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QUIZ 2 Kalkulus

1) b.) $\lim_{x \to 1^+} \frac{\cos^2(x-1) - 1}{e^{(x-1)^2} - 1}$ uji : $\frac{\cos^2 0 - 1}{e^{(0)^2} - 1} = \frac{1-1}{e^0 - 1} = \frac{0}{1-1} = \frac{0}{0}$

$\overset{L}{=} \lim_{x \to 1^+} \frac{\cancel{2}\sin(x-1)\cos(x-1)}{\cancel{2}(x-1)\, e^{(x-1)^2} \ln e} = \lim_{x \to 1^+} \frac{-\sin(x-1)}{(x-1)} \cdot \frac{\cos(x-1)}{e^{(x-1)^2}}$

$= -1 \cdot \frac{\cos(0)}{e^0} = -1 \cdot \frac{1}{1} = -1$ //

2) b.) $\lim_{x \to 0^+} \left(\frac{e^x}{x} - \frac{1}{e^x - 1} \right)$ uji : $\frac{e^0}{0} - \frac{1}{e^0 - 1} = \frac{1}{0} - \frac{1}{0} = \infty - \infty$

$\overset{L}{=} \lim_{x \to 0} \frac{e^x}{1} - \frac{1}{e^x} =$ subst: $= \frac{e^0}{1} - \frac{1}{e^0} = \frac{1}{1} - \frac{1}{1} = 0$

3) b.) $\int_{-\infty}^{\infty} \frac{x^5 \, dx}{(1+x^6)^2} = \lim_{s \to -\infty} \int_s^0 \frac{x^5 \, dx}{(1+x^6)^2} + \lim_{t \to \infty} \int_0^t \frac{x^5 \, dx}{(1+x^6)^2}$

$\int \frac{x^5}{(1+x^6)^2} dx = \int \frac{\cancel{x^5}}{(1+x^6)^2} \frac{d(1+x^6)}{6\cancel{x^5}} = \frac{1}{6} \int (1+x^6)^{-2} \, d(1+x^6)$

$= \frac{1}{6} (-1)(1+x^6)^{-1} = -\frac{1}{6(1+x^6)}$

$\lim_{s \to -\infty} \left(-\frac{1}{6(1+x^6)} \right) \Big|_s^0 + \lim_{t \to \infty} \left[-\frac{1}{6(1+x^6)} \right] \Big|_0^t$

$= -\frac{1}{6 \cdot 1} + \lim_{s \to \infty} \frac{1}{6(1+s^6)} - \lim_{t \to \infty} \frac{1}{6(1+t^6)} + \frac{1}{6 \cdot 1}$

$= -\frac{1}{6} + 0 - 0 + \frac{1}{6} = 0$ konvergen

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No. :

4) b.) $\int_{2}^{5} \frac{dx}{x^2-9x+18}$ cek $x^2-9x+18=0$

$x \quad -6 = -6x$

$x \quad -3 = -3x$

$\qquad\qquad -9x$

$\therefore (x-6)(x-3)$

$x=6 \vee x=3$ → infinite integrand dipecah

$$\int_{2}^{3^-} \frac{dx}{x^2-9x+18} + \int_{3^+}^{5} \frac{dx}{x^2-9x+18}$$

•) $\frac{1}{x^2-9x+18} = \frac{1}{(x-6)(x-3)} = \frac{A}{(x-6)} + \frac{B}{(x-3)}$

$1 = A(x-3) + B(x-6)$

– Saat $x=3$:

$1 = B(-3) \iff B = -\frac{1}{3}$

– Saat $x=6$

$1 = A(3) \iff A = \frac{1}{3}$

•) $\int \frac{1}{x^2-9x+18} = \int \frac{1}{3(x-6)} - \int \frac{1}{3(x-3)} = \frac{1}{3}\left(\ln(x-6) - \ln(x-3)\right) = \frac{1}{3}\ln\left(\frac{x-6}{x-3}\right)$

•)

$\Rightarrow = \cancel{\frac{1}{3}\ln\left(\frac{x-6}{x-3}\right)\Big|_2^{3^-}} +$

$$= \lim_{s\to 3^-} \frac{1}{3}\left(\frac{x-6}{x-3}\right)\Big|_2^{s} + \lim_{t\to 3^+} \frac{1}{3}\left(\frac{x-6}{x-3}\right)\Big|_t^{5}$$

$$= \frac{1}{3}\left(\frac{-3}{0} - \frac{-4}{-1}\right) + \frac{1}{3}\left(\frac{-1}{2} - \frac{-3}{0}\right)$$

$$= \frac{1}{3}(\infty - 4) + \frac{1}{3}\left(-\frac{1}{2} - \infty\right) = \text{Divergent } \infty$$

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5.) b.) $\int_1^{2e} \frac{1}{x\sqrt{1-\ln^2(x/2)}}\,dx$ cek saat 2e : $\frac{1}{2\sqrt{1-\ln^2 e}} = \frac{1}{2\sqrt{1-1}} = \frac{1}{0}$

infinite integrand

saat 1 : $\frac{1}{1\sqrt{1-\ln^2 1/2}}$ aman

let $\ln(x/2) = \sin\theta$ $\qquad \theta = \arcsin(\ln x/2)$

$\frac{2}{x}\,dx = \cos\theta\, d\theta$

$dx = \frac{x}{2}\cos\theta\, d\theta$

$\rightarrow \lim_{t\to 2e} \int_b^a \frac{d\theta}{\cancel{x}\sqrt{1-\sin^2\theta}}\left(\frac{\cancel{x}}{2}\right)\cos\theta = \lim_{t\to 2e} \frac{1}{2}\int_b^a \frac{\cancel{\cos\theta}\, d\theta}{\cancel{\cos\theta}} = \lim_{t\to 2e} \frac{1}{2}\,\theta\Big|_b^a$

$= \lim \frac{1}{2}\left[\arcsin(\ln(2e/2)) - \arcsin(\ln(1/2))\right]$

$= \frac{1}{2}\left(\frac{\pi}{2} - \arcsin(\ln\frac{1}{2})\right)$ //

konvergen