Locality-Sensitive Hashing (LSH)

Mining Massive Datasets

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Topic 09

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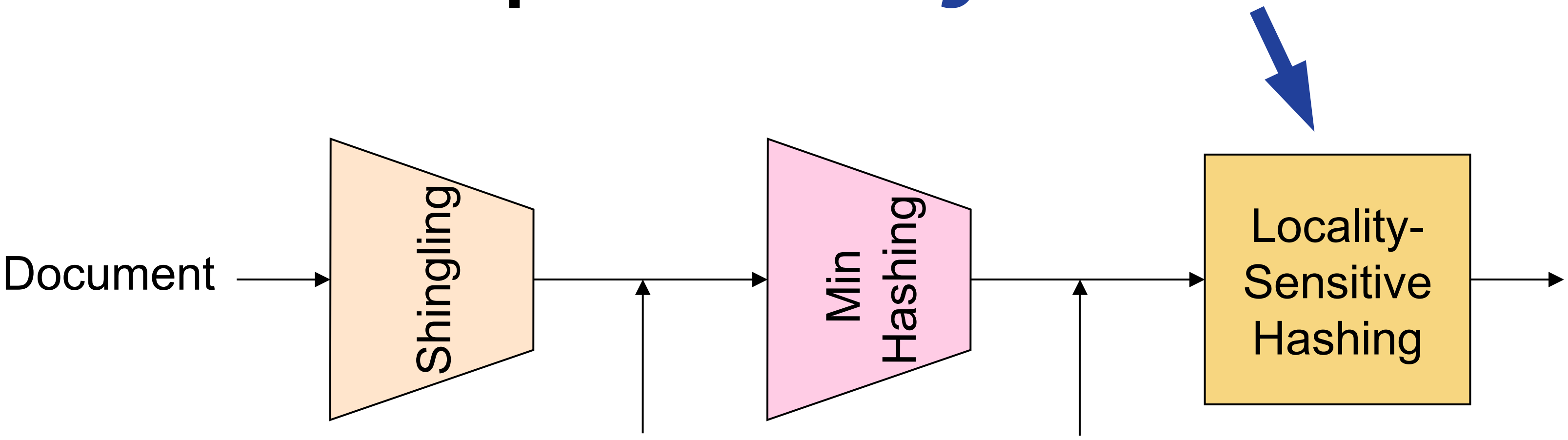
Source for this deck

- Mining of Massive Datasets 2nd edition (2014) by Leskovec et al. (Chapter 3) [slides ch3]

Locality-sensitive hashing

Final step: locality-sensitive hashing



Candidate pairs: those pairs of signatures that we need to test for similarity

Sets of ***k*** *letters or words* that appear consecutively in the document

Signatures: short integer vectors that represent the sets, and reflect their similarity

LSH: first idea

- **Goal:** Find documents with Jaccard similarity at least ***s*** (for some similarity threshold, e.g., ***s***=0.8)

- **LSH – General idea:** Use a function ***f(x,y)*** that tells whether ***(x***,***y)*** is a "***candidate pair***", *with* similarity **likely to be ≥ *s***

- We will compute an auxiliary structure over ***M***
  1) Hash each column of the signature matrix ***M*** to a bucket
  2) A pair of columns that hashes to the same bucket is a **candidate pair**

Signature matrix *M*

d1	d2	d3	d4
2	1	4	1
1	2	1	2
2	1	2	1

Selecting candidates

- **Pick a similarity threshold *s* (0 < s < 1)**
- Columns ***x*** and ***y*** of ***M*** are a **candidate pair** if their signatures agree (***M*** **(*i*, *x*)** = ***M*** **(*i*, *y*)**) on at least fraction ***s*** of their rows
- Remember we showed that documents ***x*** and ***y*** will have the same (Jaccard) similarity as their signatures

Signature matrix *M*

d1	d2	d3	d4
2	1	4	1
1	2	1	2
2	1	2	1

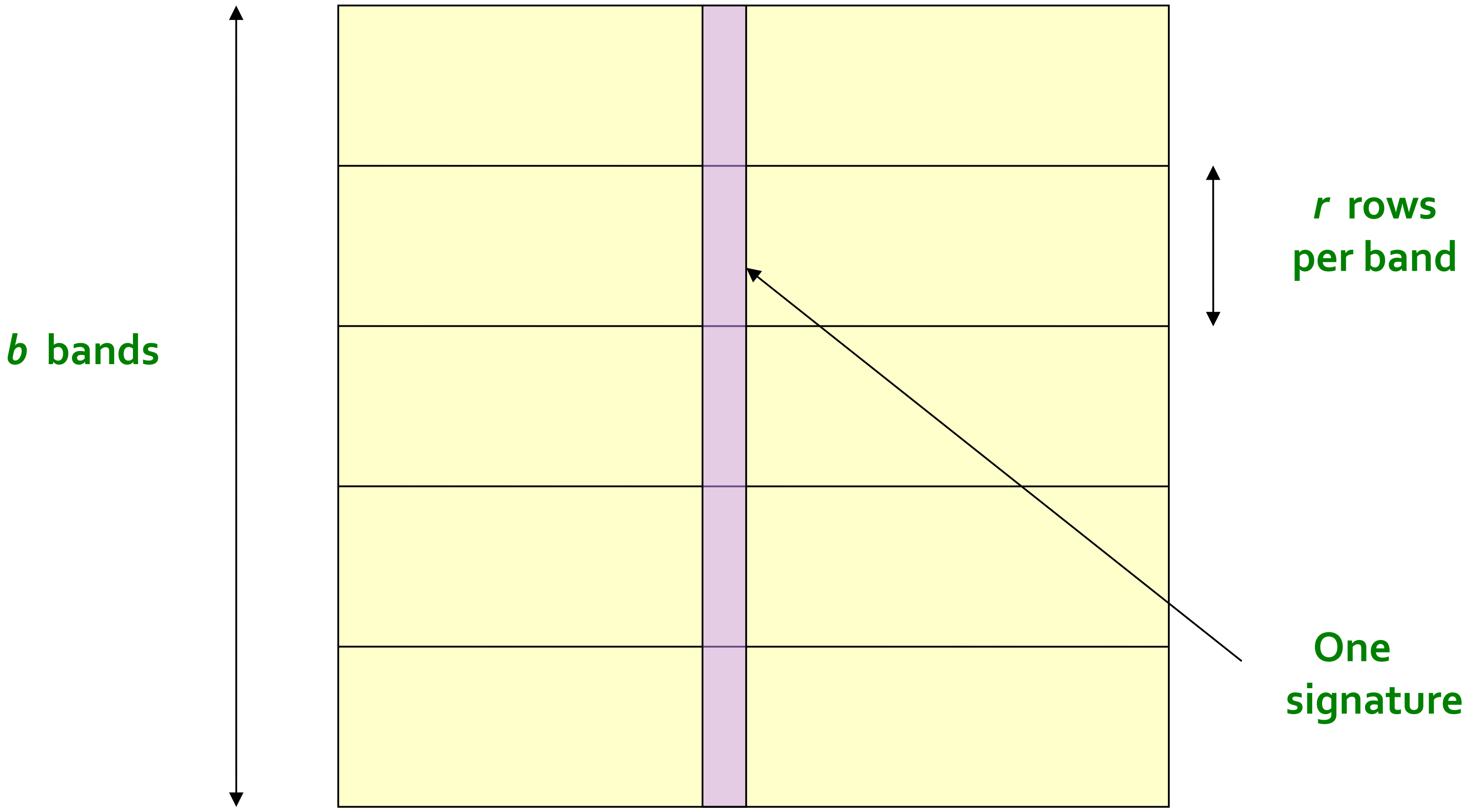
Creating buckets of similar documents

- **Hash columns of signature matrix *M***
- Make sure that (only) **similar columns** are likely to **hash to the same bucket**, with high probability
- **Candidate pairs are those that hash to the same bucket**

Signature matrix *M*

d1	d2	d3	d4
2	1	4	1
1	2	1	2
2	1	2	1

Partition M into b bands of size r



Signature matrix *M*

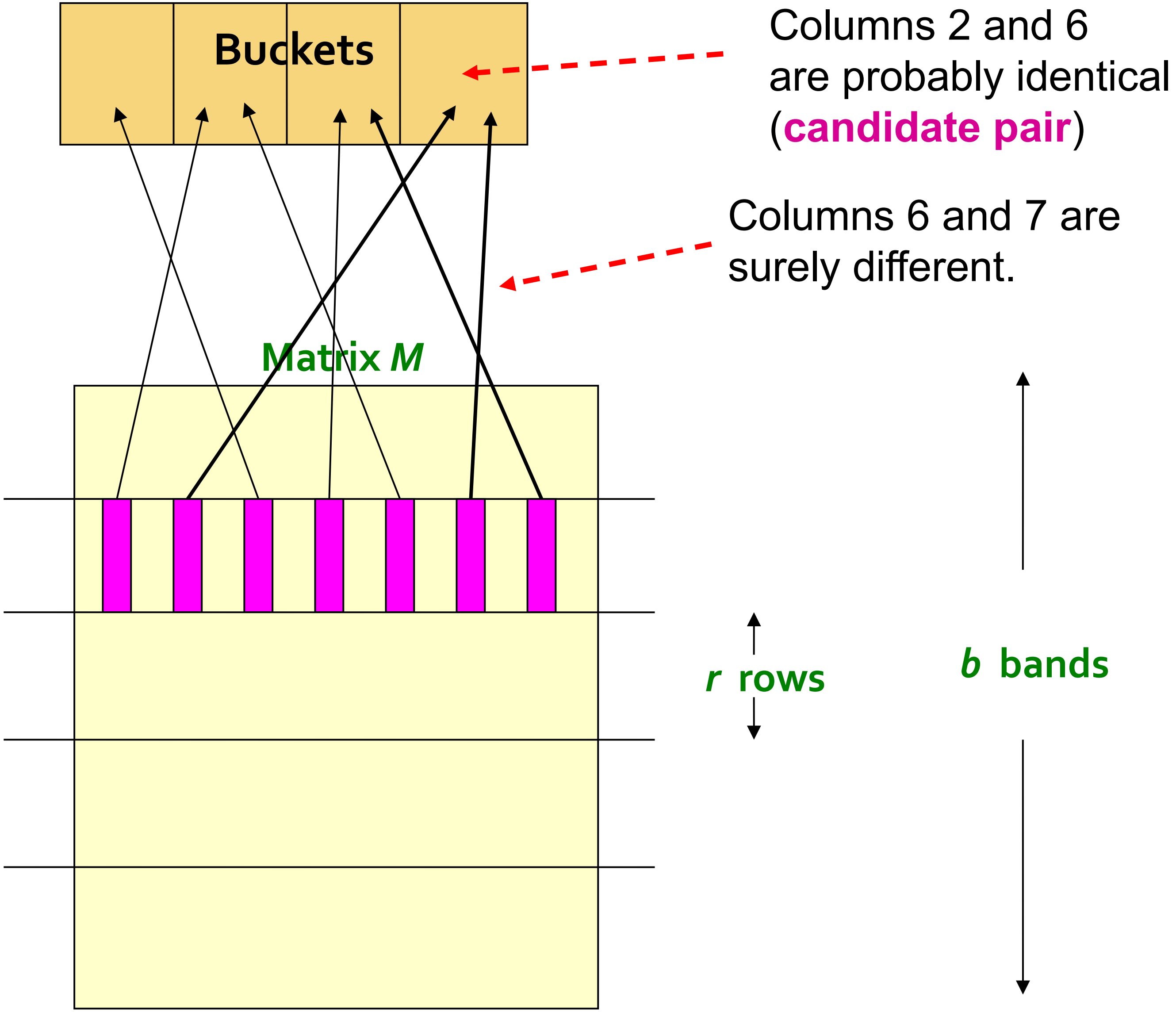
Partition M into b bands of size r (cont.)

- Remember that M has one column per document and as many rows as the signature length
- Partition matrix ***M*** into ***b*** bands of ***r*** rows
- For each band, hash its portion of each column to a hash table with ***k*** buckets
  - If ***k*** is large we use more memory but there are less spurious collisions
- ***Candidate*** column pairs are those that hash to the same bucket for **≥ 1** band
- Tune ***b*** and ***r*** to catch many similar pairs, but few non-similar pairs

Signature matrix *M*

d1	d2	d3	d4
2	1	4	1
1	2	1	2
2	1	2	1

Hashing bands



Simplifying assumption: no collisions (no false positives)

- We will assume there are **enough buckets** that columns are unlikely to hash to the same bucket unless they are **identical** in a particular band
- Hereafter, we assume that "**same bucket**" means "**identical in that band**"
- Assumption needed only to simplify analysis, not for correctness of algorithm

Example of bands

Assume the following case:

- Suppose 100,000 columns of ***M*** (100k docs)
- Signatures of 100 integers (rows)
  - Therefore, signatures take 40Mb
- Choose ***b*** = 20 bands of ***r*** = 5 integers/band
- **Goal:** Find pairs of documents that are at least ***s*** = *0.8* similar

Suppose sim(C_1, C_2) = 0.8

- **Find pairs of** $\geq$ ***s***=0.8 similarity, set **b**=20, **r**=5
- Since sim(C_1, C_2) $\geq$ **s**, we want C_1, C_2 to be a **candidate pair**
  - We want them to hash to **at least 1 common bucket** (at least one band is identical)
- **Probability C_1, C_2 identical in one particular band:** $(0.8)^5 = 0.328$
- Probability C_1, C_2 are ***not*** similar in all of the 20 bands:

  $(1\text{-}0.328)^{20} = 0.00035$

  - i.e., about 1/3000th of the 80%-similar column pairs are **false negatives** (we will miss them)
- **We would find 99.965% pairs of truly similar documents**

Suppose sim(C_1, C_2) = 0.3

- **Find pairs of** $\geq$ ***s***=0.8 similarity, set **b**=20, **r**=5
- Since sim(C_1, C_2) < **s**, we **do not** want C_1, C_2 to be a **candidate pair**
- **Probability C_1, C_2 identical in one particular band:**

  $(0.3)^5 = 0.00243$
- Probability C_1, C_2 identical in at least 1 of 20 bands:

  $1 - (1 - 0.00243)^{20} = 0.0474$
- In other words, approximately 4.74% pairs of docs with similarity 0.3% end up becoming **candidate pairs**
  - They are **false positives** since we will have to examine them (they are candidate pairs) but then it will turn out their similarity is below threshold **s**

LSH summary

- Tune ***K (permutations), b (bands), r (permutations/band)*** to get almost all pairs with similar signatures, but eliminate most pairs that do not have similar signatures

- After finding candidates, check in main memory that **candidate pairs** really do have **similar signatures**

Summary

Things to remember

- **Locality-Sensitive Hashing:** Focus on pairs of signatures likely to be from similar documents
  - We used hashing to find **candidate pairs** of similarity $\geq$ **s**

Exercises for TT08-TT09

- Mining of Massive Datasets 2nd edition (2014) by Leskovec et al.
  - Exercises 3.1.4 (Jaccard similarity)
  - Exercises 3.2.5 (Shingling)
  - Exercises 3.3.6 (Min hashing)
  - Exercises 3.4.4 (Locality-sensitive hashing)