Exercise Sheet 6

Question 7b

Show that if $A \in \mathcal{F}_\sigma$ then $A \cap \{\sigma \leq \tau\} \in \mathcal{F}_{\sigma \wedge \tau}$

Solution:

$$\begin{aligned} A \cap \{\sigma \leq \tau\} \cap \{\sigma \wedge \tau \leq t\} &= A \cap \{\sigma \leq \tau\} \cap (\{t \leq \tau\} \cup \{\tau < t\}) \\ &= A \cap \{\sigma \leq t\} \cap (\{\sigma \leq t \leq \tau\} \cup (\{\sigma < \tau < t\}) \\ &= A \cap \{\sigma \leq t\} \cap \{\sigma \wedge t \leq \tau \wedge t\} \end{aligned}$$

Since $\{\sigma \leq t\}$ is $\in \mathcal{F}_t$ and $\{\sigma \wedge t \leq \tau \wedge t\}$ is $\in \mathcal{F}_t$. Then $\{\sigma \leq t\} \cap \{\sigma \wedge t \leq \tau \wedge t\} = \{\sigma \leq \tau\} \cap \{\sigma \wedge \tau \leq t\}$ is $\in \mathcal{F}_t$. Hence $\{\sigma \leq \tau\}, \{\sigma = \tau\} \in \mathcal{F}_{\sigma \wedge \tau}$