Top Quiz 1 - 220682052 - Juan Maxwell Tanaya

1b. $f(x) = \cos(x)\, e^{\cos(x^2)}$, $f'(x) = ?$

$$f'(x) = \cos(x)\, d\left(e^{\cos(x^2)}\right) + e^{\cos(x^2)}\, d(\cos(x))$$
$$= \cos(x) \cdot e^{\cos(x^2)} \cdot -\sin(x^2) \cdot 2x\, dx + e^{\cos(x^2)} \cdot -\sin(x)\, dx$$
$$= \left(-2x(\cos(x))(e^{\cos(x^2)})(\sin(x^2)) - \sin(x)\, e^{\cos(x^2)}\right) dx$$
$$= -e^{\cos(x^2)}\left(2x\cos(x)\sin(x^2) + \sin(x)\right) dx$$

2b. $\frac{\ln(y^2)}{x} + e^{xy} + \csc^2(x) = 1$, $\frac{dy}{dx} = ?$

let $u = e^{xy}$

$\ln u = xy \ln e$

$\frac{du}{u} = x\,dy + y\,dx$

$$d\left(\frac{\ln(y^2)}{x}\right) + d(e^{xy}) + d(\csc^2(x)) = d(1)$$
$$\frac{d(\ln(y^2))}{x} + \ln(y^2)\, d(x^{-1}) + x\,dy + y\,dx + -2\cot(x)\csc^2(x)\,dx = 0$$
$$\frac{2}{xy^2}\,dy + -\frac{\ln(y^2)}{x^2}\,dx + x\,dy + y\,dx - 2\cot(x)\csc^2(x)\,dx = 0$$
$$\left(\frac{2}{xy} + x\right) dy = \left(\frac{\ln(y^2)}{x^2} - y + 2\cot(x)\csc^2(x)\right) dx$$
$$\frac{dy}{dx} = \frac{\left(\frac{\ln(y^2)}{x^2} - y + 2\cot(x)\csc^2(x)\right)}{\left(\frac{2}{xy} + x\right)}$$

3a. $\int \frac{\sin^3(2t)}{\csc(2t)}\, dt = \int \frac{\sin^3(2t)}{\frac{1}{\sin(2t)}}\, dt$

$$= \int \sin^4(2t)\, dt$$
$$= \int (\sin^2(2t))^2\, dt$$
$$= \int \left(\frac{1-\cos(4t)}{2}\right)^2 dt$$
$$= \int \frac{1 - 2\cos(4t) + \cos^2(4t)}{4}\, dt$$
$$= \int \frac{1}{4} - \frac{\cos(4t)}{2} + \frac{\cos^2(4t)}{4}\, dt$$
$$= \frac{1}{4}t - \frac{\sin(4t)}{8} + \int \frac{\cos(8x)+1}{4}\, dt$$
$$= \frac{1}{4}t - \frac{\sin(4t)}{8} + \frac{\sin(8t)}{32} + C$$

4a. $\int \frac{1}{x^2 - x - 2}\, dx = \int \frac{1}{(x+1)(x-2)}\, dx = \int -\frac{1}{3(x+1)} + \frac{1}{3(x-2)}\, dx$

$\frac{1}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-2)}$

$A = \frac{1}{(x+1)(x-2)}(x+1)\Big|_{x \to -1}$

$A = \frac{1}{-3}$

$B = \frac{1}{(x+1)(x-2)}(x-2)\Big|_{x \to 2}$

$B = \frac{1}{3}$

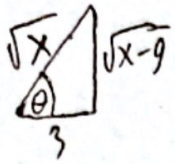
$$= \int -\frac{1}{3x+3} + \frac{1}{3x-6}\, dx$$
$$= -\frac{\ln(3x+3)}{3} + \frac{\ln(3x-6)}{3}$$
$$= \frac{1}{3}\left(\ln(3x-6) - \ln(3x+3)\right)$$
$$= \frac{1}{3}\left(\ln\left(\frac{3x-6}{3x+3}\right)\right)$$
$$= \frac{1}{3}\left(\ln\left(\frac{x-2}{x+1}\right)\right)$$

5a. $\int \frac{2x}{\sqrt{x-9}}\,dx = \int \frac{2(9\sec^2\theta)}{\sqrt{(9\sec^2\theta - 9)}}\,18\tan\theta\sec^2\theta\,d\theta$

$= \int \frac{108}{3\cancel{\tan\theta}}\,\tan\theta\sec^4\theta\,d\theta$

$\sqrt{x} = 3\sec\theta$

$x = 9\sec^2\theta$



$= \int 108\sec^4\theta\,d\theta$

$= 108\int \sec^4\theta\,d\theta = 108\int \sec^2\theta\,d\tan\theta$

$= 108\left(\int \tan^2\theta + 1\,d\tan\theta\right)$

$= 108\left(\frac{\tan^3\theta}{3} + \tan\theta\right) + C$

$dx = 18\tan\sec^2\theta\,d\theta$