# ELECTRIC CHARGES AND FIELD

## Electric charge

Electric charge is an intrinsic property of particle of matter which give rise to electric force between those particles.

Electric charge is a scalar quantity.

SI unit of charge is coulomb (c)

A proton has (+) positive charge and an electron has (-) negative charge

charge on them → $\boxed{e = 1.6 \times 10^{-19} \text{ coulomb}}$

## Basic properties of charges

1) **Additivity of charge** = Additivity of electric charge means that the total charge of a system is the algebric sum of all the individual charges located at different points.

$$\boxed{Q = q_1 + q_2 + q_3 - - - + q_n}$$

2) **Quantization of charge** = The total charge of a body is always an integral multiple of a charge.

$$\boxed{Q = ne}$$ where $n = 0, \pm 1, \pm 2 - - -$ etc

3) **Conservation of charge** = 1) The total charge of an isolated system remains constant

2) The electric charge can neither be created nor destroyed, they can only be transferred from one body to another.

## Coulomb's law of electric force

The force of attraction or repulsion between two charges is

1) directly proportional to the product of magnitude of charges

$$F \propto q_1 \times q_2$$

2) inversly proportional to the square of distance between them.


$$F \propto 1/r^2$$

Combining both

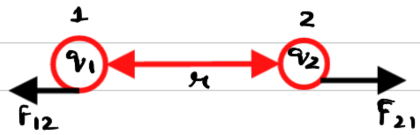
$$F \propto \frac{q_1 \times q_2}{r^2}$$

Now $\boxed{F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}}$ here $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$

& $\varepsilon_0$ = permittivity of free space

## Coulomb's law in vector form

Let $F_{21}$ = force on charge 2 due to charge 1
$F_{12}$ = force on charge 1 due to charge 2




Now from coulomb's law

$$F_{21} = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}$$

In vector form

$$\boxed{\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{12}}$$

or

$$\boxed{\vec{F}_{21} = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^3}\vec{r}_{12}}$$

$\Rightarrow$ here $\hat{r}_{12}$ is a unit vector. It tells the direction of force. The direction of force is from charge 1 to 2

Similarly

$$\boxed{\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{21}} \quad \text{or} \quad \boxed{\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^3}\vec{r}_{21}}$$

## Principle of Superposition

It states that when a number of charge are present, the total force on a given charge is the vector sum of the force exerted on it due to all other charges.

The force between two charges is not affected by the presence of other charges.

## Force between multiple charges

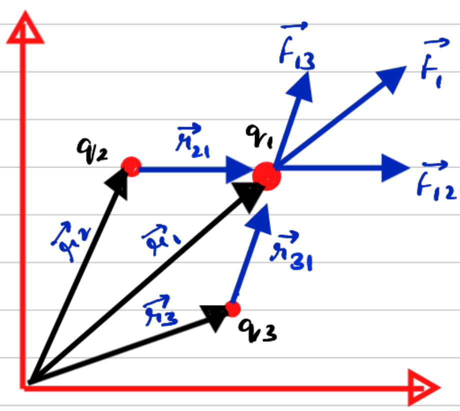
According to principle of superposition, the total force on charge $q_1$ is

$$\boxed{\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots \ldots \vec{F}_{1N}}$$

here $\vec{F}_{12}$ = force on charge 1 due to charge 2
$\vec{F}_{13}$ = force on charge 1 due to charge 3

Similarly

$\vec{F}_{1N}$ = force on charge 1 due to charge N




Now In vector form:-

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{(r_{21})^3}\vec{r}_{21} \quad \text{—(1)}$$

from triangle law of addition

$$\vec{r}_1 = \vec{r}_2 + \vec{r}_{21}$$

Then $\vec{r}_{21} = \vec{r}_1 - \vec{r}_2$

Then eqn (1):-

$$\boxed{\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|r_1 - r_2|^3}\,\vec{r}_1 - \vec{r}_2} \quad \text{—(2)}$$

Similarly
$$\boxed{\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{|\vec{r}_1-\vec{r}_3|^3}\,\vec{r}_1-\vec{r}_3} \longrightarrow (3)$$

Total force on charge 1 is
$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} - - - + \vec{F}_{1N}$$
using eqn (2) & (3)

$$\vec{F}_1 = \left[\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_1-\vec{r}_2|^3}\,\vec{r}_1-\vec{r}_2\right] + \left[\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{|\vec{r}_1-\vec{r}_3|^3}\,\vec{r}_1-\vec{r}_3\right] + - - - + - - - - + \left[\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_N}{|\vec{r}_1-\vec{r}_N|^3}\,\vec{r}_1-\vec{r}_N\right]$$

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0} q_1\left\{\left[\frac{q_2}{|\vec{r}_1-\vec{r}_2|^3}\,\vec{r}_1-\vec{r}_2\right] + \left[\frac{q_3}{|\vec{r}_1-\vec{r}_3|^3}\,\vec{r}_1-\vec{r}_3\right] + - - - - + - - - + \left[\frac{q_N}{|\vec{r}_1-\vec{r}_N|^3}\,\vec{r}_1-\vec{r}_N\right]\right\}$$

$$\boxed{\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_0}\sum_{x=2}^{x=N}\frac{q_i}{|\vec{r}_1-\vec{r}_x|^3}\,\vec{r}_1-\vec{r}_x}$$

Thus force on any $a^{th}$ charge :-
$$\boxed{\vec{F}_a = \frac{q_a}{4\pi\varepsilon_0}\sum_{\substack{x=1\\ x\neq a}}^{x=N}\frac{q_x}{|\vec{r}_a-\vec{r}_x|^3}\,\vec{r}_a-\vec{r}_x}$$

**Electric field** The electric field at a point is defined as the force experienced by a unit positive test charge placed at that point.
$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

It is a vector quantity. Electric field is from +ve charge towards -ve.

**Electric field due to a point charge**

Consider a charge Q is placed at Point O. we have to find electric field at point P. Let us put a test charge $q_0$ on P:-

Now force on $q_0$:-


$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q q_0}{r^2}$$

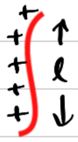
$$\text{Now } E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q q_0}{r^2}\times\frac{1}{q_0}$$

Then $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$

# Continuous charge distribution

## 1) Linear charge distribution ($\lambda$)

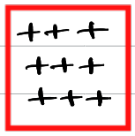
Charge stored per unit length of a wire




so $\boxed{\lambda = \frac{Q}{l}}$

## 2) Surface charge distribution ($\sigma$)

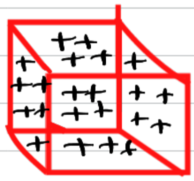
charge stored per unit Area.



so $\boxed{\sigma = \frac{Q}{A}}$

## 3) Volume charge distribution ($\rho$)

charge stored per unit volume

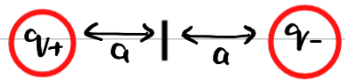


$\boxed{\rho = \frac{Q}{V}}$

## Electric dipole

A pair of equal and opposite charges separated by small distance is called electric dipole

**Dipole moment ⇒** It is equal to product of any charge with distance between the two charges.

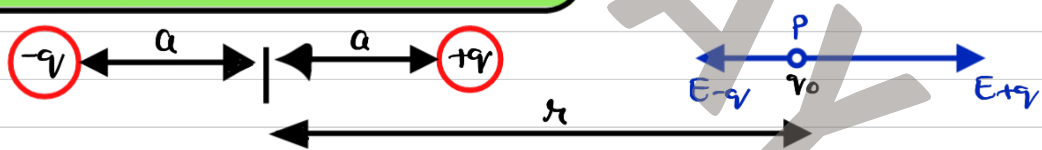



It is denoted by p.

$\boxed{p = q \times 2a}$

It is a vector quantity.
Its direction is from negative (-ve) to positive charge (+)
Its direction is opposite to that of electric field.

## Electric field at axial point due to a dipole




Consider a test charge is kept at point P.

Now $E_{+q}$ = Electric field at point P due to $+q$ charge

$E_{-q}$ = Electric field at point P due to $-q$ charge

$$\vec{E}_{axial} = \vec{E}_{+q} + \vec{E}_{-q}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{+q}{(r-a)^2} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r+a)^2}$$

$$E_{axial} = \frac{1}{4\pi\varepsilon_0} q \left\{ \frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right\}$$

$$E_{axial} = \frac{1}{4\pi\varepsilon_0} q \left[ \frac{(r+a)^2 - (r-a)^2}{(r-a)^2 (r+a)^2} \right]$$

$$E_{axial} = \frac{1}{4\pi\varepsilon_0} q \left\{ \frac{r^2+a^2+2ar - r^2 - a^2 + 2ar}{(r^2-a^2)^2} \right\}$$

$$= \frac{1}{4\pi\varepsilon_0} q \left[ \frac{4ar}{(r^2-a^2)^2} \right] = \frac{1}{4\pi\varepsilon_0} \frac{(q \times 2a) \times 2r}{(r^2-a^2)^2}$$

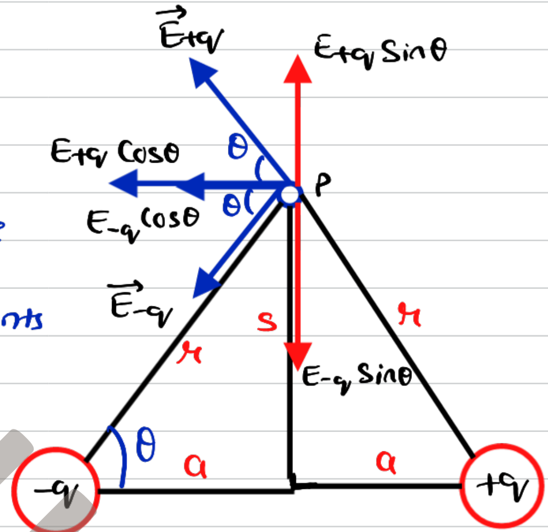
$$E_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{(p)(2r)}{(r^2-a^2)^2}$$ where $p = q \times 2a$ = dipole moment

$$\boxed{E_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{2pr}{(r^2-a^2)^2}}$$ (towards Right)

Net electric field at Point P is in the direction of dipole moment.

In vector form $$\boxed{\vec{E}_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{2pr}{(r^2-a^2)^2} \hat{p}}$$ where $\hat{p}$ is a unit vector & it is towards right.

## Electric field at equatorial point




Consider a test charge is kept at Point P

Now,

$E_{+q}$ = Electric field at Point P due to +q charge

$E_{-q}$ = Electric field at Point P due to −q charge

Now Both $E_{+q}$ and $E_{-q}$ will have two Components

for $E_{+q} \rightarrow E_{+q} \cos\theta$ & $E_{+q} \sin\theta$

for $E_{-q} \rightarrow E_{-q} \cos\theta$ & $E_{-q} \sin\theta$

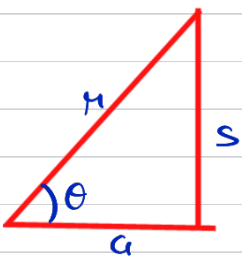
from diagram, $E_{+q} \sin\theta$ & $E_{-q} \sin\theta$ are in opposite direction. so they will cancel each other out.

Then net electric field at Point P will be

$$E_{equ} = E_{-q} \cos\theta + E_{+q} \cos\theta$$

$$E_{equ} = 2E\cos\theta$$

$$E_{equ} = 2 \times \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \cos\theta$$




$\cos\theta = \frac{B}{H} = \frac{a}{r}$

Also by Pythagoras:-

$r^2 = s^2 + a^2$

$r = \sqrt{s^2 + a^2}$

Now Putting Values of $r^2$ & $\cos\theta$

$$E_{equ} = 2 \times \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \times \frac{a}{r}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q \times 2a}{(s^2+a^2)} \frac{1}{\sqrt{s^2+a^2}}$$

$$\boxed{E_{equ} = \frac{1}{4\pi\varepsilon_0} \frac{p}{(s^2+a^2)^{3/2}}}$$ here $p$ = dipole moment, $p = q \times 2a$

here electric field at P is opposite to that the dipole moment. so In vector form

$$E_{equ} = \frac{1}{4\pi\varepsilon_0} \frac{p}{(s^2+a^2)^{3/2}} (-\hat{p})$$

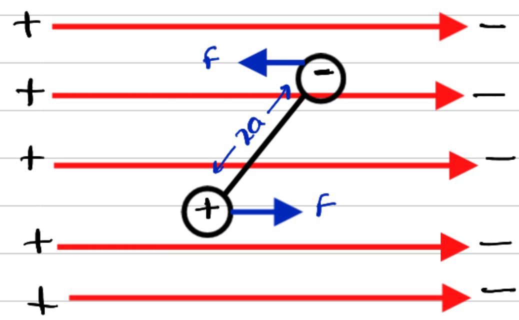
$$\boxed{E_{equ} = -\frac{1}{4\pi\varepsilon_0}\frac{p}{(s^2+a^2)^{3/2}}\hat{p}}$$

here $\hat{p}$ is a unit vector
it is toward left direction.

## Torque on a Dipole in electric field

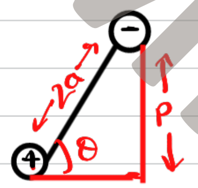
when a dipole is kept inside an electric field the dipole experience a force

$$F = qE \quad \text{--- (1)}$$




But as force expirienced by the positive charge is equal and opposite to the force experienced by a negative charge
so a Torque acts on the dipole :-

Torque = force × perpendicular distance




here $\sin\theta = \frac{P}{H} = \frac{P}{2a}$

$\boxed{P = 2a\sin\theta}$ —— (2)

$$\tau = F \times P$$

from eqn (1) & (2)

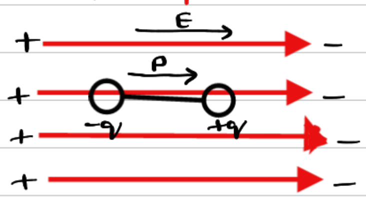
$$\tau = qE \times 2a\sin\theta$$

$$\tau = (q \times 2a)\, E\sin\theta$$

$\boxed{\tau = pE\sin\theta}$ or $\boxed{\vec{\tau} = \vec{p}\times\vec{E}}$

### Special Cases:-

**1) Stable equilibrium**




here $\vec{p}\rightarrow$ $\vec{E}\rightarrow$

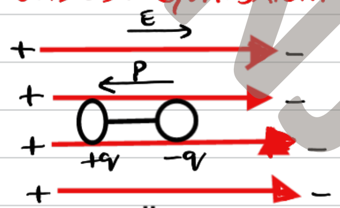
here $\theta = 0$

So, $\sin\theta = \sin 0 = 0$

$\tau = pE\sin 0 = 0$

$\boxed{\tau = 0}$ (min)

**2) Unstable equilibrium**




here $\overleftarrow{\vec{p}} \cdot \overrightarrow{\vec{E}}$ 180°

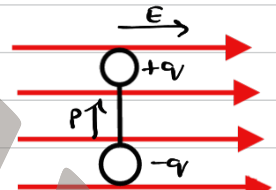
here $\theta = 180°$

So, $\sin\theta = \sin 180 = 0$

$\tau = pE\sin 180 = 0$

$\boxed{\tau = 0}$ (min)

**3) Maximum Torque**




here $\theta = 90°$

So, $\sin\theta = \sin 90 = 1$

$\tau = pE\sin 90 = pE$

$\boxed{\tau = pE}$ (max)

Note:- If dipole is placed in a uniform electric field, then it will have only rotational motion. (only Torque will act)

If the dipole is placed in a non-uniform electric field, then it will have both rotational as well as linear motion.
(Both Torque & force will act).

## Electric field lines

### Properties of Electric Lines of Force

1. The lines of force are continuous smooth curves without any breaks.
2. The lines of force start at positive charges and end at negative charges – they cannot form closed loops. If there is a single charge, then the lines of force will start or end at infinity.
3. The tangent to a line of force at any point gives the direction of the electric field at that point.
4. No two lines of force can cross each other.
5. The lines of force are always normal to the surface of a conductor on which the charges are in equilibrium.

   Reason. If the lines of force are not normal to the conductor, the component of the field $\vec{E}$ parallel to the surface would cause the electrons to move and would set up a current on the surface. But no current flows in the equilibrium condition.
6. The lines of force have a tendency to contract lengthwise. This explains attraction between two unlike charges.
7. The lines of force have a tendency to expand laterally so as to exert a lateral pressure on neighbouring lines of force. This explains repulsion between two similar charges.
8. The relative closeness of the lines of force gives a measure of the strength of the electric field in any region. The lines of force are
   (i) close together in a strong field.
   (ii) far apart in a weak field.
   (iii) parallel and equally spaced in a uniform field.
9. The lines of force do not pass through a conductor because the electric field inside a charged conductor is zero.

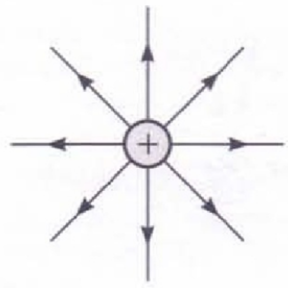



Fig. 1.74 Field lines of a positive point charge.

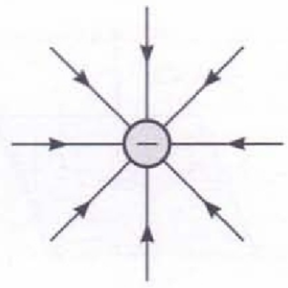



Fig. 1.75 Field lines of a negative point charge.

Electric field due to a point charge

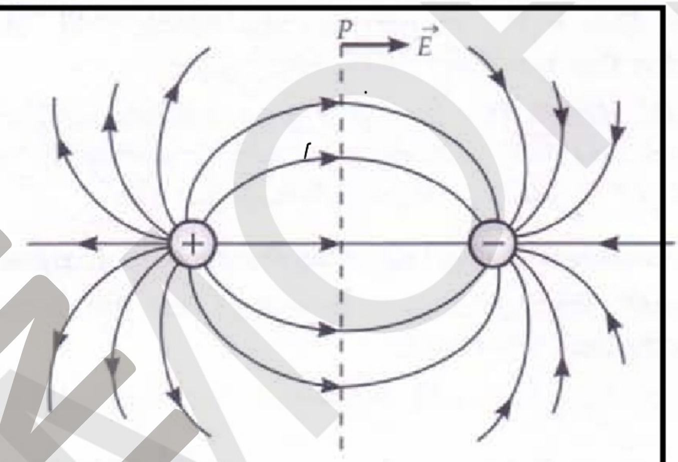



Fig. 1.76 Field lines of an electric dipole.

Electric field due to a dipole

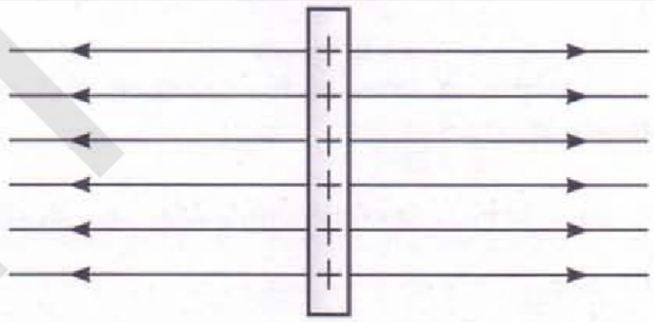


Fig. 1.78 Field pattern of a positively charged plane conductor.

Uniform electric field.

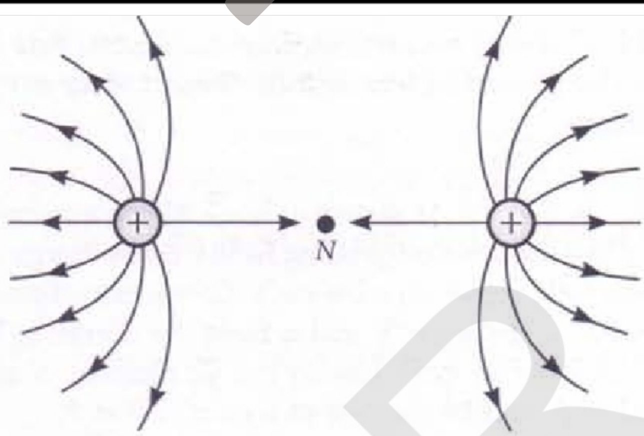
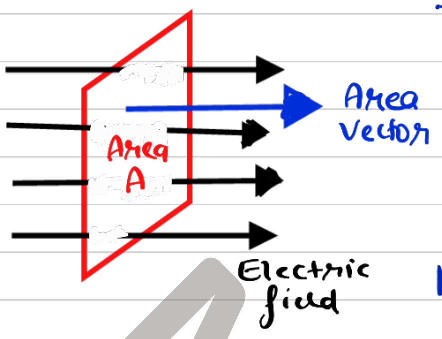



Fig. 1.77 Field lines of two equal positive charges.

Electric field due to 2 positive charges

## Electric flux

Electric flux through a given area is the measure of the total number of electric field lines passing normally through that area.



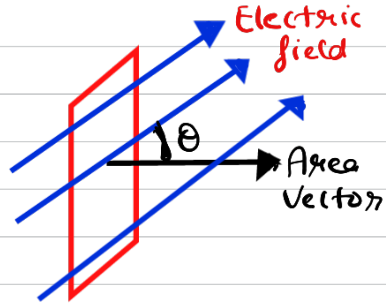

It is denoted by $\phi$

Let $E$ = Electric field

$A$ = Area

$\phi = E\cos\theta \cdot A$

$$\boxed{\phi = EA\cos\theta}$$




here $\theta$ is the angle between electric field & Area vector

Note = Area vector is any vector which is perpendicular to that Area.

$$\boxed{\vec{\phi} = \vec{E} \cdot \vec{A}}$$

$$\boxed{\text{SI unit} = Nm^2 C^{-1}}$$

## Gauss Theorem

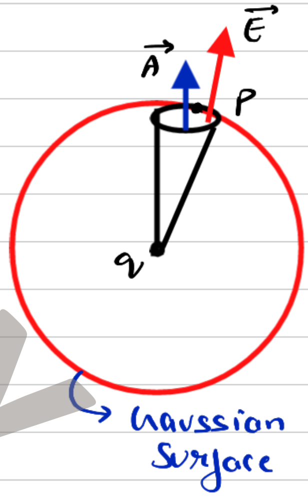
Gauss theorem states that the total flux through a closed surface is $1/\varepsilon_0$ times the net charge enclosed by the closed surface.

$$\boxed{\phi = \oint \vec{E} \cdot \vec{ds} = q/\varepsilon_0}$$

**Proof:-**

Consider a charge $q$ is placed in the centre of the Gaussian Surface. The value of electric field at a distance $r$ is

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$




Now total electric field over whole Gaussian Surface is given by. flux over whole Surface

$$\phi = \oint E.ds = E\oint ds = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \int ds$$

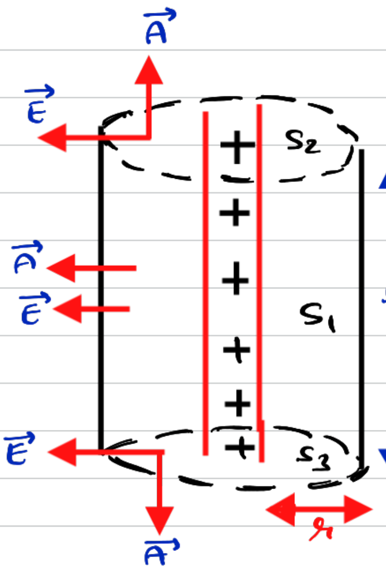
$$\phi = \oint E.ds = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \left[\text{Surface area of sphere}\right]$$

$$\phi = \oint E.ds = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \times 4\pi r^2$$

$$\boxed{\phi = \oint E.ds = q/\varepsilon_0}$$

This proves Gauss law.

# Field due to uniformly long charged wire




Consider a long charged wire of length $l$. Now we want to find electric field due to the wire at a distance $r$ from the wire.

So we draw a gaussian surface (shaped cylinder) to cover the wire completely.

Now applying Gauss law for whole surface

$$\oint_S E \cdot ds = q/\varepsilon_0$$

$$\oint_{S_1} E \cdot ds + \oint_{S_2} E \cdot ds + \oint_{S_3} E \cdot ds = q/\varepsilon_0$$

$$\oint_{S_1} E\,ds\cos\theta + \oint_{S_2} E\,ds\cos\theta + \oint_{S_3} E\,ds\cos\theta = q/\varepsilon_0$$

$$\oint_{S_1} E\,ds\cos 0 + \oint_{S_2} E\,ds\cos 90 + \oint_{S_3} E\,ds\cos 0 = q/\varepsilon_0$$

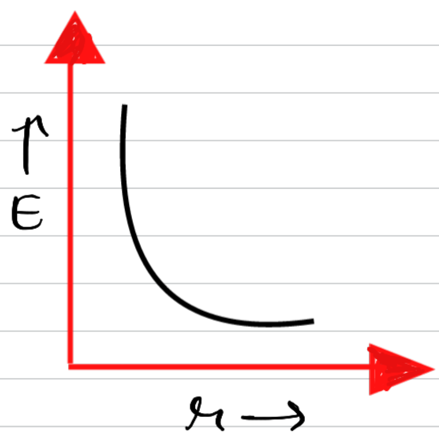
$$\oint_{S_1} E\,ds\,(1) + 0 + 0 = q/\varepsilon_0 \qquad (\because \cos 90 = 0)$$

$$\oint_{S_1} E\,ds = q/\varepsilon_0$$

$$E \oint ds = q/\varepsilon_0$$

$$E(\text{Curved Surface Area of Cylinder}) = q/\varepsilon_0$$

$$E \times 2\pi r l = q/\varepsilon_0$$




$$\boxed{E = \frac{q}{2\pi\varepsilon_0 r l}} \quad \text{—— } (1)$$

Now Linear charge density = $\frac{\text{charge}}{\text{length}}$

$\lambda = \frac{q}{l}$ Then $\boxed{q = \lambda l}$ —— (2)

from (1) & (2)

$$E = \frac{\lambda l}{2\pi\varepsilon_0 r l}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$ here $E \propto \frac{1}{r}$

## Electric field due to uniformly charged sheet

Consider an infinite charged sheet
Now we want to find electric field due to this charged sheet at a distance r from the sheet.
Now we draw a gaussian surface (cylindrical shape) as shown in the figure.

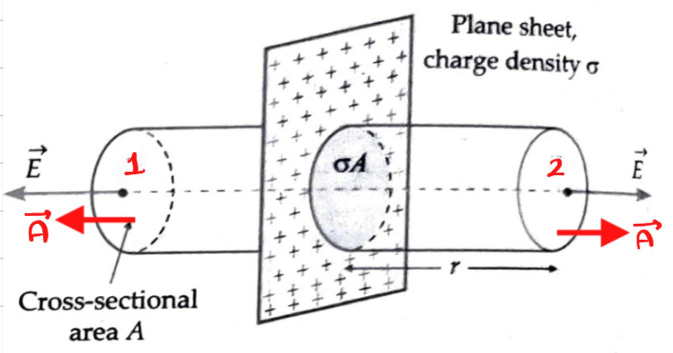



Fig. 1.98 Gaussian surface for a uniformly charged infinite plane sheet.

From Gauss law :-

$$\oint E.ds = q/\varepsilon_0$$

As there are two surfaces (surface 1 & surface 2) from which the electric field is passing out. so gauss law will be

$$\oint_1 E.ds + \oint_2 E.ds = q/\varepsilon_0$$

$$E\oint_1 ds + E\oint_2 ds = q/\varepsilon_0$$

$$ES + ES = q/\varepsilon_0 \quad \text{here } S = \text{Surface Area of Circle}$$

$$\boxed{2ES = q/\varepsilon_0} \text{ — } (1)$$

Now, we know, surface charge density = $\frac{\text{charge}}{\text{Surface Area}}$

$$\sigma = q/S$$

Then $\boxed{q = \sigma S}$ — (2)

Now from (1) & (2) :-

$$2ES = \frac{\sigma S}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

This is the value of electric field due to a sheet.

Note;- The electric field due to a sheet does not depend upon the distance.

# Electric field due to two charged sheet

## Electric field due to two positive sheet:-

Note → here charge density of both the plates is different $\sigma_1 \neq \sigma_2$

Consider two sheets with charge density $\sigma_1$ and $\sigma_2$. Now let $\sigma_1 > \sigma_2$.

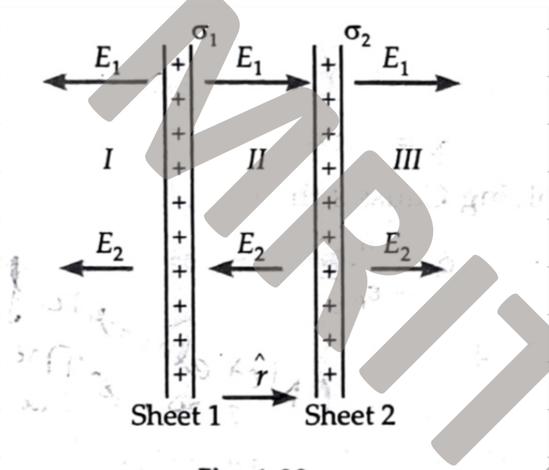



Fig. 1.99

In region I :-

$$\vec{E}_{Net} = (-\vec{E}_1) + (-\vec{E}_2)$$

$$\vec{E}_{Net} = -\frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}$$

$$\boxed{\vec{E}_{Net} = \frac{-1}{2\varepsilon_0}(\sigma_1 + \sigma_2)}$$

In region II :

$$\vec{E}_{net} = \vec{E}_1 - \vec{E}_2$$

$$= \frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}$$

$$\boxed{E_{Net} = \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2)}$$

In region III :-

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2$$

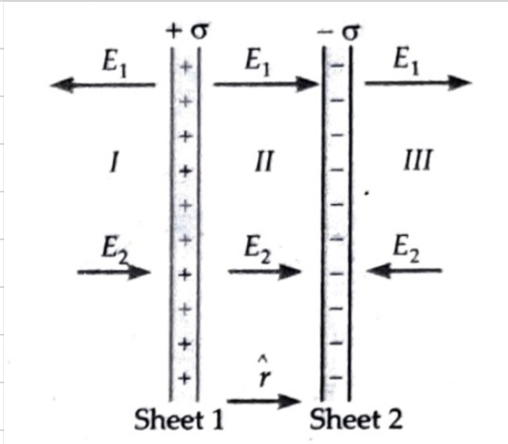
$$= \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0}$$

$$\boxed{E_{net} = \frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2)}$$

## Electric field due to one positive and one negative plate

Note → charge density is same $\sigma_1 = \sigma_2$.

Consider two sheets with charge density $+\sigma$ and $-\sigma$




In region I :-

$$\vec{E}_{Net} = (-\vec{E}_1) + (\vec{E}_2)$$

$$= \frac{-\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$\boxed{\vec{E}_{Net} = 0}$$

In region II :-

$$\vec{E}_{Net} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$= \frac{2\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

$$\boxed{E_{Net} = \frac{\sigma}{\varepsilon_0}}$$

In region III :-

$$\vec{E}_{net} = \vec{E}_1 + (-\vec{E}_2)$$

$$= \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$$

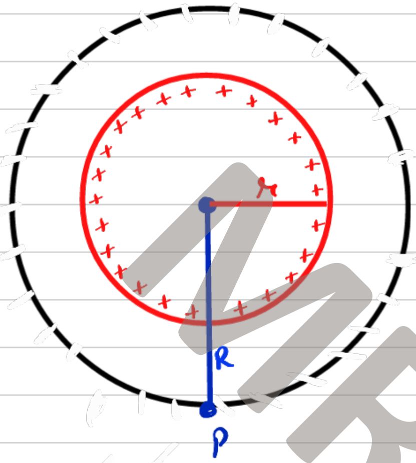
$$\boxed{E_{Net} = 0}$$

So electric field only exist between the plates.

## Electric field due to uniformly charged thin sherical shell

Consider a spherical shell of radius $r$ with charge '$q$' present on it. Now we have to find electric field at:-

### i) outside the sherical shell (At point P)




To find electric field at Point P, let us draw a gaussian surface (sperical in shape) of radius R.

Now from gauss law:.

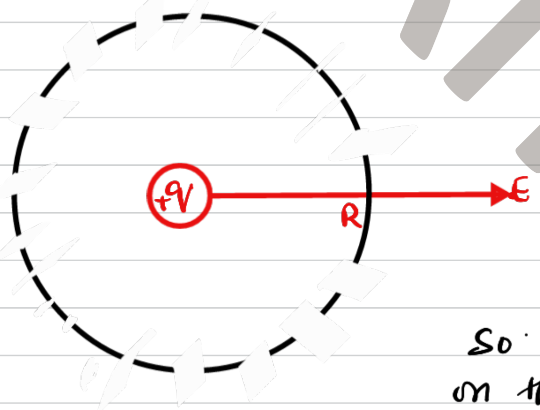
$$\oint E.ds = q/\varepsilon_0$$

$$E\oint ds = q/\varepsilon_0$$

$$E \times \text{Surface area of sphere} = q/\varepsilon_0$$

$$E \times 4\pi R^2 = q/\varepsilon_0$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}}$$




This formula is same as the electric field due to a point charge $q$ at a distance R from it. (as shown in the figure).

So we can assume the charge present on the spherical shell to be concentrated on the centre of the shell.

### b) Electric field on the sherical shell:-


Now to find electric field on the sphere, draw an gaussian surface of radius $r$.

Now from gauss law:-

$$\oint E.ds = q/\varepsilon_0$$

$$E\oint ds = q/\varepsilon_0$$

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

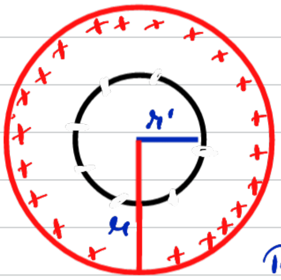
$$\boxed{E = \frac{1}{\varepsilon_0}\frac{q}{4\pi r^2}} \rightarrow \text{here } q = \text{charge and, } 4\pi r^2 = \text{Area}$$

$$\boxed{E = \frac{1}{\varepsilon_0}\frac{\text{charge}}{\text{Area}}} \rightarrow \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

where $\sigma$ = surface charge density

$\sigma$ = charge/Area

## c) Electric field inside the spherical shell:-




To find electric field inside the spherical shell let us draw a gaussian surface of radius r'

Then Acc. to gauss law:-

$$\oint E.ds = q/\varepsilon_0$$

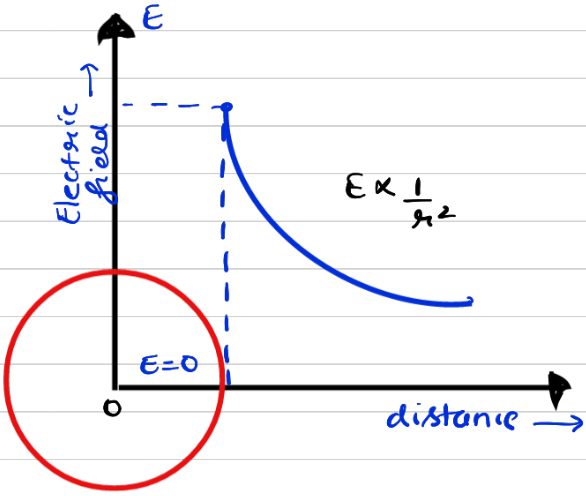
But there is No charge inside the gaussian surface

so $q = 0$ Then,

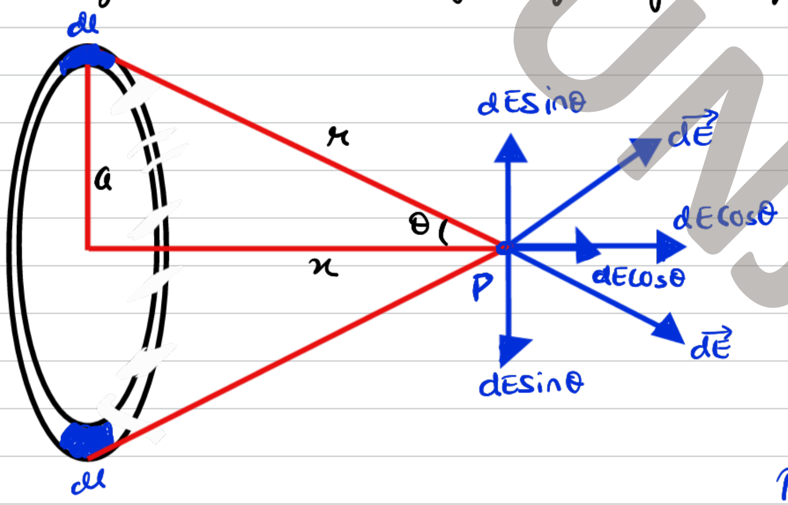
$\oint E.ds = 0$ Then $\boxed{E = 0}$

No electric field is present inside the shell

**Plot between electric field and distance**




## Electric field on the axis of uniformly charged ring




Cosider a ring of radius a have uniform distributed charge q.

Now using concept of linear charge density :-

$$\lambda = \frac{q}{l}$$

Then $q = \lambda l$

for small charge stored in length dl

$$\boxed{dq = \lambda\, dl}$$

Now the electric field at Point P due to dq stored at dl is

$$\vec{dE} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} \longrightarrow (1)$$

Now this $\vec{dE}$ will have two Component :-

1) $dE \cos\theta$ ( Both $dE\cos\theta$ are same direction)
2) $dE \sin\theta$ ( Both $dE \sin\theta$ are in opposite direction)

Both $dE\sin\theta$ will cancel each other out as they are in opposite direction with equal magnitude.

Now total electric field due to two $dl$ component of ring :-

$$dE_{axial} = dE\cos\theta + dE\cos\theta$$

$$dE_{axial} = 2dE\cos\theta$$

Integrating both sides :-

$$\int dE_{axial} = \int_0^{\pi a} 2\,dE\cos\theta$$

$$E_{axial} = 2\cos\theta \int_0^{\pi a} dE$$

Using eqn ① in above :-

$$E_{axial} = 2\cos\theta \int_0^{\pi a} \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

$$E_{axial} = \frac{2\cos\theta}{4\pi\varepsilon_0 r^2}\int_0^{\pi a} dq \qquad (\text{Put } dq = \lambda\, dl)$$

$$E_{axial} = \frac{2\cos\theta}{4\pi\varepsilon_0 r^2}\int_0^{\pi a} \lambda\, dl$$

$$E_{axial} = \frac{2\cos\theta}{4\pi\varepsilon_0 r^2}\lambda\int_0^{\pi a} dl$$

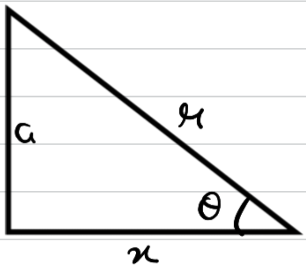
$$E_{axial} = \frac{2\cos\theta}{4\pi\varepsilon_0 r^2}\frac{q}{l}\int_0^{\pi a} dl$$

$$E_{axial} = \frac{2\cos\theta}{4\pi\varepsilon_0 r^2}\frac{q}{2\pi a}\int_0^{\pi a} dl$$

$$E_{axial} = \frac{2\cos\theta}{4\pi\varepsilon_0 r^2}\frac{q}{2\pi a}\left[l\right]_0^{\pi a}$$

$$E_{axial} = \frac{2\cos\theta}{4\pi\varepsilon_0 r^2}\frac{q}{2\pi a}\times \pi a$$

$$E_{axial} = \frac{\cos\theta}{4\pi\varepsilon_0}\frac{q}{r^2}$$




here $\cos\theta = \frac{\text{Base}}{\text{hypo}} = \frac{x}{r}$

By phythagoras theorem :-

$r^2 = a^2 + x^2$

Putting value of $\cos\theta$ :- $E_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\times\frac{x}{r}$

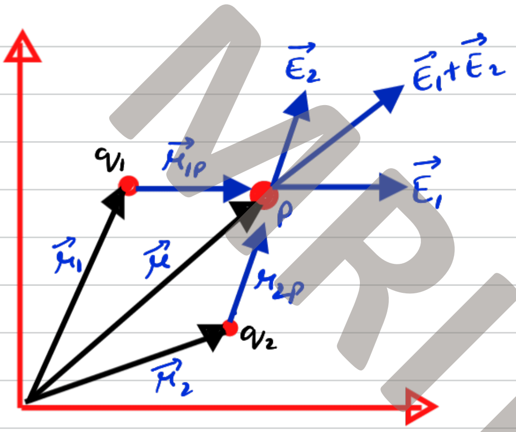
$$E_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{qx}{r^3}$$

$$\boxed{E_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(a^2+x^2)^{3/2}}}$$

# Electric field due to a System of charges

Let us assume two charges $q_1$ and $q_2$ kept at position vector $r_1$ and $r_2$ respectively

here $\vec{E_1}$ = Electric field due to charge $q_1$ at Point P
$\vec{E_2}$ = Electric field due to charge $q_2$ at Point P
Similarly
$\vec{E_N}$ = Electric field due to charge $q_N$ at Point P




Now In vector form:-

$$\vec{E_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{(r_{1P})^3}\vec{r}_{1P} \quad \text{—(1)}$$

from triangle law of addition

$$\vec{r} = \vec{r_1} + \vec{r}_{1P}$$

Then $\vec{r}_{1P} = \vec{r} - \vec{r_1}$

Then eq^n (1):-

$$\boxed{\vec{E_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{|r-r_1|^3}(\vec{r}-\vec{r_1})} \quad \text{—(2)}$$

Similarly

$$\boxed{\vec{E_2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{|r-r_2|^3}(\vec{r}-\vec{r_2})} \longrightarrow (3)$$

Total electric field at Point P due to N charges:-

$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} - - - + \vec{E_N}$$

using eq^n (2) & (3)

$$\vec{E} = \left[\frac{1}{4\pi\varepsilon_0}\frac{q_1}{|\vec{r}-\vec{r_1}|^3}(\vec{r}-\vec{r_1})\right] + \left\{\frac{1}{4\pi\varepsilon_0}\frac{q_2}{|\vec{r}-\vec{r_2}|^3}(\vec{r}-\vec{r_2})\right] + - - -$$
$$+ - - - - + \left[\frac{1}{4\pi\varepsilon_0}\frac{q_N}{|\vec{r}-\vec{r_N}|^3}(\vec{r}-\vec{r_N})\right]$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{|\vec{r}-\vec{r_i}|^3}(\vec{r}-\vec{r_i})}$$

Another form :-

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{(r_{iP})^2}\hat{r}_{iP}}$$

**MOST IMPORTANT THEORY QUESTIONS**

**CHPATER – ELECTRIC FIELD AND CHARGES**

## 1. *Why must electrostatic field lines do not form closed loop?*

Electrostatic field lines do not form closed loops because they represent the direction of the electric field at every point in space. If field lines formed closed loops, it would imply that the electric field do not has a starting and ending point, which contradicts the nature of electric field lines.

Instead, electric field lines always originate from positive charges and terminate at negative charges. This unidirectional flow of electric field lines reflects the direction in which a positive test charge would move if placed in the electric field.

## 2. *Why do electric field lines never cross each other?*

Electric field lines never cross each other because they represent the direction of the electric field at every point in space. If two field lines were to cross, it would imply that at the point of intersection, there are two different directions for the electric field, which is not physically possible.

Because, if two field lines were to intersect, it would mean that at that point, a positive test charge would experience two different directions of force simultaneously. But the positive charge cannot move in two different directions at the same time, so intersection of electric field lines is never possible.

## 3. *Why must electrostatic field at the surface of a charged conductor be perpendicular to every point on it?*

The electric field at the surface of a charged conductor must be perpendicular to every point on it due to electrostatic equilibrium and the properties of conductors.

In electrostatic equilibrium, charges on a conductor redistribute themselves such that the electric field inside the conductor is zero. This redistribution of charges ensures that there is no net force on any charge within the conductor, leading to a stable equilibrium state.

Now, consider the electric field just outside the surface of the conductor. If the electric field were not perpendicular to the surface at some point, there would be a component of the field tangent to the surface. This tangent component would exert a force on the charges on the surface, causing them to move. However, in electrostatic equilibrium, there can be no net force on charges within a conductor. Therefore, to maintain equilibrium, the electric field at the surface of the conductor must be perpendicular to every point on it.

In summary, the perpendicular nature of the electric field at the surface of a charged conductor ensures that the charges remain in equilibrium and there is no net force on them, consistent with the principles of electrostatics and conductors.

## 4. *Why the flux linked with a dipole enclosed inside a closed surface is zero?*

The flux linked with a dipole enclosed inside a closed surface is zero because the net electric flux through any closed surface enclosing the dipole is always zero, according to Gauss's law.

Gauss's law states that the net electric flux through any closed surface is equal to the total charge enclosed by that surface divided by the permittivity of the medium. Since a dipole consists of equal and opposite charges separated by a small distance, the net charge enclosed by any closed surface that contains the dipole is zero.

Therefore, according to Gauss's law, the electric flux linked with the dipole enclosed inside the closed surface is also zero.

In other words, the electric field lines originating from the positive charge of the dipole terminate on the negative charge within the dipole itself, resulting in equal outward and inward flux through any closed surface enclosing the dipole. As a result, the net electric flux through the closed surface enclosing the dipole is zero.

---

## 5. Explain all the types of fundamental forces and compare there strengths?

The four fundamental forces in nature are:

1. Gravitational Force: This force acts between any two masses and is responsible for the attraction between objects with mass. It is the weakest of the fundamental forces but has infinite range.

2. Electromagnetic Force: This force acts between charged particles (like electrons and protons) and is responsible for phenomena such as electric and magnetic interactions. It is significantly stronger than the gravitational force and also has infinite range.

3. Strong Nuclear Force: This force acts between particles within the atomic nucleus (protons and neutrons) and is responsible for holding the nucleus together. It is the strongest of the fundamental forces but has a very short range, typically only acting over distances comparable to the size of an atomic nucleus.

4. Weak Nuclear Force: This force is responsible for certain types of radioactive decay and plays a role in the interactions of subatomic particles. It is much weaker than the strong nuclear force but stronger than gravity. It also has a short range, typically acting over distances smaller than the size of an atomic nucleus.

Comparing their strengths:

- The strong nuclear force is the strongest of the fundamental forces but acts over a very short range, only within the atomic nucleus.
- The electromagnetic force is significantly stronger than gravity but weaker than the strong nuclear force. It has infinite range and acts between charged particles.
- Gravity is the weakest of the fundamental forces but has infinite range. It acts between any two masses and is responsible for the attraction between objects with mass.
- The weak nuclear force is much weaker than the strong nuclear force but stronger than gravity. It also has a short range and is responsible for certain types of radioactive decay and particle interactions.

In summary, while the strong nuclear force is the strongest, it is limited to very short distances within the atomic nucleus. The electromagnetic force is stronger than gravity and acts over infinite range. Gravity is the weakest but has infinite range, and the weak nuclear force is weaker than the strong nuclear force but stronger than gravity and also has a short range.

## 6. Explain uniform and non-uniform electric field and effect on the dipole placed inside them.

Sure, let's delve into the concepts of uniform and non-uniform electric fields and their effects on dipoles:

1. Uniform Electric Field:

- In a uniform electric field, the magnitude and direction of the electric field are constant throughout space.
- This means that the field lines are evenly spaced and parallel to each other.
- For example, the electric field between two parallel plates with constant potential difference creates a uniform electric field.
- Effect on a Dipole:
    - In a uniform electric field, a dipole experiences a torque that tends to align it with the field lines. The torque is maximum when the dipole moment is perpendicular to the electric field lines and zero when it is parallel.
    - Additionally, a dipole placed in a uniform electric field may experience translational motion if there is an external force acting on it.

2. Non-uniform Electric Field:

- In a non-uniform electric field, the magnitude or direction of the electric field varies from point to point.
- This can occur due to the presence of different charges or the geometry of the electric field source.
- For example, the electric field around a point charge or near the edges of irregularly shaped conducting objects creates a non-uniform electric field.
- Effect on a Dipole:
    - In a non-uniform electric field, a dipole experiences both translational and rotational motion.
    - The unequal forces exerted by the non-uniform electric field on the positive and negative charges of the dipole create a net force and torque, causing the dipole to both translate and rotate.
    - The translational motion occurs due to the net force acting on the dipole, while the rotational motion occurs due to the torque exerted on it.
    - The dipole tends to align itself in a direction where the electric field is weaker or stronger, depending on its orientation relative to the field lines.

In summary, a uniform electric field has a constant magnitude and direction, leading to predictable effects on dipoles, such as torque-induced alignment. In contrast, a non-uniform electric field varies in magnitude or direction, resulting in more complex motion of dipoles, including both translation and rotation.

7. ***Define an ideal electric diploe. Give an example.***

## Ideal Dipole

**Definition**: An ideal dipole is a theoretical construct in which:

1. The size (separation) of the dipole ($2a$) is infinitesimally small.
2. The charges ($+q$ and $-q$) forming the dipole are infinitely large.
3. Despite these conditions, the dipole moment ($\mathbf{p}$) remains finite.

**Dipole Moment**: The dipole moment $\mathbf{p}$ is defined as:

$\mathbf{p} = q \cdot 2a$

where $q$ is the charge and $2a$ is the separation between the charges.

## Explanation

1. **Infinitesimally Small Size**: The separation between the two charges ($2a$) tends towards zero.

   $2a \to 0$

2. **Infinitely Large Charges**: The magnitude of the charges ($q$) tends towards infinity.

   $q \to \infty$

3. **Finite Dipole Moment**: Despite the above conditions, the product of charge and separation, which gives the dipole moment ($\mathbf{p}$), remains finite.

   $\mathbf{p} = q \cdot 2a$

   For an ideal dipole:

   $\lim_{q \to \infty, 2a \to 0} q \cdot 2a = \text{finite value}$

. Here are a few examples:

1. Water Molecule (H2O): In a water molecule, the oxygen atom attracts electrons more strongly than the hydrogen atoms, creating a partial negative charge (δ-) on the oxygen atom and partial positive charges (δ+) on the hydrogen atoms. This charge separation gives rise to a dipole moment, making water a polar molecule.

2. Ammonia Molecule (NH3): Similar to water, the ammonia molecule exhibits a dipole moment due to the unequal sharing of electrons between nitrogen and hydrogen atoms. The nitrogen atom attracts electrons more strongly, resulting in a partial negative charge (δ-) on nitrogen and partial positive charges (δ+) on the hydrogen atoms.

8. ***Can Gauss theorem be applied to non-uniform electric field?***

Yes, Gauss's theorem can be applied to non-uniform electric fields. Gauss's theorem, also known as Gauss's law, states that the total electric flux through a closed surface is equal to the total charge enclosed by the surface divided by the permittivity of the medium.

In the case of non-uniform electric fields, the electric field may vary in magnitude or direction at different points within the region of interest. However, Gauss's law still holds true, and the theorem can be applied to determine the total electric flux through a closed surface enclosing a certain charge distribution, regardless of the uniformity of the electric field.
When applying Gauss's theorem to non-uniform electric fields, it's essential to consider the distribution of charges within the enclosed volume and how the electric field varies spatially. By carefully selecting a Gaussian surface and understanding the symmetry of the charge distribution, one can calculate the electric flux and infer information about the electric field configuration, even in non-uniform cases.

---

## 9. The electric field over the gaussian surface remains continuous and uniform at every point?

Yes, in many cases, the electric field over the Gaussian surface remains continuous and uniform at every point, especially when dealing with symmetrical charge distributions.

When applying Gauss's law to calculate electric flux, we often select a Gaussian surface that exhibits symmetry with respect to the charge distribution. In such cases, the electric field has a uniform magnitude and direction over the entire surface.

For example, consider a uniformly charged spherical shell. If we choose a spherical Gaussian surface concentric with the shell, the electric field is radially outward at every point on the surface, and its magnitude remains constant due to the symmetry of the charge distribution. Similarly, if we choose a cylindrical Gaussian surface around an infinitely long charged wire, the electric field is perpendicular to the surface and has a constant magnitude along its length.

However, it's important to note that in more complex charge distributions or non-symmetric situations, the electric field may not be uniform over the entire Gaussian surface. In such cases, the electric field may vary in magnitude and direction across the surface, but Gauss's law can still be applied by integrating over the entire surface to calculate the total electric flux.

## 10. Does gauss law depend on the distance? Explain in short.

No, Gauss's law does not directly depend on the distance from the charge distribution. Instead, it depends on the total charge enclosed by a closed surface and the electric flux through that surface.

Gauss's law states that the total electric flux through a closed surface is equal to the total charge enclosed by the surface divided by the permittivity of the medium. This relationship remains true regardless of the distance from the charge distribution.

## 11. Is electric field inside the sphere with no charge inside depends on whether the sphere is hollow or solid? Why is it said the electric field is always zero inside the metallic body.

The electric field inside a sphere with no charge inside does not depend on whether the sphere is hollow or solid. This is because, in both cases, according to Gauss's law, the net charge enclosed by any Gaussian surface placed inside the sphere is zero. Therefore, the total electric flux through the Gaussian surface is also zero, implying that the electric field inside the sphere is zero.

Regarding the statement that the electric field is always zero inside a metallic body, it pertains specifically to conductors and is a consequence of electrostatic equilibrium. In an electrostatically equilibrium situation, the charges within a conductor redistribute themselves on the surface in such a way that the electric field inside the conductor becomes zero. This redistribution of charges ensures that any excess charge resides on the surface, and the electric field within the bulk of the conductor is neutralized.

This phenomenon holds true regardless of whether the conductor is hollow or solid. In the case of a solid conductor, charges redistribute uniformly throughout the material, cancelling out the electric field inside. In the case of a hollow conductor, charges redistribute on the inner and outer surfaces, also resulting in an electric field of zero within the cavity.

In summary, the electric field inside a sphere with no charge inside (whether hollow or solid) is zero due to Gauss's law, and the electric field is always zero inside a metallic body (conductor) due to electrostatic equilibrium and charge redistribution.

## 12. Explain Relative permittivity or Dielectric constant in terms of electric field and permittivity ratio with the formula?

Relative permittivity, also known as the dielectric constant, is a measure of a material's ability to reduce the electric field inside it compared to a vacuum. It quantifies how much the electric field strength decreases when a material is placed in an electric field.

Relative permittivity ($\varepsilon_r$) is defined as the ratio of the electric field ($E$) in a vacuum to the electric field ($E_0$) in the material:

$$\varepsilon_r = \frac{E}{E_0}$$

Alternatively, it can be expressed in terms of the permittivity of the material ($\varepsilon$) and the permittivity of vacuum ($\varepsilon_0$):

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

Here, $\varepsilon$ represents the permittivity of the material, which is a measure of how much the material can store electrical energy in an electric field. $\varepsilon_0$ is the permittivity of vacuum, which is a physical constant representing the ability of vacuum to permit the propagation of electric field.

A material with a high relative permittivity (dielectric constant) means it can greatly reduce the electric field inside it, making it more effective at storing electrical energy. This property is widely used in capacitors, where dielectric materials are inserted between the plates to increase the capacitance.

The concept of relative permittivity allows us to compare the effectiveness of different materials in reducing the electric field and storing electrical energy, providing insights into their electrical properties and applications.

## 13. Explain Electrostatic Induction

Electrostatic induction is a process where a charged object induces a temporary separation of charge in a neutral object, without direct contact between the two objects. This occurs due to the repulsion or attraction of charges within the neutral object in response to the presence of the charged object.

**Here's how electrostatic induction works:**

**Initial Setup:** Initially, we have a neutral object, meaning it has an equal number of positive and negative charges, resulting in a net charge of zero.

**Approach of Charged Object:** When a charged object is brought near the neutral object, the electric field of the charged object exerts a force on the charges within the neutral object. For instance, if a positively charged object is brought near, it repels positive charges and attracts negative charges in the neutral object.

**Separation of Charges:** As a result of the influence of the electric field from the charged object, the charges in the neutral object redistribute themselves. The side of the neutral object closer to the charged object will have an excess of opposite charges, while the side farther away will have an excess of like charges. This separation of charges creates an induced dipole moment in the neutral object.

**Polarization:** Although the neutral object remains overall neutral, it becomes polarized due to the separation of charges. This means one side becomes temporarily positive (where negative charges have been attracted) and the other side becomes temporarily negative (where positive charges have been repelled).

**Separation of Charged Object:** If the charged object is then removed, the induced charges in the neutral object return to their original distribution, and the object returns to being neutral.

14. ***Effect on Force between charges when these charges are placed in a medium of dielectric constant K.***

### Coulomb's Law in Vacuum

In a vacuum, the electrostatic force ($F_0$) between two point charges $q_1$ and $q_2$ separated by a distance $r$ is given by Coulomb's law:

$F_0 = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$

where:

- $\epsilon_0$ is the permittivity of free space (vacuum).

### Coulomb's Law in a Medium

When the charges are placed in a medium with a dielectric constant $k$, the permittivity of the medium becomes:

$\epsilon = k\epsilon_0$

The force ($F$) between the charges in the medium is given by:

$F = \frac{1}{4\pi\epsilon}\frac{q_1q_2}{r^2}$

Substituting $\epsilon = k\epsilon_0$:

$F = \frac{1}{4\pi k\epsilon_0}\frac{q_1q_2}{r^2}$

### Relationship Between Forces in Vacuum and Medium

We can compare the force in the medium ($F$) with the force in a vacuum ($F_0$) by considering the ratio:

$F = \frac{F_0}{k}$