Association Rules Mining:

Reducing Running Time

Mining Massive Datasets

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Sources

- Data Mining, The Textbook (2015) by Charu Aggarwal (Chapters 4, 5) – slides by Lijun Zhang
- Mining of Massive Datasets 2nd edition (2014) by Leskovec et al. (Chapter 6) - slides
- Data Mining Concepts and Techniques, 3rd edition (2011) by Han et al. (Chapter 6)
- Introduction to Data Mining 2nd edition (2019) by Tan et al. (Chapters 5, 6) – slides ch5, slides ch6

Speeding up candidate generation

Speeding-up candidate generation: level-wise pruning trick

- Let F_k be the set of frequent k-itemsets [we know they are frequent]
- Let C_{k+1} be the set of (k+1)-candidates [we do not know their frequency]
- $I \in C_{k+1}$ is frequent only if all the k-subsets of I are frequent
- Pruning
  - Generate all the k-subsets of I
  - If any one of them does not belong to F_k, then remove I

Candidates generation

- A Naïve Approach
  - Check all the possible combinations of frequent itemsets
- An Example of the Naïve Approach
  - itemsets: {abc} {bcd} {abd} {cde}
  - {abc} + {bcd} = {abcd}
  - {bcd} + {abd} = {abcd}
  - {abd} + {cde} = {abcde}
  - ....

Candidates generation (cont.)

- Introduction of **ordering**
  - Items in U can be sorted in lexicographic ordering
  - Items in each itemset can be sorted in lexicographic ordering
  - Itemsets can be ordered as strings
- The improved approach:
  - Order the frequent k-itemsets
  - Merge two itemsets if and only if the first k-1 items of them are equal

Candidates generation (cont.)

Note: We are writing {xyz} to mean the set {x, y, z}

- Example 1:
  - k-itemsets: {abc} {abd} {acd} {bcd}
  - (k+1)-itemsets: {abc} + {abd} = {abcd}
  - No other pair shares a prefix of size k-1, no need to check other combinations
- Example 2:
  - k-itemsets: {abc} {acd} {bcd}
  - No (k+1) -candidates
  - Did we miss {abcd}?
    - **No**, due to the Downward Closure Property: every subset of a frequent itemset is also frequent, and {abd} is not frequent

Improving computation of support

Naïve support counting

- **Naïve counting:**

  For each candidate $I_i \in C_{k+1}$

  For each transaction T_j in T

  Check whether I_i appears in T_j

- This is very slow if both $|C_{k+1}|$ and $|T|$ are large

Support counting with a data structure

- A Better Approach
  - Organize the candidate patterns in C_{k+1} in a data structure
- Use the data structure to accelerate counting
  - Each transaction in T_i examined against the subset of candidates in C_{k+1} that *might* contain T_i

Support counting based on hashing

Naïve counting:

For each $I_i \in C_{k+1}$

For all $T_j \in T$

If $I_i \subseteq T_j$

Add to $\sup(I_i)$

Hashed counting:

For each $T_j \in T$

For $I_i \in \text{hashbucket}(T_j, C_{k+1})$

If $I_i \subseteq T_j$

Add to $\sup(I_i)$

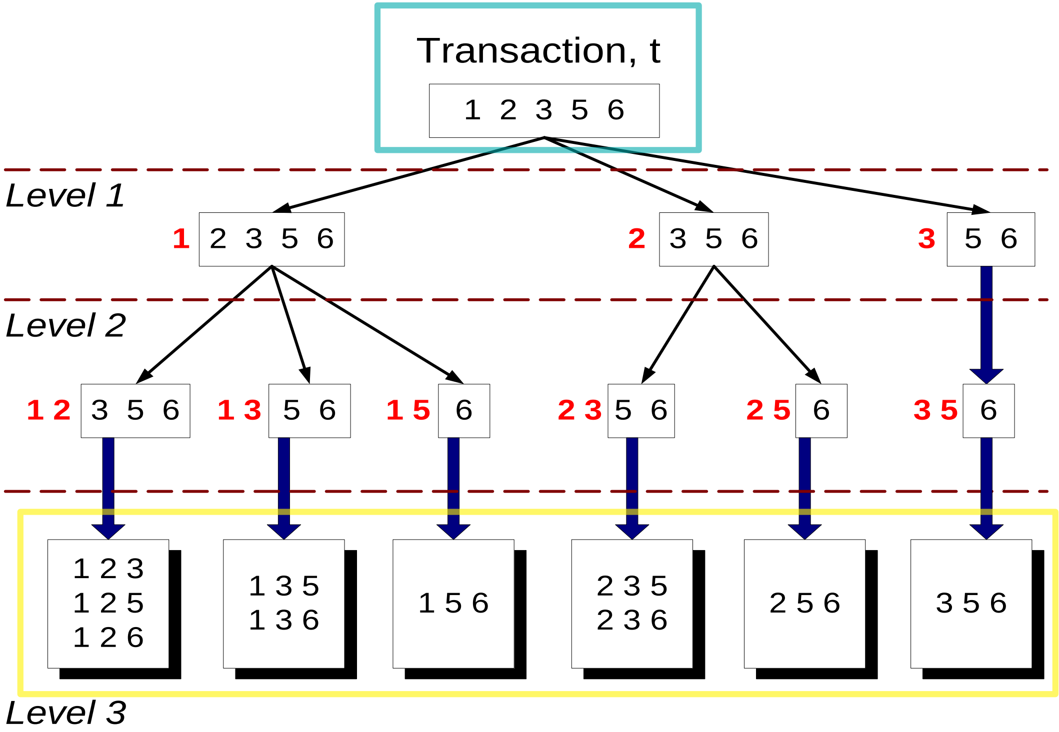
Which candidates are relevant?

Imagine 15 candidates itemsets of length 3:

```
{1 4 5}, {1 2 4}, {4 5 7},
{1 2 5}, {4 5 8}, {1 5 9},
{1 3 6}, {2 3 4}, {5 6 7},
{3 4 5}, {3 5 6}, {3 5 7},
{6 8 9}, {3 6 7}, {3 6 8}
```

Now, suppose we look for this transaction:

```
{1 2 3 5 6}
```



Here we depict only the candidates that appear in the transaction (10 out of 15)

Hash tree for itemsets in C_{k+1}

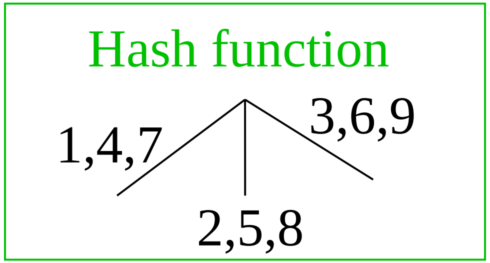
- A tree with fixed degree r
- Each itemset in C_{k+1} is stored in a leaf node
- All internal nodes use a hash function to map items to one of the r branches (can be the same for all internal nodes)
- All leaf nodes contain a lexicographically sorted list of up to `max_leaf_size` itemsets

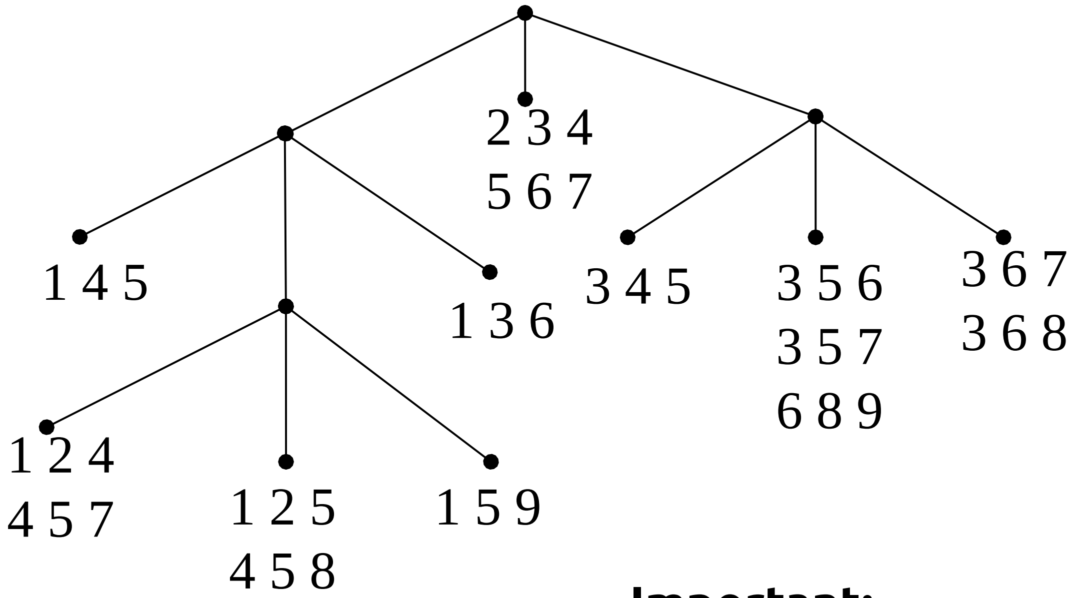
Example hash tree

`r=3  max_leaf_size=3`

Candidate itemsets

```
{1 4 5}, {1 2 4}, {4 5 7},
{1 2 5}, {4 5 8}, {1 5 9},
{1 3 6}, {2 3 4}, {5 6 7},
{3 4 5}, {3 5 6}, {3 5 7},
{6 8 9}, {3 6 7}, {3 6 8}
```

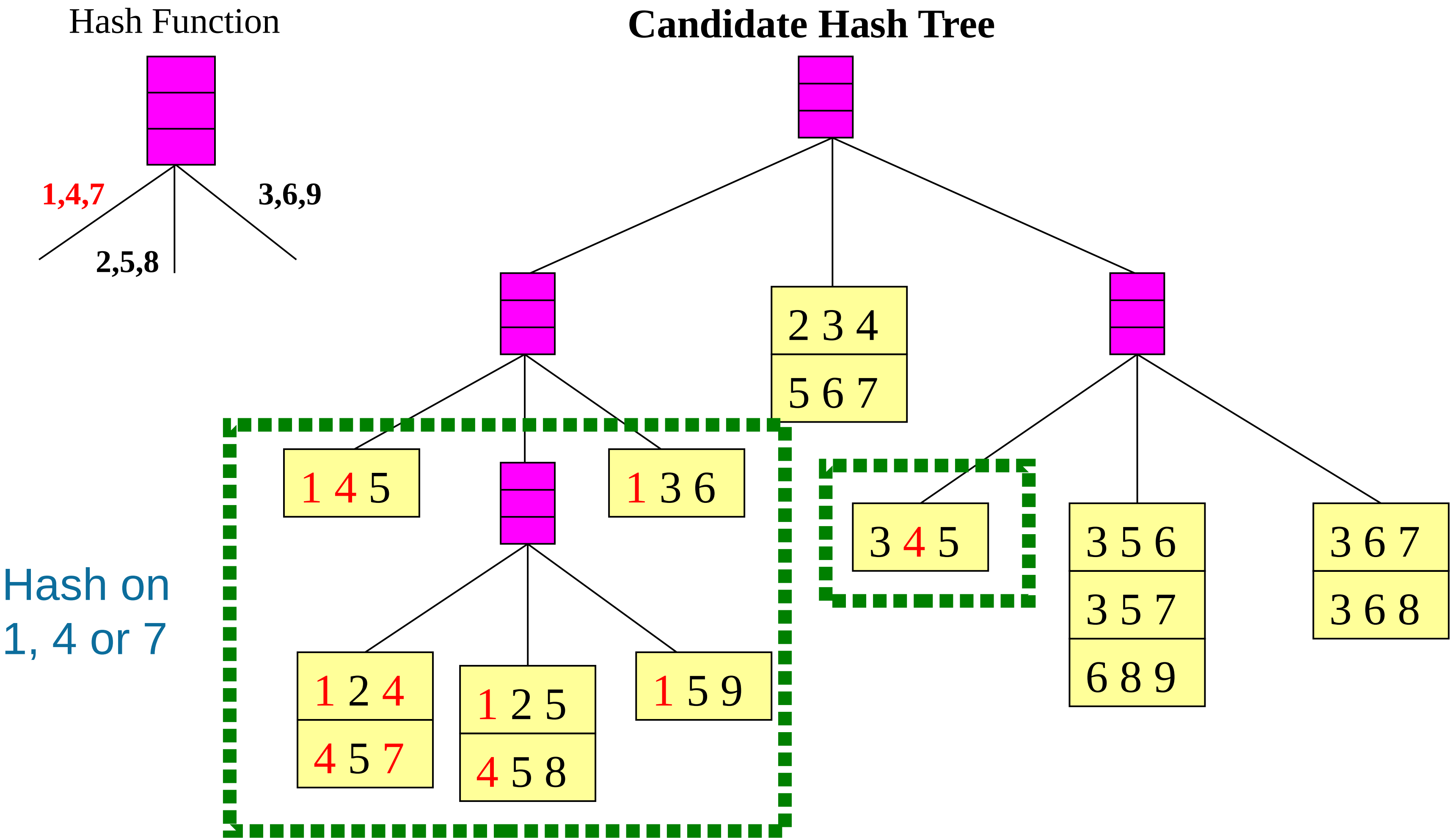




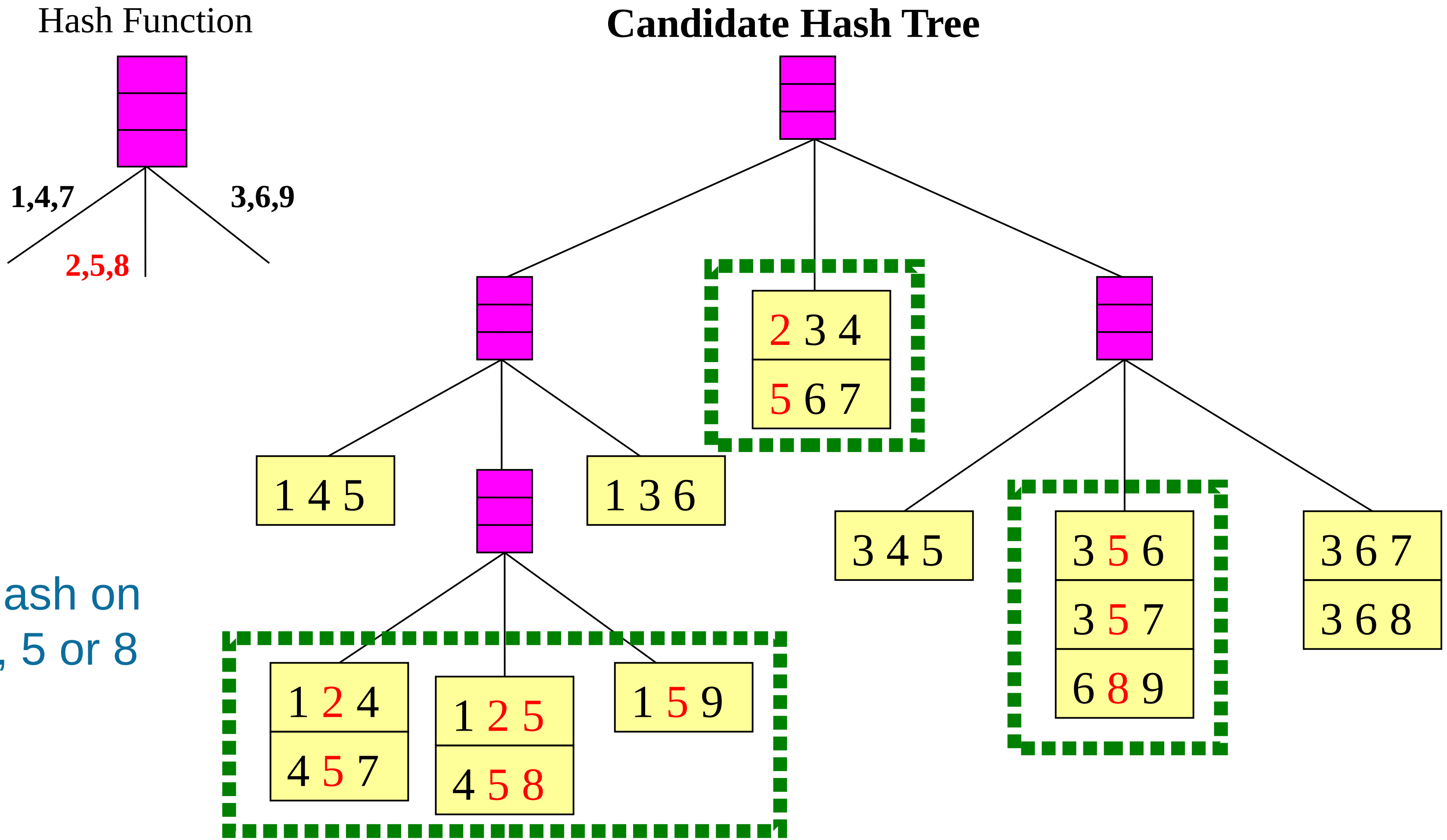
Important:
itemsets are sorted!

This example from: Introduction to Data Mining 2nd edition (2019) by Tan et al. (Chapter 5) – slides ch5

Example hash tree (cont.)

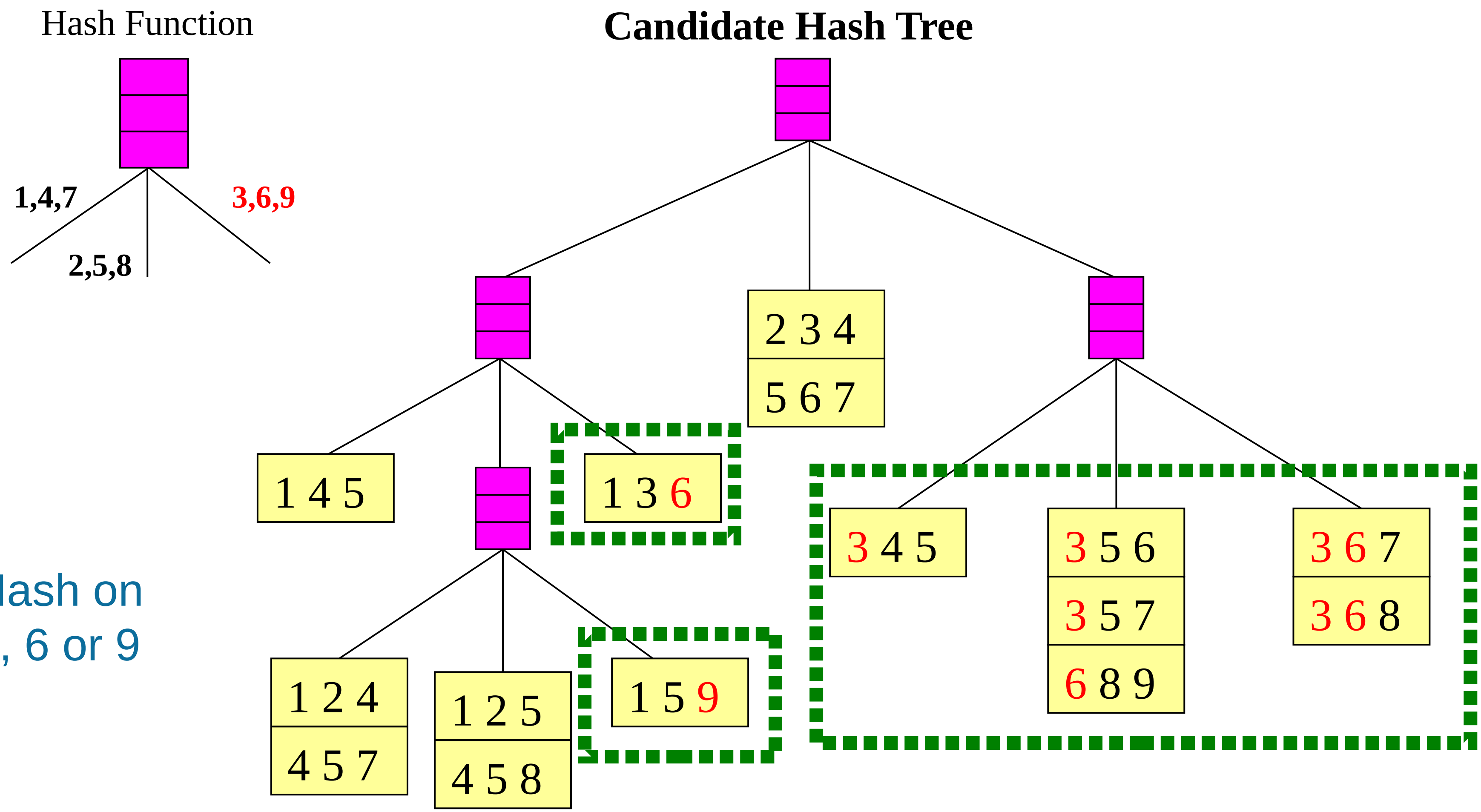


Example hash tree (cont.)



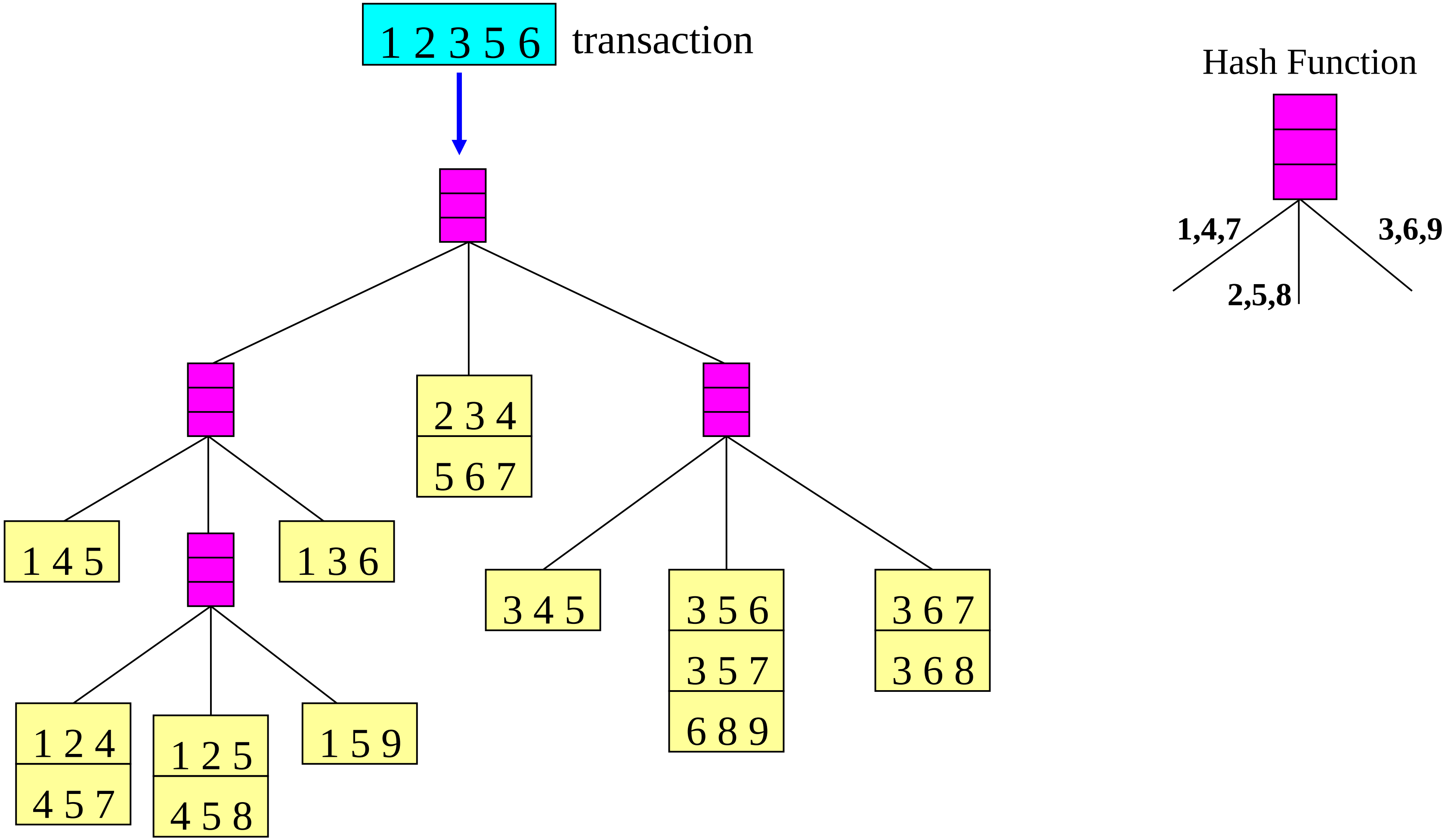
Hash on
2, 5 or 8

Example hash tree (cont.)

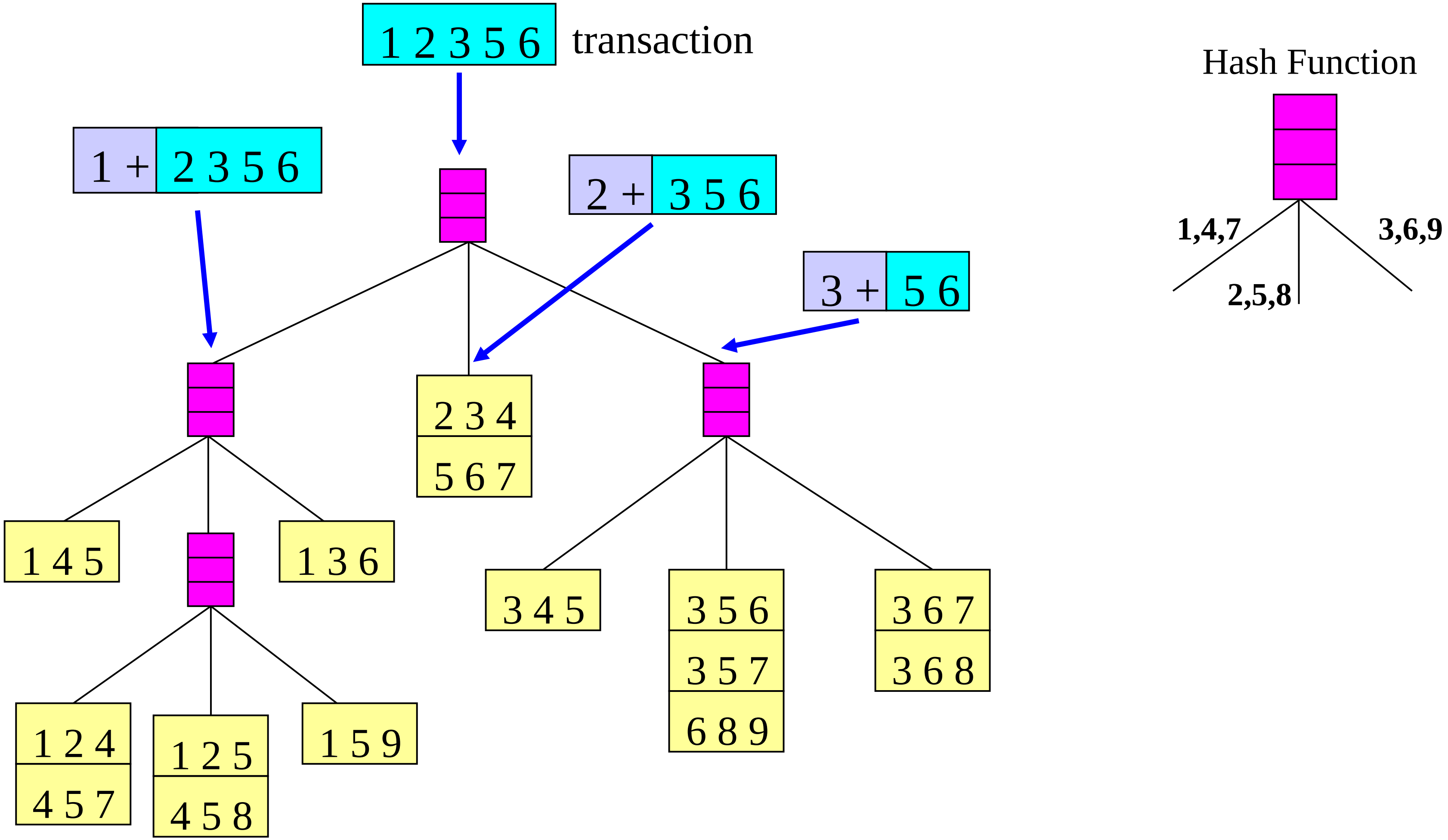


Hash on
3, 6 or 9

Checking which candidates might be in a transaction



Checking which candidates might be in a transaction



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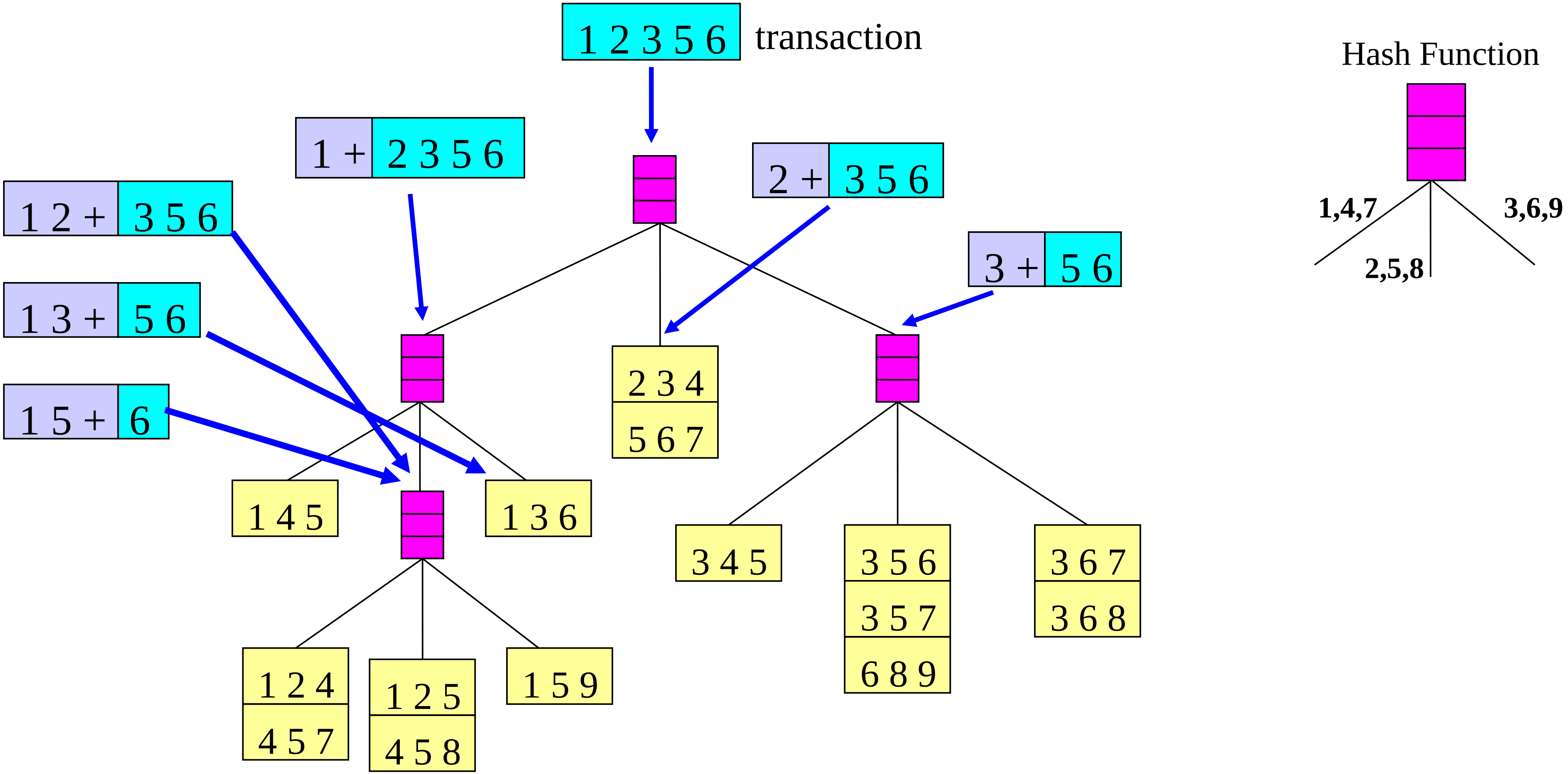
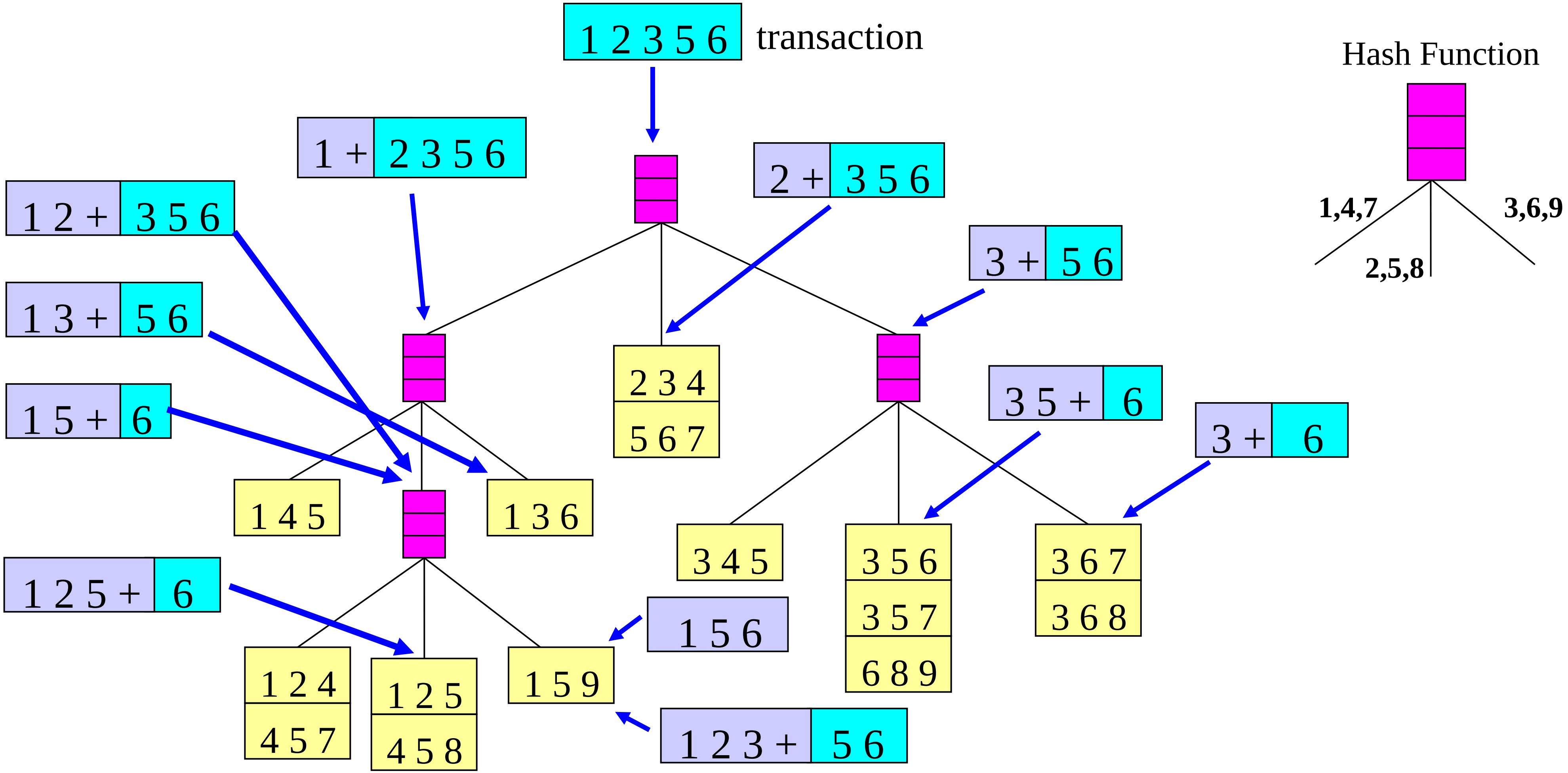
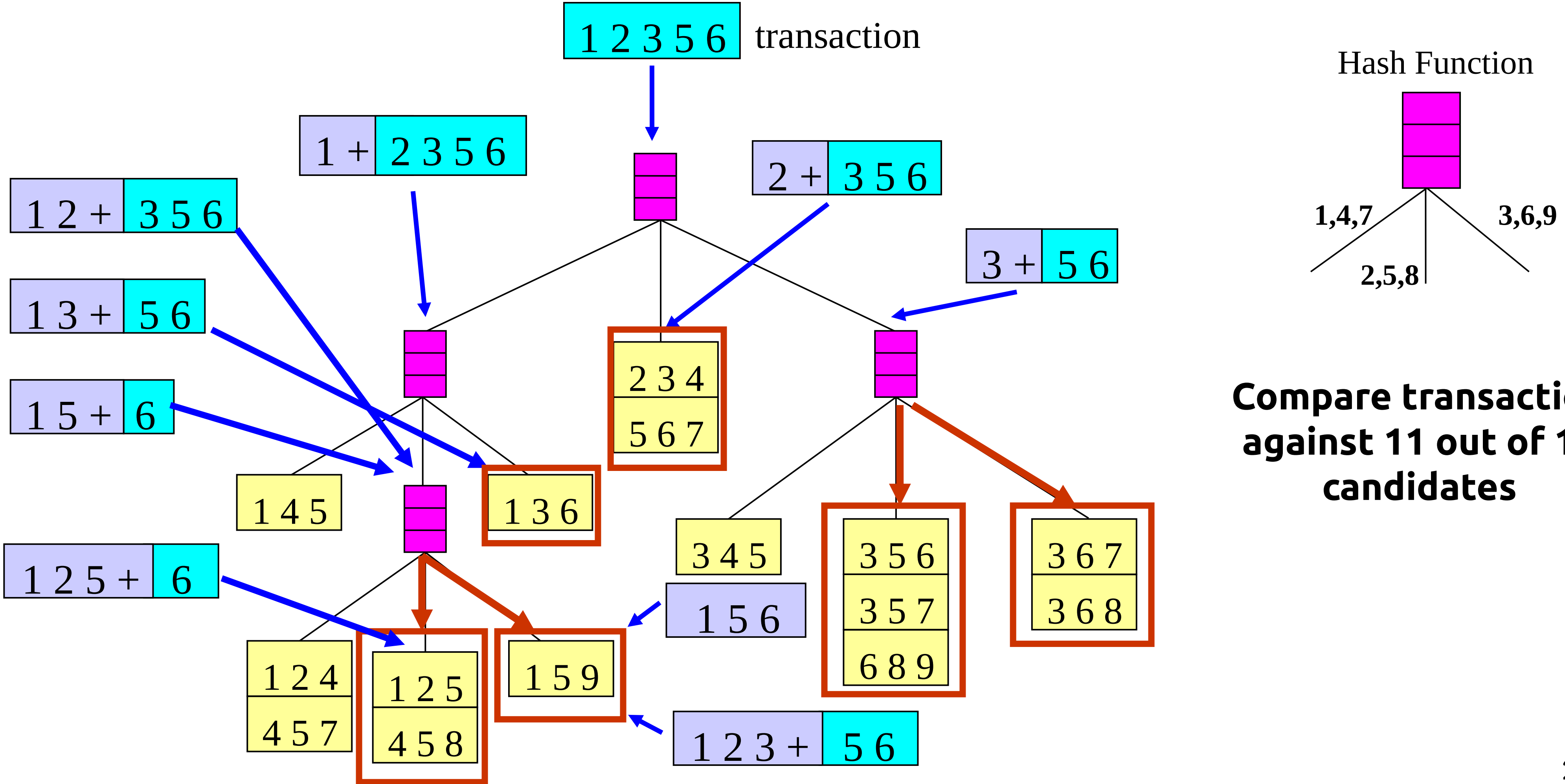


Checking which candidates might be in a transaction

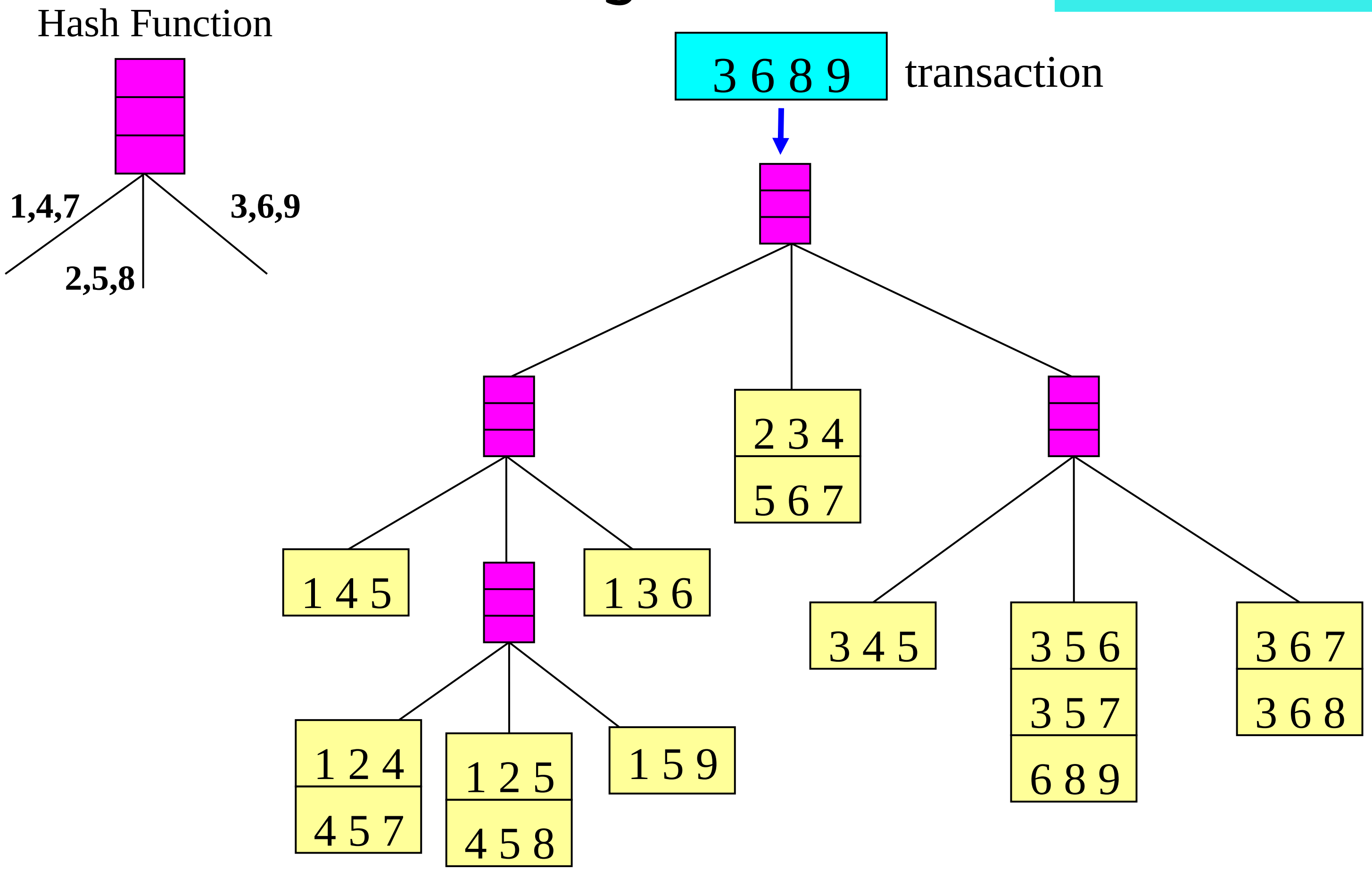


Checking which candidates might be in a transaction



Compare transaction against 11 out of 15 candidates

Exercise: Use the hash tree to determine which candidates might be in this transaction



Improved algorithm for frequent itemsets

- $C_1 \leftarrow$ singletons, lexicographically sorted
- $F_1 \leftarrow$ elements in C_1 with support ≥ minsup, obtained by direct counting
- $k \leftarrow 1$
- While F_k is not empty
  - Generate C_{k+1} by merging elements in F_k sharing a prefix of size k-1
  - Remove from C_{k+1} elements that do not have all of their subsets in F_k
  - Create hash tree for C_{k+1}
  - Pass all transactions in T by the hash tree to compute support for elements of C_{k+1}
  - $F_{k+1} \leftarrow$ elements in C_{k+1} with support ≥ minsup, lexicographically sorted
- Return the union of F_1, F_2, …, F_k

Summary

Things to remember

- Lexicographic candidate generation
- Level pruning
- Hash-tree method

Exercises for this topic

- Data Mining, The Textbook (2015) by Charu Aggarwal
  - Exercises 4.9 $\rightarrow$ 9-10
- Mining of Massive Datasets 2nd edition (2014) by Leskovec et al.
  - Exercises 6.2.7 $\rightarrow$ 6.2.5 and 6.2.6
- Introduction to Data Mining 2nd edition (2019) by Tan et al.
  - Exercises 5.10 $\rightarrow$ 9-12

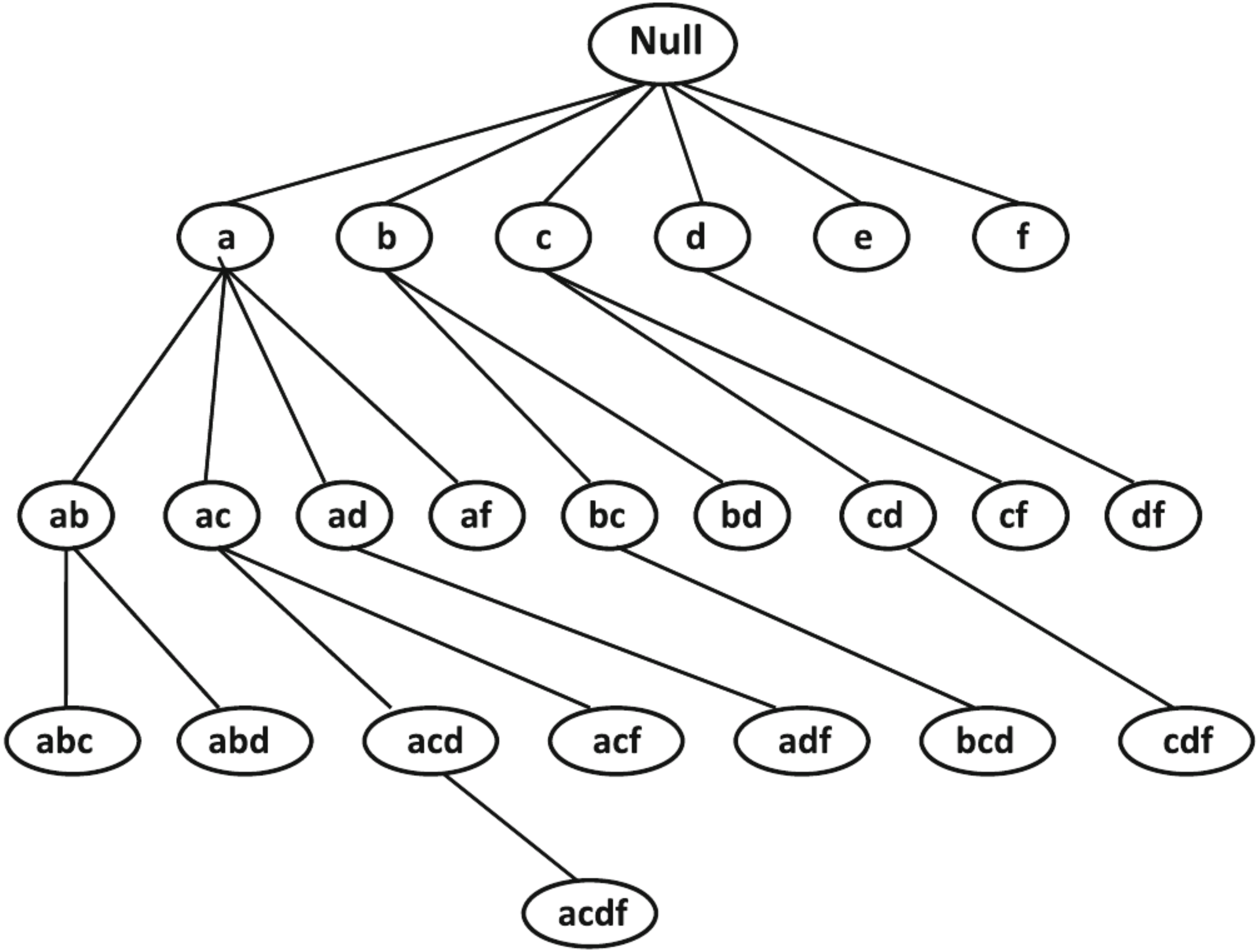
Additional contents
(not included in exams)

Enumeration-tree algorithms: Lexicographic tree

- There is a node in the tree for each frequent itemset
- The root of the tree contains the null itemset
- If $I = \{i_1, i_2, \ldots, i_k\}$ then the parent of I in the tree is $\{i_1, i_2, \ldots, i_{k-1}\}$

Example

Note that, unlike the lattice, a parent can only be extended with an item that is lexicographically larger



Enumeration tree algorithm

Algorithm *GenericEnumerationTree*(Transactions: $\mathcal{T}$,
Minimum Support: *minsup*)
begin
Initialize enumeration tree $\mathcal{ET}$ to single $Null$ node;
while any node in $\mathcal{ET}$ has not been examined **do begin**
Select one of more unexamined nodes $\mathcal{P}$ from $\mathcal{ET}$ for examination;
Generate candidates extensions $C(P)$ of each node $P \in \mathcal{P}$;
Determine frequent extensions $F(P) \subseteq C(P)$ for each $P \in \mathcal{P}$ with support counting;
Extend each node $P \in \mathcal{P}$ in $\mathcal{ET}$ with its frequent extensions in $F(P)$;
end
return enumeration tree $\mathcal{ET}$;
end

Enumeration-tree-based implementation of Apriori

- Apriori constructs the enumeration tree in a breadth-first manner
- Apriori generates candidate (k+1)-itemsets by merging two frequent k-itemsets of which the first k-1 items are the same ⇒ extension in the enumeration-tree