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POP QUIZ 9

1] b.) titik kritis $f(x,y) = x^3 + y^3 - 300x - 75y - 3$

* mencari turunan pertama

$f_x(x,y) = 3x^2 - 300$

$f_y(x,y) = 3y^2 - 75$

* mencari x_0 dan y_0

$\Rightarrow 3x^2 - 300 = 0$
$x^2 = 100$
$x = \pm 10$

$\Rightarrow 3y^2 - 75 = 0$
$y^2 = 25$
$y = \pm 5$

* titik kritis

$(10, 5)$ $(-10, 5)$ $(10, -5)$ $(-10, -5)$

* tes diturunan kedua (D)

$D = F_{xx} F_{yy} - F_{xy}^2$

$F_{xx} = 6x$
$F_{yy} = 6y$
$F_{xy} = 0$

$D = 6x \cdot 6y$
$= 36xy$

$\Rightarrow (10, 5)$ dan $(-10, -5)$

$D = 36 \cdot 10 \cdot 5$
$= 1800 > 0$

$F_{xx}(10, 5)$
$= 6 \cdot 10$
$= 60$
minimum //

$F_{xx}(-10, -5)$
$= 6 \cdot (-10)$
$= -60$
maximum //

$\Rightarrow (-10, 5)$ dan $(10, -5)$

$D = -36 \cdot 10 \cdot 5$
$= -1800 < 0$
saddle point //

$\therefore$ Titik kritis :
- $(10, 5)$ sebagai t. minimum
- $(-10, -5)$ sebagai titik maximal
- $(-10, 5)$ dan $(10, -5)$ sbg t. saddle //

2] b.)

$f(x,y) = 4x^3 + y^2$ dgn kendala $g(x,y) = 2x^2 + y^2 - 1 = 0$

* $\nabla f = \lambda \nabla g$

cari turunan pertama

$f_x = 12x^2$
$f_y = 2y$

$g_x = 4x$
$g_y = 2y$

* $\begin{pmatrix} 12x^2 \\ 2y \end{pmatrix} = \lambda \begin{pmatrix} 4x \\ 2y \end{pmatrix}$

(1) ... $12x^2 = \lambda 4x$

(2) ... $2y = \lambda 2y$

(3) ... $2x^2 + y^2 - 1 = 0$

(2) ... $2y = \lambda 2y$
$\lambda = 1$

(1) ... $12x^2 = 1 \cdot 4x$
$x = 1/3$

(3) ... $\frac{2}{9} + y^2 = 1$
$y^2 = \frac{1}{3}\sqrt{7}$

$\left(\frac{1}{3}, \frac{1}{3}\sqrt{7}\right)$

Saat $x = 0$

(1) ... $0 = 0$

(3) ... $0 + y^2 = 1$
$y = \pm 1$

(2) ... $2(\pm 1) = \lambda 2(\pm 1)$
$\lambda = 1$

$(0, \pm 1)$

Saat $y = 0$

(3) ... $2x^2 = 1$
$x = \pm\sqrt{\frac{1}{2}}$

(2) $0 = 0$

(1) $6 = \lambda \frac{4}{\sqrt{2}}$
$\frac{3}{2}\sqrt{2} = \lambda$

$\left(\pm\sqrt{\frac{1}{2}}, 0\right)$

x	y	f
$\frac{1}{3}$	$\frac{\sqrt{7}}{3}$	$\frac{25}{27}$
0	1	1
0	-1	1
$\sqrt{\frac{1}{2}}$	0	$\frac{1}{\sqrt{2}}$
$-\sqrt{\frac{1}{2}}$	0	$-\frac{1}{\sqrt{2}}$

max //

min //

$\left(-\sqrt{\frac{1}{2}}, 0\right)$

$\therefore$ Max di $(0, \pm 1)$

Min di $\left(-\frac{1}{\sqrt{2}}, 0\right)$

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3] b) $A = 192$

$g(p,l,t) = p \cdot l + 2lt + 2pt = 192$

$f(p,l,t) = V = p \cdot l \cdot t$

$\nabla f = \lambda \nabla g$

$$\begin{pmatrix} f_p \\ f_l \\ f_t \end{pmatrix} = \lambda \begin{pmatrix} g_p \\ g_l \\ g_t \end{pmatrix}$$

$$\begin{pmatrix} l+2t \\ p+2t \\ 2p+2l \end{pmatrix} = \lambda \begin{pmatrix} lt \\ pt \\ pl \end{pmatrix}$$

* $l + 2t = \lambda lt \rightarrow$

$p + 2t = \lambda pt$

$2p + 2l = \lambda pl$

* titik kritisnya $(8, 8, 4)$ (p l t)

~~counter example~~

$\therefore$ Vol max $= 8 \cdot 8 \cdot 4 = 256$ //

counter example:

$(2, 2, \frac{47}{2})$

$V = 2 \cdot 2 \cdot \frac{47}{2} = 94$

* $pl + 2lt + 2pt = 192$

$3p + 3l + 4t = \lambda(pl + pt + lt)$

$3p + 3l + 4t = \lambda(\frac{1}{2} 192 - pl)$

asumsi $p = 2x$, $l = 2x$, $t = x$ } agar proporsional

$pl + 2lt + 2pt$

$= 192$

$4x^2 + 4x^2 + 4x^2 = 192$

$12x^2 = 192$

$x^2 = 16$

$x = 4$

$p = 8$, $l = 8$, $t = 4$

4] $\displaystyle\int_1^2 \int_0^1 x e^{x-y} \, dy \, dx$

$\displaystyle\int_0^1 x e^{x-y} \, dy$

$= x e^{x-y}(-1) \Big|_{y=0}^{y=1}$

$= -x e^{x-1} - (-x e^{x-0})$

$= x e^x - x e^{x-1}$

$\displaystyle\int_1^2 x e^x - x e^{x-1} \, dx$

D	I
$+x$	e^x
-1	e^x
0	e^x

$= x e^x - e^x$

D	I
$+x$	e^{x-1}
-1	e^{x-1}
0	e^{x-1}

$= x e^{x-1} - e^{x-1}$

$= \left[x e^x - e^x - x e^{x-1} + e^{x-1} \right]_1^2$

$= e^2 - e = e(e-1)$ //