Pop Quiz 8 - 2206820352 - Juan Maxwell Tanaya
6 7 8 9 10

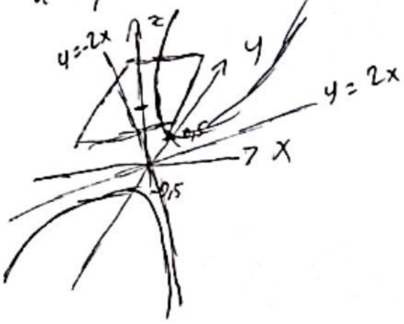
1a. $f(x,y) = 4y^2 - x^2$, $c = 1 = z$

Karena $z = f(x,y)$,

$1 = 4y^2 - x^2$

$\frac{y^2}{\frac{1}{4}} - x^2 = 1 \to$ Pers. Hyperbole Vertical

$a = 1$, $b = \frac{1}{2}$



2a. $\lim_{(x,y)\to(0,0)} \frac{e^x - e^y}{e^x + e^y} = \frac{e^0 - e^0}{e^0 + e^0}$

$= \frac{1-1}{1+1} = \frac{0}{2} = 0$

3a. Turunan Berarah

$f(x,y) = x^2 \ln(y) + xy^2$ pada $\vec{P_0} = (-1, 2)$

dengan arah $\vec{w} = \vec{i} + \vec{j}$

$\vec{u} = \frac{\vec{w}}{\|\vec{w}\|} = \frac{(1,1)}{\sqrt{1^2+1^2}} = \frac{(1,1)}{\sqrt{2}}$

$= (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$= (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

$\cancel{D_{\vec{u}} f(\vec{p})}$ $f_x(x,y) = 2x\ln(y) + y^2$

$f_y(x,y) = \frac{x^2}{y} + 2xy$

$D_{\vec{u}} f(\vec{p}) = u_1 f_x(\vec{p}) + u_2 f_y(\vec{p})$

$= \frac{\sqrt{2}}{2}(2(-1)\ln(2) + (2)^2) + \frac{\sqrt{2}}{2}\left(\frac{(-1)^2}{2} + 2(-1)(2)\right)$

$= \cancel{\frac{\sqrt{2}}{2}} - \sqrt{2}\ln(2) \cancel{+ 2\sqrt{2}} + \frac{\sqrt{2}}{4} \cancel{- 2\sqrt{2}}$

$= \frac{\sqrt{2}}{4} - \sqrt{2}\ln(2)$

4a. Tangent Plane dari f pada $\vec{P_0}$

$f(x,y) = x^3 \sin(y) + \frac{\cos(2y)}{x}$, $\vec{P_0} = (\frac{1}{2}, \pi)$

$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix} = \begin{pmatrix} 3x^2\sin(y) - \frac{\cos(2y)}{x^2} \\ x^3\cos(y) - \frac{2\sin(2y)}{x} \end{pmatrix}$

$\nabla f(\frac{1}{2}, \pi) = \begin{pmatrix} 3(\frac{1}{2})^2 \sin(\pi) - \frac{\cos(2\pi)}{(\frac{1}{2})^2} \\ (\frac{1}{2})^3\cos(\pi) - \frac{2\sin(2\pi)}{(\frac{1}{2})} \end{pmatrix}$

$= \begin{pmatrix} 0 - \frac{1}{\frac{1}{4}} \\ \frac{1}{8}(-1) - 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -\frac{1}{8} \end{pmatrix}$

$z_0 = f(\frac{1}{2}, \pi) = (\frac{1}{2})^3 \sin(\pi) + \frac{\cos(2\pi)}{\frac{1}{2}}$

$= 0 + (-2)$

$= -2$

$z - z_0 = \nabla f(x_0, y_0) \cdot \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix}$

$z + 2 = \begin{pmatrix} -4 \\ -\frac{1}{8} \end{pmatrix} \cdot \begin{pmatrix} x - \frac{1}{2} \\ y - \pi \end{pmatrix}$

$z \cancel{+2} = -4x \cancel{+2} - \frac{y}{8} + \frac{\pi}{8}$

$4x + \frac{y}{8} + z = \frac{\pi}{8}$

5a. $w = xe^{\frac{\sqrt{y}}{z}}$, $x = \sqrt{t}$, $y = t\ln(t)$, $z = \sin(t^2)$

$\frac{dw}{dt} = \frac{dw}{dx}\frac{dx}{dt} + \frac{dw}{dy}\frac{dy}{dt} + \frac{dw}{dz}\frac{dz}{dt}$

$dx = d(t^{\frac{1}{2}}) = \frac{1}{2\sqrt{t}}\, dt \to \frac{dx}{dt} = \frac{1}{2\sqrt{t}}$

$dy = d(t\ln(t)) = 1 + \ln(t)\, dt \to \frac{dy}{dt} = 1 + \ln(t)$

$dz = d(\sin(t^2)) = 2t\cos(t^2) \to \frac{dz}{dt} = 2t\cos(t^2)$

$\frac{dw}{dx} = e^{\frac{\sqrt{y}}{z}}$ | $\frac{dw}{dy} = \frac{xe^{\frac{\sqrt{y}}{z}}}{2z\sqrt{y}}$ | $\frac{dw}{dz} = \frac{x\sqrt{y}e^{\frac{\sqrt{y}}{z}}}{z^2}$

$\frac{dw}{dt} = \left(e^{\frac{\sqrt{y}}{z}}\right)\left(\frac{1}{2\sqrt{t}}\right) + \left(\frac{xe^{\frac{\sqrt{y}}{z}}}{2z\sqrt{y}}\right)(1 + \ln(t)) + \left(\frac{x\sqrt{y}e^{\frac{\sqrt{y}}{z}}}{z^2}\right)(2t\cos(t^2))$

$= \frac{e^{\frac{\sqrt{y}}{z}}}{2\sqrt{t}} + \frac{xe^{\frac{\sqrt{y}}{z}}}{2z\sqrt{y}} + xe^{\frac{\sqrt{y}}{z}}\ln(t)$

$+ \frac{x\sqrt{y}e^{\frac{\sqrt{y}}{z}}\, 2t\cos(t^2)}{z^2}$

$\frac{dw}{dt} = e^{\frac{\sqrt{y}}{z}}\left(\frac{1}{2\sqrt{t}} + \frac{x}{2z\sqrt{y}} + x\ln(t) + \frac{x\sqrt{y}\,2t\cos(t^2)}{z^2}\right)$