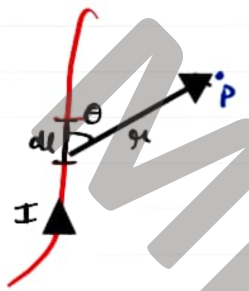
MAGNETIC EFFECT OF CURRENT

Concept of magnetic field The space around a magnet within which its influence can be experienced is called magnetic field.

A moving charge or a current carrying wire can create magnetic field around it.

Biot - Savart law Consider a current carrying wire as shown in figure



Let I = current flowing through wire

dl = current element of the wire

r = distance of Point P from dl

θ = angle b/w θ & dl

Now acc. to biot Savart law, magnitude of small magnetic field (dB)

1) Is directly proportional to the current

$$dB \propto I$$

2) Is directly proportional to the current element

$$dB \propto dl$$

3) Is directly proportional to $\sin\theta$

$$dB \propto \sin\theta$$

4) Is inversely proportional to the square of distance r from the Point P.

$$dB \propto 1/r^2$$

Then

$$dB \propto \frac{I\,dl\sin\theta}{r^2}$$

$$\boxed{dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}}$$

here μ_0 = permeability of the medium

And value of $\frac{\mu_0}{4\pi} = 10^{-7}$

In vector form

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}\times\frac{r}{r}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,r\sin\theta}{r^3}$$

$$\boxed{\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}}{r^3}}$$

Special Cases:-

1) If $\theta = 0°$, $\sin\theta = \sin 0 = 0$ Then

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\sin 0 = 0$$

$\boxed{dB = 0}$ (minimum value)

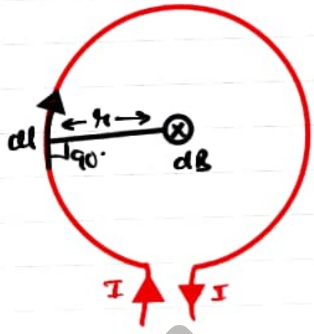
2) If $\theta = 90°$, $\sin\theta = \sin 90 = 1$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin 90}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$$

$\boxed{dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}}$ (max value)

magnetic field with distance

Magnetic field at the centre of current carrying loop



Let us suppose a current is flowing through a circular conductor as shown in the figure.

Then Acc. to Biot Savart law magnetic field at centre will be given by

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}$$

here θ is angle b/w dl and r. So $\theta = 90$

Then $\sin 90° = 1$. So

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2}$$

Integrating both sides

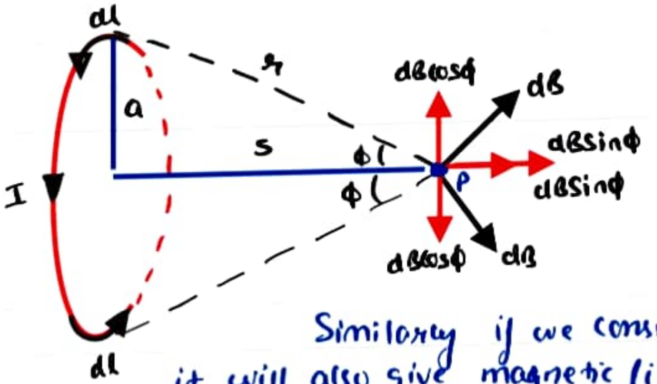
$$\int dB = \int \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2}$$

$$B = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int dl = \frac{\mu_0}{4\pi} \frac{I}{r^2} \times 2\pi r$$

$$\boxed{B = \frac{\mu_0 I}{2r}}$$

for N turns $\boxed{B = \frac{\mu_0 N I}{2r}}$

Magnetic field at the axis of a circular current loop



Consider a circular loop carrying current I.

Now consider a current element dl at the top of the loop. Now the magnetic field dB due to this length element will have two components $dB\cos\phi$ & $dB\sin\phi$

Similarly if we consider current element at the bottom, it will also give magnetic field dB at Point P. This will also have two components $dB\cos\phi$ & $dB\sin\phi$.

Now both $dB\cos\phi$ will cancel each other out

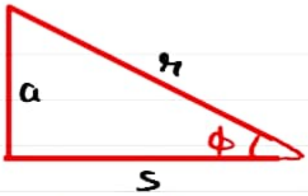
Then, total magnetic field $= dB\sin\phi + dB\sin\phi$

$$(dB)_T = 2dB\sin\phi$$

$$(dB)_T = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2} \sin\phi$$

here θ is angle b/w r and dl. $\theta = 90°$. so $\sin 90 = 1$

$$(dB)_T = \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2} \sin\phi$$



here $\sin\phi = \frac{P}{H} = \frac{a}{r}$

Also from phythagoras:

$$r^2 = a^2 + s^2$$

$$r = \sqrt{a^2+s^2}$$

Now Integrating both sides:-

$$\int (dB)_T = \int \frac{\mu_0}{4\pi} \frac{I\,dl \sin\phi}{r^2}$$

$$B = \frac{\mu_0}{4\pi} \frac{I \sin\phi}{r^2} \int dl$$

$$B = \frac{\mu_0}{4\pi} \frac{I \sin\phi}{r^2} (2\pi a)$$

$$B = \frac{\mu_0}{2} \frac{I}{r^2} \sin\phi \,(a)$$

Putting values of $\sin\phi$ & r

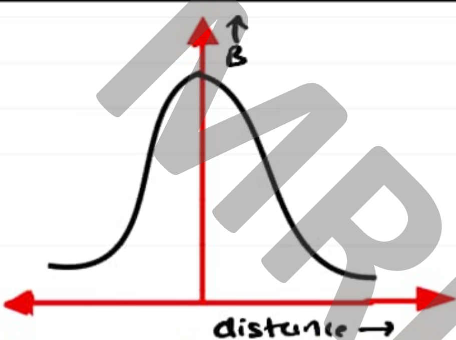
$$B = \frac{\mu_0}{2} \frac{I}{r^2} \left(\frac{a}{r}\right) a$$

$$B = \frac{\mu_0}{2} \frac{I}{a^2+s^2} \frac{a^2}{\sqrt{a^2+s^2}}$$

$$B = \frac{\mu_0 I a^2}{2(a^2+s^2)^{3/2}}$$

for N turns.

$$\boxed{B = \frac{\mu_0 N I a^2}{2(a^2+s^2)^{3/2}}}$$



Magnetic field with Distance

Special cases:-

1) At Centre of loop $s=0$ Then $B = \frac{\mu_0 N I a^2}{2(a^2)^{3/2}} = \frac{\mu_0 N I a^2}{2a^3}$

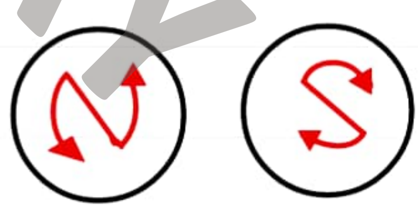
$$\boxed{B = \frac{\mu_0 N I}{2a}}$$

2) when $s \gg a$ Then
Neglect a from formula

$$B = \frac{\mu_0 N I a^2}{2(s^2)^{3/2}} = \frac{\mu_0 N I a^2}{2s^3}$$

Clock Rule

If current is in anticlockwise direction, it behaves like a North Pole



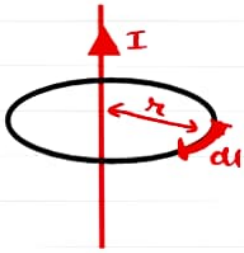
If the current flows in clockwise direction it behaves as South Pole

Ampere Circuital Law

The line integral of the magnetic field B around any closed circuit is equal to μ_0 times the total current I passing through this closed circuit. Mathematically,

$$\boxed{\oint B.dl = \mu_0 I}$$

Proof:- Consider a long wire carrying current I. The value of magnetic field at any point which is at a distance r is given by



$$B = \frac{\mu_0 I}{2\pi r}$$

Now calculate total magnetic field, we take complete line integral of magnetic field.

$$\oint B \cdot dl = \oint \frac{\mu_0 I}{2\pi r} \cdot dl = \frac{\mu_0 I}{2\pi r} \oint dl = \frac{\mu_0 I}{2\pi r} \times 2\pi r$$

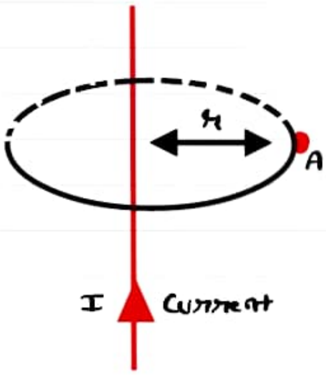
$$\boxed{\oint B \cdot dl = \mu_0 I}$$

Ampere law is proved.

Magnetic field due to infinite long wire (By Ampere law)

Let us consider a long infinite wire carrying current I. Now we have to find the value of magnetic field at point A which is at a distance r.

Now we draw circular amperian loop around the wire as shown in the figure



Now from Amphere law:-

$$\oint B \cdot dl = \mu_0 I$$

$$B \oint dl = \mu_0 I$$

$$B l = \mu_0 I$$

$$B \times \text{length of circular loop} = \mu_0 I$$

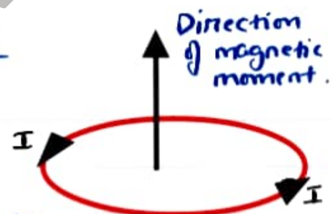
$$B \times 2\pi r = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

This is the value of magnetic field due to the wire at distance r.

Current loop as a magnetic dipole

A current carrying circular loop has magnetic field around it therefore its one face acts as a north pole & the other face acts as a south pole. There this loop can be taken as magnetic dipole.



As shown in diagram a current loop, its upper face will behave as a North Pole where as downward face acts as South pole.

Magnetic dipole moment:- It is equal to product of current & Area of the loop

$$\boxed{m = IA}$$

It is a vector quantity & its direction is from South to North.

Force on a moving charge in a magnetic field

When a charged particle is moving inside a magnetic field with some velocity at angle θ with magnetic field.
Then force experienced by charged particle is
1) Directly proportional to the magnetic field
$$F \propto B$$
2) Directly proportional to $v\sin\theta$ (velocity which is is $\perp$ to field)
$$F \propto v\sin\theta$$
3) directly proportional to the value of charge
$$F \propto q$$

Combining all factors
$$F \propto Bvq\sin\theta$$
$$F = kBvq\sin\theta$$
here k is a constant & value of $k=1$

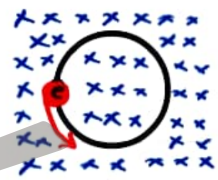
so $\boxed{F = Bvq\sin\theta}$ or $\boxed{\vec{F} = q(\vec{v} \times \vec{B})}$

This force is called magnetic lorentz force.

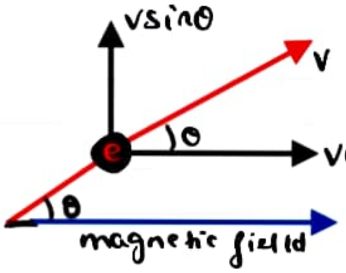
Special Cases:-
1) when $\theta = 0$ then $\sin\theta = \sin 0 = 0$
Then, $F = Bvq\sin 0 = 0$ (minimum)
2) when $\theta = 90°$, then $\sin\theta = \sin 90 = 1$
Then, $F = Bvq\sin 90 = Bvq$ (max)
3) when $\theta = 180°$ then $\sin\theta = \sin 180 = 0$
Then, $F = Bvq\sin 180 = 0$ (minimum)

Important Points:-

1) when a charged particle is thrown in a magnetic field at 90° then charged particle experience a force $F = Bvq\sin\theta$ and charged particle starts moving in a circular motion.



2) when a charged particle at angle θ with the magnetic field, then velocity of particle will have two components ($v\sin\theta$) & ($v\cos\theta$). There will be no effect on $v\cos\theta$ as it is along the magnetic field.
But $v\sin\theta$ will experience a force as it is perpendicular to magnetic field. so $v\sin\theta$ will move the charged in circular motion & $v\cos\theta$ will move the charged in forward direction.
Thus combining both motion, the particle will move in helical path.



Lorentz force

The total force experienced by a charged particle moving in both electric & magnetic field is called Lorentz force

force experienced by charge in electric field

$$F_E = qE$$

force experienced by charge in magnetic field

$$F_B = Bvq \sin\theta$$

Total force, $F = F_E + F_B$

$$F = qE + Bvq \sin\theta$$

$$\boxed{F = q(E + Bv\sin\theta)} \quad \text{or} \quad \boxed{\vec{F} = q(E + \vec{v} \times \vec{B})}$$

Velocity selector

A beam of electrons are passed through a region in which both electric field & magnetic field are. The electric field & magnetic field are perpendicular to each other as shown in the figure

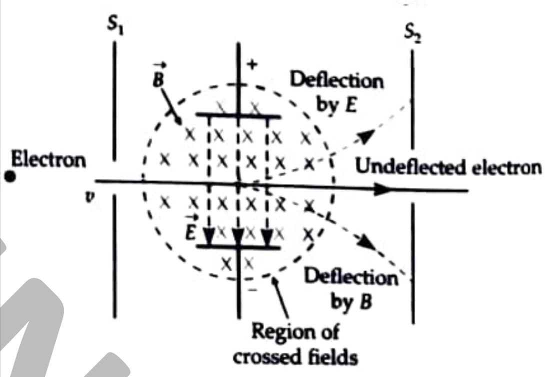


Fig. 4.64 Motion of an electron in a region of crossed magnetic and electric fields.

Electric field is in downward direction ↓ and magnetic field going into the paper ×

Now Electric field forcing electron to move upwards whereas magnetic field forcing electron to move downward.

If a electron travels straight then,

upward force on it = downward force on it

Electric force = magnetic force

$$qE = Bvq$$

$$E = Bv$$

Thus $\boxed{v = \dfrac{E}{B}}$

Thus only those electron can pass whose velocity is equal to E/B

Force experienced by a Conductor in magnetic field

Let a wire carrying current I is placed inside the magnetic field. Then force experienced by the electrons of the conductor

$F = Bvq\sin\theta$ here q = charge of electron = e

So $F = Bve\sin\theta$ —— ①

Now

Total force on conductor = (force on 1 electron) × (No. of electrons)

$F_T = F \times N$ —— ②

where

N = Total no. of electrons.

Let n = No. of electrons per unit volume

$n = \frac{N}{V}$ then $N = nV$

$N = nAl$ —— ③ (here V = Volume = Area × length)

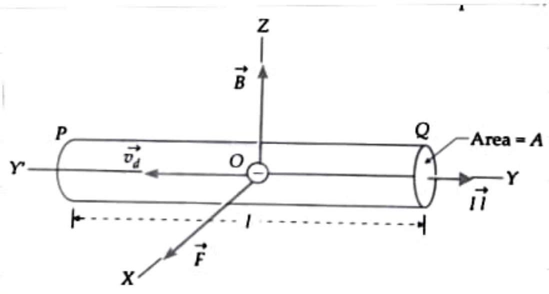


Fig. 4.70 Force on a current in a magnetic field.

From ①, ② & ③ eqn :-

$F_T = Bve\sin\theta \times nAl$

$F_T = (neAv)Bl\sin\theta$

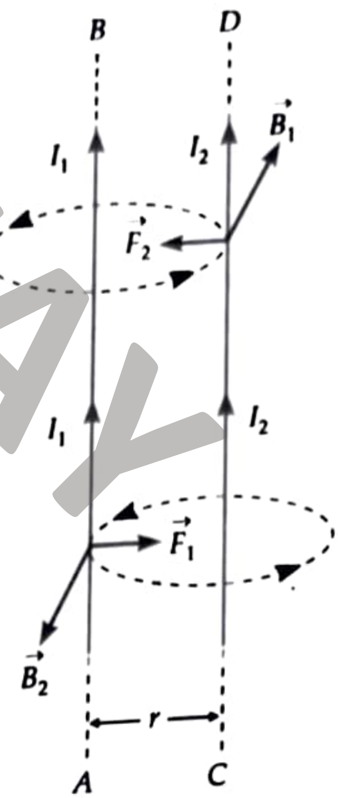
$F_T = IBl\sin\theta$ (here I = current = $neAv$)

So, $\boxed{F_T = IBl\sin\theta}$ or $\boxed{\vec{F}_T = I(\vec{l} \times \vec{B})}$

Force between two parallel current carrying conductors

When two conductors carrying current in the same direction then they will attract each other.

If the two conductor carrying current in the opposite direction then they will repel each other.



Let I_1 = current in wire A

I_2 = current in wire 2

F_2 = force experienced by wire 2

r = distance b/w the wire

B_1 = magnetic field due to wire 1.

Now value of magnetic field due to wire 1 on wire 2 :-

$B_1 = \frac{\mu_0 I_1}{2\pi r}$ —— ①

Now, due to this magnetic field wire 2 experience a force

$F_2 = I_2 l B_1 \sin\theta$ here $\theta = 90°$

$F_2 = I_2 l B_1$ ($\because \sin 90 = 1$)

from eqn ①

$F_2 = I_2 l\left(\frac{\mu_0 I}{2\pi r}\right) = \frac{\mu_0 I_1 I_2}{2\pi r} l$

So force, $\boxed{F_2 = \frac{\mu_0 I_1 I_2 l}{2\pi r}}$ and force per unit length, $\boxed{f = \frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}}$

Similarly force experienced by F_1 wire due to the magnetic field of wire 2 :-

$\boxed{F_1 = \frac{\mu_0 I_2 I_1 l}{2\pi r}}$ and $\boxed{f = \frac{\mu_0 I_1 I_2}{2\pi r}}$

Definition of 1 ampere :-

We know $f = \frac{\mu_0 I_1 I_2}{2\pi r}$. when $I_1 = I_2 = 1$ Ampere
$r = 1$ metre

Then $f = \frac{\mu_0}{2\pi} = 2 \times 10^{-7}$

So, one ampere is that value of current, which flowing in each of the two parallel conductors placed at a distance of 1m from each other, and experience a force 2×10^{-7} newton per metre of their length.

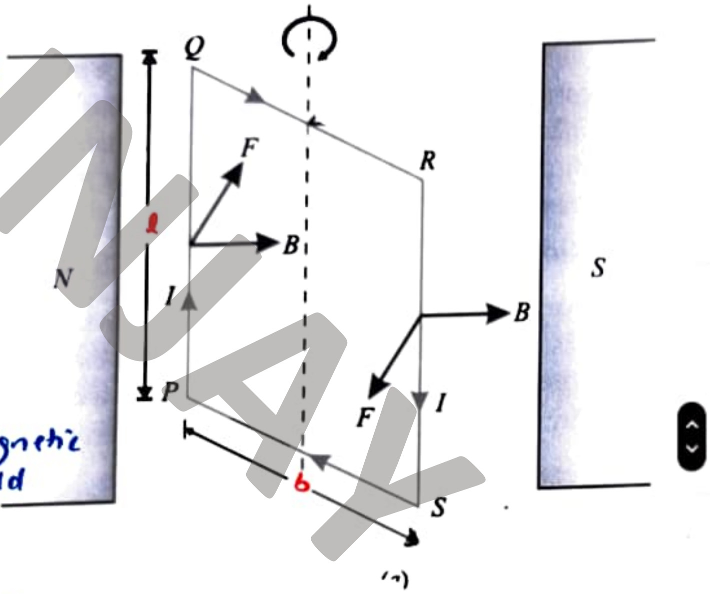
Torque experienced by a current loop in a magnetic field

Consider a rectangular coil PQRS placed in a magnetic field as shown in figure. Let

I = current flowing in PQRS

a, b = sides of PQRS

A = area of PQRS = $l \times b$



The force experienced by wire PQ

$F = IlB \sin\phi$

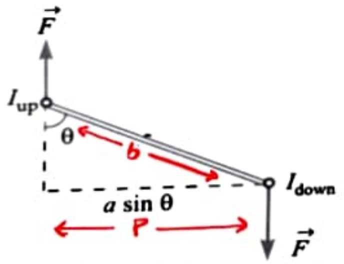
where ϕ angle b/w length of wire & magnetic field

So $\phi = 90$ (always)

Then $F = IlB \sin 90°$

$\boxed{F = IlB}$

From fleming left hand rule this force is in downward direction.



here $\sin\theta = \frac{P}{H} = \frac{P}{a}$

$\boxed{P = b\sin\theta}$

Perpendicular distance

Similarly side RS will experience a force $F = IlB$ in upward direction.

So whole loop PQRS will experience a Torque

τ = force × perpendicular distance

here force = IlB

and Perpendicular distance = $b \sin\theta$

So $\tau = I l B \times b \sin\theta$

$\tau = IB\,(l \times b) \sin\theta$

$\tau = IBA \sin\theta$ (here A = Area = $l \times b$)

for N turns Torque will be

$$\boxed{\tau = NIBA \sin\theta}$$

where θ is angle b/w Area vector & magnetic field

Now, Put $NIA = m$ (magnetic moment)

$$\boxed{\tau = mB\sin\theta} \quad \text{or} \quad \boxed{\vec{\tau} = \vec{m} \times \vec{B}}$$

Torque will be max when $\theta = 90^\circ \longrightarrow \tau = NIBA$ (max)

Torque will be minimum when $\theta = 0 \longrightarrow \tau = 0$ (min)

Moving Coil Galvanometer

A galvanometer is a device to detect current in a circuit.

Principle :- A current carrying coil placed in a magnetic field experience a Torque, which rotates the coil according to the value of current.

Construction :-

A galvanometer consist of a rectangular coil of insulated copper wire wound on a metallic frame. One ends of frame is connected with the hair spring & other end is connected with a Pointer. The motion of frame is controlled by the spring. The spring provide the restoring Torque where as the Pointer measures the deflection produced on a scale.

The coil is placed between two cylindrical strong permanent magnets having concave shaped poles. A soft iron core is also placed inside the metallic frame to intensify the magnetic field.

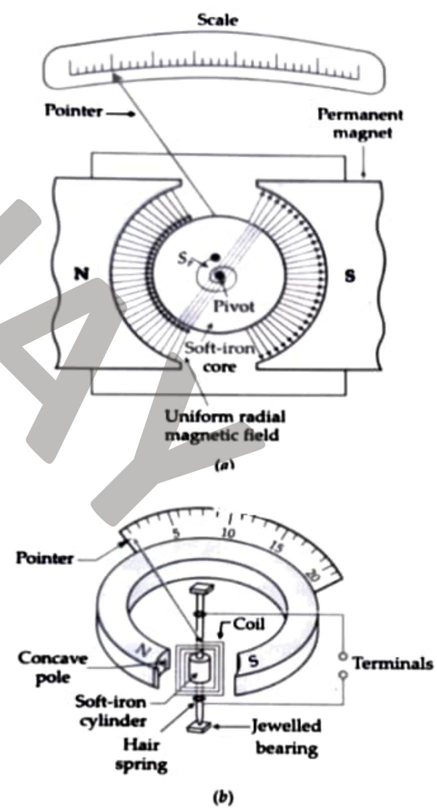
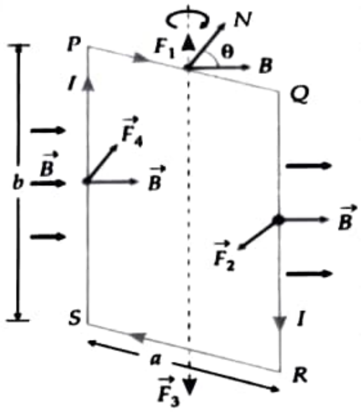


Fig. 4.94 (a) Top view (b) Front view of a pivoted-type galvanometer.

Working :- As the magnetic poles are concave shaped, the magnetic field given by them is radial, so the plane of coil always remain parallel to the magnetic field.

So the value of $F = IlB\sin\theta$ does not depend on the value of θ. as it is fixed, $\theta = 90^\circ$ at all time during rotation. So force experienced by wire PS & RQ is equal to $F = IlB$

Now according to fleming left rule,
the side PS experience an inward force whereas side QR experience an outward force.
These two forces are equal & opposite hence they exert a Torque on the coil.



$$\tau = NIBA\sin\theta$$

where N = No. of turns of the coil
I = current flowing the coil
B = Radial magnetic field
A = Area of the coil
θ = Angle b/w magnetic field and Area vectors

here $\theta = 90$ at all time of rotation. so $\sin\theta = \sin 90 = 1$

Now

$$\boxed{\tau = NIBA} \longrightarrow \text{Deflecting Torque}$$

Now a restoring Torque also acts on the coil due to the spring.

$$\boxed{\tau = K\alpha} \longrightarrow \text{Restoring Torque}$$

here K = spring constant
α = Angular Deflection

Now when pointer stops at a certain position on the scale. Then,
Restoring Torque = Deflecting Torque

$$K\alpha = NIBA$$

$$\alpha = \frac{NBA}{K} I$$

Then,
$$I = \frac{K}{NBA}\alpha$$

$$\boxed{I = G\alpha}$$

here $G = \frac{K}{NBA}$ = Galvanometer Constant

Figure of merit of Galvanometer → It is defined as the current which produce a deflection of one scale division in the galvanometer

$$G = \frac{I}{\alpha} = \frac{K}{NBA}$$

Sensitivity of Galvanometer

A galvanometer is said to be sensitive if it shows large scale deflection even when a small current is passed through it.

Current sensitivity ⇒ It is defined as the deflection produced in the galvanometer per unit current flows through it.

$$\boxed{I_s = \frac{\alpha}{I} = \frac{NBA}{K}}$$

Voltage Sensitivity

It is defined as the deflection produced in the galvanometer when a unit potential difference is applied across its ends.

$$\boxed{V_s = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NBA}{K}}$$

Factors on which Sensitivity depends :-

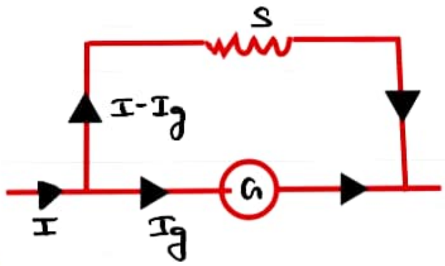
1) No. of turns (N) of the coil
2) Magnetic field B
3) Area A of the coil
4) Spring constant K of the spring.

Sensitive can be increased by :-

1) By increasing the no of turns N of the coil.
2) By increasing the value of magnetic field
3) By increasing the Area A of the coil.
4) By decreasing the value of spring constant.

Conversion of galvanometer into Ammeter.

To convert a galvanometer into ammeter a very low valued resistance is connected in parallel with the galvanometer. This low valued resistance is known as shunt.



G = Resistance of Galvanometer
I_g = Current flowing through Galvanometer
S = shunt resistance
I = Total value of current
$I - I_g$ = current flowing through shunt wire

As Potential (voltage) remains same in parallel so
Potential across G = Potential across S

$$V_G = V_S$$

$$I_g G = (I - I_g) S \longrightarrow (1)$$

$$\boxed{S = \frac{I_g}{I - I_g} \times G} \longrightarrow \text{Value of shunt.}$$

Again from eqn (1) :-

$$I_g G = (I - I_g) S$$

$$I_g G = IS - I_g S$$

$$I_g G + I_g S = IS$$

$$I_g (G + S) = IS$$

$$\boxed{I_g = \frac{S}{(G+S)} I} \longrightarrow \text{Value of current}$$

effective Resistance

$$\frac{1}{R} = \frac{1}{G} + \frac{1}{S} \Rightarrow \frac{S+G}{GS}$$

$$\boxed{R = \frac{GS}{S+G}}$$

Conversion of Galvanometer into Voltmeter

To convert a galvanometer into voltmeter by connecting a high resistance in series with it. The value of this Resistance is adjusted such that only a small current I_g passes through the Galvanometer.

G = Resistance of galvanometer

I_g = current passing through Galvanometer

R = high Resistance connected in Series.

here effective resistance = $R+G$

By ohms law,

Voltage = Current × Total Resistance

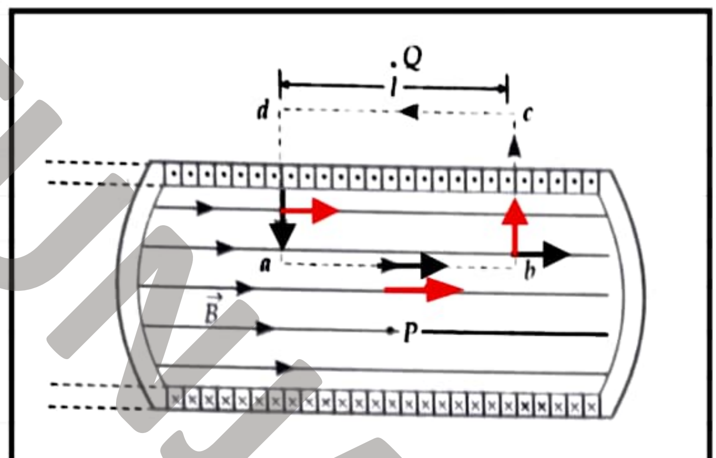
$$V = I_g \times (R+G)$$

$$\frac{V}{I_g} = R+G$$

$$\boxed{R = \frac{V}{I_g} - G}$$

Magnetic field inside a Solenoid

Let us draw an ampere loop abcd to calculate magnetic field inside the Solenoid



Acc. to Ampere circuital law

$$\boxed{\oint B\cdot dl = \mu_0 \left(\text{Total current through loop abcd}\right)}$$

Now, Taking L.H.S only:-

$$\oint B\cdot dl = \oint_{ab} B\cdot dl + \oint_{bc} B\cdot dl + \oint_{cd} B\cdot dl + \oint_{da} B\cdot dl$$

$$\oint B\cdot dl = \oint_{ab} B\,dl\cos 0^\circ + \oint_{bc} B\,dl\cos 90^\circ + \oint_{cd} 0\,dl\cos\theta + \oint_{da} B\,dl\cos 90^\circ$$

$$\oint B\cdot dl = \oint_{ab} B\,dl + 0 + 0 + 0 = \oint_{ab} B\cdot dl = B\int_{ab} dl = Bl$$

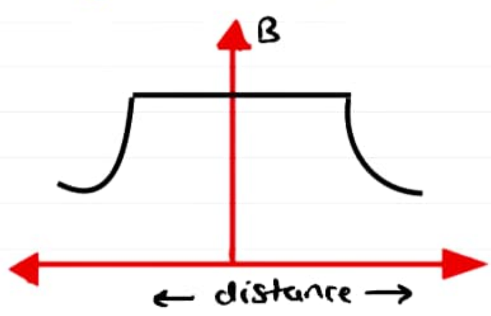
So $\boxed{\oint B\cdot dl = Bl}$ —— (1)

Now $\oint B\cdot dl = \mu_0$ (Total current through loop abcd)

$\oint B\cdot dl = \mu_0 NI$ where N is total no. of turns through loop abcd

Let n = No. of turns per unit length

$n = \frac{N}{l}$ then $N = nl$



Then, $\oint B\cdot dl = \mu_0 n l I$ —— (2)

Comparing eqn (1) & (2)

$$Bl = \mu_0 n l I$$

$$\boxed{B = \mu_0 n I}$$