ELECTRIC CHARGES AND FIELD

Electric charge

Electric charge is an intrinsic property of particle of matter which give rise to electric force between these particles.

Electric charge is a scalar quantity.

SI unit of charge is coulomb (c)

A proton has (+) positive charge and an electron has (-) negative charge

charge on them → $\boxed{e = 1.6\times10^{-19} \text{ coulomb}}$

Basic properties of charge.

1) **Additivity of charge** = Additivity of electric charge means that the total charge of a system is the algebraic sum of all the individual charges located at different points

$$\boxed{Q = q_1 + q_2 + q_3 - - - + q_n}$$

2) **Quantization of charge** = The total charge of a body is always an integral multiple of a charge.

$\boxed{Q = ne}$ where $n = 0, \pm1, \pm2 - - -$ etc

3) **Conservation of charge** = 1) The total charge of an isolated system remains constant

2) The electric charge can neither be created nor destroyed, they can only be transferred from one body to another.

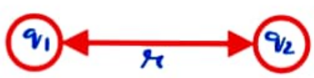
Coulomb's law of electric force

The force of attraction or repulsion between two charges is

1) directly proportional to the product of magnitude of charges

$$F \propto q_1 \times q_2$$

2) inversely proportional to the square of distance between them.



$$F \propto 1/r^2$$

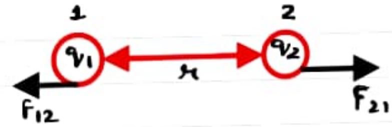
Combining both

$$F \propto \frac{q_1 \times q_2}{r^2}$$

Now $\boxed{F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}}$ here $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$

& ε_0 = permittivity of free space

Coulomb's law in vector form



Let F_{21} = force on charge 2 due to charge 1
F_{12} = force on charge 1 due to charge 2

Now from coulomb's law

$$F_{21} = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2}$$

In vector form

$$\boxed{\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}}$$

⇒ here $\hat{r}_{12}$ is a unit vector. It tells the direction of force. The direction of force is from charge 1 to 2

or

$$\boxed{\vec{F}_{21} = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^3} \vec{r}_{12}}$$

Similarly

$$\boxed{\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}} \quad \text{or} \quad \boxed{\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{21}}$$

Principle of Superposition

It states that when a number of charge are present, the total force on a given charge is the vector sum of the force exerted on it due to all other charges.

The force between two charges is not affected by the presence of other charges.

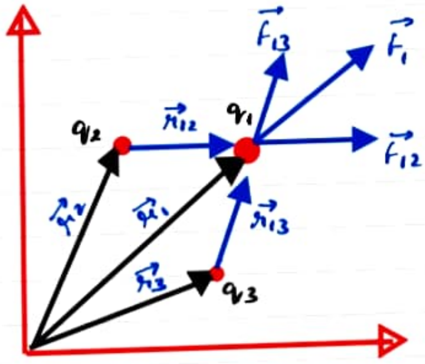
Force between multiple charges

According to principle of superposition, the total force on charge q_1 is

$$\boxed{\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \text{---} \vec{F}_{1N}}$$

here $\vec{F}_{12}$ = force on charge 1 due to charge 2
$\vec{F}_{13}$ = force on charge 1 due to charge 3

Similarly
$\vec{F}_{1N}$ = force on charge 1 due to charge N



Now In vector form:-

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{(r_{12})^3} \vec{r}_{12} \quad \text{—} (1)$$

from triangle law of addition

$$\vec{r}_1 = \vec{r}_2 + \vec{r}_{12}$$

Then $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$

Then eqn (1):-

$$\boxed{\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)} \quad (2)$$

Similarly
$$\boxed{\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_3}{|\vec{r}_1 - \vec{r}_3|^3} \, \vec{r}_1 - \vec{r}_3} \longrightarrow (3)$$

Total force on charge 1 is
$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} - - - + \vec{F}_{1N}$$
using eq^n (2) & (3)

$$\vec{F}_1 = \left[\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} \, \vec{r}_1 - \vec{r}_2\right] + \left[\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_3}{|\vec{r}_1 - \vec{r}_3|^3} \, \vec{r}_1 - \vec{r}_3\right] + - - - + - - - - + \left[\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_N}{|\vec{r}_1 - \vec{r}_N|^3} \, \vec{r}_1 - \vec{r}_N\right]$$

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0} q_1 \left\{\left[\frac{q_2}{|\vec{r}_1 - \vec{r}_2|^3} \, \vec{r}_1 - \vec{r}_2\right] + \left\{\frac{q_3}{|\vec{r}_1 - \vec{r}_3|^3} \, \vec{r}_1 - \vec{r}_3\right\} + - - - - + - - - + \left\{\frac{q_N}{|\vec{r}_1 - \vec{r}_N|^3} \, \vec{r}_1 - \vec{r}_N\right\}\right\}$$

$$\boxed{\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_0} \sum_{x=2}^{x=N} \frac{q_i}{|\vec{r}_1 - \vec{r}_x|^3} \, \vec{r}_1 - \vec{r}_x}$$

Thus force on any a^{th} charge :-

$$\boxed{\vec{F}_a = \frac{q_a}{4\pi\varepsilon_0} \sum_{\substack{x=1 \\ x \neq a}}^{x=N} \frac{q_x}{|\vec{r}_a - \vec{r}_x|^3} \, \vec{r}_a - \vec{r}_x}$$

Electric field The electric field at a point is defined as the force experienced by a unit positive test charge placed at that point.

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

It is a vector quantity. Electric field is from +ve charge towards −ve.

Electric field due to a point charge

Consider a charge Q is placed at Point O. we have to find electric field at point P. Let us put a test charge q_0 on P:-

Now force on q_0:-

$$F = \frac{1}{4\pi\varepsilon_0} \frac{Q q_0}{r^2}$$

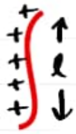
$$\text{Now } E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0} \frac{Q q_0}{r^2} \times \frac{1}{q_0}$$

Then $$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}}$$

Continuous charge distribution

1) Linear charge distribution (λ)

Charge stored per unit length of a wire



so $\boxed{\lambda = \frac{Q}{l}}$

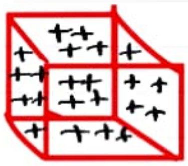
2) Surface charge distribution (σ)

Charge stored per unit Area.

so $\boxed{\sigma = \frac{Q}{A}}$

3) Volume charge distribution (ρ)

Charge stored per unit volume

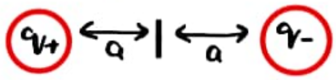


$\boxed{\rho = \frac{Q}{V}}$

Electric dipole

A pair of equal and opposite charges separated by small distance is called electric dipole

Dipole moment ⇒ It is equal to product of any charge with distance between the two charges. It is denoted by p.



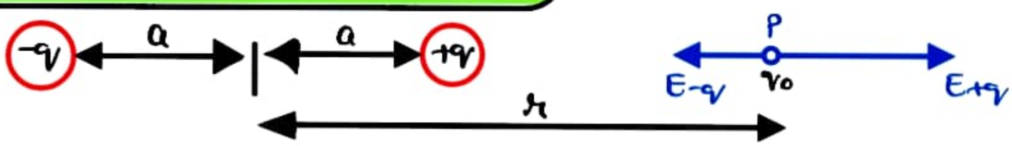
$$\boxed{p = q \times 2a}$$

It is a vector quantity.

Its direction is from negative (-ve) to positive charge (+)

Its direction is opposite to that of electric field.

Electric field at axial point due to a dipole



Consider a test charge is kept at point P.

Now E_{+q} = Electric field at point P due to +q charge

E_{-q} = Electric field at point P due to −q charge

$$\vec{E}_{axial} = \vec{E}_{+q} + \vec{E}_{-q}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{+q}{(r-a)^2} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r+a)^2}$$

$$E_{axial} = \frac{1}{4\pi\varepsilon_0} q \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$E_{axial} = \frac{1}{4\pi\varepsilon_0} q \left[\frac{(r+a)^2 - (r-a)^2}{(r-a)^2 (r+a)^2}\right]$$

$$E_{axial} = \frac{1}{4\pi\varepsilon_0} q \left\{ \frac{r^2+a^2+2ar - r^2-a^2+2ar}{(r^2-a^2)^2} \right\}$$

$$= \frac{1}{4\pi\varepsilon_0} q \left\{ \frac{4ar}{(r^2-a^2)^2} \right\} = \frac{1}{4\pi\varepsilon_0} \frac{(q \times 2a) \times 2r}{(r^2-a^2)^2}$$

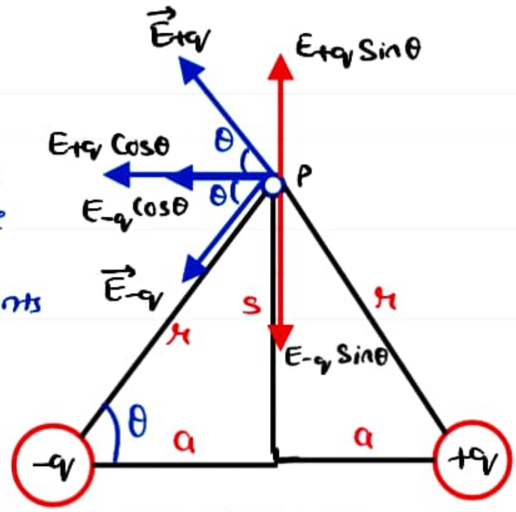
$$E_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{(p)(2r)}{(r^2-a^2)^2}$$ where $p = q \times 2a$ = dipole moment

$$\boxed{E_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{2pr}{(r^2-a^2)^2}}$$ (towards Right)

Net electric field at Point P is in the direction of dipole moment.

In vector form $$\boxed{\vec{E}_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{2pr}{(r^2-a^2)^2} \hat{p}}$$ where $\hat{p}$ is a unit vector & it is towards right.

Electric field at equatorial point



Consider a test charge is kept at Point P

Now,

E_{+q} = Electric field at Point P due to +q charge

E_{-q} = Electric field at Point P due to −q charge

Now Both E_{+q} and E_{-q} will have two Components

for $E_{+q} \rightarrow E_{+q} \cos\theta$ & $E_{+q} \sin\theta$

for $E_{-q} \rightarrow E_{-q} \cos\theta$ & $E_{-q} \sin\theta$

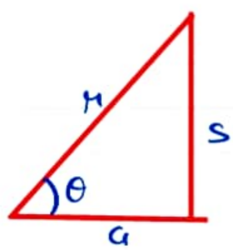
From diagram, $E_{+q} \sin\theta$ & $E_{-q} \sin\theta$ are in opposite direction. So they will cancel each other out.

Then net electric field at Point P will be

$$E_{equ} = E_{-q} \cos\theta + E_{+q} \cos\theta$$

$$E_{equ} = 2E\cos\theta$$

$$E_{equ} = 2 \times \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \cos\theta$$



$\cos\theta = \frac{B}{H} = \frac{a}{r}$

Also by Pythagoras:-

$r^2 = s^2 + a^2$

$r = \sqrt{s^2+a^2}$

Now Putting Values of r^2 & $\cos\theta$

$$E_{equ} = 2 \times \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \times \frac{a}{r}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q \times 2a}{(s^2+a^2)} \frac{1}{\sqrt{s^2+a^2}}$$

$$\boxed{E_{equ} = \frac{1}{4\pi\varepsilon_0} \frac{p}{(s^2+a^2)^{3/2}}}$$ here p = dipole moment $p = q \times 2a$

here electric field at P is opposite to that the dipole moment. So In vector form

$$E_{equ} = \frac{1}{4\pi\varepsilon_0} \frac{p}{(s^2+a^2)^{3/2}} (-\hat{p})$$

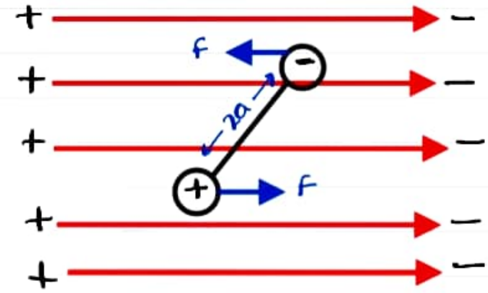
$$\boxed{E_{equ} = -\frac{1}{4\pi\varepsilon_0}\frac{p}{(s^2+a^2)^{3/2}}\,\hat{p}}$$

here $\hat{p}$ is a unit vector
it is toward left direction.

Torque on a Dipole in electric field

when a dipole is kept inside an electric field the dipole experience a force

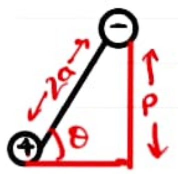
$$F = qE \quad \text{—— (1)}$$



But as force experienced by the positive charge is equal and opposite to the force experienced by a negative charge so a Torque acts on the dipole:-

Torque = force × perpendicular distance

$$\tau = F \times P$$



here $\sin\theta = \frac{P}{H} = \frac{P}{2a}$

$$\boxed{P = 2a\sin\theta} \quad \text{—— (2)}$$

from eqn (1) & (2)

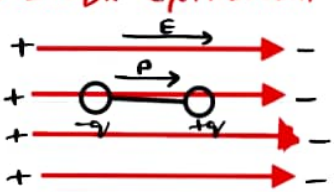
$$\tau = qE \times 2a\sin\theta$$

$$\tau = (q \times 2a)\, E\sin\theta$$

$$\boxed{\tau = pE\sin\theta} \quad \text{or} \quad \boxed{\vec{\tau} = \vec{p}\times\vec{E}}$$

Special Cases:-

1) Stable equilibrium



here $\vec{P} \rightarrow$ $\vec{E} \rightarrow$

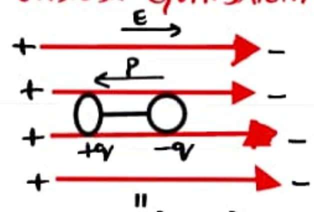
here $\theta = 0$

So, $\sin\theta = \sin 0 = 0$

$\tau = pE\sin 0 = 0$

$\boxed{\tau = 0}$ (min)

2) Unstable equilibrium



here $\vec{P} \leftarrow \bullet \rightarrow \vec{E}$ 180°

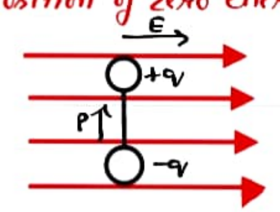
here $\theta = 180°$

So, $\sin\theta = \sin 180 = 0$

$\tau = pE\sin 180 = 0$

$\boxed{\tau = 0}$ (min)

3) Position of zero energy



here $\theta = 90°$ P ↑ → E

So, $\sin\theta = \sin 90 = 1$

$\tau = pE\sin 90 = pE$

$\boxed{\tau = pE}$ (max)

Note:- If dipole is placed in a uniform electric field, then it will have only rotational motion. (only Torque will act)

If the dipole is placed in a non-uniform electric field, then it will have both rotational as well as linear motion.
(Both Torque & force will act).

Electric field lines

Properties of Electric Lines of Force

1. The lines of force are continuous smooth curves without any breaks.
2. The lines of force start at positive charges and end at negative charges – they cannot form closed loops. If there is a single charge, then the lines of force will start or end at infinity.
3. The tangent to a line of force at any point gives the direction of the electric field at that point.
4. No two lines of force can cross each other.
5. The lines of force are always normal to the surface of a conductor on which the charges are in equilibrium.

   Reason. If the lines of force are not normal to the conductor, the component of the field $\vec{E}$ parallel to the surface would cause the electrons to move and would set up a current on the surface. But no current flows in the equilibrium condition.
6. The lines of force have a tendency to contract lengthwise. This explains attraction between two unlike charges.
7. The lines of force have a tendency to expand laterally so as to exert a lateral pressure on neighbouring lines of force. This explains repulsion between two similar charges.
8. The relative closeness of the lines of force gives a measure of the strength of the electric field in any region. The lines of force are
   (i) close together in a strong field.
   (ii) far apart in a weak field.
   (iii) parallel and equally spaced in a uniform field.
9. The lines of force do not pass through a conductor because the electric field inside a charged conductor is zero.

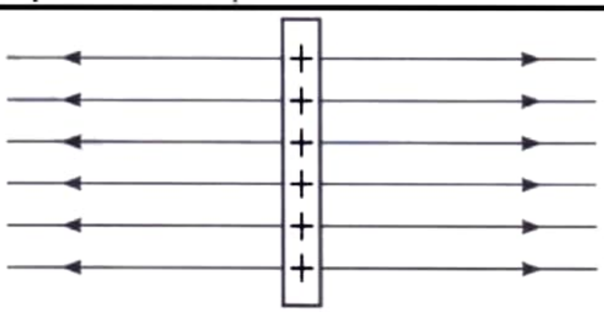


Fig. 1.78 Field pattern of a positively charged plane conductor.

Uniform electric field.

Fig. 1.74 Field lines of a positive point charge.

Fig. 1.75 Field lines of a negative point charge.

Electric field due to a point charge

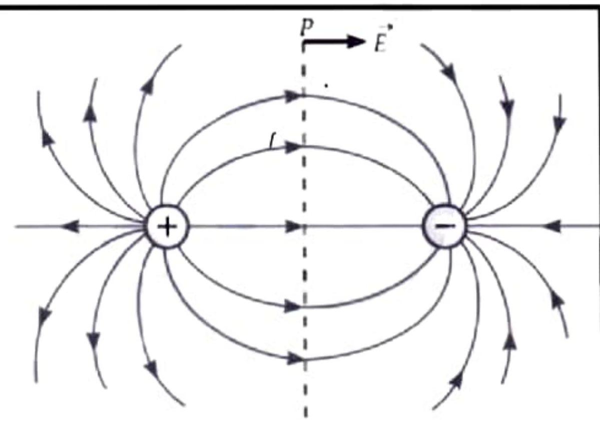


Fig. 1.76 Field lines of an electric dipole.

Electric field due to a dipole

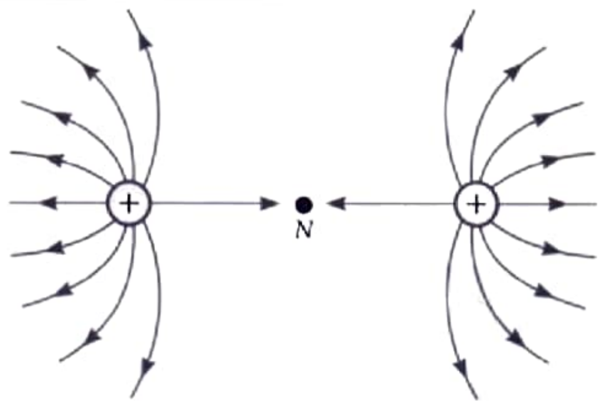


Fig. 1.77 Field lines of two equal positive charges.

Electric field due to 2 positive charges

Electric flux

Electric flux through a given area is the measure of the total number of electric field lines passing normally through that area.

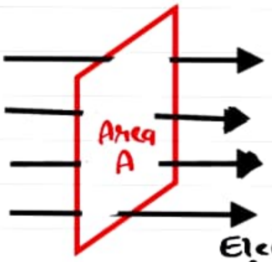
It is denoted by ϕ

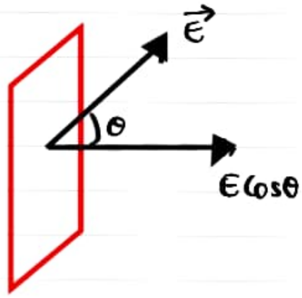
Let E = Electric field

A = Area

$\phi = E\cos\theta \cdot A$

$$\boxed{\phi = EA\cos\theta}$$





here θ is the angle between electric field & Area vector

Note = Area vector is any vector which is perpendicular to that Area.

$$\boxed{\vec{\phi} = \vec{E} \cdot \vec{A}}$$

Gauss Theorem

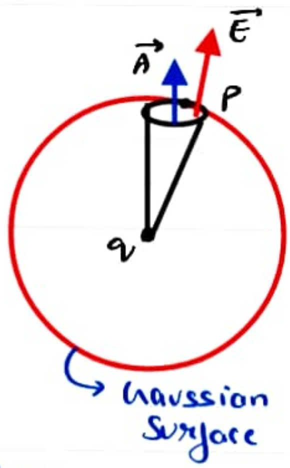
Gauss theorem states that the total flux through a closed surface is $1/\varepsilon_0$ times the net charge enclosed by the closed surface.

$$\boxed{\phi = \oint \vec{E} \cdot \vec{ds} = q/\varepsilon_0}$$

Proof:

Consider a charge q is placed in the centre of the Gaussian surface. The value of electric field at a distance r is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$



Now total electric field over whole Gaussian surface is given by. flux over whole surface

$$\phi = \oint E.ds = E\oint ds = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\int ds$$

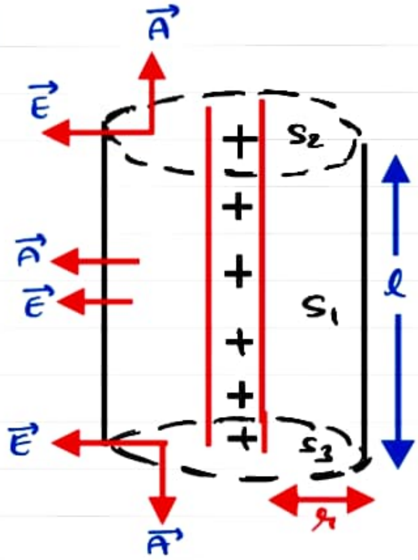
$$\phi = \oint E.ds = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\left[\text{Surface area of sphere}\right]$$

$$\phi = \oint E.ds = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \times 4\pi r^2$$

$$\boxed{\phi = \oint E.ds = q/\varepsilon_0}$$

This proves Gauss law.

Field due to uniformly long charged wire



Consider a long charged wire of length l. Now we want to find electric field due to the wire at a distance r from the wire.
So we draw a gaussian surface (shaped cylinder) to cover the wire completely.

Now applying Gauss law for whole surface

$$\oint_S E \cdot ds = q/\varepsilon_0$$

$$\oint_{S_1} E \cdot ds + \oint_{S_2} E \cdot ds + \oint_{S_3} E \cdot ds = q/\varepsilon_0$$

$$\oint_{S_1} E\, ds \cos\theta + \oint_{S_2} E\, ds \cos\theta + \oint_{S_3} E\, ds \cos\theta = q/\varepsilon_0$$

$$\oint_{S_1} E\, ds \cos 0 + \oint_{S_2} E\, ds \cos 90 + \oint_{S_3} E\, ds \cos 0 = q/\varepsilon_0$$

$$\oint_{S_1} E\, ds\,(1) + 0 + 0 = q/\varepsilon_0 \qquad (\because \cos 90 = 0)$$

$$\oint_{S_1} E\, ds = q/\varepsilon_0$$

$$E \oint ds = q/\varepsilon_0$$

$$E(\text{Curved Surface Area of Cylinder}) = q/\varepsilon_0$$

$$E \times 2\pi r l = q/\varepsilon_0$$

$$\boxed{E = \frac{q}{2\pi\varepsilon_0 r l}} \quad \text{—— (1)}$$

Now linear charge density $= \frac{\text{charge}}{\text{length}}$

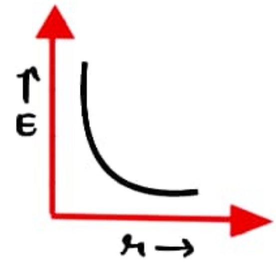
$\lambda = \frac{q}{l}$ Then $\boxed{q = \lambda l}$ —— (2)

from (1) & (2)

$$E = \frac{\lambda l}{2\pi\varepsilon_0 r l}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

here $E \propto \frac{1}{r}$



Electric field due to uniformly charged sheet

Consider an infinite charged sheet
Now we want to find electric field due to this charged sheet at a distance r from the sheet.
Now we draw a gaussian surface (cylindrical shape) as shown in the figure.

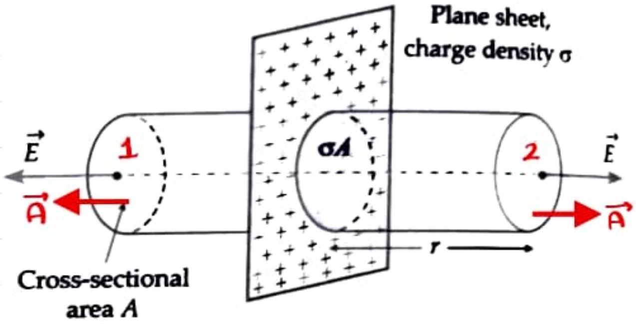


Fig. 1.98 **Gaussian surface for a uniformly charged infinite plane sheet.**

From Gauss law :-

$$\oint E \cdot ds = q/\varepsilon_0$$

As there are two surfaces (surface 1 & surface 2) from which the electric field is passing out. so gauss law will be

$$\oint_1 E \cdot ds + \oint_2 E \cdot ds = q/\varepsilon_0$$

$$E\oint_1 ds + E\oint_2 ds = q/\varepsilon_0$$

$$ES + ES = q/\varepsilon_0$$ here S = Surface Area of Circle

$$\boxed{2ES = q/\varepsilon_0} \text{ — (1)}$$

Now, we know, surface charge density = $\frac{\text{charge}}{\text{Surface Area}}$

$$\sigma = q/S$$

Then $$\boxed{q = \sigma S} \text{ — (2)}$$

Now from (1) & (2) :-

$$2ES = \frac{\sigma S}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

This is the value of electric field due to a sheet.

Note;- The electric field due to a sheet does not depend upon the distance.

Electric field due to two charged sheet

Electric field due to two positive sheet:-

Consider two sheets with charge density σ_1 and σ_2. Now let $\sigma_1 > \sigma_2$.

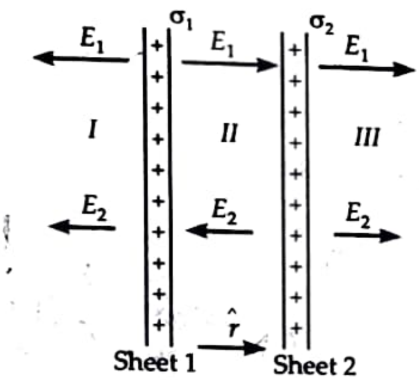


Fig. 1.99

In region I :-

$$\vec{E}_{Net} = (-\vec{E}_1) + (-\vec{E}_2)$$

$$\vec{E}_{Net} = -\frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}$$

$$\boxed{\vec{E}_{Net} = \frac{-1}{2\varepsilon_0}(\sigma_1 + \sigma_2)}$$

In region II :

$$\vec{E}_{net} = \vec{E}_1 - \vec{E}_2$$

$$= \frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}$$

$$\boxed{E_{Net} = \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2)}$$

In region III :-

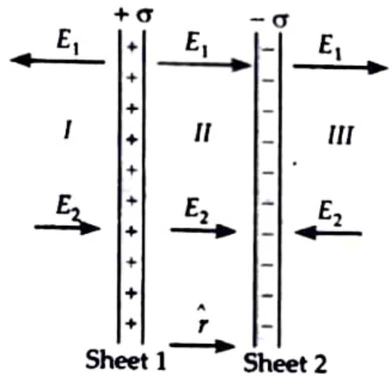
$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0}$$

$$\boxed{E_{net} = \frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2)}$$

Electric field due to one positive and one negative plate

Consider two sheets with charge density $+\sigma$ and $-\sigma$.



In region I :-

$$\vec{E}_{Net} = (-\vec{E}_1) + (\vec{E}_2)$$

$$= \frac{-\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$\boxed{\vec{E}_{Net} = 0}$$

In region II :-

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$= \frac{2\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

$$\boxed{\vec{E}_{Net} = \frac{\sigma}{\varepsilon_0}}$$

In region III :-

$$\vec{E}_{net} = \vec{E}_1 + (-\vec{E}_2)$$

$$= \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$$

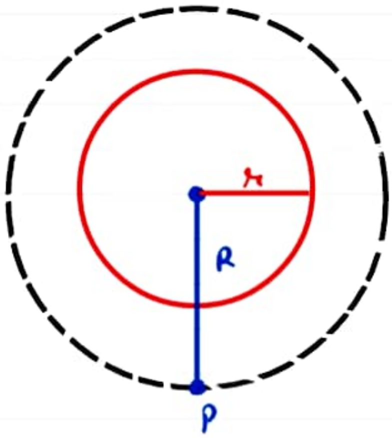
$$\boxed{E_{Net} = 0}$$

So electric field only exist between the plates.

Electric field due to uniformly charged thin spherical shell

Consider a spherical shell of radius r with charge 'q' present on it. Now we have to find electric field at:-

i) outside the sherical shell (At point P)



To find electric field at Point P, let us draw a gaussian surface (sperical in shape) of radius R.

Now from gauss law:-

$$\oint E.ds = q/\varepsilon_0$$

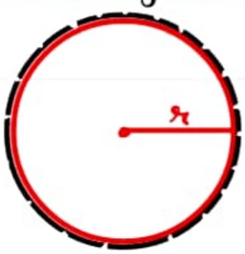
$$E\oint ds = q/\varepsilon_0$$

$$E \times \text{Surface area of sphere} = q/\varepsilon_0$$

$$E \times 4\pi R^2 = q/\varepsilon_0$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

b) Electric field on the sperical shell:-



Now to find electric field on the sphere, draw an gaussian surface of radius r.

Now from gauss law:-

$$\oint E.ds = q/\varepsilon_0$$

$$E\oint ds = q/\varepsilon_0$$

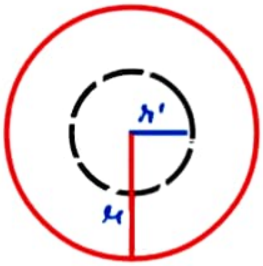
$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{\varepsilon_0}\frac{q}{4\pi r^2}}$$

here q = charge
and, $4\pi r^2$ = Area

So $E = \frac{1}{\varepsilon_0}\frac{\text{charge}}{\text{Area}}$ Then, $\boxed{E = \frac{\sigma}{\varepsilon_0}}$ where σ = Surface charge density
σ = charge/Area

c) Electric field inside the spherical shell:-



To find electric field inside the spherical shell let us draw a gaussian surface of radius r'.

Then Acc. to gauss law:-

$$\oint E.ds = q/\varepsilon_0$$

But there is No charge inside the gaussian surface so $q = 0$ Then,

$$\oint E.ds = 0$$ Then $\boxed{E = 0}$

No electric field is present inside the shell