Outlier Detection:

Probabilistic / Clustering-Based

Mining Massive Datasets

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Sources

- Data Mining, The Textbook (2015) by Charu Aggarwal (chapter 8) – slides by Lijun Zhang

Probabilistic methods

Related to probabilistic model-based clustering

- Assume data is generated from a mixture-based generative model
- Learn the parameters of the model from data
  - EM algorithm
- Evaluate the probability of each data point being generated by the model
  - Points with low values are outliers

Mixture-based generative model

- Data is generated by a mixture of k distributions with probability distributions $G_1, \ldots, G_k$
- Each point $\overline{X}$ is generated as follows:
  1) Select a mixture component with probability α_i
     - Suppose it's component r
  2) Sample a data point from distribution G_r

Learning parameters from data

- Probability of generating a point

$$
\begin{aligned}
f^{\text{point}}\left(\overline{X_j}|\mathcal{M}\right) &= \sum_{i=1}^{k} P\left(\mathcal{G}_i, \overline{X_j}\right) \\
&= \sum_{i=1}^{k} P(\mathcal{G}_i) P(\overline{X_j}|\mathcal{G}_i) \\
&= \sum_{i=1}^{k} \alpha_i f^i(\overline{X_j})
\end{aligned}
$$

Learning parameters from data

- Probability of generating a point

$$\mathrm{f}^{\mathrm{point}}\left(\overline{X_j}|\mathcal{M}\right) = \sum_{i=1}^{k} \alpha_i f^i(\overline{X_j})$$

- Probability of generating a dataset

$$f^{\mathrm{data}}(\mathcal{D}|\mathcal{M}) = \prod_{j=1}^{n} f^{\mathrm{point}}(\overline{X_j}|\mathcal{M})$$

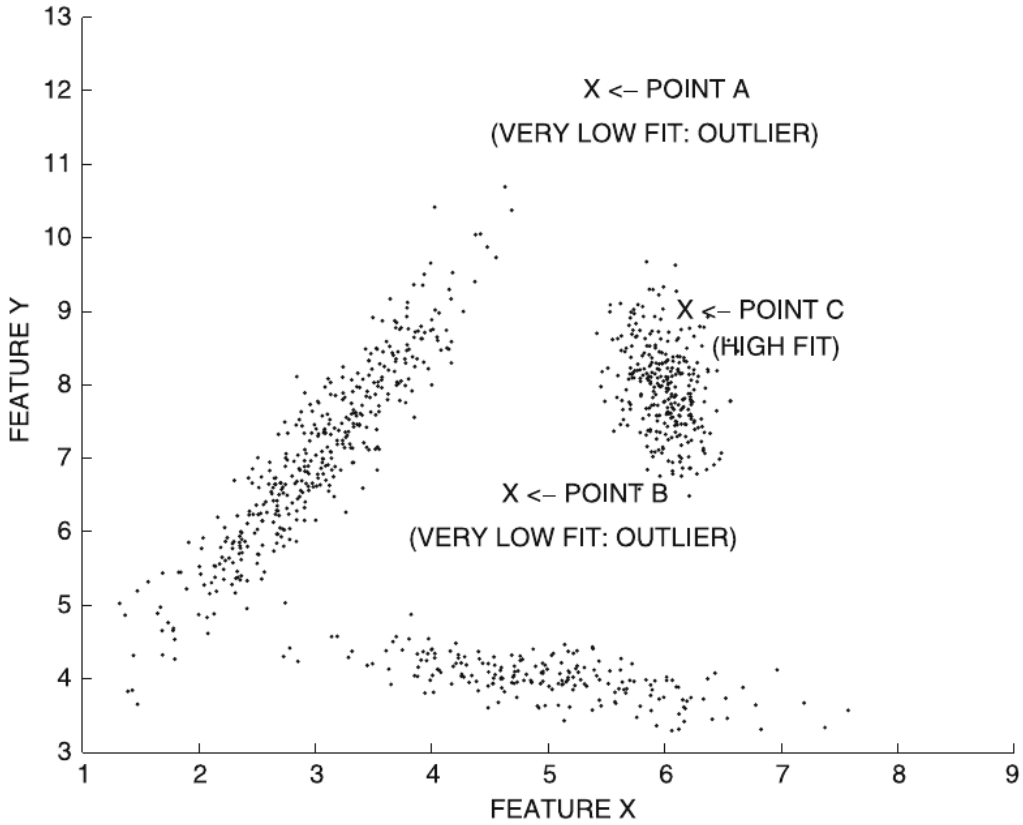
- Learning: maximize log likelihood

$$\max \mathcal{L}\left(\mathcal{D}|\mathcal{M}\right) = \log\left(\prod_{j=1}^{n} f^{\mathrm{point}}\left(\overline{X_j}|\mathcal{M}\right)\right) = \sum_{j=1}^{n} \log\left(\sum_{i=1}^{k} \alpha_i f^i\left(\overline{X_j}\right)\right)$$

Identifying an outlier

Outlier score:

$$\mathrm{f}^{\mathrm{point}}\left(\overline{X_j}|\mathcal{M}\right) = \sum_{i=1}^{k} \alpha_i f^i(\overline{X_j})$$



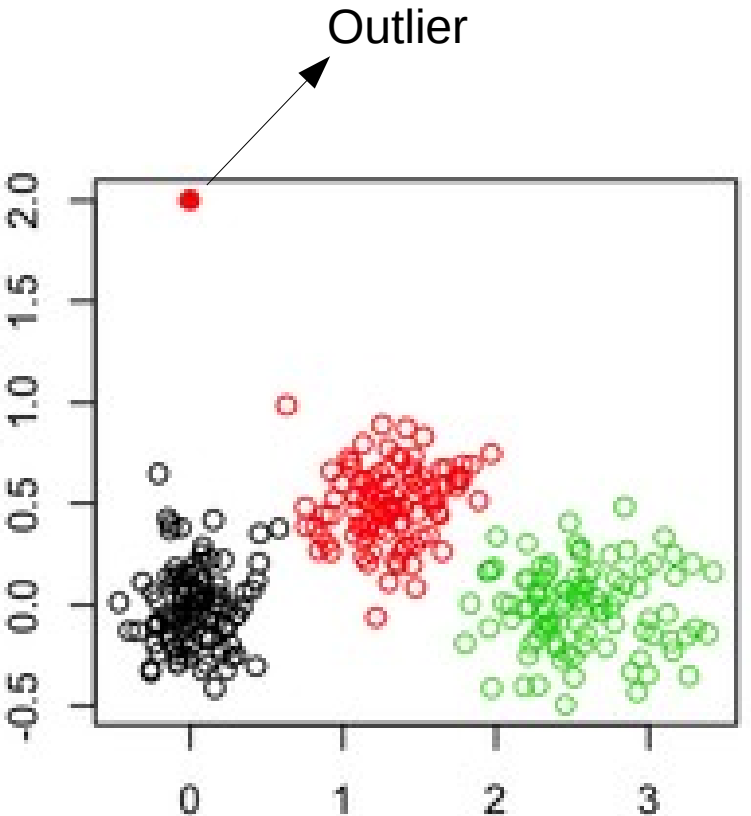
Clustering-based methods

Clustering for outlier analysis

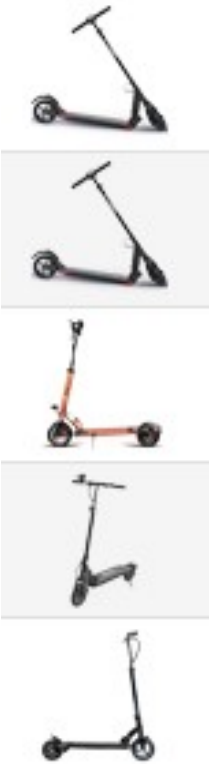
- Clustering associate points to similar points
- Points either clearly belong to a cluster or are outliers
- Some clustering algorithms also detect outliers
  - Examples: DBSCAN, DENCLUE

Simple method

- Cluster data, associating each point to a centroid, e.g., using k-means
- Outlier score = distance of point to its centroid



Exercise: outliers through clustering



Spreadsheet does k-means to cluster the electric scooter database

1)Re-run with a new initial clustering

2)Do you see any interesting pattern in the final clustering assignment?

3)Find outliers according to the method from the previous slide

Spreadsheet links: https://upfbarcelona.padlet.org/chato/hogch321o6pws1fd

Improved method

- Cluster data
- Outlier score = local Mahalanobis distance with respect to center of cluster r

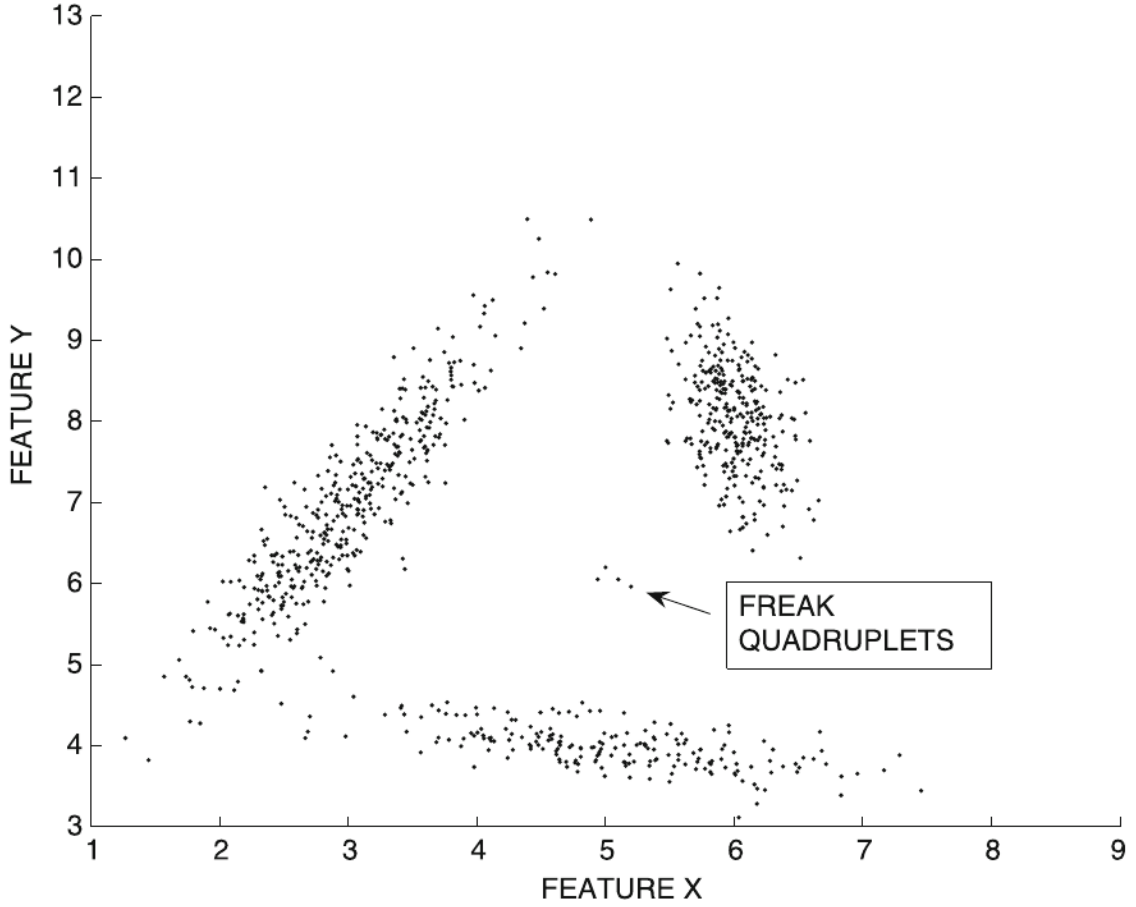
$$\mathrm{Maha}(\overline{X}, \overline{\mu_r}, \Sigma_r) = \sqrt{(\overline{X} - \overline{\mu_r})\Sigma_r^{-1}(\overline{X} - \overline{\mu_r})^T}$$

$\overline{\mu_r}$ is the mean of the cluster r

$\sum_r$ is the covariance matrix of cluster r

Improved method (cont.)

- Remove tiny clusters



Summary

Things to remember

- Probabilistic methods
- Clustering-based methods

Exercises for TT19-TT21

- Data Mining, The Textbook (2015) by Charu Aggarwal
  - Exercises 8.11 $\rightarrow$ all except 10, 15, 16, 17