Rep QUIZ 3

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Kalkulus 2 - B

1 (a) $a_n = \frac{2^n+1}{2^{2n}-1}$, $n \geq 1$

$\lim_{n\to\infty} \frac{2^n+1}{2^{2n}-1}$ use l'Hopital $\Rightarrow \lim_{x\to\infty} \frac{2^x+1}{2^{2x}-1} \overset{l'Hopital}{=} \lim_{x\to\infty} \frac{2^x \ln(2)}{2 \cdot 2^{2x} \ln(2)}$

$\Rightarrow \lim_{x\to\infty} \frac{2^x}{2 \cdot 2^{2x}} \Rightarrow \frac{1}{2} \lim_{x\to\infty} \cdot \frac{\cancel{2^x}}{(2^x)^2} \Rightarrow \frac{1}{2} \lim_{x\to\infty} \frac{1}{2^x} = 0$

maka, berdasarkan $\lim_{x\to\infty} f(x) = L$ maka $\lim_{n\to\infty} f(n) = L$

$\lim_{n\to\infty} \frac{2^n+1}{2^{2n}-1} = 0$, yang barisan tsb konvergen ke 0

(b) a. rumus eksplisit $a_n = \frac{1}{n(1+\ln(n^2))} \Rightarrow a_n = \frac{1}{n(1+2\ln|n|)}$

use limit comparison test

$a_n = \frac{1}{n(1+2\ln|n|)}$ ✓ dugaan divergen by p series karena $p \leq 1$

let $b_n = \frac{1}{n}$ ✓ divergen

$\Rightarrow$ use limit comparison test $\lim_{n\to\infty} \frac{a_n}{b_n} \Rightarrow \lim_{n\to\infty} \frac{\frac{1}{n(1+2\ln|n|)}}{\frac{1}{n}}$

$\Rightarrow \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}(1+2\ln|n|)} \Rightarrow \lim_{n\to\infty} \frac{1}{1+2\ln|n|} \Rightarrow 0$

Karena limit menghasilkan 0 dimana 0 adalah finite, maka deret divergen

3 a. $\sum_{n=1}^{\infty} \frac{\sqrt{\sqrt{n}+n^2}}{\sqrt{n}-2n} \Rightarrow$ gunakan uji suku ke-n $\Rightarrow \lim_{n\to\infty} a_n$

$\Rightarrow \lim_{n\to\infty} \frac{\sqrt{\sqrt{n}+n^2}}{\sqrt{(\sqrt{n}-2n)^2}} \Rightarrow \frac{\sqrt{\sqrt{n}+n^2}}{\sqrt{n-4n\sqrt{n}+4n^2}}$

$\Rightarrow$ bagi dan pangkat tertinggi $\Rightarrow \frac{\sqrt{\frac{\sqrt{n}}{n^2}+\frac{n^2}{n^2}}}{\sqrt{\frac{n}{n^2}-\frac{4n\sqrt{n}}{n^2}+\frac{4n^2}{n^2}}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$ karena $\lim_{n\to\infty} a_n \neq 0$

maka deret tsb

Divergen

4. $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^2\sqrt{n+1}}$ ✓ dugaan konvergen by p-series

Use limit comparison test

$$\frac{\frac{\ln(n)}{n^2\sqrt{n+1}}}{\frac{1}{n^2}} \quad \lim_{n\to\infty} \frac{\cancel{n^2}\ln(n)}{\cancel{n^2}\sqrt{n+1}} \Rightarrow \lim_{n\to\infty} \frac{\ln(n)}{\sqrt{n+1}} \Rightarrow 0$$

$\frac{1}{n^2}$ ← konvergen

maka deret dapat disimpulkan adalah konvergen

5. a. $\sum_{n=4}^{\infty} (-1)^n \frac{\sqrt{n^2-9}}{(3n)!}$

uji dengan uji rasio mutlak mutlak

$$\cancel{\sum_{n=4}^{\infty}} \cancel{(-1)^n} \frac{\sqrt{n^2-9}}{3n!} \Rightarrow \frac{\sqrt{(n+1)^2-9}}{(3n+1)!} \times \frac{3n!}{\sqrt{n^2-9}}$$

jika diaurutkan

$$\Rightarrow \lim_{n\to\infty} \frac{\sqrt{n^2+2n+1-9}}{(3n+1)\cancel{(3n!)}} \cdot \frac{\cancel{3n!}}{\sqrt{n^2-9}} \Rightarrow \frac{\sqrt{n^2+2n-8}}{(3n+1)\sqrt{n^2-9}} \Rightarrow \frac{\sqrt{n^2+2n-8}}{\sqrt{(3n+1)^2}\sqrt{n^2-9}}$$

$$\lim_{n\to\infty} \frac{\sqrt{n^2+2n-8}}{\sqrt{9n^4+6n^3-81n^2-54n-9}} \Rightarrow$$

bagi dan pangkat terbesar

$$\frac{\sqrt{\frac{n^2}{n^4}+\frac{2n}{n^4}-8}}{\sqrt{\frac{9n^4}{n^4}+\frac{6n^3}{n^4}-\frac{81n^2}{n^4}-\frac{54n}{n^4}-\frac{9}{n^4}}}$$

$$\Rightarrow \frac{0}{\sqrt{9}} \Rightarrow 0$$

Karena hasil $p < 1$, maka dpt disimpulkan deret konvergen secara mutlak