Rules for Finding Derivatives

Constant and Power Rules

Theorem A **Constant Function Rule**

If $f(x) = k$, where k is a constant, then for any x, $f'(x) = 0$; that is,

$$D_x(k) = 0$$

Theorem B **Identity Function Rule**

If $f(x) = x$, then $f'(x) = 1$; that is,

$$D_x(x) = 1$$

Theorem C **Power Rule**

If $f(x) = x^n$, where n is a positive integer, then $f'(x) = nx^{n-1}$; that is,

$$D_x(x^n) = nx^{n-1}$$

Theorem D **Constant Multiple Rule**

If k is a constant and f is a differentiable function, then $(kf)'(x) = k \cdot f'(x)$; that is,

$$D_x\left[k \cdot f(x)\right] = k \cdot D_x f(x)$$

In words, *a constant multiplier k can be passed across the operator D_x.*

Theorem E **Sum Rule**

If f and g are differentiable functions, then $(f + g)'(x) = f'(x) + g'(x)$; that is,

$$D_x\left[f(x) + g(x)\right] = D_x f(x) + D_x g(x)$$

In words, *the derivative of a sum is the sum of the derivatives.*

Theorem F **Difference Rule**

If f and g are differentiable functions, then $(f - g)'(x) = f'(x) - g'(x)$; that is,

$$D_x\left[f(x) - g(x)\right] = D_x f(x) - D_x g(x)$$

The proof of Theorem F is left as an exercise (Problem 54).

Theorem G **Product Rule**

If f and g are differentiable functions, then

$$(f \cdot g)'(x) = f(x)g'(x) + g(x)f'(x)$$

That is,

$$D_x\left[f(x)g(x)\right] = f(x)D_x g(x) + g(x)D_x f(x)$$

Theorem H **Quotient Rule**

Let f and g be differentiable functions with $g(x) \neq 0$. Then

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$$

That is,

$$D_x\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)D_x f(x) - f(x)D_x g(x)}{g^2(x)}$$

The Derivative Formulas

Theorem A

The functions $f(x) = \sin x$ and $g(x) = \cos x$ are both differentiable and,

$$D_x(\sin x) = \cos x \qquad D_x(\cos x) = -\sin x$$

Theorem B

For all points x in the function's domain,

$$D_x \tan x = \sec^2 x \qquad D_x \cot x = -\csc^2 x$$
$$D_x \sec x = \sec x \tan x \qquad D_x \csc x = -\csc x \cot x$$

Chain Rule

Theorem A **Chain Rule**

Let $y = f(u)$ and $u = g(x)$. If g is differentiable at x and f is differentiable at $u = g(x)$, then the composite function $f \circ g$, defined by $(f \circ g)(x) = f(g(x))$, is differentiable at x and

$$(f \circ g)'(x) = f'(g(x))g'(x)$$

That is,

$$D_x(f(g(x))) = f'(g(x))g'(x)$$

or

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

EXAMPLE 1 If $y = (2x^2 - 4x + 1)^{60}$, find $D_x y$.

EXAMPLE 4 If $y = \sin 2x$, find $\frac{dy}{dx}$.

EXAMPLE 7 Find $\frac{d}{dx}\frac{1}{(2x-1)^3}$.

Notations for Derivatives of $y = f(x)$

Derivative	f' Notation	y' Notation	D Notation	Leibniz Notation
First	$f'(x)$	y'	$D_x y$	$\frac{dy}{dx}$
Second	$f''(x)$	y''	$D_x^2 y$	$\frac{d^2y}{dx^2}$
Third	$f'''(x)$	y'''	$D_x^3 y$	$\frac{d^3y}{dx^3}$
Fourth	$f^{(4)}(x)$	$y^{(4)}$	$D_x^4 y$	$\frac{d^4y}{dx^4}$
⋮	⋮	⋮	⋮	⋮
nth	$f^{(n)}(x)$	$y^{(n)}$	$D_x^n y$	$\frac{d^ny}{dx^n}$

Implicit Differentiation

• The method for finding dy/dx without first solving the given equation for y explicitly in terms of x is called implicit differentiation.

EXAMPLE 1 Find dy/dx if $4x^2y - 3y = x^3 - 1$.

Dua cara, explicit dan implicit

Aturan pangkat (power rule)

Let r be any nonzero rational number. Then, for $x > 0$,

$$D_x(x^r) = rx^{r-1}$$

If r can be written in lowest terms as $r = p/q$, where q is odd, then $D_x(x^r) = rx^{r-1}$ for all x.

EXAMPLE 4 If $y = 2x^{5/3} + \sqrt{x^2 + 1}$, find $D_x y$.

Memakai teorema A dan Chain Rule (aturan rantai)