Exercise Sheet 6

Question 3

(a) Determine the risk-free interest rate, given that there is no arbitrage.

Solution:

$$\begin{pmatrix} 1 \\ 2.9 \\ 1.1 \end{pmatrix} = \frac{1}{1+r} \begin{pmatrix} 1+r & 1+r \\ 3 & 4 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

Then

$$\begin{aligned} 1 &= q_1 + q_2 \\ 2.9 &= \frac{3}{1+r} q_1 + \frac{4}{1+r} q_2 \\ 1.1 &= \frac{2}{1+r} q_1 + \frac{1}{1+r} q_2 \end{aligned}$$

Hence

$$\begin{aligned} r &= \frac{1}{4} \\ q_1 &= \frac{3}{8} \\ q_2 &= \frac{5}{8} \end{aligned}$$

(b) Is the market complete?

Solution:

Yes, there exist a unique equivalent martingale measure.

(c) A contingent claim pays out 2.00 in state ω_1 and 3.00 in state ω_2. What is the $t = 0$ value of this claim?

Solution:

The value of the claim is

$$\frac{2}{1+r} q_1 + \frac{3}{1+r} q_2 = \frac{21}{10}$$