Pop Quiz 4

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Kalkulus 2 - B

digit 8 ganjil, 9 genap

1. a. $\frac{3}{4}x + \frac{5}{9}x^2 + \frac{7}{16}x^3 + \frac{9}{25}x^4 + \frac{11}{36}x^5 + \dots$

Jawab :

Rumus powerseries :

$$\sum_{n=1}^{\infty} \frac{(2n+1) \cdot x^n}{\cancel{(2n)^2}\,(n+1)^2}$$

$\Rightarrow$ use ratio test $\Rightarrow \frac{(2n+2) \cdot x^{n+1}}{(n+2)^2} \cdot \frac{(n+1)^2}{(2n+1)\, x^n}$

$$\Rightarrow \frac{\cancel{x^n}\, x^1 (2n+2)(n+1)^2}{\cancel{x^n}\,(2n+1)(n+2)^2} \Rightarrow \rho = |x| \lim_{n\to\infty} \frac{(2n+2)(n+1)^2}{(2n+1)(n+2)^2} \Rightarrow |x| \lim_{n\to\infty} \frac{2n^3 + \dots}{2n^3 + \dots}$$

$\Rightarrow \rho = |x| < 1 \Rightarrow x < 1$ dan $x > -1$

Pada $x = 1$

$\lim_{n\to\infty} \frac{2n+1}{(n+1)^2} \Rightarrow \lim_{n\to\infty} \frac{2n+1}{n^2+2n+1}$ $\rightarrow$ can use integral test

$\int_1^{\infty} \frac{2n+1}{(n+1)^2}$ $u = n+1$ $du = dn$, $2u-1 = 2n+1$ $\Rightarrow \int_1^{\infty} \frac{2u-1}{u^2}\, du$

$\Rightarrow \int_1^{\infty} \frac{2}{u} - \frac{1}{u^2} \Rightarrow 2\ln|n+1| + \frac{1}{n+1} \Big|_1^{\infty} \Rightarrow \infty$ $\rightarrow$ divergen

Pada $x = -1$

$\lim_{n\to\infty} \frac{(-1)^n (2n+1)}{(n+1)^2} \Rightarrow$ we absolute after nothing seems

If $\lim_{n\to\infty} |a_n|$ conv, $\lim_{n\to\infty} a_n$ conv

$\Rightarrow \lim_{n\to\infty} \frac{2n+1}{(n+1)^2} = 0 \rightarrow$ konvergen

maka Convergence set : $-1 \leq x < 1$

2. a. $\sum_{n=3}^{\infty} \frac{(x-2)^n (n-1)^2}{(2n)!}$ use ratio test $\Rightarrow \frac{(x-2)^{n+1} \cdot (n)^2}{(2n+2)!} \cdot \frac{2n!}{(x-2)^n (n-1)^2}$

$(2(n+1))!$

$$\Rightarrow \sum_{n=3}^{\infty} \frac{\cancel{(x-2)^n} \cdot (x-2) \cdot n^2 \cdot \cancel{2n!}}{(2n+2)(2n+1)\,\cancel{2n!} \cdot (n-1)^2 \cdot \cancel{(x-2)^n}}$$

$$\Rightarrow \sum_{n=3}^{\infty} \frac{(x-2) \cdot n^2}{(2n+2)(2n+1)(n-1)^2}$$

$|x-2| \lim_{n\to\infty} \frac{n^2}{4n^4 + \dots} \Rightarrow (x-2) \cdot (0) = 0$

maka deret konvergen di hanya 1 titik

yaitu $x = 2$

3\. a. $\sum_{n=2}^{\infty} (-1)^n \frac{x^n \ln(n)}{n^2}$ uie absolute ratio test $\Rightarrow \sum_{n=2}^{\infty} \frac{x^{n+1} \cdot \ln(n+1)}{(n+1)^2} \cdot \frac{n^2}{x^n \cdot \ln(n)}$

$\Rightarrow \sum_{n=2}^{\infty} \frac{\cancel{x^n} \cdot x}{\cancel{x^n}} \cdot \frac{n^2}{(n+1)^2} \cdot \frac{\ln|n+1|}{\ln|n|}$

$\Rightarrow |x| \cdot \lim_{n\to\infty} \frac{n^2}{(n+1)^2} \cdot \frac{\cancel{\ln|n|} \cdot \ln|a|}{\cancel{\ln|n|}} \to 0$

$|x| \cdot 0 \Rightarrow$ deret konvergen di satu titik saja yaitu $x = 0$

4\. a. $2 - \frac{8x^2}{3!} + \frac{32x^4}{5!} - \frac{128x^6}{7!} + \dots$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$

$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \frac{x^8}{9!} + \dots$

$\frac{2 \cdot \sin x}{x} = 2 - \frac{2x^2}{3!} + \frac{2x^4}{5!} - \frac{2x^6}{7!} \dots$

$\frac{2 \cdot \sin(2x)}{x} = 2 - \frac{2(2x)^2}{3!} + \frac{2(2x)^4}{5!} - \frac{2(2x)^6}{7!}$

$\frac{2 \cdot \sin(2x)}{x} = 2 - \frac{8x^2}{3!} + \frac{32x^4}{5!} - \frac{128x^6}{7!}$

5\. a. $\boxed{\frac{\ln(1-2x^2)}{x}}$

Cari bentuk $\ln(1-2x^2)$ dahulu dan menurunkannya bentuk yg ingin dicari

$\frac{d}{dx} \ln|1-2x^2| \Rightarrow -\frac{4x}{1-2x^2} \Rightarrow$

$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \dots$

$\frac{1}{1-(2x^2)} = 1 + 2x^2 + 4x^4 + 8x^6 + 16x^8 + \dots$

$-\frac{4x}{1-2x^2} \Rightarrow -4x - 8x^3 - 16x^5 - 32x^7 - \dots$

lalu integralkan

$\int -\frac{4x}{1-2x^2} = \int -4x - 8x^3 - 16x^5 - 32x^7 - \dots$

$\ln|1-2x^2| = -2x^2 - 2x^4 - \frac{16}{6}x^6 - \frac{32}{8}x^8 - \dots$

bagi dengan x

$\frac{\ln|1-2x^2|}{x} = -2x - 2x^3 - \frac{16}{6}x^5 - \frac{32}{8}x^7 - \dots$