CURRENT ELECTRICITY

Electric Current

Rate of flow of charge gives the value of electric current.

$$I = \frac{Q}{T}$$

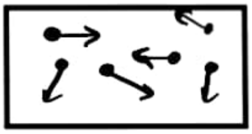
Current is denoted by I
Charge is denoted by Q
Time is denoted by T
SI unit of current is Ampere (A)

$$\text{Current} = \frac{\text{Charge}}{\text{Time}}$$

Flow of electric charges

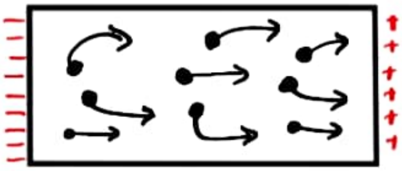
The electrons travels from the negative terminal of the battery towards the positive terminal. This type of current is known as electronic current.
The current which flows from positive terminal of the battery towards negative terminal is known as conventional current.

Drift velocity



All conductors have large no. of free electrons. But in the absence of the electric field, the electrons are moving in all random directions, thus there is no net flow of charge in any particular direction.
Let $u_1, u_2, u_3 - - -, u_n$ be the initial velocities of n electrons.
As each electron is moving in a different direction, thus average initial velocity is zero.

$$\vec{u}_{avg} = \frac{\vec{u}_1 + \vec{u}_2 + \vec{u}_3 - - - \vec{u}_n}{n} = 0 \quad \text{—} (1)$$



Now as we apply electric field to the conductor each electron start drifting towards the positive terminal of the Battery.
Now each electron will experience a force towards positive terminal

From newtons law of motion, $F = ma$
Also from coulombs law, $F = qE$ where E = Electric field
on comparing:-

$$ma = qE \quad \text{here } q = \text{charge of electron} = -e$$
$$ma = -eE$$
$$a = -\frac{eE}{m} \quad \text{—} (2)$$

Now, let $v_1, v_2, v_3 - - - t_n$ be drift velocities of electron & $t_1, t_2, t_3 - - - t_n$ be the relaxation time of n electrons. Then using $v = u + at$ for each electron.

for 1st electron → $v_1 = u_1 + at_1$
for 2nd electron → $v_2 = u_2 + at_2$
for 3rd electron → $v_3 = u_3 + at_3$
for n^{th} electron → $v_n = u_n + at_n$

Now adding all terms:-

$$v_1 + v_2 + v_3 \text{ --- } v_n = (u_1 + u_2 + u_3 \text{ --- } u_n) + (at_1 + at_2 + at_3 \text{ --- } at_n)$$
$$= (u_1 + u_2 + u_3 \text{ --- } u_n) + a(t_1 + t_2 + t_3 \text{ --- } t_n)$$

Dividing by n both sides:-

$$\frac{v_1 + v_2 + v_3 \text{ --- } v_n}{n} = \frac{(u_1 + u_2 + u_3 \text{ --- } u_n)}{n} + a\frac{(t_1 + t_2 + t_3 \text{ --- } t_n)}{n}$$

$$V_d = 0 + a\tau \quad (\because \text{ using eq}^n \text{ ①})$$
$$V_d = a\tau$$

Putting value of a from eq^n ② :-

$$\boxed{V_d = \frac{-eE\tau}{m}}$$

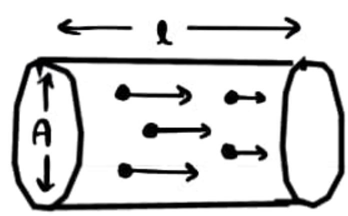
here $V_d = \frac{v_1 + v_2 + v_3 \text{ --- } v_n}{n}$ = Average drift velocity

$\tau = \frac{t_1 + t_2 + t_3 \text{ --- } t_n}{n}$ = Average Relaxation time

E = Electric field   m = mass of electron
e = charge of electron

Relaxation time The time period between two successive collision of electron

Relation between current & drift velocity



Let N = Total no. of electrons inside the conductor
A = Area of conductor   V = Volume of conductor
l = length of conductor

n = no. of electrons per unit volume
$n = \frac{N}{V}$
Then $N = nV$

Now,
Total charge = charge of 1 electron × no. of electron
$Q = e \times N$
$Q = enV$
$Q = enAl$ ($\because$ Volume = Area × length)

Now
$$\text{Current} = \frac{\text{charge}}{\text{Time}} = \frac{Q}{T} = \frac{enAl}{T} = enA\frac{l}{T}$$

$I = enA\frac{l}{T} = enAV_d$ $\quad (\because \text{ velocity} = \frac{\text{Distance}}{\text{Time}})$

so $\boxed{I = neAV_d}$

here n = no. of electrons per unit volume
e = charge of electron
A = Area of conductor
V_d = drift velocity.

Mobility

It is defined as drift velocity acquired by electron per unit value of electric field.
It is denoted by μ

$$\mu = \frac{\text{Drift velocity}}{\text{Electric field}} \qquad \boxed{\mu = \frac{V_d}{E}}$$

Also we know $V_d = \frac{eE\tau}{m}$ therefore $\mu = \frac{eE\tau}{m} \times \frac{1}{E} = \frac{e\tau}{m}$

so $\boxed{\text{mobility, } \mu = \frac{e\tau}{m}}$

where e = charge of electron
τ = Relaxation time
m = mass of electron

Relation b/w current & drift velocity

we know, $I = neAV_d$ —①

Also mobility, $\mu = \frac{V_d}{E}$ Then $V_d = \mu E$ —②

so from eqn ① & ② $\boxed{I = neA\mu E}$

here μ = mobility & I = current

Ohm's law

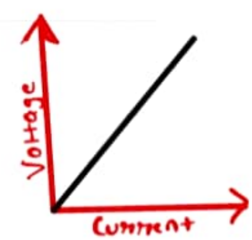
If physical condition of a wire remains same, then current flowing through the wire is directly proportional to the voltage applied across it.

Voltage $\propto$ current

$$V \propto I$$

$$\boxed{V = IR}$$

where R is known as Resistance of the wire



Resistance

Resistance is the opposition to the flow of charges.

Resistance is mainly due to the collisions of electrons with the positive ions.

SI unit of Resistance is ohm.

From observations, we get:-

Resistance $\propto$ length and Resistance $\propto \frac{1}{\text{Area}}$

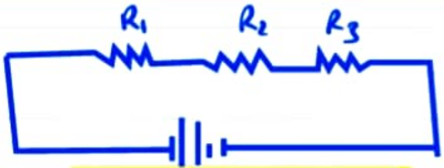
Combining Both, Resistance $\propto \frac{\text{length}}{\text{Area}} \rightarrow R \propto \frac{l}{A}$

$$\boxed{R = \rho \frac{l}{A}}$$

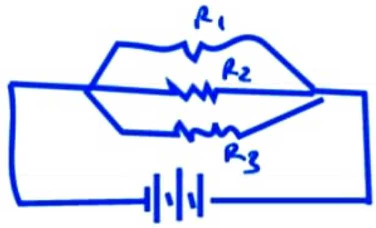
where ρ is called as Resistivity of the material

Resistivity depends upon the nature of the wire & temperature of wire.

Resistance in Parallel & Series



$$\boxed{R_{Net} = R_1 + R_2 + R_3}$$



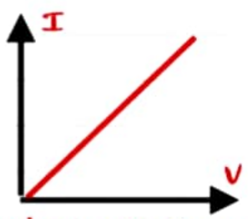
$$\boxed{\frac{1}{R_{Net}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

ohmic & non-ohmic Conductors

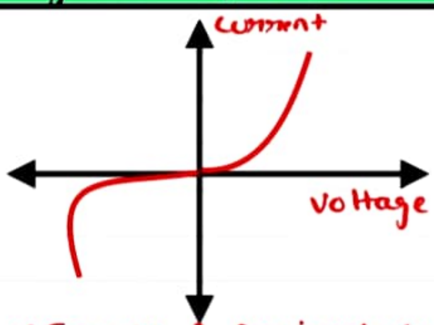
ohmic conductors : The conductors which obeys ohms law are called as ohmic conductors.

Non-ohmic conductors:- The conductors which do not obeys ohm's law are non-ohmic conductors.

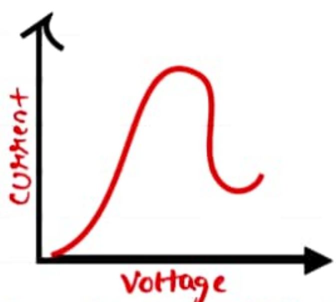
V-I characteristics of ohmic & Non-ohmic Conductors



ohmic Conductors



V-I graph of Semiconductor



V-I graph of gallium arsenide

Current :- Scalar or Vector

Current is a Scalar quantity because current doesnot follow vector law of addition

Current density

Current flowing through a conductor per unit area held normal to the direction of current.

It is denoted by J.

$J = \frac{\text{Current}}{\text{Area}}$ or $\boxed{J = \frac{I}{A}}$

Resistivity, Conductivity & Conductance

Resistivity:- It is the nature of material due to which it opposes the flow of charges.
It is denoted by ρ
SI unit of resistivity is ohm meter

$$\rho = \frac{R \times A}{l}$$

Conductivity:- It is the reciprocal of Resistivity
It is denoted by σ
SI unit of conductivity is $ohm^{-1}m^{-1}$ or $mho\, m^{-1}$ or Sm^{-1}

$$\sigma = \frac{1}{\rho}$$

Conductance:- It is the ease with which charges flow through a wire
It is reciprocal of resistance
It is denoted by G.
SI unit of conductance is ohm^{-1} or mho or siemens.

$$G = \frac{1}{R}$$

Temperature dependence of Resistivity

For metals :- As temp. increases, the amplitude of vibration of the metal ions increases. Due to which, free electron collide more frequently with the metal ions. The electron experience more opposition to its flow. Hence the resistivity increases & the conductivity decreases with increase in temperature.
For most metals, resistivity increases linearly with increase in temperature. So Resistivity ρ at any Temperature T is given by

$$\rho = \rho_0 \left[1 + \alpha (T - T_0) \right]$$

here ρ_0 = initial resistivity
ρ = final resistivity
T_0 = Initial Temperature
α = Temperature coefficient
T = final Temperature

Note:- Metals have high Temperature coefficient (α) value

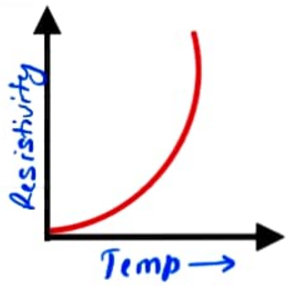
For alloys:- Alloy have high Resistivity. Alloy have weak temperature dependence. The values of Resistivity is not easily effected by the change in temperature. Alloy have low Temperature coefficient. for alloys we can use :-

$$R = R_0 \left[1 + \alpha (T - T_0) \right]$$

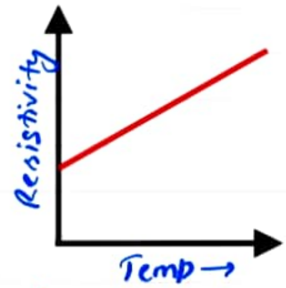
R = final Resistance at temp. T
R_0 = Initial Resistance at temp T_0
α = Temp. coefficient.

Note:- Alloy are used to make standard resistors.

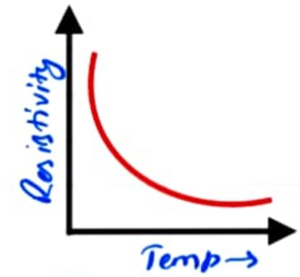
For Semiconductors :- Semiconductors do not have free electrons at low temperature. Hence they have high Resistivity at low temp. But as the temperature rises, the electrons get sufficient ionisation energy to behave as free electrons. Thus with increase in temp, more free electrons are produced. Hence resistivity decreases with increase in temperature.



For metals



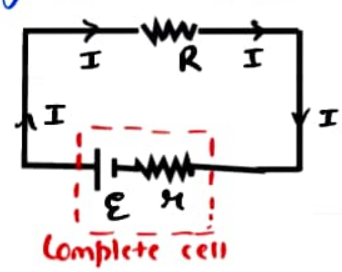
For Alloys



For Semiconductor

Internal Resistance of a cell

The resistance offered by the electrolyte of the cell to the flow of current is called internal resistance



Let ε = emf of the cell

V = voltage drop outside the cell

V' = voltage drop inside the cell

I = current flowing through wire

r = internal resistance

R = External resistance

Then

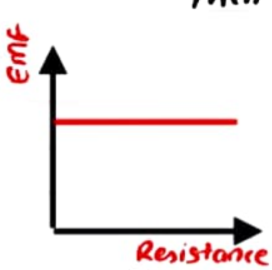
$$\varepsilon = V + V'$$

$$\varepsilon = IR + Ir' \quad \text{(using ohm's law)}$$

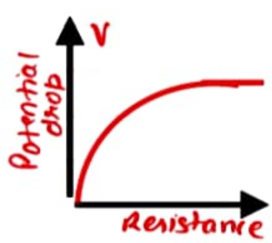
$$\boxed{\varepsilon = I(R + r')}$$

Note:- while discharging emf is always greater than terminal potential.

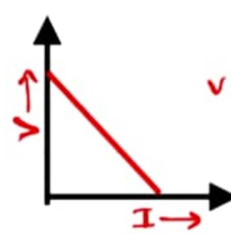
while charging the terminal potential difference is greater then the emf.



Emf is not effected by External Resistance



More Resistance means more potential drop

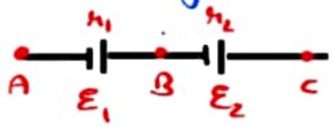


here
V = Voltage across cell
I = Current.

V-I graph for a cell

Cells in Series

Cells can be arranged in series by connecting negative terminal of one battery to the positive terminal of other battery.



Let $\varepsilon_1, \varepsilon_2 \rightarrow$ emf of the two cells
$r_1, r_2 \rightarrow$ Internal resistance of two cells

Now

$V_{AB} = \varepsilon_1 - Ir_1$ & $V_{BC} = \varepsilon_2 - Ir_2$

$V_A - V_B = \varepsilon_1 - Ir_1$ & $V_B - V_C = \varepsilon_2 - Ir_2$

Then

$V_{AC} = V_A - V_C$ Now add & substract V_B

$= (V_A - V_B) + (V_B - V_C)$

$V_{AC} = (\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2)$ —(1)

here V_{AC} = voltage across A & C

here $V_{AC} = \varepsilon_{net} - Ir_{net}$ —(2)

Comparing (1) & (2)

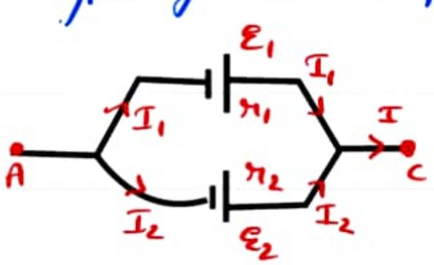
$$\varepsilon_{net} - Ir_{net} = (\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2)$$
$$\varepsilon_{net} - Ir_{net} = (\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2)$$

On comparing $\boxed{\varepsilon_{net} = \varepsilon_1 + \varepsilon_2}$ & $\boxed{r_{net} = r_1 + r_2}$

For n cells: $\varepsilon_{net} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 - - - + \varepsilon_n$
$r_{net} = r_1 + r_2 + r_3 - - - + r_n$

Cells in Parallel:-

Cells can be arranged parallely by connecting same types of terminals parallely as shown in figure



Let $\varepsilon_1, \varepsilon_2 \rightarrow$ Emf of two cells
$r_1, r_2 \rightarrow$ Internal resistance of two cells
$I_1, I_2 \rightarrow$ Current flowing through ε_1 & ε_2
$I_{net} = I_1 + I_2$

Now As we know voltage across parallel circuit remains same so

$V_{AC} = \varepsilon_1 - I_1 r_1$ & $V_{AC} = \varepsilon_2 - I_2 r_2$

$I_1 r_1 = \varepsilon_1 - V_{AC}$ $\qquad$ $I_2 r_2 = \varepsilon_2 - V_{AC}$

$I_1 = \frac{\varepsilon_1}{r_1} - \frac{V_{AC}}{r_1}$ $\qquad$ $I_2 = \frac{\varepsilon_2}{r_2} - \frac{V_{AC}}{r_2}$

Now $I_{net} = I_1 + I_2$

$$I_{net} = I_1 + I_2$$

$$I_{net} = \left(\frac{\varepsilon_1}{r_1} - \frac{V_{AC}}{r_1}\right) + \left(\frac{\varepsilon_2}{r_2} - \frac{V_{AC}}{r_2}\right)$$

$$\left(\frac{\varepsilon_{net}}{r_{net}} - \frac{V_{AC}}{r_{net}}\right) = \left(\frac{\varepsilon_1}{r_1} - \frac{V_{AC}}{r_1}\right) + \left(\frac{\varepsilon_2}{r_2} - \frac{V_{AC}}{r_2}\right)$$

$$\left(\frac{\varepsilon_{net}}{r_{net}} - \frac{V_{AC}}{r_{net}}\right) = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V_{AC}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

On comparing

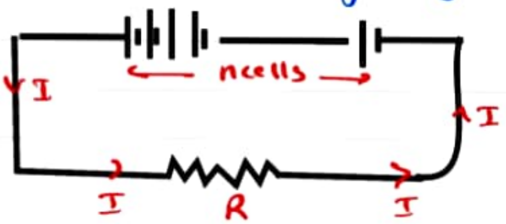
$$\boxed{\frac{\varepsilon_{net}}{r_{net}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \quad \& \quad \frac{1}{r_{net}} = \frac{1}{r_1} + \frac{1}{r_2}}$$

For n-cells

$$\boxed{\frac{\varepsilon_{net}}{r_{net}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \frac{\varepsilon_3}{r_3} - - - \frac{\varepsilon_n}{r_n} \quad \& \quad \frac{1}{r_{net}} = \frac{1}{r_1} + \frac{1}{r_2} - - + \frac{1}{r_n}}$$

Condition for maximum current (In series)

Suppose n cells each of emf 'ε' & internal resistance 'r' are connected in Series



Total emf $= \varepsilon + \varepsilon + \varepsilon - - -$ n times

$= n\varepsilon$

Total internal resistance $= r + r + - -$ n times

$= nr$

Total resistance $= nr + R$

Now using ohm's law

$$n\varepsilon = I(nr + R)$$

Then $\boxed{I = \frac{n\varepsilon}{nr + R}}$

Now when $R >>> nr$ Then neglect nR from formula

$$I = \frac{n\varepsilon}{R} = n\left(\frac{\varepsilon}{R}\right) = nI'$$

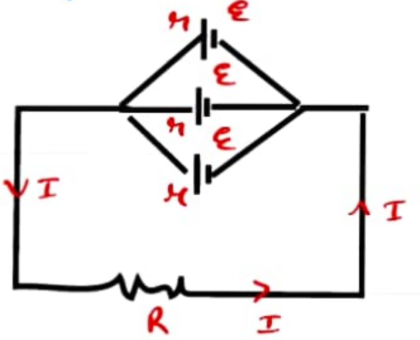
$I = n \times$ current due to one cell

So this case will give n times the current which is produced by one cell.

When external resistance is much higher than the internal resistance, cells should be connected in Series to get maximum current.

Condition for maximum current (In Parallel)

Let m cells, each of emf ε & internal resistance be r
They are connected parallely as shown in figure



we know in parallel voltage remains same. so emf of whole combination will be ε.

Total emf $= \varepsilon$

Total internal resistance.

$$\frac{1}{R'} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} - - - + \frac{m}{r} = \frac{m}{r}$$

$$R' = \frac{r}{m}$$

Total resistance $= \frac{r}{m} + R$

Now using ohm's law $\varepsilon = I\left(\frac{r}{m} + R\right)$

$$\boxed{I = \frac{\varepsilon}{\left(\frac{r}{m} + R\right)}}$$

Now when $R <<< \frac{r}{m}$ then neglect R. Then

$$I = \frac{\varepsilon}{\frac{r}{m}} = \frac{\varepsilon m}{r} = m\left(\frac{\varepsilon}{r}\right) = mI'$$

$I = m \times$ (current due to one cell)

So this case will give m times the current produced by one cell

When external resistance is much lower than the internal resistance, cells should be connected in parallel to get maximum current.

Joule's law of heating

The phenomenon of production of heat in a resistor by the flow of electric current through it is called Joule heating effect of current

Acc. to joule, the heat produced is given by

$$\boxed{H = VIT}$$

where H = Heat produced, I = Current

V = voltage or Potential, T = Time

Other formulas

$H = I^2RT$

or

$H = \frac{V^2T}{R}$

Electrical Energy

The total work done by the source emf to maintain the electric current in the circuit for a given time is called electrical energy

SI unit of energy is Joule

$$\boxed{\text{Energy} = VIT \text{ or } I^2RT \text{ or } \frac{V^2R}{T}}$$

Commercial unit of energy is Kilowatt hour (KWh)
Also $\boxed{1\,\text{KWh} = 3.6 \times 10^6 \text{ Joule}}$

Electric Power

The rate at which work is done by the source emf in maintaining the electric current through the circuit.

$$\text{Work} = \text{Energy} = VIT$$

Then $\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Energy}}{\text{Time}}$

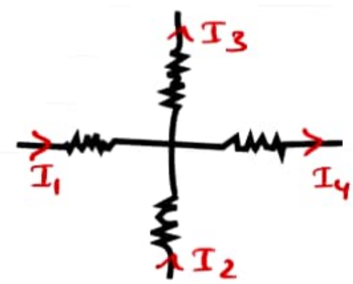
SI unit of Power is watt

$$\text{Power} = \frac{VIT}{T} = VI$$

Thus $\boxed{P = VI \text{ or } \frac{V^2}{R} \text{ or } I^2R}$

Kirchoff laws

Kirchhoff's first law ⇒ The sum of current entering a junction is equal to the sum of current leaving the junction



$$\boxed{I_1 + I_2 = I_3 + I_4}$$

This law is called Kirchoff's current law.
This law is based on law of conservation of charge

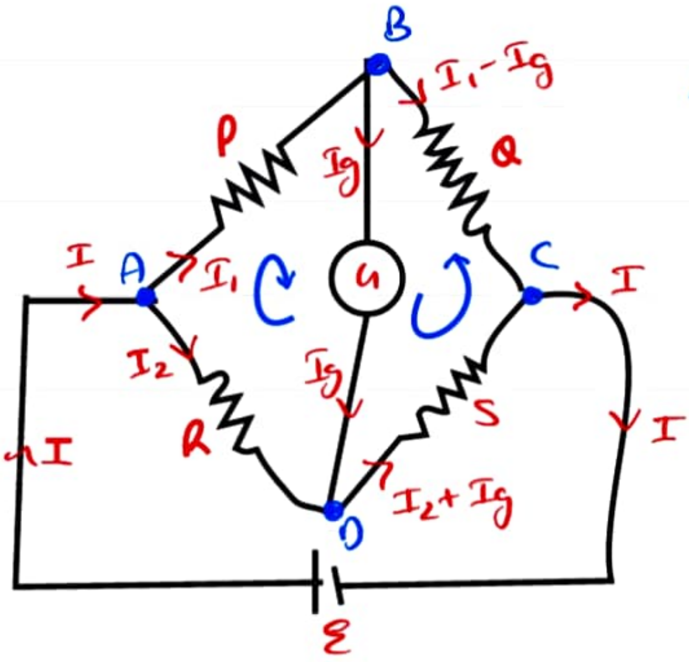
Kirchoff's second law or loop rule :- The algebraic sum of the emf in any loop of a circuit is equal to the sum of the product of current & resistances in it.

This law is called as Kirchoff's voltage law.
It is based on law of conservation of energy.

Wheatstone Bridge

It is the arrangement of four resistors used to determine one of these resistances quickly.

A wheatstone Bridge consists of four resistors P, Q, R & S connected in the form as shown in figure. A galvanometer is also attached across BD. An external Potential is applied across AC as shown in figure.



Acc. to wheatstone Bridge, If $\frac{P}{Q} = \frac{R}{S}$ then no current will flow across BD & Galvanometer shows no deflection.
In this state wheat stone Bridge is said to be in Balanced Condition.

Proof:- Apply KVL across DABD

$$-I_2R + I_1P + I_gG = 0 \quad \text{--- (1)}$$

Similarly apply KVL across DCBD

$$(I_2 + I_g)S - (I_1 - I_g)Q + I_gG = 0 \quad \text{--- (2)}$$

Now let assume $I_g = 0$ then

eqn (1) $\quad -I_2R + I_1P + 0 = 0$

$$I_1P = I_2R \quad \text{--- (3)}$$

eqn (2) $\quad (I_2 + 0)S - (I_1 - 0)Q + 0 = 0$

$$I_2S - I_1Q = 0$$

$$I_1Q = I_2S \quad \text{--- (4)}$$

Dividing (3) & (4)

$$\frac{I_1P}{I_1Q} = \frac{I_2R}{I_2S}$$

Then $\boxed{\frac{P}{Q} = \frac{R}{S}}$

This proves Balanced Condition of wheat stone Bridge.