Pop Quiz 4 – 2206820352 – Juan Maxwell Tanaya

1 2 3 4 5 6 7 8 9 10

1a. $\frac{2x+4}{3} - \frac{(2x+4)^2}{9\cdot 3} + \frac{(2x+4)^3}{27\cdot 5} - \frac{(2x+4)^4}{81\cdot 7} + \frac{(2x+4)^5}{243\cdot 9} - \dots$

$\cancel{\sum_{n=0}^{\infty} (-1)^{n+1} \frac{(2x+4)^n}{(3^n\cdot(2n+1))}}$ $\quad \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(2x+4)^n}{(3^n\cdot(2n-1))}$

$\lim_{n\to\infty} \left| \frac{(2x+4)^{n+1}}{(3^{n+1}\cdot(2(n+1)-1))} \times \frac{(3^n\cdot(2n-1))}{(2x+4)^n} \right|$

$= \lim_{n\to\infty} \left| 2x+4 \ \frac{(2n-1)}{(6n+3)} \right|$

$= |2x+4|$

Convergence set

$|2x+4| < 1$

$-1 < 2x+4 < 1$

$-5 < 2x < -3$

$-\frac{5}{2} < x < -\frac{3}{2}$

$\cancel{\left(-\frac{5}{2}, -\frac{3}{2}\right)}$

Uji titik $x = -\frac{5}{2}$

$2x+4 = -5+4$
$= -1$

$\lim_{n\to\infty} \left| \frac{1}{3^{n+1}(2n+1)} \times \frac{3^n(2n-1)}{1} \right|$

$= \lim_{n\to\infty} \left| \frac{2n-1}{6n+3} \right| = \lim_{n\to\infty} \frac{2}{6}$

$= \frac{1}{3}$

$\therefore x = -\frac{5}{2}$ ~~tidak~~ termasuk

Uji titik $x = -\frac{3}{2}$

$2x+4 = -3+4 = 1$

$\lim_{n\to\infty} \left| \frac{1}{3^{n+1}(2n+1)} \times \frac{3^n(2n-1)}{1} \right| = \lim_{n\to\infty} \left| \frac{2n-1}{6n+3} \right| = \lim_{n\to\infty} \frac{2}{6} = \frac{1}{3}$

$\therefore x = -\frac{3}{2}$ ~~tidak~~ termasuk $\left[-\frac{5}{2}, -\frac{3}{2}\right]$

2a. $\sum_{n=2}^{\infty} (x+2)^n \cdot n! \cdot \ln(n)$

$\lim_{n\to\infty} \left| \frac{(x+2)^{n+1}\cdot(n+1)!\cdot\ln(n+1)}{(x+2)^n \cdot n! \cdot \ln(n)} \right|$

$= \lim_{n\to\infty} |x+2| \cdot (n+1) \cdot \frac{\ln(n+1)}{\ln(n)}$

$= |x+2|$

Convergence set $= (-3, -1)$

$|x+2| < 1$

$-1 < x+2 < 1$

$-3 < x < -1$

Uji titik $x = -3$

$x+2 = -3+2 = -1$

$\lim_{n\to\infty} \left| \frac{(n+1)!\ \ln(n+1)}{n!\ \ln(n)} \right|$

$= \lim_{n\to\infty} \left| (n+1) \frac{\ln(n+1)}{\ln(n)} \right|$

$= \infty$

$\therefore$ Titik $x = -3$ tidak termasuk

Uji titik $x = -1$

$x+2 = -1+2 = 1$

$\lim_{n\to\infty} n! \cdot \ln(n)$

$= \infty$

$\therefore$ Titik $x = -1$ tidak termasuk

3a. $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n^2 2^n}$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)^2 2^{n+1}} \times \frac{n^2 2^n}{x^n} \right|$

$= \lim_{n\to\infty} \left| \frac{x \cdot n^2}{n^2+2n+1} \right|$

$= |x|$

Convergence set $= [-1, 1]$

$|x| < 1$

$-1 < x < 1$

Uji titik $x = -1$

$\lim_{n\to\infty} \left| \frac{1}{(n+1)^2 2^{n+1}} \times n^2 2^n \right|$

$= \lim_{n\to\infty} \left| \frac{n^2}{2n^2+4n+2} \right|$

$= \frac{1}{2}$

Karena $p < 1$, $x = -1$ termasuk

Uji titik $x = 1$

$\lim_{n\to\infty} \left| \frac{n^2 2^n}{(n+1)^2 2^{n+1}} \right|$

$= \lim_{n\to\infty} \left| \frac{n^2}{2n^2+4n+2} \right|$

$= \frac{1}{2}$

Karena $p < 1$, $x = 1$ termasuk

4a. $1 - 2x + \frac{4x^2}{2!} - \frac{8x^3}{3!} + \frac{16x^4}{4!} - \frac{32x^5}{5!} + \dots$

Kita tahu bahwa

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$

Jika kita substitusikan $x = -2x$

$e^{-2x} = 1 - 2x + \frac{4x^2}{2!} - \frac{8x^3}{3!} + \frac{16x^4}{4!} - \frac{32x^5}{5!} + \dots$

Maka $f(x)$ yang merepresentasikan deret tersebut adalah

$f(x) = e^{-2x}$

5a. $f(x) = \frac{x^2}{(1-2x^3)^3}$

$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \dots$

$\frac{2}{(1-x)^3} = \cancel{1}\,2 + 6x + 12x^2 + \dots$

$\frac{1}{(1-x)^3} = 1 + 3x + 6x^2 + \dots$

$\frac{1}{(1-2x^3)^3} = 1 + 6x^4 + 12x^5 + \dots$

$f(x) = x^2 (1 + 6x^4 + 12x^5 + \dots)$

$= x^2 + 6x^6 + 12x^7 + \dots$