

Pop Quiz 9

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1. a. $f(x, y) = x^3 + y^3 - 15x^2 - 60y + 25$

$\nabla f(x, y) = f_x(x, y) = f_y(x, y) = 0$

$f_x(x, y) = 3x^2 - 30 = 0$

$3x^2 = 30$

$x^2 = 10$

$x = \pm\sqrt{10}$

$f_y(x, y) = 3y^2 - 60 = 0$

$3y^2 = 60$

$y^2 = 20$

$y = \pm\sqrt{20}$

Semua titik kritis yang mungkin adalah

$(\sqrt{10}, \sqrt{20}), (-\sqrt{10}, -\sqrt{20})$

$(\sqrt{10}, -\sqrt{20}), (-\sqrt{10}, \sqrt{20})$

$f_{xx} = 6x$

$f_{yy} = 6y \Rightarrow D^2 f(x, y) = 6x \cdot 6y = 36xy$

$f_{xy} = 0$

cek semua titik kritis

a. $D(\sqrt{10}, \sqrt{20}) \Rightarrow \overset{f_{xx} > 0}{\underbrace{6\sqrt{10} \cdot 6\sqrt{20}}_{> 0}} = 36\sqrt{200} > 0$

↳ Karena $D(\sqrt{10}, \sqrt{20}) > 0$ dan $f_{xx} > 0$, maka titik $P(\sqrt{10}, \sqrt{20})$ adalah titik minimum lokal

b. $D(\sqrt{10}, -\sqrt{20}) \Rightarrow 6\sqrt{10} \cdot (-6\sqrt{20}) = -360$

↳ Karena $D(\sqrt{10}, -\sqrt{20}) < 0$ maka titik $P(\sqrt{10}, -\sqrt{20})$ adalah titik saddle point

c. $D(-\sqrt{10}, \sqrt{20}) \Rightarrow -6\sqrt{10} \cdot 6\sqrt{20} = -360$

↳ Karena $D(-\sqrt{10}, \sqrt{20}) < 0$, maka titik $P(-\sqrt{10}, \sqrt{20})$ adalah titik saddle point

d. $D(-\sqrt{10}, -\sqrt{20}) \Rightarrow \overset{f_{xx} < 0}{\underbrace{-6\sqrt{10} \cdot -6\sqrt{20}}_{> 0}} = 360$

↳ Karena $D(-\sqrt{10}, -\sqrt{20}) > 0$ dan $f_{xx} < 0$, maka $P(-\sqrt{10}, -\sqrt{20})$ adalah titik maksimum lokal



② ③ $f(x,y) = x^2y$ $g(x,y) = x^2 + 2y^2 - 6 = c$ ④

$$\hookrightarrow \begin{pmatrix} 2xy \\ x^2 \end{pmatrix} = \lambda \begin{pmatrix} 2x \\ 4y \end{pmatrix}$$

$$\rightarrow 2xy = \lambda 2x \quad \rightarrow \lambda = y$$

$$\rightarrow x^2 = \lambda 4y \rightarrow \text{substitusi } \lambda = y$$

$$\hookrightarrow x^2 = \lambda 4\lambda$$

$$x^2 = 4\lambda^2$$

substitusi ke persamaan batas/constant

$$4x^2 + 2\lambda^2 = 6 : c$$

$$6\lambda^2 = 6$$

$$\lambda^2 = 1$$

$$\lambda = \pm 1$$

Ketika $\lambda = 1$

$$\hookrightarrow y = 1$$

$$\hookrightarrow x = \pm 2$$

$$f(\pm 2, 1) = 4 \text{ (maksimum)}$$

Ketika $\lambda = -1$

$$\hookrightarrow y = -1$$

$$\hookrightarrow x = \pm 2$$

$$f(\pm 2, -1) = -4 \text{ (minimum)}$$

maka nilai minimum fungsi f adalah -4 ; dan nilai maksimum dari f adalah 4

fungsi objektif

P.Lt

$$\begin{pmatrix} Lt \\ Pt \\ PL \end{pmatrix} = \lambda \begin{pmatrix} L + 2t \\ P + 2t \\ 2P + 2L \end{pmatrix}$$

fungsi Constraint

$$PL + 2Pt + 2Lt = 300$$

$$Lt = \lambda L + 2\lambda t$$

$$Pt = \lambda P + 2\lambda t$$

$$PL = 2\lambda P + 2\lambda L$$

$$Lt = \lambda L + 2\lambda t$$

$$Pt = \lambda P + 2\lambda t$$

$$t(L-P) = \lambda(L-P)$$

$$t = \lambda$$

$$Pt = P + 2t \quad \times 2$$

$$PL = 2P + 2L$$

$$\therefore 2Pt = 2P + 4t$$

$$PL = 2P + 2L$$

$$P = 2$$

$$2Pt - PL = 4t - 2L$$

$$P(2t - L) = 2(2t - L)$$

$$4. a. \int_0^1 \int_1^2 x e^{x+4y} dy dx$$

$$\Rightarrow \int_0^1 \left[4x e^{x+4y} \Big|_1^2 \right] dx$$

$$\Rightarrow \int_0^1 4x e^{x+8} - 4x e^{x+4} dx$$

$$\Rightarrow \int_0^1 4x e^{x+8} - \int_0^1 4x e^{x+4}$$

	D	I
+	4x	e^{x+8}
-	4	e^{x+8}
+	0	e^{x+8}

	D	I
+	4x	e^{x+4}
-	4	e^{x+4}
+	0	e^{x+4}

$$\Rightarrow 4x e^{x+8} - 4e^{x+8} \Big|_0^1 - 4x e^{x+4} - 4e^{x+4} \Big|_0^1$$

$$\Rightarrow 4e^9 - 4e^9 - (0 - 4e^8) - ((4e^5 - 4e^5) - (0 - 4e^4))$$

$$\Rightarrow 0 - (-4e^8) - (0 - (-4e^4))$$

$$\Rightarrow 4e^8 - 4e^4$$

$$= 4(e^8 - e^4)$$