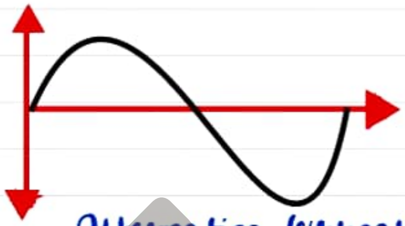
ALTERNATING CURRENT

Alternating Current

An alternating current is that current whose magnitude changes continuously with time direction reverses periodically.



Alternating current

Emf → $\mathcal{E} = \mathcal{E}_0 \sin\omega t$

Current → $I = I_0 \sin\omega t$

Direct current

Emf → $\mathcal{E}$

Current → I

here $\mathcal{E}_0$ = max Emf & I_0 = max Current.

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Important terms

Amplitude ⇒ The maximum value attained by alternating current.

Time period ⇒ The time taken by Ac current to complete one cycle

$$\text{Time taken} = \frac{\text{Angular displacement}}{\text{Angular velocity}}$$

$$\boxed{T = \frac{2\pi}{\omega}} \rightarrow \boxed{\omega = \frac{2\pi}{T}}$$

Frequency :- The no. of cycles completed in 1 sec. It is denoted by f

$$\boxed{f = \frac{1}{T} = \frac{\omega}{2\pi}} \rightarrow \boxed{\omega = 2\pi f}$$

$$\mathcal{E} = \mathcal{E}_0 \sin\omega t \xrightarrow{\omega = 2\pi f} \mathcal{E}_0 \sin 2\pi f t$$

$$\mathcal{E} = \mathcal{E}_0 \sin\omega t \xrightarrow{\omega = \frac{2\pi}{T}} \mathcal{E}_0 \sin \frac{2\pi}{T} t$$

$$I = I_0 \sin\omega t \xrightarrow{\omega = 2\pi f} I_0 \sin 2\pi f t$$

$$I = I_0 \sin\omega t \xrightarrow{\omega = \frac{2\pi}{T}} I_0 \sin \frac{2\pi}{T} t$$

Relation b/w average value & Peak value for half cycle (T/2)

We know = $I = I_0 \sin\omega t$

In small time dt, a small charge dQ will flow into wire :-

$$I = \frac{dQ}{dt}$$

So $dQ = I\,dt$

$dQ = I_0 \sin\omega t\, dt$

Integrating both sides for half cycle

$$\int dQ = \int_0^{T/2} I_0 \sin\omega t \, dt$$

$$Q = I_0 \left[-\frac{\cos\omega t}{\omega} \right]_0^{T/2} = -\frac{I_0}{\omega} \left[\cos\omega\frac{T}{2} - \cos\omega(0) \right]$$

$$Q = -\frac{I_0}{\omega} \left[\cos\frac{2\pi}{T}\frac{T}{2} - \cos 0 \right] \quad \left(\text{Put } \omega = \frac{2\pi}{T} \right)$$

$$Q = -\frac{I_0}{\omega} \left[\cos\pi - \cos 0 \right] = -\frac{I_0}{\omega}\left[-1-1\right] = -\frac{I_0}{\omega}\left[-2\right]$$

$$Q = +\frac{2I_0}{\omega} = \frac{2I_0}{2\pi} \times T \quad \left(\text{Put } \omega = \frac{2\pi}{T} \right)$$

$$Q = \frac{I_0 T}{\pi} \longrightarrow \text{value of charge for time } T$$

Now current $= \dfrac{Q}{T/2} = \dfrac{I_0 T}{\pi} \times \dfrac{1}{T/2} = \dfrac{2I_0}{\pi}$

$$\boxed{I_{avg} = \frac{2}{\pi} I_0 = 0.637 I_0}$$

Now multiplying both sides with R we get

$$(I_{avg})R = \left(\frac{2}{\pi} I_0\right) R$$

we get

$$\boxed{\mathcal{E}_{avg} = \frac{2}{\pi}\mathcal{E}_0 = 0.637\,\mathcal{E}_0}$$

Root mean square current (Rms)

Suppose an alternating current $I = I_0 \sin\omega t$ is given to a heating element. Then heat produced in small time dt is

$$dH = I^2 R\, dt$$

Then total heat produced in one cycle of AC:

$$\int dH = \int_0^T I^2 R\, dt \longrightarrow H = \int_0^T I^2 R\, dt \quad \text{--- (1)}$$

Now I_{eff} be effective value of current which will produce the same heating effect

$$H = I_{eff}^2 R T \quad \text{--- (2)}$$

Now comparing (1) & (2):-

$$I_{eff}^2 RT = \int_0^T I^2 R\, dt$$

$$I_{eff}^2 = \frac{1}{RT} R \int_0^T I^2 dt$$

$$\boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T I^2 dt}}$$

This is known as effective current or Root mean Square current

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Now $I_{rms} = I_{eff} = \sqrt{\frac{1}{T}\int_0^T I^2 dt}$

Now Putting $I = I_0 \sin \omega t$ & solving it:-

$$I_{rms}^2 = \frac{1}{T}\int_0^T (I_0 \sin\omega t)^2 dt = \frac{1}{T}\int_0^T I_0^2 \sin^2 \omega t \, dt$$

$$I_{rms}^2 = \frac{I_0^2}{T}\int_0^T \sin^2 \omega t \, dt$$

$$I_{rms}^2 = \frac{I_0^2}{T}\int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt$$

$$I_{rms}^2 = \frac{I_0^2}{2T}\left\{\int_0^T 1\, dt - \int_0^T \cos 2\omega t \, dt\right\}$$

$$I_{rms}^2 = \frac{I_0^2}{2T}\left\{\left[t\right]_0^T - \left[\frac{\sin 2\omega t}{2\omega}\right]_0^T\right\}$$

Side Note for Biology students:-

$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$2\sin^2\theta = 1 - \cos 2\theta$$
$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

If $\theta = \omega t$ Then

$$\sin^2 \omega t = \frac{1-\cos 2\omega t}{2}$$

$$I_{rms}^2 = \frac{I_0^2}{2T}\left\{(T-0) - \frac{1}{2\omega}\left(\sin 2\omega T - \sin 2\omega(0)\right)\right\}$$

$$= \frac{I_0^2}{2T}\left\{T - \frac{1}{2\omega}\left(\sin 2\times\frac{2\pi}{T}T - \sin 0\right)\right\} \quad \left(\text{here } \omega = \frac{2\pi}{T}\right)$$

$$= \frac{I_0^2}{2T}\left\{T - \frac{1}{2\omega}\left(\sin 4\pi - \sin 0\right)\right\}$$

$$= \frac{I_0^2}{2T}\left\{T - \frac{1}{2\omega}(0-0)\right\}$$

$$= \frac{I_0^2}{2T}\left\{T - 0\right\} = \frac{I_0^2}{2T}\times T = \frac{I_0^2}{2}$$

So

$$I_{rms}^2 = \frac{I_0^2}{2}$$

Then $I_{rms} = \sqrt{\frac{I_0^2}{2}} = \frac{I_0}{\sqrt{2}}$

$$\boxed{I_{rms} = \frac{1}{\sqrt{2}} I_0 = 0.707\, I_0}$$

This is the value of Rms current.

Root mean Square Emf (Rms)

Suppose an A.C emf is given to heating element. Then heat produced in time dt

$$dH = \frac{\varepsilon^2}{R}dt = \frac{(\varepsilon_0 \sin \omega t)^2}{R}dt$$

Then heat produced in one cycle

$$\int dH = \int_0^T \frac{\varepsilon_0^2 \sin^2 \omega t}{R} dt$$

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$$H = \frac{1}{R}\int_0^T \varepsilon_0 \sin^2\omega t \, dt \quad —(1)$$

Now if ε_{eff} is the effective value of emf which will produce same heating then,

$$H = \frac{\varepsilon_{eff}^2}{R} T \quad —(2)$$

Now comparing eqn (1) & (2) we get.

$$\frac{\varepsilon_{eff}^2}{R} T = \frac{1}{R}\int_0^T \varepsilon_0^2 \sin^2\omega t \, dt$$

Then

$$\varepsilon_{eff}^2 = \frac{1}{T}\int_0^T \varepsilon_0^2 \sin^2\omega t \, dt$$

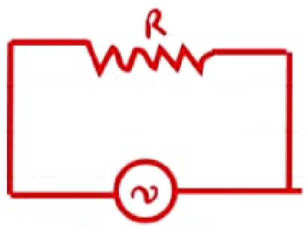
$$\boxed{\varepsilon_{eff} = \sqrt{\frac{1}{T}\int_0^T \varepsilon_0^2 \sin^2\omega t \, dt}}$$

This emf is known as rms or effective Emf

Note → You have solve this derivation.

Final Ans ⇒ $\boxed{\varepsilon_{eff} = \frac{\varepsilon_0}{\sqrt{2}} = 0.707\,\varepsilon_0}$

A.C Circuit Containing Resistor only



$\varepsilon = \varepsilon_0 \sin\omega t$

Suppose an A.C Source is connected to a Resistor as shown. Then Emf is

$$\varepsilon = \varepsilon_0 \sin\omega t$$

Using Ohm's law $\varepsilon = IR$ then

$$IR = I_0 R \sin\omega t$$

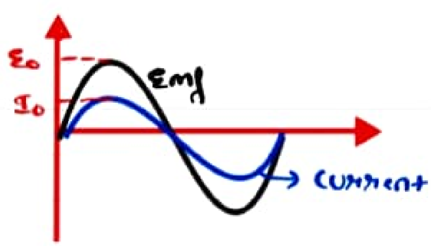
$$I = I_0 \sin\omega t$$

here I_0 is the max current.

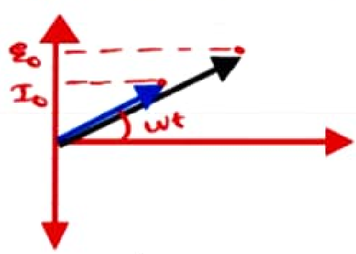
So for an Pure Resistive circuit

$\varepsilon = \varepsilon_0 \sin\omega t$ & $I = I_0 \sin\omega t$

There is no phase difference between Emf & current



Graphical diagram



Phasor diagram

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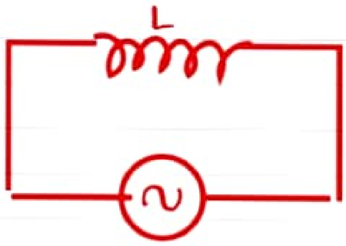
Pure Inductive circuit



$\mathcal{E} = \mathcal{E}_0 \sin\omega t$

Suppose we applied an AC emf to a pure Inductor. Now a inductor always opposes the applied Emf.

For a Pure inductive Circuit.

Applied AC Emf = Back Emf by Inductor

$$\mathcal{E}_0 \sin\omega t = L\frac{dI}{dt}$$

Then $dI = \frac{1}{L}(\mathcal{E}_0 \sin\omega t\, dt)$

Integrating both Sides

$$\int dI = \frac{1}{L}\int \mathcal{E}_0 \sin\omega t\, dt$$

$$I = \frac{1}{L}\mathcal{E}_0 \int \sin\omega t\, dt = \frac{\mathcal{E}_0}{L}\left[-\frac{\cos\omega t}{\omega}\right]$$

$$I = -\frac{\mathcal{E}_0}{\omega L}\cos\omega t$$

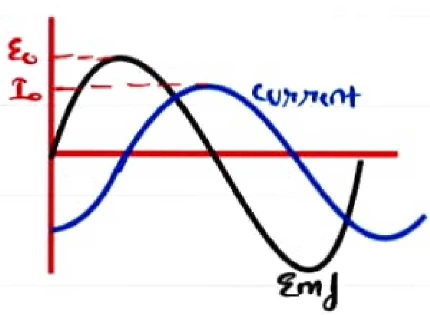
Now using $\cos\theta = \sin(90-\theta) = \sin\left(\frac{\pi}{2}-\theta\right)$

$$I = -\frac{\mathcal{E}_0}{\omega L}\left[\sin\left(\frac{\pi}{2}-\omega t\right)\right]$$

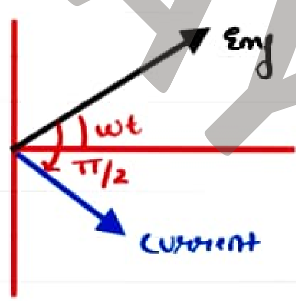
$$I = \frac{\mathcal{E}_0}{\omega L}\left[\sin\left(\omega t-\frac{\pi}{2}\right)\right]$$

$$\boxed{I = I_0\left[\sin\left(\omega t-\frac{\pi}{2}\right)\right]}$$ where $I_0 = \frac{\mathcal{E}_0}{\omega L}$

For Pure Inductive circuit $\mathcal{E} = \mathcal{E}_0 \sin\omega t$ & $I = I_0\left[\sin\left(\omega t-\frac{\pi}{2}\right)\right]$ on comparing current with Emf, we noticed current lags behind the voltage by 90°.



Graphical diagram



Phasor diagram

Inductive reactance :- we know

$$I = I_0\left[\sin\left(\omega t-\frac{\pi}{2}\right)\right]$$

where $I_0 = \frac{\mathcal{E}_0}{\omega L}$

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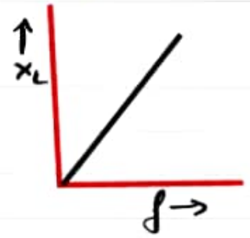
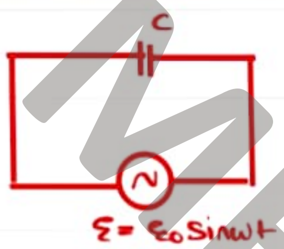
On comparing $I_0 = \frac{\varepsilon_0}{\omega L}$ with $I = \frac{\varepsilon}{R}$ (ohm law) we find that here ωL plays the role of Resistance. This type of resistance is known as Reactance. It is denoted by X_L

$$\boxed{X_L = \omega L}$$

Also $\omega = 2\pi f$ so $\boxed{X_L = 2\pi f L}$ so $X_L \propto f$



Pure Capacitive Circuit



Suppose we applied an AC Emf to Capacitor

$$\varepsilon = \varepsilon_0 \sin\omega t$$

Now we know

$$C = \frac{Q}{\varepsilon}$$

Then $Q = C\varepsilon$

To find current we apply. $I = \frac{Q}{t}$ in differential form

$$I = \frac{dQ}{dt} = \frac{d}{dt}(C\varepsilon)$$

$$I = \frac{d}{dt}[C\varepsilon_0 \sin\omega t] = C\varepsilon_0 \frac{d}{dt}(\sin\omega t)$$

$$I = C\varepsilon_0(\omega\cos\omega t) = C\omega\varepsilon_0 \cos\omega t$$

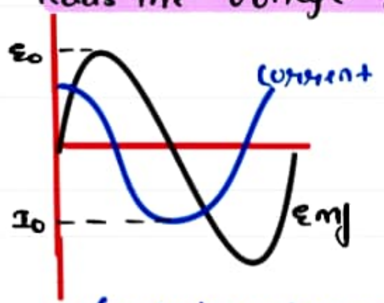
Now using $\cos\theta = \sin(90+\theta)$

so $I = C\omega\varepsilon_0 \sin(90 + \omega t)$

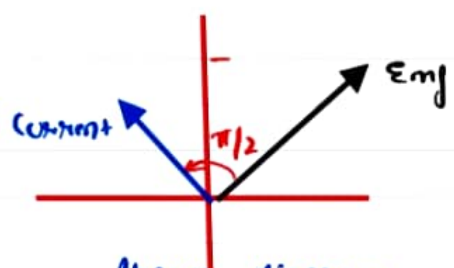
or $I = C\omega\varepsilon_0 \sin(\omega t + \frac{\pi}{2})$

$$\boxed{I = I_0 \sin(\omega t + \frac{\pi}{2})}$$ where $I_0 = C\omega\varepsilon_0$

For pure capacitive circuit, $\varepsilon = \varepsilon_0 \sin\omega t$ and $I = I_0 \sin(\omega t + \frac{\pi}{2})$ on comparing emf with current, we find that current leads the voltage emf by 90°.



Graphical diagram



Phasor diagram

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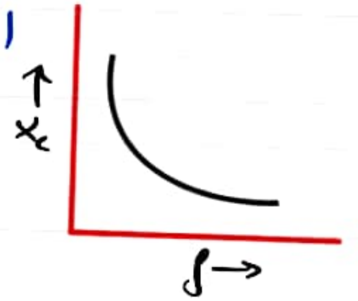
Capacitive Reactance :-

We know
$$I = I_0 \sin(\omega t + \frac{\pi}{2})$$
here $I_0 = \omega C \mathcal{E}_0 = \frac{\mathcal{E}_0}{\frac{1}{\omega C}}$

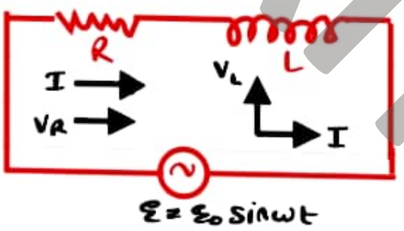
On comparing $I_0 = \frac{\mathcal{E}_0}{1/\omega C}$ with $I = \frac{\mathcal{E}}{R}$ (ohm law) we find that here $1/\omega C$ plays the role of Resistance. This type of resistance is known as reactance. It is denoted by X_C.

$$\boxed{X_C = \frac{1}{\omega C}} \quad \text{or} \quad \boxed{X_C = \frac{1}{2\pi f C}} \quad (\because \omega = 2\pi f)$$

here $X_C \propto \frac{1}{f}$

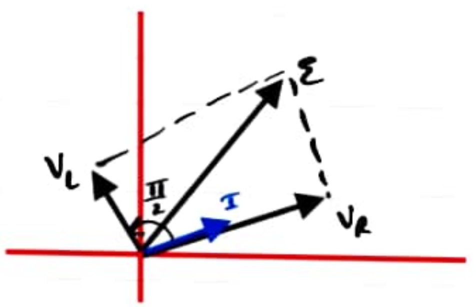


AC Circuit with Resistance and Inductance



Let us apply an A.C emf to a Inductor & Resistor connected in series.
$$\mathcal{E} = \mathcal{E}_0 \sin\omega t$$
1) Now voltage across Resistor
$$V_R = IR$$
will be in phase with current I.

2) Voltage across Inductor
$$V_L = I X_L$$
is ahead of current by $\pi/2$



Now
from parallelogram law of vector addition
$$\vec{\mathcal{E}} = \vec{V_R} + \vec{V_L}$$
$$\mathcal{E} = \sqrt{(V_R)^2 + (V_L)^2}$$
$$\mathcal{E} = \sqrt{I^2R^2 + I^2X_L^2} = \sqrt{I^2(R^2 + X_L^2)}$$
$$\mathcal{E} = I\sqrt{R^2 + X_L^2}$$
$$\boxed{I = \frac{\mathcal{E}}{\sqrt{R^2 + X_L^2}}}$$

On comparing it with $I = \mathcal{E}/R$ (ohm's law) here $\sqrt{R^2 + X_L^2}$ plays the role of effective resistance. It is known as Impedance. It is denoted by Z.

$$\boxed{Z = \sqrt{R^2 + X_L^2}} \quad \text{or} \quad \boxed{Z = \sqrt{R^2 + \omega^2 L^2}}$$

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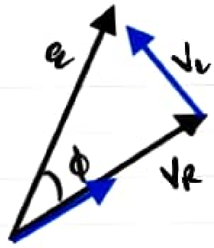


here phase angle b/w emf & current is

$$\tan\phi = \frac{P}{B} = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$$

$$\boxed{\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R}}$$

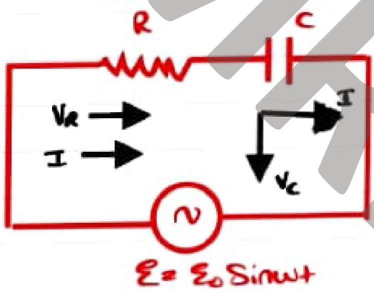
So from phasor diagram it is clear that current lags behind emf by phase angle ϕ.

$$\boxed{I = I_0 \sin(\omega t - \phi)}$$

AC Circuit with Resistor & Capacitor

Let us apply an AC emf to a Resistor & Capacitor connected in series

$$\varepsilon = \varepsilon_0 \sin\omega t$$

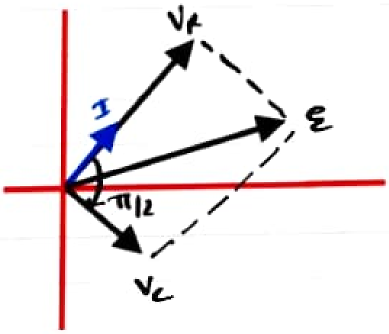


1) Now voltage across Resistor is in phase with current

$$V_R = IR$$

2) Voltage across Capacitor lags behind the current by $\pi/2$ radian

$$V_C = IX_C$$



Now from parallelogram law of addition

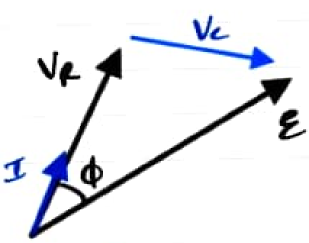
$$\varepsilon = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$\varepsilon = \sqrt{I^2(R^2 + X_C)^2} = I\sqrt{R^2 + X_C^2}$$

Now $\boxed{I = \dfrac{\varepsilon}{\sqrt{R^2 + X_C^2}}}$

Now comparing it with $I = \varepsilon / R$ (ohms law), here $\sqrt{R^2 + X_C^2}$ plays the role of effective resistance. This is known as Impedence. It is denoted by Z

here $\boxed{Z = \sqrt{R^2 + X_C^2}}$ or $\boxed{Z = \sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}}$



here phase angle b/w current & emf is

$$\tan\phi = \frac{P}{B} = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R}$$

$$\boxed{\tan\phi = \frac{X_C}{R}}$$

from phasor diagram it is clear that current leads the emf by angle ϕ

$$\boxed{I = I_0 \sin(\omega t + \phi)}$$

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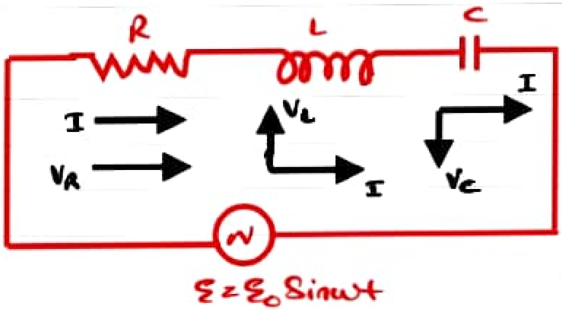
LCR Circuit



Let us apply an AC emf to RLC circuit

$$\mathcal{E} = \mathcal{E}_0 \sin\omega t$$

1) Now voltage across R will remain in phase with current

$$V_R = IR$$

2) Voltage across inductor L is ahead of current by $\pi/2$

$$V_L = I X_L$$

3) Voltage across capacitor C lags behind the current by $\pi/2$

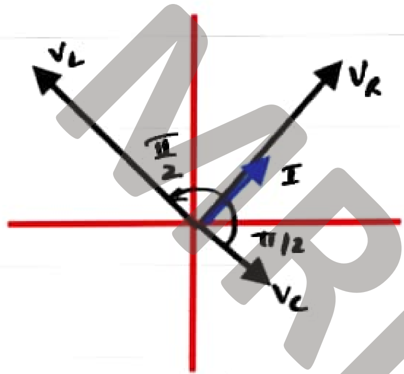
$$V_C = I X_C$$

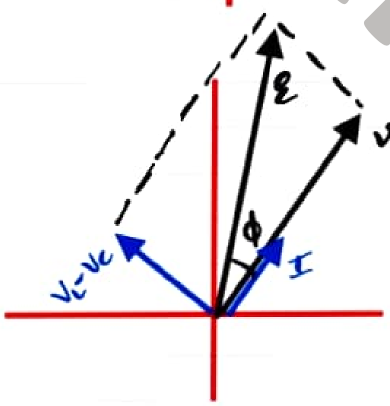
Let us suppose

$$V_L > V_C$$

Voltage across L > Voltage across C

Then phasor diagram will drawn as shown in figure.





Now from parallelogram law of addition

$$\mathcal{E} = \sqrt{(V_L - V_C)^2 + V_R^2} = \sqrt{(IX_L - IX_C)^2 + I^2R^2}$$

$$\mathcal{E} = \sqrt{I^2(X_L - X_C)^2 + I^2R^2}$$

$$\mathcal{E} = I\sqrt{(X_L - X_C)^2 + R^2}$$

$$\boxed{I = \frac{\mathcal{E}}{\sqrt{(X_L - X_C)^2 + R^2}}}$$

On comparing it with $I = \mathcal{E}/R$ (Ohm law) here $\sqrt{(X_L - X_C)^2 + R^2}$ plays the role of effective resistance. It is called Impedance. It is denoted by Z.

$$\boxed{Z = \sqrt{(X_L - X_C)^2 + R^2}}$$

OR

$$\boxed{Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}}$$

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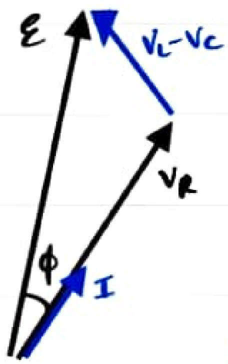


here phase angle b/w Emf & current will be

$$\tan\phi = \frac{P}{B} = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR}$$

$$\boxed{\tan\phi = \frac{X_L - X_C}{R}}$$

Impedance Triangle

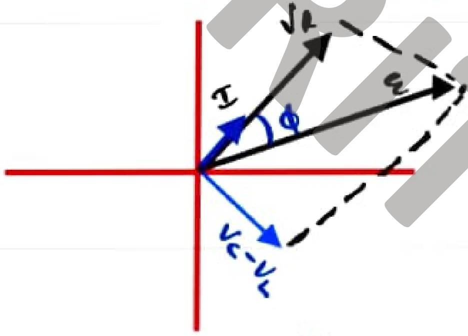
The impedance Triangle is drawn on Right Side

from phasor diagram it is clear that current lags behind Emf by angle ϕ

$$\boxed{I = I_0 \sin(\omega t - \phi)}$$

Note: when you take $V_C > V_L$ than diagram will be changed

Voltage across C > voltage across L



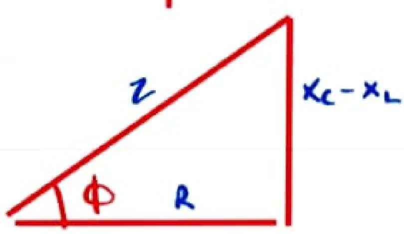
here current

$$\boxed{I = \frac{\mathcal{E}}{\sqrt{R^2 + (X_C - X_L)^2}}}$$

& Impendance is given by

$$\boxed{Z = \sqrt{R^2 + (X_C - X_L)^2}}$$

Phase angle $\boxed{\tan\phi = \frac{X_C - X_L}{R}}$



Impedance Triangle

and current leads Emf by angle ϕ

$$\boxed{I = I_0 \sin(\omega t + \phi)}$$

Resonance Condition

A LCR circuit is said to be in the resonance condition when the current through it has maximum value

we know $I = \dfrac{\mathcal{E}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

Current will be maximum when $\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ has minimum value. Then, $\left(\omega L - \frac{1}{\omega L}\right)^2 = 0$ Because $R^2 \neq 0$ (cannot)

Then $I = \dfrac{\mathcal{E}_0}{\sqrt{R^2}} = \dfrac{\mathcal{E}_0}{R}$. At this instant circuit will be pure resistive.

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And the current through the circuit will be maximum. This condition is known as Resonance condition.

We know $\left(\omega L - \frac{1}{\omega C}\right)^2 = 0 \longrightarrow \omega L - \frac{1}{\omega C} = 0$

Then, $\omega L = \frac{1}{\omega C} \longrightarrow \omega^2 = \frac{1}{LC}$

$\boxed{\omega = \frac{1}{\sqrt{LC}}}$ or $2\pi f = \frac{1}{\sqrt{LC}}$

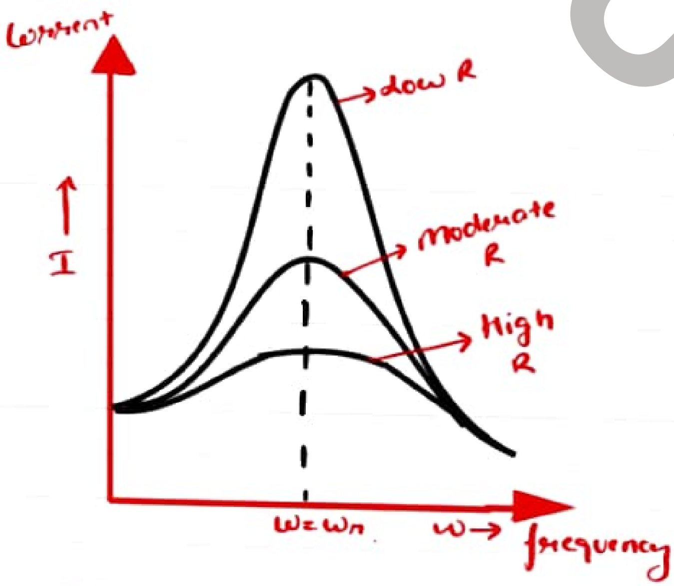
$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$

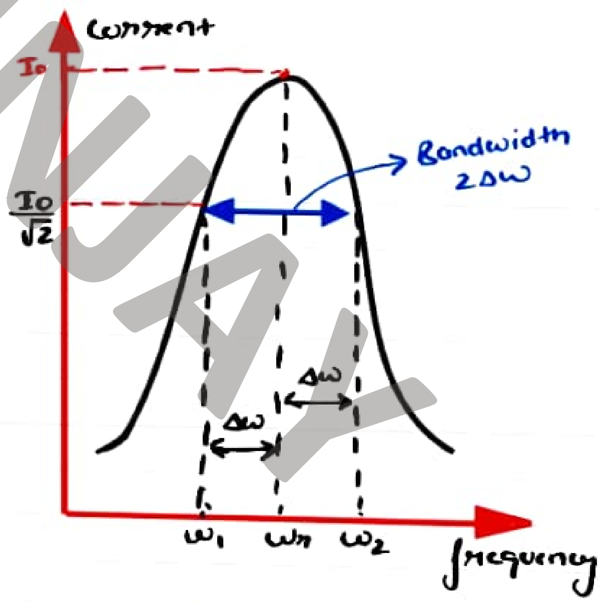
This frequency is called as resonance frequency.

Sharpness of Resonance (Quality factor)

The Quality factor is defined as the ratio of the resonant frequency to the difference in two frequencies taken on both sides of the resonant frequency such that at each frequency, the current amplitude becomes $\frac{1}{\sqrt{2}}$ times the value of current at resonant frequency.

$$\boxed{Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{2\Delta\omega} = \frac{\text{Resonant frequency}}{\text{Bandwidth}}}$$





The resonant frequency doesnot depend upon R
But the sharpness or quality depends upon R
The Peak is higher for smaller values of R (Less Noise)
The Peak is wider (lower) for high values of R (more Noise)

To get sharpness or quality of the signal, a very less value of R must be taken to get high Peak & less Noise Signal.

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Power in an AC Circuit

The rate at which electric energy is consumed in an electric circuit is called Power.

Suppose we apply an a.c Emf to a circuit.

$$\mathcal{E} = \mathcal{E}_0 \sin\omega t$$

Now let us suppose current at that time is $I = I_0 \sin(\omega t - \phi)$ where current lags behind emf by phase angle ϕ.

Now

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} = \frac{\mathcal{E} I t}{t}$$

At any instant, small Power dP is given for small time dt

$$dP = \frac{\mathcal{E} I\, dt}{dt}$$

Now total Power consumed for one cycle of A.C.

$$\int dP = \frac{\int_0^T \mathcal{E} I\, dt}{\int_0^T dt}$$

$$P = \frac{\int_0^T (\mathcal{E}_0 \sin\omega t)\left[I_0 \sin(\omega t - \phi)\right] dt}{\int_0^T dt} = \frac{\int_0^T \mathcal{E}_0 I_0 \sin\omega t \sin(\omega t - \phi)\, dt}{[t]_0^T}$$

$$P = \mathcal{E}_0 I_0 \int_0^T \frac{\sin\omega t \sin(\omega t - \phi)\, dt}{(T - 0)}$$

Now multiplying & dividing eqn by ②:-

$$P = \frac{\mathcal{E}_0 I_0}{2T} \int_0^T 2 \sin\omega t \sin(\omega t - \phi)\, dt$$

Now using $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$$P = \frac{\mathcal{E}_0 I_0}{2T} \int_0^T \left\{\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)\right\} dt$$

$$P = \frac{\mathcal{E}_0 I_0}{2T} \int_0^T \left\{\cos\phi - \cos(2\omega t - \phi)\right\} dt$$

$$P = \frac{\mathcal{E}_0 I_0}{2T} \left\{\int_0^T \cos\phi\, dt - \int_0^T \cos(2\omega t - \phi)\, dt\right\}$$

$$P = \frac{\mathcal{E}_0 I_0}{2T} \left\{\cos\phi \int_0^T dt - \int_0^T \cos(2\omega t - \phi)\, dt\right\}$$

$$P = \frac{\mathcal{E}_0 I_0}{2T} \left\{\cos\phi\, (t)_0^T - \left[\frac{\sin(2\omega t - \phi)}{2\omega}\right]_0^T\right\}$$

$$P = \frac{\mathcal{E}_0 I_0}{2T} \left\{\cos\phi\, (T - 0) - \frac{1}{2\omega}\left[\sin(2\omega T - \phi) - \sin(0 - \phi)\right]\right\}$$

$$P = \frac{\mathcal{E}_0 I_0}{2T} \left\{T \cos\phi - \frac{1}{2\omega}\left[\sin\left(2 \times \frac{2\pi}{T} \times T - \phi\right) - (\sin -\phi)\right]\right\}$$

$$P = \frac{\mathcal{E}_0 I_0}{2T} \left\{T \cos\phi - \frac{1}{2\omega}\left[\sin(4\pi - \phi) + \sin\phi\right]\right\}$$

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Now using $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$$P = \frac{\varepsilon_0 I_0}{2T}\left\{T\cos\phi - \frac{1}{2\omega}\left[\sin 4\pi\cos\phi - \cos 4\pi\sin\phi + \sin\phi\right]\right\}$$

$$P = \frac{\varepsilon_0 I_0}{2T}\left\{T\cos\phi - \frac{1}{2\omega}\left[0\times\cos\phi - 1\times\sin\phi + \sin\phi\right]\right\}$$

$$P = \frac{\varepsilon_0 I_0}{2T}\left\{T\cos\phi - \frac{1}{2\omega}\left[0 - \sin\phi + \sin\phi\right]\right\}$$

$$P = \frac{\varepsilon_0 I_0}{2T}\left\{T\cos\phi - \frac{1}{2\omega}\times 0\right\} = \frac{\varepsilon_0 I_0}{2T}\times T\cos\phi$$

$$P = \frac{\varepsilon_0 I_0}{2}\cos\phi = \frac{\varepsilon_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}}\cos\phi$$

$$\boxed{P = \varepsilon_{rms} I_{rms}\cos\phi}$$

Special Cases:-

1) **Pure Resistive circuit** = for pure resistive circuit phase difference = 0

$\phi = 0$ so $P = I_{rms}\,\varepsilon_{rms}\cos 0$

$$\boxed{P = I_{rms}\,\varepsilon_{rms}}$$ $(\because \cos 0 = 1)$

2) **Pure Inductive circuit** = for pure Inductive circuit, there is a phase difference of $\pi/2$ so $\phi = \pi/2$

$P = I_{rms}\,\varepsilon_{rms}\cos 90$

$$\boxed{P = 0}$$ $(\because \cos 90 = 0)$

3) **Pure capacitive circuit** = for pure capacitive circuit, there is a phase difference of $\pi/2$, so $\phi = \pi/2$

$P = I_{rms}\,\varepsilon_{rms}\cos 90$

$$\boxed{P = 0}$$ $(\because \cos 90 = 0)$

4) **LCR circuit** = If there is a phase difference of ϕ.

Then $$\boxed{P = \varepsilon_{rms} I_{rms}\cos\phi}$$ where $\phi = \tan^{-1}\frac{X_L - X_C}{R}$

5) **In Resonance condition** = In resonance $X_L = X_C$

Then $\phi = \tan^{-1}\frac{(X_L - X_C)}{R} = \tan^{-1}\frac{(0)}{R} = \tan^{-1}(0)$

Then $\phi = 0$

So Power

$P = \varepsilon_{rms} I_{rms}\cos 0$

$$\boxed{P = \varepsilon_{rms} I_{rms}}$$ $(\because \cos 0 = 1)$

This entire Power is dissipated across the Resistance R.

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Wattless Current

The current in an AC circuit is said to be wattless current when the average power consumed in such a circuit is zero. Such current is also called as Idle current.

The formula for calculating wattless current is

$$\boxed{P = \mathcal{E} I \cos\phi}$$

here P = Power consumed by circuit
I = current flowing through circuit.
$\mathcal{E}$ = emf applied to the circuit.
ϕ = phase angle b/w emf & current.

when the circuit is only Inductor or capacitor, in that condition phase difference between voltage & current is 90°.

So power consumption is

$P = \mathcal{E} I \cos 90°$ But $\cos 90 = 0$

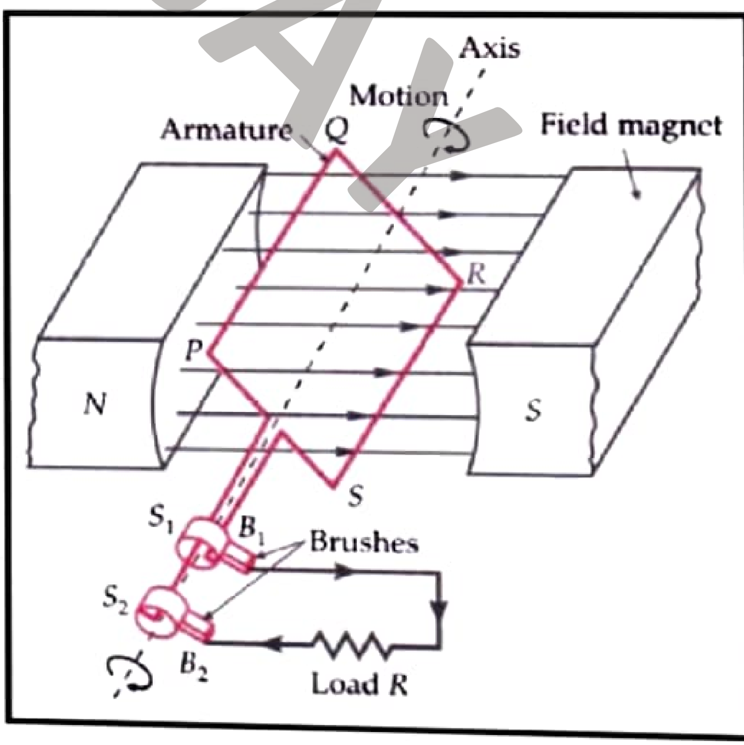
$\boxed{P = 0}$ watt

So clearly there is no power consumption.
This is called as wattless current

AC Generator

A generator (dynamo) is a device which convert mechanical energy into electrical energy.

Principal :- It works on the principal of electromagnetic Induction. when a closed coil is rotated in a uniform magnetic field the magnetic flux linked with the coil changes and an induced emf is set up in it.



Construction :-

1) **Field magnet :-** It is a permanent magnet. It produce strong magnetic field which passes through the coil.

2) **Armature :-** It consist of a rectangular coil PQRS having a large no. of turns of insulated copper wire wound on a soft iron core. The armature can be rotated inside the magnetic field

3) **Slip rings :-** The two ends of the armature coil are connected to two coaxial brass rings S_1 and S_2 called slip rings. As the armature rotates, the slip rings also rotate about the same axis of rotation.

4) **Brushes :-** Two flexible graphite brushes are slightly pressed against the two slip rings. They help to maintain the electrical connection between the coil & the external supply.

5) **Source of energy :-** The armature coil is rotated about its axis with the help of turbine or any other device connected to it.

Working :- As the armature rotates, the magnetic flux linked with it changes so an induced current flows through it. Suppose initially coil PQRS is kept in vertical position in the magnetic field & it is rotated in clockwise direction. The side PQ moves downward and SR moves upward. Acc. to fleming right hand rule, the induced current flows from Q to P and from S to R. So in first half rotation current flows in direction SRQP.

During second half rotation, the side PQ moves upward & SR moves downward. Then acc. to fleming right hand rule, the induced current flows from P to Q and from R to S. Thus the direction of the current is reversed in each half cycle & we get an Alternating current.

Mathematical Expression :-

Let N = No. of turns of coil

A = Area of each coil

B = magnetic field

Now we know $\phi = BA\cos\theta$

where θ is angle between magnetic field and Area vector

Now

$$\phi = BA\cos\theta \quad \text{Then} \quad \varepsilon = -N\frac{d\phi}{dt} = -N\frac{d}{dt}(BA\cos\theta)$$

$$\varepsilon = -NBA\frac{d}{dt}(\cos\theta) = -NBA\frac{d}{dt}(\cos\omega t) \qquad (\because \theta = \omega t)$$

$$\varepsilon = -NBA\omega(-\sin\omega t) = +NBA\omega\sin\omega t$$

$$\boxed{\varepsilon = \varepsilon_0 \sin\omega t}$$ where $\varepsilon_0 = NBA\omega$ = max value of emf

Dividing both by R :- $\frac{\varepsilon}{R} = \frac{\varepsilon_0}{R}\sin\omega t$

$$\boxed{I = I_0 \sin\omega t}$$ This is the expression for AC current.

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TRANSFORMER

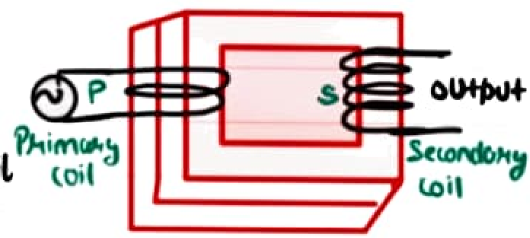
A transformer is an electrical device used for converting low AC voltage into high AC voltage and vice versa.
If the output voltage increases, it is called step up transformer.
If the output voltage decreases, it is called a step down transformer

Principal-

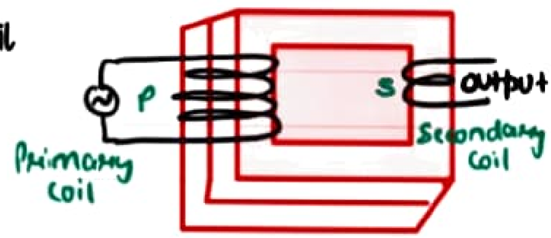
it work on the principle of mutual induction, when a changing current is passed through the primary coil and induced EMF is set up in the secondary coil

Construction-

A transformer consist of two coils of insulated copper wire having different number of turns and wound on the same a soft iron core The coil P to which electrical energy supplied is called primary coil and the coil S from which the energy is taken is called secondary coil. Because of high permeability of soft iron core the entire magnetic flux produced by the primary coil passes through the secondary coil and an induced EMF is set up in the secondary coil



Step-up Transformer

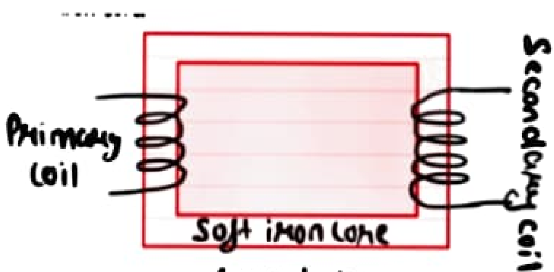


Step down Transformer

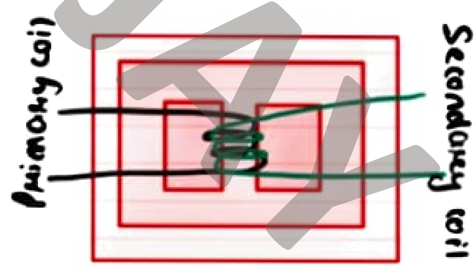
Types of transformer-

Core type- in the core type transformer the primary and the secondary coil are wound on Opposite sides of the soft iron core.

Shell type- in the shell type transformer the primary and the secondary coil are wound one over the another on the same side of the iron core.



Core type



Shell type

Working-

As the alternating current flows through the primary coil, it generates an alternative magnetic flux in the core which also passes through the secondary coil. This changing flux set up an induced EMF in the secondary coil. If there is no leakage of the magnetic flux, then Flux linked with the primary coil will always equal to The flux which passes through secondary coil.

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Mathematical Part

Let N_1 & N_2 be the number of turns in the primary and secondary coil respectively. Then at any instant, flux through primary will always equal to flux through secondary coil. So let flux ϕ.

Induced emf in primary coil, $\varepsilon_1 = -N_1 \frac{d\phi}{dt}$

Induced emf in secondary coil, $\varepsilon_2 = -N_2 \frac{d\phi}{dt}$

on dividing both $\boxed{\frac{\varepsilon_1}{\varepsilon_2} = \frac{N_1}{N_2}}$ or $\boxed{\frac{\varepsilon_2}{\varepsilon_1} = \frac{N_2}{N_1}}$

Then the ratio $\frac{N_2}{N_1}$ is called the transformer ratio.

In a step up transformer, $N_2 > N_1$

Then $\varepsilon_2 = \varepsilon_1 \frac{N_2}{N_1}$ as $N_2 > N_1$, then fraction $\frac{N_2}{N_1} > 1$

Thus ε_2 will always greater then ε_1 $(\varepsilon_2 > \varepsilon_1)$

In a step down transformer, $N_1 > N_2$

Then $\varepsilon_2 = \varepsilon_1 \frac{N_2}{N_1}$ But here $\frac{N_2}{N_1} < 1$

Thus ε_2 will be less then ε_1 $(\varepsilon_2 < \varepsilon_1)$

Now from law conservation of energy

Input Power = output Power

So, $\varepsilon_1 I_1 = \varepsilon_2 I_2$

$\frac{\varepsilon_2}{\varepsilon_1} = \frac{I_1}{I_2}$ which gives $\varepsilon_2 \propto \frac{1}{I_2}$

So a step up transformer step up the voltage but step down the current by same ratio. So total power remain conserved

Similarly, step down transformer, step down the voltage but step up the current by same ratio.

The efficiency of transformer is defined as

$$\boxed{\eta = \frac{\text{Power output}}{\text{Power input}} \times 100\%}$$

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Losses in transformer

Copper loss-some energy is lost due to the heating of copper wires as per joules law of heating. This power loss can be minimised by using thick copper wire of low resistance.

Eddie current loss-the alternative magnetic flux induces Eddy current in the iron core Which leads to some energy losses in the form of heat. This loss can be reduced by using laminated core

Hysteresis loss-the alternative current Magnetise and demagnetise the iron core many times due to which some energy is lost in the form of heat. This is called hysteresis loss and can be minimised by using material of Low hysteresis loss

Flux leakage-the magnetic flux produced by the primary coil may not fully pass through the secondary coil. some of the flux may leak into the air. This loss can be prevented by using a good iron core for both the loop

Humming loss-as the transformer works, its core length and and shortens during each cycle of alternative voltage. this phenomena is called magnetostriction which gives rise to a humming sound so some energy is lost in the form of sound

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