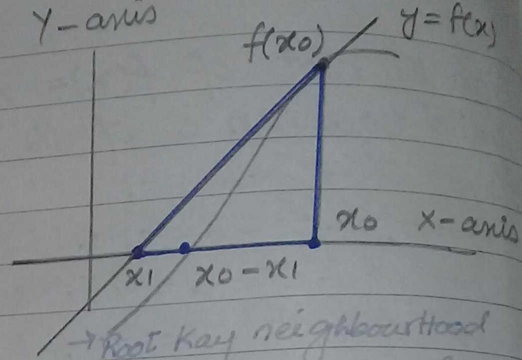
$E = |2.83984375 - 2.8359375|$

$E =$

* Newton Method Extremely Powerful Method Hota Hai
* But, we have to take out derivative (dis-advantage)

NEWTON METHOD





→ Root Kay neighbourHood May Straigh line Banygi
→ Function should be linear
→ function is linear in the Neighbourhood of root

Derivation:

$$\tan\theta = \frac{Per}{Base} = \frac{f(x_0)}{x_0 - x_1}$$

$$f'(x_0) = \frac{f(x_0)}{x_0 - x_1}$$

$$x_0 - x_1 = \frac{f(x_0)}{f'(x_0)}$$

$$x_0 - \frac{f(x_0)}{f'(x_0)} = x_1$$

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

Find the root of $f(x) = x^2 - e^x - 2$ by Newton Method with tollerance of 10^{-2}

$$x_{n+1} = x_n - \frac{f(x_n)}{f(x_n)}$$

$$f(x) = x^2 - e^x - 2$$

$$f'(x) = 2x - e^x$$

$$x_{n+1} = x_n - \left[\frac{x_n^2 - e_n^x - 2}{2x_n - e^{x_n}}\right]$$

Let $x_0 = 1$

$$x_{n+1} = 1 - \left[\frac{1^2 - e^1 - 2}{2(1) - e^1}\right]$$

$$= 1 - [-4.577422]$$

$$x_1 = 5.577422$$

Let $x_1 = 5.577422$

$$x_2 = 5.577422 - \left[\frac{(5.577422)^2 - e^{5.577422} - 2}{2(5.577422) - e^{5.577422}}\right]$$

$$x_2 = 5.577422 - [-1.148721271]$$

$$x_2 = 6.7261437$$

$$x_3 = 6.7261437 - \left[\frac{(6.7261437)^2 - e^{6.7261437} - 2}{2(6.7261437) - e^{6.7261437}}\right]$$

$$x_3 = 6.7261437 - [-1.148721271]$$

$$x_3 = 7.874864971$$

$$x_4 = 7.874864971 - \left[\frac{(7.8748649)^2 - e^{7.8748649} - 2}{2(7.8748649) - e^{7.8748649}}\right]$$

$$x_4 = 7.874864971 - [-1.148721271]$$

$$x_4 = 9.023586242$$

$\vdots$

$x_{10} = ?$

$E = |x_4 - x_3|$

$E = 1.14872127 < toll$

error is not < tollerance

* Initial Condition
* Formula
* Swapping

1- $|f(x_0)| > |f(x_1)|$

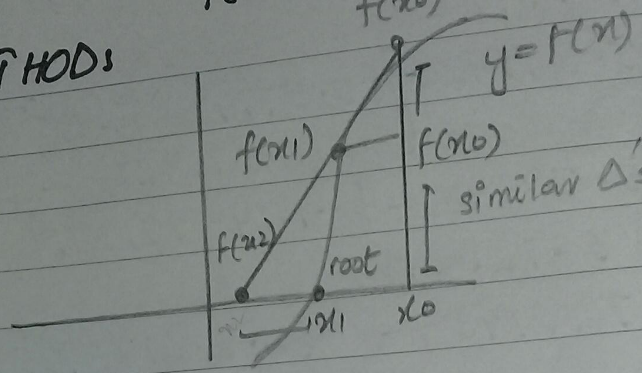
2- $x_2 = x_1 - \dfrac{f(x_1)(x_0 - x_1)}{f(x_2) - f(x_1)}$

3-

x_0	x_1	$f(x_0)$	$f(x_1)$	x_2	$f(x_2)$

$x_0 = x_1$
$x_1 = x_2$

SECANT METHOD:



Derivation:

Formula: $x - \dfrac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$

$\tan\alpha = \dfrac{f(x_0)}{x_0 - x_2}$

$\tan\alpha = \dfrac{f(x_0) - f(x_1)}{x_0 - x_1}$

$\dfrac{f(x_0)}{x_0 - x_2} = \dfrac{f(x_0) - f(x_1)}{x_0 - x_1}$

$x_0 - x_2 = f(x_0)\dfrac{(x_0 - x_1)}{f(x_0) - f(x_1)}$

$x_2 = x_1 - \dfrac{f(x_1)(x_1 - x_2)}{f(x_1) - f(x_2)}$

→ Find the root of $f(x) = e^x - 2x - 3$ by "Secant Method" with tollerance of 10^{-2}

Method of False Position (Regula Falsi)

1- $f(a) * f(b) < 0$

2- $r \in (a, b)$ then $\rightarrow$ bisection

3- $$x_2 = x_1 - \frac{f(x_1)(x_0 - x_1)}{f(x_1) - f(x_0)}$$

4-

x_0 (-ve)	x_1 (+ve)	$f(x_0)$	$f(x_1)$	x_2	$f(x_2)$

5- $E = |x_{10} - x_9|$

$E < \text{toll}$

Find the root of $f(x) = 3x + \sin x - e^x$ by 'Secant Method'

$f(0) = 3(0) + \sin(0) - e^0 = -1$

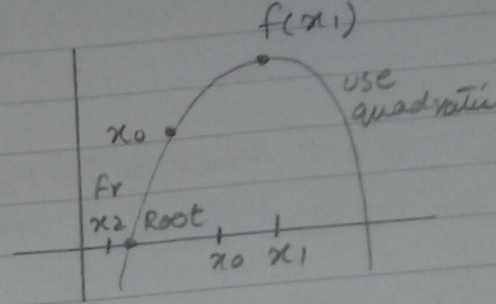
$f(1) = 3(1) + \sin(1) - e^1 = 1.123189$

$|f(x_0)| > |f(x_1)| \Rightarrow \underbrace{|-1| > |1.123189|}_{\text{False}}$

swap

x_0	x_1	$f(x_0)$	$f(x_1)$	x_2	$f(x_2)$
1	0	1.123189	-1	0.470989	0.2651574
0	0.470989	-1	0.265158	0.3712067	-0.02785
0.470989	0.3712067	0.265758	0.02785	0.359597	-0.0026628
0.359597	0.3712067	0.02785	0.0020628	0.374129409	0.034125262

MULLER METHOD:



$h_1 = x_1 - x_0$

$h_2 = x_0 - x_2$

$r = h_1 / h_2$

$c = f(x_2)$

$$a = \frac{r f_1 - f_0 (1+r) + f_2}{r h_1^2 (1+r)}$$

$$b = \frac{f_1 - f_0 - a h_1^2}{h}$$

① $x_2 < x_0 < x_1$

$$x_r = x_0 - \frac{2c}{b \pm \sqrt{b^2 - 4ac}}$$

① $x_2 < x_0 < x_1$

② x_r

③ $f x_r > x_0$

x_0, x_1, x_r

else

x_0, x_2, x_r

Question # 01:

Find the root of $f(x) = 3x + \sin x - e^x$ b/w 0 & 1 with toll of 10^{-2}

$x_2 = 0.0 \quad f(x_2) = f(0) = 3(0) + \sin(0) - e^0 = -1$

$x_0 = 0.5 \quad f(x_0) = f(0.5) = 3(0.5) + \sin(0.5) - e^{0.5} = 0.330704$

$x_1 = 1.0 \quad f(x_1) = f(1) = 3(1.0) + \sin(1.0) - e^{1.0} = 1.123189$

$h_1 = x_1 - x_0 = 1 - 0.5 = 0.5$

$h_2 = x_0 - x_2 = 0.5 - 0 = 0.5$

$r = h_1/h_2 = 0.5/0.5 = 1$

c[illegible]) = 0.330704

$$a = \frac{\text{[illegible]}(1+r) + f_2}{r h_1^2 (1+r)}$$

$$a = \frac{(1)(1.123189) - (0.330704)(1+1) + (-1)}{(1)(0.5)^2 (1+1)}$$

$a = -1.076438$

$b = \frac{1.123189 - 0.330704 - (-1.076455)(0.5)}{0.5}$

$b = +2.1281865 \rightarrow$ b ki value dekho agr +ve hoto +ve wala formula lagao agr n(-ve) hoto -ve formula

$x_r = x_0 - \frac{2c}{b \pm \sqrt{b^2 - 4ac}}$

$x_r = 0.5 - \frac{2(0.330704)}{2.2181865 + \sqrt{(2.21281865)^2 - 4(-1.076438)(0.330704)}}$

$x_r = 0.3555211$

if $x_r > x_0$

x_0, x_1, x_r

else

x_0, x_2, x_r

$0.35211 > 0.5$

$\underset{0.5}{x_0}, \underset{0.0}{x_2}, \underset{0.355211}{x_r}$

$x_2 = 0.0$ & $x_0 = 0.355211$ & $x_1 = 0.5$

FOR X_2:

$f(x_2) = -1$

$f(x_0) = -0.020855$

$f(x_1) = 0.330704$

$h_1 = x_1 - x_0 = 0.14789$

$h_2 = x_0 - x_2 = 0.355211$

$r = 2.401859$

$c = f(x_0) = -0.020855$

$a = \frac{rf_1 - f_0(1+r) + f_2}{rh_1^2(1+r)}$

$a = \frac{(2.401859)(0.330704) - (-0.020855)(1+2.401859) + (-1)}{(2.401859)(0.1644789)^2(1+2.401859)}$

$a = -0.754026$

$b = \frac{f_1 - f_0 - ah_1^2}{h_1} = \frac{0.330704 - (-0.020855) - (-0.754026)(0.14789)^2}{0.14789}$

$b = 2.488678366$

$$x_r = x_0 - \frac{2c}{b+\sqrt{b^2-4ac}} = -0.020855 - \frac{2c}{(2.488678)+\sqrt{(2.488678)^2-4(-0.0754026)(-0.020855)}}$$

* derivation:
· Newton method · Muller Method · Secant Method
· Algorithm
· codes

Computer Produces Truncation error:
→ Gauss's Elimination } Not iterative
→ Gauss's Jordan

SYSTEM OF LINEAR EQUATION:

Solve the following equation
$x+2y=5$ & $x+y=3$

$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \Rightarrow (1\times1)-(2\times1) \Rightarrow 1-2=-1$

$A_1 = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix} \Rightarrow |A_1| = 5-6=-1$

$A_2 = \begin{bmatrix} 1 & 5 \\ 1 & 3 \end{bmatrix} \Rightarrow |A_2| = 3-5=-2$

$x = \frac{|A_1|}{|A|} \rightarrow \frac{\cancel{-}1}{\cancel{-}1} = 1$

$y = \frac{|A_2|}{|A|} \Rightarrow \frac{\cancel{-}2}{\cancel{-}1} = 2$

Solution Set $\{1,2\}$

Gauss's Elimination:

$\begin{bmatrix} x+2y=5 \\ x+y=3 \end{bmatrix}$ → Elimentary row operation $R_1 \leftrightarrow R_2$
→ Two row's of matrix can be interchange

$\begin{bmatrix} x+y=3 \\ x+2y=5 \end{bmatrix}$ → A row can be multiply by Non-Zero element $2R_1$

$\begin{bmatrix} 2x+4y=10 \\ x+y=3 \end{bmatrix}$ → A row can replace by it's linear Combination

$\begin{bmatrix} 2x+3y=8 \\ x+y=3 \end{bmatrix}$ $R_1+R_2 \rightarrow R_1$
add or subtract
aR_1+bR_2

2nd Step
$\begin{cases} R_2 - R_2 \text{ ka pehla digit or } * R_1 \text{ divided by } R_1 \text{ ka Pehla digit} \\ R_3 - R_1 \end{cases}$

Question
Solve it by Gauss's Elimination

$$x + y + z = 6$$
$$2x + 2y + z = 9$$
$$x - y + z = 2$$

BY GAUSS'S ELIMINATIONS

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 2 & 1 & 9 \\ 1 & -1 & 1 & 2 \end{array}\right] \quad \text{Argumented 'Matrix'}$$

lower Δ must be 'o'

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 2-2(1) & 2-2(1) & 1-2(1) & 9-2(6) \\ 1-1 & 1-(-1) & 1-1 & 2-6 \end{bmatrix} \begin{matrix} \\ R_2 - 2R_1 \\ R_3 - R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 0 & -1 & -3 \\ 0 & 2 & 0 & -4 \end{bmatrix}$$

یہاں سے

جب 2nd روکا 2nd ڈجیٹ زیرو
ہوجائے تو انٹرچینج کردیں۔

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -4 \\ 0 & 0 & -1 & -3 \end{bmatrix} \quad R_2 \leftrightarrow R_3$$

$$x + y + z = 6$$
$$0x - 2y + 0z = -4$$
$$0x + 0y - z = -3$$

Equate $\boxed{z = 3}$

$$\cancel{-}2y = \cancel{-}4 \Rightarrow y = 4/2 \quad \boxed{y \to 2}$$

$$x + 2 + 3 = 6 \Rightarrow x + 5 = 6 \rightarrow x = 6 - 5 \Rightarrow \boxed{x = 1}$$

'Jordan Method' may upper or lower dono Triangle 'o' Hotay Hain.

Gauss's Jordans

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 2 & -2 & 1 & 9 \\ 1 & -1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 2-2(1) & 2-2(1) & 1-2(1) & 9-2(6) \\ 1-1 & -1-1 & 1-1 & 2-6 \end{bmatrix} \begin{matrix} \\ R_2-2R_1 \\ R_3-R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 0 & -1 & -3 \\ 0 & -2 & 0 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -4 \\ 0 & 0 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1\times 1/2\,(0) & 1\times 1/2(-2) & 1\times 1/2\,(0) & \overset{3}{\cancel{6}}\times 1/\cancel{2}(-4) \\ 0 & -2 & 0 & -4 \\ 0 & 0 & -1 & -3 \end{bmatrix} R_1 + 1/2\, R_2$$

$$\begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & -2 & 0 & -4 \\ 0 & 0 & -1 & -3 \end{bmatrix}$$

$\boxed{x=1}$, $\boxed{y=2}$, $\boxed{z=3}$

$3x+y+z=5$, $x+y-z=1$, $2x-y-z=0$

Argumented Matrix $\begin{bmatrix} 3 & 1 & 1 & 5 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & -1 & 0 \end{bmatrix}$

$\begin{bmatrix} 3 & 1 & 1 & 5 \\ 1-1/3(3) & 1-1/3(1) & -1-1/3(1) & 1-1/3(1) \\ 2-2/3(3) & -1-2/3(1) & -1-\frac{2}{3}(1) & 0-2/3(5) \end{bmatrix} \begin{matrix} \\ R_2 - 1/3R_1 \\ R_3 - 2/3R_1 \end{matrix}$

$\begin{bmatrix} 3 & 1 & 1 & 5 \\ 0 & 2/3 & -4/3 & -2/3 \\ 0 & -5/3 & -5/3 & -10/3 \end{bmatrix}$

$\begin{bmatrix} 3 & 1 & 1 & 5 \\ 0 & 2/3 & -4/3 & -2/3 \\ 0 & 0 & -5/3+\left(\frac{5/3}{2/3}\right)(-4/3) & -10/3+\left(\frac{5/3}{2/3}\right)(-2/3) \end{bmatrix} \begin{matrix} \\ \\ R_3 + \frac{5/3}{2/3} R_2 \end{matrix}$

$\begin{bmatrix} 3 & 1 & 1 & 5 \\ 0 & 2/3 & -4/3 & -2/3 \\ 0 & 0 & -5 & -5 \end{bmatrix}$

$3x+y+z=5$

$0x+2/3y-4/3z=-2/3$

$0x+0y-5z=-5$

Equates $\quad -5z=-5 \rightarrow \boxed{z=1}$

$2/3y-4/3(1)=-2/3 \Rightarrow 2/3y=-2/3+4/3$

$2/3y=\frac{-2+4}{3} \rightarrow 2/3y=2/3 \Rightarrow \boxed{y=1}$

$3x+y+z=5 \Rightarrow 3x+1+1=5 \Rightarrow 3x+2=5$

$3x=5-2 \Rightarrow 3x=3 \Rightarrow \boxed{x=1}$

Solution set $\{1,1,1\}$

$-8-\frac{8}{3}(4) \quad -8-\frac{32}{3} \Rightarrow \frac{-24-32}{3} \Rightarrow -\frac{56}{3} \qquad -6-\frac{8}{3}(9) \Rightarrow -6-24=-30$

$3+\frac{1}{3}(4) \Rightarrow 3+\frac{4}{3} \Rightarrow \frac{9+4}{3} \Rightarrow \frac{13}{3}$

Question # 01:

$3x+2y+4z=9$

$8x-6y-8z=-6$

$-x+2y+3z=4$

a) Solve by Gauss's Elimination (Use fractions through out)

b) Solve by Gauss's Elimination (Use 3 Significant Figures) Observe that Solution is different

c) Compute the solution when System is changed Only Slightly, change the first co-efficient of first column & first row to 3.1

$$\begin{bmatrix} 3 & 2 & 4 & 9 \\ 8 & -6 & -8 & -6 \\ -1 & 2 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 4 & 9 \\ 8-8/3(3) & -6-8/3(2) & -8-8/3(4) & -6-8/3(9) \\ -1+\frac{1}{3}(3) & 2+1/3(2) & 3+1/3(4) & 4-1/3(4) \end{bmatrix} \begin{matrix} \\ R_2-8/3R_1 \\ R_3+1/3R_1 \end{matrix}$$

$$\begin{bmatrix} 3 & 2 & 4 & 9 \\ 0 & -34/3 & -56/3 & -30 \\ 0 & 8/3 & 13/3 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 4 & 9 \\ 0 & -34/3 & 56/3 & 30 \\ 0-\frac{4}{17}(0) & 8/3+4/17(-\frac{34}{3}) & 13/3+4/17(-56/3) & 7+4/17(-30) \end{bmatrix} \begin{matrix} \\ R_3-[\frac{8/3}{-34/3}]R_2 \\ R_3+\frac{4}{17}(R_2) \end{matrix}$$

$$\begin{bmatrix} 3 & 2 & 4 & 9 \\ 0 & -34/3 & -56/3 & -30 \\ 0 & 0 & -1/7 & -1/17 \end{bmatrix}$$

$3x+2y+4z=9$

$-\frac{34}{3}y-\frac{56}{3}z=-30$

$-\frac{1}{7}z=-\frac{1}{17}$

$\boxed{z=1}$

$-6-\frac{8}{3}(2) \Rightarrow -6-\frac{16}{3} \Rightarrow \frac{-18-16}{3}=-34/3 \qquad 2+\frac{1}{3}(2) \Rightarrow 2+\frac{2}{3} \Rightarrow \frac{6+2}{3} \Rightarrow \frac{8}{3}$

(2) $\Rightarrow$ $-\frac{34}{3}y - \frac{56(1)}{3} = -30$

$-\frac{34}{3}y - \frac{56}{3} = -30$

$-\frac{34}{3}y = -30 + \frac{56}{3} \Rightarrow -\frac{34}{3}y = \frac{-90+56}{3} \Rightarrow \frac{\cancel{-34}}{\cancel{3}}y = \frac{\cancel{-34}}{\cancel{3}}$

$\boxed{y=1}$

(1) $\Rightarrow$ $3x + 2(1) + 4(1) = 9$

$3x + 2 + 4 = 9 \Rightarrow 3x + 6 = 9 \Rightarrow 3x = 9-6$

$3x = 3$ or $\boxed{x=1}$

Solution Set $\{1,1,1\}$

(b)
$$\begin{bmatrix} 3 & 2 & 4 & 9 \\ 8 & -6 & -8 & -6 \\ -1 & 2 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 4 & 9 \\ 0 & -6-2.66(2) & -8-2.66(4) & -6-2.666(9) \\ -1 & 2+0.33(2) & 3+0.333(4) & 4+0.333(9) \end{bmatrix} \begin{matrix} \\ R_2 \div 2.66R_1 \\ R_3+0.333R_1 \end{matrix}$$

$$\begin{bmatrix} 3 & 2 & 4 & 9 \\ 0 & -11.332 & -18.664 & -29.994 \\ 0 & 2.666 & 4.332 & 6.997 \end{bmatrix}$$

$R_2 \Rightarrow$
$$\begin{bmatrix} 3 & 2 & 4 & 9 \\ 0 & -11.3 & -18.6 & -29.9 \\ 0 & 2.666+0.235(-11.3) & 4.332+0.235(-18.6) & 6.997+0.235(-29.9) \end{bmatrix} \begin{matrix} \\ R_3 + \frac{2.666}{11.33}R_2 \\ R_3+0.235R_2 \end{matrix}$$

$$\begin{bmatrix} 3 & 2 & 4 & 9 \\ 0 & -11.3 & -12.6 & -29.9 \\ 0 & 0 & -0.041 & -0.036 \end{bmatrix}$$

$3x + 2y + 4z = 9$

$0x - 11.3y - 12.6z = -29.9$

$0x + 0y + 0.041z = +0.036$

$z = \frac{0.036}{0.041}$

$z = 0.87804$

$$-11.3y - 18.6(0.878) = -29.9$$
$$-11.3y - 16.3308 = -29.9$$
$$-11.3y = -29.9 + 16.3308$$
$$-11.3y = -13.5692$$
$$+11.3y = +13.5692$$
$$y = \frac{13.5692}{11.3}$$
$$\boxed{y = 1.2008}$$

$$3x + 2(1.2) + 4(0.87) = 9$$
$$3x + 2.4 + 3.48 = 9$$
$$3x + 5.88 = 9$$
$$x = \frac{9 - 5.88}{3}$$
$$\boxed{x = 1.04}$$

PARTIAL PIVOTING:

$$3x + 2y + 5z = 9$$
$$5x + 3y + 2z = 11$$
$$x - 11y + z = 12$$

Gauss's elimination by partial Pivoting

$$\begin{bmatrix} 3 & 2 & 5 & 9 \\ 5 & 3 & 2 & 11 \\ 1 & -11 & 1 & 12 \end{bmatrix}$$

See Max element & take magnitude

Max = [|3|, |5|, |1|] = |5| or 5

interchange

$$\begin{bmatrix} 5 & 3 & 2 & 11 \\ 3 & 2 & 5 & 9 \\ 1 & -11 & 1 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 3 & 2 & 11 \\ 0 & 2-0.6(3) & 5-0.6(2) & 9-0.6(11) \\ 0 & -11-0.2(3) & 1-0.2(2) & 12-0.2(11) \end{bmatrix} \begin{matrix} R_2 - 3/5R_1 = R_2 - 0.6(R_1) \\ R_3 - 1/5R_1 = R_3 - 0.2(R_1) \end{matrix}$$

$$\begin{bmatrix} 5 & 3 & 2 & 11 \\ 0 & 0.2 & 3.8 & 2.4 \\ 0 & -11.6 & 0.6 & 9.8 \end{bmatrix}$$

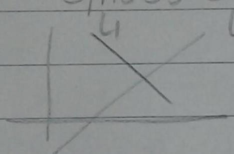
$\text{Max}\,[\,|0.21|, |-11.6|\,] = 11.6$

$$\begin{bmatrix} 5 & 3 & 2 & 11 \\ 0 & -11.6 & 0.6 & 9.8 \\ 0 & 0.2 & 3.8 & 2.4 \end{bmatrix} \quad R_2 \leftrightarrow R_3$$

$$\begin{bmatrix} 5 & 3 & 2 & 11 \\ 0 & -11.6 & 0.6 & 9.8 \\ 0 & 0 & 3.8+0.0174(0.6) & 2.4+0.0174(9.8) \end{bmatrix} \quad \begin{array}{l} R_3 + 0.2/11.6\,(R_2) \\ R_3 + 0.017241(R_2) \end{array}$$

$$\begin{bmatrix} 5 & 3 & 2 & 11 \\ 0 & -11.6 & 0.6 & 9.8 \\ 0 & 0 & 3.81034 & 2.568 \end{bmatrix}$$

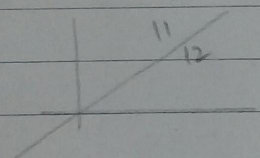
GAUSS'S ELIMINATION METHOD:



$x+y=2$

$x-y=0$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$



$x+y=2$

$x+2y=4$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x+y=2$

$\boxed{y=2-x}$

$V=\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 2-x \end{pmatrix}$

$x=1, \quad V=\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$x=0.5 \quad V=\begin{pmatrix} 0.5 \\ 1.5 \end{pmatrix}$

$x+y=5$

$x+y=9$

$$\begin{bmatrix} 1 & 1 & 5 \\ 0 & 0 & 4 \end{bmatrix}$$

$x+y=2$

$2x+2y=4$

$x+y+z=3$

$x-y+z=1$

Sum of 2 above $2x+0y+2z=4$

$x+y+z=3$

$y+z=5$

$x+y=3$

$x+z=4$

LU DECOMPOSITION METHOD:

$A = LU$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

Solve by LU decomposition Method

$$x + y + z = 6$$
$$2x + y + z = 8$$
$$x - y + z = 2$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \xrightarrow{\text{Next Step}} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & -2 & 0 \end{bmatrix} \begin{matrix} \\ R_2 - 2R_1 \\ R_3 - R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & -2-(2)(-1) & 0-2(-1) \end{bmatrix} \xrightarrow{\text{Next Step}} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

This matrix is called U - Matrix

$$U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 2 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$AX = b$, where we put $A = LU$

$LUX = b$ $\quad \therefore UX = y$

$\therefore y = UX \rightarrow (1)$

$Ly = b \rightarrow (2)$

$(2) \Rightarrow Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix}$$

$\boxed{y_1 = 6}$

$2y_1 + y_2 = 8$

$2(6) + y_2 = 8$

$12 + y_2 = 8$

$\boxed{y_2 = -4}$

$y_1 + 2y_2 + y_3 = 2$

$6 + 2(-4) + y_3 = 2$

$6 - 8 + y_3 = 2$

$\boxed{y_3 = 4}$

سسٹم کو دو حصوں میں بریک کر دیتے ہیں
اپر ٹرائینگل اور لوور ٹرائینگل

① ⇒ $y = Ux$

$$\begin{pmatrix} 6 \\ -4 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$6 = x + y + z$$
$$-4 = -y - z$$
$$4 = 2z$$

$$-4 = -y - 2$$
$$-4 + 2 = -y$$
$$\cancel{-}+2 = \cancel{-}+y$$
$$\boxed{y = 2}$$

$$\cancel{6 = x + y + z}$$
$$z = \cancel{4}^{2} / \cancel{2}_{1}$$
$$\boxed{z = 2}$$

$$6 = x + 2 + 2$$
$$6 = x + 4$$
$$6 - 4 = x$$
$$\boxed{x = 2}$$

as same as Gauss's elimination $x_1^0, x_2^0, x_3^0,$ x_1^1, x_2^1, x_3^1

System of Linear Equation

GAUSS'S JACOBIAN METHOD: (iterative Method)

(Initial Method of Jacobi & serial are Same)

$3x + y + z = 5$

$x + 4y + z = 10$

$2x + y - 5z = 3$

$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 4 & 1 \\ 2 & 1 & -5 \end{bmatrix}$$

sum, sum, sum < diagonal

Initial Condition

diagonally dominate $|3| > |1| + |1|$

$|4| > |1| + |1|$

$|-5| > |2| + |1|$

Diagonal Dominate

$3x + y + z = 5 \Rightarrow x = \frac{5 - y - z}{3}$

$x + 4y + z = 10 \Rightarrow y = \frac{10 - x - z}{4}$

$2x + y - 5z = 2 \Rightarrow z = \frac{-2 + 2x + y}{5}$

JACOBI METHOD:

$x_1^{n+1} = \frac{5 - x_2^n - x_3^n}{3} =$

$x_2^{n+1} = \frac{10 - x_1^n - x_3^n}{4} =$

$x_3^{n+1} = \frac{-2 + 2x_1^n + x_2^n}{5} =$

Let $\underset{\text{vector}}{X^0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \underset{\text{component}}{\begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{pmatrix}}$

$\boxed{n=0}$

$x_1^1 = \frac{5 - x_2^0 - x_3^0}{3} = \frac{5 - 0 - 0}{3} = \frac{5}{3} = 1.666666$

$x_2^1 = \frac{10 - x_1^0 - x_3^0}{4} = \frac{10 - 0 - 0}{4} = \frac{10}{4} = 2.5$

$x_3^1 = \frac{-2 + 2x_1^0 + x_2^0}{5} = \frac{-2 + 2(0) + 0}{5} = \frac{-2}{5} = -0.4$

$\boxed{n=1}$

$x_1^2 = \frac{5 - x_2^1 - x_3^1}{3} = \frac{5 - 2.5 - (-0.4)}{3} = 0.966666$

$x_2^2 = \frac{10 - x_1^1 - x_3^1}{4} = \frac{10 - 1.66666 - (-0.4)}{4} = 2.1833335$

$x_3^2 = \frac{-2 + 2x_1^1 + x_2^1}{5} = \frac{-2 + 2(1.666666) + 2.5}{5} = 0.7666664$

Magnitude [illegible] Method Norm Holy Hall

→ One Norm

→ Vector Norm

n	0	1	2	3	4	5	6
x^n_1	0	1.666666	0.966666	0.6833333	0.8368892	0.829944367	0.80047765
x^n_2	0	2.5	2.1833335	2.066666	2.22350015	2.2911.35	2.197649966
x^n_3	0	−0.4	0.7666664	0.4226664	0.28666654	0.3794557	0.3758000168

$$E = \frac{|x^n - x^{n-1}|_\infty}{|x^n|_\infty}$$

$$E = \frac{|x^6 - x^5|_\infty}{|x^6|_\infty}$$ Vector Norm

$$x^6 = \begin{pmatrix} 0.80047765 \\ 2.197649966 \\ 0.3758000168 \end{pmatrix}$$

$$|x^6|_\infty = 2.197649966$$

$$x^6 - x^5 = \begin{pmatrix} 0.80047765 \\ 2.197649966 \\ 0.3758000168 \end{pmatrix} - \begin{pmatrix} 0.829944367 \\ 2.2911135 \\ 0.3794557 \end{pmatrix}$$

$$x^6 - x^5 = \begin{pmatrix} -0.029466717 \\ -0.021461384 \\ -0.0036 5532 \end{pmatrix}$$

$$|x^6 - x^5|_\infty = 0.029466717$$

$$E = \frac{0.029466717}{2.197649966}$$

$$E = 0.0134082 8497$$

$tol = 10^{-2}$

$0.0134082 < 0.01$

Fast converge ...

SEIDAL METHOD

$3x_1 + x_2 + x_3 = 5$

$x_1 + 4x_2 + x_3 = 10$

$2x_1 + x_2 - 5x_3 = 2$

$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 4 & 1 \\ 2 & 1 & -5 \end{bmatrix}, \quad \begin{matrix} |3| > |1| + |1| \\ |4| > |1| + |1| \\ |-5| > |2| + |1| \end{matrix}$$

$3x_1 + x_2 + x_3 = 5 \Rightarrow x_1 = 5 - x_2 - x_3/3$

$x_1 + 4x_2 + x_3 = 10 \Rightarrow x_2 = 10 - x - x_3/4$

$2x_1 + x_2 - 5x_3 = 2 \Rightarrow x_3 = -2 + 2x_1 + x_2/5$

Seidal Method

$$x_1^{n+1} = \frac{5 - x_2^n - x_3^n}{3} =$$

$$x_2^{n+1} = \frac{10 - x_1^{n+1} - x_3^n}{4}$$

$$x_3^{n+1} = \frac{-2 + 2x_1^{n+1} + x_2^{n+1}}{5}$$

let $x^0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{pmatrix}$

$\boxed{n=0}$

$$x_1^1 = \frac{5 - x_2^0 - x_3^0}{3} = \frac{5-0-0}{3} = \frac{5}{3} = 1.666666$$

$$x_2^1 = \frac{10 - x_1^1 - x_3^0}{4} = \frac{10 - 1.66666 - 0}{4} = 2.0833335$$

$$x_3^1 = \frac{-2 + 2x_1^1 + x_2^1}{5} = \frac{-2 + 2(1.66666) + 2.0833335}{5} = 0.683331$$

$\boxed{n=1}$

$$x_1^2 = \frac{5 - x_2^1 - x_3^1}{3} = \frac{5 - 2.0833335 - 0.6833331}{3} = 2.6888888$$

$$x_2^2 = \frac{10 - x_1^2 - x_3^1}{4} = \frac{10 - 2.688888 - 0.683333}{4} = 1.65694455$$

$$x_3^2 = \frac{-2 + 2x_1^2 + x_2^2}{5} \quad \frac{-2 + 2(2.6888888) + (1.6694455)}{5} = 1.00694443$$

$6x_1 - 2x_2 + x_3 = 11$
$x_1 + 2x_2 - 5x_3 = -1$
$-2x_1 + 7x_2 + 2x_3 = 5$
by Gauss's Seidal Method with Toll $= 10^{-2}$

$$\begin{bmatrix} 6 & -2 & 1 \\ 1 & 2 & -5 \\ -2 & 7 & 2 \end{bmatrix} \quad \begin{array}{l} |6| > |2| + |1| \\ |2| > |1| + |-5| \\ |2| > |2| + |-2| \end{array}$$

$$\begin{bmatrix} 6 & -2 & 1 \\ -2 & 7 & 2 \\ 1 & 2 & -5 \end{bmatrix} \quad \begin{array}{l} |6| > |-2| + |1| \\ |7| > |-2| + |2| \\ |-5| > |2| + |1| \end{array}$$

$6x_1 - 2x_2 + x_3 = 11$
$-2x_1 + 7x_2 + 2x_3 = 5$
$x_1 + 2x_2 - 5x_3 = -1$

$$x_1 = \frac{2x_2 - x_3 + 11}{6}$$

$$x_2 = \frac{2x_1 - 2x_3 + 5}{7}$$

$$x_3 = \frac{2x_2 + x_1 + 1}{5}$$

Iterative Forms

$$x_1^{n+1} = \frac{11 + 2x_2^n - x_3^n}{6}$$

$$x_2^{n+1} = \frac{5 + 2x_1^{n+1} - 2x_3^n}{7}$$

$$x_3^{n+1} = \frac{1 + 2x_2^{n+1} + x_1^{n+1}}{5}$$

Let $X_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{pmatrix}$

$$x_1' = \frac{11 + 2x_2^0 - 2x_3^0}{6} = \frac{11 + 0 - 0}{6} = \frac{11}{6} = 1.8333333$$

$$x_2' = \frac{5 + 2(1.8333333) - 2(0)}{7} = 1.238095229$$

$$x'_3 = \frac{2(1.23895229) + 1.8333333 + 1}{5} = 1.061904752$$

n	0	1	2	3	4	5	6
x_1	0	1.8333333	2.069047761	1.9998242627	1.998810334	2.00011845	2.000000748
x_2	0	1.2388898523	1.00201488	0.995319082	1.000295999	1.000068455	1.000006997
x_3	0	1.064904735	1.0416625584	0.99977764580	0.9999988258	1.0000009512	1.000080859

- curve fitting
- interpolation
- Extrapolation

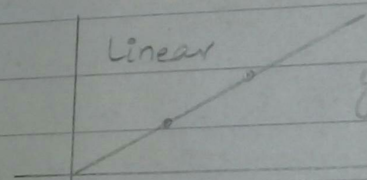
INTERPOLATION:

- ★ Lagrange Interpolation
- ★ Newton divided difference Interpolation
- ★ Curve fitting

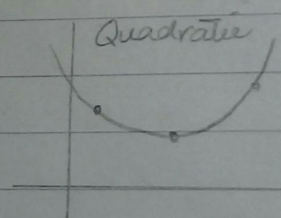
t	Population
1951	33.4
1961	52.2
1972	88.2
1981	100.2
1998	155.6
2017	222.5

If data set is given then we use curve fitting

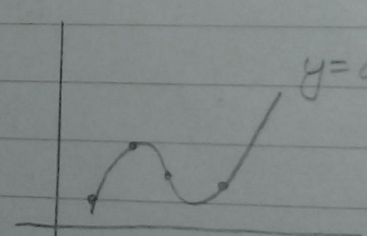
	Pakistan	Sweden
0	8	
1–4	4	
5–9	1	
10–14	0	
15–19	0	
20–24	0	
25–29	1	
30–34		
35–39		
40–44		
45–49		



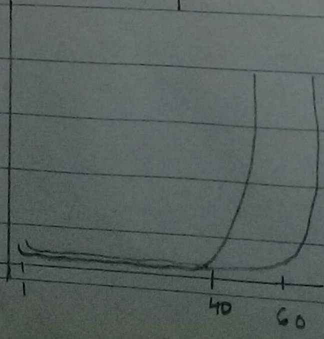
$y = ax + b$

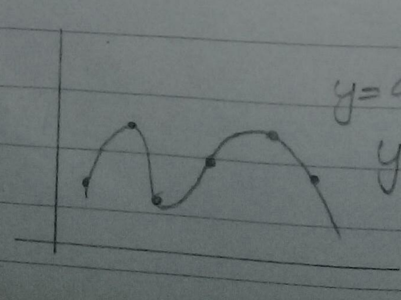


$y = ax^2 + bx + c$



$y = ax^3 + bx^2 + cx + d$





$y = ae^{bx}$

$y = \dfrac{a}{b + e^{cx}}$

جتنا ڈگری کی Polynomial [illegible] ڈگری زیادہ

Time does'nt Have absolute value

$P_2(x) = L_0y_0 + L_1y_1 + L_2y_2$

$L_0 = \dfrac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$

$L_1 = \dfrac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$

$L_2 = \dfrac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$

x	y
x_0	y_0
x_1	y_1
x_2	y_2

$P_3(x) = L_0y_0 + L_1y_1 + L_2y_2 + L_3y_3$

$L_0 = \dfrac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$

$L_1 = \dfrac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$

$L_2 = \dfrac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}$

$L_3 = \dfrac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$

x	y
x_0	y_0
x_1	y_1
x_2	y_2
x_3	y_3

Find the Langrauge

subtract

t	y
1981	10.2
1998	15.6
2017	22.5

t	y
0	10.2
17	15.6
36	22.5

Time k case may hum scaling krty Hain

$P_2 = L_0y_0 + L_1y_1 + L_2y_2$

$L_0 = \dfrac{(x-17)(x-36)}{(0-17)(0-36)} = \dfrac{x^2-53x+612}{612}$

$L_1 = \dfrac{(x-0)(x-36)}{(17-0)(17-36)} = \dfrac{x^2-36x}{-323}$

$L_2 = \dfrac{(x-0)(x-17)}{(36-0)(36-17)} = \dfrac{x^2-17x}{684}$

⊖ Heavy Computation

$$P_2 = \frac{x^2 - 53x + 612}{612}(10.2) + \frac{x^2 - 36x}{-323}(15.6) + \frac{(x^2 - 17x)}{6684}(22.5)$$

$$P_2 = (0.016666x^2 - 0.088329x + 10.01498592) + (-0.048297x^2 + 1.738692x) + (0.032894x^2 - 0.559198x)$$

$$P_2(x) = 0.0012088x^2 + 0.2961552x + 10.2$$

b) Find y at x = 1999

$t = 18$

$$P_2(18) = 0.0012088(18)^2 + 0.2961552(18) + 10.2$$

$$\boxed{P_2(18) = 15.92244448}$$

ⓒ Find y at x = 2030

$t = 2030 - 1981 = 49$

$$P_2(49) = 0.0012088(49)^2 + 0.2961552(49) + 10.2$$

$$\boxed{P_2(49) = 27.6139336}$$

اگر ڈائریکٹلی کوئی پوائنٹ پوچھا ہو تو ہم پولی نومیل فنکشن نہیں لگائیں گے
اسکی وجہ 't' کی وہ 'value' ہے جو فنکشن کرتی ہوگی وہی پلاٹ کر دینگے
پولی نومیل کی جگہ -

• Alternative form of Langrange

form of Langrange

NEWTON DIVIDED INTERPOLATION:

→ If New points are inserted than no need to work in from initial stage in this technique

$P_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2) + \ldots$

x	y	IDD
x_0	y_0	
		$\frac{y_1-y_0}{x_1-x_0}$
x_1	y_1	
		$\frac{y_2-y_1}{x_2-x_1}$
x_2	y_2	
		$\frac{y_3-y_2}{x_3-x_2}$

x	t	1DD	2DD	3DD
0	10.2 a_0			
		$\frac{15.6-10.2}{17-0} =$ 0.317647 a_1		
			$\frac{0.363157-0.317647}{36-0} =$	
17	15.6		a_2 0.00126416	$\frac{0.1266018-0.00126416}{40-0}$
		$\frac{22.5-15.6}{36-17} =$ 0.363157		$a_3 =$ 0.00313344
			$\frac{3.275-0.363157}{40-17} =$	
36	22.5		0.1266018	
		$\frac{35.6-22.5}{40-36} =$ 3.275		
40	35.6			

$P_3(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2)$

$= 10.2 + 0.317647(x-0) + 0.00126416(x-0)(x-17) + 0.00313344(x-0)(x-17)(x-36)$

$P_3(x) =$

ⓒ interrupt at $x = 37$

ⓓ Extrapulate $x = 45$

Q#1: Construct the difference table.

x	1	1.2	1.9	3.2	4.2
y	2	6.2	7.1	5.3	2.9

ⓑ Construct NOIP (Newton Division interpulation)

ⓒ Interpulate at $x = 3.1$

ⓓ Extrapulate at $x = 5.2$

x	t	1DD	2DD	3DD
x_0 1	2 a_0			
		$\frac{6.2-2}{1.2-1}$ 21 a_1		
			$\frac{1.285714-21}{1.9-1} =$ -21.904766 a_2	
x_1 1.2	6.2			
		$\frac{7.1-6.2}{1.9-1.2} =$ 1.285714		$\frac{-1.33516465+21.904766}{1.9-1} =$ 22.855511261 a_3
			$\frac{-1.38461153-1.285714}{3.2-1.2} =$ -1.33516465	
x_2 1.9	7.1			
		$\frac{5.3-7.1}{3.2-1.9} =$ -1.3846153		$\frac{-0.441471608+1.33516465}{4.2-1.9} =$ 0.388562192
			$\frac{-2.4+1.3846153}{4.2-1.9} =$ -0.441471608	
x_3 3.2	5.3			
		$\frac{2.9-5.3}{4.2-3.2} =$ -2.4		
x_4 4.2	2.9			