

CHEAT SHEET

* Explicit Implicit Derivative perfection

$$\text{ex: } x^2 y^3 \rightarrow 2xy^3 + x^2 3y^2 \frac{dy}{dx}$$

$$\sin(x^2 y) \rightarrow \cos(x^2 y) (2xy + x^2 \frac{dy}{dx})$$

* Integration by part

$$\int x^L \sin x \, dx$$

$$\begin{array}{r} + \\ - \\ + \\ - \end{array} \begin{array}{l} D \\ x^2 \\ 2x \\ 2 \\ 0 \end{array} \begin{array}{l} \sin x \\ \cos x \\ -\sin x \\ \cos x \end{array} \rightarrow -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

* Integral perfection

$$\int \frac{1}{x} = \ln|x| + C$$

$$\int a^x = \frac{a^x}{\ln a} + C$$

$$\int \ln x = x \ln x - x + C$$

$$\int \tan x = \ln|\sec x| = -\ln|\cos x| + C$$

$$\int \cot x = \ln|\sin x| + C$$

$$\int \sec x = \ln|\sec x + \tan x| + C$$

$$\int \csc x = \ln|\csc x - \cot x| + C$$

$$\int \sec^2 x = \tan x + C$$

$$\int \sec x \tan x = \sec x + C$$

$$\int \csc^2 x = -\cot x + C$$

$$\int \tan^2 x = \tan x - x + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{ax+b} = \frac{1}{a} \ln|ax+b| + C$$

$$\int \frac{dx}{ax^2+b} = \frac{1}{2a} \ln|ax^2+b| + C$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$$

* Teknik

$$\int u \, dv = uv - \int v \, du$$

$$\begin{array}{c} \frac{d}{dx} \leftarrow \rightarrow \frac{d}{dy} \\ \text{L I P E T} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \end{array}$$

* Case root

$$\sqrt{x^2 + a^2} \rightarrow x = a \tan \theta$$

$$\sqrt{x^2 - a^2} \rightarrow x = a \sec \theta$$

$$\sqrt{a^2 - x^2} \rightarrow x = a \sin \theta$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\sec^2 \theta - 1 = \tan^2 \theta$$

* Both infinite

$$\text{Başlı 2 } \int_{-\infty}^0 \text{ dan } \int_0^{\infty}, \text{ if converge both}$$

* Integrand infinite at end point

$$\lim_{t \rightarrow b^-} \int_a^t f(x) \, dx$$

* Interior

$$\int_c^b f(x) \, dx + \int_b^a f(x) \, dx$$

$$* F. genap \rightarrow 2 F(x)$$

$$F. ganjil \rightarrow 0$$

* INFINITE SERIES

* P-series test

$$\sum_{k=1}^{\infty} \frac{1}{k^p} \begin{cases} \text{conv if } p > 1 \\ \text{div if } p \leq 1 \end{cases}$$

* Ordinary comp test

$$0 \leq a_n \leq b_n$$

$$\begin{cases} \text{if } \sum b_n \text{ converge} \rightarrow \sum a_n \text{ converge} \\ \text{if } \sum a_n \text{ diverge} \rightarrow \sum b_n \text{ diverge} \end{cases}$$

* Limit comp test

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L \begin{cases} 0 < L < \infty, \sum a_n \text{ both conv/div together} \\ L = 0 \text{ and } \sum b_n \text{ conv, } \rightarrow \sum a_n \text{ conv} \end{cases}$$

* Ratio test

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \rho \begin{cases} \rho < 1 \text{ conv} \\ \rho > 1 \text{ div} \\ \rho = 1 \text{ inconclusive} \end{cases}$$

* Integral

$$\int_0^{\infty} \text{ conv} \Leftrightarrow \sum_{n=1}^{\infty} \text{ conv}$$

* Alternating series test

$$\boxed{\lim_{n \rightarrow \infty} a_n = 0 \rightarrow \text{conv}}$$

* Abs. Ratio Test

$$\lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} = \rho$$

* Conditional converge

$$\text{when } \sum a_n \text{ converge while } \sum |a_n| \text{ div}$$

* Taylor & Maclaurin

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

$$\begin{aligned} (1) \frac{1}{1-x} &= 1 + x + x^2 + x^3 + \dots \\ (2) \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \\ (3) \tan^{-1}(x) &= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \\ (4) e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \\ (5) \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \\ (6) \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \\ (7) \sinh x &= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots \\ (8) \cosh x &= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots \\ (9) (1+x)^p &= 1 + \binom{p}{1}x + \binom{p}{2}x^2 + \binom{p}{3}x^3 + \dots \quad -1 < x < 1 \end{aligned}$$

CHEAT SHEET

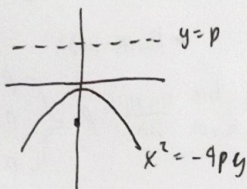
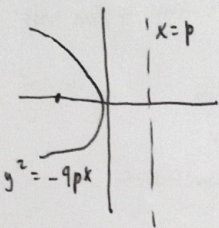
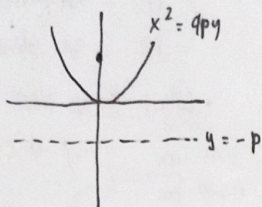
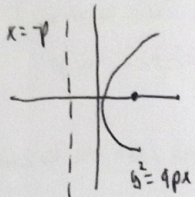
CONIC

$$|PF| = e|PL| \begin{cases} 0 < e < 1 & \text{ellipse} \\ e = 1 & \text{parabola} \\ e > 1 & \text{hyperbola} \end{cases}$$

A) PARABOLA ($e=1$)

standard: $y^2 = 4px$

$F(p, 0)$



B) ELLIPSE ($0 < e < 1$)

standard $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

horizontal
 $a > b$

vertical
 $a < b$

$c^2 = a^2 - b^2$ $e = \frac{c}{a}$ $F(\pm c, 0)$ Direct: $x = \pm \frac{a^2}{c}$

* berpusat di (x_p, y_p)

$F(x_p \pm c, y_p)$

$F(x_p, y_p \pm c)$

Dir: $x = x_p \pm \frac{a^2}{c}$

$y = y_p \pm \frac{b^2}{c}$

Lgr: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

PGS: $\frac{(x-x_p)(x_1-x_p)}{a^2} + \frac{(y-y_p)(y_1-y_p)}{b^2} = 1$

C) HYPERBOLA ($e > 1$)

Standard

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

foci di sb. x, horizontal

$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

foci di sb. y, vertical

asymptote: $y = \pm \frac{b}{a}x$; $y = y_p \pm \frac{b}{a}(x - x_p)$

$c^2 = a^2 + b^2$ $e = \frac{c}{a}$ $F(\pm c, 0)$ Direct: $x = \pm \frac{a^2}{c}$

asymptote: $y = \pm \frac{a}{b}x$

* TRANSLATION

$u = x - h$ $v = y - k$

* Rotation

$x = u \cos \theta - v \sin \theta$
 $y = u \sin \theta + v \cos \theta$
 $\cot(2\theta) = \frac{A - C}{B}$



$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$\begin{matrix} 0^\circ & 30^\circ & 45^\circ & 60^\circ & 90^\circ \\ \cot & \infty & \sqrt{3} & 1 & \frac{\sqrt{3}}{3} & 0 \end{matrix}$
 $\begin{pmatrix} A' \\ B' \\ C' \end{pmatrix} = \begin{pmatrix} \cos^2 \theta & \sin \theta \cos \theta & \sin^2 \theta \\ 0 & 0 & 0 \\ \sin^2 \theta & -\sin \theta \cos \theta & \cos^2 \theta \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix}$
 $\begin{pmatrix} D' \\ E' \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} D \\ E \end{pmatrix}$

- Latihan Soal 10.1

1. $y^2 = 4x$ $F(1, 0)$, dir: $y = -4$

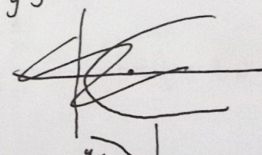
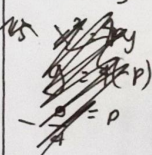
2. $x^2 = -12y$ $F(0, -3)$, dir: $x = 3$

3. $F(2, 0)$ $y^2 = 4 \cdot 2 \cdot x$ $y^2 = 8x$

4. Dir: $x = 3$ $F(0, -3)$ $x^2 = -12y$

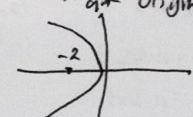
5. Dir: $y = 2$ $F(-2, 0)$ $y^2 = -8x$

6. $F(0, -\frac{1}{9})$ $x^2 = -\frac{4}{9}y$



7. through $(-2, 4)$, vertex at origin along x axis

$y^2 = 4px$
 $2^2 = 4p(-2)$
 $p = -1$



- Latihan Soal 10.2

9. $\frac{x^2}{16} + \frac{y^2}{9} = 1$ $c = \sqrt{16-9} = 2\sqrt{5}$

$a = 4, b = 3$, vert $(\pm 4, 0)$, $F(\pm 2\sqrt{5}, 0)$, dir: $x = \pm \frac{16}{2\sqrt{5}} = \pm \frac{8\sqrt{5}}{5}$

10. $\frac{x^2}{16} - \frac{y^2}{9} = 1$ $c = \sqrt{16+9} = 5$

$a = 4, b = 3$, vert $(\pm 4, 0)$, $F(\pm 5, 0)$, dir: $x = \pm \frac{16}{5}$

$y = \pm \frac{3}{4}x$

11. $-\frac{x^2}{9} + \frac{y^2}{4} = 1$ $c = \sqrt{9+4} = \sqrt{13}$

$a = 3, b = 2$, vert $(0, \pm 2)$, $F(0, \pm \sqrt{13})$, dir: $y = \pm \frac{16}{\sqrt{13}} = \pm \frac{16\sqrt{13}}{13}$

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* 10.3

$$10. x^2 - 4y^2 - 2x + 2y + 1 = 0$$

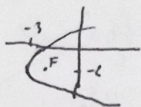
$$(4x^2 + 2x) + (-4y^2 + 2y) + 1 = 0$$

$$4(x^2 + \frac{1}{2}x + \frac{1}{16}) + (-4)(y^2 - \frac{1}{2}y + \frac{1}{4}) = -1$$

$$4(x + \frac{1}{4})^2 - 4(y - \frac{1}{4})^2 = -1$$

$$18. 4(x+3) = (y+2)^2$$

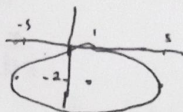
$$f(-3+1, -2)$$



$$23. x^2 + 4y^2 - 2x + 16y + 1 = 0$$

$$x^2 - 2x + 1 + 4(y^2 + 4y + 4) = -1 + 1 + 16$$

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{4} = 1$$



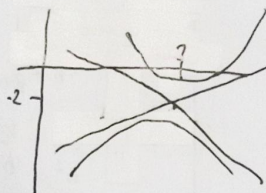
$$26. x^2 - 4y^2 - 14x - 32y - 11 = 0$$

$$(x-7)^2 - 4(y+4)^2 = 11 + 94 - 64 = -1$$

$$\frac{(y+4)^2}{1} - \frac{(x-7)^2}{4} = 1$$

$$\text{vert} \rightarrow y = \frac{a}{b}x$$

$$f($$



$$29. 2(y^2 - 1)^2 = 10x + 2$$

$$2(y-1)^2 = 10(x + \frac{1}{5})$$

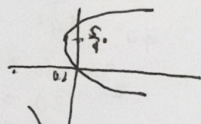
$$(y-1)^2 = 5 \frac{x + \frac{1}{5}}{2}$$

$$f(\frac{5}{2}, 1)$$

$$x = -\frac{5}{2} - \frac{1}{5}$$

$$f(\frac{21}{20}, 1)$$

$$x = \frac{29}{20}$$



$$33. x^2 + xy + y^2 = 6 \quad A = \frac{c}{b} = 6 \quad \theta = 45^\circ$$

$$x = \frac{1}{2}\sqrt{2}u - \frac{1}{2}\sqrt{2}v$$

$$y = \frac{1}{2}\sqrt{2}u + \frac{1}{2}\sqrt{2}v$$

$$\begin{cases} \frac{1}{2}u^2 + \frac{1}{2}v^2 + \frac{1}{2}u^2 - \frac{1}{2}v^2 + \frac{1}{2}u^2 + \frac{1}{2}v^2 = 6 \\ \frac{3}{2}u^2 + \frac{1}{2}v^2 = 6 \quad \frac{u^2}{4} + \frac{v^2}{12} = 1 \end{cases}$$

LAT

$$1. \lim_{x \rightarrow \infty} \frac{x^{3/2} \ln(x)}{(x+1)^2} : x^2$$

$$2. \lim_{x \rightarrow \infty} \frac{\ln(x)}{x^{1/2}} : x^2$$

$$\lim_{x \rightarrow \infty} \frac{\ln(x)}{x^{1/2}} = \lim_{x \rightarrow \infty} \frac{1/x}{1/2 x^{-1/2}} = \lim_{x \rightarrow \infty} \frac{1}{2\sqrt{x}} = 0$$

$$b. \lim_{x \rightarrow \infty} \ln\left(\frac{\ln(x)}{\ln(x)} + 1\right) \ln(x)$$

$$y = \frac{\ln(x)}{\ln(x)} \ln(x)$$

$$5) 5x^2 - 19x + 5y^2 - 2y + 6xy - 19 = 0$$

$$5(x-a)^2 + 5(y-b)^2 + 6(x-a)(y-b) = 19$$

$$-10a - 6b = -19 \quad (x-3)$$

$$-10b - 6a = -19 \quad (x-5)$$

$$30a + 13b = 42 \quad -12b = 37$$

$$50b - 30a = 10 \quad b = -1$$

Titik pusat (2, -1)

$$3. 0, 1, 2, 3, 1, 2, 3$$

$$\sum_{n=1}^{\infty} \frac{123}{1000^n}$$

$$1. \int_{\sqrt{3}}^{\sqrt{6}} \frac{x}{\sqrt{4x^2 + 29}} dx$$

$$\text{cek penyebut } 2/3 \Rightarrow$$

$$(4x^2 - 29) \Rightarrow$$

$$4x^2 = 29$$

$$x^2 = \frac{29}{4}$$

$$x = \frac{\sqrt{29}}{2}$$

$$\int_{\sqrt{3}}^{\sqrt{6}} f(x) dx + \int_{\sqrt{6}}^{\sqrt{3}} f(x) dx$$

$$2) \frac{3}{8} (4x^2 - 29)^{1/2} \Big|_{\sqrt{3}}^{\sqrt{6}}$$

$$\frac{3}{8} (4 \cdot 8 - 29)^{1/2}$$

$$\frac{3}{8} \sqrt{3}$$

$$\frac{3}{8} \sqrt{12}$$

$$2. \lim_{x \rightarrow 0} \cos x \frac{1}{x^2} \rightarrow 1^\infty$$

$$\ln y = \frac{1}{x^2} \ln(\cos x)$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{\cos x} \cdot (-\sin x)}{2x}$$

$$= \frac{1}{2} - (\sec^2 x)$$

$$= -\frac{1}{2}$$

$$e^{-1/2}$$

$$4) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{e^n}$$

$$5) \lim_{n \rightarrow \infty} \left| \frac{(n+1)^2}{e^{n+1}} \cdot \frac{e^n}{n^2} \right|$$

$$= \frac{1}{e} < 1 \text{ converge absolutely}$$

$$5) \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\lim_{n \rightarrow \infty} \frac{x}{(n+1)!} \cdot \frac{n!}{x} = 0 \quad x \in \mathbb{R}$$

$$6) \text{Order } f(0,13); f(x) = x e^{3x}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$$

$$e^{3x} = 1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{6} + \frac{81x^4}{24} + \frac{243x^5}{120}$$

$$x e^{3x} = x + 3x^2 + \frac{9x^3}{2} + \frac{27x^4}{2} + \frac{81x^5}{8} + \frac{243x^6}{40}$$

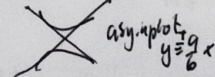
$$= 0.13 + 0.0507 + 0.0022 + \dots$$

$$7) \text{vert } (0, -9), f(0, -5)$$

vertex di y \rightarrow hyperbola vertikal

$$b^2 = c^2 - a^2 = 25 - 16 = 9$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad \frac{y^2}{16} - \frac{x^2}{9} = 1$$



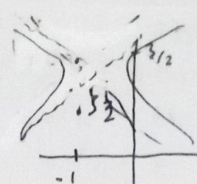
$$8) 4x^2 - 4y^2 + 8x + 12y - 6 = 0$$

$$4(x+2)^2 - 4(y-\frac{3}{2})^2 = 6 \cdot \frac{1}{4} = \frac{3}{2}$$

$$\frac{4(x+2)^2}{12} - \frac{4(y-\frac{3}{2})^2}{6} = 1$$

$$c = \frac{1}{2}$$

$$y = \pm x$$



$$y = 3 - 2 \cos t$$

$$y = -1 + 6 \sin t$$

$$\frac{dx}{dt} = 2 \sin t$$

$$\frac{dy}{dt} = 6 \cos t$$

$$\frac{dy}{dx} = 3 \cot t$$

$$dy$$

ex: $x^2 y^3 \rightarrow 2xy^3 + x^2 3y^2 \frac{dy}{dx}$

CHEATSHEET

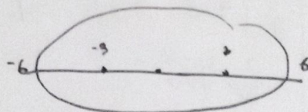
17. $F(-3,0)$ vert $(6,0)$

$c=3$ $a=6$

$3^2 = 6^2 - b^2$

$b^2 = 27$

$\frac{x^2}{36} + \frac{y^2}{27} = 1$



18. $F(6,0)$ $e = \frac{2}{3}$

$c^2 = a^2 - b^2$

$36 = 81 - b^2$

$b^2 = 45$

$\frac{x^2}{81} + \frac{y^2}{45} = 1$

19. $F(0,-5)$ $e = \frac{1}{3}$

$c^2 = a^2 - b^2$

$25 = 225 - b^2$

$b^2 = 200$

$\frac{x^2}{225} + \frac{y^2}{200} = 1$

20. $F(0,3)$, minor $b=8$

$b=4$

$c=3$

$a^2 = b^2 + c^2$

$= 25$

$\frac{x^2}{16} + \frac{y^2}{25} = 1$

21. Vert $(5,0)$, pass $(2,3)$

$a=5$

$\frac{4}{25} + \frac{9}{b^2} = 1$
 $\frac{9}{b^2} = \frac{21}{25}$
 $b^2 = \frac{75}{7}$

22. [Hyp] $F(3,0)$, ver $(4,0)$

$25 = 16 + b^2$

$b^2 = 9$

$\frac{x^2}{16} - \frac{y^2}{9} = 1$

dir: $x = \frac{16}{5}$

23. [Hyp] Vert $(0,-4)$, $F(0,-5)$

$25 = 16 + a^2$

$a^2 = 9$

dir: $\frac{9}{5}$

$\frac{y^2}{16} - \frac{x^2}{9} = 1$

24. [Hyp] Vert $(0,-3)$, $e = \frac{3}{2}$

$e = \frac{c}{b}$

$c = \frac{3}{2} \cdot 3 = \frac{9}{2}$

$\frac{81}{9} = \frac{36}{4} + a^2$

$a^2 = \frac{15}{4}$

$\frac{y^2}{9} - \frac{4x^2}{15} = 1$

25. $y = \pm \frac{1}{2}x$

$a=8$

$\frac{x^2}{64} - \frac{y^2}{16} = 1$

26. [V. Hyp] $e = \frac{\sqrt{6}}{2}$, pass $(2,1)$

$c = \frac{\sqrt{6}}{2}a$

$\frac{3}{2}a^2 = a^2 + b^2$

$\frac{1}{2}a^2 = b^2 \Rightarrow b^2 = \frac{1}{2}a^2$

$\frac{y^2}{a^2} - \frac{x^2}{\frac{1}{2}a^2} = 1$

$\frac{16}{a^2} - \frac{4}{\frac{1}{2}a^2} = 1$

$a^2 = 8$

$\frac{y^2}{8} - \frac{x^2}{4} = 1$

* INFINITE SERIES
* P-SERIES TEST

27. [Eli] $F(\pm 2,0)$, dir: $x = \pm \frac{8}{a^2}$

$\frac{a^2}{2} = 8$

$a^2 = 16$

$2^2 = 16 - b^2$

$b^2 = 12$

$\frac{x^2}{16} + \frac{y^2}{12} = 1$

28. [Hyp] $F(4,0)$, dir: $x = \pm \frac{1}{a^2}$

$\frac{a^2}{4} = 1$

$a^2 = 4$

$16 = 4 + b^2$

$b^2 = 12$

$\frac{x^2}{4} - \frac{y^2}{12} = 1$

29. [Hyp] As: $x = \pm 2y$, pass $(4,1)$

$y = \pm \frac{1}{2}x$

$\frac{16}{a^2} - \frac{9}{4a^2} = 1$

$b^2 = 20$

$16 = a^2$

Vertical

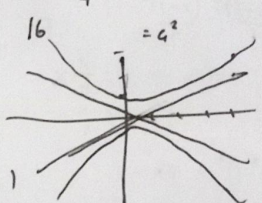
$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$\frac{9}{a^2} - \frac{16}{4a^2} = 1$

$5 = a^2$

$b^2 = 20$

$\frac{y^2}{5} - \frac{x^2}{20} = 1$



30. pass $(-5,1)$, pass $(-4,-2)$

$\frac{25}{a^2} + \frac{1}{b^2} = \frac{16}{a^2} + \frac{4}{b^2}$

$\frac{9}{a^2} = \frac{3}{b^2}$

$a^2 = 3b^2$

$\frac{16}{3b^2} + \frac{12}{3b^2} = 1$

$28 = 3b^2$

$\frac{28}{3} = b^2$

$\frac{x^2}{28} + \frac{y^2}{\frac{28}{3}} = 1$

31. Sum p. num $(0, \pm 9) = (26)$

Foci major 2a

32. diff P $(\pm 7,0) = (12)$

Foci 2a

33. $\frac{x^2}{27} + \frac{y^2}{9} = 1$ at $(3, \sqrt{6})$

$\frac{x}{9} + \frac{y\sqrt{6}}{9} = 1$

$x + y\sqrt{6} = 9$

36. $\frac{x^2}{29} + \frac{y^2}{16} = 1$ at $(3\sqrt{2}, -2)$

$\frac{18}{29} - \frac{4}{16} = 1$

$18\sqrt{2} - 9 = 8$