Mining Time Series:

Forecasting

Mining Massive Datasets

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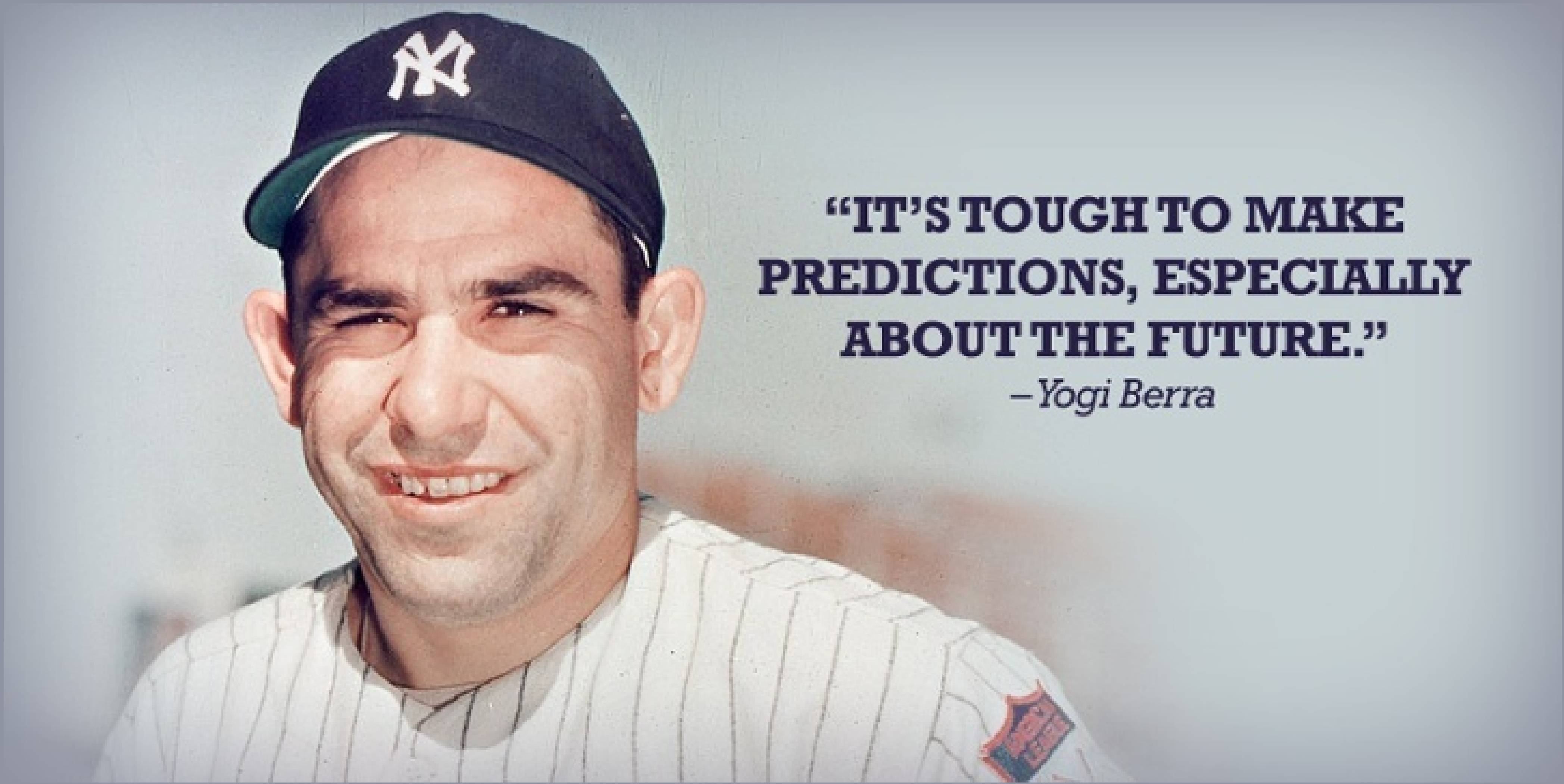
Sources

- Data Mining, The Textbook (2015) by Charu Aggarwal (chapter 14)
- Introduction to Time Series Mining (2006) tutorial by Keogh Eamonn [alt. link]
- Time Series Data Mining (2006) slides by Hung Son Nguyen



(A similar phrase is attributed to Niels Bohr, Danish physicist and winner of the Nobel Prize in 1922)

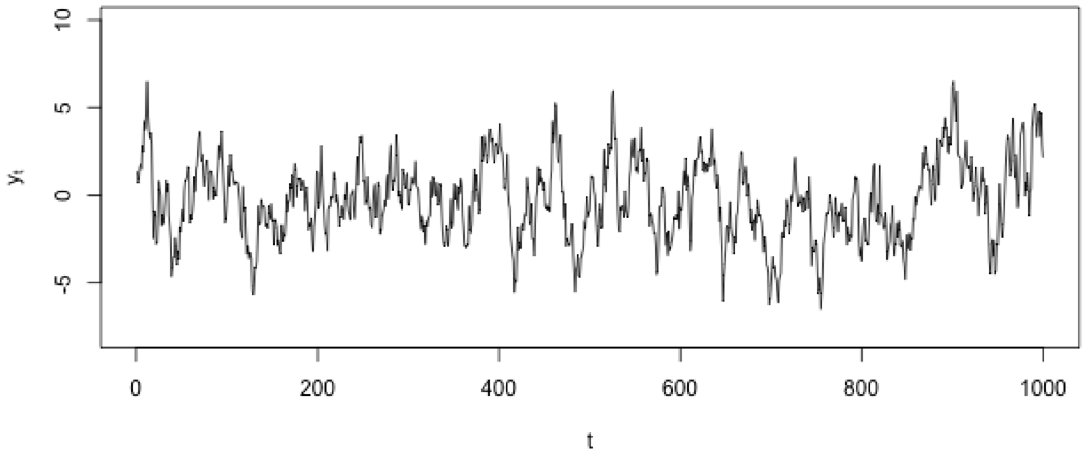
Forecasting

(AR, MA, ARMA, ARIMA, ...)

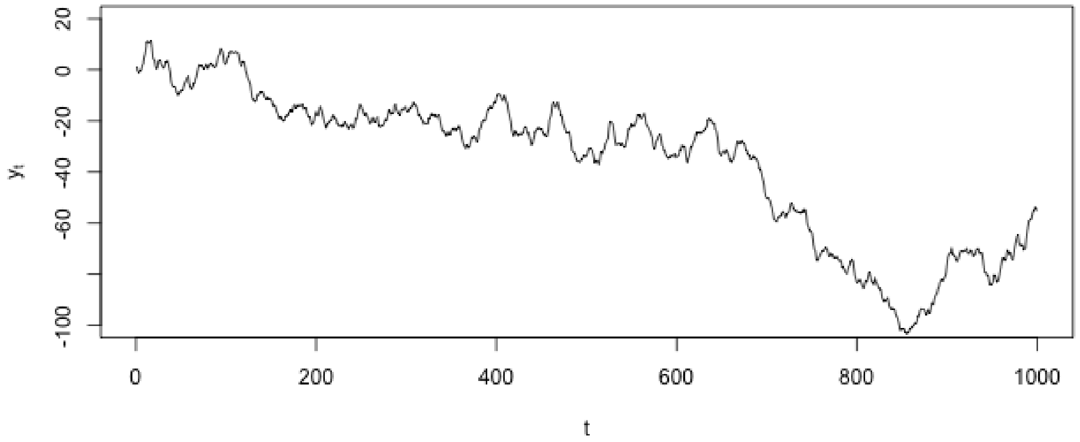
Stationary vs Non-Stationary processes

- **Stationary process**
  - Parameters do not change over time
  - E.g., *White noise* has zero mean, fixed variance, and zero covariance between y_t and y_{t+L} for any lag L
- **Non-stationary process**
  - Parameters change over time
  - E.g., price of oil, height of a child, glucose level of a patient, ...

Stationary process



Non-stationary process



Strictly stationary time series

A **strictly stationary time series** is one in which the distribution of values in any time interval $[a,b]$ is identical to that in $[a+L, b+L]$ for any value of time shift (lag) L

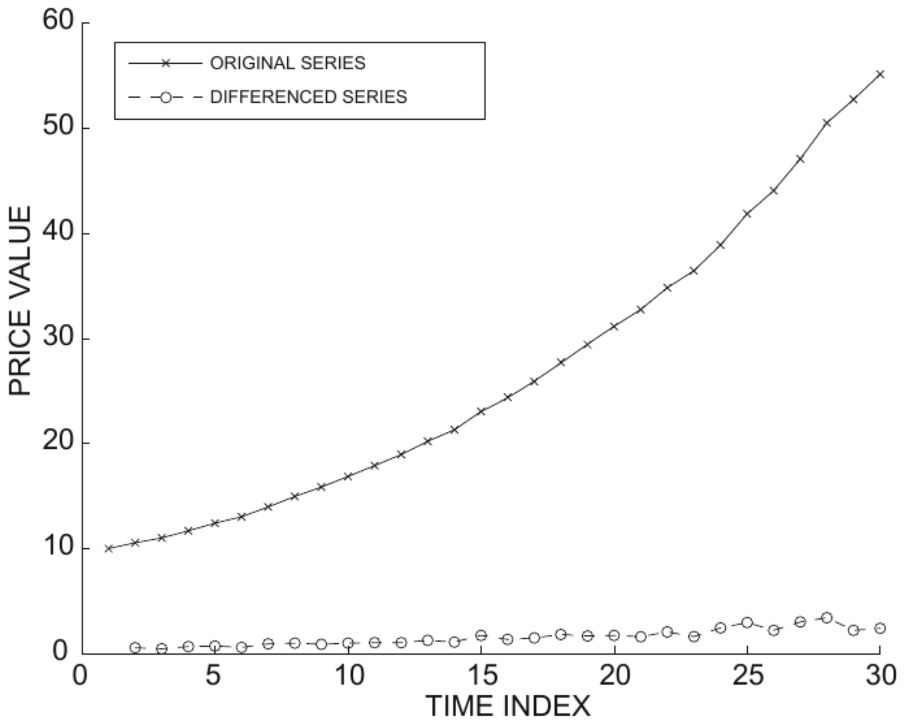
- In this case, current parameters (e.g., mean) are good predictors of future parameters

Differencing

First order differencing

$$y_i' = y_i - y_{i-1}$$

In this first example, if the original series is superlinear, the differenced series is stationary or non-stationary?

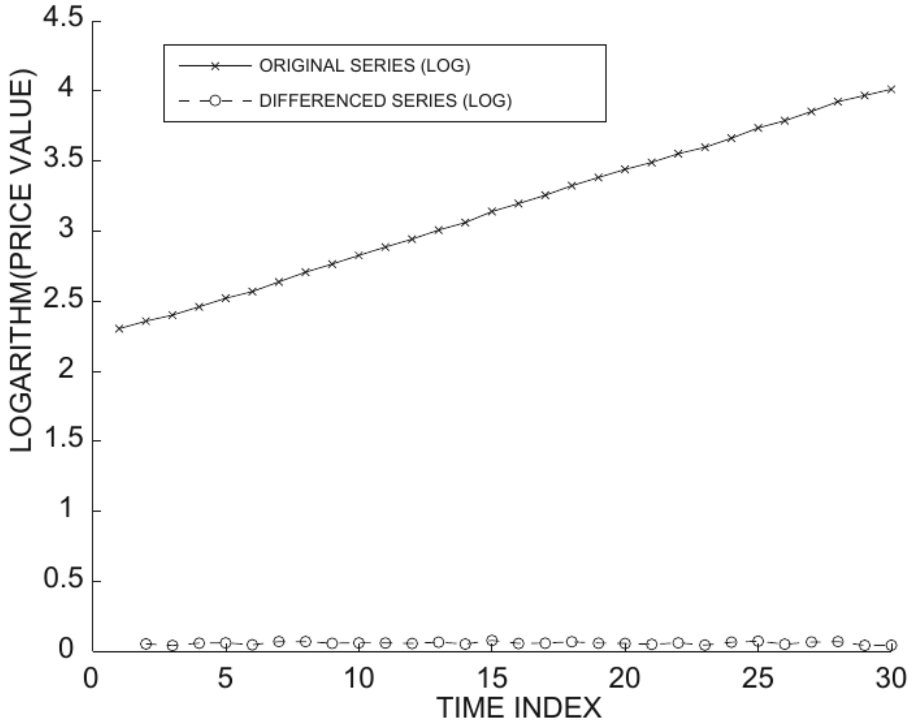


Differencing (cont.)

First order differencing

$$y_i' = y_i - y_{i-1}$$

In this second example, where the series is linear, the differenced series is stationary or non-stationary?



Other differencing operations

- Second-order differencing
$$y_i'' = y_i' - y_{i-1}'$$
$$= y_i - 2 \cdot y_{i-1} + y_{i-2}$$
- Seasonal differencing (m = 24 hours, 7 days, …)
$$y_i' = y_i - y_{i-m}$$

If you find a differencing that yields a stationary series, the forecasting problem is basically solved.

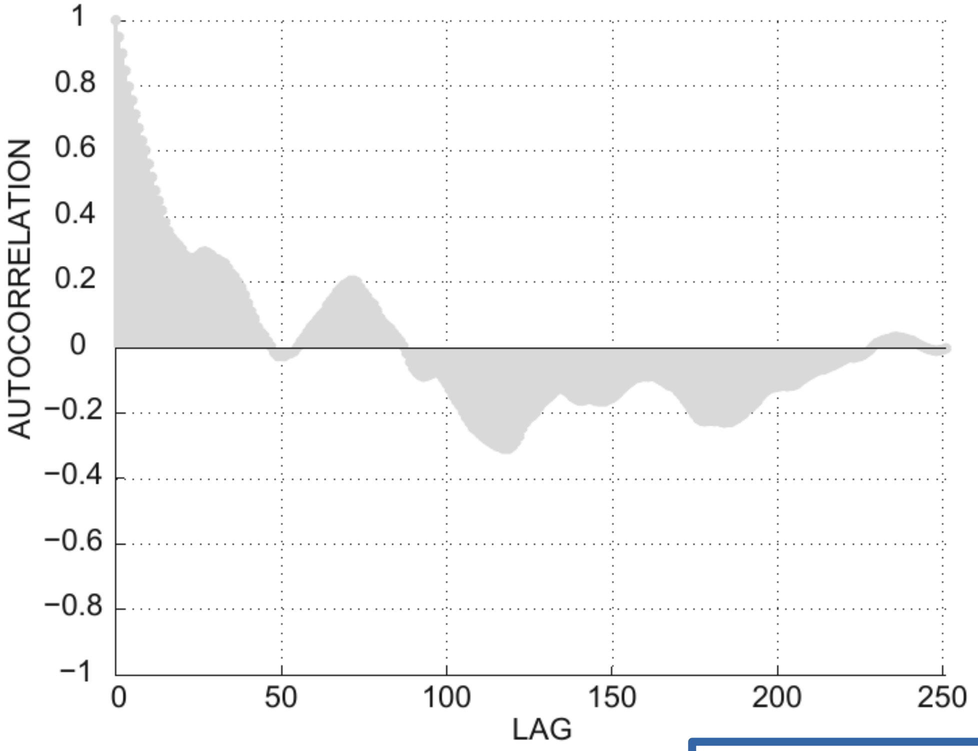
Autoregressive model AR(p)

$$\text{Autocorrelation}(L) = \frac{\text{Covariance}_t(y_t, y_{t+L})}{\text{Variance}_t(y_t)}$$

- Autocorrelation lines in [-1,1]
- High absolute values ⇒ predictability
- Autoregressive model of order p, AR(p):

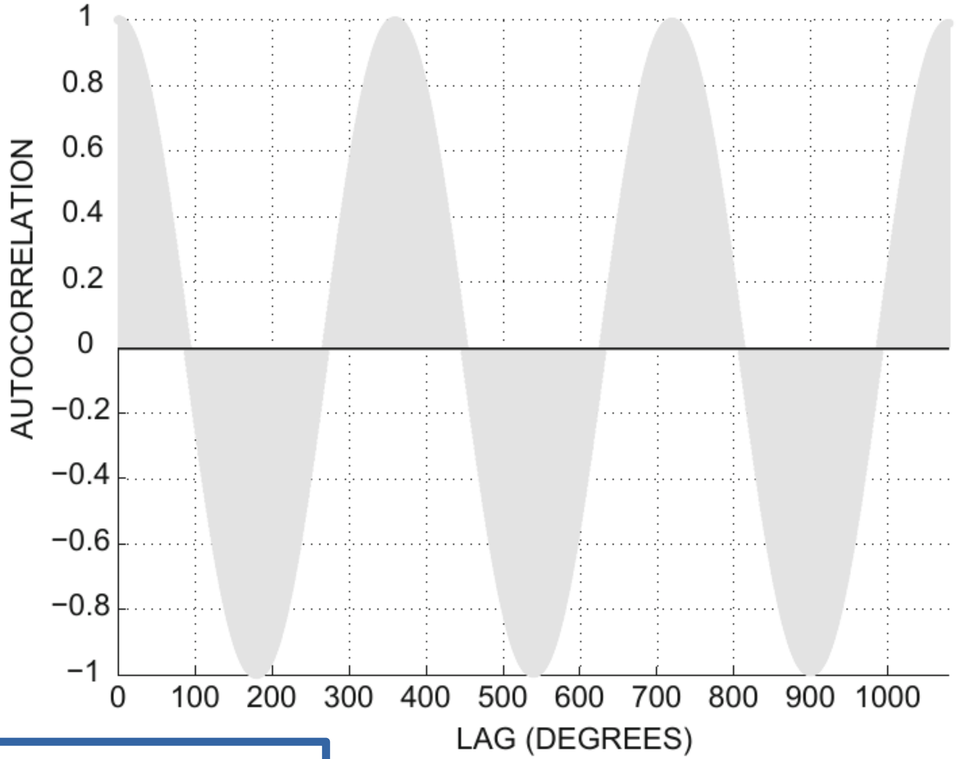
$$y_t^{\text{AR}} = \sum_{i=1}^{p} a_i \cdot y_{t-i} + c + \epsilon_t$$

How to decide p? Autocorrelation plots



(a) IBM stock

What is a reasonable range for p in these cases?



(b) Sine wave

Finding coefficients and evaluating

- Each data point is a training element

$$y_t^{\mathrm{AR}} = \sum_{i=1}^{p} a_i \cdot y_{t-i} + c + \epsilon_t$$

- Coefficients found by least-squares regression
- Best models have $\mathrm{R}^2 \to 1$

$$R^2 = 1 - \frac{\mathrm{Mean}_t(\epsilon_t^2)}{\mathrm{Variance}_t(y_t)}$$

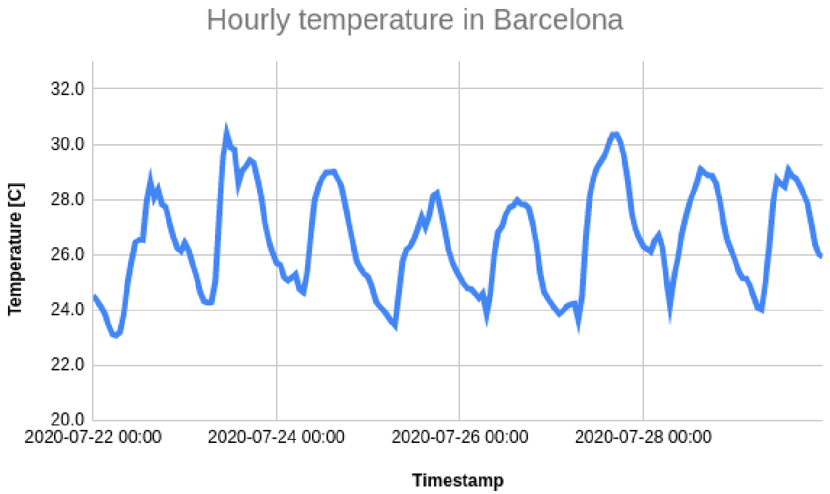
Exercise: simple auto-regressive model

- Create a simple auto-regressive model for temperature in a city
- Use two lags:

  1 hour

  24 hours
- Compute the predicted series

  (optionally: include it in the plot)
- Compute the maximum error



Spreadsheet links: https://upfbarcelona.padlet.org/chato/hogch321o6pws1fd

Moving average model MA(q)

- Focus on the variations (shocks) of the model, i.e., places where change was unexpected
- AR(p) model: $$y_t^{\mathrm{AR}} = \sum_{i=1}^{p} a_i \cdot y_{t-i} + c + \epsilon_t$$
- MA(q) model: $$y_t^{\mathrm{MA}} = \sum_{i=1}^{q} b_i \cdot \epsilon_{t-i} + c + \epsilon_t$$

Autoregressive moving average model ARMA(p,q)

- Combines both the autoregressive and the moving average model

$$y_t^{\mathrm{ARMA}} = \sum_{i=1}^{p} a_i \cdot y_{t-i} + \sum_{i=1}^{q} b_i \cdot \epsilon_{t-i} + c + \epsilon_t$$

- Select small p, q, to avoid overfitting

Autoregressive integrated moving average model ARIMA(p,q)

- Combines both the autoregressive and the moving average model on differenced series

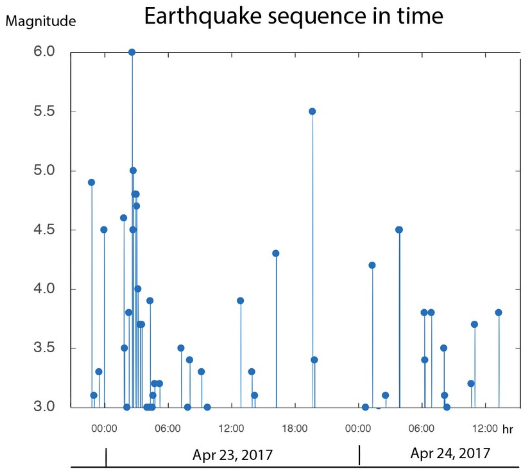
$$y_t^{\text{ARIMA}} = \sum_{i=1}^{p} a_i \cdot (y_{t-i} - y_{t-i-1}) + \sum_{i=1}^{q} b_i \cdot \epsilon_{t-i} + c + \epsilon_t$$

Note: this is an ARIMA(p,1,q) model as we're using first order differencing

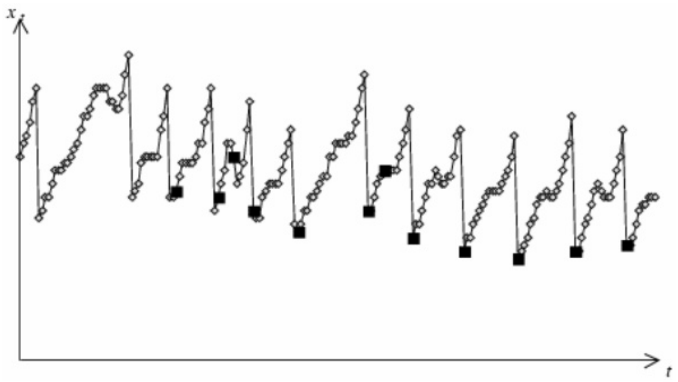
See also: ARIMA end-to-end project in Python by Susan Li (2018)

Event detection
(a simple framework)

Event: an important occurrence



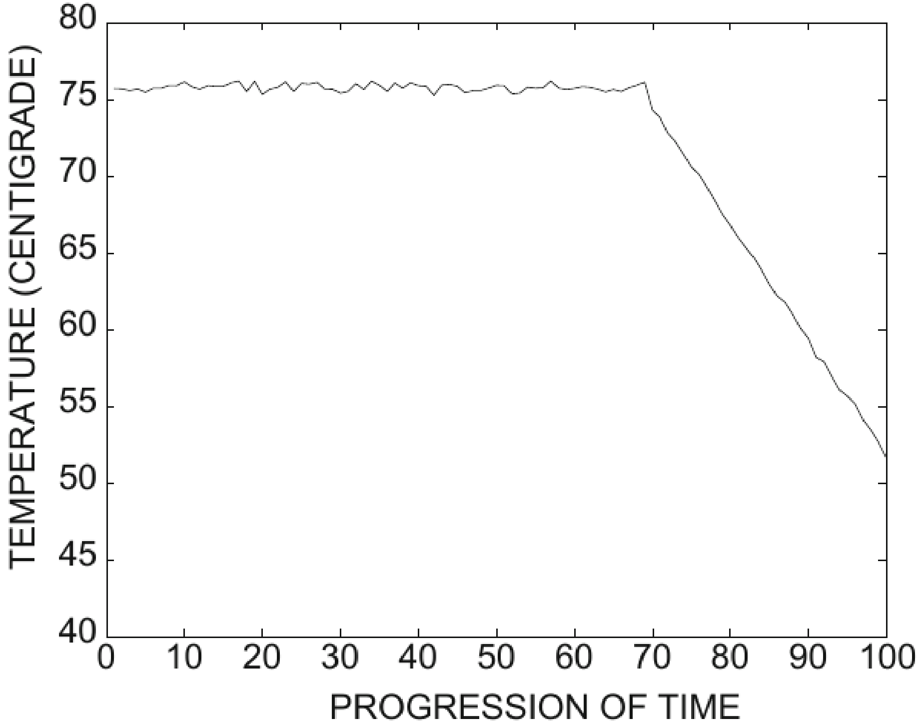
Earthquake or aftershock



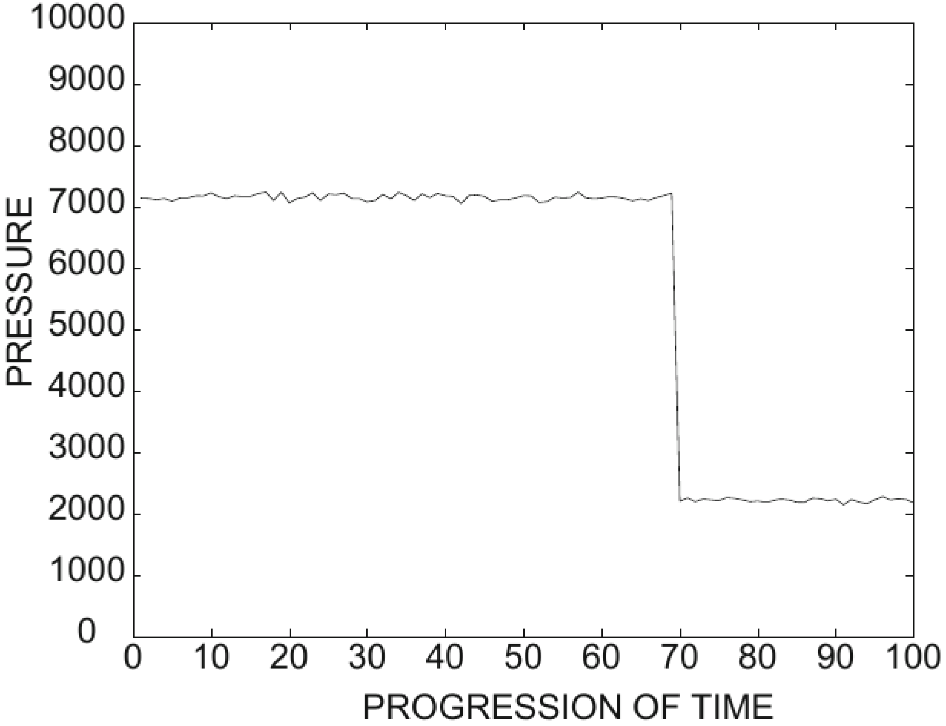
Droplet release

Sudden price change

Example: pipe rupture



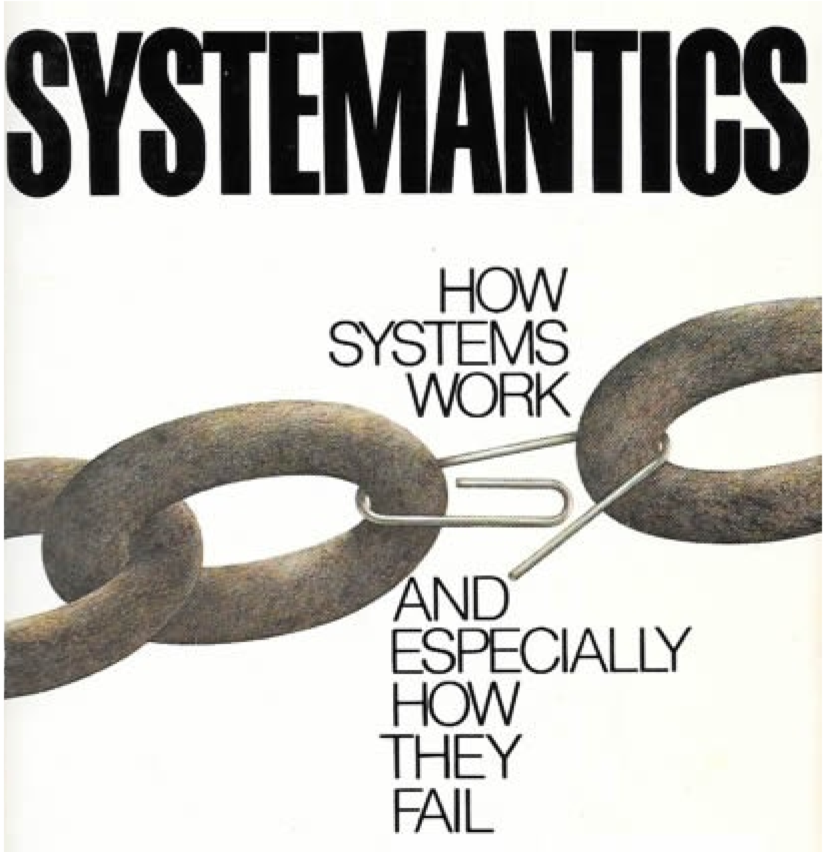
(a) Temperature (pipe rupture scenario)



(b) Pressure (pipe rupture scenario)

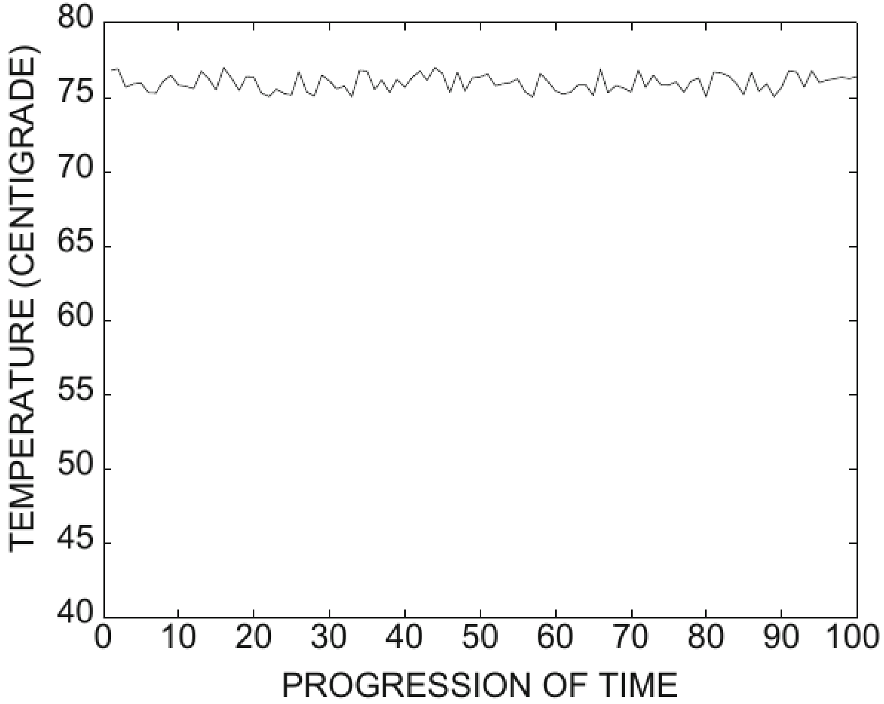
(But what if sensors fail? ...

- "Systems in general work poorly or not at all"
- "In complex systems, malfunction and even total non-function may not be detectable for long periods, if ever"

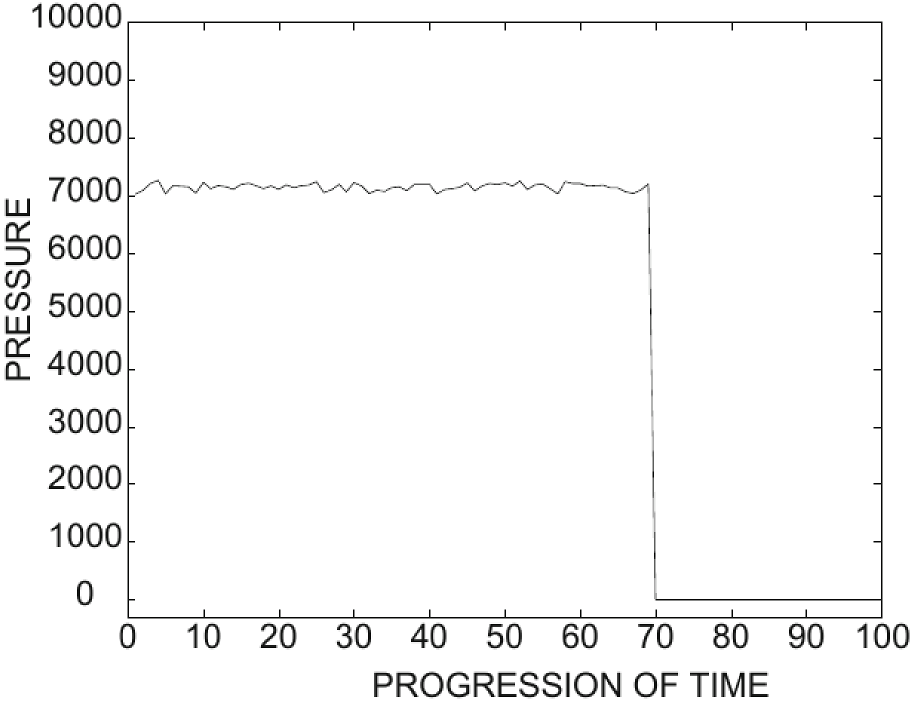


Gall, John. Systemantics: the underground text of systems lore:
how systems really work and especially how they fail. Ann Arbor, MI, 1975.

… Can we still detect failure?)



(c) Temperature (sensor failure scenario)



(d) Pressure (sensor failure scenario)

A general scheme for event detection in multivariate time series

- Let $T_1, T_2, \ldots, T_r$ be times at which an event has been observed in the past
- (Offline) Learn coefficients $\alpha_1, \alpha_2, \ldots, \alpha_d$ to distinguish between event times and non-event times
- (Online) Observe series and determine deviation of every stream i at timestamp t as z_t^i
- (Online) Compute composite alarm level $Z_t = \sum_{i=1}^{d} \alpha_i \cdot z_t^i$

Learning discrimination coefficients α_1, α_2, … , α_d

$$\boxed{Z_t = \sum_{i=1}^{d} \alpha_i \cdot z_t^i}$$

- Average alarm level for events

$$Q^{\text{event}}(\alpha_1, \ldots, \alpha_d) = \frac{1}{r} \sum_{i=1}^{r} Z_{T^i}$$

- Average alarm level for non-events
  (we assume most points are non-events)

$$Q^{\text{normal}}(\alpha_1, \ldots, \alpha_d) = \frac{1}{N} \sum_{i=1}^{N} Z_t$$

Learning discrimination coefficients $\alpha_1, \alpha_2, \ldots, \alpha_d$ (cont.)

- For events $$Q^{\text{event}}(\alpha_1, \ldots, \alpha_d) = \frac{1}{r}\sum_{i=1}^{r} Z_{T^i}$$

- For non-events $$Q^{\text{normal}}(\alpha_1, \ldots, \alpha_d) = \frac{1}{N}\sum_{i=1}^{N} Z_t$$

Maximize $$Q^{\text{event}}(\alpha_1, \ldots, \alpha_d) - Q^{\text{normal}}(\alpha_1, \ldots, \alpha_d)$$

subject to $$\sum_{i=1}^{d} \alpha_i^2 = 1$$

Use any off-the-shelf iterative optimization solver

Summary

Things to remember

- Time series forecasting
- Event detection

Exercises for TT27-TT29

- Data Mining, The Textbook (2015) by Charu Aggarwal
  - Exercises 14.10 $\rightarrow$ 1-6