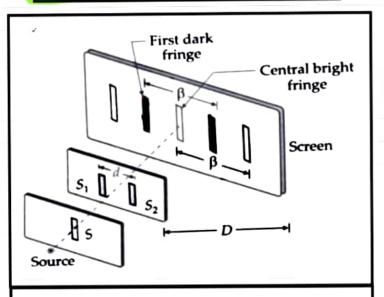
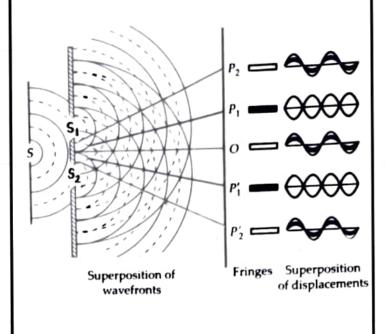
WAVEOPTICS

Young's Double slit Experiment





ton Constructive interference:

phase difference, \$ = 0,2TT, 4TT---

 $\phi = 2n\pi$

Where 120,1,2 ---

path difference, pz 0, 1, 21, 31-PZNd

Equation of light wave Coming through s, y, = a, sinut

Equation of light wave Coming through sz y = a Sin (w+++) where of is the phase difference.

Men, Resultant wave; -

$$y^{2} y_{1} + y_{2}$$

$$\Rightarrow y = A Sin (\omega \epsilon + \Theta)$$

where A = Amplitude of Resultant wave

$$\rightarrow A^{2} \alpha_{1}^{2} + \Omega_{2}^{2} + 2\alpha_{1}C_{2}(\cos\phi)$$

A = Amplitude of Resultont wave a, = Amplitude of wave through Si a, 2 Amplitude of wave muough S2 4 = phase difference

Intensity = K (Amplitude) IZ KAZ

where K = (onstant

I = Intensity of Resultant wave 2/III (05 = Antenference term

ton destructive anterjenence phase difference, \$ = TT, STT, STT

φ = (2n-1) TT where n = 1,43...

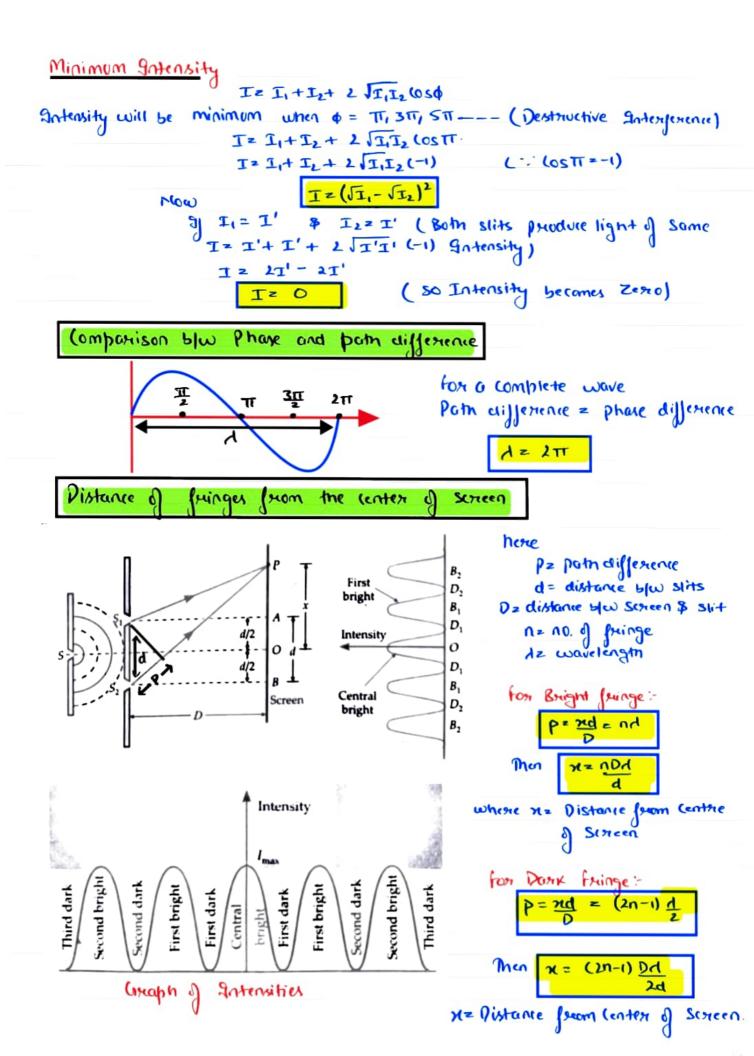
poin difference, pz 1/3/2, 3/2, 5/2-

Pz (20-1) d where nz 1,2,3 --

Maximum and minimum amplitude

```
We know, A= a2+ a2+ 20,000
   Maximum amblitude:
Amplitude will be maximum when \phi = 0,2\pi, 4\pi = - (Constructive Interference)
            A^2 = q_1^2 + q_2^2 + 2q_1q_1 \cos q
            A^2 = a_1^2 + a_2^2 + 2a_1a_2 (1) (050=1)
             g aza & azza (Both slits produce light of same
                                     amplitude)
              A2 = a4 a4 200 = a4 a4 24 2a2
                A2 2 402
                 A = 1492 = 29
                            ( Total amplitude = 2 x Amplitude of any slit)
   Minimum Amblitude:-
                   A= a1+ a1+ 2010, (050
     Amplitude will be minimum when $ = TI, 3TI, STI - - (2n-1)TT
                   A2= Q1+ Q1+ 2010, (OSTI
                   A2 = 912+ 62 + 26162 (-1) = 612+ 62 - 26162
                   a = a and a = a ( Both slits produce light of Some amplitude)
                 A= 02+01- 200 = 201-201
                                        ( Total amplitude = 0)
                           AZO
 maximum and minimum antensity
   Maximum intensity.

Iz I,+Iz+ 2 JI, Iz (050)
antensity will be maximum when \phi = 0,2\pi, 4\pi - - ( (constructive Antengenence)
                     I= I1+I2+ 2 /III2 (050
                                            ( : (OSO=1)
                     I = I,+ I, + 2 (I, I, (1)
                        T= (\(\bar{1}_1 + \bar{1}_2\)^2
                of II= I' & Iz= I' ( Both slits produce light of Some
                                           Intensity )
                   I= I'+ I'+ 2 \1'1'
                    T 2 21 + 21'
                                   ( so Intensity becomes 4 times)
```



fringe width: - 3t is the separation between two successive bright on dank lainge. It is denoted by B.



A wavefront is defined as the continuous locus of all such particles of the medium which are vibrating in the same phase at any instant.

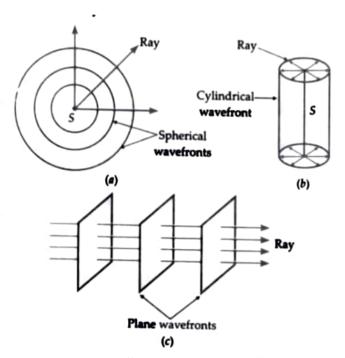


Fig. 9.1 Different types of wavefronts.

Relation Hw Ray and wave front

An arrow drawn perpendicular to a wavefront in the direction of propagation of a wave is called a ray.

A ray of light represents the path along which light travels.

This illustrates two general principles:

- 1. Rays are perpendicular to wavefronts.
- 2. The time taken for light to travel from one wavefront to another is the same along any ray.

In case of a plane wavefront, the rays are parallel [Fig. 9.2(a)]. A group of parallel rays is called a beam of light. In case of a spherical wavefront, the rays either converge to a point [Fig. 9.2(b)] or diverge from a point [Fig. 9.2(c)].

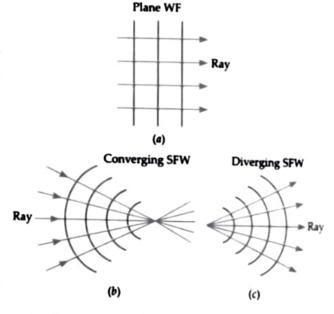
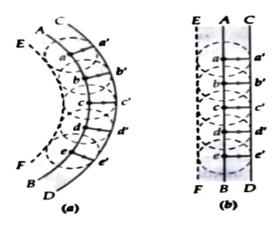


Fig. 9.2 Wavefronts and corresponding rays in three cases (a) plane, (b) converging spherical and (c) diverging spherical.

Huygen's Phinciple

This principle is based on the following assumptions:

- Leach point on a wavefront acts as a fresh source of
- new disturbance, called secondary waves or wavelets.
 The secondary wavelets spread out in all directions with the speed of light in the given medium.
- The new wavefront at any later time is given by the forward envelope (tangential surface in the forward direction) of the secondary wavelets at that time.



Reflection on the basis of Huygen's Meorey

Laws of reflection on the basis of Huygens' wave theory. As shown in Fig. 9.4, consider a plane wavefront AB incident on the plane reflecting surface XY, both the wavefront and the reflecting surface being perpendicular to the plane of paper.

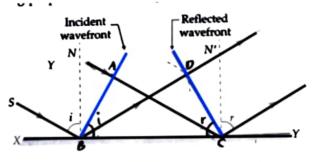


Fig. 9.4 Wavefronts and corresponding rays for reflection from a plane surface.

First the wavefront touches the reflecting surface at B and then at the successive points towards C. In accordance with Huygens' principle, from each point on BC, secondary wavelets start growing with the speed c. During the time the disturbance from A reaches the point C, the secondary wavelets from B must have spread over a hemisphere of radius BD = AC = ct, where t is the time taken by the disturbance to travel from A to C. The tangent plane CD drawn from the point C over this hemisphere of radius ct will be the new reflected wavefront.

Let angles of incidence and reflection be i and r respectively. In $\triangle ABC$ and $\triangle DCB$, we have

$$\angle BAC = \angle CDB$$
 [Each is 90°]
$$BC = BC$$
 [Common]
$$AC = BD$$
 [Each is equal to ct]
$$\triangle ABC \cong \triangle DCB$$
Hence
$$\angle ABC = \angle DCB$$

$$\angle i = \angle t$$

i.e., the angle of incidence is equal to the angle of reflection.

This proves the first law of reflection.

Refraction on the bours of huygen's

theory. Consider a plane wavefront AB incident on a plane surface XY, separating two media 1 and 2, as shown in Fig. 9.5. Let v_1 and v_2 be the velocities of light in the two media, with $v_2 < v_1$.

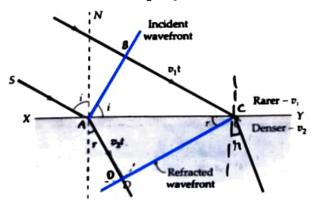


Fig. 9.5 Wavefronts and corresponding rays for refraction by a plane surface separating two media.

The wavefront first strikes at point A and then at the successive points towards C. According to Huygens' principle, from each point on AC, the secondary wavelets start growing in the second medium with speed v_2 . Let the disturbance take time t to travel from B to C, then $BC = v_1 t$. During the time the disturbance from B reaches the point C, the secondary wavelets from point A must have spread over a hemisphere of radius $AD = v_2 t$ in the second medium. The tangent plane CD drawn from point C over this hemisphere of radius $v_2 t$ will be the new refracted wavefront.

Let the angles of incidence and refraction be i and r respectively.

From right $\triangle ABC$, we have

$$\sin \angle BAC = \sin i = \frac{BC}{AC}$$

From right $\triangle ADC$, we have

or

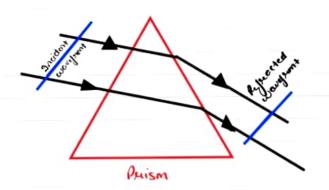
$$\sin \angle DCA = \sin r = \frac{AD}{AC}$$

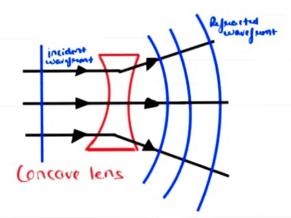
$$\therefore \frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{v_1 t}{v_2 t}$$

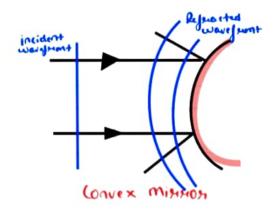
$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = {}^{1}\mu_2 \quad (a \text{ constant})$$

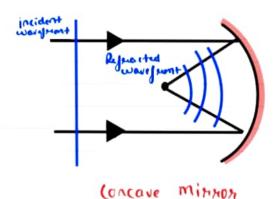
This proves Snell's law of refraction. The constant $^{1}\mu_{2}$ is called the refractive index of the second medium with respect to first medium.

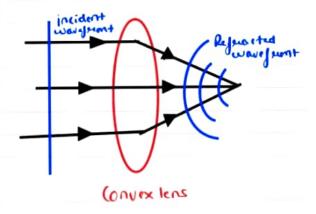
Behaviour of lens, Prism and Mither





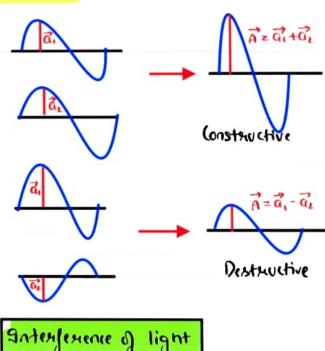






Painciple of Superposition

When a number of waves travelling through a medium superpose on each other, the resultant displacement at any point at a given instant is equal to the vector sum of the displacements due to the individual waves at that point.



Interference of light. When two light waves of the same frequency and having zero or constant phase difference travelling in the same direction superpose each other, the intensity in the region of superposition gets redistributed, becoming maximum at some points and minimum at others. This phenomenon is called interference of light.

Consent and Incohement Sources

Coherent and incoherent sources Two sources of light which continuously emit light waves of same frequency (or wavelength) with a zero or constant phase difference between them, are called coherent sources.

Two sources of light which do not emit light waves with a constant phase difference are called incoherent sources.

Conditions for obtaining two coherent sources of light:

- The two sources of light must be obtained from a single source by some method. Then the relative phase difference between the two light waves from the sources will remain constant with time.
- 2. The two sources must give monochromatic light. Otherwise, different colours will produce different interference patterns and fringes of different colours will overlap.
- 3. The path difference between the waves arriving on the screen from the two sources must not be large. It should not exceed 30 cm. Then the phase difference produced due to path difference will not be constant. There will be general illumination on the screen.

Sustained Intenjenence

The interference pattern, in which the positions of maxima and minima of intensity on the observation screen do not change with time, is called a sustained or permanent interference pattern.

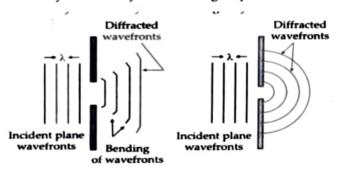
Conditions for sustained interference. The necessary conditions for obtaining a sustained and observable interference pattern of light are as follows:

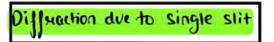
- The two sources should continuously emit waves of same frequency or wavelength
- The two sources of light should be coherent, i.e., they must vibrate either in the same phase or with a constant phase difference between them.
- For a better contrast between maxima and minima of intensity, the amplitudes of the interfering waves should be equal.
- The two sources should be narrow, otherwise interference will occur between waves of different parts of the same source and contrast will be poor.
- The interfering waves must travel nearly along the same direction.
- The sources should be monochromatic, otherwise fringes of different colours will overlap just to give a few observable fringes.



The phenomenon of bending of light around the corners of small obstacles or apertures and its consequent spreading into the regions of geometrical shadow is called diffraction of light.

Size of aperture or obstacle for observing diffraction. Suppose plane waves are made to fall on a screen having a small aperture. The waves emerging out of the aperture are observed to be slightly curved at the edges. This is diffraction. If the size of the aperture is large compared to the wavelength of the waves, the amount of bending is small [Fig. 9.20(a)]. If the size of the aperture is small, comparable to the wavelength λ of the waves, then the diffracted waves are almost spherical [Fig. 9.20(b)]. Hence the diffraction effect is more pronounced if the size of the aperture or the obstacle is of the order of the wavelength of the waves.





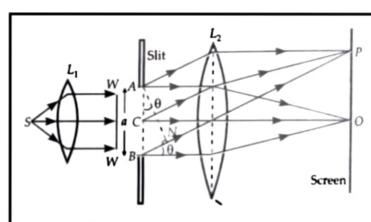


Fig. 9.21 Diffraction at a single slit.

Explanation of diffraction fringes

Central maximum. All the wavelengths going straight $(\theta = 0^{\circ})$ across the slit are focussed at the central point O of the screen, as shown in Fig. 9.23. The wavelets from any two corresponding points such as (0, 12), (2, 10), (4, 8) etc. from the two halves of the slit have zero path difference. They undergo constructive interference to produce central bright fringe.

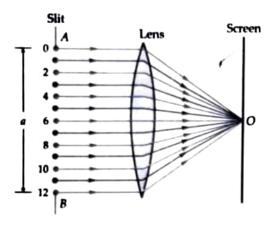
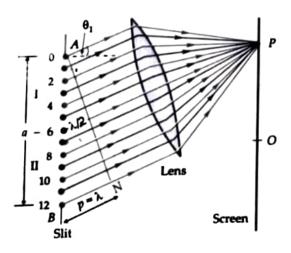


Fig. 9.23 Diffraction at angle $\theta = 0^\circ$.

First dark fringe. If angle θ is such that the path difference, $p = a \sin \theta = \lambda$, then the path difference between the rays from A and B when they reach P is λ , as shown in Fig. 9.24. If we divide the slit into two halves I and II, of 6 parts each, then obviously the wavelets form 0 and 6 will have a path difference of $\lambda/2$ or a phase difference of π . They interfere destructively. Similarly, the wavelet pairs (1, 7), (2, 8), (3, 9), (4, 10), (5, 11) and (6, 12) of the two halves will interfere destructively. Hence the condition for first dark fringe is

$$a \sin \theta = \lambda$$



First secondary maximum. Suppose the angle θ is such that the path difference, $p = a \sin \theta = 3\lambda/2$

We can divide the slit into three equal regions I, II and III, as shown in Fig. 9.25. The path difference between any two corresponding points of regions I and II will be $\frac{\lambda}{2}$ or phase difference will be π . The wavelets from

these points will interfere destructively. The wavelets from III region of the slit will contribute to some intensity forming a secondary maximum. The intensity of this maximum is much less than the central maximum. The condition for the first secondary maximum can be written as

$$a \sin \theta = \frac{3}{2} \lambda$$

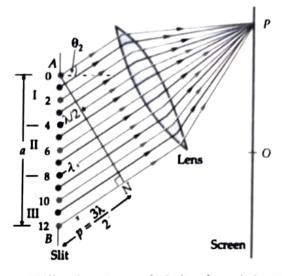


Fig. 9.25 Diffraction at an angle θ given by $a \sin \theta = 3/2 \lambda$.



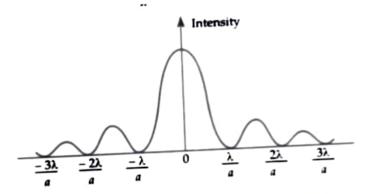
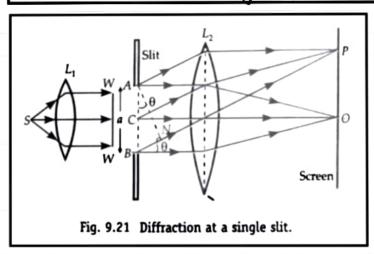


Fig. 9.22 Variation of intensity with angle θ in single slit diffraction.

Mathematical Expression of Diffraction



From above diagram, BN = Poth difference

Sine =
$$\frac{BN}{AB}$$

Then, $BN = ABSIND$
 $P = aSIND$ (: $BN = P$)

Angular position of dank suinge (minima)
As, for dank suinge
asing = nd

when Oa sind
Then,
$$\Theta = \frac{nd}{a}$$

for lentual maxima

Messe will be no path difference P=0

For first Secondary maxima
$$(P = \frac{3}{2}A)$$

$$\frac{\text{QSin}\theta}{2} = \frac{3}{2}A$$

for Second Secondary maxima (2= 5 1)

general formula for Secondary

$$\frac{\text{asin}\theta = (2n+1)d}{2}$$

maxima
When Da Sint. Then, 0= (20+1) d

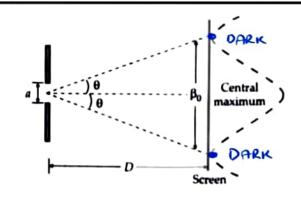


Fig. 9.26 Angular and linear widths of central maximum.

a 0 = A

$$\theta = \frac{\Lambda}{Q}$$

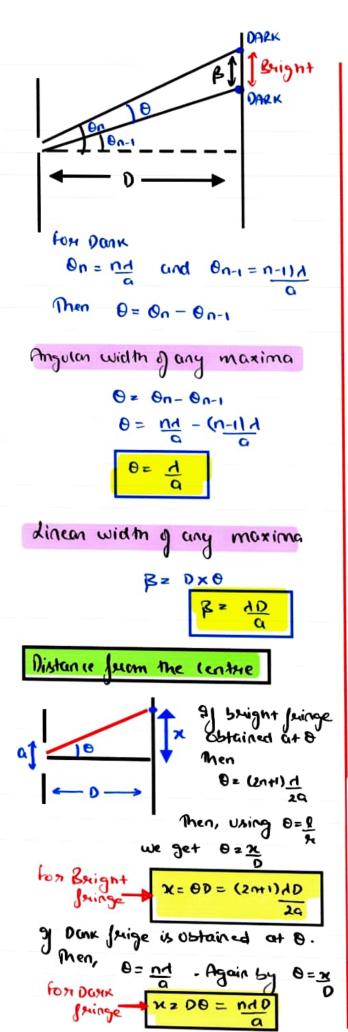
Angulan width of central maxima = 20

$$3\theta = \frac{\alpha}{34}$$

Linear width of lentral maxima= 70

$$\beta_0 = D \times 2\theta$$

$$\beta_0 = \frac{2DA}{\alpha}$$



Diffraction vs Interference

	••	
	Interference	Diffraction
1.	Interference is the result of superposition of secondary waves starting from two different wavefronts originating from two coherent sources.	Diffraction is the result of superposition of secondary waves starting from different parts of the same wavefront.
2.	All bright and dark fringes are of equal width.	The width of central bright fringe is twice the width of any secondary maximum.
3.	All bright fringes are of same intensity.	Intensity of bright fringes decreases as we move away from central bright fringe on either side.
4.	Regions of dark fringes are perfectly dark. So there is a good contrast between bright and dark fringes.	Regions of dark fringes are not perfectly dark. So there is a poor contrast between bright and dark fringes.
5	At an angle of λ/d, we get a bright fringe in the interference pattern of two narrow slits sepa- rated by a distance d.	At an angle of λ/a , we get the first dark fringe in the diffraction pattern of a single slit of width a .

fresnel and traunhoffer Diffraction

Two types of diffraction. The diffraction phenomena can be divided into two categories:

 Fresnel's diffraction. In Fresnel's diffraction, the source and screen are placed close to the aperture or the obstacle and light after diffraction appears

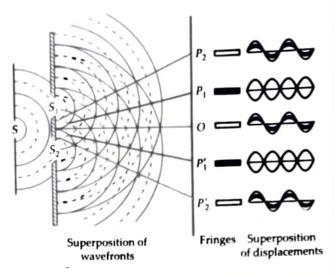
converging towards the screen and hence no lens is required to observe it. The incident wave fronts are either spherical or cylindrical.

2. Fraunhoffer's diffraction. In Fraunhoffer's diffraction, the source and screen are placed at large distances (effectively at infinity) from the aperture or the obstacle and converging lens is used to observe the diffraction pattern. The incident wavefront is planar one.

Young's Double Stit Experiment

In this experiment, a source of monochromatic light (e.g., a sodium vapour lamp) illuminates a rectangular narrow slit S, about 1 mm wide, as shown in Fig. 9.11. S_1 and S_2 are two parallel narrow slits which are arranged symmetrically and parallel to the slit S at a distance of about 10 cm from it. The separation between S_1 and S_2 is ≈ 2 mm and width of each slit is ≈ 0.3 mm. An observation screen is placed at a distance of ≈ 2 m from the two slits. Alternate bright and dark bands appear on the observation screen. These are called interference fringes. When one of the slits, S_1 or S_2 is closed, bright and dark fringes disappear and the intensity of light becomes uniform.

Explanation. Fig. 9.12 shows a section of Young's experiment in the plane of paper. According to Huygens' principle, cylindrical wavefronts emerge out from slit S, whose sections have been shown by circular arcs. The solid curves represent crests and the dotted curves represent troughs. As $SS_1 = SS_2$, these waves fall on the slits S_1 and S_2 simultaneously so that the waves spreading out from S_1 and S_2 are in the same phase. Thus S_1 and S_2 act as two coherent sources of monochromatic light. Interference takes place between the waves diverging from these sources.



At the lines leading to O, P_2 and P_2' , the crest of one wave falls over the crest of other wave or the trough of one wave falls over the trough of other wave, the amplitudes of the two waves get added up and hence the intensity ($I \propto a^2$) becomes maximum. This is called constructive interference. At the lines leading to P_1 and P_1' , the crest of one wave falls over the trough of other or the trough of one wave falls over the crest of other wave, the amplitudes of the two waves get subtracted and hence the intensity becomes minimum. This is called destructive interference. So on the observation screen, we obtain a number of alternate bright and dark fringes, parallel to the two slits.

Diffuaction Museyh Single Stit

Diffraction at a single slit. As shown in Fig. 9.21, a source S of monochromatic light is placed at the focus of a convex lens L_1 . A parallel beam of light and hence a plane wavefront WW gets incident on a narrow rectangular slit AB of width a.

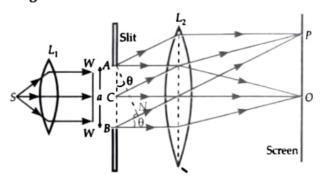


Fig. 9.21 Diffraction at a single slit.

Central maximum. All the secondary wavelets going straight across the slit AB are focussed at the central point O of the screen. The wavelets from any two corresponding points of the two halves of the slit reach the point O in the same phase, they add constructively to produce a central bright fringe. For detailed explanation of diffraction fringes, see For Your Knowledge box on page 9.32.

Calculation of path difference. Suppose the secondary wavelets diffracted at an angle θ are focussed at point P. The secondary wavelets start from different parts of the slit in same phase but they reach

the point P in different phases. Draw perpendicular AN from A on to the ray from B. Then the path difference between the wavelets from A and B will be

$$p = BP - AP = BN = AB \sin \theta = a \sin \theta$$
.

Positions of minima. Let the point *P* be so located on the screen that the path difference, $p = \lambda$ and angle $\theta = \theta_1$. Then from the above equation, we get

$$a \sin \theta_1 = \lambda$$

We can divide the slit AB into two halves AC and CB. Then the path difference between the wavelets from A and C will be $\frac{\lambda}{2}$. Similarly, corresponding to every point in the upper half AC, there is a point in the lower half CB for which the path difference is $\frac{\lambda}{2}$. Hence the wavelets from the two halves reach the point P

always in opposite phases. They interfere destructively so as to produce a minimum. Thus the condition for first dark fringe is

$$a \sin \theta_1 = \lambda$$

Similarly, the condition for second dark fringe will be

$$a \sin \theta_2 = 2\lambda$$

Hence the condition for nth dark fringe can be written as

$$a\sin\theta_n=n\lambda$$
, $n=1,2,3,...$

The directions of various minima are given by

$$\theta_n \approx \sin \theta_n = n \frac{\lambda}{a}$$

[As
$$\lambda \ll a$$
, so $\sin \theta_n \approx \theta_n$]

Positions of secondary maxima. Suppose the point P is so located that $p = \frac{3\lambda}{2}$

When
$$\theta = \theta'_1$$
, then $a \sin \theta'_1 = \frac{3}{2} \lambda$

We can divide the slit into three equal parts. The path difference between two corresponding points of the first two parts will be $\frac{\lambda}{2}$. The wavelets from these

points will interfere destructively. However, the wavelets from the third part of the slit will contribute to some intensity forming a secondary maximum. The intensity of this maximum is much less than that of the central maximum.

Thus the condition for the first secondary maximum is

$$a \sin \theta_1' = \frac{3}{2} \lambda$$

Similarly, the condition for the second secondary maximum is

$$a \sin \theta_2' = \frac{5}{2} \lambda$$

Hence the condition for nth secondary maximum can be written as

a sin
$$\theta'_n = (2n+1)\frac{\lambda}{2}$$
, $n = 1, 2, 3,$

The directions of secondary maxima are given by

$$\theta'_n \simeq \sin \theta'_n = (2n+1)\frac{\lambda}{2a}$$

The intensity of secondary maxima decreases as n increases.