
AIFRM

STOCHASTIC CALCULUS ASSIGNMENT 3

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Exercise sheet 6 Solutions

Question 10

$$\begin{aligned}\mathbb{E}[(M_n - M_m)^2 | \mathcal{F}_k] &= \mathbb{E}[M_n^2 - 2M_n M_m + M_m^2 | \mathcal{F}_k] \\&= \mathbb{E}[M_n^2 | \mathcal{F}_k] - 2\mathbb{E}[M_n M_m | \mathcal{F}_k] + \mathbb{E}[M_m^2 | \mathcal{F}_k] \\&= \mathbb{E}[M_n^2 | \mathcal{F}_k] - 2\mathbb{E}[\mathbb{E}[M_n M_m | \mathcal{F}_m] | \mathcal{F}_k] + \mathbb{E}[M_m^2 | \mathcal{F}_k] \\&= \mathbb{E}[M_n^2 | \mathcal{F}_k] - 2\mathbb{E}[M_m \mathbb{E}[M_n | \mathcal{F}_m] | \mathcal{F}_k] + \mathbb{E}[M_m^2 | \mathcal{F}_k] \\&= \mathbb{E}[M_n^2 | \mathcal{F}_k] - 2\mathbb{E}[M_m^2 | \mathcal{F}_m] | \mathcal{F}_k] + \mathbb{E}[M_m^2 | \mathcal{F}_k] \\&= \mathbb{E}[M_n^2 | \mathcal{F}_k] - \mathbb{E}[M_m^2 | \mathcal{F}_k] \\&= \mathbb{E}[(M_n^2 - M_m^2) | \mathcal{F}_k]\end{aligned}$$