

# 12

## THERMODYNAMICS

### 12.1 THERMODYNAMICS AND RELATED TERMS

**✓ 1.** Define the term thermodynamics. How does it differ from kinetic theory of gases and mechanics?

**Thermodynamics.** Thermodynamics is the branch of science that deals with the concepts of heat and temperature and the inter-conversion of heat and other forms of energy. It mainly deals with the transformation of heat into mechanical work and vice versa. Its great success lies in the fact that it explains the bulk properties of matter in terms of few macroscopic variables such as pressure, volume, temperature, mass and composition which are easily observable and directly measurable.

**Thermodynamics versus kinetic theory of gases.** Thermodynamics is a macroscopic science. It deals with bulk systems without going into the molecular constitution of matter. On the other hand, kinetic theory of gases deals with the molecular distribution of velocities in any gas.

**Thermodynamics versus mechanics.** Mechanics deals with motion of a system as a whole (in fact with its centre of mass) under the action of several forces and torques. Thermodynamics is not concerned with the motion of a system as a whole. It is concerned with the internal macroscopic state of the body. When a bullet is fired, its mechanical state changes due to the increase in the K.E. of its centre of mass but its temperature is not affected. But when the bullet pierces a wood and stops, its kinetic energy changes into heat increasing the temperature of the bullet and surround-

ing wooden layers. Thus temperature is related to the internal disordered motion of the molecules of the bullet, not to the motion of the bullet as a whole.

**✓ 2.** Define the terms thermodynamic system, surroundings, thermodynamic variable and equation of state.

(i) **Thermodynamic system.** An assembly of a very large number of particles having a certain value of pressure, volume and temperature is called a thermodynamic system.

(ii) **Surroundings.** Everything outside the system which can have a direct effect on the system is called its surroundings.

(iii) **Thermodynamic variables.** The quantities like pressure ( $P$ ), volume ( $V$ ), and temperature ( $T$ ) which help us to study the behaviour of a thermodynamic system are called thermodynamic variables.

(iv) **Equation of state.** The mathematical relation between the pressure, volume and temperature of a thermodynamic system is called its equation of state. For example, the equation of state for  $n$  moles of an ideal gas can be written as

$$PV = nRT$$

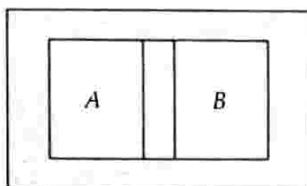
### 12.2 THERMAL EQUILIBRIUM

**✓ 3.** Define thermal equilibrium. How is it attained?

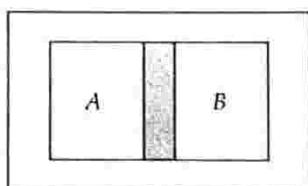
**Thermal equilibrium.** Two systems are said to be in thermal equilibrium with each other if they have the same temperature.

Consider two gases  $A$  and  $B$  contained in two different vessels. Let the pressure and volume of the

gases by  $(P_A, V_A)$  and  $(P_B, V_B)$  respectively. As shown in Fig. 12.1(a), if the two vessels are separated by an *adiabatic wall* (an insulating wall that does not allow the flow of heat), then any possible pair of values  $(P_A, V_A)$  will be in equilibrium with any possible pair of values  $(P_B, V_B)$ .



(a) Gases A and B separated by an adiabatic wall.



(b) Gases A and B separated by a diathermic wall.

**Fig. 12.1**

As shown in Fig. 12.1(b), if A and B are now separated by a *diathermic wall* (a conducting wall that allows heat to flow through it), then the pressure and volume variables of the two gases change to  $(P'_A, V'_A)$  and  $(P'_B, V'_B)$  such that the new states of A and B are in equilibrium with each other. There is no more flow of heat. The two systems attain equal temperature and we say they are in equilibrium with each other.

### 12.3 THERMODYNAMIC EQUILIBRIUM

✓ 4. When is a system said to be in the state of thermodynamic equilibrium?

**Thermodynamic equilibrium.** A system is said to be in the state of thermodynamic equilibrium if the macroscopic variables describing the thermodynamic state of the system do not change with time. Consider a gas inside a closed rigid container completely insulated from the surroundings. If the pressure, volume, temperature, mass and composition of the gas do not change with time, then it is in a state of thermodynamic equilibrium.

A system in the state of thermodynamic equilibrium possesses the following equilibria simultaneously:

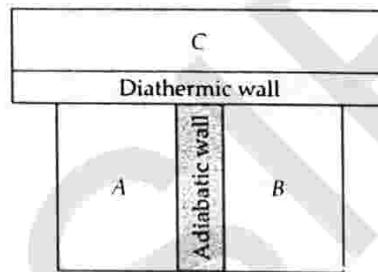
- Mechanical equilibrium.** There is no unbalanced force in its interior or between the system and the surroundings.
- Thermal equilibrium.** All parts of the system and the surroundings are at the same temperature.
- Chemical equilibrium.** The system does not undergo any spontaneous change in its internal structure due to chemical reaction, diffusion, etc.

### 12.4 ZEROTH LAW OF THERMODYNAMICS

✓ 5. State and explain zeroth law of thermodynamics. How does it lead to the concept of temperature?

**Zeroth law of thermodynamics.** It states that if two systems A and B are separately in thermal equilibrium with

a third system C, then A and B are also in thermal equilibrium with each other. Fig. 12.2 shows two systems A and B separated by an adiabatic wall (through which heat does not flow). They are separated from system C by a diathermic wall (through which heat can flow). The systems A and C and B and C will reach thermal equilibrium separately. If adiabatic wall between A and B is removed, there will be no exchange of heat showing that A and B are already in thermal equilibrium. □



**Fig. 12.2** Zeroth law of thermodynamics

◀ **Concept of temperature.** Zeroth law of thermodynamics implies that *temperature is a physical quantity which has the same value for all systems which are in thermal equilibrium with each other*. Temperature of a system determines whether it is in thermal equilibrium or not with another system. Thus if A and B are separately in thermal equilibrium C,  $T_A = T_C$  and  $T_B = T_C$ . This shows that  $T_A = T_B$  i.e., the systems A and B are also in thermal equilibrium. □

As the modern concept of temperature follows from the zeroth law of thermodynamics, so this law may be stated in a more fundamental way as follows:

*There exists a scalar quantity called temperature which is a property of all thermodynamic systems such that temperature equality is a necessary and sufficient condition for thermal equilibrium.*

### For Your Knowledge

- ▲ The temperature which was first defined as the *degree of hotness* and later on the *condition determining the flow of heat*, is now regarded as *one of the seven fundamental quantities* like mass, length, time, etc.
- ▲ The zeroth law of thermodynamics was formulated by R.H. Fowler in 1931 long after the First and Second laws of thermodynamics were stated. But as this law leads to the concept of the fundamental quantity temperature, so this law was called the zeroth law.

### 12.5 HEAT, INTERNAL ENERGY AND WORK

6. What do you mean by internal energy of a system? Is it a state variable? What is the nature of the internal energy of an ideal gas?

**Internal energy.** The internal energy of a system is the sum of molecular kinetic and potential energies in the frame of reference relative to which the centre of mass of the system is at rest.

The molecules of a real gas exert mutual force of attraction on one another. Hence they possess intermolecular potential energy. If the volume of the gas increases, work is done by the gas against intermolecular attraction and so its potential energy increases. Thus *intermolecular potential energy of a real gas is a function of its volume*.

The molecules of a gas are always in a state of random motion. The motion may be translational, rotational and vibrational. Hence the molecules possess kinetic energy. As the temperature increases, the average kinetic energy of the gas molecules also increases. Thus *the internal kinetic energy of a gas is a function of its temperature*.

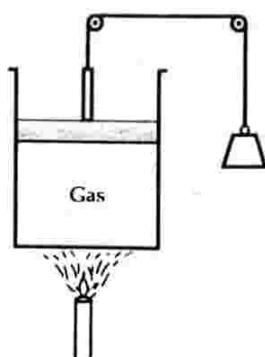
The internal energy does not include the over-all kinetic energy of the system as a whole. It includes only the (disordered) energy associated with the random motion of the molecules of the system. We denote it by  $U$ .

**Internal energy of a system is a thermodynamic state variable.** That is its value depends only on the state of existence of the system and not on the path along which that state has been brought about. Thus the internal energy of a given mass of a gas depends only on its state described by the specific values of pressure, volume and temperature.

**Internal energy of an ideal gas is purely kinetic in nature.** In an ideal gas, there are no molecular forces of attraction. So the gas does not possess intermolecular potential energy. Its internal energy is just the sum of kinetic energies associated with various random (translational, rotational and vibrational) motions of its molecules. Thus the internal energy of an ideal gas is wholly kinetic in nature and depends only on its temperature.

7. Describe the two ways of changing the internal energy of a system. Are heat and work state variables?

**Heat and work as two distinct modes of energy transfer.** As shown in Fig. 12.3, consider a fixed mass of gas in a cylinder provided with a movable piston. The internal energy of the gas can be increased in two ways:



**Fig. 12.3** Heat and work as two different modes of energy transfer to a system.

(i) Place the cylinder over a hot body. Heat energy will flow from the hotter body to the gas due to temperature difference. This increases the internal energy of the gas.

(ii) Push the piston down by raising some weight attached to it. Work is done on the gas. This also increases the internal energy of the gas.

No, heat and work are not state variables.

### 8. Give difference between heat and work.

#### Difference between heat and work.

(i) Heat is a mode of energy transfer due to temperature difference between the system and the surroundings. Work is the mode of energy transfer brought about by means that do not involve temperature difference such as moving the piston of a cylinder containing the gas, by raising or lowering the weight connected to it.

(ii) When heat is supplied to a gas, its molecules move faster in all directions at random. So heat is a mode of energy transfer that produces random motion, when a piston compresses a gas to do work on it, it forces the molecules to move in the direction of piston's motion. So work may be regarded as the mode of energy transfer that produces organised motion.

9. State the sign conventions used in the measurement of heat, work and internal energy.

#### Sign conventions used :

- (i) Heat absorbed by a system is positive. Heat given out by a system is negative.
- (ii) Work done by a system is positive. Work done on a system is negative.
- (iii) The increase in internal energy of a system is positive. The decrease in internal energy of a system is negative.

► The thermodynamic state of a system is characterised by its internal energy, not heat. That is, internal energy is a state variable. For this reason, the statement like 'a gas in given state has a certain amount of energy' is meaningful.

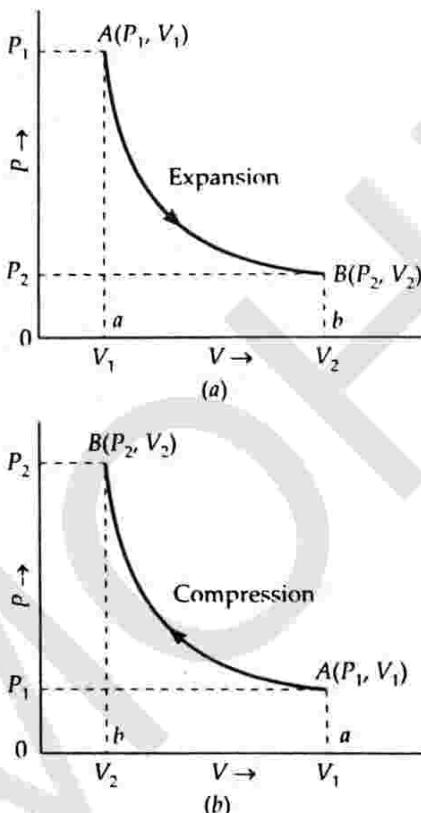
► In thermodynamics, **heat and work are not state variables**. These are the modes of energy transfer to a system resulting in the change in its internal energy. Thus the statement like 'a gas in a given state has certain amount of heat or work' is meaningless, on the other hand, the statement like 'a certain amount of heat is supplied to the system or a certain amount of work is done by the system' is meaningful.

## 12.6 INDICATOR DIAGRAM

**10.** What is an indicator diagram? What is its importance?

**Indicator diagram.** The state of a thermodynamic system can be completely described if only two thermodynamical variables are known because the third variable gets automatically fixed by the equation of the state of the system. A graphical representation of the state of system with the help of two thermodynamical variables is called an indicator diagram. A graph drawn between the pressure and volume of a gas under thermodynamic operation is called P-V diagram. Such diagrams are drawn with the help of a device called indicator which records the changes in volume and pressure accompanying the movement of the piston in the cylinder.

Fig. 12.4(a) shows P-V diagram for a system undergoing expansion from the state  $A(P_1, V_1)$  to  $B(P_2, V_2)$ , while Fig. 12.4(b) shows the P-V diagram for a system undergoing compression from the state  $A(P_1, V_1)$  to  $B(P_2, V_2)$ .



**Fig. 12.4** Indicator diagrams.

**Importance of P-V diagram.** The area under the P-V diagram is numerically equal to the work done by a system or on the system.

## 12.7 WORK DONE DURING EXPANSION

**11.** Describe an analytical method for determining the work done during the expansion of gas.

**Analytical method for the work done during expansion.** Consider a gas contained in a cylinder of cross-sectional area  $A$  and provided with a frictionless movable piston. Let  $P$  be the pressure of the gas.

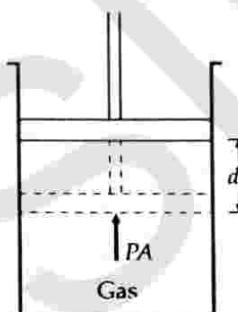
Force exerted by the gas on the piston,

$$F = P \times A$$

Suppose the gas expands a little and pushes out the piston through a small distance  $dx$ . The work done by the gas is

$$dW = F dx = PA dx = P dV$$

where  $dV = Adx$ , is the change in volume of the gas.



**Fig. 12.5** Work done during expansion.

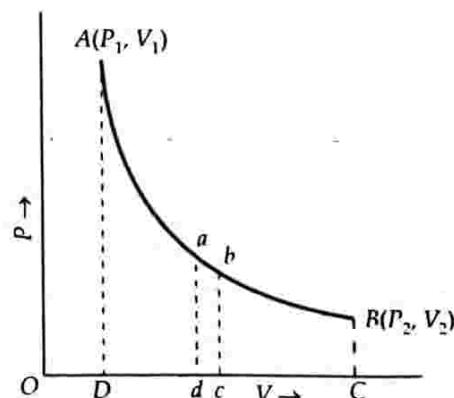
The total work done by the gas when its volume increases from  $V_1$  to  $V_2$  will be

$$W = \int dW = \int_{V_1}^{V_2} P dV.$$

**12.** What is non-cyclic process? Show that the area under the P-V diagram gives the work done by a system in a non-cyclic process.

**Non-cyclic process.** A non-cyclic process is one in which the system does not return to its initial state.

**Indicator diagram method for the work done during expansion.** In Fig. 12.6, the points  $A$  and  $B$  represent the initial state  $(P_1, V_1)$  and final state  $(P_2, V_2)$  respectively of a system on a P-V diagram. At any point  $a$ , let  $P$  and  $V$  be the pressure and volume



**Fig. 12.6** Indicator diagram method

respectively. Suppose that the volume increases from  $V$  to  $V + dV$  corresponding to a point  $b$  on the indicator diagram such that the pressure remains constant. Then

$$ad = bc = P \quad \text{and} \quad cd = dV$$

The small work done when the system changes from state  $a$  to state  $b$ ,

$$dW = PdV = ad \times cd \\ = \text{area of shaded strip } abcd$$

The total work done by the gas during the expansion from the initial state  $A(P_1, V_1)$  to the final state  $B(P_2, V_2)$  can be obtained by adding the areas of all such strips formed between  $AD$  and  $BC$  under the  $P$ - $V$  diagram. Clearly, then the total work done will be

$$W = \text{area } ABCDA \\ \text{or } W = \int_{V_1}^{V_2} P dV = \text{area under } P-V \text{ diagram}$$

Hence the work done by a system is numerically equal to the area enclosed between the  $P$ - $V$  diagram and the volume-axis.

During expansion, the area under the  $P$ - $V$  diagram is traced in the clockwise direction. Work is done by the gas and is taken positive.

During compression, the area under the  $P$ - $V$  diagram is traced in the anticlockwise direction. Work is done on the gas and is taken negative.

## 12.8 WORK DONE DURING A CYCLIC PROCESS

**13. What is cyclic process? Prove that the net work done during a cyclic process is numerically equal to the area of the loop representing the cycle.**

**Cyclic process.** Any process in which the system returns to its initial state after undergoing a series of changes is known as a cyclic process.

**Work done during a cyclic process.** Suppose a gas expands from the initial state  $A$  to the final state  $B$  after undergoing a series of changes of pressure and volume, along the path  $AXB$ .

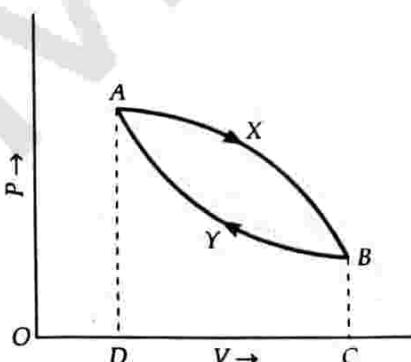


Fig. 12.7  $P$ - $V$  diagram for a cyclic process

Work done by the gas during the expansion is

$$W_1 = + \text{area } A \times BCDA$$

Now the gas is subjected to a different series of changes of pressure and volume so that it returns to its initial state  $A$  via the path  $BYA$ .

Work done on the gas during the compression is

$$W_2 = - \text{area } BYADCB$$

During compression, work is done on the gas which is taken negative.

The net work done during the cyclic process is

$$W = W_1 + W_2 \\ = \text{area } AXBCDA - \text{area } BYADCB \\ = + \text{area } AXBYA$$

**Conclusion.** For a cyclic process :

- (i) Work done per cycle is numerically equal to the area of the loop representing the cycle.
- (ii) If the closed loop is traced in the clockwise direction, the expansion curve lies above the compression curve ( $W_1 > W_2$ ), the area of the loop is positive, indicating that the net work is done by the system.
- (iii) If the closed loop is traced in the anticlockwise direction, the expansion curve lies below the compression curve, ( $W_1 < W_2$ ), the area of the loop is negative, indicating that the net work is done on the system.

### Examples based on Work Done during a Cyclic Process

#### CONCEPTS USED

1. Work done during the expansion or compression of a gas is equal to the area enclosed between the  $P$ - $V$  curve and the volume axis.
2. Work done per cycle  
= Area of the loop representing the cycle.
3. If the loop is traced clockwise, the work done is positive and work is done by the system.
4. If the loop is traced anticlockwise, the work done is negative and work is done on the system.

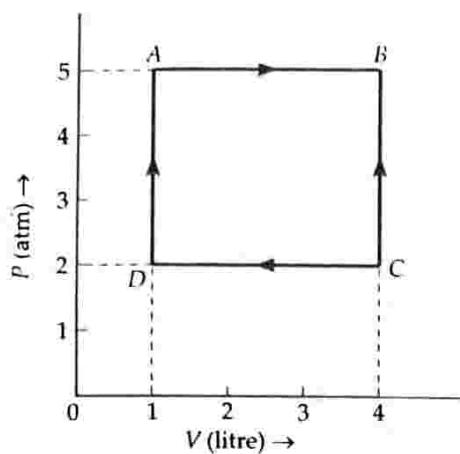
#### UNITS USED

When pressure  $P$  is in  $\text{Nm}^{-2}$  and volume  $V$  in  $\text{m}^3$ , the work done  $W$  is in joule.

**EXAMPLE 1.** One mole of an ideal gas undergoes a cyclic change  $ABCD$ . From the given diagram (Fig. 12.8), calculate the net work done in the process.

$$1 \text{ atm} = 10^6 \text{ dyne cm}^{-2}$$

**Solution.** The work done in a cyclic process is equal to the area of the loop.


**Fig. 12.8**

As the loop  $ABCD$  is traced in the clockwise direction, the work done is positive.

$$\therefore W = + \text{Area } ABCD = DC \times AD$$

$$\text{Now } DC = 4 - 1 = 3 \text{ litre} = 3 \times 10^3 \text{ cm}^3$$

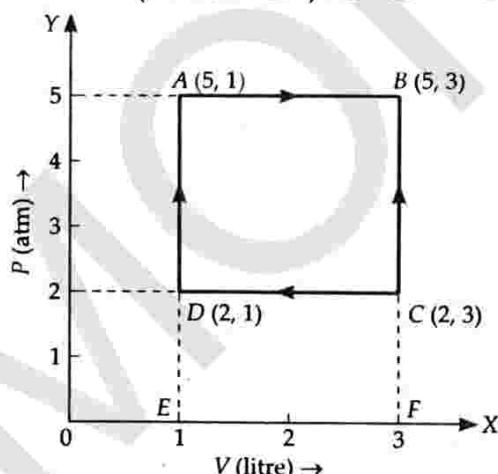
$$AD = 5 - 2 = 3 \text{ atm} = 3 \times 10^6 \text{ dyne cm}^{-2}$$

$$\therefore W = 3 \times 10^3 \times 3 \times 10^6 = 9 \times 10^9 \text{ erg.}$$

**EXAMPLE 2.** One mole of an ideal gas undergoes a cyclic change  $ABCD$  where the  $(P, V)$  co-ordinates are  $A(5, 1)$ ,  $B(5, 3)$ ,  $C(2, 3)$  and  $D(2, 1)$ .  $P$  is in atmosphere and  $V$  is in litre. Calculate work done along  $AB$ ,  $BC$ ,  $CD$  and  $DA$  and also net work done in the process. Given 1 atmosphere  $= 1.01 \times 10^5 \text{ Nm}^{-2}$ .

**Solution.** (i) Work done along  $AB$

$$W_1 = \text{area } ABFEA = EA \times EF \\ = (5 \times 1.01 \times 10^5) \times 2 \times 10^{-3} = 1010 \text{ J.}$$


**Fig. 12.9**

(ii) Work done along  $BC$ ,  $W_2 = PdV = 0$ .

(iii) Work done along  $CD$

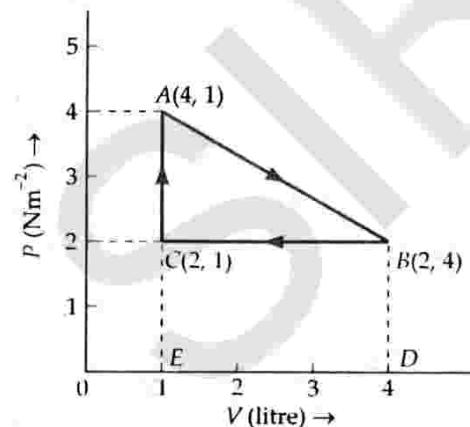
$$W_3 = -\text{area } CDEF = -EF \times CF \\ = -(2 \times 1.01 \times 10^5) \times 2 \times 10^{-3} = -404 \text{ J.}$$

(iv) Work done along  $DA$ ,  $W_4 = PdV = 0$ .

Net work done in the process

$$W = W_1 + W_2 + W_3 + W_4 \\ = 1010 + 0 - 404 + 0 = 606 \text{ J.}$$

**EXAMPLE 3.** The  $P$ - $V$  diagram (Fig. 12.10), for a cyclic process is a triangle  $ABC$  drawn in order. The coordinates of  $A, B, C$  are  $(4, 1)$ ,  $(2, 4)$  and  $(2, 1)$ . The coordinates are in the order  $P, V$ . Pressure is in  $\text{Nm}^{-2}$  and volume is in litre. Calculate the work done during the process from  $A$  to  $B$ ,  $B$  to  $C$  and  $C$  to  $A$ . Also calculate the work done in the complete cycle.


**Fig. 12.10**

**Solution.** (i) Work done during the process from  $A$  to  $B$  (expansion) is

$$W_{AB} = + \text{Area of trapezium } ABDEA \\ = \text{Area of } \triangle ABC + \text{Area of rectangle } BDEC \\ = \frac{1}{2} BC \times AC + DE \times CE \\ = \frac{1}{2} (4 - 1) \text{ litre} \times (4 - 2) \text{ Nm}^{-2} + (4 - 1) \text{ litre} \\ \times (2 - 0) \text{ Nm}^{-2} \\ = \frac{1}{2} \times 3 \times 10^{-3} \text{ m}^3 \times 2 \text{ Nm}^{-2} + 3 \times 10^{-3} \text{ m}^3 \\ \times 2 \text{ Nm}^{-2} \\ = 9 \times 10^{-3} \text{ J.}$$

As the gas is expanding, so the work done is positive.

(ii) Work done during the process from  $B$  to  $C$  (compression) is

$$W_{BC} = -BCDE = -DE \times CE \\ = -3 \times 10^{-3} \times 2 = -6 \times 10^{-3} \text{ J.}$$

As the gas is compressed, so the work done is negative.

(iii) Work done during the process from  $C$  to  $A$  is

$$W_{CA} = 0$$

This is because there is no change in volume in going from  $C$  to  $A$ .

(iv) Work done in the complete cycle,

$$W = + \text{Area } ABC = W_{AB} + W_{BC} + W_{CA} \\ = 3 \times 10^{-3} \text{ J.}$$

Here  $W$  is positive because the cycle  $ABCA$  is traced in the clockwise direction.

## X PROBLEMS FOR PRACTICE

- An ideal monoatomic gas is taken round the cycle ABCDA, where co-ordinates of A, B, C, D on P-V diagram are A( $p, V$ ), B( $2p, V$ ), C( $2p, 2V$ ) and D( $p, 2V$ ). Calculate work done during the cycle.  
**(Ans.  $pV$ )**
- Calculate net work done by the gas whose thermodynamical behaviour is represented by right angled triangle ABC on P-V diagram. The P-V co-ordinates are A(20, 6), B(10, 12) and C(10, 6), where  $P$  is in  $\text{Nm}^{-2}$  and  $V$  is in  $\text{m}^3$ .  
**(Ans. 30 J)**

## 12.9 FIRST LAW OF THERMODYNAMICS

- ✓ 14.** State and explain first law of thermodynamics.

**First law of thermodynamics.** It is simply the law of conservation of energy applied to any system in which energy transfer (in the form of heat or work) from or to the surroundings is taken into consideration. According to the first law of thermodynamics, *if some heat is supplied to a system which is capable of doing work, then the quantity of heat absorbed by the system will be equal to the sum of the increase in its internal energy and the external work done by the system on the surroundings.*

Let

$\Delta Q$  = Heat supplied to the system by the surroundings

$\Delta W$  = Work done by the system on the surroundings

$\Delta U$  = Change in internal energy of the system

Then according to the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

As shown in Fig. 12.11, suppose the system is a gas contained in a cylinder provided with a movable piston. Then the gas does work in moving the piston. The work done by the system against a constant pressure  $P$  is

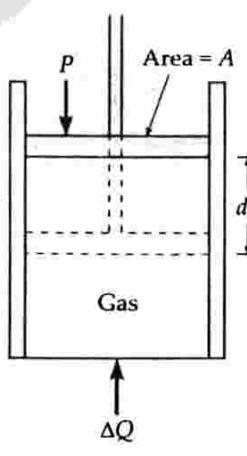
$$\begin{aligned}\Delta W &= \text{Force} \times \text{Distance} \\ &= \text{Pressure} \times \text{Area} \times \text{Distance} \\ &= PA dx\end{aligned}$$

$$\text{or } \Delta W = P \Delta V$$

where  $\Delta V = Adx$  = the change in the volume of the gas.

So the first law of thermodynamics takes the form,

$$\Delta Q = \Delta U + P \Delta V$$



**Fig. 12.11** Absorption of heat by a gas in a cylinder

## Examples based on

### FORMULAE USED

- According to first law of thermodynamics  
 $dQ = dU + dW = dU + PdV$
- For change of state,  $dQ = mL$
- For rise in temperature,  $dQ = mC \Delta T$
- Change in internal energy,  $dU = U_f - U_i$

### UNITS USED

$dQ$ ,  $dU$  and  $dW$  are all in joule. Pressure  $P$  is in  $\text{Nm}^{-2}$  and the change in volume  $dV$  in  $\text{m}^3$ .

**EXAMPLE 4.** 1 g of water at  $100^\circ\text{C}$  is converted into steam at the same temperature. If the volume of steam is  $1671 \text{ cm}^3$ , find the change in the internal energy of the system. Latent heat of steam =  $2256 \text{ J g}^{-1}$ . Given 1 atmospheric pressure =  $1.013 \times 10^5 \text{ N m}^{-2}$ .

**Solution.** Mass of water,  $m = 1 \text{ g} = 10^{-3} \text{ kg}$

Latent heat of steam

$$L = 2256 \text{ J g}^{-1} = 2256 \times 10^3 \text{ J kg}^{-1}$$

Atmospheric pressure,  $P = 1.013 \times 10^5 \text{ N m}^{-2}$

Volume of steam,  $V_s = 1671 \text{ cm}^3 = 1671 \times 10^{-6} \text{ m}^3$

$$\text{Volume of water, } V_w = \frac{\text{Mass}}{\text{Density}} = \frac{10^{-3}}{10^3} = 10^{-6} \text{ m}^3$$

According to first law of thermodynamics,

$$dQ = dU + PdV$$

$$\text{or } mL = dU + P(V_s - V_w)$$

$\therefore$  Change in internal energy is

$$\begin{aligned}\Delta U &= mL - P(V_s - V_w) \\ &= 10^{-3} \times 2256 \times 10^3 \\ &\quad - 1.013 \times 10^5 (1671 \times 10^{-6} - 10^{-6}) \\ &= 2256 - 1.013 \times 10^5 \times 10^{-6} \times 1670 \\ &= 2256 - 0.1013 \times 1670 \\ &= 2256 - 169.171 = 2086.829 \text{ J.}\end{aligned}$$

**EXAMPLE 5.** The volume of steam produced by 1 g of water at  $100^\circ\text{C}$  is  $1650 \text{ cm}^3$ . Calculate the change in internal energy during the change of state. Given  $J = 4.2 \times 10^7 \text{ erg cal}^{-1}$ ,  $g = 981 \text{ cm s}^{-2}$ . Latent heat of steam =  $540 \text{ cal g}^{-1}$ .

**Solution.** Here  $J = 4.2 \times 10^7 \text{ erg cal}^{-1}$

Latent heat of steam,  $L = 540 \text{ cal g}^{-1}$

Mass of water = 1 g

Temperature of water =  $100^\circ\text{C}$

Initial volume,  $V_1 = 1 \text{ cm}^3$

Final volume,  $V_2 = 1650 \text{ cm}^3$

A. Change in volume,

$$dV = V_2 - V_1 = 1650 - 1 = 1649 \text{ cm}^3$$

When 1 g of water at  $100^\circ\text{C}$  is changed to steam at  $100^\circ\text{C}$ , temperature remains constant. So the heat supplied is

$$dQ = mL = 1 \times 540 = 540 \text{ cal} = 540 \times 4.2 \times 10^7 \text{ erg}$$

$$\text{Pressure, } P = 1 \text{ atm} = 76 \times 13.6 \times 981 \text{ dyne cm}^{-2}$$

From first law of thermodynamics,

$$\begin{aligned} dU &= dQ - PdV \\ &= 540 \times 4.2 \times 10^7 - 76 \times 13.6 \times 981 \times 1649 \\ &= 22.68 \times 10^9 - 1.67 \times 10^9 \\ &= 21.01 \times 10^9 \approx 2.1 \times 10^{10} \text{ erg.} \end{aligned}$$

**EXAMPLE 6.** 1.0 m<sup>3</sup> of water is converted into 1671 m<sup>3</sup> of steam at atmospheric pressure and  $100^\circ\text{C}$  temperature. The latent heat of vaporisation of water is  $2.3 \times 10^6 \text{ J kg}^{-1}$ . If 2.0 kg of water be converted into steam at atmospheric pressure and  $100^\circ\text{C}$  temperature, then how much will be the increase in its internal energy? Density of water  $1.0 \times 10^3 \text{ kg m}^{-3}$ , atmospheric pressure  $= 1.01 \times 10^5 \text{ Nm}^{-2}$ .

**Solution.** Heat gained by 2.0 kg of water in changing into steam,

$$Q = mL = 2.0 \text{ kg} \times 2.3 \times 10^6 \text{ J kg}^{-1} = 4.6 \times 10^6 \text{ J}$$

Volume of 2.0 kg of water

$$= \frac{\text{Mass}}{\text{Density}} = \frac{2.0 \text{ kg}}{10^3 \text{ kg m}^{-3}} = 2.0 \times 10^{-3} \text{ m}^3$$

Volume of steam formed by 1.0 m<sup>3</sup> of water  
 $= 1671 \text{ m}^3$

$$\therefore \text{Volume of steam formed by } 2.0 \times 10^{-3} \text{ m}^3 \text{ of water} \\ = 1671 \times 2.0 \times 10^{-3} = 3342 \times 10^{-3} \text{ m}^3$$

When 2.0 kg of water changes into steam, the increase in volume is

$$dV = 3342 \times 10^{-3} - 2.0 \times 10^{-3} = 3340 \times 10^{-3} \text{ m}^3.$$

The external work done against the atmospheric pressure,

$$\begin{aligned} dW &= PdV = 1.01 \times 10^5 \times 3340 \times 10^{-3} \\ &= 0.337 \times 10^6 \text{ J} \end{aligned}$$

From first law of thermodynamics, the increase in internal energy is

$$\begin{aligned} dU &= Q - dW = 4.6 \times 10^6 - 0.337 \times 10^6 \\ &= 4.263 \times 10^6 \text{ J.} \end{aligned}$$

The positive value of  $dU$  indicates that the internal energy of ice increases on melting.

**EXAMPLE 7.** At  $0^\circ\text{C}$  and normal atmospheric pressure, the volume of 1 g of water increases from  $1 \text{ cm}^3$  to  $1.091 \text{ cm}^3$  on freezing. What will be the change in its internal energy? Normal atmospheric pressure is  $1.013 \times 10^5 \text{ Nm}^{-2}$  and latent heat of melting of ice  $= 80 \text{ cal g}^{-1}$ .

**Solution.** Heat lost by water on freezing,

$$Q = -mL = -1 \text{ g} \times 80 \text{ cal g}^{-1} = -80 \text{ cal}$$

The negative sign shows that heat is given out by water.

During freezing water expands against the atmospheric pressure. Hence external work done by the water is

$$\begin{aligned} dW &= PdV \\ &= 1.013 \times 10^5 \text{ Nm}^{-2} \times [(1.091 - 1) \times 10^{-6} \text{ m}^3] \\ &= 0.0092 \text{ joule} = \frac{0.0092}{4.18} \text{ cal} = 0.0022 \text{ cal} \end{aligned}$$

As this work is done by ice, it is taken as positive.

According to first law of thermodynamics, the change in the internal energy will be

$$\begin{aligned} dU &= Q - dW \\ &= -80 \text{ cal} - 0.0022 \text{ cal} = -80.0022 \text{ cal.} \end{aligned}$$

The negative sign shows the internal energy of water decreases on freezing.

**EXAMPLE 8.** 5 moles of oxygen are heated at constant volume from  $10^\circ\text{C}$  to  $20^\circ\text{C}$ . What will be the change in the internal energy of the gas? The gram molecular specific heat of oxygen at constant pressure,  $C_p = 8 \text{ cal mole}^{-1} \text{ }^\circ\text{C}^{-1}$  and  $R = 8.36 \text{ J mole}^{-1} \text{ }^\circ\text{C}^{-1}$

**Solution.** Here  $C_p = 8 \text{ cal mole}^{-1} \text{ }^\circ\text{C}^{-1}$ ,

$$R = 8.36 \text{ J mole}^{-1} \text{ }^\circ\text{C}^{-1}$$

$$= \frac{8.36}{4.18} = 2 \text{ cal mole}^{-1} \text{ }^\circ\text{C}^{-1}$$

$$\text{As } C_p - C_v = R$$

$$\therefore C_v = C_p - R = 8 - 2 = 6 \text{ cal mole}^{-1} \text{ }^\circ\text{C}^{-1}$$

Heat gained by 5 moles of oxygen when heated from  $10^\circ\text{C}$  to  $20^\circ\text{C}$ ,

$$Q = nC_v \Delta T = 5 \times 6 \times (20 - 10) = 300 \text{ cal.}$$

As the gas is heated at constant volume, the external work done,

$$dW = PdV = P \times 0 = 0$$

From first law of thermodynamics, the increase in internal energy is

$$dU = Q - dW = 300 - 0 = 300 \text{ cal.}$$

**EXAMPLE 9.** A metal of mass 1 kg at constant atmospheric pressure and at initial temperature  $20^\circ\text{C}$  is given a heat of 20000 J. Find (i) change in temperature, (ii) work done and (iii) change in internal energy. Given

specific heat,  $c = 400 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$ ,

coefficient of cubical expansion,  $\gamma = 9 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$

density,  $\rho = 9000 \text{ kg m}^{-3}$

atmospheric pressure,  $P = 10^5 \text{ Nm}^{-2}$ .

**Solution.** (i) As  $\Delta Q = mc\Delta T$

∴ Rise in temperature,

$$\Delta T = \frac{\Delta Q}{mc} = \frac{20000 \text{ J}}{1 \text{ kg} \times 400 \text{ J kg}^{-1} \text{ }^{\circ}\text{C}^{-1}} = 50 \text{ }^{\circ}\text{C}.$$

(ii) Density,  $\rho = \frac{M}{V}$

$$\therefore \text{Volume, } V = \frac{M}{\rho} = \frac{1 \text{ kg}}{9000 \text{ kg m}^{-3}} = \frac{1}{9000} \text{ m}^3$$

Change in volume,

$$\Delta V = \gamma V \Delta T = 9 \times 10^{-5} \times \frac{1}{9000} \times 50 \\ = 5 \times 10^{-7} \text{ m}^3$$

Work done,

$$W = P\Delta V = 10^5 \times 5 \times 10^{-7} = 0.05 \text{ J.}$$

(iii) The change in internal energy,

$$\Delta U = \Delta Q - \Delta W \\ = 20000 - 0.05 = 19999.95 \text{ J.}$$

### X PROBLEMS FOR PRACTICE

1. Calculate the change in internal energy of a block of copper of mass 200 g when it is heated from 25°C to 75°C. Take specific heat of copper = 0.1 cal g<sup>-1</sup> °C<sup>-1</sup>, and assume the change in volume as negligible.

(Ans. 4200 J)

2. 1 kg of water at 373 K is converted into steam at the same temperature. The volume of 1 cm<sup>3</sup> of water becomes 1671 cm<sup>3</sup> on boiling. Calculate change in the internal energy of the system, if heat of vaporisation is 540 cal g<sup>-1</sup>. Given standard atmospheric pressure = 1.013 × 10<sup>5</sup> Nm<sup>-2</sup>.

(Ans. 499.84 kcal)

3. Calculate the change in internal energy when 5 g of air is heated from 0° to 4°C at constant volume. The specific heat of air at constant volume is 0.172 cal g<sup>-1</sup> °C<sup>-1</sup>.

(Ans. 14.4 J)

4. A volume of 10 m<sup>3</sup> of a liquid is supplied with 100 kcal of heat and expands at a constant pressure of 10 atm to a final volume of 10.2 m<sup>3</sup>. Calculate the work done and the change in internal energy.

(Ans. 48 kcal, 52 kcal)

5. The internal energy of a monoatomic gas is 1.5 nRT. One mole of helium is kept in a cylinder of cross-section 8.5 cm<sup>2</sup>. The cylinder is closed by a light frictionless piston. The gas is heated slowly in a process during which a total of 42 J heat is given to the gas. If the temperature rises through 2°C, find the distance moved by the piston. Atmospheric pressure = 100 kPa.

(Ans. 20 cm)

### X HINTS

1.  $dQ = mc\Delta T = 200 \times 0.1 \times (75 - 25) = 100 \text{ cal}$   
 $dW = PdV = P \times 0 = 0$   
 $dU = dQ - dW = 1000 - 0 = 1000 \text{ cal}$   
 $= 1000 \times 4.2 = 4200 \text{ J.}$

2. Here  $m = 1 \text{ kg} = 10^3 \text{ g}$   
Initial volume,  $V_1 = 10^3 \text{ cm}^3$ ;  
Final volume,  $V_2 = 1671 \times 10^3 \text{ cm}^3$   
 $P = 1 \text{ atom} = 1.013 \times 10^6 \text{ dyne cm}^{-2}$   
 $dQ = mL = 10^3 \times 540 \text{ cal} = 540 \text{ kcal}$   
 $dW = PdV = P(V_2 - V_1)$   
 $= 1.013 \times 10^6 (1671 - 1) \text{ erg}$   
 $= \frac{1.013 \times 10^6 \times 1670}{4.2 \times 10^7} \text{ cal}$   
 $= 40.16 \times 10^3 \text{ cal} = 40.16 \text{ kcal.}$

From first law of thermodynamics,

$$dU = dQ - dW = 540 - 40.16 \\ = 499.84 \text{ kcal.}$$

3. At constant volume,  $dW = PdV = P \times 0 = 0$

$$dQ = mC_v \Delta T = 5 \times 0.172 \times 4 \text{ cal} \\ = 5 \times 0.172 \times 4 \times 4.2 \text{ J} = 14.4 \text{ J}$$

$$\therefore dU = dQ - dW = 14.4 - 0 = 14.4 \text{ J}$$

4.  $dW = PdV = 10 \times 1.013 \times 10^5 \times (10.2 - 10) = 2 \times 10^5 \text{ J}$   
 $= \frac{2 \times 10^5 \text{ J}}{4186 \text{ J kcal}^{-1}} = 48 \text{ kcal}$

$$dQ = 100 \text{ kcal}$$

$$\therefore dU = dQ - dW = 100 - 48 = 52 \text{ kcal.}$$

5.  $dU = 1.5 nR \Delta T = 1.5 \times 1 \times 8.3 \times 2 = 24.9 \text{ J}$  [∴ n = 1]

$$dQ = 42 \text{ J}$$

$$dW = dQ - dU = 42 - 24.9 = 17.1 \text{ J}$$

$$P = 100 \text{ kPa} = 10^5 \text{ Pa,}$$

$$A = 8.5 \text{ cm}^2 = 8.5 \times 10^{-4} \text{ m}^2$$

As  $dW = PdV = PA dx$

$$\therefore dx = \frac{dW}{PA} = \frac{17.1}{1 \times 10^5 \times 8.5 \times 10^{-4}} \\ = 0.2 \text{ m} = 20 \text{ cm.}$$

### 12.10 SPECIFIC HEATS OF A GAS

15. Why does a gas not possess a unique or single specific heat? Discuss the limits of the specific heat of a gas.

**Specific heats of a gas.** When a gas is heated, its volume and pressure change with the increase in temperature. So the amount of heat required to raise the temperature of 1 gram of gas through 1°C is not fixed. That is a gas does not possess a unique or single

specific heat. A gas can have any value of specific heat depending on the conditions under which it is heated. This can be clearly seen in the following examples.

**Limits of specific heat of a gas.** Consider  $m$  mass of a gas enclosed in a cylinder fitted with an air-tight and frictionless piston.

- (i) Suppose the gas is suddenly compressed. No heat is supplied to the gas i.e.,  $\Delta Q = 0$ . But the temperature of the gas rises due to compression.

$$\therefore c = \frac{\Delta Q}{m \Delta T} = \frac{0}{m \Delta T} = 0.$$

i.e., specific heat of the gas is zero.

- (ii) Now the gas is heated and allowed to expand such that the rise in temperature of the gas due to the heat supplied is equal to the fall in temperature of the gas due to the expansion of the gas itself. Then the net rise in temperature is zero i.e.,  $\Delta T = 0$ .

$$\therefore c = \frac{\Delta Q}{m \Delta T} = \frac{\Delta Q}{m \times 0} = \infty$$

i.e., the specific heat of the gas is infinite.

- (iii) Again, the gas is heated and allowed to expand at such a rate that the fall in temperature due to expansion is less than the rise in temperature due to heat supplied. The temperature of the gas will rise i.e.,  $\Delta T$  is positive.

$$\therefore c = \frac{\Delta Q}{m \Delta T} = \text{a positive value}$$

i.e., specific heat of the gas is positive.

- (iv) Finally, the gas is heated and allowed to expand at such a rate that the fall in temperature due to expansion is more than the rise in temperature due to heat supplied. The temperature of gas will decrease i.e.,  $\Delta T$  is negative.

$$\therefore c = \frac{\Delta Q}{m \Delta T} = \text{a negative value}$$

i.e., the specific heat of the gas is negative.

Thus the specific heat of a gas may have any positive or negative value ranging from zero to infinity. The exact value depends on the conditions of pressure and volume when the gas is being heated.

## 12.11 TWO PRINCIPAL SPECIFIC HEATS OF A GAS

16. Define the two principal specific heats of a gas. Explain why molar specific heat at constant pressure ( $C_p$ ) is greater than the molar specific heat at constant volume ( $C_v$ )?

**Two principal specific heats of a gas.** Out of the many specific heats of a gas, two are of special

significance : one when the gas is heated at *constant volume* and another when the gas is heated at *constant pressure*. These are known as the two *principal specific heats* of the gas and may be defined as follows :

(i) **Molar specific heat at constant volume.** It is defined as the amount of heat required to raise the temperature of 1 mole of a gas through  $1^\circ\text{C}$  at constant volume. It is denoted by  $C_V$ .

(ii) **Molar specific heat at constant pressure.** It is defined as the amount of heat required to raise the temperature of 1 mole of a gas through  $1^\circ\text{C}$  at constant pressure. It is denoted by  $C_p$ .

$C_p$  is greater than  $C_V$ . When a gas is heated at constant volume, all the heat supplied is used to increase the temperature or internal energy of the gas.

When a gas is heated at constant pressure, the gas expands. It does work against the external pressure. The heat supplied is used in two ways : (i) partly to increase the internal energy and (ii) partly to do work against external pressure. So at constant pressure, an additional amount of heat equivalent to work done is utilised.

Hence more amount of heat is required to increase the temperature of 1 mole of a gas at constant pressure than that at constant volume i.e.,  $C_p > C_V$ . The difference between the two specific heats is equal to the amount of heat equivalent to the work performed by the gas during expansion at constant pressure.)

## 12.12 SPECIFIC HEAT RELATION : MAYER'S FORMULA

17. Applying the first law of thermodynamics, obtain the relation between the two specific heats of a gas.

**Relation between  $C_p$  and  $C_V$  : Mayer's formula.**

Consider one mole of an ideal gas. Heat the gas to raise its temperature by  $\Delta T$ . According to the first law of thermodynamics, the heat supplied  $\Delta Q$  is used partly to increase the internal energy and partly in doing the work of expansion. That is,

$$\Delta Q = \Delta U + P \Delta V$$

If the heat  $\Delta Q$  is absorbed at constant volume, then  $\Delta V = 0$  and we have

$$C_V = \left( \frac{\Delta Q}{\Delta T} \right)_V = \left( \frac{\Delta U}{\Delta T} \right)_V = \left( \frac{\Delta U}{\Delta T} \right)$$

We have dropped the subscript  $V$  because the internal energy  $U$  of an ideal gas depends only on its temperature  $T$ .

If now the heat  $\Delta Q$  is absorbed at constant pressure, then

$$\begin{aligned} C_p &= \left( \frac{\Delta Q}{\Delta T} \right)_p = \left( \frac{\Delta U}{\Delta T} \right)_p + P \left( \frac{\Delta V}{\Delta T} \right)_p \\ &= \left( \frac{\Delta U}{\Delta T} \right)_p + P \left( \frac{\Delta V}{\Delta T} \right)_p \end{aligned}$$

Again, we have dropped the subscript  $P$  from the first term because  $U$  of an ideal gas depends only on  $T$ . Clearly

$$C_p - C_V = P \left( \frac{\Delta V}{\Delta T} \right)_p$$

But for one mole of an ideal gas,  $PV = RT$

Differentiating both sides w.r.t.  $T$  for constant pressure  $P$ ,

$$\frac{\Delta(PV)}{\Delta T} = \frac{\Delta(RT)}{\Delta T}$$

$$\text{or } P \left( \frac{\Delta V}{\Delta T} \right)_P = R$$

$$\text{Hence } C_p - C_V = R$$

This is the required relation between  $C_p$  and  $C_V$ . It is also known as *Mayer's formula*.



### For Your Knowledge

#### ▲ For one mole of a gas :

$$\Leftrightarrow C_p - C_V = R. \quad (\text{When } C_p, C_V \text{ are in units of work})$$

$$\Leftrightarrow C_p - C_V = \frac{R}{J} \quad (\text{When } C_p, C_V \text{ are in units of heat})$$

where  $R$  is universal gas constant for one mole of a gas.

▲ The above relations will remain same even if we consider any number of moles of a gas because in that case both sides will get multiplied by the same number.

▲ As  $R$  is always positive, it follows that  $C_p > C_V$ .

#### ▲ For one gram of a gas :

$$\Leftrightarrow c_p - c_v = r.$$

(When  $c_p, c_v$  are in units of work)

$$\Leftrightarrow c_p - c_v = \frac{r}{M}$$

(When  $c_p, c_v$  are in units of heat)

where  $r = \frac{R}{M}$  = gas constant for 1 g of a gas.

#### ▲ Heat lost or gained by $n$ moles of a gas,

$$\Leftrightarrow Q = nC_p \Delta T \quad (\text{At constant pressure})$$

$$\Leftrightarrow Q = nC_V \Delta T \quad (\text{At constant volume})$$

▲ The ratio of the two principal specific heats is represented by  $\gamma$ .

$$\therefore \gamma = \frac{C_p}{C_V}$$

▲ The value of  $\gamma$  depends on the atomicity of the gas.

### Examples based on

#### Relation between Two Specific Heats of a Gas

##### FORMULAE USED

1. For one mole of a gas,

$$(i) C_p - C_V = R \quad (\text{When } C_p, C_V \text{ are in units of work})$$

$$(ii) C_p - C_V = \frac{R}{J} \quad (\text{When } C_p, C_V \text{ are in units of heat})$$

2. For 1 g of a gas

$$(i) c_p - c_v = r \quad (\text{when } c_p, c_v \text{ are in units of work})$$

$$(ii) c_p - c_v = \frac{r}{J} \quad (\text{when } c_p, c_v \text{ are in units of heat})$$

where  $r = \frac{R}{M}$  = gas constant for 1 g of a gas

3. Heat lost or gained by a gas,

$$(i) Q = nC_p \Delta T \quad (\text{At constant pressure})$$

$$(ii) Q = nC_V \Delta T \quad (\text{At constant volume})$$

where  $n$  = Number of moles of gas

$$= \frac{\text{Mass of gas}}{\text{Molecular mass}}$$

##### CONSTANTS USED

$$1. J = 4.18 \text{ J cal}^{-1}$$

$$2. R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1} = 1.98 \text{ cal mol}^{-1} \text{ K}^{-1}$$

**EXAMPLE 10.** Calculate the specific heat at constant volume for a gas. Given specific heat at constant pressure is  $6.85 \text{ cal mol}^{-1} \text{ K}^{-1}$ ,  $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$  and  $J = 4.18 \text{ J cal}^{-1}$ .

**Solution.** Here  $C_p = 6.85 \text{ cal mol}^{-1} \text{ K}^{-1}$ ,

$$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}, J = 4.18 \text{ J cal}^{-1}$$

$$\text{As } C_p - C_V = \frac{R}{J}$$

$$\therefore C_V = C_p - \frac{R}{J} = 6.85 - \frac{8.31}{4.18}$$

$$= 6.85 - 1.988 = 4.862 \text{ cal mol}^{-1} \text{ K}^{-1}$$

**EXAMPLE 11.** Calculate the difference between the two principal specific heats of 1 g of helium gas at S.T.P. Given atomic weight of helium = 4 and  $J = 4.186 \text{ J cal}^{-1}$  and  $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$ .

**Solution.** For 1 g of helium gas,

$$\begin{aligned} c_p - c_v &= \frac{r}{J} = \frac{R}{MJ} \\ &= \frac{8.31}{4 \times 4.186} = 0.496 \text{ cal g}^{-1} \text{ K}^{-1} \end{aligned}$$

**EXAMPLE 12.** The difference between two specific heats of a gas is  $5000 \text{ J kg}^{-1} \text{ K}^{-1}$  and the ratio of specific heats is 1.6. Find the two specific heats.

**Solution.** Here  $c_p - c_V = 5000 \text{ J kg}^{-1} \text{ K}^{-1}$

$$\text{and } \frac{c_p}{c_V} = 1.6 \quad \text{or} \quad c_p = 1.6 c_V$$

$$\therefore 1.6 c_p - c_V = 5000 \quad \text{or} \quad 0.6 c_V = 5000$$

$$\text{or } c_V = \frac{5000}{0.6} = 8333.33 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$c_p = c_V + 5000 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$= 13333.33 \text{ J kg}^{-1} \text{ K}^{-1}.$$

**EXAMPLE 13:** Specific heat of argon at constant pressure is  $0.125 \text{ cal g}^{-1} \text{ K}^{-1}$  and at constant volume is  $0.075 \text{ cal g}^{-1} \text{ K}^{-1}$ . Calculate the density of argon at S.T.P. Given  $J = 4.18 \times 10^7 \text{ erg cal}^{-1}$  and normal pressure  $= 1.01 \times 10^6 \text{ dyne cm}^{-2}$ .

**Solution.** Here  $c_p = 0.125 \text{ cal g}^{-1} \text{ K}^{-1}$ ,

$$c_V = 0.075 \text{ cal g}^{-1} \text{ K}^{-1}$$

$$J = 4.18 \times 10^7 \text{ erg cal}^{-1}$$

Standard pressure,  $P = 1.01 \times 10^6 \text{ dyne cm}^{-2}$

$$\text{For 1 g of a gas, } c_p - c_V = \frac{r}{J}$$

$$\therefore r = J(c_p - c_V) = 4.18 \times 10^7 \times (0.125 - 0.075)$$

$$= 4.18 \times 10^7 \times 0.050 = 2.09 \times 10^7 \text{ erg g}^{-1} \text{ K}^{-1}$$

But for 1 g of a gas,  $PV = rT$

$\therefore$  Volume of 1 g of argon at S.T.P. is

$$V = \frac{rT}{P} = \frac{2.09 \times 10^7 \times 273}{1.01 \times 10^6} = 564.92 \text{ cm}^3$$

Density of argon,

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{1}{564.92} = 1.77 \times 10^{-3} \text{ g cm}^{-3}.$$

**EXAMPLE 14.** Calculate the value of  $c_V$  for air, given that  $c_p = 0.23 \text{ calorie g}^{-1} \text{ K}^{-1}$ . Density of the air at S.T.P. is  $1.293 \text{ g litre}^{-1}$  and  $J = 4.2 \times 10^7 \text{ erg calorie}^{-1}$ .

**Solution.** Sp. heat of air at constant pressure,

$$c_V = 0.23 \text{ cal g}^{-1} \text{ K}^{-1}$$

Density of air at S.T.P.  $= 1.293 \text{ g litre}^{-1}$

$$\therefore \text{Volume of 1 g of air at S.T.P., } V = \frac{1000}{1.293} \text{ cm}^3$$

Normal pressure,  $P = 76 \times 13.6 \times 980 \text{ dyne cm}^{-2}$

Normal temperature,  $T = 273 \text{ K}$

$$J = 4.2 \times 10^7 \text{ erg cal}^{-1}$$

$\therefore$  Gas constant for 1 g of air,

$$r = \frac{PV}{T} = \frac{76 \times 13.6 \times 980 \times 1000}{1.293 \times 273}$$

$$= 2.87 \times 10^6 \text{ erg g}^{-1} \text{ K}^{-1}$$

$$\text{Also for 1 g of a gas, } c_p - c_V = \frac{r}{J}$$

$$\therefore c_V = c_p - \frac{r}{J} = 0.23 - \frac{2.87 \times 10^6}{4.2 \times 10^7}$$

$$= 0.23 - 0.068 = 0.162 \text{ cal g}^{-1} \text{ K}^{-1}.$$

**EXAMPLE 15.** For air, specific heat at constant pressure is  $0.237 \text{ cal g}^{-1} \text{ C}^{-1}$  and specific heat at constant volume is  $0.169 \text{ cal g}^{-1} \text{ C}^{-1}$ , density of air  $= 0.001293 \text{ g cm}^{-3}$  at S.T.P. Calculate the value of  $J$ .

**Solution.** Here  $c_p = 0.237 \text{ cal g}^{-1} \text{ C}^{-1}$ ,

$$c_V = 0.169 \text{ cal g}^{-1} \text{ C}^{-1}$$

Density of air at S.T.P.,

$$\rho = 0.001293 \text{ g cm}^{-3}$$

$\therefore$  Volume of 1 g air at S.T.P.,

$$V = \frac{1}{0.001293} \text{ cm}^3$$

Normal pressure,  $P = 76 \times 13.6 \times 980 \text{ dyne cm}^{-2}$

Normal temperature,  $T = 273 \text{ K}$

Gas constant for 1 g of gas,

$$r = \frac{PV}{T} = \frac{76 \times 13.6 \times 980 \times 1}{273 \times 0.001293}$$

$$= 2.87 \times 10^6 \text{ erg g}^{-1} \text{ K}^{-1}$$

$$\therefore J = \frac{r}{c_p - c_V} = \frac{2.869 \times 10^6}{0.237 - 0.169}$$

$$= \frac{2.87 \times 10^6}{0.06} = 4.219 \times 10^7 \text{ erg cal}^{-1}.$$

**EXAMPLE 16.** An ideal gas has a specific heat at a constant pressure,  $C_p = (5/2) R$ . The gas is kept in a closed vessel of volume  $0.0083 \text{ m}^3$  at a temperature of  $300 \text{ K}$  and a pressure of  $1.6 \times 10^6 \text{ Nm}^{-2}$ . An amount of  $2.49 \times 10^4 \text{ J}$  of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas.

**Solution.** Here  $P = 1.6 \times 10^6 \text{ Nm}^{-2}$ ,

$$V = 0.0083 \text{ m}^3, T = 300 \text{ K}$$

As  $PV = nRT$

$$\therefore n = \frac{PV}{RT} = \frac{1.6 \times 10^6 \times 0.0083}{8.3 \times 300} = \frac{16}{3}$$

$$\text{Now } C_V = C_P - R = \frac{5}{2} R - R = \frac{3}{2} R$$

When  $2.49 \times 10^4 \text{ J}$  of heat energy is supplied to the gas, suppose its temperature increases by  $\Delta T$ . Then

$$n C_V \Delta T = Q$$

$$\text{or } \frac{16}{3} \times \frac{5}{2} R \times \Delta T = 2.49 \times 10^4$$

$$\text{or } \Delta T = \frac{2.49 \times 10^4 \times 6}{80 R} = \frac{2.49 \times 10^4 \times 6}{80 \times 8.3} = 375 \text{ K}$$

Final temperature,

$$T' = T + 375 = 300 + 375 = 675 \text{ K}$$

As the gas is heated at constant volume, so

$$\frac{P'}{P} = \frac{T'}{T}$$

$$\text{or } P' = \frac{T'}{T} \times P = \frac{675 \times 1.6 \times 10^6}{300} \\ = 3.6 \times 10^6 \text{ Nm}^{-2}.$$

### X PROBLEMS FOR PRACTICE

1. Calculate the difference between two specific heats of 1 g of nitrogen. Given molecular weight of nitrogen = 28 and  $J = 4.2 \times 10^7 \text{ erg cal}^{-1}$ .

$$(\text{Ans. } 0.0706 \text{ cal g}^{-1} \text{ K}^{-1})$$

2. Calculate the gas constant for 1 g of a gas. Given that  $c_p = 0.245 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$ ,  $c_v = 0.165 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$  and  $J = 4.2 \times 10^7 \text{ erg cal}^{-1}$ .

$$(\text{Ans. } 3.36 \times 10^6 \text{ erg g}^{-1} \text{ }^\circ\text{C}^{-1})$$

3. Calculate difference in specific heats for 1 g of air at S.T.P. Given density of air at S.T.P. is  $1.293 \text{ g litre}^{-1}$ ,  $J = 4.2 \times 10^7 \text{ erg cal}^{-1}$ . ( $\text{Ans. } 0.068 \text{ cal g}^{-1} \text{ K}^{-1}$ )

4. The specific heats of air at constant pressure and constant volume are  $0.237 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$  and  $0.169 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$  respectively and  $J = 4.2 \text{ J cal}^{-1}$ . Calculate density of air. ( $\text{Ans. } 1.293 \times 10^{-3} \text{ g cm}^{-3}$ )

5. Calculate the ratio of specific heats for nitrogen, given that specific heat at constant pressure is  $0.236 \text{ cal g}^{-1} \text{ K}^{-1}$  and density at S.T.P. is  $0.001234 \text{ g cm}^{-3}$ . ( $\text{Ans. } 1.434$ )

6. For hydrogen gas,  $c_p = 3.409 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$ ,  $c_v = 2.409 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$  and molecular weight of hydrogen = 2. Calculate the value of  $J$ .

$$(\text{Ans. } 4.156 \times 10^7 \text{ erg cal}^{-1})$$

7. The specific heat of argon at constant pressure is 0.127 and ratio of specific heats is 1.667. Calculate the value of  $J$ . One litre of argon weighs 1.786 g at N.T.P. ( $\text{Ans. } 4.07 \times 10^7 \text{ erg cal}^{-1}$ )

8. One mole of oxygen is heated at a constant pressure from  $0^\circ\text{C}$ . What must be the quantity of heat that should be supplied to the gas for the volume to be doubled? The specific heat of oxygen under these conditions is  $0.218 \text{ cal g}^{-1} \text{ K}^{-1}$ . ( $\text{Ans. } 1904 \text{ cal}$ )

### X HINTS

1. Here  $J = 4.2 \times 10^7 \text{ erg cal}^{-1} = 4.2 \text{ J cal}^{-1}$

$$\therefore c_p - c_v = \frac{r}{J} = \frac{R}{M J} = \frac{8.3}{28 \times 4.2} \\ = 0.0706 \text{ cal g}^{-1} \text{ K}^{-1}.$$

$$2. r = J(c_p - c_v) = 4.2 \times 10^7 \times (0.245 - 0.165)$$

$$= 3.36 \times 10^6 \text{ erg g}^{-1} \text{ }^\circ\text{C}^{-1}.$$

$$3. \text{Here } \rho = 1.293 \text{ g litre}^{-1} = 1.293 \times 10^{-3} \text{ g cm}^{-3}$$

$$\text{Now } c_p - c_v = \frac{r}{J} = \frac{Pv}{TJ} = \frac{P}{\rho TJ}$$

$$[v = \text{volume of } 1 \text{ g gas} = \frac{1}{\rho}]$$

$$= \frac{1.013 \times 10^6}{1.293 \times 10^{-3} \times 273 \times 4.2 \times 10^7} = 0.068 \text{ cal g}^{-1} \text{ K}^{-1}.$$

$$4. \rho = \frac{P}{TJ(c_p - c_v)} = \frac{1.01 \times 10^6}{273 \times 4.2 \times 10^7 \times (0.237 - 0.169)} \\ = 1.293 \times 10^{-3} \text{ g cm}^{-3}.$$

5. For 1 g of nitrogen,

$$c_p - c_v = \frac{r}{J} = \frac{Pv}{TJ} = \frac{P}{\rho TJ} \\ = \frac{1.01 \times 10^6}{0.001234 \times 234 \times 4.2 \times 10^7} \\ = 0.0714 \text{ cal g}^{-1} \text{ K}^{-1}$$

$$c_v = c_p - 0.0714 = 0.236 - 0.0714 = 0.1646 \\ \therefore \gamma = \frac{c_p}{c_v} = \frac{0.236}{0.1646} = 1.434$$

$$7. c_v = \frac{c_p}{\gamma} = \frac{0.127}{1.667} = 0.076$$

$$\therefore c_p - c_v = 0.127 - 0.076 = 0.051$$

$$\text{For } 1 \text{ g of gas, } c_p - c_v = \frac{P}{\rho TJ}$$

$$\therefore J = \frac{P}{\rho T}(c_p - c_v) = \frac{1.01 \times 10^6}{1.786 \times 10^{-3} \times 273 \times 0.051} \\ = 4.07 \times 10^7 \text{ erg cal}^{-1}.$$

8. Initial temperature,  $T_1 = 0 + 273 = 273 \text{ K}$

$$\text{At constant pressure, } \frac{V_2}{V_1} = \frac{T_2}{T_1} \quad \text{or} \quad \frac{2V_1}{V_1} = \frac{T_2}{273}$$

$$\therefore T_2 = 2 \times 273 = 546 \text{ K}$$

Rise in temperature,

$$\Delta T = T_2 - T_1 = 546 - 273 = 273 \text{ K}$$

Mass of 1 mole of oxygen,  $m = 32 \text{ g}$

Heat required,

$$Q = mc_p \Delta T = 32 \times 0.218 \times 273 = 1904 \text{ cal.}$$

### 12.13 THERMODYNAMIC PROCESSES

18. What is a thermodynamic process? Mention its different types.

**Thermodynamic process.** A thermodynamic process is said to occur if the thermodynamic variables of a system undergo a change with time.

Different types of thermodynamic processes are as follows :

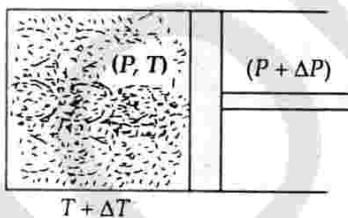
- Isothermal process.** It is a thermodynamic process which occurs at a constant temperature.
- Isobaric process.** It is a thermodynamic process which occurs at a constant pressure.
- Isochoric process.** It is a thermodynamic process which occurs at a constant volume.
- Adiabatic process.** It is a thermodynamic process in which there is no exchange of heat energy between system and surroundings.

**Tablet 12.1 Some typical thermodynamic processes**

Type of process	Special feature
Isothermal	Constant temperature
Isobaric	Constant pressure
Isochoric	Constant volume
Adiabatic	No heat flow between the system and surroundings ( $\Delta Q = 0$ )

### 19. What is a quasi-static process ? Briefly explain.

**Quasi-static process.** A quasi-static process is an infinitely slow process such that the system remains in thermal and chemical equilibrium with the surroundings. In a quasi-static (meaning nearly static) process, at every stage, the temperature and pressure of the surroundings differ only infinitesimally from those of the system.



**Fig. 12.12** A quasi-static process.

As illustrated in Fig. 12.12, suppose we wish to take a gas from the state  $(P, T)$  to another state  $(P', T')$  via a quasi-static process. For this we first change the external pressure by a very small amount  $\Delta P$ , allow the system to equalise its pressure with that of the surroundings and continue the process infinitely slowly until the pressure of the gas becomes  $P'$ . Now to change the temperature, we create a very small temperature difference  $\Delta T$  between the system and surrounding reservoir and continue the process by choosing reservoirs of progressively different temperatures from  $T$  to  $T'$ . The system finally attains the temperature  $T'$ .

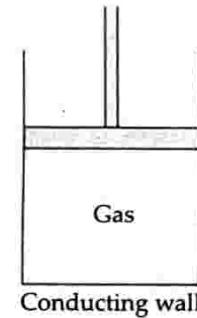
Practically, the processes that are sufficiently slow and do not involve accelerated motion of the piston, large temperature gradients, etc., are nearly quasi-static processes.

### 12.14 ISOTHERMAL PROCESS

20. What is an isothermal process ? Give an example. What are the essential conditions for an isothermal process to take place ? Write the equation for an isothermal process.

**Isothermal process.** An isothermal process is one in which the pressure and volume of the system change but temperature remains constant.

As shown in Fig. 12.13, consider an ideal gas enclosed in a cylinder provided with a piston and having conducting walls. If the gas is slowly compressed, the heat produced due to the work done on the gas is transferred to the surroundings so that temperature of the gas remains constant. Similarly, when the gas is allowed to expand slowly, its temperature tends to fall but some heat from the surroundings is conducted to the gas, keeping the temperature constant.



**Fig. 12.13** Apparatus for isothermal process.

**Essential conditions for an isothermal process to take place :**

- The walls of the container must be perfectly conducting to allow free exchange of heat between the system and the surroundings.
- The process of compression or expansion should be very slow, so as to provide sufficient time for the exchange of heat.

**Equation of isothermal process.** The ideal gas equation for  $n$  moles of a gas is

$$PV = nRT$$

For a fixed mass ( $n$  fixed) of a gas undergoing an isothermal process ( $T$  fixed), the above equation gives

$$PV = \text{constant}$$

This equation is the **equation of state** of an isothermal process. It is nothing but Boyle's law according to which the pressure of a given mass of a gas varies inversely as its volume.

### 12.15 FIRST LAW OF THERMODYNAMICS APPLIED TO ISOTHERMAL PROCESS

21. Discuss the application of the first law of thermodynamics to an isothermal process.

**First law of thermodynamics applied to an isothermal process.** The internal energy of an ideal gas depends only on its temperature. As temperature remains constant in an isothermal process, there is no change in internal energy of the gas i.e.,  $\Delta U = 0$ .

Applying first law of thermodynamics to an isothermal process,

$$\Delta Q = \Delta U + P \Delta V = 0 + P \Delta V \text{ or } \Delta Q = P \Delta V$$

(i) **When a gas expands isothermally,**  $\Delta V$  and hence  $P \Delta V$  is positive and so  $\Delta Q$  will also be positive. Therefore, when a gas expands isothermally, *an amount of heat equivalent to the work done by the gas has to be supplied from an external source.*

(ii) **When a gas is compressed isothermally,**  $\Delta V$  and hence  $P \Delta V$  is negative and  $\Delta Q$  will also be negative. Therefore, *when a gas is compressed isothermally, an amount of heat equivalent to the work done on the gas has to be removed from the gas.*

(iii) In an isothermal expansion or compression, the internal energy of the gas remains unchanged.

## 12.16 WORK DONE IN AN ISOTHERMAL PROCESS

22. Derive an expression for the work done during the isothermal expansion of an ideal gas.

**Work done in an isothermal expansion.** Consider  $n$  moles of an ideal gas contained in a cylinder having conducting walls and provided with frictionless and movable piston, as shown in Fig. 12.14. Let  $P$  be the pressure of the gas.

Work done by the gas when the piston moves up through a small distance  $dx$  is given by

$$dW = PA dx = PdV$$

where  $A$  is the cross-sectional area of the piston and  $dV = Adx$ , is the small increase in the volume of the gas. Suppose the gas expands isothermally from initial state  $(P_1, V_1)$  to the final state  $(P_2, V_2)$ . The total amount of work done will be

$$W_{iso} = \int_{V_1}^{V_2} P dV$$

For  $n$  moles of a gas,  $PV = nRT$  or  $P = \frac{nRT}{V}$

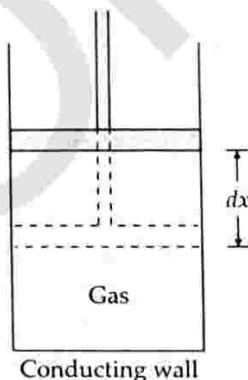


Fig. 12.14 Isothermal expansion

$$\therefore W_{iso} = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \int_{V_1}^{V_2} \frac{1}{V} dV = nRT [\ln V]_{V_1}^{V_2}$$

$$= nRT [\ln V_2 - \ln V_1] = nRT \ln \frac{V_2}{V_1}$$

$$\text{or } W_{iso} = 2.303 nRT \log \frac{V_2}{V_1} = 2.303 nRT \log \frac{P_1}{P_2}$$

This is the expression for the work done during the isothermal expansion of  $n$  moles of an ideal gas.

## 12.17 ADIABATIC PROCESS

23. What is an adiabatic process? Give an example. What are the essential conditions for an adiabatic process to occur?

**Adiabatic process.** An adiabatic process is one in which the pressure, volume and temperature of the system change but there is no exchange of heat between the system and surroundings (a = not, dia = through, bates = heat, so the Greek word adiabatic means heat not passing through).

Consider a gas enclosed in a cylinder having perfectly insulated walls. Suppose the gas is allowed to expand very quickly. Work is done by the gas during its expansion, so its internal energy decreases. As the heat cannot enter the system from the surroundings, so the temperature of the gas falls.

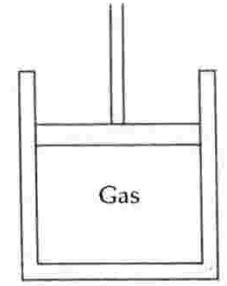


Fig. 12.15 Apparatus for adiabatic process.

Similarly, when the gas is suddenly compressed, work is done on the gas. This increases the internal energy of the gas. As heat cannot escape to the surroundings, the temperature of the gas increases.

**Essential conditions for an adiabatic process to take place :**

- The walls of the container must be perfectly insulated so that there cannot be any exchange of heat between the gas and the surroundings.
- The process of compression or expansion should be sudden, so that heat does not get time to get exchanged with the surroundings.

## 12.18 ADIABATIC RELATIONS BETWEEN P, V AND T

24. Applying first law of thermodynamics, obtain an adiabatic relation between pressure and volume. Hence write relation between  $P$  and  $T$  and also between  $V$  and  $T$ .

(i) Adiabatic relation between  $P$  and  $V$ .

According to the first law of thermodynamics,

$$dQ = dU + dW$$

For one mole of a gas,

$$dU = C_V dT \text{ and } dW = P dV$$

Also, for an adiabatic process,  $dQ = 0$

$$\therefore C_V dT + P dV = 0$$

According to the ideal gas equation,

$$PV = RT$$

Differentiating both sides, we get

$$P dV + V dP = R dT$$

$$\text{or } dT = \frac{P dV + V dP}{R}$$

$$\therefore C_V \cdot \frac{P dV + V dP}{R} + P dV = 0$$

$$\text{or } C_V P dV + C_V V dP + RP dV = 0$$

$$\text{or } C_V V dP + (C_V + R) P dV = 0$$

$$\text{or } C_V V dP + C_P P dV = 0$$

$$[\because C_P = C_V + R]$$

Dividing both sides by  $C_V PV$ , we get

$$\frac{dP}{P} + \frac{C_P}{C_V} \frac{dV}{V} = 0$$

$$\text{or } \frac{dP}{P} + \gamma \frac{dV}{V} = 0 \quad [\because \gamma = \frac{C_P}{C_V}]$$

Integrating both sides, we get

$$\int \frac{dP}{P} + \gamma \int \frac{dV}{V} = C$$

where  $C$  is constant of integration

$$\therefore \log_e P + \gamma \log_e V = C$$

$$\text{or } \log_e PV^\gamma = C \text{ or } PV^\gamma = e^C$$

$$\text{or } PV^\gamma = K$$

where  $K$  is another constant. This is the adiabatic relation between pressure  $P$  and volume  $V$  of an ideal gas.

(ii) Adiabatic relation between  $P$  and  $T$ . For one mole of a gas  $PV = RT$ , therefore

$$V = \frac{RT}{P}$$

Putting in  $PV^\gamma = K$ , we get

$$P \left( \frac{RT}{P} \right)^\gamma = K$$

$$\text{or } P^{1-\gamma} T^\gamma = \frac{K}{R^\gamma} = \text{another constant}$$

$$\text{i.e., } P^{1-\gamma} T^\gamma = \text{constant}$$

This is the adiabatic relation between pressure  $P$  or and temperature  $T$  of an ideal gas.

(iii) Adiabatic relation between  $V$  and  $T$ . Again, for one mole of a gas  $PV = RT$ , therefore

$$P = \frac{RT}{V}$$

Putting in  $PV^\gamma = K$ , we get

$$\frac{RT}{V} \cdot V^\gamma = K \text{ or } TV^{\gamma-1} = \frac{K}{R} = \text{another constant}$$

$$\text{i.e., } TV^{\gamma-1} = \text{constant}$$

This is the adiabatic relation between volume  $V$  and temperature  $T$  of an ideal gas.

### 12.19 WORK DONE IN AN ADIABATIC PROCESS

25. Derive an expression for the work done during the adiabatic expansion of an ideal gas.

**Work done in an adiabatic expansion.** Consider  $n$  moles of an ideal gas contained in a cylinder having insulating walls and provided with frictionless and insulating piston. Let  $P$  be the pressure of the gas. When the piston moves up through a small distance  $dx$ , the work done by the gas will be

$$dW = PA dx = P dV$$

where  $A$  is the cross-sectional area of the piston and  $dV = A dx$  is the increase in the volume of the gas.

Suppose the gas expands adiabatically and changes from the initial state  $(P_1, V_1, T_1)$  to the final state  $(P_2, V_2, T_2)$ . The total work done by the gas will be

$$W_{\text{adia}} = \int_{V_1}^{V_2} P dV.$$

For an adiabatic change

$$PV^\gamma = K \text{ or } P = KV^{-\gamma}$$

$$\therefore W_{\text{adia}} = \int_{V_1}^{V_2} KV^{-\gamma} dV.$$

$$= K \int_{V_1}^{V_2} V^{-\gamma} dV = K \left[ \frac{V^{1-\gamma}}{1-\gamma} \right]_{V_1}^{V_2}$$

$$= \frac{K}{1-\gamma} [V_2^{1-\gamma} - V_1^{1-\gamma}] = \frac{1}{\gamma-1} [KV_1^{1-\gamma} - KV_2^{1-\gamma}]$$

$$\text{But } K = P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\therefore W_{\text{adia}} = \frac{1}{\gamma-1} [P_1 V_1^\gamma V_1^{1-\gamma} - P_2 V_2^\gamma V_2^{1-\gamma}]$$

$$W_{\text{adia}} = \frac{1}{\gamma-1} [P_1 V_1 - P_2 V_2]$$

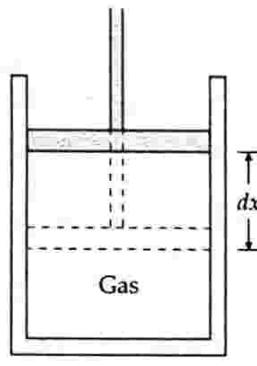


Fig. 12.16 Adiabatic expansion.

Also,  $P_1 V_1 = nRT_1$  and  $P_2 V_2 = nRT_2$

$$\therefore W_{\text{adia}} = \frac{1}{\gamma - 1} [nRT_1 - nRT_2]$$

$$\text{or } W_{\text{adia}} = \frac{nR}{\gamma - 1} [T_1 - T_2]$$

This equation gives the work done during the adiabatic expansion of  $n$  moles of an ideal gas.

Clearly, when work is done by the gas during its adiabatic expansion,  $W_{\text{adia}} > 0$  and  $T_2 < T_1$ , i.e., temperature of the gas decreases. When work is done on the gas during its adiabatic compression,  $W_{\text{adia}} < 0$  and  $T_2 > T_1$ , i.e., temperature of the gas rises.

## 12.20 FIRST LAW OF THERMODYNAMICS APPLIED TO ISOCHORIC PROCESS

**26.** Apply first law of thermodynamics to show that the change in the internal energy of a system is equal to the heat given to or taken from the system in an isochoric process.

**First law of thermodynamics applied to an isochoric process.** In an isochoric process, the volume  $V$  of the system remains constant. As there is no change in volume ( $\Delta V = 0$ ), no work is done on or by the system ( $W = P \Delta V = 0$ ). According to the first law of thermodynamics,

$$Q = \Delta U + W = \Delta U + 0 = \Delta U$$

Hence in an isochoric process, the entire heat given to or taken from the system goes to change its internal energy and temperature of the system. The change in temperature can be determined from the equation

$$Q = nC_V \Delta T.$$

## 12.21 FIRST LAW OF THERMODYNAMICS APPLIED TO A CYCLIC PROCESS

**27.** Apply first law of thermodynamics to show that the entire heat absorbed by a system is equal to the work done by the system in a cyclic process.

**First law of thermodynamics applied to a cyclic process.** In a cyclic process, the system returns to its initial state after undergoing a series of changes. As internal energy is a state function, so  $\Delta U = 0$  for a cyclic process. From first law of thermodynamics,

$$Q = \Delta U + W = 0 + W = W$$

Hence in a cyclic process, the total heat absorbed by a system is equal to the work done by the system.

## 12.22 FIRST LAW OF THERMODYNAMIC APPLIED TO ISOBARIC PROCESS

**28.** Discuss the application of first law of thermodynamics to an isobaric process.

**First law of thermodynamics applied to an isobaric process.** A thermodynamic process which

occurs at a constant pressure is called an isobaric process. For example, freezing of water, formation of steam, etc.

Suppose the pressure  $P$  of a gas remains constant and its volume changes from  $V_1$  to  $V_2$ , then the work done by the gas is

$$W = \int_{V_1}^{V_2} P dV = P \int_{V_1}^{V_2} dV = P (V_2 - V_1) \\ = nR (T_2 - T_1)$$

As the temperature of the gas changes, so its internal energy also changes. Hence in an isobaric process, the absorbed heat goes partly to increase internal energy and partly to do work.

**29.** Apply the first law of thermodynamics to determine the change in internal energy during the boiling process.

**Boiling process.** Suppose  $m$  mass of a liquid is heated at the temperature of its boiling point so that it changes into vapour at a pressure  $P$ . Let  $V_i$  be the volume of the liquid and  $V_f$  that of vapour. The work done by the liquid during its expansion at temperature  $T$  is given by

$$\Delta W = P \Delta V = P (V_f - V_i)$$

Let  $L$  be the heat of vaporisation of the liquid. It represents the heat needed per unit mass to change from liquid to vapour phase at constant temperature and pressure. Then the amount of heat required for vaporisation of  $m$  mass of liquid will be

$$\Delta Q = mL$$

Let  $U_i$  and  $U_f$  be the initial and final values of internal energy. According to the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

$$\text{or } mL = U_f - U_i + P (V_f - V_i)$$

$$\text{or } U_f - U_i = mL - P (V_f - V_i)$$

Knowing  $m$ ,  $L$ ,  $P$ ,  $V_i$  and  $V_f$ , the change in internal energy can be determined.

**30.** Apply the first law of thermodynamics to determine the change in internal energy during the melting process.

**Melting process.** Suppose  $m$  mass of a solid is heated at its melting point. When the solid melts, the change in its volume  $\Delta V$  is negligibly small. So work done by the solid is

$$\Delta W = P \Delta V = P \times 0 = 0.$$

Let  $L$  be the latent heat of fusion. It represents the heat needed per unit mass to change from solid to liquid phase at constant temperature and pressure.

Then the amount of heat required for fusion of  $m$  mass of a solid will be

$$\Delta Q = mL$$

Let  $U_i$  and  $U_f$  be the initial and final values of internal energy. According to the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W \quad \text{or} \quad mL = U_f - U_i + 0$$

$$\therefore U_f - U_i = mL$$

Hence the internal energy of a system increases by  $mL$  during the melting process.

### Examples based on

#### Isothermal and Adiabatic Processes

##### FORMULAE USED

1. Equation for isothermal process,

$$PV = \text{constant} \quad \text{or} \quad P_1 V_1 = P_2 V_2$$

2. Equations for adiabatic processes,

$$(i) P_1 V_1^\gamma = P_2 V_2^\gamma \quad (ii) T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$(iii) \frac{P_1^{\gamma-1}}{T_1^\gamma} = \frac{P_2^{\gamma-1}}{T_2^\gamma}, \quad \text{where } \gamma = C_p / C_v$$

3. Work done when 1 mole of a gas expands isothermally,

$$(i) W_{\text{iso}} = 2.303 RT \log \frac{V_2}{V_1}$$

$$(ii) W_{\text{iso}} = 2.303 RT \log \frac{P_1}{P_2}$$

4. Work done when 1 mole of a gas expands adiabatically and its temperature falls from  $T_1$  to  $T_2$ ,

$$(i) W_{\text{adi}} = \frac{R}{\gamma-1} [T_1 - T_2]$$

$$(ii) W_{\text{adi}} = \frac{1}{\gamma-1} [P_1 V_1 - P_2 V_2]$$

##### UNITS USED

All pressures are in  $\text{Nm}^{-2}$ , volumes in  $\text{m}^3$ , temperatures in K, work done W in joule.

**Example 17.** The compression ratio of a certain diesel engine is 15. This means that air in the cylinder is compressed to  $1/15$  of its initial volume. If the initial pressure is  $1.0 \times 10^5 \text{ Pa}$  and the initial temperature is  $300 \text{ K}$ , find the final pressure and temperature after compression. Air is mostly a mixture of oxygen and nitrogen and  $\gamma = 1.4$ .

**Solution.** Here  $\frac{V_1}{V_2} = 15$ ,  $P_1 = 1.0 \times 10^5 \text{ Pa}$ ,

$$T_1 = 300 \text{ K}, \quad P_2 = ?, \quad T_2 = ?$$

For an adiabatic process,  $P_1 V_1^\gamma = P_2 V_2^\gamma$

$$\therefore P_2 = P_1 \left( \frac{V_1}{V_2} \right)^\gamma \\ = 1.0 \times 10^5 \times (15)^{1.4} = 44.3 \times 10^5 \text{ Pa.}$$

Again, for an adiabatic process,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\therefore T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1}$$

$$= 300 \times (15)^{0.4} = 886 \text{ K.}$$

**EXAMPLE 18.** If, at  $50^\circ\text{C}$  and  $75 \text{ cm of mercury pressure}$ , a definite mass of a gas is compressed (i) slowly, (ii) suddenly, then what will be the final pressure and temperature of the gas in each case if the final volume is one-fourth of the initial volume? ( $\gamma = 1.5$ ).

**Solution.** Here  $V_2 = \frac{1}{4} V_1$ ,  $P_1 = 75 \text{ cm of Hg}$ ,

$$T_1 = 50 + 273 = 323 \text{ K}$$

(i) When the gas is compressed suddenly, the process is *isothermal*.

$$\therefore P_1 V_1 = P_2 V_2 \quad \text{or} \quad 75 \times V_1 = P_2 \times \frac{1}{4} V_1$$

$$\text{or} \quad P_2 = 75 \times 4 = 300 \text{ cm of Hg.}$$

As the process is isothermal, so  $T_2 = 50^\circ\text{C}$ .

(ii) When the gas is compressed suddenly, the process is *adiabatic*.

$$\therefore P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\text{or} \quad P_2 = P_1 \left( \frac{V_1}{V_2} \right)^\gamma = 75 \left( \frac{V_1}{V_1/4} \right)^{1.5} = 75 \times 4^{1.5}$$

$$= 75 \times 4 \times 4^{1/2} = 75 \times 4 \times 2 = 600 \text{ cm of Hg.}$$

$$\text{Also, } T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\text{or} \quad T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1}$$

$$= 323 \times (4)^{0.5} = 323 \times 2$$

$$= 646 \text{ K} = 373^\circ\text{C.}$$

**EXAMPLE 19.** A tyre pumped to a pressure of  $3.375 \text{ atmosphere}$  and at  $27^\circ\text{C}$  suddenly bursts. Calculate the temperature of escaping air. Given  $\gamma = 1.5$ .

**Solution.** Here  $P_1 = 3.375 \text{ atm}$ ,  $P_2 = 1 \text{ atm}$ ,  $\gamma = 1.4$

$$T_1 = 27 + 273 = 300 \text{ K}, \quad T_2 = ?$$

When tyre bursts suddenly, its air expands adiabatically. So

$$\frac{P_1^{\gamma-1}}{T_1^\gamma} = \frac{P_2^{\gamma-1}}{T_2^\gamma} \quad \text{or} \quad \left( \frac{T_1}{T_2} \right)^\gamma = \left( \frac{P_1}{P_2} \right)^{\gamma-1}$$

$$\text{or } \left( \frac{300}{T_2} \right)^{1.5} = \left( \frac{3.375}{1} \right)^{1.5-1}$$

$$\text{or } \left( \frac{300}{T_2} \right)^{3/2} = \left( \frac{3.375}{1} \right)^{1/2}$$

$$\text{or } \frac{300}{T_2} = (3.375)^{1/3} = 1.5$$

$$\text{or } T_2 = \frac{300}{1.5} = 200 \text{ K} = -73^\circ\text{C}.$$

**EXAMPLE 20.** Calculate the fall in temperature of helium initially at  $15^\circ\text{C}$ , when it is suddenly expanded to 8 times its volume. Given  $\gamma = 5/3$ .

**Solution.** Here  $T_1 = 15 + 273 = 288 \text{ K}$ ,

$$V_2 = 8 V_1, \quad \gamma = 5/3, \quad T_2 = ?$$

As the expansion of helium is sudden, so the process is adiabatic. For such a process, we have

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\text{or } T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1} = 288 \left( \frac{1}{8} \right)^{5/3-1} \\ = 288 \times \left( \frac{1}{8} \right)^{2/3} = 288 \times \frac{1}{4} = 72 \text{ K}$$

Fall in temperature of helium

$$= T_1 - T_2 = 288 - 72 = 216 \text{ K or } 216^\circ\text{C}.$$

**EXAMPLE 21.** Two different adiabatic parts for the same gas intersect two isothermals at  $T_1$  and  $T_2$  as shown in P-V diagram, Fig. 12.17. How does the ratio  $(V_a / V_d)$  compare with the ratio  $(V_b / V_c)$ ?

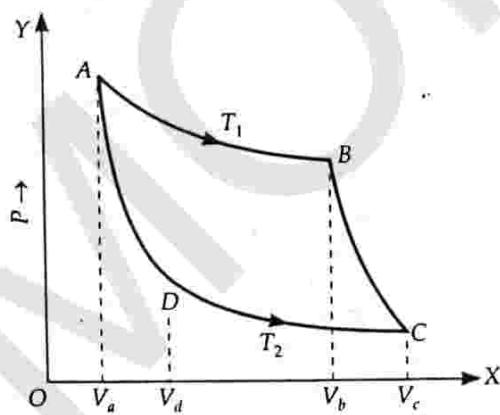


Fig. 12.17

**Solution.** For adiabatic curve BC, we have

$$T_1 V_b^{\gamma-1} = T_2 V_c^{\gamma-1} \quad \dots(i)$$

For adiabatic curve AD, we have

$$T_1 V_a^{\gamma-1} = T_2 V_d^{\gamma-1} \quad \dots(ii)$$

Dividing (ii) by (i), we get :

$$\left( \frac{V_a}{V_b} \right)^{\gamma-1} = \left( \frac{V_d}{V_c} \right)^{\gamma-1} \text{ or } \frac{V_a}{V_b} = \frac{V_d}{V_c}$$

$$\therefore \frac{V_a}{V_d} = \frac{V_b}{V_c}, \quad \text{i.e., the two ratios are equal.}$$

**EXAMPLE 22.** Two samples of a gas initially at same temperature and pressure are compressed from a volume  $V$  to  $V/2$ . One sample is compressed isothermally and the other adiabatically. In which sample is the pressure greater?

**Solution.** Here  $V_1 = V, V_2 = V/2$

$$\therefore V_1 / V_2 = 2$$

In isothermal compression,  $P_1 V_1 = P_2 V_2$

$$\therefore P_2 = P_1 \times \frac{V_1}{V_2} = P_1 \times 2 = 2 P_1$$

In adiabatic compression,

$$P'_2 = P'_1 \left( \frac{V_1}{V_2} \right)^\gamma = P_1 \times (2)^\gamma$$

As  $\gamma > 1$ , so  $2^\gamma > 2$  and hence  $P'_2 > P_2$  i.e., the pressure is greater in the sample compressed adiabatically.

**EXAMPLE 23.** Three moles of an ideal gas kept at a constant temperature of  $300 \text{ K}$  are compressed from a volume of 4 litre to 1 litre. Calculate the work done in the process. Given  $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$ .

**Solution.** Here  $n = 3, T = 300 \text{ K}, V_1 = 4 \text{ litre}$ ,

$$V_2 = 1 \text{ litre,}$$

Work done in isothermal process is given by

$$W = 2.303 nRT \log_{10} \frac{V_2}{V_1} \\ = 2.303 \times 3 \times 8.31 \times 300 \log_{10} \frac{1}{4} \\ = 2.303 \times 3 \times 8.31 \times 300 \times (-2 \log_{10} 2) \\ = -2.303 \times 3 \times 8.31 \times 300 \times 2 \times 0.3010 \\ = -1.037 \times 10^4 \text{ J.}$$

**EXAMPLE 24.** A cylinder containing one gram molecule of the gas was compressed adiabatically until its temperature rose from  $27^\circ\text{C}$  to  $97^\circ\text{C}$ . Calculate the work done and heat produced in the gas. Given  $\gamma = 1.5$ .

**Solution.** Here  $T_1 = 27 + 273 = 300 \text{ K}$ ,

$$T_2 = 97 + 273 = 370 \text{ K}$$

Work done in adiabatic compression of the gas is given by

$$W = \frac{R}{1-\gamma} (T_2 - T_1) = \frac{8.3 \times (370 - 300)}{1-1.5} = -1162 \text{ J}$$

$$\text{Heat produced, } H = \frac{W}{J} = \frac{1162}{4.2} = 276.67 \text{ cal.}$$

**EXAMPLE 25.** One gram molecule of an ideal gas at S.T.P. is subjected to a reversible adiabatic expansion to double its volume. Find the change in internal energy in the process. Given  $\gamma = 1.4$ .

**Solution.** Here  $T_1 = 273\text{ K}$ ,  $V_2 = 2 V_1$

$$\text{or } V_2/V_1 = 2, \gamma = 1.4$$

For an adiabatic change,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\therefore T_2 = T_1 \times \left( \frac{V_1}{V_2} \right)^{\gamma-1} = 273 \times \left( \frac{1}{2} \right)^{1.4-1} \\ = \frac{273}{2^{0.4}} = \frac{273}{1.319} = 207\text{ K}$$

For an adiabatic process,  $dQ = 0$

From first law of thermodynamics, change in internal energy is given by

$$dU = dQ - dW = 0 - dW \\ = -\frac{R}{\gamma-1} (T_2 - T_1) = \frac{R}{\gamma-1} (T_1 - T_2) \\ = \frac{8.3 \times (273 - 207)}{1.4 - 1} = \frac{8.3 \times 66}{0.4} = 1369.5\text{ J.}$$

**EXAMPLE 26.** A sample of gas ( $\gamma = 1.5$ ) is compressed adiabatically from a volume of  $1600\text{ cm}^3$  to  $400\text{ cm}^3$ . If the initial pressure is  $150\text{ kPa}$ , what is the final pressure and how much work is done on the gas in the process?

**Solution.** Here  $\gamma = 1.5$ ,  $V_1 = 1600\text{ cm}^3$ ,

$$V_2 = 400\text{ cm}^3, P_1 = 150\text{ kPa}, P_2 = ?$$

For an adiabatic process,

$$P_2 V_2^\gamma = P_1 V_1^\gamma$$

$$\therefore P_2 = P_1 \left( \frac{V_1}{V_2} \right)^\gamma = 150 \left( \frac{1600}{400} \right)^{1.5} = 1200\text{ kPa.}$$

Work done in the adiabatic compression,

$$W_{\text{adi}} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} \\ = \frac{150 \times 10^3 \times 1600 \times 10^{-6} - 1200 \times 10^3 \times 400 \times 10^{-6}}{1.5 - 1} \\ = \frac{240 - 480}{0.5} = -480\text{ J.}$$

### X PROBLEMS FOR PRACTICE

1. A certain gas at atmospheric pressure is compressed adiabatically so that its volume becomes half of its original volume. Calculate the resulting pressure in  $\text{Nm}^{-2}$ . Given  $\gamma$  for air = 1.4.

$$(\text{Ans. } 2.674 \times 10^5 \text{ Nm}^{-2})$$

2. A gas is suddenly compressed to one-fourth of its original volume. Calculate the rise in temperature, the original temperature being  $27^\circ\text{C}$  and  $\gamma = 1.5$ .

$$(\text{Ans. } 300^\circ\text{C})$$

3. A tyre pumped to a pressure of 6 atmosphere, suddenly bursts. Room temperature is  $15^\circ\text{C}$ . Calculate the temperature of escaping air.  $\gamma = 1.4$ .

$$(\text{Ans. } -94.4^\circ\text{C})$$

4.  $200\text{ cm}^3$  of a gas is compressed to  $100\text{ cm}^3$  at the atmospheric pressure ( $10^6$  dyne  $\text{cm}^{-2}$ ). Find the resultant pressure if the change is (i) slow (ii) sudden. Given  $\gamma = 1.4$ . [Ans. (i) 2 atm (ii) 2.638 atm]

5. A quantity of air at  $27^\circ\text{C}$  and atmospheric pressure is suddenly compressed to half its original volume. Find the final (i) pressure and (ii) temperature. Given  $\gamma$  for air = 1.42

$$[\text{Ans. (i) } 2.675\text{ atm (ii) } 128.3^\circ\text{C}]$$

6. A quantity of air at normal temperature is compressed (i) slowly (ii) suddenly to one third of its volume. Find the rise in temperature, if any in each case,  $\gamma = 1.4$ . [Ans. (i) Zero (ii)  $150.6^\circ\text{C}$ ]

7. A quantity of air is kept in a container having walls which are slightly conducting. The initial temperature and volume are  $27^\circ\text{C}$  and  $800\text{ cm}^3$  respectively. Find the rise in temperature if the gas is compressed to  $200\text{ cm}^3$  (i) in a short time and (ii) in a long time. Given  $\gamma = 1.4$ .

$$[\text{Ans. (i) } 222\text{ K (ii) Zero}]$$

8. Dry air at 10 atm pressure and  $15^\circ\text{C}$  is suddenly compressed to atmospheric pressure. Find the new temperature. Given  $\gamma = 1.41$ ,  $\log 2.88 = 0.4594$ ,  $\log 1.474 = 0.1686$ . [Ans.  $-125.1^\circ\text{C}$ ]

9. Calculate the rise in temperature when a gas, for which  $\gamma = 1.5$  is compressed to 27 times its original pressure, assuming the initial temperature to be  $27^\circ\text{C}$ . [Ans.  $599.9^\circ\text{C}$ ]

10.  $1000\text{ cm}^3$  of argon at  $27^\circ\text{C}$  is adiabatically compressed so that the temperature is  $127^\circ\text{C}$ . Calculate the resulting volume. Given  $\gamma = 5/3$ . [Ans.  $649.4\text{ cm}^3$ ]

11. Find the final volume of a gram molecule of a gas after an isothermal expansion at  $127^\circ\text{C}$ , if the original volume is  $400\text{ cm}^3$ . Given the amount of work done by a gram molecule of a gas during expansion is 2302.6 joule,  $R = 8.3\text{ joule mole}^{-1}\text{ K}^{-1}$ . [Ans.  $800\text{ cm}^3$ ]

12. Calculate work done to compress isothermally 1 g of hydrogen gas at N.T.P. to half its initial volume. Find the amount of heat evolved and change in internal energy. Given  $R = 8.31\text{ J mol}^{-1}\text{ K}^{-1}$ .

$$(\text{Ans. } -786.2\text{ J, } 187.2\text{ cal., zero})$$

13. A sample of hydrogen of mass 6 g is allowed to expand isothermally at 27°C, till its volume is doubled.

- (a) How many moles of H<sub>2</sub> do we have?  
 (b) What is the final temperature of the H<sub>2</sub>?  
 (c) Calculate work done during expansion?  
 Given, R = 8.31 J mol<sup>-1</sup> K<sup>-1</sup>.

[Ans. (i) 3 moles (ii) 27°C (iii) 5184 J]

14. 50 g of oxygen at N.T.P. is compressed adiabatically to a pressure of 5 atmosphere. Calculate the work done on the gas, if  $\gamma = 1.4$  and R = 8.31 J mol<sup>-1</sup> K<sup>-1</sup>.

(Ans. - 5167 J)

### Hints

1.  $P_1 = 1 \text{ atm} = 1.013 \times 10^5 \text{ Nm}^{-2}$ ,  $V_1 / V_2 = 2$

$$P_2 = P_1 \left( \frac{V_1}{V_2} \right)^{\gamma} = 1.013 \times 10^5 \times (2)^{1.4}$$

$$= 1.013 \times 10^5 \times 2.64 = 2.694 \times 10^5 \text{ Nm}^{-2}.$$

2.  $T_1 = 27 + 273 = 300 \text{ K}$ ,  $V_1 / V_2 = 4$ ,  $\gamma = 1.5$ ,  $T_2 = ?$

$$T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1} = 300 (4)^{1.5-1}$$

$$= 300 (4)^{1/2} = 300 \times 2 = 600 \text{ K}$$

or  $T_2 = 600 - 273 = 327^\circ\text{C}$

Rise in temperature =  $327 - 27 = 300^\circ\text{C}$ .

3. Here  $P_1 = 6 \text{ atm}$ ,  $T_1 = 15 + 273 = 288 \text{ K}$ ,  $\gamma = 1.4$ ,  $P_2 = 1 \text{ atm}$

$$\text{As } \frac{P_1^{\gamma-1}}{T_1^{\gamma}} = \frac{P_2^{\gamma-1}}{T_2^{\gamma}}$$

$$\therefore T_2^{\gamma} = T_1^{\gamma} \left( \frac{P_2}{P_1} \right)^{\gamma-1} = (298)^{1.4} \left( \frac{1}{6} \right)^{1.4-1}$$

$$\text{or } T_1^{1.4} = (298)^{1.4} \left( \frac{1}{6} \right)^{0.4}$$

Taking log of both sides, we get

$$1.4 \log T_2 = 1.4 \log 298 - 0.4 \log 6$$

$$= 1.4 (2.4742) - 0.4 (0.7782) = 3.1526$$

$$\text{or } \log T_2 = \frac{3.1526}{1.4} = 2.2519$$

$$\therefore T_2 = \text{Antilog}(2.2519) = 178.6 \text{ K}$$

$$= 178.6 - 273 = -94.4^\circ\text{C}.$$

4. Here  $V_1 = 200 \text{ cm}^3$ ,  $V_2 = 100 \text{ cm}^3$ ,  $P_1 = 1 \text{ atm}$

(i) When the change is slow, it is isothermal change.

So we have

$$P_2 = \frac{P_1 V_1}{V_2} = \frac{1 \times 200}{100} = 2 \text{ atm.}$$

(ii) When the change is sudden, it is adiabatic change. So we have

$$P_2 = P_1 \left( \frac{V_1}{V_2} \right)^{\gamma} = 1 \times \left( \frac{200}{100} \right)^{1.4} = (2)^{1.4} = 2.638 \text{ atm.}$$

5. Here  $T_1 = 27 + 273 = 300 \text{ K}$ ,  $P_1 = 1 \text{ atm}$ ,  $V_2 = V_1 / 2$ ,  $\gamma = 1.42$

(i) As  $P_2 V_2^{\gamma} = P_1 V_1^{\gamma}$

$$\therefore P_2 = P_1 \left( \frac{V_1}{V_2} \right)^{\gamma} = 1 (2)^{1.42} = 2.675 \text{ atm}$$

(ii) As  $T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$

$$\therefore T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1}$$

$$= 300 (2)^{1.42-1} = 401.3 \text{ K}$$

or  $T_2 = 401.3 - 273 = 128.3^\circ\text{C}$ .

6. Here  $V_2 = V_1 / 3$ ,  $T_1 = 273 \text{ K}$ ,  $\gamma = 1.4$ ,  $T_2 - T_1 = ?$

(i) When the air is compressed slowly, temperature remains constant. So

$$T_2 - T_1 = 0.$$

(ii) When the air is compressed suddenly, the change is adiabatic.

$$\therefore T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1}$$

$$= 273 (3)^{1.4-1} = 273 (3)^{0.4} = 423.6 \text{ K}$$

Rise in temperature

$$= 423.6 - 273 = 150.6 \text{ K} = 150.6^\circ\text{C}.$$

7. Here  $T_1 = 27 + 273 = 300 \text{ K}$ ,  $V_1 = 800 \text{ cm}^3$ ,  $V_2 = 200 \text{ cm}^3$

(i) When the gas is compressed in a short time, the process is adiabatic.

$$\therefore T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1}$$

$$= 300 \left( \frac{800}{200} \right)^{1.4-1} = 300 (4)^{0.4} = 522 \text{ K}$$

Rise in temperature =  $522 - 300 = 222 \text{ K}$ .

(ii) When the gas is compressed in long time, the process is isothermal.

$$\therefore \text{Rise in temperature} = 0.$$

8. Here  $T_1 = 15 + 273 = 288 \text{ K}$ ,  $P_1 / P_2 = 10$

$$\text{For adiabatic process, } \left( \frac{T_1}{T_2} \right)^{\gamma} = \left( \frac{P_1}{P_2} \right)^{\gamma-1}$$

$$\therefore \left( \frac{288}{T_2} \right)^{1.41} = (10)^{1.41-1} = (10)^{0.41}$$

Taking log of both sides

$$1.41(\log 288 - \log T_2) = 0.41 \log 10 = 0.41 \times 1$$

$$\log T_2 = \frac{1.41 \log 288 - 0.41}{1.41}$$

$$= \frac{1.41 \times 2.4594 - 0.41}{1.41} = 2.1701$$

$$T_2 = \text{Antilog}(2.1701) = 147.9 \text{ K or } -125.1^\circ\text{C}.$$

9. Here  $T_1 = 27 + 273 = 300 \text{ K}$ ,  $P_1 / P_2 = 1/27$ ,  $\gamma = 1.5$

$$\text{As } \left(\frac{T_1}{T_2}\right)^\gamma = \left(\frac{P_1}{P_2}\right)^{\gamma-1}$$

$$\therefore \left(\frac{300}{T_2}\right)^{1.5} = \left(\frac{1}{27}\right)^{1.5-1}$$

Taking log of both sides,

$$1.5(\log 300 - \log T_2) = 0.5(\log 1 - \log 27)$$

$$= -0.5 \log 27$$

$$\text{or } \log T_2 = \frac{1.5 \log 300 + 0.5 \log 27}{1.5}$$

$$= \frac{1.5 \times 2.4771 + 0.5 \times 1.4314}{1.5} = 2.9542$$

$$\therefore T_2 = \text{Antilog}(2.9542)$$

$$= 899.9 \text{ K or } 626.9^\circ\text{C}$$

Rise in temperature =  $626.9 - 27 = 599.9^\circ\text{C}$ .

10. Here  $V_1 = 1000 \text{ cm}^3$ ,  $T_1 = 27 + 273 = 300 \text{ K}$ ,

$$T_2 = 127 + 273 = 400 \text{ K}, \gamma = 5/3$$

$$\text{As } T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\therefore \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \text{ or } \frac{300}{400} = \left(\frac{V_2}{100}\right)^{5/3-1}$$

$$\text{or } \left(\frac{V_2}{100}\right)^{2/3} = \frac{3}{4} \text{ or } \frac{V_2}{100} = \left(\frac{3}{4}\right)^{3/2}$$

$$\text{or } V_2 = 100 \times \left(\frac{3}{4}\right)^{3/2} = 649.4 \text{ cm}^3.$$

11. Here  $T_1 = 127 + 273 = 400 \text{ K}$ ,  $V_1 = 400 \text{ cm}^3$ ,

$$W = 2306.6 \text{ J}, R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}, V_2 = ?$$

$$\text{As } W = 2.3026 RT \log_{10} \left(\frac{V_2}{V_1}\right)$$

$$\therefore 2302.6 = 2.3026 \times 8.3 \times 400 \log_{10} \left(\frac{V_2}{V_1}\right)$$

$$\text{or } \log_{10} \left(\frac{V_2}{V_1}\right) = \frac{2302.6}{2.3026 \times 8.3 \times 400} \\ = 0.3012 = \log_{10} 2$$

$$\text{Hence } \frac{V_2}{V_1} = 2, V_2 = 2V_1 = 2 \times 400 = 800 \text{ cm}^3.$$

$$12. W = 2.303 \left(\frac{R}{M}\right) T \log_{10} \left(\frac{V_2}{V_1}\right)$$

$$= 2.303 \times \frac{8.31}{2} \times 273 \log \left(\frac{1}{2}\right) = -786.2 \text{ J.}$$

$$\text{Amount of heat evolved} = \frac{786.2}{4.2} = 187.2 \text{ cal.}$$

As the change is isothermal, temperature remains constant. Internal energy of the gas also remains constant.

$\therefore$  Change in internal energy = 0.

13. Here  $T = 27 + 273 = 300 \text{ K}$ ,  $V_2 / V_1 = 2$ ,

mass  $m = 6 \text{ g}$

(i) No. of moles of hydrogen

$$= \frac{\text{Mass of hydrogen}}{\text{Molecular mass}} = \frac{6}{2} = 3.$$

(ii) As the expansion is isothermal, so final temperature =  $27^\circ\text{C}$ .

$$(iii) W_{\text{iso}} = 2.303 nRT \log \frac{V_2}{V_1}$$

$$= 2.303 \times 3 \times 8.31 \times 300 \log 2$$

$$= 2.303 \times 3 \times 8.31 \times 300 \times 0.3010 = 5184 \text{ J.}$$

14. Here  $T_1 = 273 \text{ K}$ ,  $P_1 = 1 \text{ atm}$ ,  $P_2 = 5 \text{ atm}$ ,

no. of moles of oxygen =  $50/32$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 273 \left(\frac{5}{1}\right)^{\frac{14-1}{14}}$$

$$= 273 \times (5)^{2/7} = 273 \times 1.584 = 432.37 \text{ K}$$

$$W = \frac{nR}{\gamma-1} [T_1 - T_2]$$

$$= \frac{50}{32} \times \frac{8.31}{1.4-1} (273 - 432.37) = -5167 \text{ J.}$$

## 12.23 HEAT ENGINE

31. What is a heat engine? Explain its working principle. Define its efficiency.

**Heat engine.** It is a device which converts continuously heat energy into mechanical energy in a cyclic process.

As shown in Fig. 12.18, a heat engine has the following essential parts :

(i) **Source.** It is a heat reservoir at higher temperature  $T_1$ . It is supposed to have infinite thermal capacity so that any amount of heat can be drawn from it without changing its temperature.

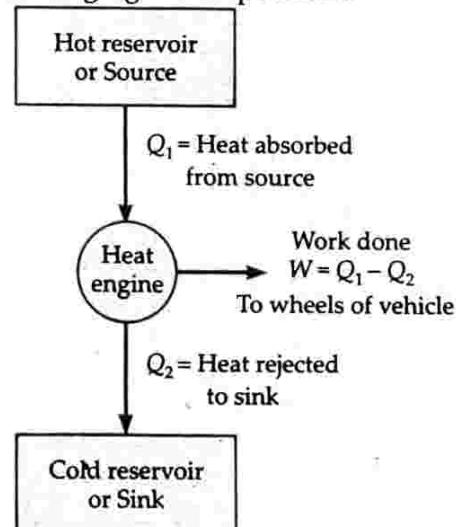


Fig. 12.18 Heat engine.

(ii) **Sink.** It is a heat reservoir at a lower temperature  $T_2$ . It has also infinite thermal capacity so that any amount of heat can be added to it without changing its temperature.

(iii) **Working substance.** Working substance is any material (solid, liquid or gas) which performs mechanical work when heat is supplied to it.

For example, a mixture of fuel vapour and air is used in a gasoline or diesel engine or steam in a steam engine.

**Working.** In every cycle of operation, the working substance absorbs a definite amount of heat  $Q_1$  from the source at higher temperature  $T_1$ , converts a part of this heat energy into mechanical work  $W$  and rejects the remaining heat  $Q_2$  to the sink at lower temperature  $T_2$ . The work done  $W$  in a cycle is transferred to the environment by some arrangement e.g., the working substance may be in a cylinder with a moving piston that transfers mechanical energy to the wheels of a vehicle via a shaft.

**Efficiency of a heat engine.** The efficiency of a heat engine is defined as the ratio of the net work done by the engine in one cycle to the amount of heat absorbed by the working substance from the source.

As the working substance returns to its initial state after completing one cycle, there is no change in its internal energy. Hence by first law of thermodynamics,

Net heat absorbed in a cycle = Work done

i.e.,

$$Q_1 - Q_2 = W$$

The efficiency of heat engine is given by

$$\eta = \frac{\text{Work done by engine (output)}}{\text{Heat absorbed from the source (input)}}$$

$$= \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} \quad \text{or, } \eta = 1 - \frac{Q_2}{Q_1}$$

**Efficiency of a heat engine is always less than unity.** Clearly, when  $Q_2 = 0$ ,  $\eta = 1$  or 100%. But any working substance working in a cycle cannot convert all the heat extracted from the source into work. It has to reject some amount of heat to the sink. That is why the efficiency of a heat engine is always less than unity. The efficiency of a steam engine varies from 12 to 16%. The maximum efficiency of a petrol engine is 26% and that of a diesel engine is 40%.

**32. What are the two basic types of heat engines ? Give examples.**

**Types of heat engines.** The heat engines are of two types :

(i) **External combustion engine.** In such a heat engine, the heat needed for the working substance is

produced by burning the fuel outside the cylinder and piston arrangement of the engine. A steam engine is an external combustion engine.

(ii) **Internal combustion engine.** In such a heat engine, the heat needed for the engine is produced by burning the fuel inside the main cylinder. The petrol and diesel engines are internal combustion engines.

## 12.24 LIMITATIONS OF THE FIRST LAW OF THERMODYNAMICS

**33. State the limitation of the first law of thermodynamics.**

**Limitations of the first law of thermodynamics.** These are as follows :

(i) **It does not indicate the direction of transfer of heat.** Heat always flows from a hot body to a cold one. First law does not give any reason as to why heat cannot flow from a cold body to a hot one.

(ii) **It does not tell anything about the conditions under which heat can be converted into mechanical work.** First law explains the stopping of a revolving wheel due to conversion of its kinetic energy into heat due to friction. But it fails to explain as to why the heat energy cannot be converted into kinetic energy of rotation of the wheel and put it back into rotation.

(iii) **It does not indicate the extent to which heat energy can be converted into mechanical work continuously.** No heat engine can convert all the heat extracted from the source into mechanical work continuously without rejecting a part of it to the surrounding. First law has no explanation for this fact.

## 12.25 SECOND LAW OF THERMODYNAMICS

**34. State and explain second law of thermodynamics. What is its significance ?**

**Second law of thermodynamics.** There are many processes in which energy is conserved and yet they are never observed. The principle which disallows such phenomena (as discussed in limitations of first law of thermodynamics) consistent with the first law of thermodynamics is called second law of thermodynamics. It can be stated in a number of ways as follows :

(i) **Kelvin-Planck statement.** It is impossible to construct an engine, which will produce no effect other than extracting heat from a reservoir and performing an equivalent amount of work.

This is applicable to a heat engine. It indicates that a working substance, operating in a cycle, cannot convert all the heat extracted from the source into mechanical work. It must reject some heat to the sink at a lower temperature.

(ii) **Clausius statement.** It is impossible for a self-acting machine, unaided by any external agency, to transfer heat from a body to another at higher temperature.

This is applicable to a refrigerator. The working substance can absorb heat from a cold body only if work is done on it. The work is done by an electric compressor. If no external work is done, the refrigerator will not work.

**Significance of second law.** The second law of thermodynamics puts a fundamental limit to the efficiency of a heat engine and the coefficient of performance of a refrigerator.

- (i) According to second law, *the efficiency of a heat engine can never be unity*. This in turn, implies that the heat released to the cold reservoir can never be made zero.
- (ii) According to second law, *the coefficient of performance of a refrigerator can never be infinite*. This implies that the external work ( $W$ ) can never be zero.

**35. State the limitations of the second law of thermodynamics.**

**Limitations of the second law of thermodynamics :**

- (i) The second law of thermodynamics cannot be proved directly. But its validity has not been contradicted by any machine designed so far.
- (ii) It is applicable to a cyclic process in which the system returns to its original state after a complete cycle of changes.
- (iii) It makes no predictions as to what will happen under certain conditions but simply states what will happen under a given set of conditions.

## 12.26 REVERSIBLE AND IRREVERSIBLE PROCESSES

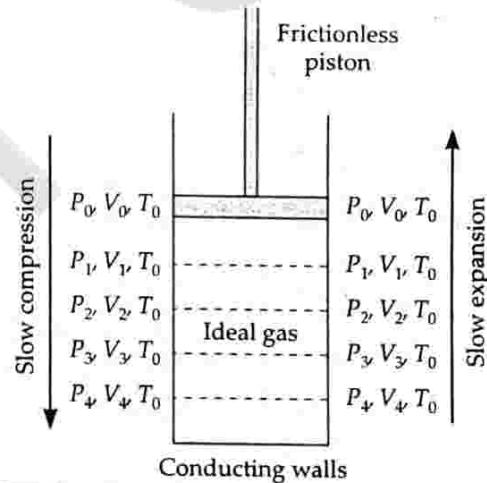
**36. What do you understand by reversible and irreversible processes ? Give examples. What are the necessary conditions for a process to be reversible ?**

(a) **Reversible process.** Any process which can be made to proceed in the reverse direction by variation in its conditions such that any change occurring in any part of the direct process is exactly reversed in the corresponding part of reverse process is called a reversible process. Thus if some work is done by the system in the direct process, an equal amount of work must be done on the system in the reverse process. If some heat is absorbed by the system in the direct process, it must release an equal amount of heat to the surroundings in the reverse process. At the end of the reversible process, both the system and surroundings must return to their initial states.

**Necessary conditions for a reversible process :**

- (i) The process must be *quasi-static*. For this, the process must be carried out infinitesimally slowly so that the system remains in thermal and mechanical equilibrium with the surroundings throughout.
- (ii) The dissipative forces such as viscosity, friction, inelasticity, etc. should be absent.

**Examples.** (i) *An infinitesimally slow compression and expansion of an ideal gas at constant temperature.* Consider some ideal gas in a cylinder having conducting walls and provided with a frictionless piston. Increase the pressure of the gas very slowly. The volume of the gas decreases isothermally. The temperature remains constant because heat is continuously rejected by the gas to the surroundings. Now decrease the pressure of the gas very slowly. The volume of the gas increases isothermally. The temperature remains constant because the gas continuously absorbs heat from the surroundings. The process is illustrated in Fig. 12.19.



**Fig. 12.19 A reversible process**

- (ii) The process of gradual compression and extension of an elastic spring is approximately reversible.
- (iii) A working substance taken along the complete Carnot's cycle.
- (iv) The process of electrolysis is reversible if the resistance offered by the electrolyte is negligibly small.

A complete reversible process is an idealised concept as it can never be realised because dissipative forces cannot be completely eliminated.

(b) **Irreversible process.** Any process which cannot be retraced in the reverse direction exactly is called an irreversible process. Most of the processes occurring in the nature are irreversible processes.

- Examples.** (i) Diffusion of gases.
- (ii) Dissolution of salt in water.
- (iii) Rusting of iron.
- (iv) Sudden expansion or contraction of a gas.

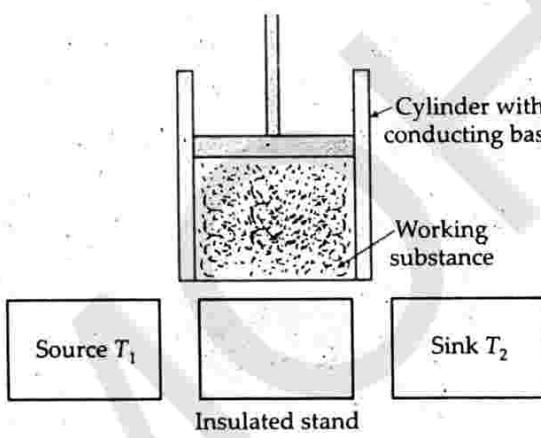
**37. Why is the concept of a reversible process a basic concept of thermodynamics?**

**Reversibility—a basic concept of thermodynamics.** Thermodynamics mainly deals with the efficiency with which a heat engine converts heat into work. According to the second law of thermodynamics, no heat engine can have an efficiency of 100%. But then what is the maximum efficiency possible for a heat engine working between two temperatures  $T_1$  and  $T_2$ ? It is found that an idealised heat engine based on a reversible cyclic process can have the highest possible efficiency. All practical engines (involving irreversible processes) have efficiency lower than this limiting efficiency. Hence the reversibility is an important concept.

## 12.27 CARNOT ENGINE

**38. Describe the operation of a Carnot's engine. Calculate the efficiency of a Carnot's engine and explain why the efficiency of an irreversible engine is small.**

**Carnot engine.** It is an ideal reversible heat engine that operates between two temperatures  $T_1$  (source) and  $T_2$  (sink). It was first conceived by a French engineer, Sadi Carnot in 1824. It operates through a series of two isothermal and two adiabatic processes called Carnot cycle. It is a theoretical heat engine with which the efficiency of practical engines is compared.



**Fig. 12.20** Carnot engine.

**Construction.** As shown in Fig. 12.20, a Carnot engine has the following main parts:

(i) **Cylinder.** This main part of the engine has a conducting base and insulating walls. It is fitted with an insulating and frictionless piston.

(ii) **Source.** It is a heat reservoir at a higher temperature  $T_1$  from which the engine draws heat. It is supposed that the source has an infinite thermal capacity and so any amount of heat can be drawn from it without changing its temperature.

(iii) **Sink.** It is a heat reservoir at a lower temperature  $T_2$  to which any amount of heat can be rejected by the engine. It has also infinite thermal capacity and so any amount of heat can be added to it without changing its temperature.

(iv) **Working substance.** The working substance is an ideal gas contained in the cylinder.

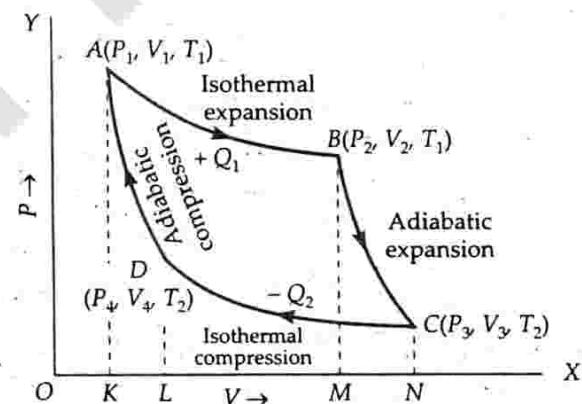
(v) **Insulating stand.** When the base of the cylinder is attached to the insulating stand, the working substance gets isolated from the surroundings.

**Carnot cycle.** The working substance is carried through a reversible cycle of the following four steps:

**Step 1. Isothermal expansion.** Place the cylinder on the source so that the gas acquires the temperature  $T_1$  of the source. The gas is allowed to expand by slow outward motion of the piston. The temperature of the gas falls. As the gas absorbs the required amount of heat from the source, it expands isothermally.

If  $Q_1$  heat is absorbed from the source and  $W_1$  work is done by the gas in isothermal expansion which takes its state from  $(P_1, V_1, T_1)$  to  $(P_2, V_2, T_1)$ , then

$$W_1 = Q_1 = nRT_1 \ln\left(\frac{V_2}{V_1}\right) = \text{area } ABMKA.$$



**Fig. 12.21** Carnot cycle.

**Step 2. Adiabatic expansion.** The gas is now placed on the insulating stand and allowed to expand slowly till its temperature falls to  $T_2$ .

If  $W_2$  work is done by the gas in the adiabatic expansion which takes its state from  $(P_2, V_2, T_1)$  to  $(P_3, V_3, T_2)$ , then

$$W_2 = \frac{nR(T_1 - T_2)}{\gamma - 1} = \text{area } BCNMB.$$

**Step 3. Isothermal compression.** The gas is now placed in thermal contact with the sink at temperature  $T_2$ . The gas is slowly compressed so that as heat is produced, it easily flows to the sink. The temperature of the gas remains constant at  $T_2$ .

If  $Q_2$  heat is released by the gas to the sink and  $W_3$  work is done on the gas by the surroundings in the isothermal compression which takes its state from  $(P_3, V_3, T_2)$  to  $(P_4, V_4, T_2)$ , then

$$W_3 = Q_2 = nRT_2 \ln\left(\frac{V_3}{V_4}\right) = \text{area } CNLDC.$$

**Step 4. Adiabatic compression.** The cylinder is again placed on the insulating stand. The gas is further compressed slowly till it returns to its initial state  $(P_1, V_1, T_1)$ .

If  $W_4$  is the work done in the adiabatic compression from  $(P_4, V_4, T_2)$  to  $(P_1, V_1, T_1)$ , then

$$W_4 = \frac{nR(T_1 - T_2)}{\gamma - 1} = \text{area } DAKLD.$$

**Net work done by the gas per cycle.**

$$\text{Total work done by the gas} = W_1 + W_2 \\ (\text{in steps 1 and 2})$$

$$\text{Total work done on the gas} = W_3 + W_4 \\ (\text{in steps 3 and 4})$$

$\therefore$  Net work done by the gas in one complete cycle,

$$W = W_1 + W_2 - (W_3 + W_4)$$

$$\text{But } W_2 = W_4$$

$$\therefore W = W_1 - W_3 = Q_1 - Q_2$$

$$\text{Also, } W = \text{area } ABMKA + \text{area } BCNMB \\ - \text{area } CNLDC - \text{area } DAKLD$$

$$\text{or } W = \text{area } ABCDA.$$

Hence in a Carnot engine, the mechanical work done by the gas per cycle is numerically equal to the area of the Carnot cycle.

**Efficiency of Carnot engine.** It is defined as the ratio of the net work done per cycle by the engine to the amount of heat absorbed per cycle by the working substance from the source.

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\text{or } \eta = 1 - \frac{nRT_2 \ln(V_3/V_4)}{nRT_1 \ln(V_2/V_1)}$$

Now step 2 is an adiabatic process, therefore

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1} \quad \dots(i)$$

Similarly, step 4 is an adiabatic compression, therefore

$$T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1} \quad \dots(ii)$$

On dividing (i) by (ii), we get

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

or

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

Hence

$$\eta = 1 - \frac{T_2}{T_1}$$

### Important Knowledge

▲ The efficiency of a Carnot engine

❖ depends upon the temperatures of the source and the sink.

❖ is independent of the nature of the working substance.

❖ is the same for all reversible engines working between the same two temperatures.

❖ is directly proportional to the temperature difference  $(T_1 - T_2)$ .

❖ is always less than 100% because  $Q_2 < Q_1$ .

▲ The efficiency of a Carnot engine will be unity or 100% if  $T_1 = \infty$  or  $T_2 = 0$  K. As 0 K or infinite temperature cannot be realised, hence a Carnot engine working on reversible cycle cannot have 100% efficiency.

▲ If  $T_1 = T_2$ , then  $\eta = 0$ . This means that the conversion of heat into mechanical work is impossible without having the source and sink at different temperatures.

$$\Delta \text{ As } \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$\therefore$  If  $T_2 = 0$  K, then  $Q_2 = 0$ .

Since  $T_2 = 0$  K cannot be realised, so  $Q_2 = 0$  is also not possible. This means that it is not possible to convert whole of the heat energy absorbed from the source into mechanical work continuously, without rejecting a part of it to the sink.

**39. Can a Carnot engine be realised in practice?**

**Non-practicability of Carnot engine.** Carnot engine is an ideal engine. It cannot be realised in practice due to the following reasons :

(i) It is difficult to realise source and sink of infinite thermal capacity.

(ii) The working substance should be an ideal gas. But no real gas fulfills the ideal gas behaviour.

(iii) The cylinder cannot be provided perfect frictionless piston.

(iv) It is difficult to attain the conditions of reversibility because the processes of expansion and compression have to be carried out very slowly.

### Examples based on Carnot Engine

#### FORMULAE USED

1. Efficiency of a heat engine,

$$\eta = \frac{\text{Work output}}{\text{Heat input}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

2. Efficiency of a Carnot's engine (an ideal heat engine),

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

where  $Q_1$  = heat extracted from the source

$Q_2$  = heat rejected to the sink

$T_1$  = temperature of the source

$T_2$  = temperature of the sink

#### UNITS USED

Heats  $Q_1$  and  $Q_2$  are in joule and temperatures  $T_1$  and  $T_2$  are in kelvin. Efficiency  $\eta$  has no units.

**EXAMPLE 27.** What is the efficiency of a Carnot engine working between ice point and steam point?

**Solution.** Temperature of source,

$$T_1 = \text{Steam point} = 373 \text{ K}$$

Temperature of sink,  $T_2 = \text{Ice point} = 273 \text{ K}$

$$\text{Efficiency, } \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{273}{373} = 0.268 = 26.8\%.$$

**EXAMPLE 28.** One of the most efficient engines ever developed operated between 2100 K and 700 K. Its actual efficiency is 40%. What percentage of its maximum possible efficiency is this?

**Solution.**  $T_1 = 2100 \text{ K}$ ,  $T_2 = 700 \text{ K}$ ,  $\eta_{\text{actual}} = 40\%$

Maximum possible efficiency is

$$\eta_{\text{max}} = 1 - \frac{T_2}{T_1} = 1 - \frac{700}{2100} = 0.666 = 66.6\%.$$

Actual efficiency as the percentage of the maximum possible efficiency is

$$\frac{\eta_{\text{actual}}}{\eta_{\text{max}}} \times 100 = \frac{40}{66.6} \times 100 \approx 60\%.$$

**EXAMPLE 29.** In a heat engine, the temperature of the source and sink are 500 K and 375 K. If the engine consumes  $25 \times 10^5 \text{ J}$  per cycle, find (i) the efficiency of the engine, (ii) work done per cycle, and (iii) heat rejected to the sink per cycle.

[Central Schools 12]

**Solution.** Here  $T_1 = 500 \text{ K}$ ,  $T_2 = 375 \text{ K}$ ,  $Q_1 = 25 \times 10^5 \text{ J}$

$$(i) \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{375}{500} = 0.25 = 25\%.$$

$$(ii) W = \eta Q_1 = 0.25 \times 25 \times 10^5 = 6.25 \times 10^5 \text{ J.}$$

$$(iii) Q_2 = Q_1 - W = (25 - 6.25) \times 10^5 = 18.75 \times 10^5 \text{ J.}$$

**EXAMPLE 30.** A carnot engine takes  $3 \times 10^6 \text{ cal}$  of heat from a reservoir at  $627^\circ\text{C}$  and gives it to a sink at  $27^\circ\text{C}$ . Find the work done by the engine.

[AIEEE 03]

**Solution.** Here  $T_1 = 627 + 273 = 900 \text{ K}$ ,

$$T_2 = 27 + 273 = 300 \text{ K}, Q_1 = 3 \times 10^6 \text{ cal}$$

$$\text{As } \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$Q_2 = \frac{T_2}{T_1} \times Q_1 = \frac{3 \times 10^6 \times 300}{900} = 10^6 \text{ cal}$$

Work done by the engine,

$$W = Q_1 - Q_2 = 3 \times 10^6 - 10^6 = 2 \times 10^6 \text{ cal}$$

$$= 2 \times 10^6 \times 4.2 \text{ J} = 8.4 \times 10^6 \text{ J.}$$

**EXAMPLE 31.** The efficiency of a Carnot cycle is  $1/6$ . If on reducing the temperature of the sink by  $65^\circ\text{C}$ , the efficiency becomes  $1/3$ , find the initial and final temperatures between which the cycle is working.

**Solution.** Given  $\eta_1 = \frac{1}{6}$ ,  $\eta_2 = \frac{1}{3}$

If the temperatures of the source and the sink between which the cycle is working are  $T_1$  and  $T_2$ , then the efficiency in the first case will be

$$\eta_1 = 1 - \frac{T_2}{T_1} \text{ or } \frac{1}{6} = 1 - \frac{T_2}{T_1} \quad \dots(1)$$

In the second case, the temperature of the sink is reduced by  $65^\circ\text{C}$ . Hence

$$\eta_2 = 1 - \frac{T_2 - 65}{T_1} \text{ or } \frac{1}{3} = 1 - \frac{T_2 - 65}{T_1} \quad \dots(2)$$

Solving equations (1) and (2), we get

$$T_1 = 390 \text{ K} = 117^\circ\text{C}, T_2 = 325 \text{ K} = 52^\circ\text{C.}$$

**EXAMPLE 32.** A reversible engine converts one fifth of heat which it absorbs from source into work. When the temperature of the sink is reduced by  $70^\circ$ , its efficiency is doubled. Calculate the temperature of the source and the sink.

**Solution.** Here  $W = \frac{1}{5} Q_1$

$$Q_2 = Q_1 - W = Q_1 - \frac{1}{5} Q_1 = \frac{4}{5} Q_1$$

$$\text{Hence } \frac{Q_2}{Q_1} = \frac{4}{5} \text{ and } \frac{T_2}{T_1} = \frac{Q_2}{Q_1} = \frac{4}{5} \quad \dots(1)$$

$$\text{Efficiency, } \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{4}{5} = \frac{1}{5}$$

On reducing the temperature of the sink by  $70^\circ$ , the efficiency is doubled.

$$2\eta = 1 - \frac{T_2 - 70}{T_1}$$

$$\text{or } 2 \times \frac{1}{5} = 1 - \frac{T_2}{T_1} + \frac{70}{T_1} = 1 - \frac{4}{5} + \frac{70}{T_1}$$

[Using equation (1)]

$$\text{or } \frac{2}{5} = \frac{1}{5} + \frac{70}{T_1} \quad \text{or} \quad \frac{70}{T_1} = \frac{1}{5}$$

$$\text{or } T_1 = 350 \text{ K.}$$

$$\text{and } T_2 = \frac{4}{5} T_1 = \frac{4}{5} \times 350 = 280 \text{ K.}$$

EXAMPLE 33. A Carnot engine has the same efficiency (i) between  $100 \text{ K}$  and  $500 \text{ K}$  and (ii) between  $T \text{ K}$  and  $900 \text{ K}$ . Calculate the temperature  $T \text{ K}$  of the sink.

**Solution.** (i) Here  $T_1 = 500 \text{ K}$ ,  $T_2 = 100 \text{ K}$

$$\therefore \text{Efficiency, } \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{100}{500} = \frac{400}{500} = \frac{4}{5}$$

(ii) Here  $T'_1 = 900 \text{ K}$ ,  $T'_2 = T \text{ K}$

$$\therefore \text{Efficiency, } \eta' = 1 - \frac{T'_2}{T'_1} = 1 - \frac{T}{900}$$

$$\text{But } \eta' = \eta \quad \therefore 1 - \frac{T}{900} = \frac{4}{5} \quad \text{or} \quad T = 180 \text{ K.}$$

EXAMPLE 34. A Carnot engine whose heat sink is at  $27^\circ\text{C}$  has an efficiency of 40%. By how many degrees should the temperature of the source be changed to increase the efficiency by 10% of the original efficiency?

**Solution.** Here  $T_2 = 27 + 273 = 300 \text{ K}$ ,  $\eta = 40\%$ ,

$$T_1 = ?$$

$$\text{As } \eta = 1 - \frac{T_2}{T_1}$$

$$\therefore \frac{T_2}{T_1} = 1 - \eta = 1 - \frac{40}{100} = \frac{60}{100} = \frac{3}{5}$$

$$\text{or } T_1 = \frac{5}{3} \times T_2 = \frac{5}{3} \times 300 = 500 \text{ K.}$$

Increase in efficiency = 10% of 40 = 4%.

$\therefore$  New efficiency,  $\eta' = 40 + 4 = 44\%$

Let new temperature of the source =  $T'_1 \text{ K}$ . Then

$$\eta' = 1 - \frac{T'_2}{T'_1} \quad \text{or} \quad \frac{44}{100} = 1 - \frac{300}{T'_1}$$

$$\text{or } \frac{300}{T'_1} = 1 - \frac{44}{100} = \frac{56}{100}$$

$$\therefore T'_1 = \frac{100 \times 300}{56} = 535.7 \text{ K}$$

Increase in temperature of the source

$$= 535.7 - 500 = 35.7 \text{ K or } 35.7^\circ\text{C.}$$

EXAMPLE 35. An ideal engine operates by taking in steam from a boiler at a temperature of  $327^\circ\text{C}$  and rejecting heat to the sink at a temperature of  $27^\circ\text{C}$ . The engine runs at  $500 \text{ rpm}$  and the heat taken is  $600 \text{ kcal}$  in each revolution. Calculate (i) the Carnot efficiency of the engine (ii) the work done in each cycle (iii) the heat rejected in each revolution and (iv) the power output of this engine.

**Solution.** Here  $T_1 = 327 + 273 = 600 \text{ K}$ ,

$$T_2 = 273 + 27 = 300 \text{ K}, Q_1 = 600 \text{ kcal}$$

$$(i) \text{Efficiency, } \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{600} = 0.5 = 50\%.$$

$$(ii) \text{As, } \eta = \frac{W}{Q_1}$$

$$\therefore W = \eta Q_1 = 0.5 \times 600 = 300 \text{ kcal} \\ = 300 \times 4.2 \times 10^3 \text{ J} = 1.26 \times 10^6 \text{ J.}$$

(iii) Heat rejected,

$$Q_2 = Q_1 - W = 600 - 300 = 300 \text{ kcal.}$$

(iv) Power output,

$$P = \frac{W}{t} = \frac{500 \times 1.26 \times 10^6 \text{ J}}{60 \text{ s}} = 1.05 \times 10^7 \text{ W} \\ = 1.05 \times 10^4 \text{ kW.}$$

EXAMPLE 36. Two Carnot engines A and B are operated in series. The first one A receives heat  $800 \text{ K}$  and rejects to a reservoir at temperature  $T \text{ K}$ . The second engine B receives the heat rejected by the first engine and in turn rejects to a heat reservoir at  $300 \text{ K}$ . Calculate the temperature  $T \text{ K}$  for the following cases :

(i) When the outputs of the two engines are equal.

(ii) When the efficiencies of the two engines are equal.

**Solution.** For engine A :  $T_1 = 800 \text{ K}$ ,  $T_2 = T \text{ K}$

$$\text{Efficiency, } \eta_A = 1 - \frac{T_2}{T_1} = 1 - \frac{T}{800}$$

$$\text{Also, } \frac{Q_2}{Q_1} = \frac{T_2}{T_1} = \frac{T}{800}$$

Work output,

$$W_A = Q_1 - Q_2 = \eta_A \times Q_1 \quad \left[ \because \eta_A = 1 - \frac{Q_2}{Q_1} \right]$$

$$\text{or } W_A = \left( 1 - \frac{T}{800} \right) Q_1$$

For engine B :  $T'_1 = T \text{ K}$ ,  $T'_2 = 300 \text{ K}$

$$\text{Efficiency, } \eta_B = 1 - \frac{T'_2}{T'_1} = 1 - \frac{300}{T}$$

Work output,

$$W_B = Q'_1 - Q'_2 = \eta_B \times Q'_1 = \left( 1 - \frac{300}{T} \right) Q'_1$$

Since the engine  $B$  absorbs the heat rejected by the engine  $A$ , so

$$Q'_1 = Q_2$$

$$W_B = \left(1 - \frac{300}{T}\right) Q_2$$

**Case (i).** When outputs of the two engines are equal,

$$W_A = W_B$$

$$\text{or } \left(1 - \frac{T}{800}\right) Q_1 = \left(1 - \frac{300}{T}\right) Q_2$$

$$\text{or } 1 - \frac{T}{800} = \left(1 - \frac{300}{T}\right) \frac{Q_2}{Q_1} = \left(1 - \frac{300}{T}\right) \frac{T}{800}$$

On solving, we get :  $T = 550 \text{ K}$ .

**Case (ii).** When the efficiencies are equal,  $\eta_A = \eta_B$

$$\text{or } 1 - \frac{T}{800} = 1 - \frac{300}{T} \quad \text{or} \quad T^2 = 24 \times 10^4$$

$$T = 489.9 \text{ K.}$$

**EXAMPLE 37.** Five moles of an ideal gas are taken in a Carnot engine working between  $100^\circ\text{C}$  and  $30^\circ\text{C}$ . The useful work done in one cycle is 420 joule. Calculate the ratio of the volume of the gas at the end and beginning of the isothermal expansion:  $R = 8.4 \text{ J mole}^{-1} \text{ K}^{-1}$ . [INCERT]

**Solution.** Here  $T_1 = 100 + 273 = 373 \text{ K}$ ,

$$T_2 = 30 + 273 = 303 \text{ K}$$

Amount of ideal gas = 5 mole

Useful work done in one cycle,

$$W = Q_1 - Q_2 = 420 \text{ J} \quad \dots(i)$$

$$\text{Now, } \frac{Q_1}{Q_2} = \frac{T_1}{T_2} = \frac{373}{303} \quad \text{or} \quad Q_1 = \frac{373}{303} Q_2 \quad \dots(ii)$$

From equations (i) and (ii), we have

$$\frac{373}{303} Q_2 - Q_2 = 420 \quad \text{or} \quad \frac{70}{303} Q_2 = 420$$

$$\text{or} \quad Q_2 = \frac{420 \times 303}{70} = 1818 \text{ J.}$$

From equation (i),

$$Q_1 = Q_2 + 420 = 1818 + 420 = 2238 \text{ J.}$$

When the gas is carried through a Carnot cycle, the heat absorbed  $Q_1$  during the isothermal expansion is equal to the work done by the gas. If  $V_1$  and  $V_2$  are the volumes of the gas at the beginning and at the end of the isothermal expansion, then

$$Q_1 = 2.303 nRT_1 \log_{10} \frac{V_2}{V_1}$$

$$\text{or} \quad 2238 = 2.303 \times 5 \times 8.4 \times 373 \log_{10} \frac{V_2}{V_1}$$

$$\text{or} \quad \log_{10} \frac{V_2}{V_1} = \frac{2238}{2.303 \times 5 \times 8.4 \times 373} = 0.0620$$

$$\text{Hence } \frac{V_2}{V_1} = 1.153.$$

**EXAMPLE 38.** A Carnot cycle is performed by air initially at  $327^\circ\text{C}$ . Each stage represents a compression or expansion in the ratio 1 : 6. Calculate (i) the lowest temperature and (ii) efficiency of the cycle. Given  $\gamma = 1.4$ .

**Solution.** Here  $T_1 = 327 + 273 = 600 \text{ K}$ ,

$$V_1 / V_2 = 1/6, \quad \gamma = 1.4$$

$$(i) \text{ As } T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$$

$$\therefore T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \times T_1 \\ = \left(\frac{1}{6}\right)^{1.4-1} \times 600 = 293 \text{ K or } 20^\circ\text{C.}$$

$$(ii) \text{ Efficiency, } \eta = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{293}{600} = 0.512 = 51.2\%.$$

### X PROBLEMS FOR PRACTICE

- Efficiency of an engine is 0.4, when temperature of its sink (cold body) is 300 K. What is the temperature of the hot body ? Find its efficiency, if the temperature of the hot body is kept unchanged, while temperature of the sink (cold body) is lowered by 50 K. (Treat it as reversible engine).  
[Ans. 500 K, 0.5]
- A Carnot engine absorbs 1000 J of heat from a reservoir at  $127^\circ\text{C}$  and rejects 600 J of heat during each cycle. Calculate (i) efficiency of the engine (ii) temperature of the sink and (iii) amount of the useful work done during each cycle. [Central Schools 05]  
[Ans. (i) 40% (ii)  $-33^\circ\text{C}$  (iii) 400 J]
- A reversible heat engine operates with an efficiency of 50%. If during each cycle it rejects 150 cal to a reservoir of heat at  $30^\circ\text{C}$ , then (i) what is the temperature of the other reservoir and (ii) how much work does it carry out per cycle ?  
[Ans. (i)  $333^\circ\text{C}$  (ii) 630 J]
- A reversible engine converts one sixth of heat input into work. When the temperature of the sink is reduced by  $62^\circ\text{C}$ , its efficiency is doubled. Find the temperature of the source and the sink.  
[Ans. 372 K, 310 K]
- Calculate the difference in efficiencies of a Carnot engine working between (i) 400 K and 350 K and (ii) between 350 K and 300 K.  
[Ans. 1.8%]

6. A Carnot engine operates between  $227^{\circ}\text{C}$  and  $127^{\circ}\text{C}$ . If it absorbs  $60 \times 10^4$  calorie at higher temperature, how much work per cycle can the engine perform ?  
**(Ans.**  $5.04 \times 10^5$  J)

7. A perfect Carnot engine utilises an ideal gas as the working substance. The source temperature is  $227^{\circ}\text{C}$  and the sink temperature is  $127^{\circ}\text{C}$ . Find the efficiency of this engine, and find the heat received from the source and the heat released to the sink when 10,000 J of external work is done.  
**(Ans.** 20%,  $5 \times 10^4$  J,  $4 \times 10^4$  J)

8. A Carnot engine takes in heat from a reservoir of heat at  $427^{\circ}\text{C}$  and gives out heat to the sink at  $77^{\circ}\text{C}$ . How many calorie per second must it take from the reservoir in order to produce useful mechanical work at the rate of 357 W ?  
**(Ans.**  $170 \text{ cal s}^{-1}$ )

9. Two Carnot engines A and B are operated in series. The first one A receives heat at 900 K and rejects to a reservoir at temperature T K. The second engine B receives the heat rejected by the first engine and in turn rejects to a heat reservoir at 400 K. Calculate the temperature T for the situation when (i) the efficiencies of the two engines are equal (ii) the work outputs of the two engines are equal.

**[Ans.** (i) 600 K (ii) 650 K]

### X HINTS

1. Case (i)  $\eta = 0.4$ ,  $T_2 = 300 \text{ K}$ ,  $T_1 = ?$   
 As  $\eta = 1 - \frac{T_2}{T_1}$   $\therefore 0.4 = 1 - \frac{300}{T_1}$  or  $T_1 = 500 \text{ K}$ .  
 Case (ii)  $T_1 = 500 \text{ K}$ ,  $T_2 = 300 - 50 = 250 \text{ K}$   
 $\therefore \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{250}{500} = 0.5$ .

2. (i)  $Q_1 = 1000 \text{ J}$ ,  $Q_2 = 600 \text{ J}$ ,  $T_1 = 127 + 273 = 400 \text{ K}$   
 $\therefore \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{600}{1000} = 0.4 = 40\%$ .  
 $T_2 = \frac{Q_2}{Q_1} \times T_1 = \frac{600}{1000} \times 400 = 240 \text{ K} = -33^{\circ}\text{C}$ .

(iii)  $W = Q_1 - Q_2 = 1000 - 600 = 400 \text{ J}$ .

3. Here  $Q_2 = 150 \text{ cal}$ ,  $T_2 = 30 + 273 = 303 \text{ K}$ ,

$\eta = 50\% = 1/2$

(i) As  $\eta = 1 - \frac{T_2}{T_1}$   $\therefore \frac{1}{2} = 1 - \frac{303}{T_1}$

or  $T_1 = 606 \text{ K} = 333^{\circ}\text{C}$ .

(ii)  $Q_1 = \frac{T_1}{T_2} \times Q_2 = \frac{606}{303} \times 150 = 300 \text{ cal}$

$\therefore W = Q_1 - Q_2 = 300 - 150 = 150 \text{ cal}$   
 $= 150 \times 4.2 = 630 \text{ J}$ .

4. Here  $\eta = 1 - \frac{T_2}{T_1} = \frac{1}{6}$  or  $\frac{T_2}{T_1} = \frac{5}{6}$

When the temperature of the sink is reduced by  $62^{\circ}\text{C}$  or 62 K, efficiency is doubled.

$$\therefore T'_2 = T_2 - 62, \quad \eta' = \frac{1}{6} \times 2 = \frac{1}{3}$$

$$\text{But } \eta' = 1 - \frac{T'_2}{T_1} \text{ or } \frac{1}{3} = 1 - \frac{T_2 - 62}{T_1} = 1 - \frac{T_2}{T_1} + \frac{62}{T_1}$$

$$\text{or } \frac{62}{T_1} = \frac{T_2}{T_1} - \frac{2}{3} = \frac{5}{6} - \frac{2}{3} = \frac{1}{6} \text{ or } T_1 = 372 \text{ K}$$

$$\text{and } T_2 = \frac{5}{6} \times T_1 = \frac{5}{6} \times 372 = 310 \text{ K.}$$

$$5. \eta = 1 - \frac{T_2}{T_1} = \left( 1 - \frac{350}{400} \right) \times 100 = 12.5\%$$

$$\eta' = 1 - \frac{T'_2}{T'_1} = \left( 1 - \frac{300}{350} \right) \times 100 = 14.3\%$$

Difference in efficiencies

$$= \eta' - \eta = 14.3 - 12.5 = 1.8\%.$$

6. Here  $T_1 = 227 + 273 = 500 \text{ K}$ ,  $T_2 = 127 + 273 = 400 \text{ K}$

$$\text{As } \eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1} \therefore \frac{W}{60 \times 10^4} = 1 - \frac{400}{500} = \frac{1}{5}$$

$$\text{or } W = 12 \times 10^4 \text{ cal}$$

$$= 12 \times 10^4 \times 4.2 \text{ J} = 5.04 \times 10^5 \text{ J.}$$

8. Use  $\eta = 1 - \frac{T_2}{T_1}$  and  $\eta = \frac{W}{Q_1}$ .

9. Here  $\eta_A = \frac{W_A}{Q_1} = \frac{T_1 - T_2}{T_1}$  and  $\eta_B = \frac{W_B}{Q'_1} = \frac{T'_1 - T'_2}{T'_1}$

Take (i)  $W_A = W_B$  and (ii)  $\eta_A = \eta_B$ .

### 12.28 CARNOT THEOREM

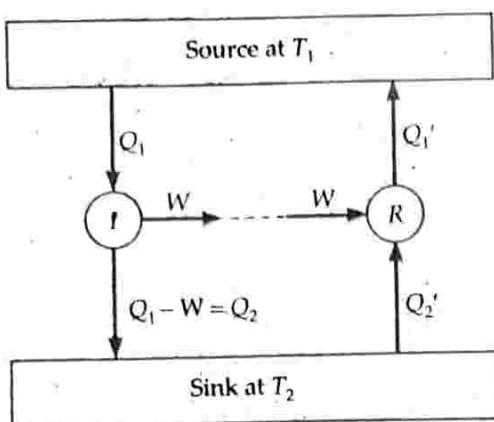
40. State Carnot theorem. Prove that the efficiency of a reversible heat engine is maximum.

**Carnot theorem.** It states that (i) no engine working between two given temperatures can have efficiency greater than that of the Carnot engine working between the same two temperatures and (ii) the efficiency of the Carnot engine is independent of the nature of the working substance.

**Proof.** As shown in Fig. 12.22, consider two engines – an irreversible engine I and a reversible engine R. The two engines are so coupled that as I runs forwards, it drives R backwards. So R works as a refrigerator driven by I.

The engine I absorbs heat  $Q_1$  from the source, performs work  $W$  and rejects heat  $Q_2$  to the sink.

$$\therefore \text{Efficiency of engine } I, \eta_I = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$



**Fig. 12.22** An irreversible engine  $I$  coupled to a reversible engine  $R$ .

The engine  $R$  absorbs heat  $Q_2'$  from the sink, work  $W$  is done on it and it rejects heat  $Q_1'$  to the source.

$$\therefore \text{Efficiency of engine } R, \eta_R = \frac{W}{Q_1'} = \frac{Q_1' - Q_2'}{Q_1'}$$

Suppose the engine  $I$  is more efficient than  $R$ . Then

$$\eta_I > \eta_R \quad \text{or} \quad \frac{W}{Q_1} > \frac{W}{Q_1'}$$

$$\therefore Q_1 < Q_1' \quad \text{i.e., } Q_1' - Q_1 \text{ is positive.}$$

The source loses heat  $Q_1$  to  $I$  and gains  $Q_1'$  from  $R$ .

$\therefore$  Net heat gained by the source per cycle

$$= Q_1' - Q_1.$$

The sink gains heat  $(Q_1 - W)$  from  $I$  and loses  $Q_2'$  to  $R$ .

$\therefore$  Net heat lost by the sink per cycle

$$\begin{aligned} &= Q_2' - (Q_1 - W) = Q_2' - Q_1 + W \\ &= Q_2' - Q_1 + (Q_1' - Q_1) = Q_1' - Q_1 \end{aligned}$$

The compound engine  $IR$  is a self-acting machine which transfers heat  $(Q_1' - Q_1)$  from the sink at lower temperature  $T_2$  to the source at higher temperature  $T_1$ , without any work being done by any external agency. This is against the second law of thermodynamics. So our assumption that  $I$  is more efficient than  $R$  is wrong. Hence no engine can have efficiency greater than that of the Carnot engine. Similarly, we can prove that a reversible engine with one working substance cannot be more efficient than the one using another working substance.

## 12.29 REFRIGERATOR OR HEAT PUMP

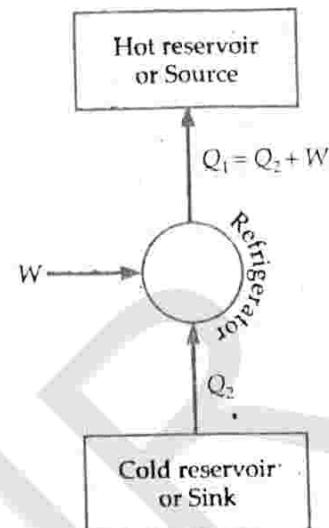
41. Describe the working of a refrigerator as a heat pump. Derive an expression for its coefficient of performance.

**Refrigerator.** A refrigerator is a Carnot's heat engine working in the reverse direction.

In a refrigerator, the working substance absorbs an amount of heat  $Q_2$  from the cold reservoir at temp-

perature  $T_2$ . An amount of work  $W$  is done on it by some external agency (a compressor pump driven by an electric motor) and rejects a larger quantity of heat  $Q_1$  to the source at temperature  $T_1$ , as shown in Fig. 12.23.

In domestic refrigerators, food and ice constitute the cold reservoir and the surroundings act as hot reservoir. Work is done by an electric motor and freon ( $CCl_2F_2$ ) is used as a working substance.



**Fig. 12.23** Refrigerator as a reversed heat engine.

The working substance is carried through a cycle of the following four steps :

- The gas is allowed to expand suddenly (adiabatically) from high to low pressure. This cools it and converts it into a vapour-liquid mixture.
- The cold fluid is allowed to absorb heat  $Q_2$  isothermally from the cold reservoir. This converts the mixture into vapour.
- Then the vapour is adiabatically compressed till it heats up to the temperature of the surroundings.
- Finally the vapour is compressed isothermally in contact with the surroundings. The vapour releases heat  $Q_1 (= Q_2 + W)$  to the surroundings and returns to the initial state. Here  $W$  is the work done on the gas per cycle.

**Coefficient of performance.** It may be defined as the ratio of the amount of heat removed ( $Q_2$ ) per cycle to the mechanical work ( $W$ ) required to be done on it.

$$\beta = \frac{Q_2}{W}$$

By first law of thermodynamics (energy conservation), the heat released to the hot reservoir is

$$Q_1 = Q_2 + W$$

$$\therefore W = Q_1 - Q_2 \quad \text{and} \quad \beta = \frac{Q_2}{Q_1 - Q_2}$$

$$\text{or} \quad \beta = \frac{1}{\left(\frac{Q_1}{Q_2} - 1\right)} = \frac{1}{\frac{T_1}{T_2} - 1} \quad \left[ \because \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \right]$$

$$\text{or} \quad \beta = \frac{T_2}{T_1 - T_2}$$

### Key Point Knowledge

- ▲ The coefficient of performance represents the efficiency of a refrigerator. Higher the value of  $\beta$ , more efficient is the refrigerator.
- ▲ In a heat engine, heat cannot be fully converted into work. Likewise a refrigerator cannot work without some external work done on the system. Hence *the coefficient of performance cannot be infinite*. Practical refrigerators have a coefficient of performance close to ten.
- ▲ Lesser the temperature difference ( $T_1 - T_2$ ) between the atmosphere and the cooling chamber, higher is the coefficient of performance of the refrigerator.
- ▲ **Why is defrosting necessary?** As the refrigerator works,  $T_2$  decreases due to formation of too much ice and  $T_1$  remains almost constant. The temperature difference ( $T_1 - T_2$ ) increases. This decreases the value of  $\beta$ . Defrosting increases  $T_2$  and hence improves the coefficient of performance.

### Examples based on Refrigerators

#### FORMULAE USED

Coefficient of performance of a refrigerator,

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

where  $Q_2$  = heat drawn per cycle from sink

$W$  = work done per cycle on refrigerator

#### UNITS USED

$Q_1$ ,  $Q_2$  and  $W$  are in joule, temperatures  $T_1$  and  $T_2$  in K.

**EXAMPLE 39.** In a refrigerator, heat from inside at 277 K is transferred to a room at 300 K. How many joules of heat shall be delivered to the room for each joule of electrical energy consumed ideally?

**Solution.** Coefficient of performance of a refrigerator,

$$\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

$$Q_2 = W \frac{T_2}{T_1 - T_2}$$

But  $W$  = Energy consumed by the refrigerator = 1 J,  
 $T_1 = 300$  K,  $T_2 = 277$  K

$$\therefore Q_2 = 1 \times \frac{277}{300 - 277} = \frac{277}{23} = 12 \text{ J}$$

Heat rejected by the refrigerator,

$$Q_1 = W + Q_2 = 1 + 12 = 13 \text{ J.}$$

**EXAMPLE 40.** (i) Calculate the least amount of work that must be done to freeze one gram of water at 0°C by means of

a refrigeration machine. The temperature of the surroundings is 27°C. (ii) How much heat is passed on to the surroundings in the process?

**Solution.** Here  $T_1 = 27^\circ\text{C} = 300 \text{ K}$ ,

$$T_2 = 0^\circ\text{C} = 273 \text{ K}$$

(i) To freeze one gram of water at 0°C, 80 cal of heat must be transferred from water at 0°C to the surroundings at 27°C.

$$\therefore Q_2 = 80 \text{ cal}$$

Coefficient of performance of a refrigerator,

$$\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

$$\text{or } \frac{80 \text{ cal}}{W} = \frac{273}{300 - 273}$$

$$\therefore W = \frac{80 \times 27}{273} = 7.91 \text{ cal.}$$

(ii) Heat transferred to the surroundings,

$$Q_1 = Q_2 + W = 80 + 7.91 = 87.91 \text{ cal.}$$

**EXAMPLE 41.** How much energy in watt hour may be required to convert 2 kg of water into ice at 0°C, assuming that the refrigerator is ideal? Given temperature of freezer is  $-15^\circ\text{C}$ , room temperature is  $25^\circ\text{C}$  and initial temperature of water is  $25^\circ\text{C}$ .

**Solution.** Here  $T_1 = 25 + 273 = 298 \text{ K}$ ,

$$T_2 = -15 + 273 = 258 \text{ K}$$

Specific heat of water,  $c = 4.2 \times 10^3 \text{ J kg}^{-1}$

Latent heat of ice,  $L = 3.36 \times 10^5 \text{ J kg}^{-1}$

Amount of heat required to be removed from 2 kg of water at  $25^\circ\text{C}$  to change it into ice at  $0^\circ\text{C}$ ,

$$\begin{aligned} Q_2 &= Mc(\theta_2 - \theta_1) + ML \\ &= 2 \times 4.2 \times 10^3 (25 - 0) + 2 \times 3.36 \times 10^5 \\ &= 2.1 \times 10^5 + 6.72 \times 10^5 \\ &= 8.82 \times 10^5 \text{ J} \end{aligned}$$

Heat rejected to the surroundings,

$$Q_1 = \frac{T_1}{T_2} \times Q_2 = \frac{298}{258} \times 8.82 \times 10^5 = 10.15 \times 10^5 \text{ J}$$

Energy supplied to convert water into ice,

$$\begin{aligned} W &= Q_1 - Q_2 = (10.15 - 8.82) \times 10^5 = 1.33 \times 10^5 \text{ J} \\ &= \frac{1.33 \times 10^5}{3600} = 36.96 \text{ Wh} \quad [\because 1 \text{ Wh} = 3600 \text{ J}] \end{aligned}$$

**EXAMPLE 42.** Ice in a cold storage melts at the rate of 2 kg per hour when the external temperature is  $20^\circ\text{C}$ . Find the minimum power output of the motor used to drive the refrigerator which just prevents the ice from melting. Take latent heat of ice =  $80 \text{ cal g}^{-1}$  and  $J = 4.2 \text{ J cal}^{-1}$ .

**Solution.** Mass of ice that melts per second

$$= \frac{2 \times 1000}{3600} = \frac{5}{9} \text{ g}$$

To prevent melting of ice, heat needed to be drawn out per second,

$$Q_2 = mL = \frac{5}{9} \times 80 = \frac{400}{9} \text{ cal}$$

Also  $T_2 = 273 \text{ K}$ ,  $T_1 = 20 + 273 = 293 \text{ K}$

$$\text{Coefficient of performance, } \beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

$$\therefore W = \frac{Q_2 (T_1 - T_2)}{T_2} = \frac{400 \times (293 - 273)}{9 \times 273}$$

$$= \frac{400 \times 20}{9 \times 273} \text{ cal s}^{-1} = \frac{8000 \times 4.2}{9 \times 273} \text{ Js}^{-1}$$

$$= 13.67 \text{ Js}^{-1}$$

∴ Minimum power of the motor used = 13.67 W.

**EXAMPLE 43.** A Carnot engine having a perfect gas as the working substance is driven backward and is used for freezing water already at 0°C. If the engine is driven by a 500 W electric motor having an efficiency of 60%, how long will it take to freeze 15 kg of water. Take 15°C and 0°C as the working temperatures of the engine and assume there are no heat losses in the refrigerating system. Latent heat of ice =  $333 \times 10^3 \text{ J kg}^{-1}$ .

**Solution.** Here  $T_1 = 273 \text{ K}$ ,  $T_2 = 15 + 273 = 288 \text{ K}$ ,

$$L = 333 \times 10^3 \text{ J kg}^{-1}$$

Efficiency of electric motor = 60%

Useful power of the engine

$$= 60\% \text{ of } 500 \text{ W} = 300 \text{ W}$$

or useful work done by the engine,  $W = 300 \text{ Js}^{-1}$

Coefficient of performance of the refrigerator,

$$\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

∴ Heat drawn from water at 0°C to freeze it into ice,

$$Q_2 = \frac{T_2}{T_1 - T_2} \times W$$

$$= \frac{273}{288 - 273} \times 300 = 5460 \text{ Js}^{-1}$$

Total heat needed to be drawn from 15 kg water to freeze into ice,

$$Q = mL = 15 \times 333 \times 10^3 \text{ J}$$

Total time taken in freezing water into ice

$$= \frac{Q}{Q_2} = \frac{15 \times 333 \times 10^3}{5460} = 914.8 \text{ s.}$$

### X PROBLEMS FOR PRACTICE

1. Refrigerator A works between  $-10^\circ\text{C}$  and  $27^\circ\text{C}$ , while refrigerator B works between  $-27^\circ\text{C}$  and  $17^\circ\text{C}$ , both removing heat equal to 2000 J from the freezer. Which of the two is the better refrigerator?

(Ans. Refrigerator A)

2. Assuming that a domestic refrigerator can be regarded as a reversible engine working between the temperature of melting ice and that of the atmosphere ( $17^\circ\text{C}$ ), calculate the energy which must be supplied to freeze one kilogram of water already at  $0^\circ\text{C}$ . (Ans.  $2.092 \times 10^4 \text{ J}$ )

3. A refrigerator whose coefficient of performance is 5 extracts heat from the cooling compartment at the rate of 250 J/cycle. How much electric energy is spent per cycle? How much heat per cycle is discharged to the room? (Ans. 50 J, 300 J)

4. A refrigerator has to transfer an average of 263 J of heat per second from temperature  $-10^\circ\text{C}$  to  $25^\circ\text{C}$ . Calculate the average power consumed, assuming no energy losses in the process. [Central Schools 04]

(Ans. 35 W)

5. A refrigerator freezes 5 kg of water at  $0^\circ\text{C}$  into ice at  $0^\circ\text{C}$  in a time interval of 20 minutes. Assume that the room temperature is  $20^\circ\text{C}$ . Calculate the minimum power needed to accomplish it.

(Ans. 102.5 W)

### X HINTS

2. Here  $T_2 = 273 \text{ K}$ ,  $T_1 = 17 + 273 = 290 \text{ K}$

Heat required to be removed to freeze 1 kg of water at  $0^\circ\text{C}$

$$Q_2 = mL = 1 \times 80,000 \text{ cal} = 80,000 \times 4.2 \text{ J}$$

$$\text{As } \beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

$$\therefore W = \frac{Q_2 (T_1 - T_2)}{T_1}$$

$$= \frac{80000 \times 4.2 \times (290 - 273)}{273} = 2.092 \times 10^4 \text{ J.}$$

3. Here  $Q_2 = 250 \text{ J / cycle}$ ,  $\beta = 5$

$$\text{As } \beta = \frac{Q_2}{W}$$

$$\therefore W = \frac{Q_2}{\beta} = \frac{250}{5} = 50 \text{ J / cycle}$$

Heat discharged to the room per cycle,

$$Q_1 = Q_2 + W = 250 + 50 = 300 \text{ J.}$$

4. Here  $T_1 = 25 + 273 = 298 \text{ K}$ ,

$$T_2 = -10 + 273 = 263 \text{ K,}$$

$$Q_2 = 263 \text{ J}$$

$$\text{As } \frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\therefore Q_1 = \frac{T_1}{T_2} \times Q_2 = \frac{298}{263} \times 263 = 298 \text{ J s}^{-1}$$

Average power consumed,

$$W = Q_1 - Q_2 = 298 - 263 = 35 \text{ Js}^{-1} = 35 \text{ W.}$$

$$5. Q_2 = mL = 5 \times 80 = 400 \text{ kcal}$$

$$T_1 = 20 + 273 = 293 \text{ K}, T_2 = 273 \text{ K}$$

$$\text{As } \beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

$$\therefore W = \frac{Q_2 (T_1 - T_2)}{T_2} = \frac{400 \times (293 - 273)}{273}$$

$$= 29.3 \text{ kcal} = 29.3 \times 4.2 \times 10^3 \text{ J} = 123 \times 10^3 \text{ J}$$

$$t = 20 \text{ min} = 1200 \text{ s}$$

$$P = \frac{W}{t} = \frac{123 \times 10^3}{1200} = 102.5 \text{ W.}$$

## Very Short Answer Conceptual Problems

**Problem 1.** What is a cyclic process ? What is the change in internal energy of a system after it completes one cycle of such a process ?

**Solution.** Any process in which the system returns to its initial state after undergoing a series of changes is known as a cyclic process. The change in internal energy after complete cycle is zero because the system returns to its initial state.

**Problem 2.** State the first law of thermodynamics.

[Delhi 1999]

**Solution.** According to the first law of thermodynamics, the amount of heat  $\Delta Q$  absorbed by a system capable of doing mechanical work is equal to the sum of the increase in internal energy  $\Delta U$  of the system and the external work  $\Delta W$  done by the system. Mathematically,

$$\Delta Q = \Delta U + \Delta W = \Delta U + P \cdot \Delta V.$$

**Problem 3.** How does the internal energy of an ideal gas differ from that of real gas ?

**Solution.** The internal energy of an ideal gas consists of only the kinetic energy of the particles. But for real gases it consists of both the kinetic as well as potential energies.

**Problem 4.** Is the internal energy of a gas a function of pressure ? Comment.

**Solution.** The internal energy of an ideal gas depends only on the temperature of the gas, while that of a real gas depends on the temperature and volume, which in turn is dependent on pressure.

**Problem 5.** Out of a solid, liquid and gas of the same mass and at the same temperature, which one has the greatest internal energy ? Which one least ? Justify.

**Solution.** The gas has greatest internal energy because the potential energy (which is negative) of the molecules is very small. On the other hand, the (negative) potential energy of the molecules of a solid is very large ; hence the internal energy of solid is least.

**Problem 6.** When is the heat supplied to a system equal to the increase in its internal energy ?

**Solution.** According to the first law of thermodynamics,

$$\Delta Q = \Delta U + P \Delta V.$$

If the heat is supplied in such a manner that the volume does not change (for isochoric change,  $\Delta V = 0$ ). Then whole of the heat energy supplied to the system will increase internal energy only.

**Problem 7.** A gas does work during isothermal expansion. What is the source of mechanical energy so produced ?

**Solution.** By first law of thermodynamics,  $\Delta Q = \Delta U + \Delta W$ . But for an isothermal process,  $\Delta U = 0$ , so  $\Delta W = \Delta Q$ . Thus the energy required for doing mechanical work during an isothermal process is obtained as heat by the gas from the surroundings.

**Problem 8.** A gas does work during adiabatic expansion. What is the source of mechanical energy so produced ?

**Solution.** By first law of thermodynamics,  $\Delta Q = \Delta U + \Delta W$ . But for an adiabatic process,  $\Delta Q = 0$ , so  $\Delta W = -\Delta U$ . Thus the source of energy required for doing mechanical work during adiabatic expansion is the internal energy of the gas itself.

**Problem 9.** The temperature of a gas rises during an adiabatic compression, although no heat is given to the gas from outside. Why ?

**Solution.** By first law of thermodynamics,  $\Delta Q = \Delta U + \Delta W$ . For an adiabatic compression,  $\Delta Q = 0$ , so  $\Delta U = -\Delta W$ . That is work is done on the gas which increases its internal energy. Hence temperature of the gas rises.

**Problem 10.** An ideal gas is compressed at constant temperature. Will its internal energy increase or decrease ?

**Solution.** It will remain same because the internal energy of a gas depends only on its temperature.

**Problem 11.** Cooling is produced when a gas at high pressure suddenly expands. Why ?

**Solution.** During its expansion, the gas does work against high pressure. This decreases the internal energy and hence the temperature of the gas.

**Problem 12.** When a gas is suddenly compressed, its temperature rises. Why?

**Solution.** Sudden compression of a gas is an adiabatic process. The work done in compressing the gas increases the internal energy of the gas. Hence the temperature of the gas rises.

**Problem 13.** If an inflated tyre bursts, the air escaping out is cooled, why?

**Solution.** When the tyre bursts, there is an adiabatic expansion of air because the pressure of the air inside is sufficiently greater than the atmospheric pressure. During the expansion, the air does some work against the surroundings, therefore, its internal energy decreases, and as such temperature falls.

**Problem 14.** Is it possible that there is change in temperature of a body without giving heat to it or taking heat from it?

**Solution.** Yes, for example, during an adiabatic compression temperature increases and in an adiabatic expansion temperature decreases, although no heat is given to or taken from the system in these changes.

**Problem 15.** Is it possible that there is no increase in the temperature of a body despite being heated?

**Solution.** Yes, for example, during a change of state (from solid to liquid or from liquid to gas), the system takes heat, but there is no rise in temperature. Internal energy of the system increases in each case.

**Problem 16.** Can the temperature of a gas be increased by keeping its pressure and volume constant?

**Solution.** No, it cannot be increased.

**Problem 17.** Why does air pressure in a car tyre increase during driving? [Delhi 09]

**Solution.** During driving, as a result of the friction between the tyre and the road, the temperature of the tyre and hence that of air inside it, increases. Since the volume of air in the tyre remains constant, pressure of the air increases due to increase of temperature (Charles's law).

**Problem 18.** Whose molecules : liquid water at 0°C or ice at 0°C have greater potential energy? Give reason.

**Solution.** Water molecules possess more potential energy. The heat given to melt the ice at 0°C is used up in increasing the potential energy of water molecules formed at 0°C.

**Problem 19.** Why mechanical energy can be completely converted into heat energy but the whole of the heat energy cannot be converted into mechanical energy?

**Solution.** The whole of the mechanical energy can be absorbed by the molecules of the system in the form of their kinetic energy. This kinetic energy gets converted into heat. But the whole of the heat energy cannot be converted into work as a part of it is always retained by the system as its internal energy.

**Problem 20.** During adiabatic changes, the volume of a gas is found to depend inversely on the square of its absolute temperature. Find how its pressure will depend on the absolute temperature.

**Solution.** Given  $V \propto \frac{1}{T^2}$

$$\text{or } V = \frac{\text{constant}}{T^2}$$

$$\text{But } \frac{PV}{T} = \text{constant}$$

$$\therefore \frac{P}{T} \cdot \frac{\text{constant}}{T^2} = \text{constant} \quad \text{or } P \propto T^3.$$

**Problem 21.** Does the mass of a body change when it is heated or cooled?

**Solution.** When a body is heated or cooled, it absorbs or releases energy and so its mass increases or decreases in accordance with the Einstein's relation :  $E = mc^2$ . As the speed of light  $c$  is very large, so the change in mass is extremely small.

**Problem 22.** When air of the atmosphere rises up, it cools. Why?

**Solution.** When the air rises up, it expands due to the decrease in atmospheric pressure. It does work at the expense of its internal energy. So its temperature falls.

**Problem 23.** Is the equation  $PV = RT$  valid for isothermal and adiabatic processes?

**Solution.** Yes, the equation  $PV = RT$  is valid for all types of the thermodynamically processes.

**Problem 24.** If  $PV = RT$  is valid for all types of thermodynamically processes, what do the relations  $PV = \text{a constant}$  and  $PV^\gamma = \text{a constant}$ , signify?

**Solution.** The relations  $PV = \text{a constant}$  and  $PV^\gamma = \text{a constant}$ , are the relations between pressure and volume in the isothermal and adiabatic processes respectively. They are derived from the equation  $PV = RT$ .

**Problem 25.** First law of thermodynamics does not forbid flow of heat from lower temperature to higher temperature. Comment.

**Solution.** First law of thermodynamics simply tells about the conversion of mechanical energy into heat energy and vice-versa. It does not put any condition as to why heat cannot flow from lower temperature to higher temperature.

**Problem 26.** Can two isothermal curves intersect?

**Solution.** No. If two isotherms intersect, then this would mean that the pressure and volume of a gas are the same at two different temperatures. This is not possible.

**Problem 27.** What is the significance of the area of closed curve on a P-V diagram?

**Solution.** The area gives the work done in a cyclic process.

**Problem 28.** Can the work done during a cyclic process be zero ?

**Solution.** Yes. The work done in a cyclic process is zero, if the process reverses exactly under similar conditions such that it retraces the same  $P$ - $V$  diagram. In such a case, the area of the curve on a  $P$ - $V$  diagram is zero and hence the work done is zero.

**Problem 29.** How many specific heats does a gas possess ?

**Solution.** A gas can possess infinite number of specific heats, depending upon the conditions of temperature and pressure. However, generally we consider only two specific heats, one at constant pressure and the other at constant volume.

**Problem 30.** Why a gas has two principal specific heat capacities ? [Delhi 97, 05]

Or

Gases have two specific heats, but the solids and liquids possess only one specific heat. Why ?

**Solution.** When the gases are heated, there occurs an appreciable change in their volume. So energy is required for expansion and we have two specific heats one at constant pressure and the other at constant volume. In case of solids and liquids, expansion is negligible, so they have only one specific heat i.e., at constant volume.

**Problem 31.** Compare the formula  $C_p - C_V = R$  for an ideal gas with the thermodynamic relation

$$\Delta U = \Delta Q - P \Delta V.$$

**Solution.** The equation  $\Delta U = \Delta Q - P \Delta V$  can be written as

$$\Delta Q = \Delta U + P \Delta V$$

This equation implies that when the heat  $\Delta Q$  is given to an ideal gas, a part of it is used in increasing the internal energy ( $\Delta U$ ) and the remaining in doing work of expansion ( $P \Delta V$ ).

Similarly, the formula  $C_p - C_V = R$  can be written as

$$C_p \Delta T = C_V \Delta T + R \Delta T$$

This equation implies that when the heat  $C_p \Delta T$  is given to an ideal gas at constant pressure, its one part is used in increasing the temperature or internal energy ( $C_V \Delta T$ ) and the other part in doing work against external pressure ( $R \Delta T$ ).

**Problem 32.** A gas has two principal specific heats. Which one is greater and why ? [Delhi 97]

**Solution.** Refer answer to Q. 16 on page 12.10.

**Problem 33.** Of what physical significance is the difference between the two principal specific heat capacities and their ratio ? [Delhi 97]

**Solution.** (i) The difference between the two principal specific heats is equal to the amount of heat equivalent to the work performed by the gas during expansion at constant pressure.

(ii) Knowing the specific heat ratio of a gas, we can determine the number of degrees of freedom and the atomicity of any gas.

$$\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f}$$

**Problem 34.** Can specific heat of a gas be negative ?

**Solution.** Yes ; negative specific heat will imply that with rise in temperature ( $\Delta T$  is +ve) heat will be released ( $Q$  is -ve). This actually happens in case of saturated vapour i.e., specific heat of saturated vapour is negative.

**Problem 35.** What is the specific heat of a gas in an isothermal process ? [Chandigarh 04 ; Central Schools 10]

**Solution.** As  $c = \frac{\Delta Q}{m \Delta T}$

For an isothermal process,  $\Delta T = 0$ , so  $c = \infty$ .

**Problem 36.** What is the specific heat of a gas in an adiabatic process ? [Chandigarh 04 ; Central Schools 10]

**Solution.** As  $c = \frac{\Delta Q}{m \Delta T}$ .

For an adiabatic process,  $\Delta Q = 0$ , so  $c = 0$ .

**Problem 37.** A liquid is being converted into steam at its boiling point. What will be the specific heat of the liquid at this time ?

**Solution.** During vaporisation, the temperature of the liquid remains constant ( $\Delta T = 0$ ). Hence the specific heat,

$$c = \frac{\Delta Q}{m \Delta T} = \infty.$$

**Problem 38.** By what methods can the internal energy of an ideal gas be changed ? Give examples.

**Solution.** The internal energy of an ideal gas can be changed by heating or cooling the gas in a closed vessel, by adiabatic compression or expansion of the gas.

**Problem 39.** Heat equivalent to 50 joule is supplied to a thermodynamic system and 10 joule work is done on the system. What is the change in the internal energy of the system in the process ?

**Solution.** Here  $\Delta Q = +50$  J,  $\Delta W = -10$  J

$$\therefore \Delta U = \Delta Q - \Delta W = 50 - (-10) = +60 \text{ J.}$$

Thus internal energy of the system increases by 60 J.

**Problem 40.** 400 J of work is done on a gas to reduce its volume by compression adiabatically. What is the change in internal energy of the gas ?

**Solution.** For an adiabatic process,

$$\Delta Q = 0$$

As work is done on the gas, so

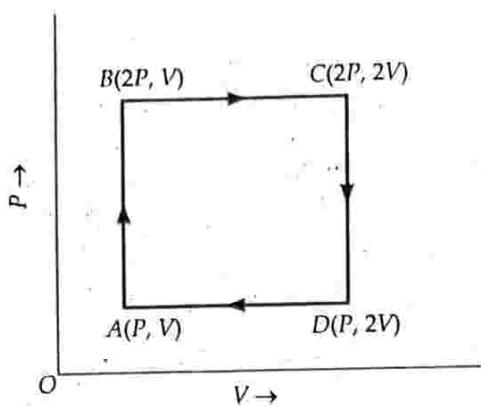
$$\Delta W = -400 \text{ J}$$

Change in internal energy,

$$\begin{aligned} \Delta U &= \Delta Q - \Delta W = 0 - (-400) \\ &= +400 \text{ J.} \end{aligned}$$

**Problem 41.** An ideal gas is taken around the cycle ABCDA as shown in the P-V diagram. What is the work done during the cycle ?

[IIT 93]

**Fig. 12.24**

**Solution.** Work done in the cyclic process

$$\begin{aligned} &= \text{Area of the loop } ABCD \\ &= (2P - P) \times (2V - V) = PV. \end{aligned}$$

**Problem 42.** What is difference between heat and work ?

**Solution.** Heat and work are two different modes of energy transfer to a system. Heat is the energy transfer that occurs due to temperature difference between the system and the surroundings. Work is the energy transfer that is brought about by other means, such as moving the piston of a cylinder containing the gas, by raising or lowering some weight connected to it.

**Problem 43.** Why is the conversion of heat into work not possible without sink at lower temperature ?

**Solution.** For converting heat energy into work continuously, a part of the heat energy absorbed from the source has to be rejected. The heat energy can be rejected only if there is a body, whose temperature is less than that of the source. This body at low temperature is called sink.

**Problem 44.** Explain why it is impossible to design a heat engine with 100% efficiency.

*Or*

Even Carnot engine cannot have 100% efficiency.  
Explain, why ?

[AIEEE 02]

**Solution.** Efficiency,  $\eta = 1 - \frac{T_2}{T_1}$

The efficiency will be 100% or 1, if  $T_2 = 0$  K. Since, temperature equal to 0 K cannot be realised, a heat engine with 100% efficiency cannot be designed.

**Problem 45.** Why can a ship not use the internal energy of sea water to operate its engine ?

**Solution.** For the operation of a heat engine we require a sink at temperature lower than the source and of sufficiently high thermal capacity, which is not possible in the sea.

**Problem 46.** In a Carnot engine, the temperature of the sink is increased. What will happen to its efficiency ?

**Solution.** Efficiency,  $\eta = 1 - \frac{T_2}{T_1}$

If the temperature ( $T_2$ ) of the sink is increased, the efficiency of the Carnot engine will decrease.

**Problem 47.** What is meant by reversible engine ? Explain why the efficiency of a reversible engine is maximum ?

**Solution.** The engine in which the process can be retraced at any stage of its operation by reversing the boundary conditions is called reversible engine. Its efficiency is maximum because in such a device no dissipation of energy takes place against friction, etc.

**Problem 48.** If you are asked to increase the efficiency of a Carnot engine by increasing the temperature of the source or by decreasing the temperature of the sink by 10 K, which method would you prefer and why ?

**Solution.** Let  $T_1$  and  $T_2$  be initial temperatures of source and sink respectively. When temperature of the source is increased by 10 K, the efficiency becomes

$$\eta_1 = 1 - \frac{T_2}{T_1 + 10}$$

When the temperature of the sink is decreased by 10 K, the efficiency becomes

$$\eta_2 = 1 - \frac{T_2 - 10}{T_1}$$

It can be easily seen that

$$\eta_2 - \eta_1 > 0 \text{ or } \eta_2 > \eta_1.$$

Hence the efficiency of a Carnot engine can be increased by a greater amount by decreasing the temperature of the sink by 10 K than by increasing the temperature of the source by 10 K.

**Problem 49.** The temperature of the surface of the sun is approximately 6000 K. If we take a big lens and focus the sun rays, can we produce a temperature of 8000 K ?

**Solution.** No. According to second law of thermodynamics, heat by itself cannot flow from a body at lower temperature to a body at higher temperature. This can only be accomplished with the help of an external agency.

**Problem 50.** Is the efficiency of a heat engine more in hilly areas than in the plains ?

**Solution.** In hilly areas, the temperature of the surroundings is lower than that in plains, so the ratio  $T_2 / T_1$  is less in hilly areas than that in plains.

$$\text{But } \eta = 1 - \frac{T_2}{T_1}$$

Hence efficiency  $\eta$  is more in hilly areas than in plains.

**Problem 51.** Is it theoretically possible to device a heat engine which will create no thermal pollution ?

**Solution.** No. According to the second law of thermodynamics, whole of the heat cannot be converted into work. So a part of the heat that is not converted into work is exhausted by the engine to the atmosphere (sink). Hence thermal pollution will always occur.

**Problem 52.** Can a kitchen be cooled by leaving the door of an electric refrigerator open ?

Or

If a door of a working refrigerator is kept open for a long time in a closed room, will it make the room warm or cool ?

[Himachal 06, 07C]

**Solution.** No. A kitchen cannot be cooled by leaving the door of a refrigerator open, rather it will get slightly heated. When the door of the refrigerator is kept open, refrigerator now extracts heat from the kitchen room (acting as cooling chamber). Work is done on it by the electric motor and the total energy is rejected to the room (now acting as surroundings). Thus the work done by the motor gets added to the room, so it gets heated.

**Problem 53.** How a refrigerator can be used as a heat pump to heat a house in winter ?

**Solution.** When the outside environment is colder than the inside of a room, we leave a refrigerator open with its radiator (backside) facing the room. The refrigerator pumps in heat from the environment to the room. This heats up the room.

## Short Answer Conceptual Problems

**Problem 1.** A thermos flask contains coffee. It is vigorously shaken. Consider the coffee as the system. (i) Has any heat been added to it ? (ii) Has any work been done on it ? (iii) Has its internal energy changed ? (iv) Does its temperature rise ?

**Solution.** (i) No. As the thermos flask is insulated, heat has not been added to the coffee ( $\Delta Q = 0$ ).

(ii) Yes. Some work is done by the man in shaking the coffee against the forces of viscosity i.e.,  $\Delta W$  is negative.

(iii) By first law of thermodynamics,  $\Delta Q = \Delta U + \Delta W$ . As  $\Delta Q = 0$  and  $\Delta W$  is negative, so  $\Delta U$  is positive i.e., internal energy of the coffee increases.

(iv) Because of the increase in internal energy of the coffee, the temperature of the coffee will also increase.

**Problem 2.** When a bore is made with a small drill in a hard board, the drill becomes very hot. But when the bore is made in a soft board, the drill does not become so hot. Why ?

**Solution.** When a bore is made in a hard board, the friction between the drill and the board is quite large. A large amount of work has to be done in making the bore

**Problem 54.** Milk is poured into a cup of tea and is mixed with a spoon. Is this an example of a reversible process ? Give reason.

**Solution.** No. When milk is poured into a cup of tea and mixed, some work is done which gets converted into heat. Heat produced cannot be converted back into work, which will separate milk from tea. Hence the mixing of milk with tea is an irreversible process.

**Problem 55.** A system goes from state A to B via two processes I and II, as shown in Fig. 12.25. How are  $\Delta U_1$  and  $\Delta U_2$  (the changes in internal energies in the processes I and II) related to each other ? [AIEEE 05]

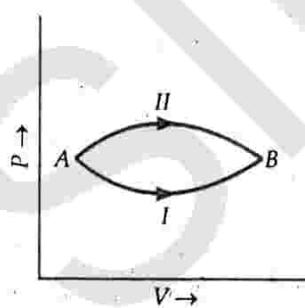


Fig. 12.25

**Solution.** Since the initial and final states are same for both processes I and II and the change in internal energy is path independent, therefore,

$$\Delta U_1 = \Delta U_2$$

which produces a large amount of heat and the drill becomes too hot to touch. In case of the soft board, the friction is less, small work has to be done in making the bore. Hence heat produced is also small.

**Problem 3.** What is an isothermal process ? What are the essential conditions for an isothermal process to take place ?

[Delhi 10]

**Solution.** A process in which temperature remains constant is called isothermal process. The essential conditions for an isothermal process to take place are :

- (i) The walls of the container must be perfectly conducting to allow free exchange of heat between the gas and the surroundings.
- (ii) The process of compression or expansion should be slow, so as to provide sufficient time for the exchange of heat.

**Problem 4.** What is an adiabatic process ? What are the essential conditions for an adiabatic process to take place ?

**Solution.** A process in which a thermally insulated system neither loses nor gains heat from the surroundings is called adiabatic process.

The essential conditions for an adiabatic process to take place are :

- (i) The walls of the container must be perfectly nonconducting to prevent any exchange of heat between the gas and the surroundings.
- (ii) The process of compression or expansion should be rapid, so that there is time for the exchange of heat.

**Problem 5. Explain why the internal energy of a compressed gas is less than that of a rarefied gas at the same temperature.**

**Solution.** When a real gas is compressed, its molecules come closer. Their mutual interactions become stronger. This increases the magnitude of potential energy. But potential energy is negative. Hence the total energy of the compressed gas decreases. However, the kinetic energy is same for both the compressed and rarefied gases as both are at the same temperatures.

**Problem 6. When you whistle out air on to your palm held close to your mouth, the air feels cold ; but when you blow out air from your mouth, keeping it wide open, the air feels hot. Why ?**

**Solution.** During whistling, we blow out air through a small opening between the lips. This is an adiabatic expansion, so there is a fall in temperature. But when we keep our mouth wide open, hot air of the mouth blows on to the palm which feels hot.

**Problem 7. Two bodies of specific heats  $c_1$  and  $c_2$  having same heat capacities are combined to form a single composite body. What is the specific heat of the composite body ?**

**Solution.** As the heat capacities are equal, so  $m_1 c_1 = m_2 c_2$ . Let  $c$  be the specific heat of the composite body. Then

$$(m_1 + m_2) c = m_1 c_1 + m_2 c_2 = m_1 c_1 + m_1 c_1 = 2m_1 c_1$$

$$\text{or } c = \frac{2m_1 c_1}{m_1 + m_2} = \frac{2m_1 c_1}{m_1 + m_1 \frac{c_1}{c_2}} = \frac{2c_1 c_2}{c_1 + c_2}$$

**Problem 8. A gas expands in such a manner that its pressure and volume comply with the condition  $PV^2 = \text{constant}$ . Will the gas cool or get heated on expansion ?**

**Solution.** For a perfect gas,  $\frac{PV}{T} = \text{constant} (K)$

Given  $PV^2 = \text{constant} (K')$

$$\therefore P = \frac{K'}{V^2}$$

$$\therefore \frac{K'}{V^2} \cdot \frac{V}{T} = K \quad \text{or} \quad VT = \frac{K'}{K} = \text{constant}$$

$$\text{Hence } V \propto \frac{1}{T}$$

That is the expansion of the gas will result in the decrease of temperature.

**Problem 9. What happens to the change in internal energy of a gas during (i) isothermal expansion and (ii) adiabatic expansion ?** [Delhi 08]

**Solution.** (i) Isothermal expansion. Temperature remains constant during an isothermal change. As internal energy is a function of temperature only, so it will remain constant during an isothermal change.

$$\text{As } \Delta T = 0, \text{ so } \Delta U = C_V \Delta T = 0.$$

(ii) Adiabatic expansion. For an adiabatic change,  $\Delta Q = 0$ , so from first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W = 0 \quad \text{or} \quad \Delta W = -\Delta U$$

During expansion, work is done by a gas i.e.,  $\Delta W$  is positive. So  $\Delta U$  must be negative. Hence internal energy of a gas decreases during an adiabatic expansion.

**Problem 10. Show that the slope of an adiabatic curve at any point is  $\gamma$  times the slope of an isothermal curve at the corresponding point.** [Chandigarh 08]

**Solution.** For an isothermal change,  $PV = K$

Differentiating both sides, we get

$$P \cdot dV + V \cdot dP = 0 \quad \text{or} \quad V \cdot dP = -PdV$$

$$\therefore \text{Slope of an isothermal curve, } \left( \frac{dP}{dV} \right)_{iso} = -\frac{P}{V}$$

For an adiabatic change,  $PV^\gamma = K'$

Differentiating both sides, we get

$$P \cdot \gamma V^{\gamma-1} \cdot dV + V^\gamma \cdot dP = 0$$

$$\text{or } PdV + VdP = -\gamma PdV$$

$$\therefore \text{Slope of an adiabatic curve, } \left( \frac{dP}{dV} \right)_{adia} = -\frac{\gamma P}{V}$$

Clearly, slope of an adiabatic curve =  $\gamma \times$  slope of an isothermal curve.

As  $\gamma > 1$ , so an adiabatic P-V curve is steeper than the corresponding isothermal P-V curve.

**Problem 11. The volume versus temperature  $T$  graphs for a certain amount of a perfect gas at two pressures  $P_1$  and  $P_2$  are shown in Fig. 12.26. Which one is greater —  $P_1$  or  $P_2$  ?**

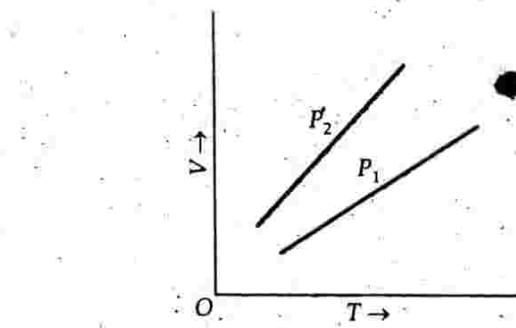


Fig. 12.26

**Solution.** For a perfect gas,

$$PV = nRT$$

$$V = \frac{nR}{P} T$$

Slope of  $V-T$  graph with  $T$ -axis =  $\frac{nR}{P}$

For a given amount of gas, slope  $\propto \frac{1}{P}$

Hence  $P_1 > P_2$ .

**Problem 12.** In Figs. 12.27(a), (b) and (c) given below, identify the isothermal and adiabatic process in each diagram.

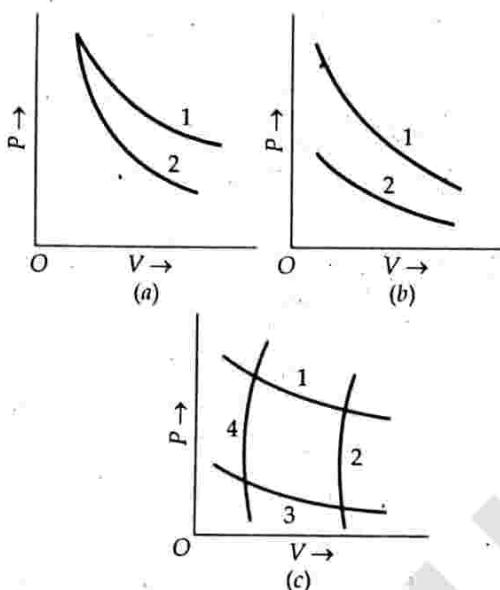


Fig. 12.27

**Solution.** For the same values of pressure and volume, the slope of an adiabatic curve is greater than the slope of an isothermal curve.

Using this fact, we find

- |                       |                   |
|-----------------------|-------------------|
| (a) 1 – isothermal    | 2 – adiabatic     |
| (b) 1 – adiabatic     | 2 – isothermal    |
| (c) 1, 3 – isothermal | 2, 4 – adiabatic. |

**Problem 13.** No real engine can have an efficiency greater than that of a Carnot engine working between the same two temperatures. Give reason.

**Solution.** A Carnot engine is an ideal engine satisfying the following conditions :

- (i) There is no friction between the walls of the cylinder and the piston.
- (ii) The working substance is an ideal gas. That is the gas molecules have point sizes and have no attractive forces between them.

Real engines Cannot fulfil these conditions.

Hence no heat engine working between the same two temperatures can have efficiency greater than that of a Carnot engine.

**Problem 14.** Can a Carnot engine be realised in practice ?

**Solution.** No. A Carnot engine should consist of

- (i) a source of infinite thermal capacity,
- (ii) a sink of infinite thermal capacity,
- (iii) an ideal gas as its work substance.

Moreover, the working substance should be contained in cylinder provided with frictionless, non-conducting movable piston. It is not possible to achieve all these conditions. Hence a Carnot engine cannot be realised in practice.

**Problem 15.** The volume of an ideal gas is  $V$  at pressure  $P$ . On increasing the pressure by  $\Delta P$ , the change in volume of the gas is  $\Delta V_1$  under isothermal conditions and  $\Delta V_2$  under adiabatic conditions. Under which of the two conditions, will the change in volume be more ? Given reason.

**Solution.** Under isothermal conditions, the bulk modulus of elasticity is

$$K_{\text{iso}} = \frac{\Delta P}{\Delta V_1 / V} = P \quad \dots(i)$$

Under adiabatic conditions, the bulk modulus of elasticity is

$$K_{\text{adia}} = \frac{\Delta P}{\Delta V_2 / V} = \gamma P \quad \dots(ii)$$

Dividing (ii) by (i), we get

$$\frac{\Delta V_1}{\Delta V_2} = \gamma$$

As  $\gamma > 1$  so  $\Delta V_1 > \Delta V_2$ .

**Problem 16.** Discuss whether the following phenomena are reversible :

- (i) Water fall
- (ii) Rusting of iron
- (iii) Electrolysis.

**Solution.** (i) Water fall. It is not a reversible process. During fall of the water, the major part of its potential energy is converted into kinetic energy of the water. However, on striking the ground, a part of it is converted into heat and sound. It is not possible to convert the heat and the sound produced alongwith the K.E. of water into potential energy so that the water may rise back to its initial height. Therefore, water fall is not a reversible process.

(ii) Rusting of iron. During rusting, iron gets oxidised by the oxygen of the air. Since it is a chemical change, it is not a reversible process.

(iii) Electrolysis. It is a reversible process, provided the electrolyte does not offer any resistance to the flow of current. If we reverse the direction of current, the direction of motion of ions is also reversed.

**HOTS****Problems on Application of Thinking Skills**

**Problem 1.** A certain amount of gas occupies volume  $V_0$  at pressure  $P_0$  and temperature  $T_0$ . It is allowed to expand (i) isobarically, (ii) adiabatically and (iii) isothermally. In which case the work done is maximum and in which case it is minimum? Explain.

**Solution.** During expansion, work is done by the gas.

The horizontal line  $AB_1$  represents the isobaric expansion ( $P = \text{constant}$ ).

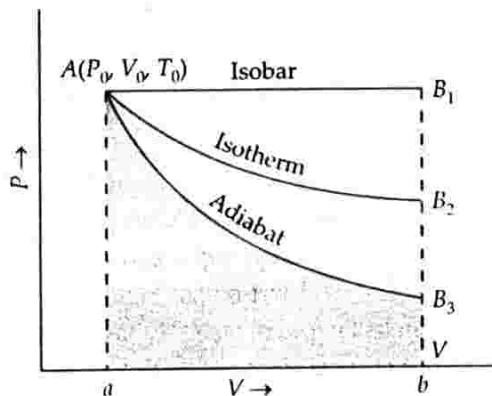


Fig. 12.28

Since an adiabatic is steeper than an isothermal, so the adiabatic expansion curve  $AB_3$  lies below the isothermal expansion curve  $AB_2$ .

Work done = Area between the  $P$ - $V$  curve and the  $V$ -axis

As the area under curve  $AB_1$  is maximum, so maximum work is done in isobaric expansion.

Again, the area under curve  $AB_3$  is minimum, so minimum work is done in adiabatic expansion.

**Problem 2.** Two samples of a gas initially at the same temperature and pressure are compressed from a volume  $V$  to  $V/2$ , one isothermally, the other adiabatically. In which sample is the final pressure greater?

**Solution.** Fig. 12.29 shows the  $P$ - $V$  diagrams for the two samples of gas compressed from volume  $V$  to

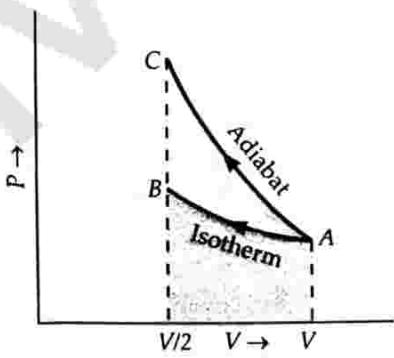


Fig. 12.29

$V/2$  isothermally and adiabatically. As an adiabatic is steeper than an isotherm, so the adiabatic compression curve  $AC$  lies above the isothermal compression curve  $AB$ .

Clearly, the final pressure represented by point  $C$  is greater than that represented by point  $B$ . Hence the pressure of the sample compressed adiabatically will be greater.

**Problem 3.** Two gases have the same initial pressure  $P_0$ , volume  $V_0$  and temperature  $T_0$ . They expand to the same volume, one adiabatically and the other isothermally.

(i) In which case is the final pressure greater?

(ii) In which case is the work done greater?

(iii) In which case is the final temperature greater?

**Solution.** Fig. 12.30 shows the  $P$ - $V$  diagrams for two gases expanded from volume  $V_0$  to  $V$ . As an

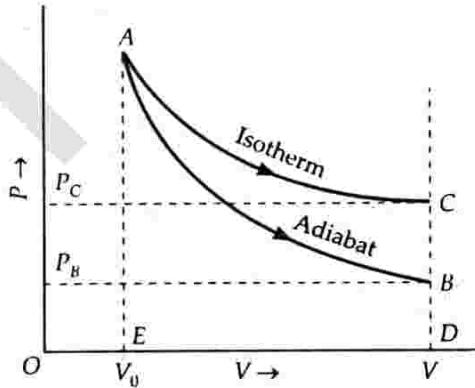


Fig. 12.30

adiabatic is steeper than an isotherm, so the adiabatic expansion curve  $AB$  lies below the isothermal expansion curve  $AC$ .

(i)  $P_B$  and  $P_C$  are the final pressures for adiabatic and isothermal expansions respectively. Clearly,  $P_C > P_B$ . Hence the final pressure is greater for the isothermal expansion.

(ii) Work done in adiabatic expansion  
= area  $ABDE$

Work done in isothermal expansion  
= area  $ACDE$

As area  $ACDE >$  area  $ABDE$

So more work is done in the isothermal expansion.

(iii) In isothermal expansion, temperature remains constant  $T_0$ . In adiabatic expansion temperature decreases below  $T_0$ . So the final temperature is greater for the isothermal expansion.

**Problem 4.** The work of 146 J is performed in order to compress one kilo mole of gas adiabatically and in this process the temperature of the gas increases by 7°C. Identify the atomicity of the gas. Given  $R = 83 \text{ J mol}^{-1} \text{ K}^{-1}$

[AIEEE 06]

**Solution.** By first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

For an adiabatic process,  $\Delta Q = 0$

$$\therefore \Delta U = \Delta W = -(-146 \text{ kJ}) = +146 \text{ kJ}$$

$$\text{As } \Delta U = nC_V \Delta T$$

$$\therefore C_V = \frac{\Delta U}{n\Delta T} = \frac{146 \times 10^3}{1 \times 10^3 \times 7} \quad [\because \Delta T = 7^\circ\text{C} = 7 \text{ K}] \\ = 20.8 \text{ J mol}^{-1} \text{ K}^{-1}$$

For a diatomic gas,

$$C_V = \frac{5}{2} R = \frac{5}{2} \times 8.3 = 20.8 \text{ J mol}^{-1} \text{ K}^{-1}.$$

Hence the gas is diatomic.

**Problem 5.** In a given process on an ideal gas,  $dW = 0$  and  $dQ < 0$ . What happens to the temperature of the gas?

[IIT Screening 01]

**Solution.** As  $dQ = dU + dW$

But  $dW = 0$  and  $dQ < 0$ , so  $dU < 0$

For an ideal gas,  $U \propto T$ , so  $dT < 0$  i.e., temperature of the gas decreases.

**Problem 6.** An ideal gas is taken through the cycle  $A \rightarrow B \rightarrow C \rightarrow A$ , as shown in Fig. 12.31. If the net heat supplied to the gas in the cycle is 5 J, what is the work done by the gas in the process  $C \rightarrow A$ ? [IIT Screening 02]

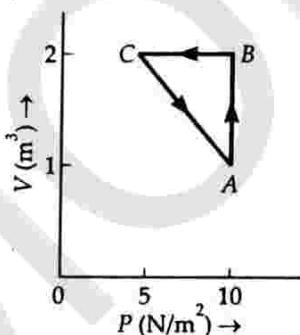


Fig. 12.31

**Solution.** For the cyclic process,

$$\Delta Q = \Delta W$$

$$\text{or } \Delta Q = W_{AB} + W_{BC} + W_{CA}$$

$$\text{or } 5 = 10(2 - 1) + 0 + W_{CA}$$

$$\text{or } W_{CA} = 5 - 10 = -5 \text{ J.}$$

**Problem 7.** A monoatomic ideal gas, initially at temperature  $T_1$ , is enclosed in a cylinder fitted with a frictionless

piston. The gas is allowed to expand adiabatically to a temperature  $T_2$  by releasing the piston suddenly. If  $L_1$  and  $L_2$  are the lengths of the gas column before and after expansion respectively then, what is the ratio  $T_1 / T_2$ ? [IIT 2K]

**Solution.** For an adiabatic process,

$$TV^{\gamma-1} = \text{constant}$$

For a monoatomic gas,  $\gamma = 5/3$

$$\therefore TV^{2/3} = \text{constant}$$

Let  $A$  = the area of cross-section of the cylinder. Then

$$T_1 (L_1 A)^{2/3} = T_2 (L_2 A)^{2/3}$$

$$\therefore \frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{2/3}.$$

**Problem 8.** A monoatomic ideal gas of two moles is taken through a cyclic process starting from  $A$  as shown in

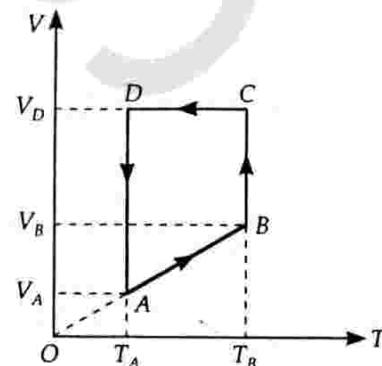


Fig. 12.32

Fig. 12.32. The volume ratios are  $\frac{V_B}{V_A} = 2$  and  $\frac{V_D}{V_A} = 4$ . If the temperature  $T_A$  at  $A$  is  $27^\circ\text{C}$ , calculate,

- (a) the temperature of the gas at point  $B$ ,
  - (b) heat absorbed or released by the gas in each process,
  - (c) the total work done by the gas during the complete cycle.
- Express your answer in terms of the gas constant  $R$ .

[IIT Mains 01]

**Solution.** (a) The process  $A \rightarrow B$  is isobaric.

$$\therefore V/T = \text{constant}$$

$$\text{Hence } \frac{V_A}{T_A} = \frac{V_B}{T_B}$$

$$\text{or } T_B = \frac{V_B}{V_A} \cdot T_A = 2 \times 300 = 600 \text{ K} = 327^\circ\text{C}$$

(b) The heat absorbed in the process  $A \rightarrow B$  is given by

$$Q_1 = n C_p dT \\ = 2 (5R/2)(600 - 300) = 1500 R.$$

The process  $B \rightarrow C$  is isothermal. The heat absorbed is given by

$$\begin{aligned} Q_2 = W_2 &= nRT_B \log(V_D/V_B) \\ &= 2 \times R \times 600 \log_e 2 = 1200 R \times 0.693 \\ &= 831.6 R \end{aligned}$$

The process  $C \rightarrow D$  is isochoric. The heat released is given by

$$\begin{aligned} Q_3 &= nC_v dT = 2 \times \frac{3}{2} R \times (300 - 600) \\ &= -900 R. \end{aligned}$$

The process  $D \rightarrow A$  is isothermal. The heat released is given by

$$\begin{aligned} Q_4 &= nRT_A \log_e \frac{V_A}{V_D} = 2 R \times 300 \log_e \frac{1}{4} \\ &= -1200 R \log_e 2 = -831.6 R. \end{aligned}$$

(c) The total work done by the gas during the complete cycle,

$$\begin{aligned} W &= Q_1 + Q_2 + Q_3 + Q_4 \\ &= 1500 R + 831.6 R - 900 R - 831.6 R \\ &= 600 R. \end{aligned}$$

**Problem 9.** An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are  $Q_1 = 5960 J$ ,  $Q_2 = -5585 J$ ,  $Q_3 = -2980 J$  and  $Q_4 = 3645 J$  respectively. The corresponding works involved are  $W_1 = 2200 J$ ,  $W_2 = -825 J$ ,  $W_3 = -1100 J$  and  $W_4$  respectively. Find (i)  $W_4$  and (ii) efficiency of the cycle. [IIT 94]

**Solution.** (i) By the first law of thermodynamics,

$$dQ = dU + dW$$

But for a cyclic process,  $dU = 0$

$$dQ = dW$$

$$\text{or } Q_1 + Q_2 + Q_3 + Q_4 = W_1 + W_2 + W_3 + W_4$$

$$\text{or } 5960 - 5585 - 2980 + 3645 = 2200 - 825 - 1100 + W_4$$

$$\text{or } W_4 = (5960 + 3645 + 825 + 1100) - (5585 + 2980 + 2200) = 11530 - 10765 = 765 J.$$

(ii) Efficiency,

$$\begin{aligned} \eta &= \frac{\text{Total work done}}{\text{Heat absorbed}} = \frac{W_1 + W_2 + W_3 + W_4}{Q_1 + Q_4} \\ &= \frac{2200 - 825 - 1100 + 765}{5960 + 3645} \\ &= \frac{1040}{9605} = 0.1083 = 10.83\%. \end{aligned}$$

**Problem 10.** Let the temperatures  $T_1$  and  $T_2$  of the two heat reservoirs in an ideal carnot engine be  $1500^\circ C$  and

$500^\circ C$  respectively. Which of these, increasing  $T_1$  by  $100^\circ C$  or decreasing  $T_2$  by  $100^\circ C$ , would result in a greater improvement in the efficiency of the engine? [Roorkee 80]

**Solution.** Efficiency of Carnot engine,

$$\eta = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1}$$

(i) When  $T_1$  is increased from  $1500^\circ C$  to  $1600^\circ C$  or  $1500 + 273 = 1773 K$  and  $T_2$  remains constant i.e.,  $500^\circ C$  or  $500 + 273 = 773 K$ , we have

$$\eta_1 = \frac{1873 - 773}{1873} = \frac{1100}{1873} = \frac{1100 \times 100}{1873}\% = 58.73\%.$$

(ii) When  $T_1$  remains constant i.e.,  $1500^\circ C$  or  $1500 + 273 = 1773 K$  and  $T_2$  is decreased by  $100^\circ C$  from  $500^\circ C$  to  $400^\circ C$  or  $400 + 273 = 673 K$ , we have

$$\eta_2 = \frac{1773 - 673}{1773} = \frac{1100}{1773} = \frac{1100 \times 100}{1773}\% = 62.04\%.$$

Thus,  $\eta_2 > \eta_1$ .

Hence efficiency will increase if  $T_2$  is decreased from  $500^\circ C$  to  $400^\circ C$ .

**Problem 11.** An ideal gas having initial pressure  $P$ , volume  $V$  and temperature  $T$  is allowed to expand adiabatically until its volume becomes  $5.66 V$  while its temperature falls to  $T/2$ . (i) How many degrees of freedom do the gas molecules have? (ii) Obtain the work done by the gas during the expansion as a function of the initial pressure  $P$  and volume  $V$ . [IIT 90]

**Solution.** (i) For an adiabatic expression :

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\therefore TV^{\gamma-1} = \frac{T}{2} \times (5.66V)^{\gamma-1}$$

$$\text{or } (5.66)^{\gamma-1} = 2$$

Taking logarithms,  $(\gamma-1) \log 5.66 = \log 2$

$$\text{or } \gamma-1 = \frac{\log 2}{\log 5.66} = \frac{0.3010}{0.7528} = 0.4$$

$$\text{or } \gamma = 1.4$$

Let  $f$  be the degrees of freedom of the gas. Then

$$\gamma = 1 + \frac{2}{f} \quad \text{or } 1.4 = 1 + \frac{2}{f}$$

$$\text{or } \frac{2}{f} = 1.4 - 1 = 0.4 \quad \text{or } f = \frac{2}{0.4} = 5.$$

(ii) Work done by the gas in adiabatic expansion from temperature  $T_1 (= T)$  to  $T_2 (= T/2)$  is

$$W = \frac{R}{\gamma-1} (T_1 - T_2) = \frac{R}{0.4} \left( T - \frac{T}{2} \right) = \frac{5}{4} RT = \frac{5}{4} PV.$$

**Problem 12.** Two different adiabatic paths for the same gas intersect two isothermals at  $T_1$  and  $T_2$ , as shown in P-V diagram of Fig. 12.33. How does the ratio  $V_a/V_d$  compare with the ratio  $V_b/V_c$ ? [Roorkee 83]

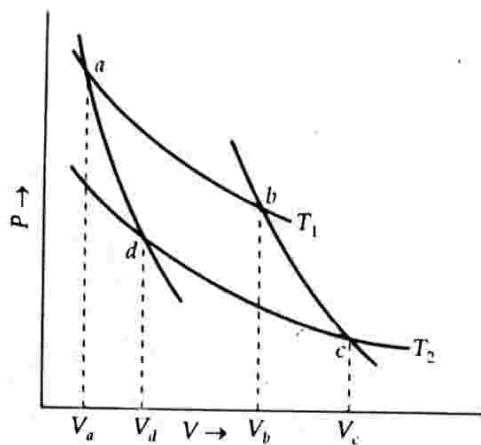


Fig. 12.33

**Solution.** Let  $P_a, P_b, P_c$  and  $P_d$  be the pressures at  $a, b, c$  and  $d$  respectively. As  $a$  and  $b$  are on the same isothermal, so

$$\therefore P_a V_a = P_b V_b \quad \dots(i)$$

Also,  $c$  and  $d$  are on the same isothermal, so

$$P_c V_c = P_d V_d \quad \dots(ii)$$

Now,  $a$  and  $d$  are on the same adiabatic and similarly  $b$  and  $c$  are on the same adiabatic, so

$$P_d V_d^\gamma = P_a V_a^\gamma \quad \dots(iii)$$

$$\text{and} \quad P_b V_b^\gamma = P_c V_c^\gamma \quad \dots(iv)$$

Multiplying equations (i), (ii), (iii) and (iv), we get

$$P_a V_a \cdot P_c V_c \cdot P_d V_d \cdot P_b V_b = P_b V_b \cdot P_d V_d \cdot P_a V_a^\gamma \cdot P_c V_c^\gamma$$

or

$$V_a V_c V_d V_b^\gamma = V_b V_d V_a^\gamma V_c$$

or

$$\frac{V_a V_d^\gamma}{V_a^\gamma V_d} = \frac{V_b V_c^\gamma}{V_c V_b}$$

or

$$\left(\frac{V_d}{V_a}\right)^{\gamma-1} = \left(\frac{V_c}{V_b}\right)^{\gamma-1}$$

or

$$\frac{V_d}{V_a} = \frac{V_c}{V_b} \quad \text{or} \quad \frac{V_a}{V_d} = \frac{V_b}{V_c}$$

**Problem 13.** In Fig. 12.34, an ideal gas changes its state from state A to C by two paths ABC and AC. By

- Find the path along which work done is the least.
- The internal energy of gas at A is 10 J and amount of heat supplied to change its state to C through the path AC is 200 J. Calculate the internal energy at C.
- The internal energy of gas at state B is 20 J. Find the amount of heat supplied to the gas to go from A to B.

[Roorkee 98]

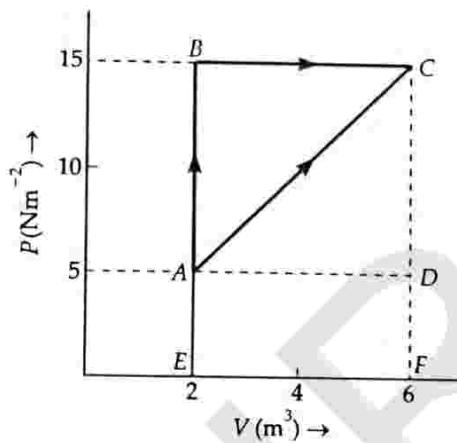


Fig. 12.34

**Solution.** (i) Along path ABC,

$$W_{AB} = PdV = 0 \quad [\because dV = 0]$$

$$W_{BC} = P_B (V_C - V_B) = 15 (6 - 2) = 60 \text{ J}$$

$$\therefore W_{ABC} = W_{AB} + W_{BC} = 0 + 60 \text{ J} = 60 \text{ J}.$$

Along path AC,

$$W_{AC} = \text{Area under the curve } AC$$

$$= \text{Area of } \Delta ACD + \text{Area of rectangle } ADFE$$

$$= \frac{1}{2} AD \times DC + AD \times FD$$

$$= \frac{1}{2} \times 4 \times 10 + 4 \times 5 = 20 + 20 = 40 \text{ J}$$

Clearly, work done is the least along path AC.

(ii) Here  $U_A = 10 \text{ J}$ ,  $\Delta Q_{AC} = 200 \text{ J}$

By the first law of thermodynamics,

$$\Delta Q_{AC} = \Delta U_{AC} + \Delta W_{AC} = U_C - U_A + \Delta W_{AC}$$

$$U_C = \Delta Q_{AC} + U_A - \Delta W_{AC}$$

$$= 200 \text{ J} + 10 \text{ J} - 40 \text{ J} = 170 \text{ J}.$$

(iii) Here  $U_B = 20 \text{ J}$ . By the first law of thermodynamics,

$$\Delta Q_{AB} = \Delta U_{AB} + \Delta W_{AB} = U_B - U_A + \Delta W_{AB}$$

$$= 20 \text{ J} - 10 \text{ J} + 0 = 10 \text{ J}.$$

**Problem 14.** A sample of 2 kg of monoatomic helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC (Fig. 12.35). Given molecular mass of Helium = 4,  $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$ .

- What is the temperature of Helium in each of the states A, B, C and D?
- How much is the heat involved in each of the processes ABC and ADC?

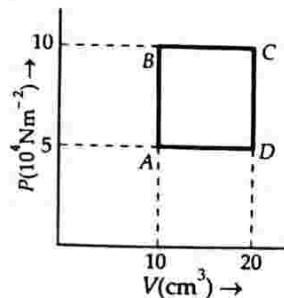


Fig. 12.35

**Solution.** (i) Number of moles of He

$$n = \frac{\text{Mass of He}}{\text{Molecular mass}} = \frac{2000}{4} = 500$$

$$\text{As } P_A V_A = n R T_A$$

$$\therefore T_A = \frac{P_A V_A}{nR} = \frac{5 \times 10^4 \times 10}{500 \times 8.3} = 120.5 \text{ K.}$$

For the isochoric process ( $V = \text{constant}$ )  $AB$ ,

$$\frac{T_B}{T_A} = \frac{P_B}{P_A} \quad \therefore T_B = \frac{P_B}{P_A} \times T_A = \frac{10}{5} \times 120.5 = 241 \text{ K.}$$

For the isobaric process ( $P = \text{constant}$ )  $BC$ ,

$$\frac{T_C}{T_B} = \frac{V_C}{V_B} \quad \therefore T_C = \frac{V_C}{V_B} \times T_B = \frac{20}{10} \times 241 = 482 \text{ K.}$$

For the isochoric process  $DC$ ,

$$\frac{T_D}{T_C} = \frac{P_D}{P_C} \quad \therefore T_D = \frac{P_D}{P_C} \times T_C = \frac{5}{10} \times 482 = 241 \text{ K.}$$

(ii) Heat involved in the process  $ABC$ ,

$$\begin{aligned} Q_{ABC} &= \Delta U_{AC} + W_{ABC} = nC_V(T_C - T_A) + W_{BC} \\ &= nC_V(T_C - T_A) + P_B(V_C - V_B) \\ &= 500 \times \frac{3}{2} \times 8.3 \times (482 - 120.5) + 10 \times 10^4 \times (20 - 10) \\ &\quad [\because C_V = 3/2 R] \\ &= 2.25 \times 10^6 + 10^6 = 3.25 \times 10^6 \text{ J} = 3.25 \text{ MJ.} \end{aligned}$$

Heat involved in the process  $ADC$ ,

$$\begin{aligned} Q_{ADC} &= \Delta U_{AC} + W_{ADC} = \Delta U_{AC} + W_{AD} \\ &= 2.25 \times 10^6 + P_A(V_D - V_A) \\ &= 2.25 \times 10^6 + 5 \times 10^4 \times (20 - 10) \\ &= 2.75 \times 10^6 \text{ J} = 2.75 \text{ MJ.} \end{aligned}$$

**Problem 15.** One mole of an ideal monoatomic gas is taken round the cyclic process  $ABCA$  as shown in Fig. 12.36. Calculate

- the work done by the gas,
- the heat rejected by the gas in the path  $CA$  and the heat absorbed by the gas in the path  $AB$ .

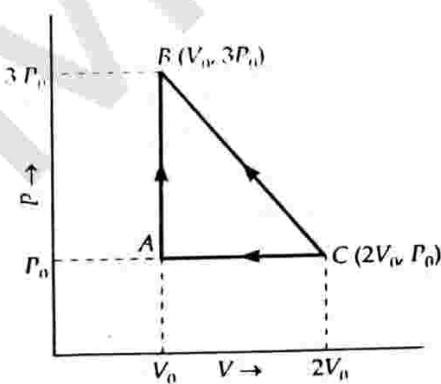


Fig. 12.36

- the net heat absorbed by the gas in the path  $BC$ , and
- the maximum temperature attained by the gas during the cycle. [IIT 98]

**Solution.** (i) Work done = Area of closed curve  $ABCA$  ( $\Delta ABC$ )

$$dW = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} V_0 \times 2P_0 = P_0 V_0.$$

(ii) Heat rejected by the gas in the path  $CA$  during Isobaric compression process,

$$dQ_{CA} = nC_p \Delta T = 1 \times (5R/2) \times (T_A - T_C)$$

$$T_C = \frac{2P_0 V_0}{1 \times R}, \quad T_A = \frac{P_0 V_0}{1 \times R}$$

$$dQ_{CA} = \frac{5R}{2} \left[ \frac{P_0 V_0}{R} - \frac{2P_0 V_0}{R} \right] = -\frac{5}{2} P_0 V_0$$

Heat absorbed by the gas in the path  $AB$  of Isobaric process,

$$dQ_{AB} = nC_v dT = 1 \times (3R/2)(T_B - T_A)$$

$$= \frac{3R}{2} \left[ \frac{3P_0 V_0}{1 \times R} - \frac{P_0 V_0}{1 \times 2} \right] = 3P_0 V_0$$

(iii) As  $dU = 0$  in cyclic process, hence  $dQ = dW$

$$dQ_{AB} + dQ_{CA} + dQ_{BC} = dW$$

$$\text{or } 3P_0 V_0 - \frac{5}{2} P_0 V_0 + dQ_{BC} = P_0 V_0 \quad \text{or } dQ_{BC} = \frac{P_0 V_0}{2}$$

As net heat is absorbed by the gas during path  $BC$ , temperature will reach maximum between  $B$  and  $C$ .

$$(iv) \text{ Equation for line } BC : P = -\left[\frac{2P_0}{V_0}\right]V + 5P_0$$

$$\text{As } PV = RT \quad \text{or } P = \frac{RT}{V} \quad [\text{For one mole}]$$

$$RT = -\frac{2P_0}{V_0} V^2 + 5P_0 V \quad \dots(1)$$

$$\frac{dT}{dV} = 0 \text{ for maximum}$$

$$\therefore -\frac{2P_0}{V_0} \times 2V + 5P_0 = 0 \quad \text{or} \quad V = \frac{5}{4} V_0 \quad \dots(2)$$

From equations (1) and (2), we get

$$\begin{aligned} RT_{\max} &= -\frac{2P_0}{V_0} \times \left(\frac{5V_0}{4}\right)^2 + 5P_0 \left(\frac{5V_0}{4}\right) \\ &= -2P_0 V_0 \times \frac{25}{16} + \frac{25P_0 V_0}{4} = \frac{25}{8} P_0 V_0 \end{aligned}$$

$$\therefore T_{\max} = \frac{25}{8} \frac{P_0 V_0}{R}.$$

# Guidelines to NCERT Exercises

**12.1.** A geyser heats water flowing at the rate of 3.0 litres per minute from  $27^\circ\text{C}$  to  $77^\circ\text{C}$ . If the geyser operates on a gas burner, what is the rate of consumption of the fuel if its heat of combustion is  $4.0 \times 10^4 \text{ J/g}$ ?

**Ans.** Volume of water heated = 3.0 litre  $\text{min}^{-1}$ .

Mass of water heated = 3000 g  $\text{min}^{-1}$

Rise in temperature,  $\Delta T = 77 - 27 = 50^\circ\text{C}$

Specific heat of water,  $c = 4.2 \text{ Jg}^{-1}$

Heat absorbed by water,

$$\begin{aligned} Q &= mc \Delta T = 3000 \times 4.2 \times 50 \\ &= 63 \times 10^4 \text{ J min}^{-1} \end{aligned}$$

Heat of combustion =  $4.0 \times 10^4 \text{ Jg}^{-1}$

Rate of combustion of fuel

$$= \frac{63 \times 10^4 \text{ J min}^{-1}}{4.0 \times 10^4 \text{ J g}^{-1}} = 15.75 \text{ g min}^{-1}.$$

**12.2.** What amount of heat must be supplied to  $2.0 \times 10^{-2} \text{ kg}$  of nitrogen at room temperature to raise its temperature by  $45^\circ\text{C}$  at constant pressure? Given molecular weight of  $N_2$  is 28 and  $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$  and  $C_V$  (diatomic gases) =  $\frac{7}{2} R$ .

**Ans.** Number of moles of gas,

$$\begin{aligned} n &= \frac{\text{Mass of gas in grams}}{\text{Molecular mass}} \\ &= \frac{2.0 \times 10^{-2} \times 10^3}{28} = \frac{20}{28} \end{aligned}$$

$$R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}, \Delta T = 45^\circ\text{C}$$

Molar sp. heat of  $N_2$  at constant pressure,

$$C_P = \frac{7}{2} R = \frac{7}{2} \times 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$$

Heat supplied to the gas,

$$Q = n C_P \Delta T = \frac{20}{28} \times \frac{7}{2} \times 8.3 \times 45 = 933.75 \text{ J.}$$

**12.3. Explain why**

(a) Two bodies at different temperatures  $T_1$  and  $T_2$  if brought in thermal contact do not necessarily settle to mean temperature  $(T_1 + T_2)/2$ .

(b) The coolant in a chemical or a nuclear plant (i.e., the liquid used to prevent the different parts of a plant from getting too hot) should have high specific heat. [Delhi 11]

(c) Air pressure in a car tyre increases during driving.

[Delhi 09, 11]

(d) The climate of a harbour town is more temperate than that of a town in a desert at the same latitude.

**Solution.** (a) The two bodies may have different masses and different materials i.e., they may have different thermal capacities. In case the two bodies have equal thermal capacities, they would settle at the mean temperature  $(T_1 + T_2)/2$ .

(b) The purpose of a coolant is to absorb maximum heat with least rise in its own temperature. This is possible only if specific heat is high because  $Q = mc \Delta T$ . For a given value of  $m$  and  $Q$  the rise in temperature  $\Delta T$  will be small if  $c$  is large. This will prevent different parts of the nuclear reactor from getting too hot.

(c) Due to the friction between the tyres and the road, the tyres get heated. The temperature of air inside the tyres increases. Consequently, the air pressure in the tyres increases slightly.

(d) The relative humidity of a harbour town is more than that of a desert town. Due to high specific heat of water, the variations in the temperature of humid air are less. Hence the climate of a harbour town is without the extremes of hot or cold.

**12.4.** A cylinder with a movable piston contains 3 moles of hydrogen at standard temperature and pressure. The walls of the cylinder are made of a heat insulator, and the piston is insulated by having a pile of sand on it. By what factor does the pressure of the gas increase if the gas is compressed to half its original volume?

**Solution.** The piston and the walls of the cylinder are insulated. No heat exchange occurs. For this adiabatic process,

$$P_1 V_1^\gamma = P_2 V_2^\gamma \quad \text{or} \quad \frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^\gamma$$

$$\text{But} \quad V_2 = V_1 / 2$$

$$\text{For a diatomic gas, } \gamma = 1.4$$

$$\therefore \frac{P_2}{P_1} = \left( \frac{V_1}{V_1/2} \right)^{1.4} = 2^{1.4} = 2.64$$

**12.5.** In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state A to B via a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case? (Take 1 cal = 4.19 J)

**Solution.** Work done on a system is negative and heat absorbed by a system is positive.

For the adiabatic process,

$$Q = 0 \quad \text{and} \quad W = -22.3 \text{ J}$$

As  $Q = \Delta U + W$

$\therefore 0 = \Delta U - 22.3 \text{ J}$  or  $\Delta U = 22.3 \text{ J}$

In the second case,

$$Q = 9.35 \text{ cal} = 9.35 \times 4.19 \text{ J} = 39.2 \text{ J}$$

$$\therefore W = Q - \Delta U = 39.2 - 22.3 = +16.9 \text{ J.}$$

The positive sign for  $W$  indicates that the work is done by the system.

**12.6.** Two cylinders A and B of equal capacity are connected to each other via a stopcock. The cylinder A contains a gas at standard temperature and pressure, while the cylinder B is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened.

Answer the following :

- (i) What is the final pressure of the gas in A and B ?
- (ii) What is the change in internal energy of the gas ?
- (iii) What is the change in temperature of the gas ?
- (iv) Do the intermediate states of the system (before settling to final equilibrium state) lie on its P-V-T surface ?

[Delhi 09]

**Solution.** (i) When the stopcock is suddenly opened, the volume available to the gas at 1 atm becomes twice the original volume and hence pressure becomes half the original volume (Boyle's law). Hence the pressure of the gas in each of the cylinders A and B is **0.5 atm**.

(ii) As the system is thermally insulated, so  $\Delta Q = 0$ . Also, the gas expands against zero pressure, so  $\Delta W = 0$ . Hence by first law of thermodynamics,  $\Delta U = 0$  i.e., there is no change in the internal energy of the gas.

(iii) As there is no change in the internal energy of the gas, so the temperature of the gas remains unchanged.

(iv) No. The free expansion of the gas is very rapid and hence cannot be controlled. The intermediate states are non-equilibrium states and do not satisfy the gas equation. In due course, the gas returns to equilibrium state which lies on P-V-T surface.

**12.7.** A steam engine delivers  $5.4 \times 10^8 \text{ J}$  of work per minute and services  $3.6 \times 10^9 \text{ J}$  of heat per minute from its boiler. What is the efficiency of the engine ? How much heat is wasted per minute ?

[Delhi 09]

**Solution.**

$$\text{Work output, } W = 5.4 \times 10^8 \text{ J}$$

$$\text{Heat input, } Q_1 = 3.6 \times 10^9 \text{ J}$$

$$\text{Efficiency, } \eta = \frac{W}{Q_1} = \frac{5.4 \times 10^8}{3.6 \times 10^9} = 0.15 = 15\%$$

Heat wasted per minute,

$$Q_2 = Q_1 - W = 3.6 \times 10^9 - 5.4 \times 10^8 \\ = 30.6 \times 10^8 \text{ J} = 3.1 \times 10^9 \text{ J.}$$

**12.8.** An electric heater supplies heat to a system at a rate of 100 W. If system performs work at a rate of 75 joules per second, at what rate is the internal energy increasing ?

**Solution.** According to first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

$$\therefore \frac{\Delta Q}{\Delta t} = \frac{\Delta U}{\Delta t} + \frac{\Delta W}{\Delta t}$$

$$\text{or } 100 \text{ W} = \frac{\Delta U}{\Delta t} + 75 \text{ W}$$

$$\text{or } \frac{\Delta U}{\Delta t} = 25 \text{ W.}$$

**12.9.** A thermodynamic system is taken from an original state to an intermediate state by the linear process shown in Fig. 12.37.

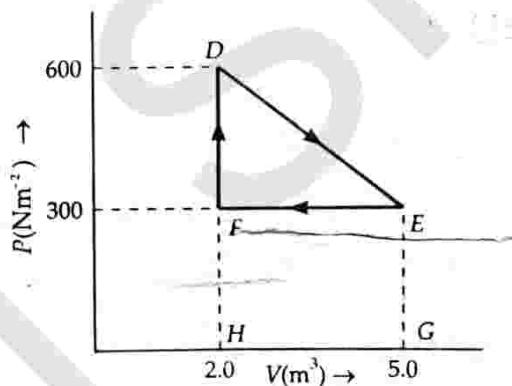


Fig. 12.37

Its volume is then reduced to the original value from E to F by an isobaric process. Calculate the total work done by the gas from D to E to F.

**Solution.** Total work done by the gas from D to E to F,

$$W = W_{DE} + W_{EF}$$

$$= \text{Area of trapezium } DEGHD$$

$$- \text{Area of rectangle } EFHG$$

$$= \text{Area of triangle } DEF$$

$$= \frac{1}{2} DF \times FE$$

$$= \frac{1}{2} (600 - 300) \text{ Nm}^{-2} \times (5.0 - 2.0) \text{ m}^3$$

$$= 450 \text{ J.}$$

**12.10.** A refrigerator is to maintain eatables kept inside at  $9^\circ\text{C}$ . If room temperature is  $36^\circ\text{C}$ , calculate the coefficient of performance.

**Solution.** Here  $T_1 = 273 + 36 = 309 \text{ K}$ ,

$$T_2 = 273 + 9 = 282 \text{ K}$$

Coefficient of performance,

$$\beta = \frac{T_2}{T_1 - T_2} = \frac{282}{309 - 282} = \frac{282}{27} = 10.4.$$

## Text Based Exercises

### Type A : Very Short Answer Questions

**1 Mark Each**

1. What is an isobaric process ?
2. What is an isochoric process ?
3. What is meant by free expansion ?
4. Write the relation among heat energy, work done and change in internal energy.
5. Is it possible to increase the temperature of a gas without giving it heat ? [Himachal 03]
6. An ideal gas is compressed at constant temperature. Will its internal energy increase or decrease ?
7. Can the temperature of an isolated system remain constant ?
8. Is the super heating of steam an isobaric process or an isothermal process, and why ?
9. If no external energy is supplied to an expanding gas, will the gas do any work ? If yes, then what will be the source of energy ?
10. Name the thermodynamical variables defined by
  - (i) Zeroth law and
  - (ii) first law of thermodynamics.
11. Is it possible to convert internal energy into work ?
12. A sample of an ideal gas in a cylinder is compressed adiabatically to one-third of its volume. Will the final pressure be more or less than three times the initial pressure ?
13. What is the specific heat of a gas
  - (i) in an isothermal process and
  - (ii) in an adiabatic process ?[Chandigarh 04; Central Schools 10]
14. Can the specific heat of a gas be infinity ?
15. Out of the parameters : temperature, pressure, work and volume, which parameter does not characterise the thermodynamic state of matter ? [AIEEE 04]
16. State two limitations of the first law of thermodynamics.
17. What is a heat engine ?
18. Which law forbids the complete conversion of heat energy into mechanical work ?
19. What type of process is Carnot's cycle ?
20. Can a Carnot engine be realized in actual practice ?
21. Name the hot reservoir in the steam engine and the petrol-engine.
22. Name the sink in case of a steam-engine.
23. Name the working substance in the steam-engine and petrol-engine.
24. What is a heat pump ? Give an example.
25. Is rusting of iron a reversible process ?
26. What is the nature of P-V diagram for isobaric and isochoric processes ?
27. How is the efficiency of a Carnot engine affected by the nature of the working substance ?
28. State two essential requirements of an ideal heat engine.
29. Can we increase the coefficient of performance of a refrigerator by increasing the amount of working substance ?
30. Refrigerator transfers heat from a cold body to a hot body. Does this not violate the second law of thermodynamics ?
31. Is coefficient of performance of a refrigerator a constant quantity ?
32. Heat cannot flow itself from a body at lower temperature to a body at higher temperature is a statement or consequence of which law of thermodynamics ? [AIEEE 03]
33. On what factors, the efficiency of a Carnot engine depends ? [Himachal 04]
34. Heat is supplied to a system, but its internal energy does not increase. What is the process involved ? [Delhi 12]

## Answers

1. A thermodynamic process which occurs at a constant pressure is called an isobaric process.
2. A thermodynamic process which occurs at a constant volume is called an isochoric process.
3. The expansion of a gas against zero external pressure is known as its *free expansion*.
4.  $\Delta Q = \Delta U + \Delta W$ .
5. Yes, it happens during an adiabatic process.
6. It will remain same because the internal energy of an ideal gas depends only upon the temperature.
7. Yes. The temperature of a system remains constant when no physical or chemical change takes place inside the system.
8. Isobaric, because during heating the temperature of the steam does not remain constant.
9. Yes, the gas will do work at the expense of its internal energy.
10. (i) Zeroth law of thermodynamics defines temperature. (ii) First law of thermodynamics defines internal energy.
11. Yes. For example, in an explosion of a bomb, chemical energy (which is a form of internal energy) is converted into kinetic energy.
12. The change in pressure will be more than three times the initial pressure.
13. (i) Infinity (ii) Zero.
14. Yes.
15. Work.
16. (i) First law does not indicate the direction of heat transfer.  
(ii) It does not indicate as to why the whole of heat energy cannot be converted into work continuously.
17. Heat engine is a device for converting heat into mechanical work.
18. Second law of thermodynamics.
19. Carnot's cycle is a reversible cyclic process.
20. No, Carnot engine is an ideal heat engine.
21. Boiler is the hot reservoir in a steam-engine. The source of heat is the combustion of petrol vapours and air in case of a petrol-engine.
22. Atmosphere is the sink in a steam-engine.
23. In the steam-engine, the working substance is the steam. In the case of a petrol-engine, the hot gas obtained from the combustion of the air-petrol mixture is the working substance.
24. Heat pump is a device which uses mechanical work to remove heat. A refrigerator is a heat pump.
25. No. Rusting of iron is an irreversible process.
26. (i) For an isobaric process, the P-V diagram is a straight line parallel to the volume-axis.  
(ii) For an isochoric process, the P-V diagram is a straight line parallel to the pressure-axis.
27. The efficiency of a Carnot engine is independent of the nature of the working substance.
28. (i) An ideal heat engine should have a source of infinite thermal capacity.  
(ii) It should have a sink of infinite thermal capacity.
29. No.
30. No. External work is done by the compressor of the refrigerator.
31. No. As the inside temperature of the refrigerator decreases, its coefficient of performance decreases.
32. Second law of thermodynamics.
33. Temperatures of source of heat and sink.
34. Isothermal expansion.

### Type B : Short Answer Questions

**2 or 3 Marks Each**

1. Define the terms thermodynamic system, surroundings, thermodynamic variables and equation of state.
2. Describe when a system is said to be in a state of thermodynamic equilibrium.
3. State Zeroth law of thermodynamics. How does it lead to the concept of temperature?
4. Define internal energy of a system. Is it a state variable or not? What is the nature of the internal energy of an ideal gas?

5. Distinguish between the terms heat and work.
6. What is an indicator diagram? What does the area between  $P-V$  curve and volume-axis signify?
7. What is a cyclic process? Prove that the net work done during a cyclic process is numerically equal to the area of the loop representing the cycle.
8. State and explain first law of thermodynamics. Discuss its use in isothermal and adiabatic processes. **[Himachal 05]**
9. What are the limitations of the first law of thermodynamics? **[Central Schools 05 ; Chandigarh 07]**
10. Establish relation between two specific heats of a gas. Which is greater and why?  
**[Himachal 05 ; Central Schools 04]**
11. Prove that :  $C_p - C_V = R / J$ , where the symbols have their usual meanings. **[Himachal 03, 08]**
12. (i) Why a gas has two principal specific heat capacities?  
 (ii) Which one is greater and why?  
 (iii) Of what significance is the difference between these two specific heat capacities and their ratio?  
**[Delhi 97, 05]**
13. State first law of thermodynamics. On its basis establish the relation between two molar specific heats for a gas. **[Himachal 05C, 06, 09C ; Delhi 06, 08]**
14. Define two principal specific heats of a gas. Which is greater and why? **[Himachal 05 ; Delhi 03]**
15. Derive the relation between specific heats of a gas at constant pressure and at constant volume, when the amount of gas is one gram molecule.  
**[Chandigarh 07, 08]**
16. State the first law of thermodynamics. Apply this to derive an expression for the change in internal energy during boiling process. **[Delhi 1998]**
17. Compare between isothermal and adiabatic processes. **[Central Schools 05]**
18. Give two statements for the second law of thermodynamics. **[Delhi 99 ; Himachal 05, 08C]**
19. What do you understand by reversible process and irreversible process? Give an example of each. **[Himachal 05C, 09 ; Central Schools 07, 12]**
20. What is an isothermal process? Derive an expression for work done during an isothermal process. **[Himachal 05, 06, 09]**
21. What is an adiabatic process? Derive expression for the work done during such a process. **[Himachal 05, 07, 09C ; Delhi 05 ; Central Schools 09]**
22. Apply first law of thermodynamics to
  - (i) an isochoric process,
  - (ii) a cyclic process and
  - (iii) an isobaric process.
 State what happens to heat absorbed in each case.
23. Draw a neat  $P-V$  diagram showing cycle of operations for an ideal heat engine. Also list the four stages of operations in proper order.  
**[Central Schools 07]**
24. What is Carnot's engine? Derive an expression for the efficiency of a Carnot's engine. On what factors does it depend?  
**[Chandigarh 07 ; Central Schools 08]**
25. Give two characteristics of Carnot engine as compared to other engines, which are sometimes known as Carnot's theorem. **[Central Schools 08]**
26. Explain briefly the working principle of a refrigerator and obtain an expression for its coefficient of performance. **[Himachal 01, 05, 06 ; Chandigarh 03, 04 ; Central Schools 04]**
27. Define the co-efficient of performance of a heat pump and obtain an expression for it in terms of temperature  $T_1$  of the source and  $T_2$  of the sink.  
**[Central School 07]**
28. State second law of thermodynamics. Write a difference between heat engine and refrigerator.  
**[Delhi 12]**

## **Answers**

1. Refer answer to Q. 2 on page 12.1.
2. Refer answer to Q. 4 on page 12.2.
3. Refer answer to Q. 5 on page 12.2.
4. Refer answer to Q. 6 on page 12.2.
5. Refer answer to Q. 8 on page 12.3.
6. Refer answer to Q. 10 on page 12.4.
7. Refer answer to Q. 13 on page 12.5.
8. Refer answer to Q. 14 on page 12.7, Q. 21 on page 12.14 and Problem 29 on page 12.39.
9. Refer answer to Q. 33 on page 12.23.
10. Refer answer to Q. 16 and Q. 17 on page 12.10.
11. Refer answer to Q. 17 on page 12.10.

12. Refer answer to Q. 16 on page 12.10. The value of  $\gamma$  indicates the atomicity of a gas.
13. Refer answer to Q. 17 on page 12.10.
14. Refer answer to Q. 16 on page 12.10.
15. Refer answer to Q. 17 on page 12.10.
16. Refer answer to Q. 29 on page 12.17.
17. Refer answer to Q. 20 on page 12.14 and Q. 23 on page 12.15.
18. Refer answer to Q. 34 on page 12.23.
19. Refer answer to Q.36 on page 12.24.
20. Refer answer to Q.22 on page 12.15.
21. Refer answer to Q.25 on page 12.16.
22. Refer answer to Q. 26, Q. 27 and Q. 28 on page 12.17.
23. See Fig. 12.21 on page 12.25.
24. Refer answer to Q. 38 on page 12.25
25. See statement of Carnot theorem in the answer of Q. 40 on page 12.30.
26. Refer answer to Q. 41 on page 12.31.
27. Refer answer to Q. 41 on page 12.31.
28. Refer to point 28 of Glimpses and answer to Q. 7(i) on page 12.52.

### Type C : Long Answer Questions

**5 Marks Each**

1. Define  $C_p$  and  $C_v$ . Why is  $C_p > C_v$ ? For an ideal gas, prove that  $C_p - C_v = R$ .

[Himachal 04, 05C, 06, 07C]

2. State mathematically, first law of thermodynamics and use it to find the expression for work done during adiabatic expansion. Write two limitations of first law of thermodynamics.

[Chandigarh 07]

3. Derive an expression for work done during an adiabatic process. Show that slope in adiabatic process is  $\gamma$  times the slope in isothermal process.

[Chandigarh 08]

4. What is an isothermal process? State two essential conditions for such a process to take place. Show analytically that work done by one mole of an ideal gas during isothermal expansion from volume  $V_1$  to volume  $V_2$  is given by

$$W = RT \log_e \frac{V_2}{V_1}$$

What is the change in internal energy of a gas, which is compressed isothermally?

[Himachal 06]

5. What is an adiabatic process? Show that its gas equation is

$$TV^{\gamma-1} = \text{constant}$$

where  $\gamma$  is specific heat constant.

[Himachal 07C]

6. Define an adiabatic process and state two essential conditions for such a process to take place. Show analytically that work done by one mole of an ideal gas during adiabatic expansion from temperature  $T_1$  to  $T_2$  is given by

$$W = \frac{R(T_1 - T_2)}{\gamma - 1}$$

[Himachal 06, 09]

7. (i) State second law of thermodynamics. How is a heat engine different from a refrigerator?

(ii) Show that for a Carnot engine, efficiency

$$\eta = 1 - \frac{T_2}{T_1} \quad (T_2 < T_1)$$

where  $T_1$  is the temperature of the source and  $T_2$  is the temperature of the sink. [Delhi 05]

8. Give Kelvin-Planck statement for second law of thermodynamics. Define Carnot engine. Draw P-V diagram for Carnot cycle. List down and mark the sequence of processes involved in Carnot cycle.

[Central Schools 2003]

9. What is meant by a thermodynamic reversible process? Write two characteristics of a reversible process.

State in brief the meaning of Carnot cycle. Write an expression for the efficiency of a Carnot engine. What is the advantage of Carnot engine in terms of efficiency? [Delhi 03C]

10. Explain the construction and various operations for Carnot's heat engine working between two temperatures. Hence derive from it the efficiency of the engine.

[Himachal 03, 07C]

11. (a) Draw P-V diagram for Carnot cycle.  
 (b) Write the name of thermodynamic process carried out by each part of the cycle.  
 (c) Label and shade the area corresponding to net work done by the engine in one cycle.

[Central Schools 08]

12. Write Kelvin-Planck and Clausius statements for second law of thermodynamics. Define coefficient of efficiency and coefficient of performance. Show the heat flow in case of an engine and refrigerator using schematic diagram.

[Central Schools 08, 09]

## Answers

1. Refer answer to Q.16 and Q.17 on page 12.10.
2. Refer answer to Q.14 on page 12.7, Q.25 on page 12.16 and Q.33 on page 12.23.
3. Refer answer to Q. 25 on page 12.16 and refer to solution of Problem 10 on page 12.39.
4. Refer answer to Q. 20 on page 12.14, Q.21 and Q.22 on page 12.15.
5. Refer answer to Q.24 on page 12.15.
6. Refer answer to Q.23 on page 12.15 and Q.25 on page 12.16.
7. (i) Refer answer to Q. 34 on page 12.23. A heat engine is a device in which the working substance undergoes a cyclic process to convert heat into work. The working substance extracts some heat  $Q_1$  from the source at a high temperature, converts a part of it into work  $W$  and rejects the rest  $Q_2$  to the sink at lower temperature. A refrigerator is a heat engine working in the reverse direction. Here the working substance extracts heat  $Q_1$  from the sink at a lower temperature. Then work  $W$  is done on it and a higher amount of heat  $Q_1 = Q_2 + W$  is rejected to the source at a higher temperature.
- (ii) Refer answer to Q. 38 on page 12.25.
8. Refer answer to Q. 34 on page 12.23 and Q. 38 on page 12.25.
9. Refer answer to Q. 36 on page 12.24 and Q. 38 on page 12.25.
10. Refer answer to Q. 38 on page 12.25.
11. See Fig. 12.21 on page 12.25. The area of the Carnot cycle gives the net work done by the engine per cycle.
12. Refer to points 27, 28 and 34 of Glimpses. See Fig. 12.18 on page 12.22 and Fig. 12.23 on page 12.31.

## Competition Section

# Thermodynamics

### GLIMPSES

- Thermodynamics.** It is the branch of science that deals with the concepts of heat and temperature and the inter-conversion of heat and other forms of energy.
- Thermodynamic system.** An assembly of a very large number of particles having a certain value of pressure, volume and temperature is called a thermodynamic system.
- Surroundings.** Everything outside the system which can have a direct effect on the system is called its surroundings.
- Thermodynamic variables.** The quantities like pressure ( $P$ ), volume ( $V$ ) and temperature ( $T$ ) which help us to study the behaviour of a thermodynamic system are called thermodynamic variables.
- Equation of state.** The mathematical relation between the pressure, volume and temperature of a thermodynamic system is called its equation of state. For example, the equation of state of  $n$  moles of an ideal gas can be written as.

$$PV = nRT$$

- Thermal equilibrium.** Two systems are in thermal equilibrium with each other if they have the same temperature.
- Thermodynamic equilibrium.** A system is said to be in the state of thermodynamic equilibrium if the macroscopic variables describing the thermodynamic state of the system do not change with time. A system in a state of thermodynamic equilibrium possesses mechanical, thermal and chemical equilibria simultaneously.
- State variables.** The macroscopic quantities which are used to describe the equilibrium states of a thermodynamic system are called state variables. The value of a state variable depends only on the particular state, not on the path used to attain that

state. Pressure ( $P$ ), volume ( $V$ ), temperature ( $T$ ) and mass ( $m$ ) are state variables. Heat ( $Q$ ) and work ( $W$ ) are not state variables.

- Zeroth law of thermodynamics.** If two systems  $A$  and  $B$  are in thermal equilibrium with a third system  $C$ , then  $A$  and  $B$  are in thermal equilibrium with each other. According to this law, temperature is a physical quantity which has the same value for all systems which are in thermal equilibrium with each other.
- Internal energy.** The internal energy of a system is the sum of molecular kinetic and potential energies in the frame of reference relative to which the centre of mass of the system is at rest. It does not include the over-all kinetic energy of the system. It is a state variable denoted by  $U$ .
- Quasi-static process.** A quasi-static process is an infinitely slow process such that system remains in thermal and mechanical equilibrium with the surroundings throughout. In such a process, the pressure and temperature of the surroundings can differ from those of the system only infinitesimally.
- Isothermal process.** A process in which temperature remains constant is called an isothermal process. For such a process,

$$PV = \text{constant} \quad \text{or} \quad P_1 V_1 = P_2 V_2$$

- Adiabatic process.** A process in which thermally insulated system neither loses nor gains heat from the surroundings is called adiabatic process.

Equations of state for adiabatic processes are :

- $PV^\gamma = \text{constant}$  or  $P_1 V_1^\gamma = P_2 V_2^\gamma$
- $TV^{\gamma-1} = \text{constant}$  or  $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$
- $\frac{P^{\gamma-1}}{T^\gamma} = \text{constant}$  or  $\frac{P_1^{\gamma-1}}{T_1^\gamma} = \frac{P_2^{\gamma-1}}{T_2^\gamma}$

where  $\gamma = C_p / C_v$ .

14. Isobaric process. A process in which volume remains constant is called isobaric process. For such a process

$$\frac{V}{T} = \text{constant} \quad \text{or} \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}.$$

15. Isochoric process. A process in which volume remains constant is called isochoric process. For such a process,

$$\frac{P}{V} = \text{constant} \quad \text{or} \quad \frac{P_1}{V_1} = \frac{P_2}{V_2}.$$

16. Indicator diagram. A graphical representation of the state of a system with the help of two thermodynamical variables is called indicator diagram of the system. The graph between pressure  $P$  and volume  $V$  is called  $P$ - $V$  diagram.

17. Work done during the expansion of a gas. When the volume of a gas changes from  $V_1$  to  $V_2$ , the work done is

$$W = \int_{V_1}^{V_2} P dV$$

= Area enclosed between the  $P$ - $V$  curve and the volume axis.

18. Work done during a cyclic process. From the  $P$ - $V$  diagram,

Work done per cycle = Area of the loop  
representing the cycle

- (i) If the loop is traced clockwise, the work done is positive and work is done by the system.
- (ii) If the loop is traced anticlockwise, the work done is negative and work is done on the system.

19. First law of thermodynamics. It states that if heat  $dQ$  is given to a system, a part of it is used in increasing the internal energy by an amount  $dU$  and the remaining energy is used in doing the external work  $dW$ . It is just a restatement of the law of conservation of energy. Thus

$$dQ = dU + dW \quad \text{or} \quad dQ = dU + PdV.$$

#### Sign conventions used.

- (i) Heat absorbed by a system is positive and heat given out by a system is negative.
  - (ii) Increase in internal energy of a system is positive and decrease in internal energy of a system is negative.
  - (iii) Work done by a system is positive and work done on a system is negative.
20. Work done in an isothermal process. Work done when 1 mole of a gas expands isothermally,

$$W_{\text{iso}} = 2.303 RT \log \frac{V_2}{V_1} = 2.303 RT \log \frac{P_1}{P_2}.$$

21. Work done in an adiabatic process. Work done when 1 mole of a gas expands adiabatically and its temperature falls from  $T_1$  to  $T_2$ ,

$$W_{\text{adia}} = \frac{R}{\gamma - 1} [T_1 - T_2] = \frac{1}{\gamma - 1} [P_1 V_1 - P_2 V_2].$$

22. Dulong and Petit's law. Near the room temperature, the molar specific heat of most of the solids at constant volume is equal to  $3R$  or  $6 \text{ cal mol}^{-1} \text{ K}^{-1}$  or  $25 \text{ J mol}^{-1} \text{ K}^{-1}$ . This statement is known as Dulong and Petit's law.

23. Molar specific heat of a gas at constant volume ( $C_v$ ). It is defined as the amount of heat required to raise the temperature of 1 mole of the gas through  $1^\circ\text{C}$  at constant volume.

If  $c_v$  is specific heat of gas for 1 g at constant volume and  $M$  is its molecular weight, then molar specific heat at constant volume,

$$C_v = Mc_v$$

24. Molar specific heat of a gas at constant pressure ( $C_p$ ). It is defined as the amount of heat required to raise the temperature of 1 mole of the gas through  $1^\circ\text{C}$  at constant pressure. Thus,

$$C_p = Mc_p$$

25. Relation between two specific heats of a gas. Specific heat of a gas at constant pressure is greater than the specific heat at constant volume.

For one mole of a gas :

$$(i) C_p - C_v = R \quad (\text{when } C_p, C_v \text{ are in units of work})$$

$$(ii) C_p - C_v = \frac{R}{J} \quad (\text{when } C_p, C_v \text{ are in units of heat})$$

where  $R$  is universal gas constant for 1 mole of a gas. For 1 g of a gas :

$$(i) c_p - c_v = r \quad (\text{when } c_p, c_v \text{ are in units of work})$$

$$(ii) c_p - c_v = \frac{r}{J} \quad (\text{when } c_p, c_v \text{ are in units of heat})$$

where  $r = \frac{R}{M}$  = gas constant for 1 g of a gas.

Clearly, heat lost or gained by  $n$  moles of a gas,

$$(i) Q = nC_p \Delta T \quad (\text{At constant pressure})$$

$$(ii) Q = nC_v \Delta T \quad (\text{At constant volume})$$

where  $n$  = number of moles of gas

$$= \frac{\text{Mass of gas}}{\text{Molecular mass}}.$$

26. Heat engine. It is a device which converts continuously heat energy into mechanical energy in a cyclic process. It essentially consists of (i) a source of heat (ii) a sink of heat and (iii) a working substance.

27. **Efficiency of a heat engine.** It is the ratio of useful work done ( $W$ ) by the engine per cycle to the heat energy ( $Q_1$ ) absorbed from the source per cycle.

$$\eta = \frac{\text{Work output}}{\text{Heat input}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

where  $Q_2$  is the heat rejected to the sink.

28. **Second law of thermodynamics.**

- (i) **Kelvin-Planck statement.** It is impossible to construct an engine, which will produce no effect other than extracting heat from a reservoir and performing an equivalent amount of work.
- (ii) **Clausius statement.** It is impossible for a self acting machine, unaided by an external agency, to transfer heat from a body to another at higher temperature.

29. **Reversible process.** A process which can be made to proceed in the reverse direction by variation in its conditions so that any change occurring in any part of the direct process is exactly reversed in the corresponding part of the reverse process is called a reversible process.

30. **Irreversible process.** A process which cannot be made to proceed in the reverse direction is called an irreversible process.

31. **Carnot Engine.** It is an ideal heat engine which is based on Carnot's reversible cycle. Its working consists of four steps viz. Isothermal expansion, adiabatic expansion, isothermal compression and

adiabatic compression. The efficiency of Carnot engine is given by

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

where  $T_1$  and  $T_2$  are the temperatures of source and sink respectively.

32. **Carnot theorem.** It states that

- (i) no engine working between two given temperatures can have efficiency greater than that of the Carnot engine working between the same two temperatures and
- (ii) the efficiency of the Carnot engine is independent of the nature of the working substance.

33. **Refrigerator.** It is a heat engine working in the reverse direction. Here a working substance absorbs heat  $Q_2$  from the sink at temperature  $T_2$ . An external agency does work  $W$  on the working substance. A larger amount of heat  $Q_1$  is rejected to source at a higher temperature  $T_1$ .

$$Q_1 = Q_2 + W.$$

34. **Coefficient of performance.** It is defined as the ratio of the amount of heat ( $Q_2$ ) removed per cycle from the contents of the refrigerator to the work done ( $W$ ) by the external agency to remove it.

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}.$$

## IIT Entrance Exam

### MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT ANSWER

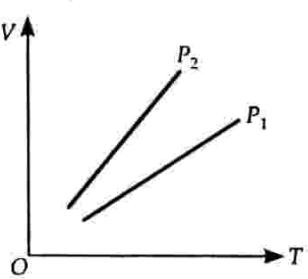
1. In a given process of an ideal gas,  $dW = 0$  and  $dQ < 0$ . Then for the gas

- (a) the temperature will decrease
- (b) the volume will increase
- (c) the pressure will remain constant
- (d) the temperature will increase.

[IIT 01]

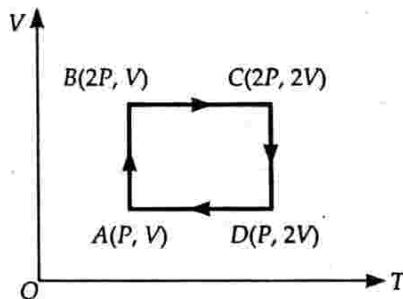
2. The volume ( $V$ ) versus temperature ( $T$ ) graphs for a certain amount of a perfect gas at two pressures  $P_1$  and  $P_2$  are shown in the figure. It follows from the graphs that

- (a)  $P_1 > P_2$
- (b)  $P_1 < P_2$
- (c)  $P_1 = P_2$



- (d) information is insufficient to draw any conclusion. [IIT 88]

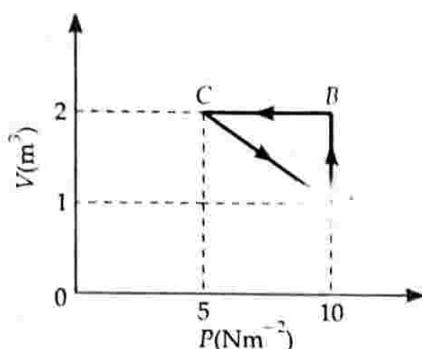
3. An ideal monoatomic gas is taken round the cycle ABCDA as shown in the P-V diagram.



The work done during the cycle is

- (a)  $PV$
- (b)  $2PV$
- (c)  $PV/2$
- (d) zero. [IIT 83]

4. An ideal gas is taken through the cycle  $A \rightarrow B \rightarrow C \rightarrow A$  as shown in the figure. If the net heat



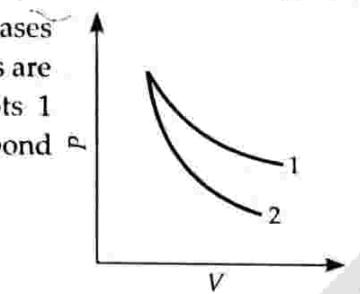
supplied to the gas in the cycle is 5 J, the work done by the gas in the process  $C \rightarrow A$  is

- (a) -5 J      (b) -10 J  
 (c) -15 J      (d) -20 J

[IIT 02]

5. P-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to

- (a) He and O<sub>2</sub>  
 (b) O<sub>2</sub> and He  
 (c) He and Ar  
 (d) O<sub>2</sub> and N<sub>2</sub>



[IIT 01]

6. An ideal gas initially at  $P_1$ ,  $V_1$  is expanded to  $P_2$ ,  $V_2$  and then compressed adiabatically to the same volume  $V_1$  and pressure  $P_3$ . If  $W$  is the net work done by the gas in complete process, which of the following is true?

- (a)  $W > 0$ ;  $P_3 > P_1$       (b)  $W < 0$ ;  $P_3 > P_1$   
 (c)  $W > 0$ ;  $P_3 < P_1$       (d)  $W < 0$ ;  $P_3 < P_1$

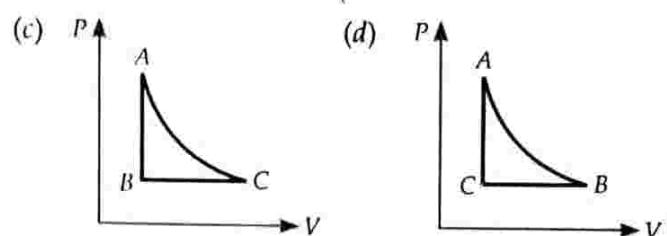
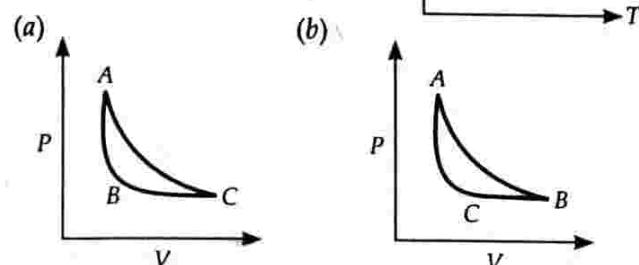
[IIT 04]

7. A monoatomic ideal gas, initially at temperature  $T_1$  is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature  $T_2$  by releasing the piston suddenly.  $L_1$  and  $L_2$  are the lengths of the gas column before and after expansion respectively, then  $T_1 / T_2$  is given by

- (a)  $(L_1 / L_2)^{2/3}$   
 (b)  $L_1 / L_2$   
 (c)  $L_2 / L_1$   
 (d)  $(L_2 / L_1)^{2/3}$

[IIT 2K]

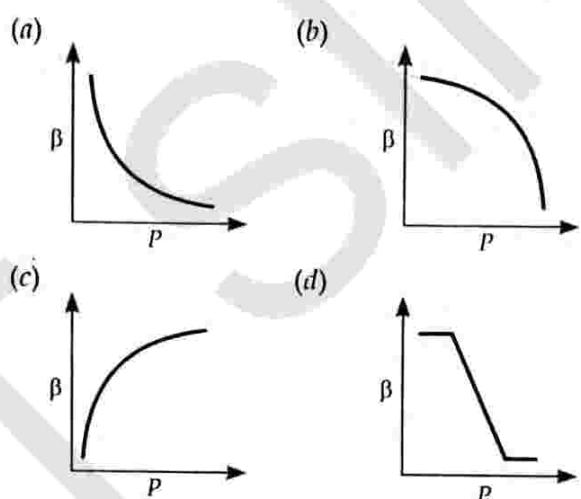
8. The PT diagram for an ideal gas is shown in the figure, where AC is an adiabatic process. Find the corresponding PV diagram.



[IIT 03]

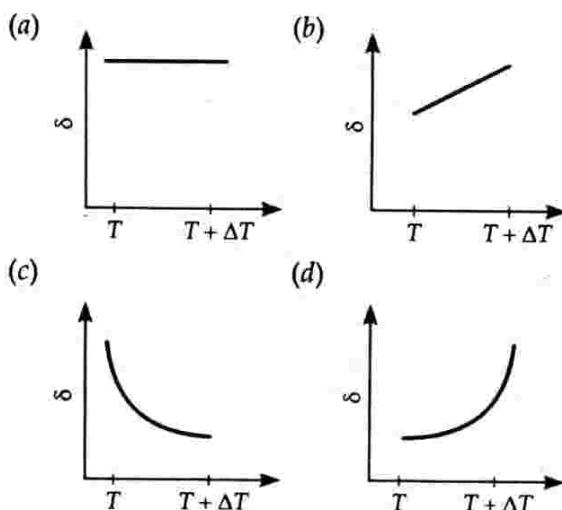
9. Which of the following graphs correctly represents the variation of  $\beta = -\frac{dV/dP}{V}$  with  $P$  for an ideal gas at constant temperature?

[IIT 02]



10. An ideal gas is initially at temperature  $T$  and volume  $V$ . Its volume is increased by  $\Delta V$  due to an increase in temperature  $\Delta T$ , pressure remaining constant. The quantity  $\delta = \Delta V / V\Delta T$  varies with temperature as

[IIT 2K]



11. One mole of a monoatomic gas is heated at a constant pressure of 1 atmosphere from 0 K to 100 K. If the gas constant  $R = 8.32 \text{ J/mol K}$ , the change in internal energy of the gas is approximately

- (a) 2.3 J      (b) 46 J  
 (c)  $8.67 \times 10^3 \text{ J}$       (d)  $1.25 \times 10^3 \text{ J}$

[IIT 98]

12. An ideal gas heat engine is operating between  $227^{\circ}\text{C}$  and  $127^{\circ}\text{C}$ . It absorbs  $10^4\text{ J}$  of heat at the higher temperature. The amount of heat converted into work is

13. 5.6 litre of helium gas at STP is adiabatically

- impressed to 0.7 litre. Taking the initial temperature to be  $T_1$ , the work done in the process is

$$(a) \frac{9}{8} RT_1 \quad (b) \frac{3}{2} RT_1$$

$$(c) \frac{15}{8} RT_1 \quad (d) \frac{9}{2} RT_1 \quad [\text{IIT 2011}]$$

**MULTIPLE CHOICE QUESTIONS WITH  
ONE OR MORE THAN ONE CORRECT  
ANSWER**

- 14.** An ideal gas is taken from the state  $A$  (pressure  $V$ ) to the state  $B$  (pressure  $P/2$ , volume  $2V$ ) along a straight line path in the  $P-V$  diagram. Select the correct statement(s) from the following :

- (a) the work done by the gas in the process  $A$  to  $B$  exceeds the work that would be done by it, if the system were taken from  $A$  to  $B$  along an isotherm

(b) in the  $T-V$  diagram, the path  $AB$  becomes a part of a parabola

(c) in the  $P-T$  diagram, the path  $AB$  becomes a part of a hyperbola

(d) in going from  $A$  to  $B$ , the temperature  $T$  of the gas first increases to a maximum value and then decreases.

15. During the melting of a slab of ice at 273 K at atmospheric pressure,

- (a) positive work is done by the ice-water system on the atmosphere
  - (b) positive work is done on the ice-water system by the atmosphere
  - (c) the internal energy of the ice-water system increases
  - (d) the internal energy of the ice-water system decreases.

- 16.** 70 calories are required to raise the temperature of 2 moles of an ideal gas at constant pressure from  $30^{\circ}\text{C}$  to  $35^{\circ}\text{C}$ . The amount of heat required (in calories) to raise the temperature of the same gas through the same range ( $30^{\circ}$  to  $35^{\circ}\text{C}$ ) at constant volume is

17. For an ideal gas

- (a) the change in internal energy in a constant pressure process from temperature  $T_1$  to  $T_2$  is equal to  $nC_V(T_2 - T_1)$ , where  $C_V$  is the molar specific heat at constant volume and  $n$ , the number of moles of the gas

(b) the change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process

(c) no heat is added or removed in an adiabatic process

(d) the internal energy does not change in an isothermal process.

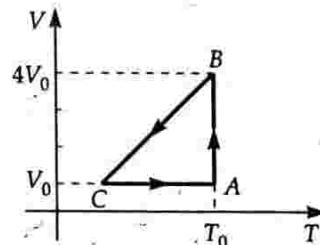
18. Two cylinders  $A$  and  $B$  fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of  $A$  is free to move, while that of  $B$  is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in  $A$  is 30 K, then the rise in temperature of the gas in  $B$  is



19. Two identical containers  $A$  and  $B$  with frictionless pistons contain the same ideal gas at the same temperature and the same volume  $V$ . The mass of the gas in  $A$  is  $m_A$  and that in  $B$  is  $m_B$ . The gas in each cylinder is now allowed to expand isothermally to the same final volume  $2V$ . The changes in the pressure in  $A$  and  $B$  are found to be  $P$  and  $1.5P$  respectively. Then

- (a)  $2m_A = 3m_B$       (b)  $3m_A = 2m_B$   
 (c)  $4m_A = 9m_B$       (d)  $9m_A = 4m_B$  [IIT 98]

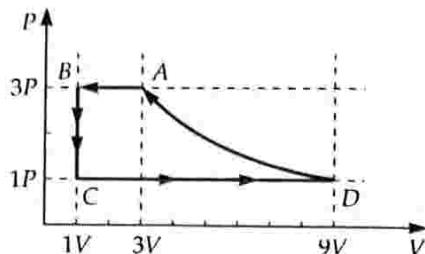
20. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA as shown in the figure. Its pressure at A is  $P_0$ . Choose the correct option(s), from the following :



- (a) Internal energies at A and B are the same  
 (b) Work done by the gas in process AB is  $P_0 V_0 \ln 4$   
 (c) Pressure at C is  $\frac{P_0}{4}$   
 (d) Temperature at C is  $\frac{T_0}{4}$

**✓ MATRIX MATCH TYPE**

21. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in the  $P$ - $V$  diagram. Column II gives the characteristics involved in the cycle. Match them with each of the processes given in Column I.



<b>Column I</b>	<b>Column II</b>
(a) Process A-B	(p) Internal energy decreases
(b) Process B-C	(q) Internal energy decreases
(c) Process C-D	(r) Heat is lost
(d) Process D-A	(s) Heat is gained
	(t) Work is done on the gas

[IIT 2011]

**✓ INTEGER ANSWER TYPE**

22. A diatomic ideal gas is compressed adiabatically to  $\frac{1}{32}$  of its initial volume. If the initial temperature of the gas is  $T_i$  (in kelvin) and the final temperature is  $aT_i$ , find the value of  $a$ .

[IIT 2010]

23. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures  $T_1$  and  $T_2$ , respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

[IIT 2010]

## Answers and Explanations

1. (a) By first law of thermodynamics,

$$dQ = dU + dW$$

As  $dW = 0$  and  $dQ < 0$ , so  $dU < 0$

But for an ideal gas,

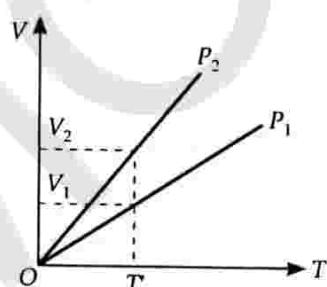
$$U \propto T$$

$$\therefore dT < 0$$

Hence temperature of the gas decreases.

2. (a) At a given temperature  $T'$ ,

$$P_1 V_1 = P_2 V_2$$



From the graph,

$$V_1 < V_2$$

$$\therefore P_1 > P_2$$

3. (a) Work done in the cyclic process

$$= \text{Area of the loop } ABCD$$

$$= (2P - P) \times (2V - V) = PV.$$

4. (a) For the cyclic process,

$$\Delta Q = \Delta W = W_{AB} + W_{BC} + W_{CA}$$

$$5 = 10(2 - 1) + 0 + W_{CA}$$

$$W_{CA} = 5 - 10 = -5 \text{ J.}$$

5. (b) Slope of an adiabatic curve,

$$\frac{dP}{dV} = -\gamma \frac{P}{V}$$

$$\therefore \text{slope} \propto \gamma \quad [\text{with a -ve sign}]$$

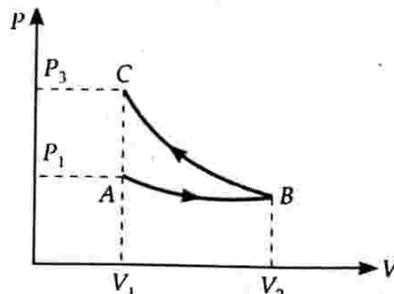
From the given graph,

$$\text{slope of plot 2} > \text{slope of plot 1}$$

$$\gamma_2 > \gamma_1$$

Hence plot 1 should correspond to diatomic  $O_2$  (small  $\gamma = 1.4$ ) and plot 2 should correspond to monoatomic He (large  $\gamma = 1.67$ ).

6. (b) The slope of an adiabatic curve is  $\gamma$  times the slope of an isothermal curve at any given point. In the figure, AB is an isotherm and BC is an adiabat.



As volume increases from A to B,  $W_{AB} = +\text{ve}$ .

As volume decreases from B to C,  $W_{BC} = -\text{ve}$ .

Area under P-V graph gives work done.

Clearly,  $|W_{BC}| > |W_{AB}|$

$$\therefore W_{AB} + W_{BC} = W < 0$$

Also, from the graph,  $P_3 > P_1$

Hence option (b) is correct.

7. (d) For an adiabatic expansion,

$$TV^{\gamma-1} = \text{constant}$$

$$\text{or } T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

For a monoatomic gas,  $\gamma = \frac{5}{3}$ .

If A is the area of cross-section of the piston, then

$$\frac{T_1}{T_2} = \left( \frac{V_2}{V_1} \right)^{\gamma-1} = \left( \frac{AL_2}{AL_1} \right)^{\frac{5}{3}-1} = \left( \frac{L_2}{L_1} \right)^{\frac{2}{3}}.$$

8. (b) From the given P-T graph, it is obvious that

- (i) AC is an adiabatic process (given)
- (ii) AB is an isothermal process, as  $T = \text{constant}$ .
- (iii) BC is an isobaric process, as  $P = \text{constant}$ .

In all the four options, BC is an isobaric process.

For AB to be an isothermal process ( $PV = \text{constant}$ ), P-V graph must be a rectangular hyperbola. This is satisfied in options (b) and (d).

In option (d), AC is not an isothermal process but it is an isochroic process ( $V = \text{constant}$ ).

Slope of adiabat  $AC >$  Slope of isotherm AB.

This is satisfied in (b). Hence only option (b) is correct.

$$9. (a) \beta = -\frac{dV/dP}{V} = -\frac{dV/V}{dP} = \frac{1}{\text{Bulk modulus}}$$

Under isothermal conditions,

Bulk modulus = Pressure  $P$  of the gas

$$\beta = \frac{1}{P}$$

Hence the graph between  $\beta$  and  $P$  is a rectangular hyperbola as in option (a).

10. (c) For an ideal gas,  $PV = nRT$

At constant pressure,  $P\Delta V = nR\Delta T$

$$\text{On dividing, } \frac{\Delta V}{V} = \frac{\Delta T}{T}$$

$$\therefore \delta = \frac{\Delta V}{V\Delta T} = \frac{1}{T}$$

Hence graph between  $\delta$  and  $T$  is a rectangular hyperbola as in option (c).

11. (d) For a monoatomic gas

$$C_V = \frac{3}{2} R$$

$$\therefore dU = C_V dT = \frac{3R}{2} dT \\ = \frac{3}{2} \times 8.32 \times (100 - 0) = 1.25 \times 10^3 \text{ J.}$$

$$12. (a) \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{273 + 127}{273 + 227} = 1 - \frac{400}{500} = \frac{1}{5}$$

$$W = \eta Q_1 = \frac{1}{5} \times 10^4 = 2000 \text{ J.}$$

13. (a) For monoatomic He gas,  $\gamma = 5/3$

$V_1 = 5.6 \text{ litre}, T_1 = 273 \text{ K}, P_1 = 1 \text{ atm}, V_2 = 0.7 \text{ litre}$

For adiabatic compression,

$$\begin{aligned} T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\ \therefore T_2 &= T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1} = T_1 \left( \frac{5.6}{0.7} \right)^{\frac{5}{3}-1} \\ &= T_1 (8)^{2/3} = 4 T_1 \\ n &= \frac{5.6 \text{ litre}}{22.4 \text{ litre}} = \frac{1}{4} \\ W_{\text{adia}} &= \frac{nR(T_2 - T_1)}{\gamma - 1} = \frac{(1/4)R(4T_1 - T_1)}{(5/3) - 1} \\ &= \frac{9}{8} RT_1 \end{aligned}$$

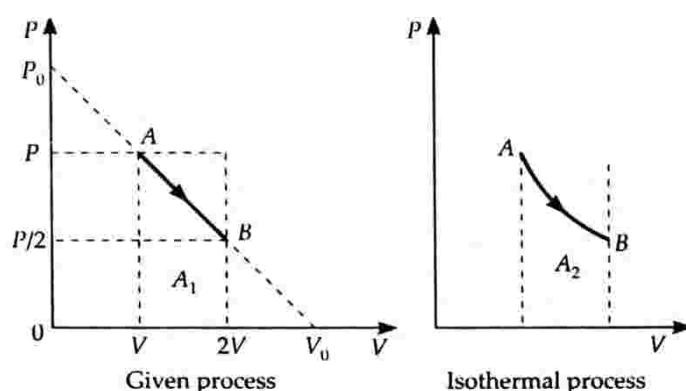
14. (a), (b), (d)

(a) Work done = Area under P-V graph

Area  $A_1 >$  Area  $A_2$

$\therefore W_{\text{given process}} > W_{\text{isothermal process}}$

Hence option (a) is correct.



(b) If  $P_0$  and  $V_0$  are the intercepts on P and V axes, then equation of line AB will be

$$P = -\frac{P_0}{V_0}V + P_0$$

## 12.60 PHYSICS-XI

or  $\frac{nRT}{V} = -\frac{P_0}{V_0}V + P_0$

or  $T = \frac{1}{nR} \left( -\frac{P_0}{V_0}V^2 + P_0V \right)$

As  $T \propto V^2$ ,

the  $T$ - $V$  graph will be a parabola. Hence option (b) is correct.

(c) Again,  $P = -\frac{P_0}{V_0} \frac{nRT}{P} + P_0$

or  $\frac{V_0 P^2}{nRP_0} = -T + \frac{V_0}{nR} P$

or  $T = -\frac{V_0}{nRP_0} P^2 + \frac{V_0}{nR} P$

As  $T \propto P^2$ ,

so,  $P$ - $T$  graph is a parabola not a hyperbola. Hence option (c) is incorrect.

(d)  $T = \frac{V_0}{nR} \left( -\frac{1}{P_0} P^2 + P \right)$

$$\frac{dT}{dP} = \frac{V_0}{nR} \left( -\frac{2}{P_0} P + 1 \right) = 0$$

or  $P = \frac{P_0}{2}$

$$\frac{d^2T}{dP^2} = -\frac{2V_0}{nRP_0} < 0$$

$\therefore T$  has a maximum value.

$T \propto PV$

$$(PV)_A = (PV)_B \quad \text{or} \quad T_A = T_B$$

In going from  $A$  to  $B$ ,  $T$  first increases to a maximum and then decreases to original value.

Hence option (d) is correct.

15. (b), (c) When ice slab melts at 273 K, its volume decreases. Therefore, negative work is done by ice-water system on the atmosphere or positive work is done by the atmosphere on ice-water system.

Hence option (b) is correct.

From first law of thermodynamics,

$$dQ = dU + dW \quad \text{or} \quad dU = dQ - dW.$$

$dQ$  is +ve as ice absorbs heat during melting.

Also,  $dW$  is -ve. Hence  $dU$  will be +ve or internal energy of ice-water system increases.

Hence option (c) is correct.

16. (b) At constant pressure,  $Q_1 = nC_p dT$

At constant volume,  $Q_2 = nC_V dT$

$$\frac{Q_2}{Q_1} = \frac{C_V}{C_p} = \frac{1}{\gamma}$$

or  $Q_2 = \frac{Q_1}{\gamma} = \frac{70}{1.4} = 50 \text{ cal.}$

17. (a), (b), (c), (d)

(a)  $\Delta U = nC_V \Delta T = nC_V (T_2 - T_1)$ , for all processes.

(b) In an adiabatic process,  $\Delta Q = 0$

$$\therefore \Delta U = -\Delta W \quad \text{or} \quad |\Delta U| = |\Delta W|$$

(c) In an isothermal process,  $\Delta T = 0$

$$\therefore \Delta U = nC_V \Delta T = 0$$

(d) In an adiabatic process,  $\Delta Q = 0$

Hence all the options are correct.

18. (d) As the piston of  $A$  is free to move, heat is supplied at constant pressure.

$$dQ = nC_p dT_A$$

As the piston of  $B$  is held fixed, heat is supplied at constant volume

$$dQ_B = nC_V dT_B$$

But  $dQ_A = dQ_B$

or  $nC_p dT_A = nC_V dT_B$

or  $dT_B = \left( \frac{C_p}{C_V} \right) dT_A = \gamma dT_A$

$$= 1.4 \times 30 \text{ K} \quad [\text{For a diatomic gas, } \gamma = 1.4]$$

$$= 42 \text{ K.}$$

19. (b)  $\Delta P = P_i - P_f = \frac{nRT}{V} - \frac{nRT}{2V} = \frac{nRT}{2V} = \frac{mRT}{2MV}$

$\Delta P \propto m$

$$\frac{m_A}{m_B} = \frac{\Delta P_A}{\Delta P_B} = \frac{P_i}{1.5 A} = \frac{2}{3}$$

or  $3m_A = 2m_B$ .

20. (a), (b), (c), (d)

Process  $A \rightarrow B$  is isothermal, so  $U_A = U_B$

$$W_{AB} = nRT \ln \frac{V_2}{V_1} = RT_0 \ln \frac{4V_0}{V_0}$$

$$= P_0 V_0 \ln 4 \quad [P_0 V_0 = RT_0 \text{ for } n=1]$$

$$\frac{P_0 V_0}{T_0} (\text{at } A) = \frac{P_B 4V_0}{T_0} (\text{at } B)$$

$$\Rightarrow P_B = \frac{P_0}{4}$$

Process  $B \rightarrow C$  is isobaric, so  $P_C = P_B = \frac{P_0}{4}$

Process  $C \rightarrow A$  is isochoric,  $\frac{P}{T} = \text{constant}$

$$\therefore T_C = \frac{T_0}{4}$$

$$\left[ \frac{P_0}{T_0} (\text{at } A) = \frac{P_0/4}{T_0/4} (\text{at } C) \right]$$

21. (a)  $\rightarrow p, r, t$ ; (b)  $\rightarrow p, r$ ; (c)  $\rightarrow q, s$ ; (d)  $\rightarrow r, t$

(a) Process  $A \rightarrow B$  is isobaric compression

$\therefore$  Volume decreases  $\Rightarrow$  Temperature decreases

Work is done on the gas,

$$W = P(3V - V) = 2PV$$

Heat is lost and internal energy decreases.

Hence, (a)  $\rightarrow p, r, t$  are correct matching.

(b) Process  $B \rightarrow C$  is isochoric process

$\therefore$  Pressure decreases  $\Rightarrow$  Temperature decreases.

Heat is lost and internal energy decreases.

Hence, (b)  $\rightarrow p, r$  are correct matching

(c) Process  $C \rightarrow D$  is isobaric expansion.

$\therefore$  Volume increases  $\Rightarrow$  Temperature increases

Heat is gained and internal energy increases.

Hence, (c)  $\rightarrow q, r$  are correct matching.

(d) Process  $D \rightarrow A$  is polytropic with  $T_A = T_D$

because

$$T_A = \frac{(3P)(3V)}{nR} = \frac{9PV}{nR}$$

$$T_D = \frac{PV}{nR} = \frac{(P)(9V)}{nR} = \frac{9PV}{nR}$$

$$T_A = T_D \Rightarrow \Delta U = 0$$

$$\therefore \Delta Q = \Delta U + W = W$$

$\Rightarrow$  Work done on the gas is lost as heat.

Hence, (d)  $\rightarrow r, t$  are correct matching.

22.

0	0	0	4
---	---	---	---

For an adiabatic process,  $TV^{\gamma-1} = \text{constant}$

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$\therefore T_f = T_i \left( \frac{V_i}{V_f} \right)^{\gamma-1}$$

$$= T_i \left( \frac{V_i}{V_i/32} \right)^{\frac{7}{5}-1}$$

[For a diatomic gas,  $\gamma = \frac{7}{5}$ ]

$$= T_i (2^5)^{2/5} = 4T_i$$

Hence,  $a = 4$

23.

0	0	0	9
---	---	---	---

As per Wein's displacement law,

$$\frac{T_A}{T_B} = \frac{\lambda_B}{\lambda_A} = \frac{1500}{1500} = 3$$

As per Stefan's law,

$$\frac{P_A}{P_B} = \frac{A_A T_A^4}{A_B T_B^4} = \frac{r_A^2}{r_B^2} \left( \frac{T_A}{T_B} \right)^4 = \frac{(6)^2}{(18)^2} \times (3)^4 = 9$$

1. Which of the following parameters does not characterize the thermodynamic state of matter?

- (a) Temperature      (b) Pressure  
(c) Work                (d) Volume

[AIEEE 03]

2. A system goes from A to B via two processes I and II as shown in the figure. If  $\Delta U_1$  and  $\Delta U_2$  are the changes in internal energies in the processes I and II respectively, then

- (a)  $\Delta U_1 = \Delta U_2$

$$(b) \Delta U_1 > \Delta U_2$$

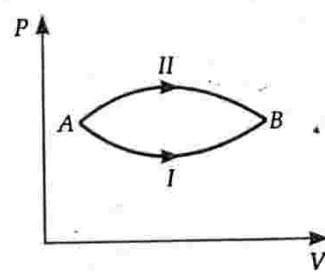
$$(c) \Delta U_1 < \Delta U_2$$

(d) relation between  $\Delta U_1$  and  $\Delta U_2$  cannot be determined. [AIEEE 05]

3. The internal energy change, when a system goes from state A and B is  $40 \text{ kJ mole}^{-1}$ . If the system goes from A to B by a reversible path and returns to state A by an irreversible path, what would be the net change in internal energy?

- (a)  $40 \text{ kJ}$       (b)  $> 40 \text{ kJ}$   
(c)  $< 40 \text{ kJ}$       (d) zero.

[AIEEE 80]



## 12.62 PHYSICS-XI

4. Which of the following is incorrect regarding the first law of thermodynamics?

- (a) It introduces the concept of the internal energy
- (b) It introduces the concept of the entropy
- (c) It is not applicable to any cyclic process
- (d) It is a restatement of the principle of conservation of energy.

[AIEEE 80]

5. When a system is taken from the initial state  $i$  to final state  $f$  along the path  $iaf$ , it is found that  $Q = 50$  cal and  $W = 20$  cal. If along the path  $ibf$ ,  $Q = 36$  cal, then  $W$  along the path  $ibf$  is

- (a) 6 cal
- (b) 16 cal
- (c) 66 cal
- (d) 14 cal

[AIEEE 07]

6. Which of the following statements is correct for any thermodynamic system?

- (a) The internal energy changes in all processes
- (b) Internal energy and entropy are state functions
- (c) The change in entropy can never be zero
- (d) The work done in an adiabatic process is always zero.

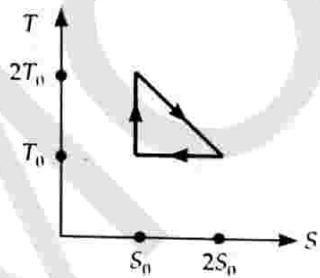
[AIEEE 04]

7. If  $c_p$  and  $c_v$  denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then

- (a)  $c_p - c_v = R/28$
- (b)  $c_p - c_v = R/14$
- (c)  $c_p - c_v = R$
- (d)  $c_p - c_v = 28R$

[AIEEE 07]

8. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is



- (a) 1/3
- (b) 1/2
- (c) 2/3
- (d) 1/4

[AIEEE 05]

9. "Heat cannot itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of

- (a) second law of thermodynamics
- (b) conservation of momentum
- (c) conservation of mass
- (d) first law of thermodynamics.

[AIEEE 03]

10. Even Carnot engine cannot give 100% efficiency, because we cannot

- (a) prevent radiation
- (b) find ideal sources
- (c) reach absolute zero temperature
- (d) eliminate friction.

[AIEEE 02]

11. Which statement is incorrect?

- (a) all reversible cycles have same efficiency.
- (b) reversible cycle has more efficiency than an irreversible one.
- (c) Carnot cycle is a reversible one.
- (d) Carnot cycle has the maximum efficiency of all the cycles.

[AIEEE 02]

12. A Carnot engine takes  $3 \times 10^6$  cal of heat from a reservoir at  $627^\circ\text{C}$  and gives it to a sink at  $27^\circ\text{C}$ . The work done by the engine is

- (a) zero
- (b)  $8.4 \times 10^6$  J
- (c)  $4.2 \times 10^6$  J
- (d)  $16.8 \times 10^6$  J

[AIEEE 03]

13. A Carnot engine, having an efficiency of  $1/10$  as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is

- (a) 1 J
- (b) 90 J
- (c) 99 J
- (d) 100 J

[AIEEE 07]

14. 100 g of water is heated from  $30^\circ\text{C}$  to  $50^\circ\text{C}$ .

Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is  $4184 \text{ J/kg/K}$ )

- (a) 4.2 kJ
- (b) 8.4 kJ
- (c) 84 kJ
- (d) 2.1 kJ

[AIEEE 2011]

15. A Carnot engine operating between temperatures  $T_1$  and  $T_2$  has efficiency  $\frac{1}{6}$ . When  $T_2$  is lowered by 62 K, its efficiency increases to  $\frac{1}{3}$ . Then  $T_1$  and  $T_2$  are, respectively

- (a) 372 K and 310 K
- (b) 372 K and 330 K
- (c) 330 K and 268 K
- (d) 310 K and 248 K

[AIEEE 2011]

16. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increased from  $V$  to  $32V$ , the efficiency of the engine is

- (a) 0.25
- (b) 0.5
- (c) 0.75
- (d) 0.99

[AIEEE 2010]

## Answers and Explanations

**1. (c)** Work is not a state variable. It is a mode of transfer of energy from system to surrounding or vice versa.

**2. (a)** Since the initial and final states are the same and the change in internal energy is path independent, therefore,  $\Delta U_1 = \Delta U_2$

**3. (d)** As the system returns to the initial state A, the change in internal energy is zero.

**4. (b)** The first law of thermodynamics does not introduce the concept of entropy.

**5. (a)** Change in internal energy is path independent,

$$\Delta U_{\text{inf}} = \Delta U_{\text{inf}}$$

$$Q - W = Q' - W'$$

$$50 - 20 = 36 - W'$$

$$W' = 36 - 30 = 6 \text{ cal.}$$

**6. (b)** Internal energy and entropy are state functions.

**7. (a)** For 1 g of a gas,

$$C_p - C_v = r = \frac{R}{M}$$

For  $N_2$ ,  $M = 28$

$$\therefore C_p - C_v = \frac{R}{28}.$$

$$8. (a) \eta = \frac{\Delta W}{Q_{RC}} = \frac{\text{Area of } \Delta ABC}{\text{Area under BC}}$$

$$= \frac{\frac{1}{2} S_0 \times T_0}{S_0 T_0 + \frac{1}{2} S_0 T_0} = \frac{\frac{1}{2} S_0 T_0}{\frac{3}{2} S_0 T_0} = \frac{1}{3}.$$

**9. (a)** It is a consequence of the second law of thermodynamics.

**10. (a)** The efficiency of a Carnot engine will be 100% when its sink is at 0 K. But the temperature 0 K cannot be realised in practice, so the efficiency is never 100%.

**11. (a)** Work done per cycle

= Area of the loop representing the cycle

As different reversible cycles may have different loop areas, their efficiencies will also be different.

**12. (b)** Refer to the solution of Example 30 on page 12.27.

**13. (b)** Coefficient of performance,

$$\beta = \frac{1-\eta}{\eta} = \frac{1-(1/10)}{1/10} = 9$$

$$\text{But } \beta = \frac{Q_2}{W}$$

$$\therefore Q_2 = \beta W = 9 \times 10 = 90 \text{ J.}$$

$$14. (b) \Delta U = mc\Delta T = \frac{100}{1000} \times 4184 \times 20 \\ = 8368 \text{ J} \approx 8.4 \text{ kJ}$$

$$15. (a) \text{Here } \eta = 1 - \frac{T_2}{T_1} = \frac{1}{6} \text{ or } \frac{T_2}{T_1} = \frac{5}{6}$$

When  $T_2$  is lowered by 62 K,

$$\frac{1}{3} = 1 - \frac{T_2 - 62}{T_1} = 1 - \frac{T_2}{T_1} + \frac{62}{T_1}$$

$$\frac{62}{T_1} = \frac{T_2}{T_1} - \frac{2}{3} = \frac{5}{6} - \frac{2}{3} = \frac{1}{6} \text{ or } T_1 = 372 \text{ K}$$

$$\text{and } T_2 = \frac{5}{6} \times T_1 = \frac{5}{6} \times 372 = 310 \text{ K}$$

**16. (c)** For adiabatic expansion,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

For a diatomic gas,  $\gamma = \frac{7}{5}$ , so

$$\frac{T_1}{T_2} = \left( \frac{V_2}{V_1} \right)^{\gamma-1} = (32)^{\frac{7}{5}-1} = (32)^{2/5} = 4$$

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{1}{4} = 0.75$$