No
Date

TURUNAN

:> $D_x(k) = 0$

:> $D_x(x) = 1$

:> $D_x(x^n) = nx^{n-1}$

:> $D_x(kf(x)) = k \cdot D_x f(x)$

:> $D_x[f(x)g(x)] = f(x) \cdot D_x g(x) + g(x) D_x f(x)$

:> $D_x\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) D_x f(x) - f(x) D_x g(x)}{g^2(x)}$

:> $D_x(\sin x) = \cos x$, $D_x(\cos x) = -\sin x$

:> $D_x(\tan x) = \sec^2 x$ $\quad D_x \cot x = -\csc^2 x$

$D_x(\sec x) = \sec x \tan x$ $\quad D_x \csc x = -\csc x \cot x$

$$\boxed{f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}}$$

contoh: $4x^2y - 3y = x^3 - 1$ $\quad dy/dx$?

$y = \frac{x^3 - 1}{4x^2 - 3}$

$y' = \frac{3x^2(4x^2-3) - 8x(x^3-1)}{(4x^2-3)^2}$

~~ekspli~~ implisit:

$4x^2y - 3y = x^3 - 1$

$4y(2x)dx - 4x^2 dy - 3dy = 3x^2 dx - 0$

$8xy\,dx - 4x^2 dy - 3dy = 3x^2 dx$

$dy(-4x^2 - 3) = (3x^2 - 8xy)dx$

$dy/dx = \frac{8xy - 3x^2}{4x^2 + 3}$

contoh:

$\frac{\ln(y^2)}{x} + e^{xy} + \sec^2(x) = 1$

$\frac{\frac{2}{xy}dy - \ln(y^2)dx}{x^2} + e^{xy} \ln e\,(x\,dy + y\,dx) + 2\tan x \sec^2 x\,dx = 0$

$\frac{2}{x^3y}dy - \frac{2\ln y}{x^2}dx + xe^{xy}dy + ye^{xy}dx + 2\tan x \sec^2 x\,dx = 0$

$\frac{dy}{dx} = \frac{\frac{2\ln y}{x^2} - ye^{xy} - 2\tan x \sec^2 x}{xe^{xy} + \frac{2}{x^3y}}$

Higher order derivative

$f^{(n)}(x)$; $y^{(n)}$; $D_x^n y$; $\frac{d^n y}{dx^n}$

ex: $f(x) = 2x^3 - 4x^2 + 7x - 8$

$f'(x) = 6x^2 - 8x + 7$

$f''(x) = 12x - 8$

$f'''(x) = 12$

$f^{(4)}(x) = 0$

Implicit Differentiation

ex: find dy/dx if $4x^2y - 3y = x^3 - 1$

$$\frac{d}{dx}(4x^2y - 3y) = \frac{d}{dx}(x^3 - 1)$$

$$4x^2\frac{dy}{dx} + y \cdot 8x - 3\frac{dy}{dx} = 3x^2 \Leftrightarrow \frac{dy}{dx}(4x^2 - 3) = 3x^2 - 8xy \Leftrightarrow \frac{dy}{dx} = \frac{3x^2 - 8xy}{4x^2 - 3}$$

Approximation

$$f(x + \Delta x) \approx f(x) + dy = f(x) + f'(x)\Delta x$$

Tangent Line

$$m_{tan} = \lim_{h\to 0} m_{sec} = \lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$$

ex: $y = f(x) = -x^2+2x+2$

$$m_{tan} = \lim_{h\to 0} \frac{-(c+h)^2+2(c+h)+2-(-c^2+2c+2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-2c-h+2)}{\cancel{h}} = -2c+2$$

Instantaneous Velocity

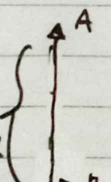
$$v = \lim_{h\to 0} v_{avg} = \lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$$

Related Rates

ex: airplane A at 640 mph (→ northbound) pass a city at noon, 15 minutes later plane B passes that city too at 600 miles (↳ east bound). How fast they be separating at 1.15 PM?

$S = \sqrt{600^2+(640+160)^2}$
$= 1000$

$640 \cdot \frac{15}{60} = 160$ mile



$$s^2 = x^2 + (y+160)^2$$

$$\cancel{2}s\, ds/dt = \cancel{2}x\, dx/dt + \cancel{2}(y+160)\, dy/dt$$

$$1000\, ds/dt = 600 \cdot 600 + (640+160)(640)$$

$$ds/dt = 872 \text{ mph} \;//$$

Approximations

$$f(x+\Delta x) \approx f(x) + dy = f(x) + f'(x)\Delta x$$

ex: $f(x) = 1+\sin 2x$ to $x = \pi/2$

$f'(x) = 2\cos 2x$

$$\to L(x) = f(\pi/2) + f'(\pi/2)(x-\pi/2)$$

$$= (1+\sin\pi) + (2\cos\pi)(x-\pi/2)$$

$$= 1 - 2(x-\pi/2) = 1+\pi-2x \;//$$

INTEAGRAL

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$$\boxed{\int f(x)\,dx = F(x) + C}$$

dgn $f(x)$ = integran

Aturan :

1. $\int k f(x)\,dx = k \int f(x)\,dx$
2. $\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
3. $\int a^x\,dx = \frac{a^x}{\ln a} + C$
4. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$
5. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$
6. $\int n x^{n-1}\,dx = x^n + C$
7. $\int \frac{1}{x}\,dx = \ln|x| + C$

Soal

1. $\int e^x \sec^2 x + e^x \tan x \; dx$ $\quad \int e^x(\tan^2 x + \tan x + 1)\,dx$

$= \int e^x \sec^2 x\,dx + \int e^x \tan x\,dx$ $\quad =$ D I

$\tan^2 x + \tan x + 1 \quad e^x$

$=$

x

D I

$\sec^2 x$

$\tan x$ $\quad \frac{y}{x} \cdot \frac{1}{x} \quad \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$

$\ln|\sec(x)|$

$-\cos x + x^4 - x^2$

2.) $\int -\csc x \cot x + x^4 - x^2\,dx$ $\quad d \csc x \cot x = \csc x \cdot \cot^2 x\,dx \; \csc^3 x\,dx$

$= \int -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} + x^4 + x^2\,dx$ $\quad \cos x$

$u = \sin^2 x$

$= \int -\frac{\cos x}{\sin^2 x} +$ $\quad du = 2\cos x \sin x\,dx$

$du = \sin 2x\,dx$

$= \frac{dx}{du}$ $\quad dx = \frac{du}{2\cos x \cdot \sin x}$

インタグラル リスト

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$\int a\,dx = ax + C$

$\int \frac{1}{x}\,dx = \ln|x| + C$

$\int e^x\,dx = e^x + C$

$\int a^x\,dx = \frac{a^x}{\ln a} + C$

$\int \ln x\,dx = x\ln x - x + C$

$\int \sin x\,dx = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int \tan x\,dx = \ln|\sec x| + C$

$= -\ln|\cos x| + C$

$\int \cot x\,dx = \ln|\sin x| + C$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$

$\int \csc x\,dx = \ln|\csc x - \cot x| + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \sec x \cdot \tan x\,dx = \sec x + C$

$\int \csc^2 x\,dx = -\cot x + C$

$\int \tan^2 x\,dx = \tan x - x + C$

$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\left(\frac{x}{a}\right) + C$

$\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln\left|\frac{x + \sqrt{a^2 + x^2}}{a}\right| + C$

$\int \frac{dx}{1 + x^2} = \tan^{-1} x + C$

$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

$\int \frac{1}{ax + b}\,dx = \frac{1}{a}\ln|ax + b| + C$

$\int \frac{x}{ax^2 + b}\,dx = \frac{1}{2a}\ln|ax^2 + b| + C$

$\int \frac{1}{x^2 - a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x - a}{x + a}\right| + C$

ノン エレメント

- $\int e^{x^2}\,dx$
- $\int e^{-x^2}\,dx$
- $\int \frac{e^x}{x}\,dx$
- $\int \frac{1}{\ln x}\,dx$
- $\int \frac{\sin x}{x}\,dx$
- $\int \frac{\cos x}{x}\,dx$
- $\int \sin(x^2)\,dx$
- $\int \cos(x^2)\,dx$
- $\int x^x\,dx$
- $\int \sqrt{1 + x^3}\,dx$

INTEGRAL TENTU

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$\sum_{i=1}^{n} i$	$\sum_{i=1}^{n} i^2$	$\sum_{i=1}^{n} i^3$
$\frac{n(n+1)}{2}$	$\frac{n(n+1)(2n+1)}{6}$	$\left[\frac{n(n+1)}{2}\right]^2$

A Jumlah Riemann

$$R_P = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x_i \qquad \Delta x = \frac{b-a}{n}$$

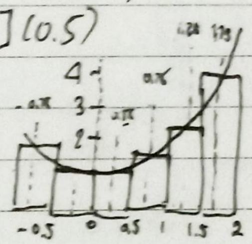
ex: $f(x) = x^2+1$ interval $[-1, 2]$, partisi sama 0,5. titik sample $\bar{x}_i$ merupakan titik" tengah pd intervalnya

$$R_P = \sum_{i=1}^{6} f(\bar{x}_i)\cdot \Delta x_i$$

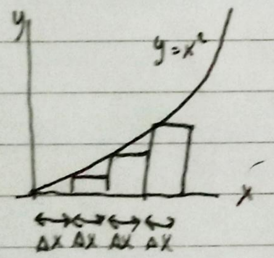
$$= [f(-0.75) + f(-0.25) + f(0.25) + f(0.75) + f(1.25) + f(1.75)]\,(0.5)$$

$$= [1.5625 + 1.0625 + 1.0625 + 1.5625 + 2.5625 + 4.0625]\,(0.5)$$

$$= 5.9375$$



★ Riemann kiri



lebar sub interval

$$\Delta x = \frac{4-0}{4} = 1$$

luas sub interval

$L\square_1 = 1\cdot 0^2 = 0$

$L\square_2 = 1\cdot 1^2 = 1$

$L\square_3 = 1\cdot 2^2 = 4$

$L\square_4 = 1\cdot 3^2 = 9$

B Turunan - Integral

1st fund theory: $\frac{d}{da}\int_c^a f(x) = f(a)$, c konstan

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

★ disebut 2nd fund. theorem

ex: $y = \int_1^{x^3} t \sin(2t)\,dt$

$$\frac{dy}{dx} = \frac{d}{dx}\int_1^{x^3} t\sin(2t)\,dt = \frac{dx^3}{dx}\,\frac{d}{dx^3}\int_1^{x^3} t\sin(2t)\,dt = 3x^2\cdot x^3\sin(2x^3)$$

$$= 3x^5\sin(2x^3)$$

C Teorema nilai Rata-Rata

Jika f terintegrasi pd interval $[a, b]$, maka nilai rata" f pd $[a,b]$ adalah

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

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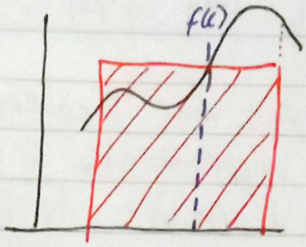
Jika f kontinu pd $[a,b]$, maka terdapat suatu bil. c antara a dan b sedemikian rupa sehingga

$$f(c) = \frac{1}{b-a} \int_a^b f(t)\,dt$$



ex: $\int f(x) = \int_0^2 \frac{1}{(x+1)^2}\,dx$, find x that satisfy the mean value

$f(c) = \frac{1}{2-0} \int_0^2 \frac{1}{(x+1)^2}\,dx$ let : $u = x+1$

$= \frac{1}{2} \int_1^3 \frac{1}{u^2}\,du = \frac{1}{2}\left(u^{-1}\right)\Big|_1^3 = \frac{1}{2}\left(-\frac{1}{3}+1\right) = \frac{1}{3}$

$f(c) = \frac{1}{(c+1)^2}$

$\frac{1}{3} = \frac{1}{(c+1)^2}$ $\therefore c_1 = -1-\sqrt{3} \vee c_2 = -1+\sqrt{3}$

$\boxed{D}$ Fungsi ganjil-genap

$\int_{-a}^{a} f(x)$ → Fungsi ganjil = 0
→ Fungsi genap = $2 \cdot F(a)$

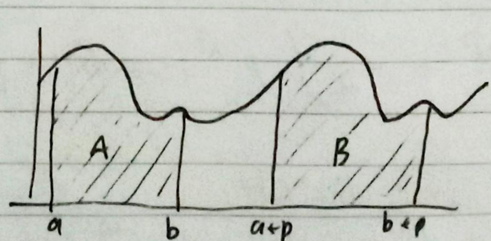
ex: $\int_{-2}^{2} f(x)\,dx = \int_{-2}^{2} (x \sin^4 x + x^3 - x^4$
ganxgen, gan, gen

$= 0 + 0 - 2 \cdot \int_0^2 x^4\,dx = -\frac{64}{5}$

$\boxed{E}$ Fungsi Periodik

Jika f periodik dgn periode p:

$$\int_{a+p}^{b+p} f(x)\,dx = \int_a^b f(x)\,dx$$



Area A = Area B

レバース・パワー・ルール
Reverse Power Rule

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$$\boxed{\int x^n\,dx = \frac{1}{n+1}x^{n+1}}$$

ユー・サブスティトゥーション
U substitution

$$\int f(x)\,u'\cdot dx \xrightarrow{u\text{ sub}} \int f(u)\cdot u'\,\frac{du}{u'}$$

ex: $\int \frac{x^3}{1+x^4}$ $\quad u = 1+x^4$, $du = \frac{dx}{4x^3}$

$$= \int \frac{x^3}{u}\cdot\frac{du}{4x^3} = \frac{1}{4}\int u^{-1}\,du = \frac{1}{4}\ln|u| = \frac{1}{4}\ln(1+x^4)$$

parenthesis cuz always positive ..

ex: $\int \frac{x}{1+x^4}$ $\quad u = x^2$, $du = \frac{dx}{2x}$

$$= \int \frac{\cancel{x}}{1+u^2}\cdot\frac{du}{2\cancel{x}} = \frac{1}{2}\int\frac{1}{1+u^2}\,du = \frac{\tan^{-1}(u)}{2} = \frac{\tan^{-1}x^2}{2} + C$$

ex: $\int \frac{1}{1+\sqrt{x}}\,dx$ $\quad u = 1+\sqrt{x} \Rightarrow \sqrt{x} = u-1$, $dx = 2\sqrt{x}\,dx$

$$= \int \frac{1}{u}\cdot 2\sqrt{x}\,du$$

$$= 2\int\frac{u-1}{u}\,du = 2\int 1-\frac{1}{u}\,du = 2(u-\ln|u|) = 2(1+\sqrt{x}-\ln(1+\sqrt{x})) = 2+2\sqrt{x}-2\ln(1+\sqrt{x})+C$$

ex: $\int \tan x\,dx$ $\quad u = \sec x \quad du = \sec x\cdot\tan x\,dx$

$$\int\frac{\sec x\cdot\tan x}{\sec x}\,dx = \int\frac{1}{u}\cdot du = \ln|\sec x| + C$$

ex: $\int \frac{1}{x^3+x}\,dx$ $\quad$ note: $x^3+x = x(x^2+1)$ ..
$x^3+x = x^3(1+x^{-2})$

$$= \int\frac{dx}{x^3(1+x^{-2})} = \int\frac{x^{-3}}{1+x^{-2}}\,dx$$

let $u = 1+x^{-2}$, $du = -2x^{-3}\,dx$

$$= \int\frac{\cancel{x^{-3}}}{u}\cdot\frac{du}{-2\cancel{x^{-3}}} = \ln|u| + C = \ln(1+x^{-2}) + C$$

インテグレーション・バイ・パート
Integration by Part

$$\boxed{\int u\,dv = uv - \int v\,du}$$

	D	I
+	$f(x)$	$F(x)$
−	$f'(x)$	$\int F(x)$
+	$f''(x)$	$\iint F(x)$
⋮		

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• 1^{st} ストップ
0 in D column

ex: $\int x^2 \sin x\, dx$

	D	I
+	x^2	$\sin x$
−	$2x$	$-\cos x$
+	2	$-\sin x$
−	0	$\cos x$

$= -x^2\cos x + 2x\sin x + 2\cos x + C$

• 2^{nd} ストップ
when we can integrate the product of a row

ex: $\int x^3 \ln x\, dx$

	D	I
+	$\ln x$	x^3
$-\int$	$\frac{1}{x}$	$\frac{1}{4}x^4$

$= \frac{1}{4}x^4 \cdot \ln x - \int \frac{1}{\cancel{x}} \cdot \frac{1}{4} x^{\cancel{4}3} dx = \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^{4} dx = \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$

• 3^{rd} ストップ

ex: $\int e^x \sin 2x\, dx$

	D	I
+	e^x	$\sin 2x$
−	e^x	$-\frac{1}{2}\cos 2x$
$+\int$	e^x	$-\frac{1}{4}\sin 2x$

Recursive

$= \int e^x \sin 2x\, dx = -\frac{1}{2}e^x\cos 2x + \frac{1}{4}e^x \sin 2x - \frac{1}{4}\int e^x \sin 2x\, dx$

$= \frac{5}{4}\int e^x \sin 2x\, dx = -\frac{1}{2}e^x\cos(2x) + \frac{1}{4}e^x \sin 2x$

$\int e^x \sin 2x\, dx = -\frac{2}{5}e^x \cos 2x + \frac{1}{5}e^x \sin 2x + C$

トリゴノメトリー・インテグラル Trigono Integral

Case: $\sin x, \cos x$

$\int (\ \ )\cos x\, dx$
expression in terms of $\sin x$
$u = \sin x$

$\int (\ \ )\sin x\, dx$
expression in terms of $\cos x$
$u = \cos x$

note: $\sin^2 x = \frac{1-\cos 2x}{2}$ $\quad \cos^2 x = \frac{1+\cos 2x}{2}$

ex: $\int \cos^4 x = \int \left(\frac{1+\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int 1 + 2\cos 2x + \cos^2 2x\, dx$

$= \frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{8}\int 1 + \cos 4x\, dx$

$= \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\int \cos 4x\, d4x = \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x$

$\int \sin^n x \, dx$ dan $\int \cos^n dx \, dx$

$\int \sin^m x \cos^n x \, dx$

→ if m or n ganjil, → $\sin^2 x + \cos^2 x = 1$

→ if m and n genap, → $\sin^2 x = \frac{1 - \cos 2x}{2}$ and $\cos^2 x = \frac{1 + \cos 2x}{2}$

ex: $\int \sin^3 x \, dx = \int \sin^2 x \sin x \, dx = \int (1 - \cos^2 x) \sin x \, dx$ $\quad u = \cos x \quad du = -\sin x \, dx$

$= \int (1 - u^2) \cancel{\sin x} \frac{du}{\cancel{-\sin x}} = -\int (1 - u^2) du = -\cos x + \frac{1}{3}\cos^3 x + C$ $\quad dx = \frac{du}{-\sin x}$

Case: tan x, sec x

$\int (\quad) \sec^2 x \, dx$ $\qquad$ $\int (\quad) \sec x \tan x$

↑ expression in terms of tan x $\qquad$ ↑ expression in terms of sec x

$u = \tan x$ $\qquad$ $u = \sec x$

note: $\tan^2\theta = \sec^2\theta - 1$, $\sec^2\theta = \tan^2\theta + 1$

ex: $\int \sec^4 x \, dx = \int \sec^2 x \cdot \sec^2 x \, dx$ $\quad u = \tan x$; $du = \sec^2 x \, dx$

$= \int (\tan^2 x + 1) \sec^2 x = \int (u^2 + 1) \, du = \frac{1}{3}u^3 + u = \frac{1}{3}\tan^3 x + \tan x + C$

ex: $\int \sec^4 \tan x \, dx = \int \sec^3 x \sec x \tan x \, dx$ $\quad u = \sec x$, $du = \sec x \tan x \, dx$

$= \frac{1}{4}\sec^4 x + C$

ex: $\int \tan^3 x \, dx = \int (\sec^2 x - 1) \tan x \, dx = \int \sec^2 x \tan x - \int \tan x \, dx$

~~for~~ $u = \tan x \quad du = \sec^2 dx$

$= \int u \, du + \ln|\cos x| = -\frac{1}{2}\tan^2 x + \ln|\cos x| + C$

Case: roots!

you see	you let	you use
$\sqrt{x^2 + a^2}$	$x = a\tan\theta$	$\tan^2\theta + 1 = \sec\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$

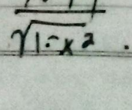
ex: $\int \frac{1}{x\sqrt{x^2 - 4}} dx$ $\quad x = \sec\theta$, $dx = 2\sec\theta \cdot \tan\theta \, d\theta$

$= \int \frac{dx}{2\sec\theta \sqrt{(2\sec\theta)^2 - 4}} = \int \frac{1}{\cancel{2\sec\theta} \cdot \cancel{2\tan\theta}} \cdot \cancel{2\sec\theta\tan\theta} \, d\theta = \frac{1}{2}\theta = \frac{1}{2}\sec^{-1}\left(\frac{x}{2}\right) + C$

ex: $\int \sqrt{1 - x^2} \, dx$ $\quad$ let: $x = \sin\theta$; $dx = \cos\theta \cdot dx$

$= \int \sqrt{1 - \sin^2\theta} \cos\theta \, d\theta = \int \cos^2\theta \, d\theta = \frac{1}{2}\int (1 + \cos 2\theta) \, d\theta = \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right)$

$= \frac{1}{2}\theta + \frac{1}{2} \cancel{\frac{1}{2}} \cancel{2} \sin\theta \cdot \cos\theta = \frac{1}{2}\sin^{-1} x + \frac{1}{2} x \sqrt{1 - x^2}$



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ex: $\int \frac{1}{x^2+a^2}\,dx$ let: $a\tan\theta = x \Rightarrow \tan\theta = \frac{x}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{x}{a}\right)$

$a\sec^2\theta\, d\theta = dx$

$$= \int \frac{1}{(a\tan\theta)^2+a^2}\, a\sec^2\theta\, \theta = \int \frac{1}{a^2\sec^2\theta} \cdot a\sec^2\theta\, d\theta = \frac{1}{a}\theta = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

パーシャル・フラクション・ディーコンポジション

Integral partial

$\int \frac{\text{Polynomial}}{\text{polynomial}}\,dx$

note: $\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + C$

$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

$\int \frac{x}{ax^2+b}\,dx = \frac{1}{2a}\ln|ax^2+b| + C$

$\int \frac{1}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

Case 1: deg (top) $\geq$ deg (bottom)

Do long division

ex: $\int \frac{x^3}{x^2+9}\,dx = \int x - \frac{9x}{x^2+9}\,dx = \frac{1}{2}x^2 - \frac{9}{2}\ln(x^2+9) + C$

$$\begin{array}{r|l} & x \\ x^2+9 & x^3 \\ & -(x^3+9x) \\ & -9x \end{array}$$

Case 2: Distinct linear factor

Do cover-up

ex: $\int \frac{8x-17}{x^2-5x+4}\,dx = \int \frac{8x-17}{(x-1)\cdot(x-4)}\,dx = \int \frac{3}{x-1} + \frac{5}{x-4}\,dx = 3\ln|x-1| + 5\ln|x-4| + C$

$\frac{8x-17}{(x-1)(x-4)} = \frac{A^{=3}}{x-1} + \frac{B^{=5}}{x-4}$

$A \overset{x=1}{=} \frac{8(1)-17}{1-4} = \frac{-9}{-3} = 3$

$B \overset{x=4}{=} \frac{8(4)-17}{4-1} = \frac{15}{3} = 5$

Case 3: Distinct irreducible quadratic Factor

ex $\int \frac{4x^2-9x+2}{(x+3)(x^2+4)}\,dx = \int \frac{A^{=5}}{x+3} + \frac{Bx+C}{x^2+4}\,dx = \int \frac{5}{x+3} - \frac{1x}{x^2+4} - \frac{6}{x^2+4} = 5\ln|x+3| - \frac{1}{2}\ln(x^2+4) - 3\tan^{-1}\left(\frac{x}{2}\right) + C$

$A \overset{x=-3}{=} \frac{4(-3)^2-9(-3)+2}{(-3)^2+4} = \frac{65}{13} = 5$

$4x^2-9x+2 = 5(x^2+4) + (Bx+C)(x+3)$

$= 5x^2 + 20 + Bx^2 + 3Bx + Cx + 3C$

$4 = 5+B \Rightarrow B = -1$

$3C + 20 = 2 \Rightarrow C = -6$

case 4: repeating factor

build up the power

ex: $\int \frac{2x-5}{x^3+x^2}\,dx = \int \frac{7}{x} - \frac{3}{x^2} - \frac{7}{x+1}\,dx = 7\ln|x| + \frac{3}{x} - 7\ln|x+1|$

$\left(\frac{2x-5}{x^2(x+1)} = \frac{\overset{=7}{A}}{x} + \frac{\overset{=-5}{B}}{x^2} + \frac{\overset{=-7}{C}}{x+1}\right) x^2(x+1) \Rightarrow 2x-3 = Ax(x+1) - 3(x+1) - 7x^2$

$= Ax^2 + Ax - 3x - 3 - 7x^2$

$B \overset{x=0}{=} \frac{2(0)-5}{0+1} = -5$

$0x^2 + 2x - 3 = (A-7)x^2 + \cdots$

$A - 7 = 0,\ A = 7$

$C \overset{x=-1}{=} \frac{2(-1)-5}{(-1)^2} = -7$

ex: $\int \frac{2x^2+8x+5}{x^2+4x+13}\,dx = \int 2 - \frac{21}{x^2+4x+13}\,dx$

$$\begin{array}{r} 2 \\ x^2+4x+13 \,\big)\overline{2x^2+8x+5} \\ 2x^2+8x+26 \quad - \\ \hline -21 \end{array}$$

$= \int 2\,dx - 21 \int \frac{1}{(x+2)^2+3^2}\,dx = 2x - \frac{21}{3}\tan^{-1}\left(\frac{x+2}{3}\right)$

$= 2x - 7\tan^{-1}\left(\frac{x+2}{3}\right) + C$

ex: $\int \frac{1}{x^2-a^2}\,dx = \int \frac{\frac{1}{2a}}{x-a} + \frac{-\frac{1}{2a}}{x+a}\,dx = \frac{1}{2a}\ln|x-a| - \frac{1}{2a}\ln|x+a|$

$= \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

A] L'Hopital

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

ex: $\lim_{x\to 0} \frac{\tan 2x}{\ln(1+x)} = \lim_{x\to 0} \frac{2\sec^2 2x}{1/(1+x)} = \frac{2}{1} = 2$

B] Cauchy's Mean Value Theorem

let f & g differentiable di (a,b) & continous di [a,b]. jika $g'(x) \neq 0$ di a-b exist c di (a,b):

$$\boxed{\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}}$$

ex: $\lim_{x\to 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right) = \lim_{x\to 1^+} \frac{x\ln x - x + 1}{(x-1)\ln x} \overset{L}{=} \lim_{x\to 1^+} \frac{x\cdot\frac{1}{x} + \ln x - 1}{(x-1)(1/x) + \ln x} = \lim_{x\to 1^+} \frac{x\ln x}{x-1+\ln x\cdot x}$

$\overset{L}{=} \lim_{x\to 1^+} \frac{1+\ln x}{2+\ln x} = \frac{1}{2}$ //

C] Indeterminate forms $0^0, \infty^0, 1^\infty$

tips: consider its logarithm!

ex: $\lim_{x\to 0^+} (x+1)^{\cot x}$ let $y = (x+1)^{\cot x}$

$\ln y = \cot x \cdot \ln(x+1) = \frac{\ln(x+1)}{\tan x}$

$\rightarrow \lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(x+1)}{\tan x} \overset{L}{=} \lim_{x\to 0^+} \frac{\frac{1}{x+1}}{\sec^2 x} = 1 \Leftrightarrow e^{\ln y} = e^1 = e$ //

D] Other: $\frac{0}{0}, \frac{\infty}{\infty}, 0\cdot\infty, \infty-\infty$

E] One Finite Limit

$\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to -\infty} \int_a^b f(x)\,dx \quad \cdot \quad \int_a^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$.

→ converge jika mendekati satu value, else: divergent

ex: $\int_{-\infty}^{-1} xe^{-x^2} = -\frac{1}{2}\int_a^{-1} e^{-x^2}(-2x\,dx) = \left[-\frac{1}{2}e^{-x^2}\right]_a^{-1} = -\frac{1}{2}e^{-1} + \frac{1}{2}e^{-a^2} = -\frac{1}{2e}$ //

$\to 0$

F] Both Limits Infinite

If BOTH $\int_{-\infty}^{0} f(x)\,dx$ and $\int_0^{\infty} f(x)\,dx$ converge, then $\int_{-\infty}^{\infty} f(x)\,dx$ converge

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_0^{\infty} f(x)\,dx$$

6| Integrands that are Infinite at an end point

Suppose $\lim\limits_{x \to b^-} |f(x)| = \infty$ then:

$$\int_a^b f(x)\,dx = \lim_{t \to b^-} \int_a^t f(x)\,dx$$ if finite : converge, else: div

ex: $\int_0^2 \frac{dx}{\sqrt{4-x^2}} = \lim\limits_{t \to 2^-} \int_0^t \frac{dx}{\sqrt{4-x^2}} = \lim\limits_{t \to 2^-} \sin^{-1}\left(\frac{x}{2}\right)\Big|_0^t = \lim\limits_{t \to 2^-} \left[\sin^{-1}\left(\frac{t}{2}\right) - \sin^{-1}\left(\frac{0}{2}\right)\right] = \frac{\pi}{2}$

H| Integrands that are infinite at interior point

$a < c < b$, Suppose $\lim\limits_{x \to c} |f(x)| = \infty$ then:

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$ If BOTH finite : converge, else : div

ex: $\int_{-2}^{1} \frac{1}{x^2}\,dx = \int_{-2}^{1} \frac{1}{x^2}\,dx = \int_{-2}^{0} \frac{1}{x^2}\,dx + \int_{0}^{1} \frac{1}{x^2}\,dx$

$= \left(-\frac{1}{x}\right)\Big|_{-2}^{0} + \left(-\frac{1}{x}\right)\Big|_{0}^{1} = -\infty - \frac{1}{2} + \left(-\frac{1}{1}\right) + \infty$

$\therefore$ Divergent