Nama : Arzana Raffan Mawardi — POP QUIZ 2

NPM : 2306152393

Kelas : B

① a. $\lim_{x\to 1^+} \dfrac{\sin 2(x-1)}{e^{(x-1)^2}-1} \Rightarrow \dfrac{\sin 2(1-1)}{e^{(1-1)^2}-1} \Rightarrow \dfrac{0}{0}$ indeterminate

use L'Hopital

$\Rightarrow \lim_{x\to 1^+} \dfrac{d(\sin 2(x-1))}{d(e^{(x-1)^2}-1)} \Rightarrow \lim_{x\to 1^+} \dfrac{2\cdot\sin(x-1)\cdot\cos(x-1)}{2(x-1)\,e^{(x-1)^2}}$

$\dfrac{1}{2}\cdot\lim_{x\to 1^+} \dfrac{\sin(2(x-1))}{(x-1)\,e^{(x-1)^2}} \Rightarrow \dfrac{1}{2}\cdot\dfrac{\sin(0)}{0\cdot e^0} \Rightarrow \dfrac{0}{0}$ still indeterminate

L'Hopital

$\dfrac{1}{2}\cdot\lim_{x\to 1^+} \dfrac{d(\sin(2x-2))}{d((x-1)\,e^{(x-1)^2})} \Rightarrow \dfrac{1}{2}\lim_{x\to 1^+} \dfrac{2\cdot\cos(2x-2)}{e^{(x-1)^2}+2(x-1)\,e^{(x-1)^2}\cdot(x-1)}$

$\Rightarrow \dfrac{1}{2}\,\dfrac{2\cos(0)}{e^0+0} \Rightarrow \dfrac{1}{2}\cdot\dfrac{2}{1} = \underline{\underline{1}}$

② a. $\lim_{x\to 1^+}\left(\dfrac{e^x}{x-1} - \dfrac{1}{e^x-e}\right) \Rightarrow$ jadikan bentuk $\dfrac{f(x)}{g(x)}$

$\Rightarrow \dfrac{e^x(e^x-e)-(x-1)}{(x-1)(e^x-e)} \Rightarrow \dfrac{e^{2x}-e^{x+1} \quad x+1}{(x-1)(e^x-e)}$

apply L'Hopital

$\Rightarrow \dfrac{2e^{2x}-e^{x+1}-1}{e^x-e+e^x(x-1)} \Rightarrow \dfrac{2e^{2x}-e^{x+1}-1}{\cancel{e^x}-e+xe^x\cancel{-e^x}} \Rightarrow \lim_{x\to 1^+}\dfrac{2e^{2x}-e^{x+1}-1}{xe^x-e}$

$\Rightarrow \dfrac{2e^2-e^2-1}{e-e} = \dfrac{e^2-1}{0}$ indeterminate

Apply L'Hopital

$\lim_{x\to 1^+}\dfrac{4e^{2x}-e^{x+1}}{e^x+xe^x} \Rightarrow \dfrac{4e^2-e^2}{e+e} \Rightarrow \dfrac{3e^{\cancel{2}}}{2\cancel{e}} \Rightarrow \dfrac{3}{2}e$

③.ⓑ. $\int_{-\infty}^{\infty} \frac{x^5}{(1+x^6)^2}\,dx$

Jawab:

$u = 1+x^6$ $\Rightarrow \int_{-\infty}^{\infty} \frac{\cancel{x^5}}{u^2} \cdot \frac{1}{6x^5}\,du \Rightarrow \frac{1}{6}\int_{-\infty}^{\infty} \frac{1}{u^2}\,du$

$du = 6x^5\,dx$

$dx = \frac{1}{6x^5}\,du$

$\Rightarrow \frac{1}{6}\left(\int_{-\infty}^{0} \frac{1}{u^2}\,du + \int_{0}^{\infty} \frac{1}{u^2}\,du\right)$

Sub kembali u

$\lim_{b\to-\infty} \int_{b}^{0} \frac{1}{u^2}\,du \Rightarrow \lim_{b\to-\infty} \left| -\frac{1}{u} \right|_b^0 \Rightarrow -\lim_{b\to-\infty} \left| \frac{1}{1+x^6} \right|_b^0$

$\Rightarrow -\left(\frac{1}{1+0} - \left(\frac{1}{1+(-\infty)^6}\right)\right) = -(1-0) = -1$

$\lim_{a\to\infty} \int_{0}^{a} \frac{1}{u^2}\,du \Rightarrow \lim_{a\to\infty} \left| -\frac{1}{u} \right|_0^a \Rightarrow -\lim_{a\to\infty} \left| \frac{1}{1+x^6} \right|_0^a$

$\Rightarrow -\lim_{a\to\infty} \left(\frac{1}{1+a^6} - \frac{1}{1+0}\right) \Rightarrow -(0-1) = +1$

$\Rightarrow \frac{1}{6}(-1+1) = 0 \rightarrow$ konvergen

④ ⓑ. $\int_{2}^{5} \frac{dx}{x^2-9x+18} \Rightarrow \int_{2}^{3} \frac{dx}{x^2-9x+18} + \int_{3}^{5} \frac{dx}{x^2-9x+18} = \lim_{a\to3^-} \int_{2}^{a} \frac{dx}{x^2-9x+18} + \lim_{b\to3^+} \int_{b}^{5} \frac{dx}{x^2-9x+18}$

hitung $\int \frac{dx}{x^2-9x+18} \Rightarrow \int \frac{dx}{(x-6)(x-3)} \rightarrow \frac{1}{(x-6)(x-3)} = \frac{A}{x-6} + \frac{B}{x-3}$

$\Rightarrow A \overset{x=6}{=} \frac{1}{(6-3)} = \frac{1}{3}$; $B \overset{x=3}{=} \frac{1}{3-6} = -\frac{1}{3}$

$\Rightarrow \int \frac{1}{3(x-6)} - \frac{1}{3(x-3)} \Rightarrow \frac{1}{3}\ln|x-6| - \frac{1}{3}\ln|x-3|$

maka,

$\lim_{a\to3^-} \left| \frac{1}{3}\ln|x-6| - \frac{1}{3}\ln|x-3| \right|_2^a + \lim_{b\to3^+} \left| \frac{1}{3}\ln|x-6| - \frac{1}{3}\ln|x-3| \right|_b^5$

$\Rightarrow \left(\frac{1}{3}\ln|3| - \frac{1}{3}\underbrace{\ln|0|}_{\infty}\right) - \left(\frac{1}{3}\ldots\right.$

$\Rightarrow$ maka integral divergen

5. (b). $\int_1^{2e} \frac{1}{x\sqrt{1-\ln^2(x/2)}}$

$\ln\left(\frac{x}{2}\right) = \sin\theta\, d\theta \to \theta = \arcsin\left(\ln\left(\frac{x}{2}\right)\right)$

$\frac{1}{\frac{x}{2}} \cdot \frac{1}{2}\, dx = \cos\theta\, d\theta$

$\frac{2}{x} \cdot \frac{1}{2} dx = \cos\theta\, d\theta$

$\frac{1}{x} dx = \cos\theta\, d\theta$

$dx = x\cos\theta\, d\theta$

$\lim_{a \to 2e} \int_1^{a} \frac{1}{x\sqrt{1-\sin^2\theta}} \cdot x\cos\theta\, d\theta$; $\cos^2\theta$

$\lim_{a \to 2e} \int_1^{a} \frac{1}{x\sqrt{\cos^2\theta}} \cancel{d\theta} \cdot x \cdot \cos\theta\, d\theta$

$\lim_{a \to 2e} \int_1^{a} \frac{\cancel{x}\cancel{\cos\theta}\, d\theta}{\cancel{x}\cancel{\cos\theta}} = \int_1^{2e} d\theta$

$\Rightarrow \lim_{a \to 2e} \arcsin\left(\ln\left(\frac{x}{2}\right)\right)\Big|_1^{a}$

$\to \ln(e) = 1$

$\Rightarrow \arcsin\left(\ln\left(\frac{2e}{2}\right)\right) - \arcsin\left(\ln\left(\frac{1}{2}\right)\right)$

$\arcsin(1) - \arcsin\left(\ln\left(\frac{1}{2}\right)\right)$

$\frac{\pi}{2} - \arcsin(-\ln(2)) \to$ Konvergen