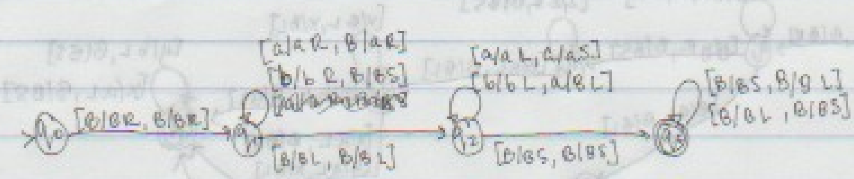
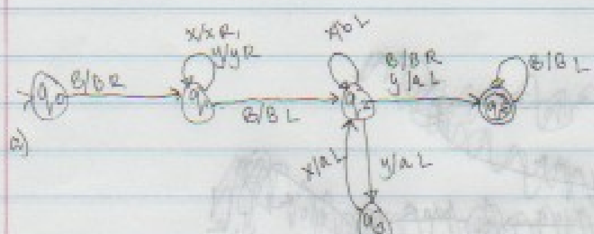
1



q_0BaB, q_0BB	q_0BabB, q_0B
⊢ Bq_1aB, Bq_1B	⊢ Bq_1abB, Bq_1B
⊢ Baq_1B, Baq_1B	⊢ Baq_1bB, Baq_1B
⊢ Bq_2aB, Bq_2aB	⊢ $Babq_1B$, Baq_1B
⊢ q_2BaB, Bq_2aB	⊢ Baq_2bB, Bq_2aB
⊢	⊢ Bq_2abB, $\cancel{B}q_2B$
	⊢ $\cancel{B}q_2BabB$, q_2B
	⊢ q_fBabB, q_fB

2

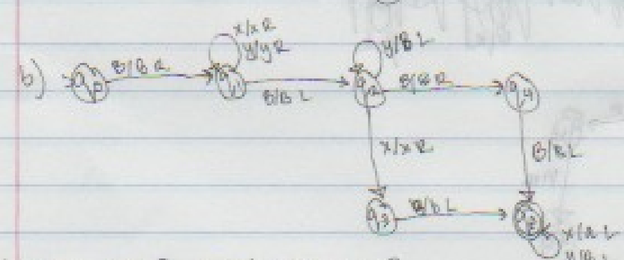
a)



a)

w∈L	in L?	r(w)∈Q	in Q?
λ	yes	λ	yes
x	no	b	no
y	no	a	no
xy	yes	aa	yes
yx	no	ab	no
xyx	no	aab	no
xyxy	yes	aaaa	yes
xxy	no	baa	no
yyx	no	bab	no

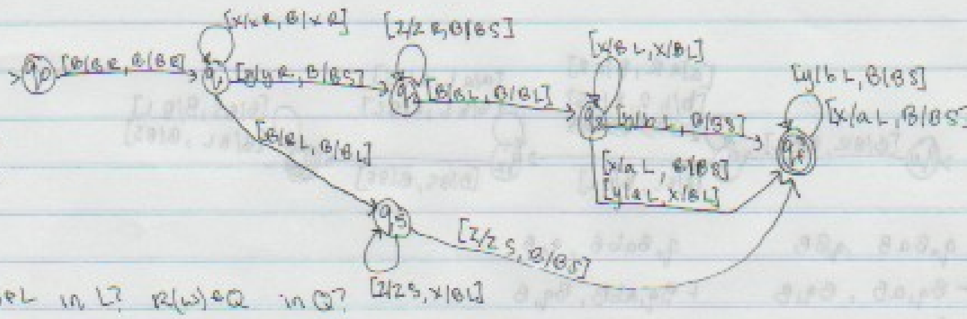
b)



b)

w∈L	in L?	$\cancel{r}$(w)∈Q	in Q?
xx	yes	aab	yes
xxyy	yes	aab	yes
x	yes	ab	yes
xyyy	yes	ab	yes
xyx	no	abab	no
y	no	b	no
yyx	no	bbab	no
λ	no	λ	no
xyxy	no	abab	no

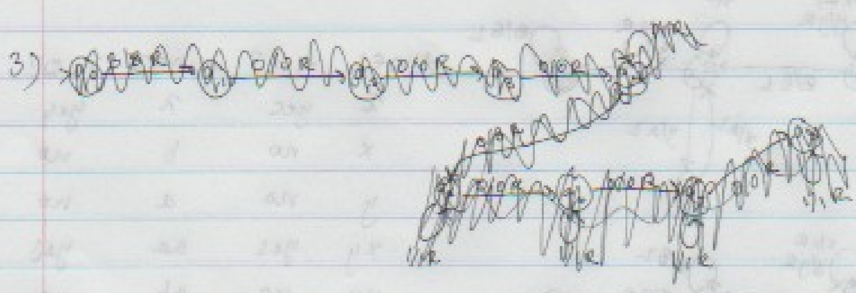
2c)

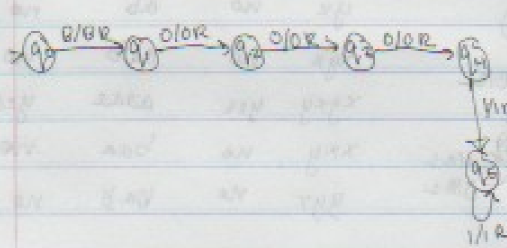


ZG[x,y]

w∈L	in L?	R(w)∈Q	in Q?
xy	no	aa	no
xyx	yes	ab	yes
y	no	b	no
xxyx	no	aaa	no
xyxx	no	aba	no
xxyyxx	yes	aabb	yes

3)





4) Regular representation of a turing machine is:

000 enc(M) 000

enc(M) consists of state transitions followed by "00". the encoding of a transition looks like this:

enc(q_i) 0 enc(x) 0 enc(q_j) 0 enc(y) 0 enc(d)

Where q_i is current state, x is current character being looked at, q_j is state transitioning to, y is new character to replace x, and d is direction to move tape head.

To extend this to include final state, an extra '0' and the unary representation of 0 and 1 could be added to the encoding of a state transition function. eg:

enc(x_i) 0 enc(x) 0 enc(x_j) 0 enc(y) 0 enc(d) 01 for not final state

and enc(x_i) 0 enc(x) 0 enc(x_j) 0 enc(y) 0 enc(d) 011 for final state.

The universal machine U_f would be similar to U, but with an added action that when there are no more available transition to take and the current state is denoted by a final state, then the input run on M is accepted by the machine.

5) The "halts on n'th transition problem" has a bound which is n. This guarantees that if n transitions have been made, but there are more left to compute, the machine halts anyways. The "halting problem" has no bound and can loop forever.

6

7

8) $g = id \quad h = p_1^{(3)} + p_3^{(3)}$

a) $f(3,0) = g(3) = id\ 3 = 3$

$f(3,1) = h(3,0,f(3,0)) = 3 + f(3,0) = 3+3 = 6$

$f(3,2) = h(3,1,f(3,1)) = 3 + f(3,1) = 3+6 = 9$

b) closed form is $3+n$ or $f(x,y) = x+y$

9) a) $f(x,y) = g(x,y,x)$

$f(x,y) = g \circ (p_1^{(2)}, p_2^{(2)}, p_1^{(2)})$

b) $f(x,y,z,2) = g(x,y,x)$

$f(x,y,z,2) = g \circ (p_1^{(4)}, p_2^{(4)}, p_1^{(4)})$

c) $f(x) = g(1,2,x)$

$f(x) = g \circ (1,2,p_1^{(1)})$

10 $gcd(x,y) = \quad gcd(x,0) = x$

$\qquad gcd(x,y+1) = gcd(y, x \bmod y)$

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11 $tc_m(0) = 2$ $\quad tc_m(x) = \begin{cases} x \text{ is even} & 2x + \frac{x}{2} + 4 \\ x \text{ is odd} & \frac{4(n+1)}{2} \end{cases}$

$tc_m(1) = 4$

$tc_m(2) = 9$

$tc_m(3) = 8$

$tc_m(4) = 14$

$tc_m(5) = 12$

12