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No. :

POP QUIZ 3

1.) b.) $\cancel{a_n = \frac{2^n+1}{2^{2n}-1}}$ ~~uji rasio~~ $\cancel{\lim_{n\to\infty} \frac{a_{n+1}}{a_n}}$

$\cancel{a)\ \lim_{n\to\infty} \frac{2^{n+1}+1}{2^{2n+2}-1} \cdot \frac{2^{2n}-1}{2^n+1}}$

L'Hopital

$\cancel{a_n = \lim_{n\to\infty} \frac{2^n+1}{2^{2n}-1} \overset{L}{=} \lim_{n\to\infty} \frac{\ln 2 \cdot 2^n}{\ln 2 \cdot 2n \cdot 2^{2n}} \quad \lim_{n\to\infty} \frac{2^n}{2n \cdot 2 \cdot 2^n}}$:

L'Hopital

$$\lim_{n\to\infty} a_n = \frac{e^{2n}+1}{2n+1} \overset{L}{=} \lim_{n\to\infty} \frac{\cancel{2} \cdot e^{2n}}{\cancel{2}} = \lim_{n\to\infty} e^{2n} = \infty \quad \text{Divergen} //$$

limit comp.

2.) a.) $1 + \frac{1}{2(1+\ln 4)} + \frac{1}{3(1+\ln 9)} + \frac{1}{4(1+\ln 16)} + \dots$ ~~Divergence~~ t

$$\text{pola} = \sum_{n=1}^{\infty} \frac{1}{n(1+\ln n^2)},\quad n \geq 1$$

bukti lain $\frac{1}{n} > \frac{1}{n(1+\ln n^2)}$

pasti $\frac{1}{n^p} \to > 1$

cek: $\lim_{n\to\infty} \frac{1}{n(1+\ln n^2)} = \frac{1}{\infty} = 0$ ~~konvergen~~

~~bukti lain dgn rasio~~ ~~konvergen~~ jika $\frac{1}{n^p} \to$ harus $p > 1$ utk konvergen

→ disini ada faktor lain $(1+\ln n^2)$ maka konvergen //

3.) a.) $\sum_{n=1}^{\infty} \frac{\sqrt{\sqrt{n}+n^2}}{\sqrt{n}-2n}$ seperti $\frac{\sqrt{n^2}}{-2n} = \frac{n}{-2n} = -\frac{1}{2}$ diambil pangkat tertinggi

$$\lim_{n\to\infty} \left\{ \frac{\sqrt{\sqrt{n}+n^2}}{\sqrt{n}-2n} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} \right.$$

$$= \lim_{n\to\infty} \frac{\sqrt{\frac{1}{\sqrt{n}}+1}}{\frac{1}{\sqrt{n}}-2} = -\frac{1}{2}$$

maka $\sum_{n=1}^{\infty} -\frac{1}{2} = -\infty$ divergen

Dengan limit comparison Test $\lim_{n\to\infty} \frac{a_n}{b_n} = L$

→ $\sqrt{\sqrt{n}+n^2} / (\sqrt{n}-2n) / -\frac{1}{2} = -\sqrt{\sqrt{n}+n^2} / (\sqrt{n}-2n) / 2$

jika b_n diverge, maka a_n yaitu persamaan di soal jg divergen //

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4) a) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^2\sqrt{n+1}}$ ~~compare dgn a_{n+1} yi p $\to \frac{a_{n+1}}{a_n}$~~

limit comparison test jika $\frac{1}{n^{1,25}}$ konvergen maka $\frac{\ln(n)}{n^2\sqrt{n+1}}$ dgn yi $\lim_{n\to\infty} \frac{a_n}{b_n}$

coba dgn compare $\frac{1}{n^{1,25}}$ yg converge → proof $\frac{1}{n^{1,25}}$ → power tertinggi $p > 1$

$\lim_{n\to\infty} \frac{\ln(n)}{n^2\sqrt{n+1}} \cdot \frac{n^{1,25}}{1} = \lim_{n\to\infty} \frac{\ln(n)}{n^{0,75}\sqrt{n+1}} \overset{①}{=} \frac{\lim_{n\to\infty} \frac{1}{n}}{0,75\, n^{-0,25} \cdot \frac{1}{2\sqrt{n+1}}} = \frac{\lim_{n\to\infty} n^{0,25} \cdot \sqrt{n+1}}{n} = 0$

$\frac{n^{0,75}}{n}$ ← lebih besar

5) b.) $\sum_{n=3}^{\infty} (-1)^{n+1} \frac{\sqrt{n^2-4}}{(2n)!}$ abs. ratio test $\frac{a_{n+1}}{a_n}$

$\lim_{n\to\infty} \left| \frac{\sqrt{n^2+2n-3}}{(2n+2)!} \cdot \frac{(2n!)}{\sqrt{n^2-4}} \right| = \lim_{n\to\infty} \left| \frac{2n!}{(2n+2)(2n+1)\,2n!} \cdot \frac{\sqrt{n^2+2n-3}}{\sqrt{n^2-4}} \right|$

L'opital $\frac{\infty}{\infty}$ $=1$ pangkat sama

$= \lim_{n\to\infty} \left| \frac{1}{(2n+2)(2n+1)} \right| = 0 \quad p<1$ //

$= \lim_{n\to\infty} \left| \frac{1}{4n^2+6n+2} \right|$ ∴ konvergen

BUKTI LAIN NO 4

4) a.) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^2\sqrt{n+1}}$ ~~coba bandingkan~~ dgn

$\frac{\ln(n)}{n^2\sqrt{n+1}} < \frac{n}{n^2\sqrt{n+1}} = \frac{1}{n\sqrt{n+1}}$ mirip $\frac{1}{n^{3/2}}$

menurut comparison test jika $\frac{1}{n^p} \to p > 1$

maka $\sum \ln(n)/n^2\sqrt{n+1}$ converge //