Dasar sekali (ingat formula dan teorema dasar integral)

Theorem A **Integrability Theorem**

If f is bounded on $[a, b]$ and if it is continuous there except at a finite number of points, then f is integrable on $[a, b]$. In particular, if f is continuous on the whole interval $[a, b]$, it is integrable on $[a, b]$.

Theorem B **Interval Additive Property**

If f is integrable on an interval containing the points a, b, and c, then

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

no matter what the order of a, b, and c.

Theorem A **First Fundamental Theorem of Calculus**

Let f be continuous on the closed interval $[a, b]$ and let x be a (variable) point in (a, b). Then

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Theorem B **Comparison Property**

If f and g are integrable on $[a, b]$ and if $f(x) \leq g(x)$ for all x in $[a, b]$, then

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$$

In informal but descriptive language, we say that the definite integral preserves inequalities.

Theorem C **Boundedness Property**

If f is integrable on $[a, b]$ and $m \leq f(x) \leq M$ for all x in $[a, b]$, then

$$m(b - a) \leq \int_a^b f(x)\,dx \leq M(b - a)$$

Theorem D Linearity of the Definite Integral

Suppose that f and g are integrable on $[a, b]$ and that k is a constant. Then kf and $f + g$ are integrable and

(i) $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$;

(ii) $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$; and

(iii) $\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$.

Theorem A Second Fundamental Theorem of Calculus

Let f be continuous (hence integrable) on $[a, b]$, and let F be any antiderivative of f on $[a, b]$. Then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Theorem B Substitution Rule for Indefinite Integrals

Let g be a differentiable function and suppose that F is an antiderivative of f. Then

$$\int f(g(x))g'(x)\,dx = F(g(x)) + C$$

EXAMPLE 12 Evaluate $\int_0^1 \frac{x+1}{(x^2+2x+6)^2}\,dx$.

Standard Integral Forms

Constants, Powers	1. $\int k\,du = ku + C$	2. $\int u^r\,du = \begin{cases} \dfrac{u^{r+1}}{r+1} + C & r \neq -1 \\ \ln\lvert u\rvert + C & r = -1 \end{cases}$
Exponentials	3. $\int e^u\,du = e^u + C$	4. $\int a^u\,du = \dfrac{a^u}{\ln a} + C, a \neq 1, a > 0$
Trigonometric Functions	5. $\int \sin u\,du = -\cos u + C$	6. $\int \cos u\,du = \sin u + C$
	7. $\int \sec^2 u\,du = \tan u + C$	8. $\int \csc^2 u\,du = -\cot u + C$
	9. $\int \sec u \tan u\,du = \sec u + C$	10. $\int \csc u \cot u\,du = -\csc u + C$
	11. $\int \tan u\,du = -\ln\lvert\cos u\rvert + C$	12. $\int \cot u\,du = \ln\lvert\sin u\rvert + C$
Algebraic Functions	13. $\int \dfrac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\dfrac{u}{a}\right) + C$	14. $\int \dfrac{du}{a^2 + u^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{u}{a}\right) + C$

EXAMPLE 2 Find $\int \dfrac{3}{\sqrt{5 - 9x^2}}\,dx$.

Gunakan substitusi dan bentuk standard

SOLUTION Think of $\int \dfrac{du}{\sqrt{a^2 - u^2}}$. Let $u = 3x$, so $du = 3\,dx$. Then

$$\int \frac{3}{\sqrt{5 - 9x^2}}\,dx = \int \frac{1}{\sqrt{5 - u^2}}\,du = \sin^{-1}\left(\frac{u}{\sqrt{5}}\right) + C$$

$$= \sin^{-1}\left(\frac{3x}{\sqrt{5}}\right) + C$$

Berikutnya, trigonometry, rational substitution, partial fraction (fungsi pecahan)

Integration by Parts: Indefinite Integrals

$$\int u\, dv = uv - \int v\, du$$

Integration by Parts: Definite Integrals

$$\int_a^b u\, dv = \big[uv\big]_a^b - \int_a^b v\, du$$

EXAMPLE 1 Find $\int x \cos x\, dx$.

Notes: salah pemisalan, bisa tambah sulit.

Reduction Formulas A formula of the form

$$\int f^n(x) g(x)\, dx = h(x) + \int f^k(x)\, g(x)\, dx$$

where $k < n$, is called a **reduction formula** (the exponent on f is reduced). Such formulas can often be obtained via integration by parts.

EXAMPLE 7 Derive a reduction formula for $\int \sin^n x\,dx$.

SOLUTION Let $u = \sin^{n-1} x$ and $dv = \sin x\,dx$. Then

$$du = (n-1)\sin^{n-2} x \cos x\,dx \quad \text{and} \quad v = -\cos x$$

from which

$$\int \sin^n x\,dx = -\sin^{n-1} x \cos x + (n-1)\int \sin^{n-2} x \cos^2 x\,dx$$

If we replace $\cos^2 x$ by $1 - \sin^2 x$ in the last integral, we obtain

$$\int \sin^n x\,dx = -\sin^{n-1} x \cos x + (n-1)\int \sin^{n-2} x\,dx - (n-1)\int \sin^n x\,dx$$

After combining the first and last integrals above and solving for $\int \sin^n x\,dx$, we get the reduction formula (valid for $n \geq 2$)

$$\int \sin^n x\,dx = \frac{-\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$