Pop Quiz 10 - 2206820852 - Juan Maxwell Tonaya

1b. $\int_0^1 \int_0^{\sqrt{15}} \frac{xy}{\sqrt{x^2+1}}\, dx\, dy$

Inner
misal $u = x^2+1$
$du = 2x\, dx$

$x=0$	$x=\sqrt{15}$
$u=1$	$u=16$

$y \int_1^{16} \frac{x}{\sqrt{u}} \frac{du}{2x} = \frac{y}{2} \int_1^{16} (u)^{-\frac{1}{2}}\, du$
$= \frac{y}{2} \left[2\sqrt{u}\right]_1^{16}$
$= \frac{y}{2} [2 \cdot 4 - 2 \cdot 1]$
$= 3y$

Outer
$\int_0^1 3y\, dy = \left[\frac{3}{2}y^2\right]_0^1$
$= \frac{3}{2} \cdot 1 - 0$
$= \frac{3}{2}$

2a. $f(x,y) = \frac{2y}{(x-1)^2} = z$

Semua titik-titik pada bidang xy yang dibatasi $x=2$, $x=3$, $y=0$, dan $\underbrace{y = x^2-2x+1}_{y=(x-1)^2}$

$2 \le x \le 3$
$0 \le y \le (x-1)^2$

$V = \int_2^3 \int_0^{(x-1)^2} \frac{2y}{(x-1)^2}\, dy\, dx$

Inner
$\int_0^{(x-1)^2} \frac{2y}{(x-1)^2}\, dy = \frac{1}{(x-1)^2} \int_0^{(x-1)^2} 2y\, dy$
$= \frac{1}{(x-1)^2} \left[y^2\right]_0^{(x-1)^2}$
$= \frac{1}{(x-1)^2} \left((x-1)^2)^2 - 0\right)$
$= (x-1)^2$

Outer
$\int_2^3 (x-1)^2\, dx = \left[\frac{1}{3}(x-1)^3\right]_2^3$
$= \frac{1}{3}(3-1)^3 - \frac{1}{3}(2-1)^3$
$= \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$

3a. $\iiint_S f(x,y,z)\, dV$, $f(x,y,z) = 4x$

$8x+4y+2z-24=0$
$4x+2y+z-12=0$

x	y	z
0	0	12
0	6	0
3	0	0

$z = 12-4x-2y$
$2y = 12-4x$
$y = 6-2x$

$0 < x < 3$
$0 < y < 6-2x$
$0 < z < 12-4x-2y$

Inner
$\int_0^{12-4x-2y} 4x\, dz = \left[4xz\right]_0^{12-4x-2y}$
$= 48x - 16x^2 - 8xy$

Middle
$\int_0^{6-2x} 48x - 16x^2 - 8xy\, dy = \left[48xy - 16x^2y - 4xy^2\right]_0^{6-2x}$
$= 288x - 96x^2 - 96x^2 + 32x^3 - 144x + 24x^2 - 4x^3$
$= 252x - 168x^2 + 28x^3$

Outer
$\int_0^3 252x - 168x^2 + 28x^3\, dx = \left[126x^2 - 56x^3 + 7x^4\right]_0^3$
$= 1134 - 1512 + 567$
$= 189$

$\iiint_S f(x,y,z)\, dV = 189$

Sketsa S