1 2 3 4 5 6 7 8 9 10

Pop Quiz 2 - 2206820352 - Juan Maxwell Tanaya

1a. $\lim\limits_{x \to \infty} \dfrac{\ln(4x)}{e^{2x}\sqrt{x}} \to \dfrac{\infty}{\infty}$

$$\lim_{x \to \infty} \frac{d(\ln(4x))}{d(e^{2x}\sqrt{x})} = \lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{e^{2x}}{2\sqrt{x}} + 2\sqrt{x} \cdot e^{2x}}$$

$$= \lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{e^{2x}(1+4x)}{2\sqrt{x}}}$$

$$= \frac{0}{\infty}$$

$$= 0$$

2a. $\lim\limits_{x \to 0^+} (1+2x)^{\frac{1}{2x}} \to 1^{\frac{1}{0}} \to 1^{\infty}$

$$e^{\ln\left(\lim\limits_{x \to 0^+} (1+2x)^{\frac{1}{2x}}\right)} = \lim_{x \to 0^+} e^{\ln\left((1+2x)^{\frac{1}{2x}}\right)}$$

$$= \lim_{x \to 0^+} e^{\left(\frac{\ln(1+2x)}{2x}\right)} \to \frac{0}{0}$$

$$= \lim_{x \to 0^+} e^{\left(\frac{d(\ln(1+2x))}{d(2x)}\right)}$$

$$= \lim_{x \to 0^+} e^{\left(\frac{\frac{\cancel{2}}{1+2x}}{\cancel{2}}\right)}$$

$$= \lim_{x \to 0^+} e^{\left(\frac{1}{1+2x}\right)}$$

$$= e^1 = e$$

3b. $\displaystyle\int_{-\infty}^{2} \frac{dx}{x^2-4x+16} = \lim_{a \to -\infty} \int_{a}^{2} \frac{dx}{x^2-4x+16}$

4b. $\displaystyle\int_{-9}^{9} \frac{3\,dx}{\sqrt{81-x^2}} = \int_{-9}^{0} \frac{3\,dx}{\sqrt{81-x^2}} + \int_{0}^{9} \frac{3\,dx}{\sqrt{81-x^2}}$

$$\int_{0}^{9} \frac{3\,dx}{\sqrt{81-x^2}} = \int_{0}^{9} \frac{3 \cdot -9\cos\theta\, d\theta}{\sqrt{81-81\sin^2\theta}}$$

$x = 9\sin\theta$

$dx = -9\cos\theta\, d\theta$

$$= -\int_{0}^{9} \frac{\cancel{27}^{3}\cancel{\cos\theta}\, d\theta}{\cancel{9}\cancel{\cos\theta}}$$

$$= -\int 3\,d\theta = -3\theta$$

$$= -3\arcsin\left(\frac{x}{9}\right)\Big]_0^9$$

$\theta = \arcsin\left(\frac{x}{9}\right)$

$$= -3\arcsin(1) + 3\arcsin(0)$$

$$= -\frac{3\pi}{2}$$

Karena $\dfrac{3}{\sqrt{81-x^2}}$ fungsi genap, maka

$$\int_{-9}^{0} \frac{3\,dx}{\sqrt{81-x^2}} = \int_{0}^{9} \frac{3\,dx}{\sqrt{81-x^2}}$$

Sehingga $\displaystyle\int_{-9}^{9} \frac{3\,dx}{\sqrt{81-x^2}} = -3\pi$ (Konvergen)

5b. $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin x\, dx}{(4-4\cos x)^{\frac{1}{3}}}$