



COMPUTER SCIENCE & MATHEMATICS

Integral Tak Wajar

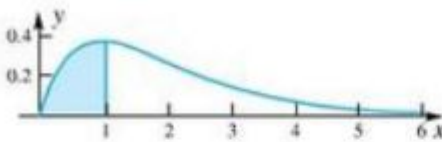
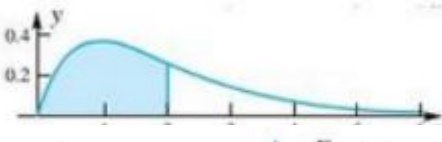
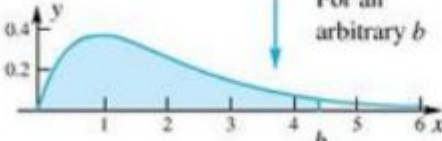
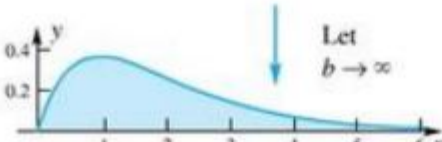
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Batasan Integral Tak Wajar

Definisi Integral Tak Wajar

Contoh:

$$F(x) = \int_0^b x e^{-x} dx$$

Integral	Picture	Exact Value	Numerical Approximation
$\int_0^1 x e^{-x} dx$		$1 - e^{-1} - 1e^{-1}$	0.2642
$\int_0^2 x e^{-x} dx$		$1 - e^{-2} - 2e^{-2}$	0.5940
$\int_0^b x e^{-x} dx$		$1 - e^{-b} - be^{-b}$	
$\int_0^{\infty} x e^{-x} dx$		$\lim_{b \rightarrow \infty} [1 - e^{-b} - be^{-b}] = 1$	

Definisi Integral Tak Wajar

Definisi Integral Tak Wajar

Misalkan $F(x)$ yaitu suatu integral $f(x)$ yang disimbolkan sebagai

$$F(x) = \int_a^b f(x)dx$$

dengan a atau b atau keduanya bernilai ∞ , maka $F(x)$ merupakan **integral tak wajar**.

Batasan Integral Tak Wajar

Limit Tak Hingga Pada Satu Sisi

Definition

$$\int_{-\infty}^b f(x) dx = \lim_{a \rightarrow -\infty} \int_a^b f(x) dx$$

$$\int_a^{\infty} f(x) dx = \lim_{b \rightarrow \infty} \int_a^b f(x) dx$$

If the limits on the right exist and have finite values, then we say that the corresponding improper integrals **converge** and have those values. Otherwise, the integrals are said to **diverge**.

Latihan Soal

Cek apakah masing-masing integral *converge* atau *diverge*

1. $\int_{-\infty}^{-1} x e^{-x^2} dx$

2. $\int_0^{\infty} \sin x dx$

Latihan Soal

Cek apakah masing-masing integral *converge* atau *diverge*

$$\begin{aligned} 1. \int_{-\infty}^{-1} x e^{-x^2} dx \quad & \int_a^{-1} x e^{-x^2} dx = -\frac{1}{2} \int_a^{-1} e^{-x^2} (-2x dx) = \left[-\frac{1}{2} e^{-x^2} \right]_a^{-1} \\ & = -\frac{1}{2} e^{-1} + \frac{1}{2} e^{-a^2} \end{aligned}$$

$$\int_{-\infty}^{-1} x e^{-x^2} dx = \lim_{a \rightarrow -\infty} \left[-\frac{1}{2} e^{-1} + \frac{1}{2} e^{-a^2} \right] = -\frac{1}{2e}$$

Latihan Soal

Cek apakah masing-masing integral *converge* atau *diverge*

1. $\int_{-\infty}^{-1} x e^{-x^2} dx$

2. $\int_0^{\infty} \sin x dx$

$$\begin{aligned}\int_0^{\infty} \sin x dx &= \lim_{b \rightarrow \infty} \int_0^b \sin x dx = \lim_{b \rightarrow \infty} [-\cos x]_0^b \\ &= \lim_{b \rightarrow \infty} [1 - \cos b]\end{aligned}$$

Batasan Integral Tak Wajar

Limit Tak Hingga Pada Kedua Batas

Definition

If both $\int_{-\infty}^0 f(x) dx$ and $\int_0^{\infty} f(x) dx$ converge, then $\int_{-\infty}^{\infty} f(x) dx$ is said to converge and have value

$$\int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^0 f(x) dx + \int_0^{\infty} f(x) dx$$

Otherwise, $\int_{-\infty}^{\infty} f(x) dx$ diverges.

Hitunglah:

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$$

Integral Tak Hingga

Integral Tak Hingga Pada Titik Ujung

Definition

Let f be continuous on the half-open interval $[a, b)$ and suppose that $\lim_{x \rightarrow b^-} |f(x)| = \infty$. Then

$$\int_a^b f(x) dx = \lim_{t \rightarrow b^-} \int_a^t f(x) dx$$

provided that this limit exists and is finite, in which case we say that the integral converges. Otherwise, we say that the integral diverges.

Latihan Soal

Temukan:

1. $\int_0^2 \frac{dx}{\sqrt{4-x^2}}$

Integran Tak Hingga

Integran Tak Hingga Pada Titik Interior

Definition

Let f be continuous on $[a, b]$ except at a number c , where $a < c < b$, and suppose that $\lim_{x \rightarrow c} |f(x)| = \infty$. Then we define

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

provided both integrals on the right converge. Otherwise, we say that $\int_a^b f(x) dx$ diverges.

Latihan Soal

Temukan:

1. $\int_{-2}^1 1/x^2 dx$ apakah diverge?



End