ELECTRIC POTENTIAL AND CAPACITANCE

Electric potential

Work done in moving a unit positive charge from infinity to that point against the electrostatic forces.

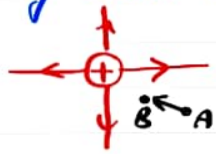
$$\boxed{\text{Electric potential} = \frac{\text{work done}}{\text{charge}}}$$

SI unit of electric potential is volt.

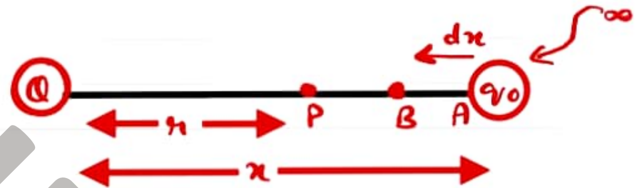
Potential difference

Work done in moving a unit positive charge from one point to other inside the electric field of other charge

$$\boxed{V_A - V_B = \frac{\text{work done}}{\text{charge}}}$$



Potential due to a point charge



Consider we bring a unit positive charge from infinity to point A which is inside the electric field of main charge Q

Now force experienced by q_0:- $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{x^2}$

Now,

Small work done to move the charge towards the charge Q

$$dw = F.dx = Fdx \cos 180$$

$$dw = -Fdx$$

Then total work done on charge q_0 to bring it to point P.

$$\int_0^w dw = \int_\infty^r -Fdx$$

$$[w]_0^w = -\int_\infty^r \frac{1}{4\pi\varepsilon}\frac{Qq_0}{x^2}dx = -\frac{1}{4\pi\varepsilon_0}Qq_0\int_\infty^r \frac{1}{x^2}dx$$

$$[w-0] = -\frac{Qq_0}{4\pi\varepsilon_0}\int_\infty^r x^{-2}dx = \frac{-Qq_0}{4\pi\varepsilon_0}\left[\frac{x^{-2+1}}{-2+1}\right]_\infty^r$$

$$w = -\frac{Qq_0}{4\pi\varepsilon_0}\left[\frac{x^{-1}}{-1}\right]_\infty^r = \frac{Qq_0}{4\pi\varepsilon_0}\left[x^{-1}\right]_\infty^r$$

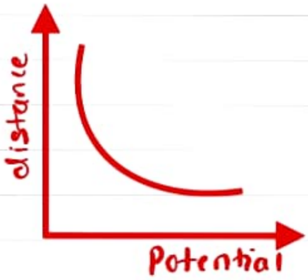
$$w = \frac{Qq_0}{4\pi\varepsilon}\left[\frac{1}{x}\right]_\infty^r = \frac{Qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$W = \frac{Qq_0}{4\pi\varepsilon}\left[\frac{1}{r} - 0\right] = \frac{1}{4\pi\varepsilon}\frac{Qq_0}{r} \quad \text{—} (1)$$

Now we know

$$\text{Potential} = \frac{\text{Work}}{\text{charge}} = \frac{W}{q_0}$$

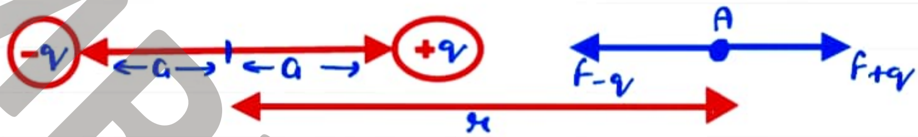
from eqn (1) $V = \frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{r} \times \frac{1}{q_0}$



$$\boxed{V = \frac{1}{4\pi\varepsilon}\frac{Q}{r}}$$

here $V \propto \frac{1}{r}$

Potential due to a dipole at axial Point



Let V_{+q} = Potential at A due to +q charge

V_{-q} = Potential at A due to −q charge

$$V_{Net} = V_{+q} + V_{-q} = \frac{1}{4\pi\varepsilon}\frac{+q}{(r-a)} + \frac{1}{4\pi\varepsilon}\frac{-q}{(r+a)}$$

$$V_{Net} = \frac{1}{4\pi\varepsilon} q \left[\frac{1}{r-a} - \frac{1}{r+a}\right] = \frac{1}{4\pi\varepsilon} q \left[\frac{(r+a)-(r-a)}{(r-a)(r+a)}\right]$$

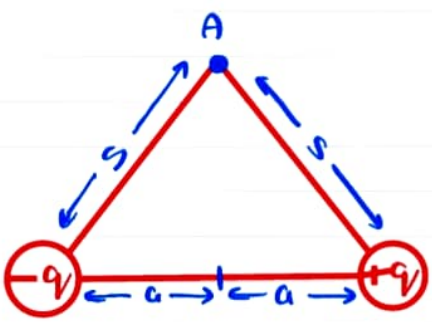
$$V_{Net} = \frac{1}{4\pi\varepsilon} q \left[\frac{r+a-r+a}{(r^2-a^2)}\right] = \frac{1}{4\pi\varepsilon} q \left[\frac{2a}{r^2-a^2}\right]$$

$$V_{Net} = \frac{1}{4\pi\varepsilon}\frac{(q \times 2a)}{r^2 - a^2}$$

here $q \times 2a$ = dipole moment

$$\boxed{V_{Net} = \frac{1}{4\pi\varepsilon}\frac{P}{r^2 - a^2}}$$

Potential at equatorial Point



Potential at Point A is given by

$$V_{Net} = V_{+q} + V_{-q}$$

$$= \left(\frac{1}{4\pi\varepsilon}\frac{+q}{s}\right) + \left(\frac{1}{4\pi\varepsilon}\frac{-q}{s}\right)$$

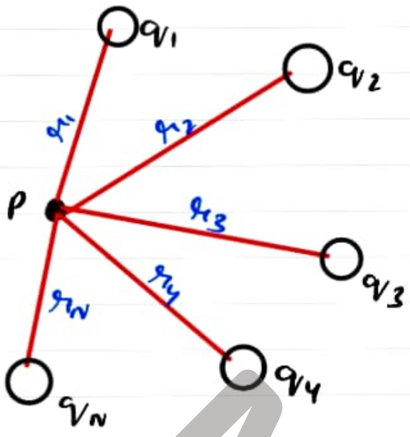
$$V_{Net} = \frac{1}{4\pi\varepsilon}\left(\frac{+q}{s} - \frac{q}{s}\right)$$

$$V_{Net} = \frac{1}{4\pi\varepsilon}[0] = 0$$

$$\boxed{V_{Net} = 0}$$

So Potential at equatorial point is zero.

Electric potential due to a system of charges



We need to find Potential at Point P due to N charges.

We know potential due to a point charge is $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Similarly for Point P:-

$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$, $V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$

$V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$ and so on for other charges

Total Potential $= V_1 + V_2 + V_3 - - - - V_N$

$V_{Net} = V_1 + V_2 + V_3 - - - V_N$

$V_{Net} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + - - - - + \frac{1}{4\pi\varepsilon_0}\frac{q_N}{r_N}$

$$\boxed{V_{Net} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q_1}{r_1} + \frac{q_2}{r_2} + - - - + \frac{q_N}{r_N}\right\}}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i}}$$

Behaviour of Conductor in Electric field

1) Net electric field inside a Conductor is always zero
2) Just outside the Surface of a charged Conductor, electric field is normal (Perpendicular) to the surface

   If electric field is not perpendicular then it will have a tangential Component along the surface which will induce Surface current. But no such current exist. So electric field is normal to the Surface.

3) Net charge inside a conductor is always zero, any excess charge always comes to the Surface of the conductor.
4) Potential inside the Conductor & on the Surface is always Constant.

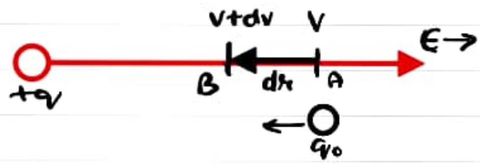
   we know $E = -\frac{dV}{dr}$ But inside Conductor $E = 0$

   So $-\frac{dV}{dr} = 0$ Or $dV = 0$

   after Integration $\boxed{V = \text{Constant}}$

5) Electric field is zero in the Cavity of a hollow Conductor.

Relation b/w electric field & Potential



Consider a charge Q located at point O.
Let A & B be two points separated by distance dr.

V_A = Potential at Point A = V

V_B = Potential at Point B = $V + dV$

Then Potential difference $V_B - V_A = V + dV - V = dV$ —— (1)

Now, Force applied on Test charge (q_0)

$F = -q_0 E$ (minus because force & Electric field are opposite)

Now work = force × displacement

$W = F\,dr$

$W = -q_0 E\,dr$ —— (2)

Also from definition of potential difference

$V_B - V_A = \frac{W}{q_0}$

$W = q_0 (V_B - V_A)$

$W = q_0\,dV$ —— (3) ($\because$ from eqn (1))

equating eqn (3) & (2)

$-q_0 E\,dr = q_0\,dV$

$$\boxed{E = -\frac{dV}{dr}}$$

Negative sign indicates that the electric field is always points towards the decreasing Potential.

Equipotential Surface

A Surface that has same potential at every point on it is called equipotential surface.

Properties:-

1) No work is done in moving a charge over an equipotential surface

we know $V_B - V_A = \frac{W}{q_0}$ But at equipotential surface $V_A = V_B$

Thus $\frac{W}{q_0} = V_B - V_B = 0$

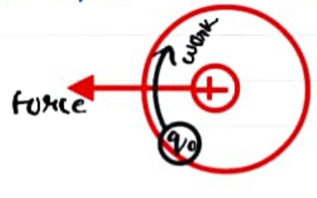
So $\boxed{W = 0}$

2) Electric field is always perpendicular to the equipotential Surface.

we know

$W = FS\cos\theta$

As $W = 0$ for moving charge at equipotential surface



so $0 = Fs\cos\theta$ (force is along electric field)

here $F \neq 0$ $s \neq 0$ so $\cos\theta = 0$ (must be)

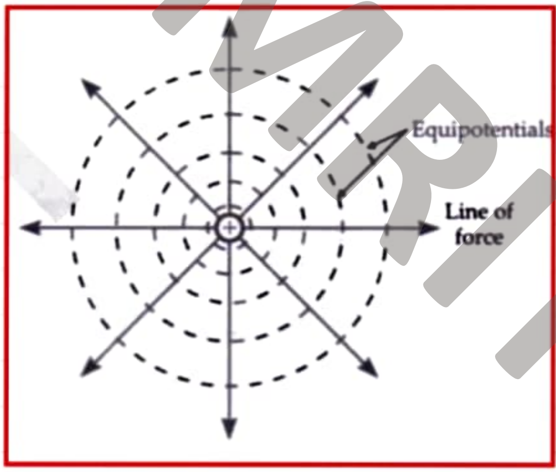
$\cos\theta = 0$ when $\theta = 90$

So $\theta = 90°$ is angle b/w force (electric field) & displacement

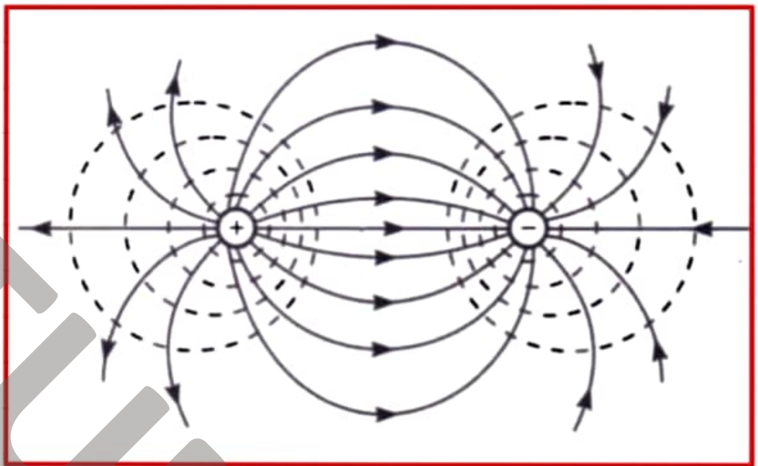
3) Equipotential surface are closer together in regions of strong field & farther apart in the regions of weak field.

4) No two equipotential surface can intersect each other.

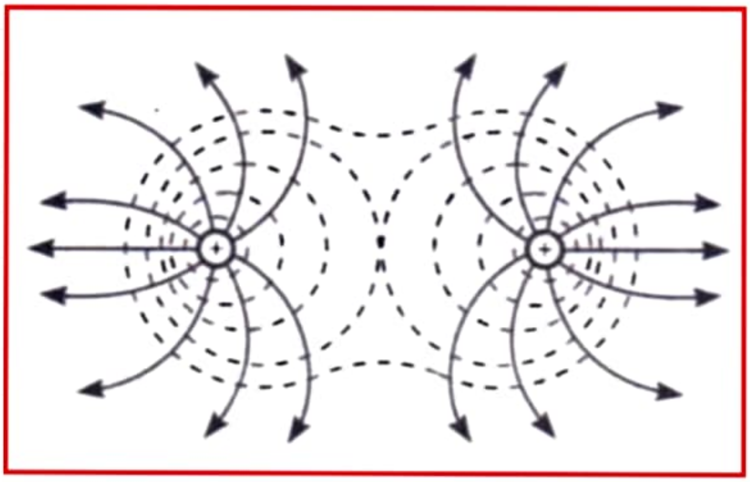
Equipotential Surface



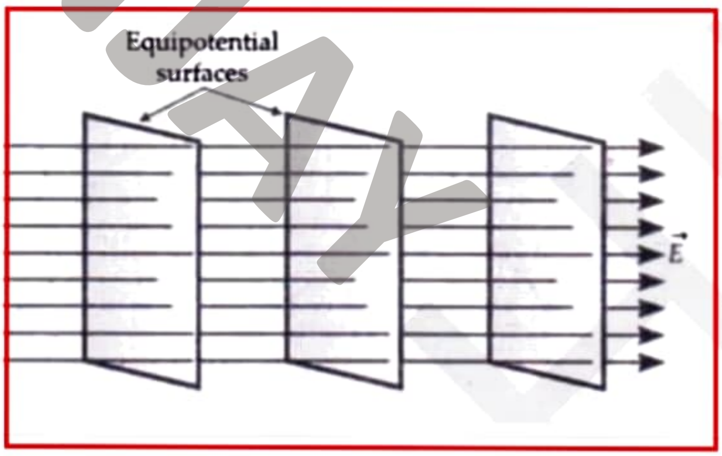
Point charge



Equipotential Surface due to a dipole



Equipotential Surface due to Same type of charges

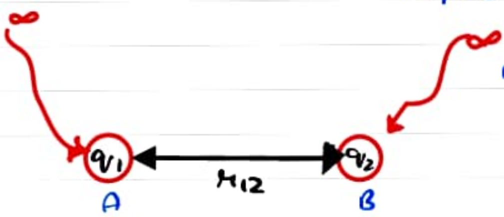


Equipotential Surface due to Uniform electric field

Potential energy due to a system of two charges

w_1 = work done to bring q_1 from infinity to point A

$w_1 = 0$ ($\because$ Because no electric field is present)



w_2 = work done to bring q_2 from infinity to point B :-

$w_2 = V_1 q_2$

here V_1 = Potential due to charge q_1

$$w_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{12}} q_2$$

Total work = $w_1 + w_2$

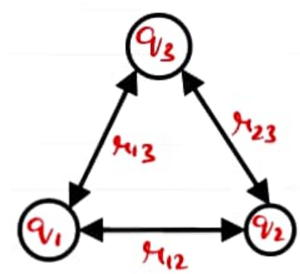
$$= 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

This work is converted into Potential energy

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}}$$

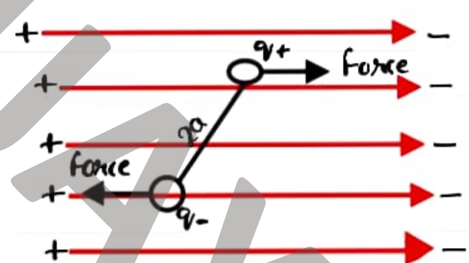
For a system of 3 charges, Potential energy is

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}}\right]}$$



Potential energy of a dipole in electric field

when a dipole is kept inside a uniform electric field, it experience a equal force in opposite directions as shown in figure. Thus Net force = 0



Now

we know $\tau = pE \sin\theta$

where p = dipole moment E = Electric field

Now using:-

Work = Torque × Angular displacement

for small displacement $d\theta$, work will be dw. so

$$dw = \tau \times d\theta$$

$$dw = pE\sin\theta \, d\theta$$

Integrating both sides :- $\int_0^w dw = \int_{\theta_1}^{\theta_2} pE\sin\theta \, d\theta$

$$[w]_0^w = pE[-\cos\theta]_{\theta_1}^{\theta_2} = -pE[\cos\theta]_{\theta_1}^{\theta_2}$$

$$[w - 0] = -pE[\cos\theta_2 - \cos\theta_1]$$

$$w = -pE[\cos\theta_2 - \cos\theta_1]$$

This work is converted into Potential energy

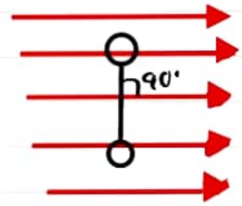
$$\boxed{U = -pE\left[\cos\theta_2 - \cos\theta_1\right]}$$

Now let initially dipole is kept perpendicular to the electric field.

Then $\theta_1 = 90°$

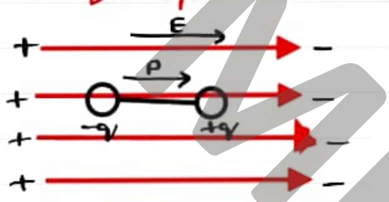
$$U = -pE\left[\cos\theta_2 - \cos 90\right] = -pE\left[\cos\theta_2 - 0\right]$$

$$\boxed{U = -pE\cos\theta_2}$$



Special cases:-

1) Stable equilibrium



here $\vec{P} \rightarrow$ $\vec{E} \rightarrow$

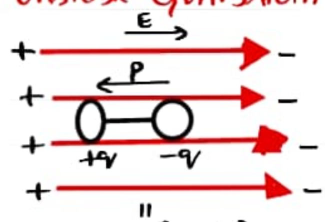
so $\theta_2 = 0$ & $\theta_1 = 90$

Thus $U = -pE(\cos\theta_2)$

$U = -pE(\cos 0)$

$$\boxed{U = -pE}$$

2) Unstable equilibrium



here $\overleftarrow{P}$ $\overrightarrow{E}$ 180°

so $\theta_2 = 180$ & $\theta_1 = 90°$

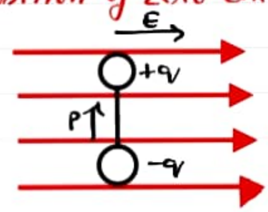
$U = -pE(\cos\theta_2)$

$U = -pE(\cos 180°)$

$U = -pE(-1)$

$$\boxed{U = pE}$$

3) Position of zero energy



here $\theta_2 = 90°$

so

$U = -pE(\cos\theta_2)$

$U = -pE(\cos 90°)$

$U = -pE(0)$

$$\boxed{U = 0}$$

Capacitance The electrical capacitance of a conductor is the capacity to hold electrical charge

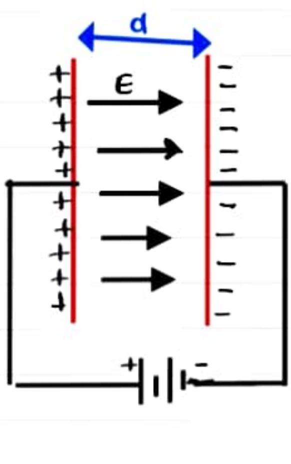
$$\boxed{\text{Capacitance} = \frac{\text{Charge}}{\text{Potential}}} \quad \text{or} \quad \boxed{C = \frac{Q}{V}}$$

Capacitance depends on:-

1) Size & shape of the conductor
2) Nature of surrounding medium
3) Presence of other conductors near it.

SI unit of Capacitance is Farad (F)

Parallel plate Capacitor



Let A = Area of plate

d = distance b/w plates

σ = Surface charge density

Q = charge on each plate

Now Capacitance = $\dfrac{\text{Charge}}{\text{Potential}}$ — (1)

Now surface charge density, $\sigma = \frac{Q}{A}$

Then $Q = \sigma A$ —— (2)

Also,

Potential = Electric field × distance

$V = Ed$

$V = \frac{\sigma}{\varepsilon_0} d$ —— (3) ($\because$ electric field inside the plates is $= \sigma/\varepsilon_0$)

Put (2), (3) in (1) eqn:-

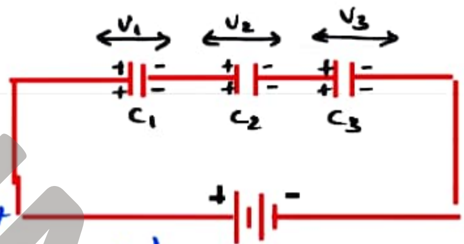
Then

$$C = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0} d} = \frac{\varepsilon_0 A}{d}$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

So Capacitance depends upon:-
1) Area of plate
2) Permittivity of the medium
3) distance b/w plates

Capacitance in Series



When capacitors are arranged in Series, charge on each capacitor is same = Q

But Potential of each capacitor is different (V_1 across C_1), (V_2 across C_2) & (V_3 across C_3)

Now using formula $C = Q/V$

for Capacitance C_1 :-

$C_1 = \frac{Q}{V_1}$

$V_1 C_1 = Q$

$V_1 = \frac{Q}{C_1}$

for capacitance C_2

$C_2 = \frac{Q}{V_2}$

$V_2 C_2 = Q$

$V_2 = \frac{Q}{C_2}$

for capacitance C_3

$C_3 = \frac{Q}{V_3}$

$V_3 C_3 = Q$

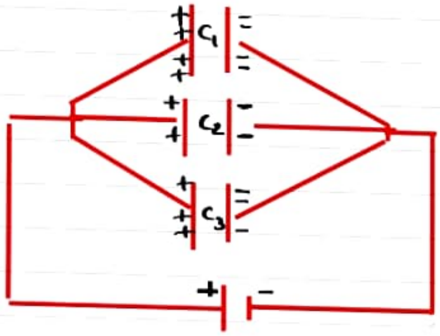
$V_3 = \frac{Q}{C_3}$

Total potential $V_{Net} = V_1 + V_2 + V_3$

$$\frac{Q}{C_{Net}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

Thus $$\boxed{\frac{1}{C_{Net}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Capacitors in parallel



When capacitors are arranged in parallel voltage across each capacitor is same But charge stored on each capacitor is different (Q_1 in C_1), (Q_2 in C_2) & (Q_3 in C_3)

Now using formula $C = \frac{Q}{V}$

For capacitance C_1:-
$C_1 = \frac{Q_1}{V}$
$Q_1 = C_1 V$

For capacitance C_2:-
$C_2 = \frac{Q_2}{V}$
$Q_2 = C_2 V$

For capacitance C_3:-
$C_3 = \frac{Q_3}{V}$
$Q_3 = C_3 V$

Total charge, $Q_{Net} = Q_1 + Q_2 + Q_3$

$C_{Net} V = C_1 V + C_2 V + C_3 V$

$$\boxed{C_{Net} = C_1 + C_2 + C_3}$$

Energy stored in a capacitor

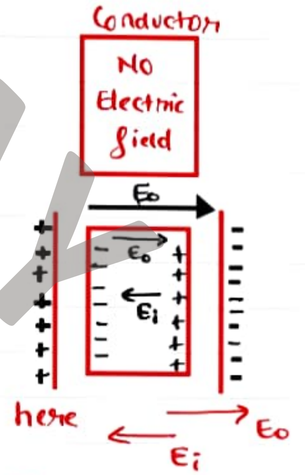
(No Derivation, only formula in syllabus)

Energy stored, $\boxed{U = \frac{1}{2} CV^2}$ or $\boxed{U = \frac{1}{2} \frac{Q^2}{C}}$ or $\boxed{U = \frac{1}{2} QV}$

Behaviour of Conductor in external field

When a conductor is placed inside an external electric field, the free electron present inside the conductor starts drifting towards the positive plate & the positive ions of the conductor starts moving towards the negative plate. Due to which the electric field is set up inside the conductor (E_i) This process continues till the internal electric field is equal to the external field.

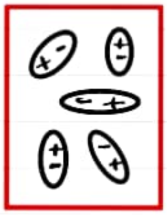
Now the two electric field (External field & internal electric field) Becomes equal inside the conductor. Thus net electric field is zero.



Thus $E_{Net} = E_0 - E_i$

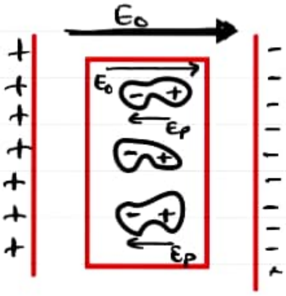
$\boxed{E_{Net} = 0}$

Behaviour of dielectric (Insulator) in external electric field



A dielectric is a type of Insulator which doesnot have free electrons but have polar or non polar molecules which can be polarised in external electric field.

When a dielectric is placed inside the external field the dipole present inside the dielectric starts rotating in such a way, that -ve part of dipole points towards positive plate & +ve part of dipole points towards negative plate.



Due to the shifting of dipole, a small electric field develops inside the dielectric. This electric field is known as Polarised Electric field (E_p). However this electric field is never able to completely cancel out the external electric field.

Thus net electric field inside the dielectric

$$\boxed{E_i = E_0 - E_p}$$

here $\overrightarrow{E_0}$

$\overleftarrow{E_p}$

Thus

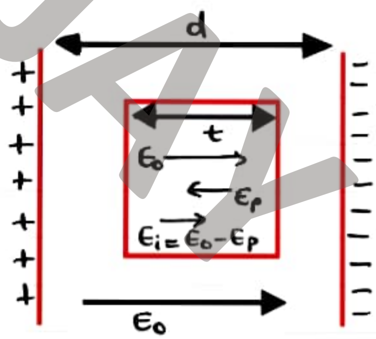
$$\boxed{E_i = E_0 - E_p}$$

Dielectric Constant

The ratio of external electric field (E_0) to the reduced electric field (Inside Dielectric) is called Dielectric constant (K) or relative permittivity.

$$\boxed{K = \frac{\vec{E_0}}{\vec{E_i}} = \frac{\vec{E_0}}{\vec{E_0} - \vec{E_p}}}$$

Parallel plate capacitor with dielectric



Let A = Area of plate

d = distance b/w plates

t = thickness of dielectric

E_0 = External electric field

E_p = polarised electric field

E_i = Reduced electric field inside dielectric

σ = Surface charge density of capacitor

Q = Charge stored in plates.

Now using Capacitance = $\dfrac{\text{Charge}}{\text{Potential}}$ —— (1)

we know surface charge density, $\sigma = \dfrac{Q}{A}$

$Q = \sigma A$ —— (2)

Now

Potential = Electric field × distance

$V = (\text{outside field} \times \text{outside distance}) + (\text{Inside field} \times \text{Inside distance})$

$$V = \{E_0 \times (d-t)\} + \{E_i \times t\}$$

Now,

using $K = \frac{E_0}{E_i}$ then $E_i = \frac{E_0}{K}$

After putting value of E_i :- $V = \{E_0 (d-t)\} + \{\frac{E_0}{K} \times t\}$

$$V = E_0 \left[d - t + \frac{t}{K} \right]$$

$$V = \frac{\sigma}{\varepsilon_0} \left[d - t + \frac{t}{K} \right] \quad \left(\because E_0 = \frac{\sigma}{\varepsilon_0} \right)$$

③

Putting eqn ② & ③ in eqn ①

$$C = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0} \left[d - t + \frac{t}{K} \right]}$$

Thus $\boxed{C = \frac{\varepsilon_0 A}{\left[d - t + \frac{t}{K} \right]}}$

here
ε_0 = Permittivity
A = Area
K = dielectric constant

Effect on Various parameters due to dielectric

Battery disconnected from the capacitor	Battery kept connected across the capacitor
$Q = Q_0$ (constant)	$Q = \kappa Q_0$
$V = \frac{V_0}{\kappa}$	$V = V_0$ (constant)
$E = \frac{E_0}{\kappa}$	$E = E_0$ (constant)
$C = \kappa C_0$	$C = \kappa C_0$
$U = \frac{U_0}{\kappa}$	$U = \kappa U_0$

here
Q = charge
V = Potential
E = Electric field
C = Capacitance
U = Energy

K = dielectric constant