

Pop Quiz 10-2206820352 - Juan Maxwell Tanaya

1b. $\int_0^1 \int_0^{\sqrt{x}} \frac{xy}{\sqrt{x^2+1}} dx dy$

Inner
misal $u = x^2 + 1$
 $du = 2x dx$

$x=0 \rightarrow u=1$
 $x=\sqrt{1} \rightarrow u=2$
 $\int_0^1 \int_0^{\sqrt{x}} \frac{xy}{\sqrt{x^2+1}} dx dy = \frac{y}{2} \int_1^2 (u)^{-\frac{1}{2}} du$
 $= \frac{y}{2} [2\sqrt{u}]_1^2$
 $= \frac{y}{2} [2 \cdot 4 - 2 \cdot 1]$
 $= 3y$

Outer
 $\int_0^1 3y dy = [\frac{3}{2} y^2]_0^1$
 $= \frac{3}{2} \cdot 1 - 0$
 $= \frac{3}{2}$

2a. $f(x,y) = \frac{2y}{(y-1)^2} = z$

5 titik titik pada bidang xy
yang dibatasi $x=2$, $x=3$, $y=0$,
dan $y = x^2 - 2x + 1$
 $y = (x-1)^2$

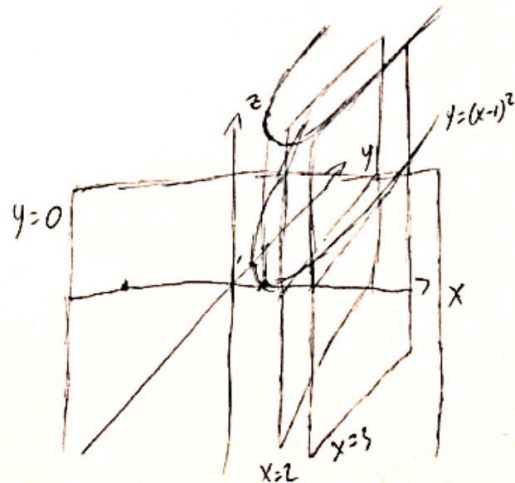
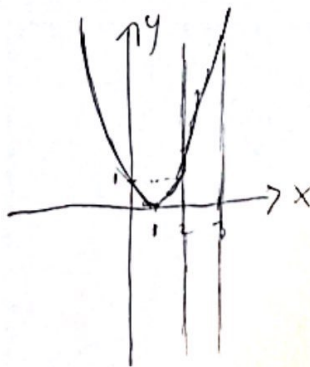
$2 \leq x \leq 3$
 $0 \leq y \leq (x-1)^2$

$V = \int_2^3 \int_0^{(x-1)^2} \frac{2y}{(y-1)^2} dy dx$

Inner
 $\int_0^{(x-1)^2} \frac{2y}{(y-1)^2} dy = \frac{1}{(x-1)^2} \int_0^{(x-1)^2} 2y dy$
 $= \frac{1}{(x-1)^2} [y^2]_0^{(x-1)^2}$
 $= \frac{1}{(x-1)^2} ((x-1)^2)^2 - 0$
 $= (x-1)^2$

Outer
 $\int_2^3 (x-1)^2 dx = [\frac{1}{3} (x-1)^3]_2^3$
 $= \frac{1}{3} (3-1)^3 - \frac{1}{3} (2-1)^3$
 $= \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$

Sketsa 5



3a. $\iiint_S f(x,y,z) dV$, $f(x,y,z) = 4x$

$8x + 4y + 2z - 24 = 0$
 $4x + 2y + z - 12 = 0$
 $z = 12 - 4x - 2y$
 $2y = 12 - 4x$
 $y = 6 - 2x$
 $0 < x < 3$
 $0 < y < 6 - 2x$
 $0 < z < 12 - 4x - 2y$

Inner
 $\int_0^{12-4x-2y} 4x dz = [4xz]_0^{12-4x-2y}$
 $= 48x - 16x^2 - 2xy$

Middle
 $\int_0^{6-2x} (48x - 16x^2 - 2xy) dy = [48xy - 16x^2y - xy^2]_0^{6-2x}$
 $= 288x - 96x^2 - 96x^2 + 32x^3 - \frac{3}{2} 36x + 24x^2 - 4x^3$
 $= 252x - 168x^2 + 28x^3$

Outer
 $\int_2^3 (252x - 168x^2 + 28x^3) dx = [126x^2 - 56x^3 + 7x^4]_2^3$
 $= 1134 - 1512 + 567$
 $= 189$

$\iiint_S f(x,y,z) dV = 189$