

**COLLEGE OF ENGINEERING, PUNE**  
(An Autonomous Institute of Government of Maharashtra)  
SHIVAJI NAGAR, PUNE - 411 005

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**END Semester Examination**  
**MA-101 Engineering Mathematics 1**

Branch Applied Science

Course: Btech

Semester: Sem I

Year: 2014-2015

Duration: 3 Hours Time:-10.00 am to 1.00 pm

**Question [I]**

1. Attempt (**any three**)

(a) Solve the following system of linear equations:

$$2x_1 - x_2 + 3x_3 = 0; 3x_1 + 2x_2 + x_3 = 0; x_1 - 4x_2 + 5x_3 = 0.$$

(b) Find all values of  $c$  for which the system of equations  $5x + 3y + 2z = 12; 2x + 4y + 5z = 2; 39x + 43y + 45z = c$  is consistent. For these values of  $c$ , solve the system.

(c) If the equation  $F(x, y, z) = 0$  determines  $z$  as a differentiable function of  $x$  and  $y$  then at points where  $F_z \neq 0$ , show that  $\frac{\partial z}{\partial x} = -\frac{\partial F_x}{\partial F_z}$ .

(d) If the equation  $F(x, y, z) = xe^y + ye^z + z\ln x - 2 - 3\ln 2 = 0$ , find  $\frac{\partial z}{\partial x}$  at  $(1, \ln 2, \ln 3)$ .

2. Describe the matrix  $A$  corresponding to the orthogonal projection of  $R^3$  onto the plane  $y = x$  with detail. Determine the principal values and corresponding principal direction of the linear transformation either algebraically or geometrically.

3. Define the null space of a linear transformation from one vector space to another. Prove that the null space of a linear transformation has dimension 0 if and only if the transformation is one-to-one.

**Question [II]**

1. Attempt (**any three**)

(a) If  $w = \tan^{-1}(\frac{y}{x})$ , verify that  $w_x y = w_y x$ .

(b) Determine the absolute extreme values of  $g(x) = \sqrt{5 - x^2}$ .

(c) Assume that  $f$  is continuous on  $[a, b]$  and differentiable on  $(a, b)$ . Also assume that  $f(a)$  and  $f(b)$  have opposite signs and that  $f' \neq 0$  between  $a$  and  $b$ . Show that  $f(x)$  is zero exactly once between  $a$  and  $b$ .

(d) For  $0 < a < b < 1$ , prove that