Locality-Sensitive Hashing (LSH) – **Additional Materials**

Mining Massive Datasets
Prof. Carlos Castillo
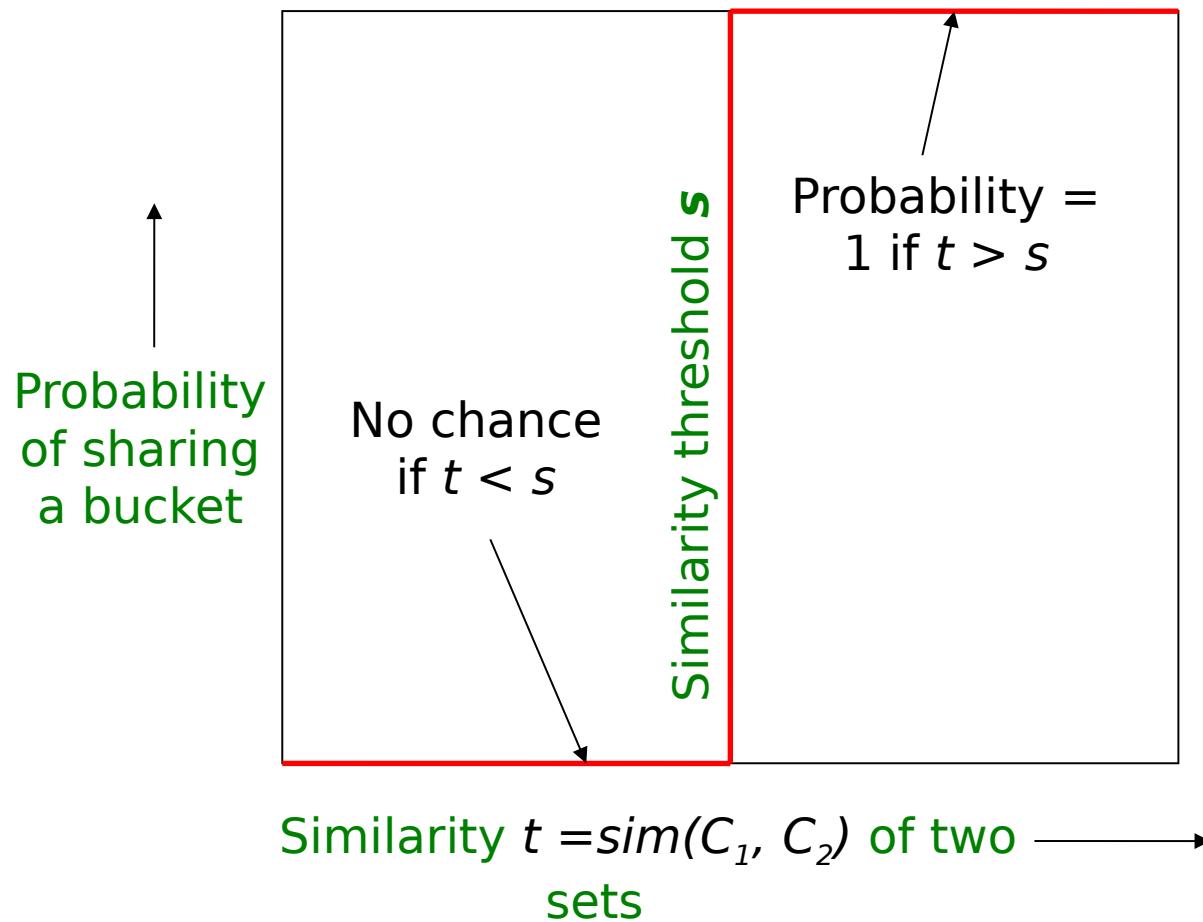Topic 10

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Source for this deck

- Mining of Massive Datasets 2nd edition (2014) by Leskovec et al. (Chapter 3) [slides ch3]
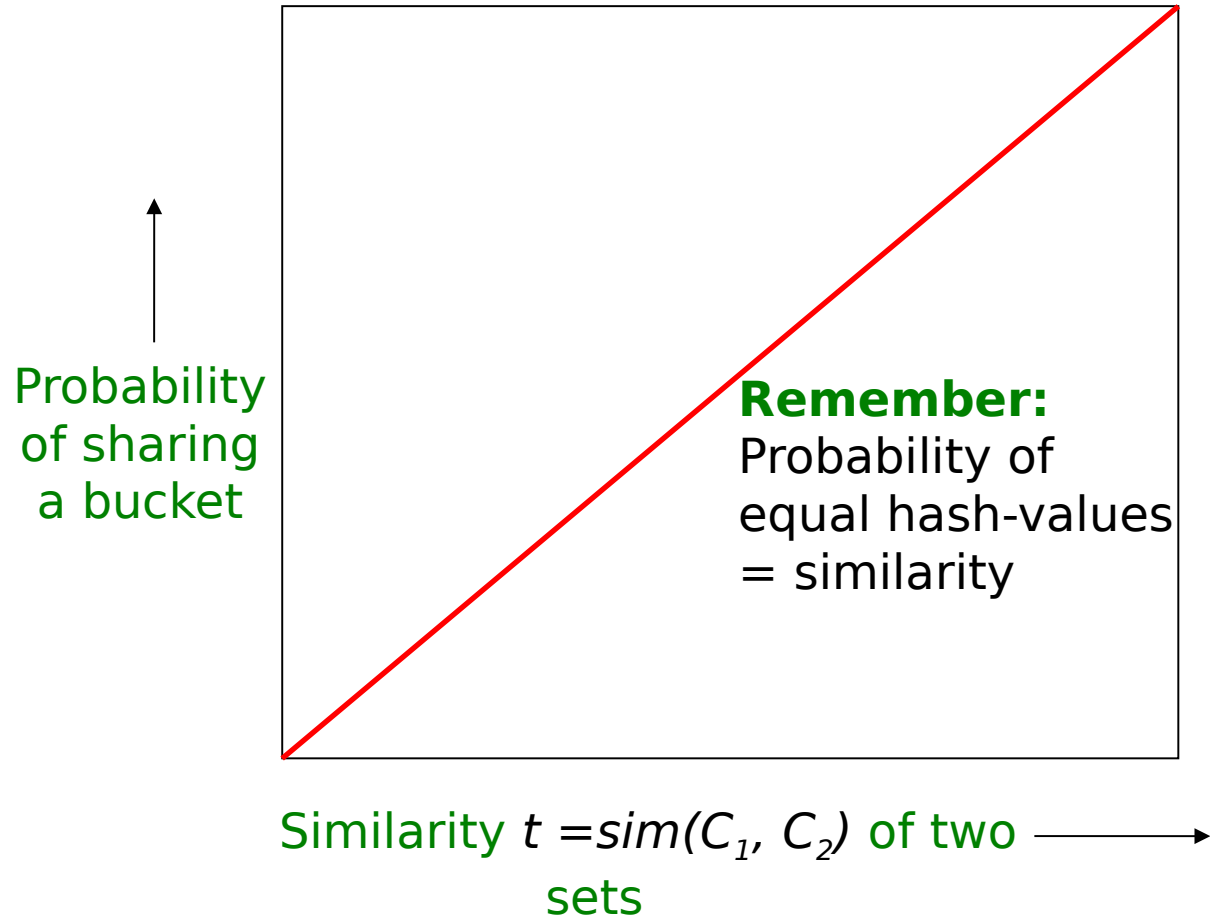
LSH involves a trade-off

- Pick:
  - The number of Min-Hashes (rows of M = K)
  - The number of bands b, and
  - The number of rows r per band to balance false positives/negatives

- Example: If we had only 15 bands of 5 rows, the number of false positives would go down, but the number of false negatives would go up
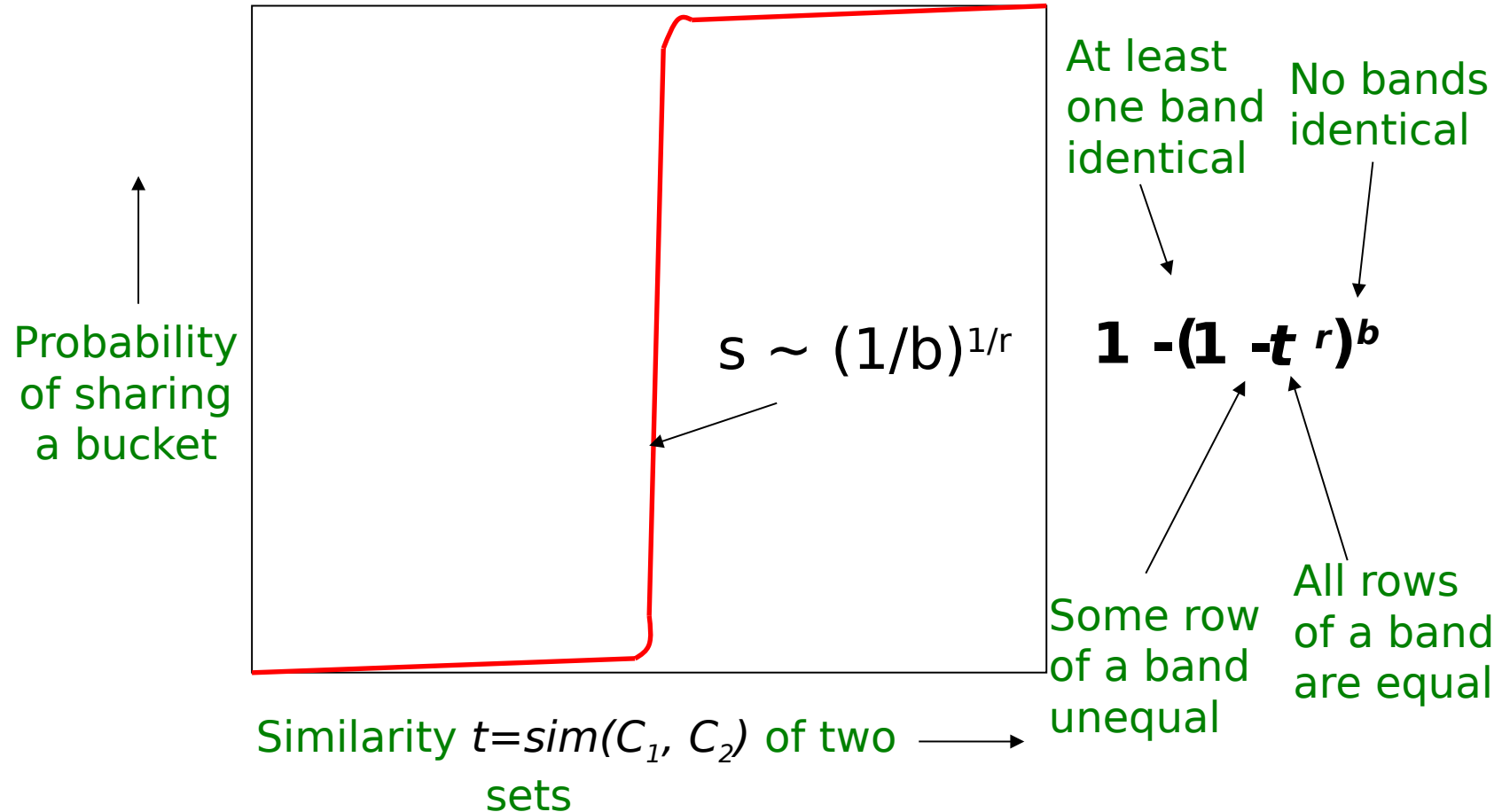
LSH: what we want

Probability = 1 if $t > s$

Probability of sharing a bucket

Similarity threshold **s**

No chance if $t < s$

Similarity $t = sim(C_1, C_2)$ of two sets

What 1 band of 1 row gives you

Remember:
Probability of
equal hash-values
= similarity

Probability
of sharing
a bucket

Similarity $t = sim(C_1, C_2)$ of two sets

b bands, r rows/band

- Columns C_1 and C_2 have similarity t
- Pick any band (r rows)

  - Prob. that all rows in band equal = t^r

  - Prob. that some row in band unequal = $1 - t^r$

- Prob. that no band identical = $(1 - t^r)^b$

- Prob. that at least 1 band identical = $1 - (1 - t^r)^b$

What b bands of r rows give you

Probability of sharing a bucket

Similarity $t=sim(C_1, C_2)$ of two sets

$s \sim (1/b)^{1/r}$

$1 - (1 - t^r)^b$

At least one band identical

No bands identical

Some row of a band unequal

All rows of a band are equal

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Example: b=20, r=5

- **Similarity threshold s**
- **Prob. that at least 1 band is identical:**
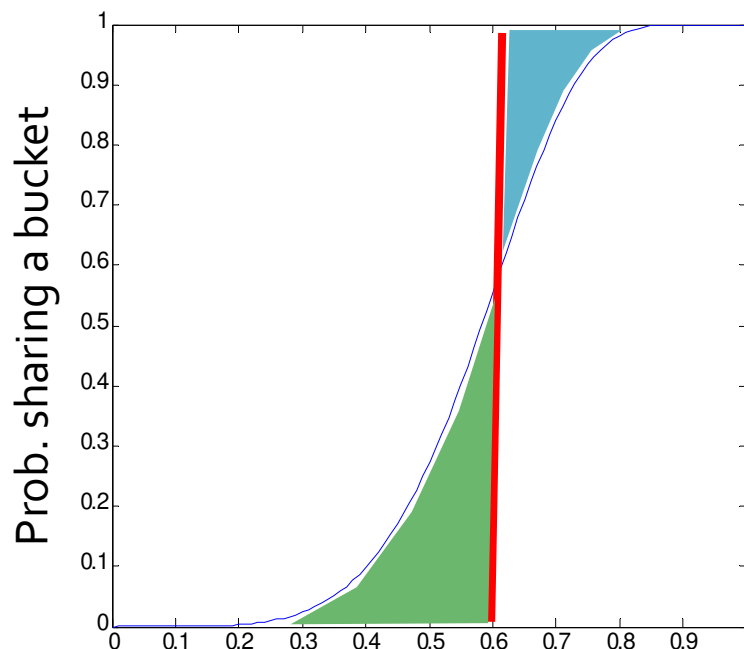
s	$1-(1-s^r)^b$
.2	.006
.3	.047
.4	.186
.5	.470
.6	.802
.7	.975
.8	.9996

Picking r and b: the S curve

Picking *r* and *b* to get the best S-curve
50 hash-functions (r=5, b=10)

Blue area: False Negative rate
Green area: False Positive rate

Summary

Things to remember

- **Locality-Sensitive Hashing:** Focus on pairs of signatures likely to be from similar documents

  - We used hashing to find **candidate pairs** of similarity $\geq$ **s**

Exercises for TT08-TT09

- Mining of Massive Datasets 2nd edition (2014) by Leskovec et al.
  - Exercises 3.1.4 (Jaccard similarity)
  - Exercises 3.2.5 (Shingling)
  - Exercises 3.3.6 (Min hashing)
  - Exercises 3.4.4 (Locality-sensitive hashing)