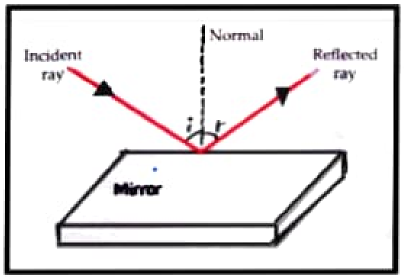
RAY OPTICS

Reflection of Light

The bouncing back of light from a reflecting surface is called as Reflection of light.

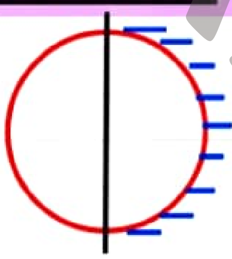
Laws of Reflection

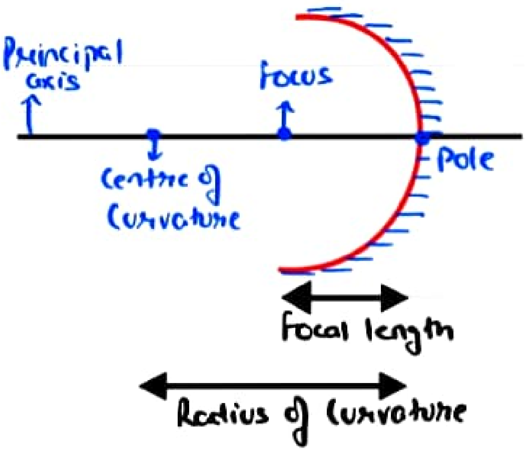


1) The incident ray, the reflected ray and normal always lie on the same point and on the same plane
2) The angle of incidence is equal to the angle of reflection [$\angle i = \angle r$]

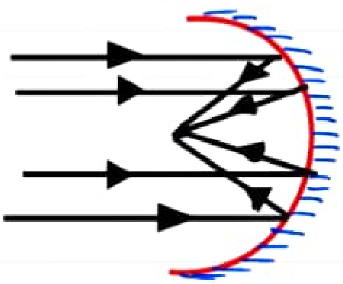
Spherical mirror

Concave mirror

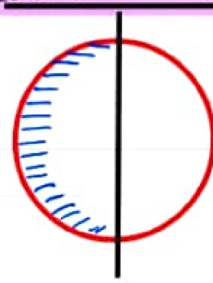
\#  Point outside

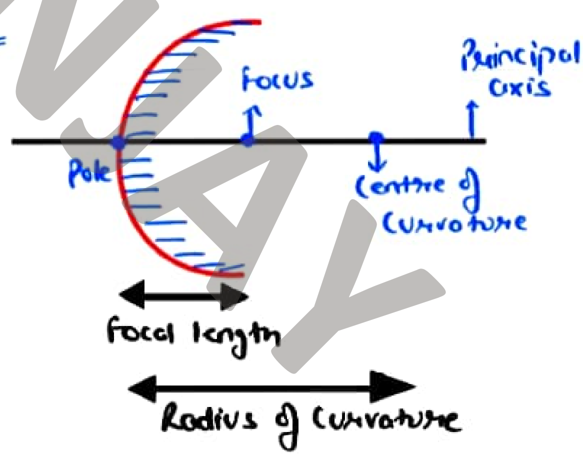
\# 

\# It is Converging mirror

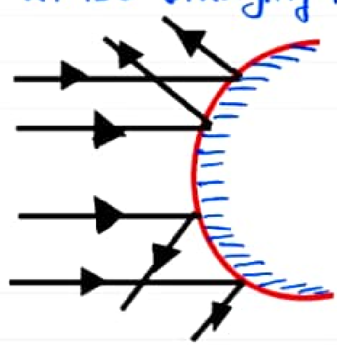


Convex Mirror

\#  Point inside

\# 

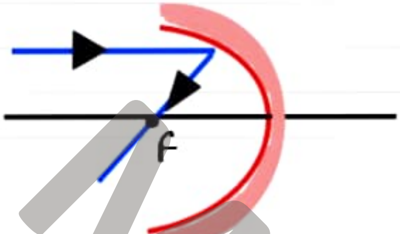
\# It is a Diverging mirror.



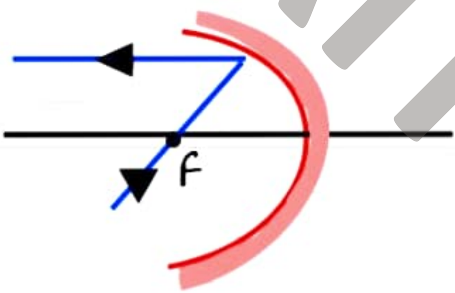
Rules for Reflection through Curved Surface

Concave mirror

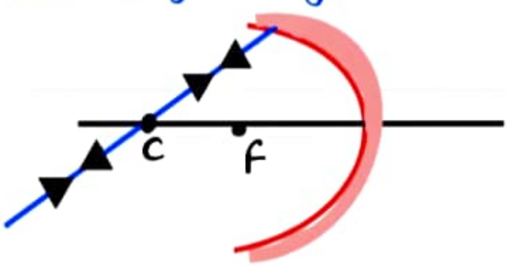
1) Any ray parallel to the principal axis will always pass through focus after reflection.



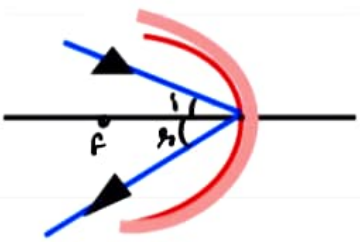
2) Any ray passing through focus becomes parallel to principal axis after reflection.



3) A ray which passes through centre of curvature will retrace its path after reflection

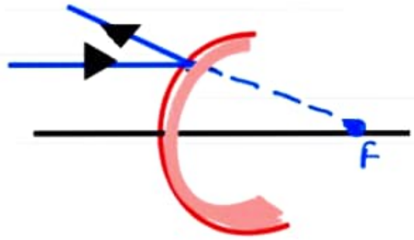


4) Angle of incidence = Angle of reflection
$\angle i = \angle r$

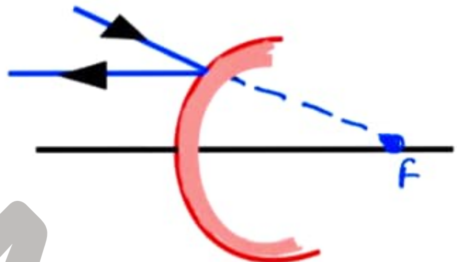


Convex mirror

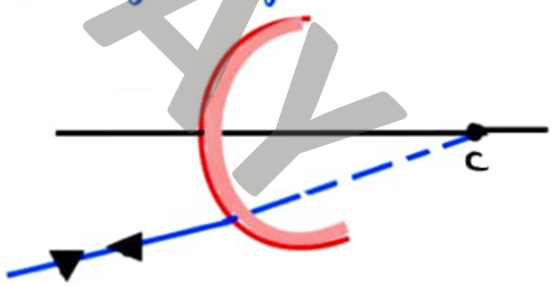
1) Any ray parallel to the principal axis will appear to coming from focus after reflection.



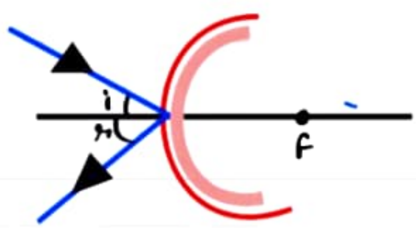
2) Any ray coming towards focus becomes parallel to principal axis after reflection



3) A ray which appear to come towards centre of curvature will retrace its path after reflection



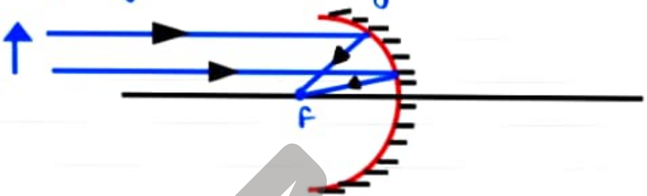
4) Angle of incidence = Angle of reflection
$\angle i = \angle r$



Mirror ray diagram

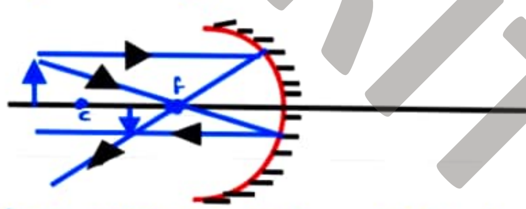
Concave mirror

1) object → At infinity



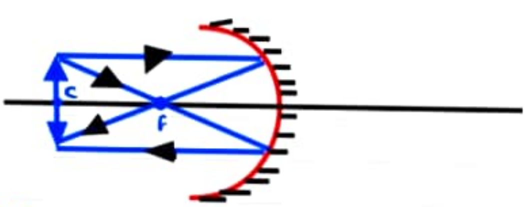
a) Image → At focus b) Diminished
c) Real d) Inverted

2) object → Beyond C



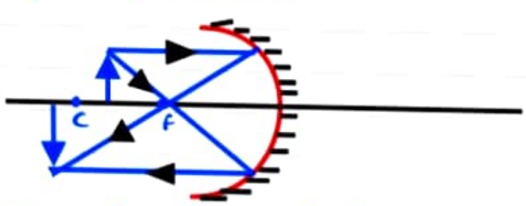
a) Image → Between C & F b) Small
c) Real d) Inverted

3) object → At C



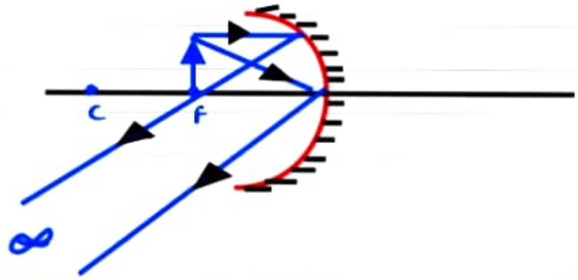
a) Image → At C b) Same size
c) Real d) Inverted

4) object → Between C & F



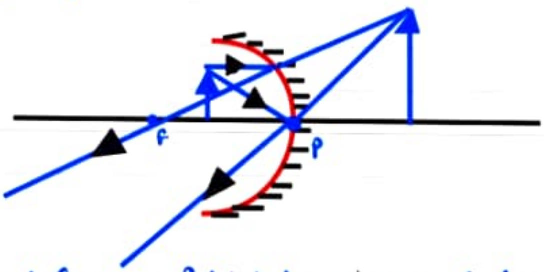
a) Image → Beyond C b) Large
c) Real d) Inverted

5) object → At F



a) Image → At infinity b) Very large
c) Real d) Inverted

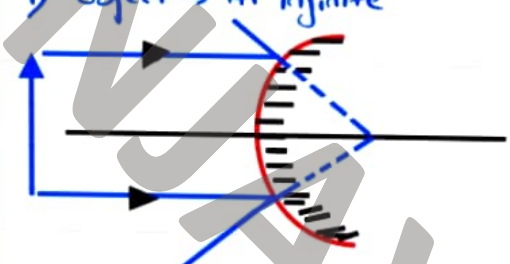
6) object → Between F & P.



a) Image → Behind the mirror b) Large
c) Virtual d) Erect.

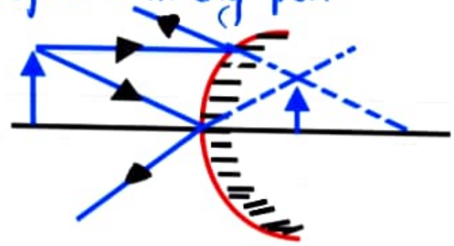
Convex mirror

1) object → At infinite



a) Image → At F b) Very small
c) Virtual d) Erect

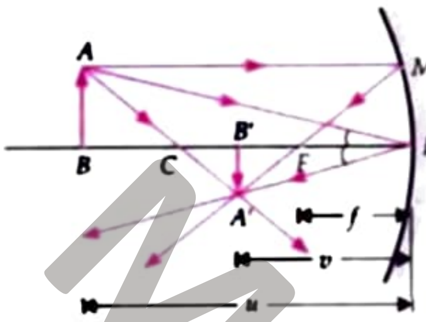
2) object → At any point



a) Image → Between F & P b) Small
c) Virtual d) Erect

Mirror formula for Concave mirror

Derivation of mirror formula for a concave mirror when it forms a real image. **Consider an object AB placed on the principal axis beyond the centre of curvature C of a concave mirror of small aperture,**



Using cartesian sign convention, we find

Object distance, $BP = -u$

Image distance, $B'P = -v$

Focal length, $FP = -f$

Radius of curvature, $CP = -R = -2f$

Now $\Delta A'B'C \sim \Delta ABC$

$$\therefore \quad \frac{A'B'}{AB} = \frac{CB'}{BC} = \frac{CP - B'P}{BP - CP} = \frac{-R + v}{-u + R} \qquad ...(1)$$

As $\angle A'PB' = \angle APB$, therefore,

$\Delta A'B'P \sim \Delta ABP$.

Consequently,

$$\frac{A'B'}{AB} = \frac{B'P}{BP} = \frac{-v}{-u} = \frac{v}{u} \qquad ...(2)$$

From equations (1) and (2), we get

$$\frac{-R + v}{-u + R} = \frac{v}{u}$$

or $$-uR + uv = -uv + vR$$

or $$vR + uR = 2uv$$

Dividing both sides by uvR, we get

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

But $$R = 2f$$

$$\therefore \quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

This proves the mirror formula for a concave mirror, when it forms a real image.

Linear magnification. ***The ratio of the height of the image to that of the object is called linear or transverse magnification or just magnification*** **and is denoted by** m.

$$m = \frac{\text{Height of image}}{\text{Height of object}} = \frac{h_2}{h_1}$$

Concave mirror. **Fig.** 9.13 **shows the ray diagram for the formation of image** $A'B'$ **of a finite object** AB **by a concave mirror.**

Now, $\Delta APB \sim \Delta A'PB'$

$$\therefore \quad \frac{A'B}{AB} = \frac{BP}{BP}$$

Applying the new cartesian sign convention, we get

$A'B' = -h_2$ (*Downward image height*)

$AB = +h_1$ (*Upward object height*)

$B'P = -v$ (*Image distance on left*)

$BP = -u$ (*Object distance on left*)

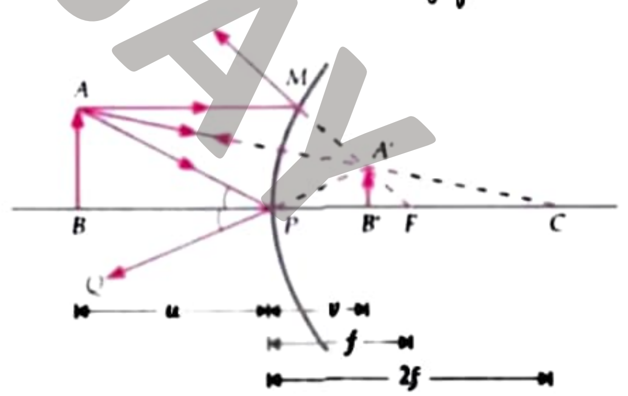
$$\therefore \quad \frac{-h_2}{h_1} = \frac{-v}{-u}$$

Magnification,

$$m = \frac{h_2}{h_1} = -\frac{v}{u}.$$

Mirror formula of Convex mirror

Consider an object AB is placed on principal axis in front of a convex mirror. as shown in the figure then.



ig. 9.15 To derive mirror formula for a convex mirror.

Using cartesian sign convention, we find

Object distance, $BP = -u$

Image distance, $PB' = +v$

Focal length, $FP = +f$

Radius of curvature, $PC = +R = +2f$

Now $\Delta A'B'C \sim \Delta ABC$

$$\therefore \quad \frac{A'B'}{AB} = \frac{B'C}{BC} = \frac{PC - PB'}{BP + PC} = \frac{R - v}{-u + R}$$

As $\angle A'PB' = \angle BPQ = \angle APB$,

Therefore, $\Delta A'B'P \sim \Delta ABP$.

Consequently,

$$\frac{A'B'}{AB} = \frac{PB'}{BP} = \frac{v}{-u}$$

From equations (1) and (2), we get

$$\frac{R - v}{-u + R} = \frac{v}{-u}$$

or $$-uR + uv = -uv + vR$$

or $$vR + uR = 2uv$$

Dividing both sides by uvR, we get

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

But $R = 2f$

$$\therefore \quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

This proves the mirror formula for a convex mirror.

Linear magnification. *The ratio of the height of the image to that of the object is called* ***linear or transverse magnification*** *or just* ***magnification*** and is denoted by m.

$$m = \frac{\text{Height of image}}{\text{Height of object}} = \frac{h_2}{h_1}$$

Convex mirror. Fig. 9.15 shows the formation of image $A'B'$ of a finite object AB by a convex mirror.

Now, $\Delta A'B'P \sim \Delta ABP$

$$\therefore \quad \frac{A'B'}{AB} = \frac{PB'}{BP}$$

Applying the new cartesian sign convention, we get

$$A'B' = +h_2, \quad AB = +h_1$$

$$PB' = +v, \quad BP = -u$$

$$\therefore \quad \frac{h_2}{h_1} = \frac{v}{-u}$$

Magnification, $$m = \frac{h_2}{h_1} = -\frac{v}{u}$$

Refraction of light

The phenomenon of the change in the path of light as it passes obliquely from one transparent medium to another is called refraction of light.

The path along which the light travels in the first medium is called *incident ray* and that in the second medium is called *refracted ray*. The angles which the incident ray and the refracted ray make with the normal at the surface of separation are called *angle of incidence* (i) and *angle of refraction* (r) respectively.

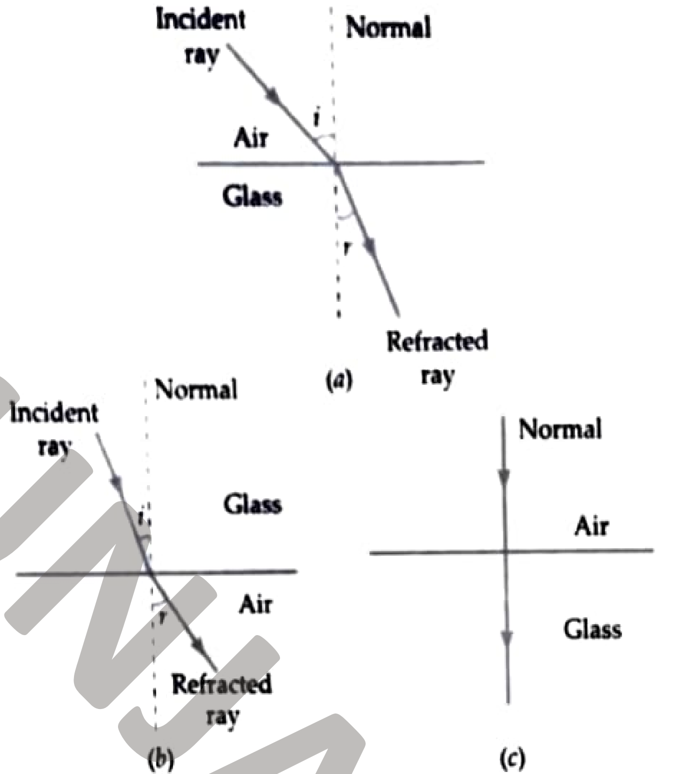


Fig. 15.24 Refraction of light (*a*) from rarer to denser medium (*b*) from denser to rarer medium (*c*) no refraction for normal incidence.

It is observed that

1. When a ray of light passes from an optically rarer medium to a denser medium, it bends towards the normal ($\angle r < \angle i$), as shown in Fig. 15.24(*a*).
2. When a ray of light passes from an optically denser to a rarer medium, it bends away from the normal ($\angle r > \angle i$), as shown in Fig. 15.24(*b*).
3. A ray of light travelling along the normal passes undeflected, as shown in Fig. 15.24(*c*). Here $\angle i = \angle r = 0°$.

Law of Reflection

1) The incident Ray, reflected Ray and normal all lie in the Same plane.
2) The ratio of sine of angle of incidence to the Sine of angle of refraction is always a constant for a pair of media.

$$\frac{\sin i}{\sin r} = {}^1\mu_2 \text{ (constant)}$$

where ${}^1\mu_2$ is called the refractive index of the Second medium with respect to the first medium.

Refractive index

Refractive index in terms of speed of light. The refractive index of a medium may be defined in terms of the speed of light as follows :

The refractive index of a medium for a light of given wavelength may be defined as the ratio of the speed of light in vacuum to its speed in that medium.

$$\text{Refractive index} = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$$

or $$\mu = \frac{c}{v}$$

Refractive index of a medium with respect to vacuum is also called ***absolute refractive index***.

Refractive index in terms of wavelength. Since the frequency (ν) remains unchanged when light passes from one medium to another, therefore,

$$\mu = \frac{c}{v} = \frac{\lambda_{vac} \times \nu}{\lambda_{med} \times \nu} = \frac{\lambda_{vac}}{\lambda_{med}}$$

The refractive index of a medium may be defined as the ratio of wavelength of light in vacuum to its wavelength in that medium.

Relative refractive index. The relative refractive index of medium 2 with respect to medium 1 is defined as the ratio of speed of light (v_1) in medium 1 to the speed of light (v_2) in medium 2 and is denoted by ${}^1\mu_2$.

Thus $${}^1\mu_2 = \frac{v_1}{v_2}$$

As refractive index is the ratio of two similar physical quantities, so it has no units and dimensions.

Total internal reflection

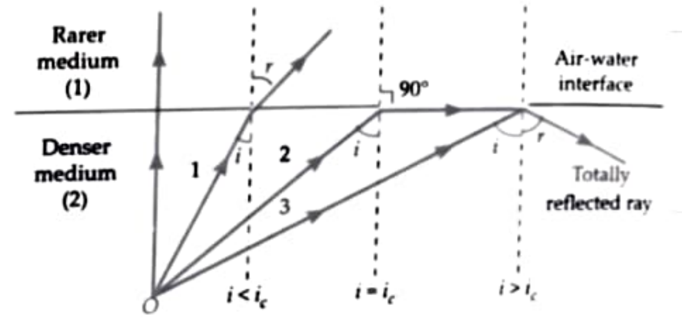


Fig. 15.34 Total internal reflection.

Critical angle :-

The angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90° is called critical angle of the denser medium and is denoted by i_c.

Total internal reflection :-

The phenomenon in which a ray of light travelling at an angle of incidence greater than the critical angle from denser to a rarer medium is totally reflected back into the denser medium is called total internal reflection.

Necessary conditions for total internal reflection :

1. *Light must travel from an optically denser to an optically rarer medium.*
2. *The angle of incidence in the denser medium must be greater than the critical angle for the two media.*

Relation between critical angle and refractive index. From Snell's law,

$$\frac{\sin i}{\sin r} = {}^2\mu_1 = \frac{1}{{}^1\mu_2}$$

When $i = i_c$, $r = 90°$. Therefore,

$$\frac{\sin i_c}{\sin 90°} = \frac{1}{{}^1\mu_2} \quad \text{or} \quad {}^1\mu_2 = \frac{1}{\sin i_c}$$

If the rarer medium is air, then $\mu_1 = 1$ and $\mu_2 = \mu$ (say) and we get

$$\mu = \frac{1}{\sin i_c}$$

Thus the refractive index of any medium is equal to the reciprocal of the sine of its critical angle.

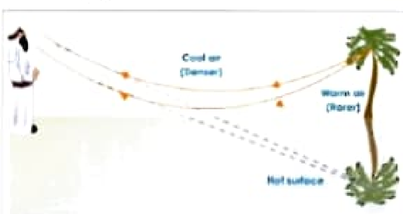
Application of TIR

1. Sparkling of diamond. The brilliancy of diamonds is due to total internal reflection. As the refractive index of diamond is very large, its critical angle is very small, about 24.4°. The faces of diamond are so cut that the light entering the crystal suffers total internal reflections repeatedly, and hence gets collected inside but it comes out through only a few faces. Hence the diamond sparkles when seen in the direction of emerging light.

2) TIR in mirage

What is a mirage?

Mirage happens due to total internal reflection of light. The figure below illustrates the mechanism of a mirage being formed in a desert. On a hot sunny day, the air near the surface is hotter and rarer than the air above it. So when light from a distant object goes towards earth, it is actually going from a denser to rarer medium. So it gets reflected back if the angle of incidence is too steep and we happen to observe mirage.



Totally reflecting Prism

Totally reflecting prism. *A right-angled isosceles prism, i.e., a 45°-90°-45° prism is called a totally reflecting prism.* Whenever a ray falls normally on any face of such a prism, it is incident on the inside face at 45°, that is at an angle greater than the critical angle of glass (about 42°); hence this ray is always totally internally reflected.

These prisms may be used in *three* ways :

(*i*) To deviate a ray through 90°. As shown in Fig. 15.36(*a*), as the light is incident normally on one of the faces containing right angle, it enters the prism without deviation. It is incident on the hypotenuse face at an angle of 45°, greater than the critical angle. The light is totally internally reflected. Having been deviated through 90°, the light passes through third face without any further deviation. Such prisms are used in *periscopes*.

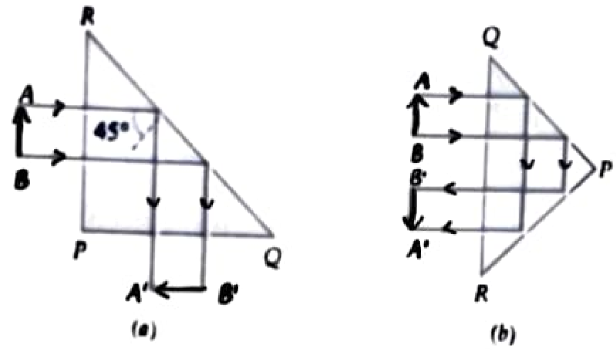


Fig. 15.36 (*a*) To deviate a ray through 90°
(*b*) To invert an image with deviation of rays through 180°.

(*ii*) To invert an image with deviation of rays through 180°. As shown in Fig. 15.36(*b*), the light is incident normally on the hypotenuse face, it first suffers total internal reflection from one shorter face and then from the other shorter face. The final beam emerges through the hypotenuse face, parallel to the incident beam. The deviation is 180°. Such a prism is called a *porroprism*.

(*iii*) To invert an image without deviation of rays. (Erecting prism). As shown in Fig. 15.37, the light enters at one shorter face at an angle. After refraction, it is totally reflected from the hypotenuse face and then refracted out of the other shorter face to become parallel to the incident beam. The rays do not suffer any deviation, only their order is reversed. The incident ray, which is on the top, emerges from the bottom of the prism. Such prisms are called *erecting prisms* and are used in binoculars and in projection lanterns.

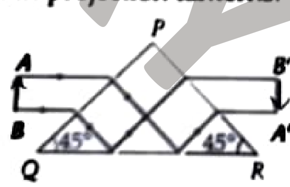


Fig. 15.37 To invert an image without deviation of rays.

Important formulas

1) ${}^1\mu_2 = \frac{\mu_2}{\mu_1}$

2) $\frac{\sin i}{\sin r} = {}^1\mu_2 = \frac{\mu_2}{\mu_1}$

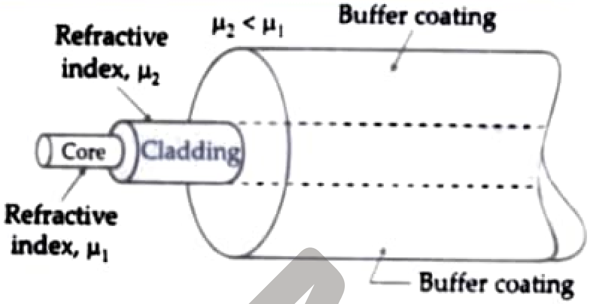
3) ${}^1\mu_2 = \frac{1}{{}^2\mu_1}$

4) ${}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w}$

5) The refractive index is always of the medium in which light going

Optical fibre

An optical fibre is a hair-thin long strand of quality glass or quartz surrounded by a glass coating of slightly lower reflective index. It is used as a guided medium for transmitting an optical signal from one place to another.



Construction. An optical fibre consists of *three* main parts :

(*i*) **Core.** The central cylindrical core is made of high quality glass/silica/plastic of refractive index μ_1 and has a diameter about 10 to 100 μm.

(*ii*) **Cladding.** The core is surrounded by a glass / plastic jacket of refractive index $\mu_2 < \mu_1$.

(*iii*) **Buffer coating.** For providing safety and strength, the core cladding of optical fibres is enclosed in a plastic jacket.

Propagation of light through an optical fibre. As shown in Fig. 15.39(*a*), when light is incident on one end of the fibre at a small angle, it goes inside and suffers repeated total internal reflections because the angle of incidence is greater than the critical angle of the fibre material with respect to its outer coating. As there is no loss of intensity in total internal reflection, the outcoming beam is of as much intensity as the incident beam. Even if the fibre is bent, light easily travels through along the fibre.

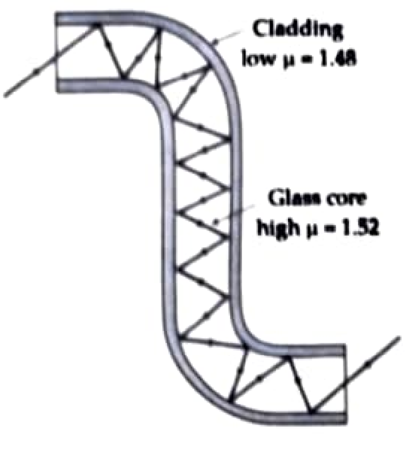


Fig. 15.39 **(a) Propagation of light through an optical fibre.**

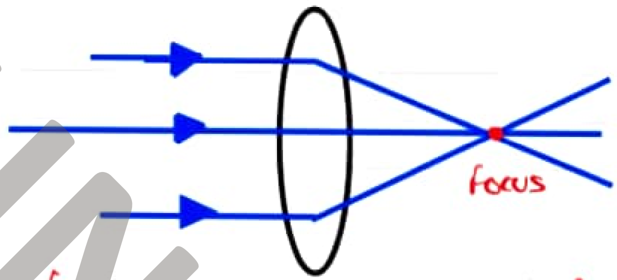
Applications of optical fibres. Some of the important applications are as follows :

1. *As a light pipe, optical fibres are used in medical and optical examination.* A light pipe is inserted into the stomach through the mouth. Light transmitted through the outer layers of the light pipe is scattered by the various parts of stomach into the central portion of the light pipe to produce a final image with excellent details. The technique is called endoscopy.

2. *They are used in transmitting and receiving electrical signals in telecommunication.* The electrical signals are first converted to light by suitable transducers. Each fibre can transmit about 2000 telephone conversations without much loss of intensity.

Lenses

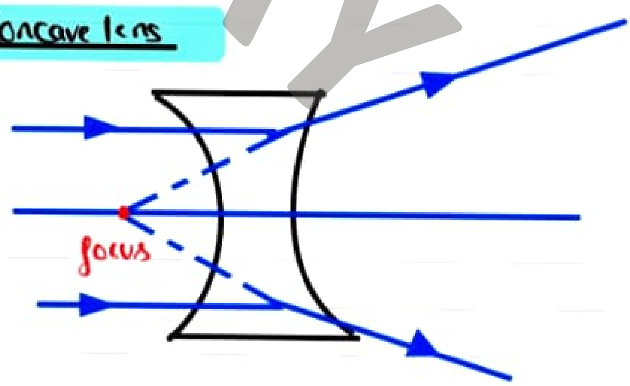
Convex lens



→ focal length of Convex lens is Positive
$f = +ve$

→ Convex lens converges the light Rays hence it behaves as Converging lens

Concave lens



→ focal length of a Concave lens is negative
$f = -ve$

→ Concave lens Diverges the light rays hence it behaves as Diverging lens.

RULES FOR DRAWING IMAGES FORMED BY SPHERICAL LENSES

Rules for drawing images formed by spherical lenses. The position of the image formed by any spherical lens can be found by considering any two of the following rays of light coming from a point on the object.

(*i*) A ray from the object parallel to the principal axis after refraction passes through the second principal focus F_2 [in a convex lens, as shown in Fig. 15.75(*a*)] or appears to diverge [in a concave lens, as shown in Fig. 15.75(*b*)] from the first principal focus F_1.

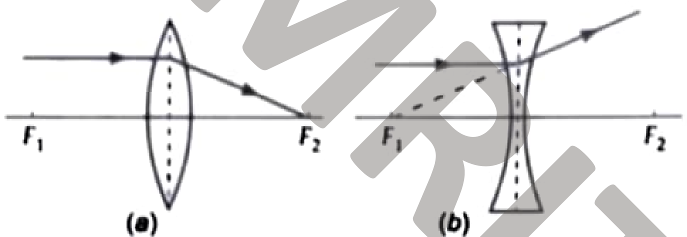


Fig. 15.75 **Path of ray incident parallel to the principal axis of (*a*) convex lens (*b*) concave lens.**

(*ii*) A ray of light passing through the first principal focus [in a convex lens, as shown in Fig. 15.76(*a*)] or appearing to meet at it [in a concave lens, as shown in Fig. 15.76(*b*)] emerges parallel to the principal axis after refraction.

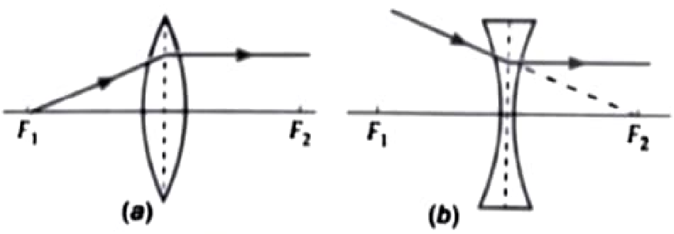


Fig. 15.76 **Path of a ray passing through focus of (*a*) convex lens (*b*) concave lens.**

(*iii*) A ray of light, passing through the optical centre of the lens, emerges without any deviation after refraction, as shown in Figs. 15.77(*a*) and (*b*).

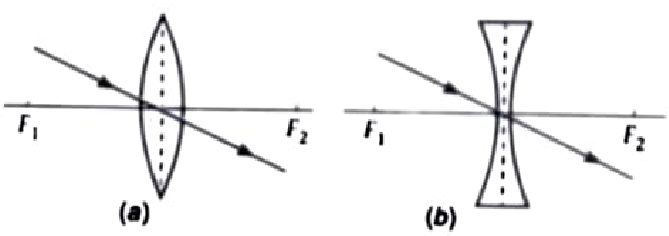
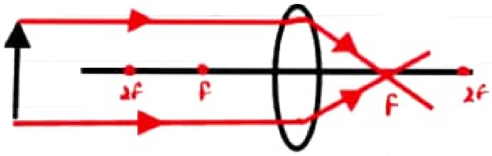


Fig. 15.77 **Path of a ray passing through the optical centre (*a*) convex lens (*b*) concave lens.**

Ray diagram for Convex lens

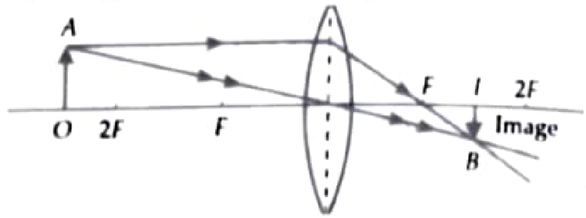
1) object = At infinity



(i) Image = At focus (ii) Diminished
(iii) Inverted (iv) Real

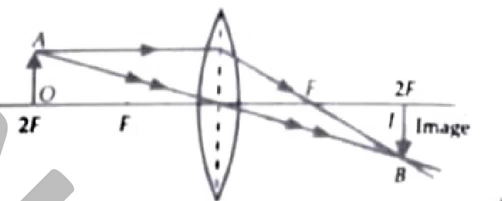
2) Object beyond 2*F*. The image is

(*i*) between *F* and 2*F* (*ii*) real
(*iii*) inverted (*iv*) smaller



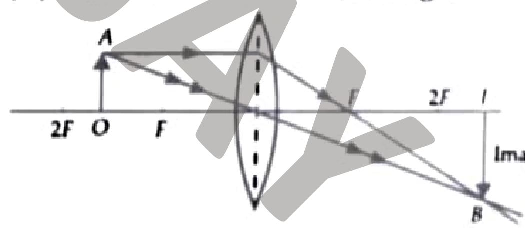
3) Object at 2*F*. The image is

(*i*) at 2*F* (*ii*) real
(*iii*) inverted (*iv*) same size



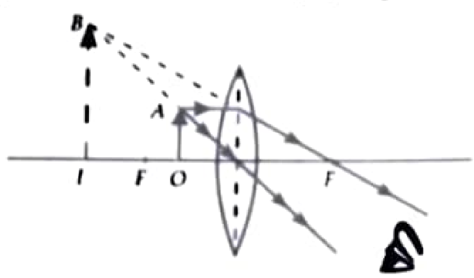
4) Object between 2*F* and *F*. The image is

(*i*) beyond 2*F* (*ii*) real
(*iii*) inverted (*iv*) larger



5) Object between *F* and *O*. The image is

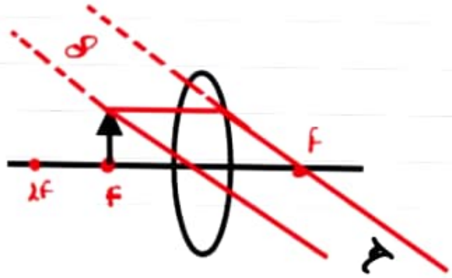
(*i*) behind object (*ii*) virtual
(*iii*) erect (*iv*) larger



Object in any position. The image is

(*i*) in front of object (*ii*) virtual
(*iii*) erect (*iv*) smaller

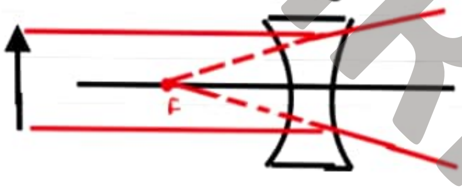
6) Object = At focus



i) Image = At infinity (ii) Virtual
(iii) Large (iv) Erect

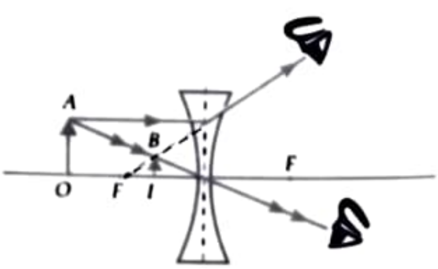
Ray diagram for Concave lens

i) Object = infinity



i) Image = At focus ii) Virtual
iii) Diminished iv) Erect

ii) Object in any position. The image is
(i) in front of object (ii) virtual
(iii) erect (iv) smaller



Thin lens formula

Thin lens formula. Thin lens formula is a mathematical relation between the object distance u, image distance v and focal length f of a spherical lens. This relation is :

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

In words, we can say that

$$\frac{1}{\text{Image distance}}-\frac{1}{\text{Object distance}}=\frac{1}{\text{Focal length}}$$

This formula is valid for both convex and concave lenses for both real and virtual images.

Thin lens formula for Convex lens

Derivation of thin lens formula for a convex lens when it forms a real image. As shown in Fig. 15.79, consider an object AB placed perpendicular to the principal axis of a thin convex lens between its F' and C'. A real, inverted and magnified image $A'B'$ is formed beyond C on the other side of the lens.

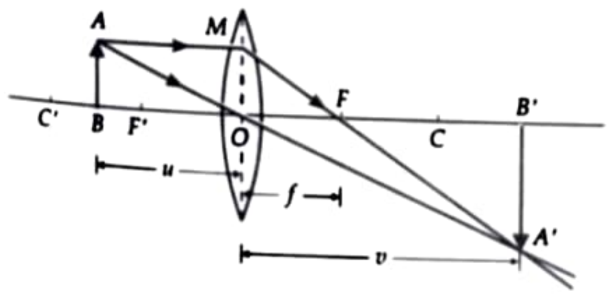


Fig. 15.79 Real image formed by a convex lens.

$\Delta A'B'O$ and ΔABO are similar,

$$\therefore \quad \frac{A'B'}{AB}=\frac{OB'}{BO} \qquad ...(1)$$

Also $\Delta A'B'F$ and ΔMOF are similar,

$$\therefore \quad \frac{A'B'}{MO}=\frac{FB'}{OF}$$

But $MO = AB$,

$$\therefore \quad \frac{A'B'}{AB}=\frac{FB'}{OF} \qquad ...(2)$$

From (1) and (2), we get

$$\frac{OB'}{BO}=\frac{FB'}{OF}=\frac{OB'-OF}{OF}$$

Using new Cartesian sign convention, we get

Object distance, $BO = -u$
Image distance, $OB' = +v$
Focal length, $OF = +f$

$$\therefore \quad \frac{v}{-u}=\frac{v-f}{f}$$

or $\quad vf = -uv + uf \quad$ or $\quad uv = uf - vf$

Dividing both sides by uvf, we get

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

This proves the lens formula for a convex lens when it forms a real image.

Linear magnification. *The linear magnification produced by a lens is defined as the ratio of the size of the image formed by the lens to the size of the object.* It is denoted by m. Thus

$$m = \frac{\text{Size of image}}{\text{Size of object}} = \frac{h_2}{h_1}$$

Convex lens. Earlier Fig. 15.79 shows a ray diagram for the formation of image $A'B'$ of a finite object AB by a convex lens.

Now $\Delta AOB \sim \Delta A'OB'$

$$\therefore \quad \frac{A'B'}{AB} = \frac{OB'}{OB}$$

Applying the new cartesian sign convention, we get

$A'B' = -h_2$ (*Downward image height*)

$AB = +h_1$ (*Upward object height*)

$OB = -u$ (*Image distance on left*)

$OB' = +v$ (*Image distance on right*)

$$\therefore \quad \frac{-h_2}{+h_1} = \frac{+v}{-u} \quad \text{or} \quad \frac{h_2}{h_1} = \frac{v}{u}$$

$$\therefore \text{ Magnification, } m = \frac{h_2}{h_1} = \frac{v}{u}.$$

Thin lens formula for Concave lens

Derivation of thin lens formula for a concave lens. As shown in Fig. 15.81, suppose O be the optical centre and F be the principal focus of concave lens of focal length f. AB is an object placed perpendicular to its principal axis. A virtual, erect and diminished image $A'B'$ is formed due to refraction through the lens.

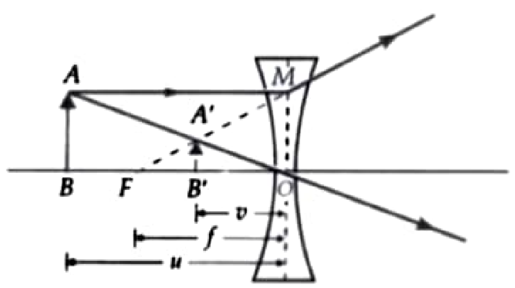


Fig. 15.81 **Virtual image formed by a concave lens.**

As $\Delta A'B'O \sim \Delta ABO$

$$\therefore \quad \frac{A'B'}{AB} = \frac{B'O}{BO} \qquad \ldots(1)$$

Also, $\Delta A'B'F \sim \Delta MOF$

$$\therefore \quad \frac{A'B'}{MO} = \frac{FB'}{FO}$$

But $MO = AB$, therefore

$$\frac{A'B'}{AB} = \frac{FB'}{FO} \qquad \ldots(2)$$

From (1) and (2), we get

$$\frac{B'O}{BO} = \frac{FB'}{FO} = \frac{FO - B'O}{FO}$$

Using new Cartesian sign convention, we get

$$BO = -u, \quad B'O = -v, \quad FO = -f$$

$$\therefore \quad \frac{-v}{-u} = \frac{-f + v}{-f}$$

or $\quad vf = uf - uv \quad$ or $\quad uv = uf - vf$

Dividing both sides by uvf, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

This proves the thin lens formula for a concave lens.

Linear magnification. *The linear magnification produced by a lens is defined as the ratio of the size of the image formed by the lens to the size of the object.* It is denoted by m. Thus

$$m = \frac{\text{Size of image}}{\text{Size of object}} = \frac{h_2}{h_1}$$

Concave lens. Fig. 15.81 shows the formation of a virtual image $A'B'$ of a finite object AB by a concave lens.

Now $\Delta AOB \sim \Delta A'OB'$

$$\therefore \quad \frac{A'B'}{AB} = \frac{OB'}{OB}$$

Applying the new cartesian sign convention, we get

$A'B' = +h_2, \quad AB = +h_1$

$OB' = -v, \quad OB = -u$

$$\therefore \quad \frac{+h_2}{+h_1} = \frac{-v}{-u}$$

$$\therefore \text{ Magnification, } m = \frac{h_2}{h_1} = \frac{v}{u}.$$

Power of lens

The power of lens is defined as inverse of focal length.

$$\boxed{\text{Power} = \frac{1}{\text{focal length}}}$$

SI unit = Dioptre or m^{-1}

Combination of lenses

Equivalent lens. *A single lens which forms the image of an object at the same position as is formed by a combination of lenses is called an equivalent lens.*

Equivalent focal length and power of two thin lenses in contact. As shown in Fig. 15.100, let L_1 and L_2 be two thin lenses of focal length f_1 and f_2 respectively, placed coaxially in contact with one another. Let O be a point object on the principal axis of the lens system.

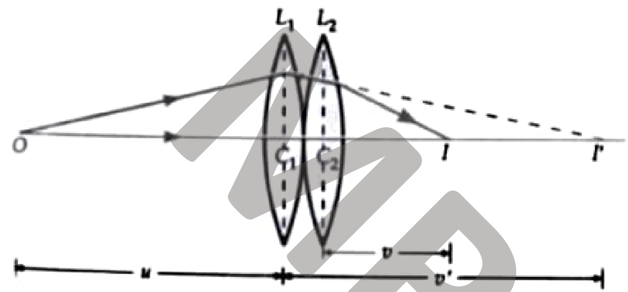


Fig. 15.100 **Two thin lenses in contact.**

Let $OC_1 = u$. In the absence of second lens L_2, the first lens L_1 will form a real image I' of O at distance $C_1I' = v'$. Using thin lens formula,

$$\frac{1}{f_1} = \frac{1}{v'} - \frac{1}{u} \qquad ...(1)$$

The image I' acts as a virtual object ($u = v'$) for the second lens L_2 which finally forms its real image I at distance v. Thus

$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v'} \qquad ...(2)$$

Adding equations (1) and (2), we get

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u} \qquad ...(3)$$

For the combination of thin lenses in contact, if f is the equivalent focal length, then

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad ...(4)$$

From equations (3) and (4), we find that

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$\therefore$ Equivalent power,

$$P = P_1 + P_2$$

For n thin lenses in contact, we have

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \dots + \frac{1}{f_n}$$

$\therefore$ Equivalent power,

$$P = P_1 + P_2 + P_3 + \dots + P_n$$

other optical instruments.

Total magnification. When lenses are used in combination, each lens magnifies the image formed by the preceding lens. Hence the total magnification m is equal to the product of the magnifications m_1, m_2 and m_3 ......, produced by the individual lenses.

$$m = m_1 \times m_2 \times m_3 \times \dots$$

Refraction through Convex Surface

(*i*) **The object lies in rarer medium and the image formed is real.** In Fig. 15.55, *APB* is a convex refracting surface which separates a rarer medium of refractive index μ_1 from a denser medium of refractive index μ_2. Let P be the pole, C be the centre of curvature and $R = PC$ be the radius of curvature of this surface.

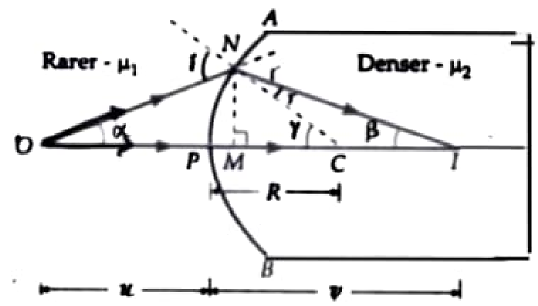


Fig. 15.55 Refraction from rarer to denser medium, when the image is real.

Draw NM perpendicular to the principal axis. Let α, β and γ be the angles, as shown in Fig. 15.55.

In ΔNOC, i is an exterior angle, therefore,

$$i = \alpha + \gamma$$

Similarly, from $\Delta\, NIC$, we have

$$\gamma = r + \beta$$

or $$r = \gamma - \beta$$

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

$$\therefore \quad \alpha \simeq \tan\alpha = \frac{NM}{OM} \simeq \frac{NM}{OP} \qquad [\because \ P \text{ is close to } M]$$

$$\beta \simeq \tan\beta = \frac{NM}{MI} \simeq \frac{NM}{PI}$$

and $$\gamma \simeq \tan\gamma = \frac{NM}{MC} \simeq \frac{NM}{PC}$$

From Snell's law of refraction,

$$\mu_1 \sin i = \mu_2 \sin r$$

As i and r are small, so

$$\sin i \simeq i \quad \text{and} \quad \sin r \simeq r$$

$$\therefore \qquad \mu_1 i = \mu_2 r$$

or $$\mu_1 [\alpha + \gamma] = \mu_2 [\gamma - \beta]$$

or $$\mu_1\left[\frac{NM}{OP}+\frac{NM}{PC}\right]=\mu_2\left[\frac{NM}{PC}-\frac{NM}{PI}\right]$$

or $$\mu_1\left[\frac{1}{OP}+\frac{1}{PC}\right]=\mu_2\left[\frac{1}{PC}-\frac{1}{PI}\right]$$

or $$\frac{\mu_1}{OP}+\frac{\mu_2}{PI}=\frac{\mu_2-\mu_1}{PC}$$

Using new Cartesian sign convention, we find

Object distance, $OP=-u$

Image distance, $PI=+v$

Radius of curvature, $PC=+R$

$$\therefore \quad \frac{\mu_1}{-u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$$

or $$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

NOTE If first medium is air, then $\mu_1=1$ and $\mu_2=\mu$, we have

$$\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{R}.$$

Case 2: when object lie in denser medium

(iii) **The object lies in the denser medium and the image formed is real.** Fig. 15.57 shows a convex refracting surface which is convex towards the rarer medium. The point object O lies in the denser medium. The two refracted rays meet at point I. So I is the real image of the point object O.

From ΔNOC, $\gamma=i+\alpha$ or $i=\gamma-\alpha$

From ΔNIC, $r=\beta+\gamma$

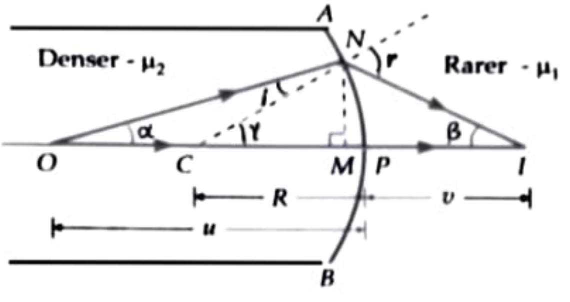


Fig. 15.57 Refraction from denser to rarer medium when the image is real.

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

$$\therefore \quad \alpha \simeq \tan\alpha=\frac{NM}{OM}\simeq\frac{NP}{OP} \qquad [\because M \text{ is close to } P]$$

$$\beta\simeq\tan\beta=\frac{NM}{MI}\simeq\frac{NM}{PI}$$

and $$\gamma\simeq\tan\gamma=\frac{NM}{CM}\simeq\frac{NM}{CP}$$

From Snell's law of refraction, for refraction from denser to rarer medium, we have

$$\mu_2 \sin i=\mu_1 \sin r$$

As i and r are small angles, so

$$\sin i \simeq i \quad \text{and} \quad \sin r \simeq r$$

$$\therefore \quad \mu_2 i=\mu_1 r$$

or $$\mu_2(\gamma-\alpha)=\mu_1(\beta+\gamma)$$

or $$\mu_2\left[\frac{NM}{CP}-\frac{NM}{OP}\right]=\mu_1\left[\frac{NM}{PI}+\frac{NM}{CP}\right]$$

or $$\mu_2\left[\frac{1}{CP}-\frac{1}{OP}\right]=\mu_1\left[\frac{1}{PI}+\frac{1}{CP}\right]$$

or $$-\frac{\mu_1}{PI}-\frac{\mu_2}{OP}=\frac{\mu_1-\mu_2}{CP}$$

Using the new Cartesian sign convention, we have

Object distance, $OP=-u$

Image distance, $PI=+v$

Radius of curvature, $CP=-R$

$$\therefore \quad -\frac{\mu_1}{v}-\frac{\mu_2}{-u}=\frac{\mu_1-\mu_2}{-R}$$

$$\frac{\mu_1}{v}-\frac{\mu_2}{u}=\frac{\mu_1-\mu_2}{R}$$

Refraction through Concave Surface

Refraction at a concave spherical surface

(*i*) **The object lies in the rarer medium.** In Fig. 15.59, *APB* is a concave refracting surface separating two media of refractive indices μ_1 and μ_2.

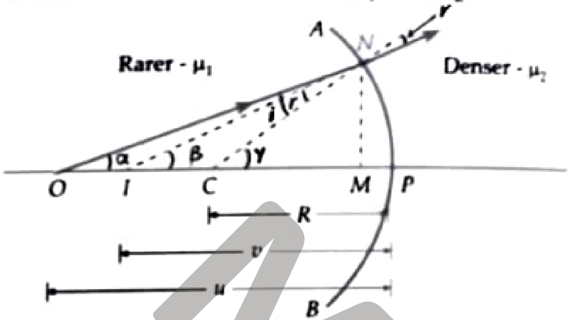


Fig. 15.59 Refraction at a concave surface when the object lies in the rarer medium.

Let

P = Pole of the concave surface APB

C = Centre of curvature of the concave surface

O = Point object placed on the principal axis

I = Virtual image of point object O

In ΔNOC, γ is an exterior angle, therefore

$$\gamma = \alpha + i \quad \text{or} \quad i = \gamma - \alpha$$

Similarly, from $\Delta\, NIC$, we have

$$\gamma = \beta + r \quad \text{or} \quad r = \gamma - \beta$$

Suppose all the rays are *paraxial*. Then the angles *i*, *r*, α, β and γ will be small.

$$\therefore \quad \alpha \simeq \tan\alpha = \frac{NM}{OM} \simeq \frac{NM}{OP} \qquad [\because M \text{ is close to } P]$$

$$\beta \simeq \tan\beta = \frac{NM}{IM} \simeq \frac{NM}{IP}$$

$$\gamma \simeq \tan\gamma = \frac{NM}{CM} \simeq \frac{NM}{CP}$$

From Snell's law of refraction,

$$\mu_1 \sin i = \mu_2 \sin r$$

As *i* and *i* are small angles, so

$$\sin i \simeq i \quad \text{and} \quad \sin r \simeq r$$

$$\therefore \quad \mu_1 i = \mu_2 r$$

$$\mu_1[\gamma - \alpha] = \mu_2[\gamma - \beta]$$

or $$\mu_1\left[\frac{NM}{CP} - \frac{NM}{OP}\right] = \mu_2\left[\frac{NM}{CP} - \frac{NM}{IP}\right]$$

or $$\mu_1\left[\frac{1}{CP} - \frac{1}{OP}\right] = \mu_2\left[\frac{1}{CP} - \frac{1}{IP}\right]$$

or $$-\frac{\mu_1}{OP} + \frac{\mu_2}{IP} = \frac{\mu_2 - \mu_1}{CP}$$

Using new Cartesian sign convention, we find

Object distance, $\quad OP = -u$

Image distance, $\quad IP = -v$

Radius of curvature, $\quad CP = -R$

$$\therefore \quad \frac{-\mu_1}{-u} + \frac{\mu_2}{-v} = \frac{\mu_2 - \mu_1}{-R}$$

or $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

(*ii*) **The object lies in the denser medium.** As shown in Fig. 15.60, when the point object *O* is placed in the denser medium, the refracted rays appear to diverge from a point *I* in the denser medium. So *I* is the virtual image of the point object *O*.

From $\Delta\, NOC$, $\quad i = \alpha + \gamma$

From $\Delta\, NIC$, $\quad r = \beta + \gamma$

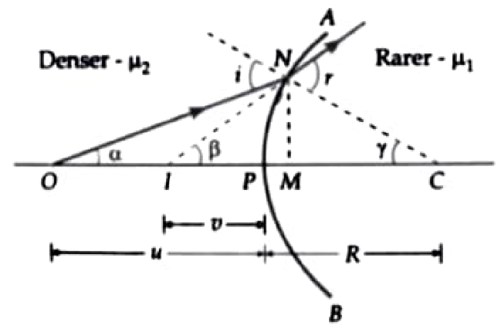


Fig. 15.60 Refraction at a concave surface when the object lies in the denser medium.

Suppose all the rays are *paraxial*. Then the angles *i*, *r*, α, β and γ will be small.

$$\therefore \quad \alpha \simeq \tan\alpha = \frac{NM}{OM} \simeq \frac{NM}{OP} \qquad [\because M \text{ is close to } P]$$

$$\beta \simeq \tan\beta = \frac{NM}{IM} \simeq \frac{NM}{IP}$$

$$\gamma \simeq \tan\gamma = \frac{NM}{MC} \simeq \frac{NM}{PC}$$

From Snell's law of refraction, for refraction from denser to rarer medium, we have

$$\mu_2 \sin i = \mu_1 \sin r$$

As *i* and *r* are small angles, so

$$\sin i \simeq i \quad \text{and} \quad \sin r \simeq r$$

$$\therefore \quad \mu_2 i = \mu_1 r$$

or $$\mu_2[\alpha + \gamma] = \mu_1[\beta + \gamma]$$

or $$\mu_2\left[\frac{NM}{OP} + \frac{NM}{PC}\right] = \mu_1\left[\frac{NM}{IP} + \frac{NM}{PC}\right]$$

or $$\mu_2\left[\frac{1}{OP} + \frac{1}{PC}\right] = \mu_1\left[\frac{1}{IP} + \frac{1}{PC}\right]$$

or $$-\frac{\mu_1}{IP} + \frac{\mu_2}{OP} = \frac{\mu_1 - \mu_2}{PC}$$

Using new Cartesian sign convention, we find

Object distance, $OP = -u$

Image distance, $IP = -v$

Radius of curvature, $PC = +R$

$$\therefore \quad \frac{-\mu_1}{-v} + \frac{\mu_2}{-u} = \frac{\mu_1 - \mu_2}{R}$$

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

Len's maker formula

Lens maker's formula for a double convex lens. As shown in Fig. 15.70, consider a thin double convex lens of refractive index μ_2 placed in a medium of refractive index μ_1. Here $\mu_1 < \mu_2$. Let B and D be the poles, C_1 and C_2 be the centres of curvature, and R_1 and R_2 be the radii of curvature of the two lens surfaces ABC and ADC, respectively.

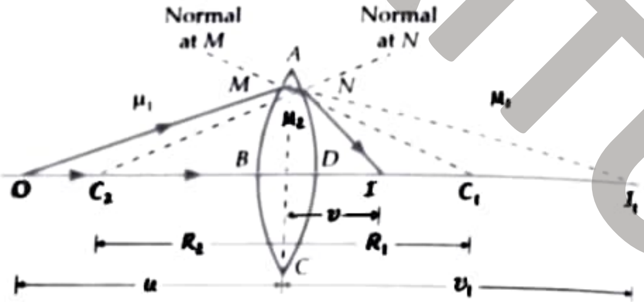


Fig. 15.70 Refraction through a double convex lens.

Suppose a point object O is placed on the principal axis in the rarer medium of refractive index μ_1. The ray OM is incident on the first surface ABC. It is refracted along MN, bending towards the normal at this surface. If the second surface ADC were absent, the ray MN would have met the principal axis at I_1. So we can treat I_1 as the real image formed by first surface ABC in the medium of refractive index μ_2.

For refraction at surface ABC, we can write the relation between the object distance u, image distance v_1 and radius of curvature R_1 as

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \qquad \text{...(1)}$$

For refraction at second surface, I_1 acts as a virtual object placed in the medium of refractive index μ_2 and I is the real image formed in the medium of refractive index μ_1. Therefore, the relation between the object distance v_1, image distance v and radius of curvature R_2 can be written as

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2} \qquad \text{...(2)}$$

Adding equations (1) and (2), we get

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or

$$\frac{1}{v} - \frac{1}{u} = \left[\frac{\mu_2 - \mu_1}{\mu_1}\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad \text{...(3)}$$

If the object is placed at infinity ($u = \infty$), the image will be formed at the focus, *i.e.*, $v = f$. Therefore,

$$\frac{1}{f} = \left[\frac{\mu_2 - \mu_1}{\mu_1}\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad \text{...(4)}$$

This is ***lens maker's formula***.

When the lens is placed in *air*, $\mu_1 = 1$, and $\mu_2 = \mu$. The lens maker's formula takes the form :

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

From equations (3) and (4), we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

This is the *thin lens formula* which gives relationship between u, v and f of a lens.

Refraction through Prism

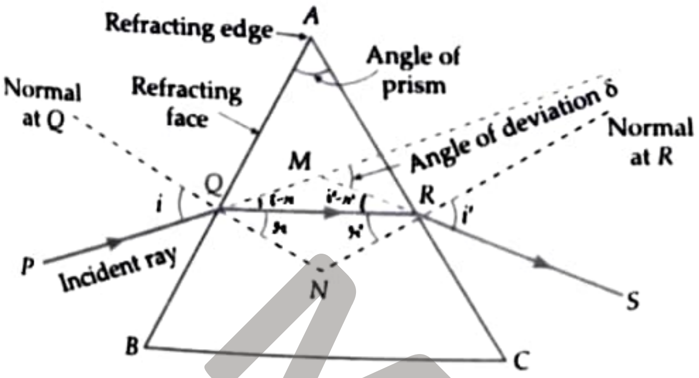


Fig. 15.116 Refraction through a prism.

An ray PQ is incident on the Prism which refract to QR & then again refracts to RS as shown in figure.

In ΔQRN:-

$$r + r' + N = 180°$$

$$r + r' = 180° - N \quad —①$$

In $\square$ AQNR:-

$$A + \angle AQN + N + \angle NRA = 360°$$

$$A + 90° + N + 90° = 360°$$

$$A + N = 360° - 180°$$

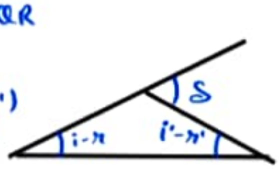
$$A = 180 - N \quad —②$$

on Comparing ① & ②

$$\boxed{r + r' = A} \quad —③$$

where A = Angle of Prism

Now, In Δ MQR



$$\delta = (i - r) + (i' - r')$$

$$\delta = i + i' - (r + r')$$

from eqn ③

$$\delta = i + i' - A$$

$$\boxed{\delta + A = i + i'}$$

here

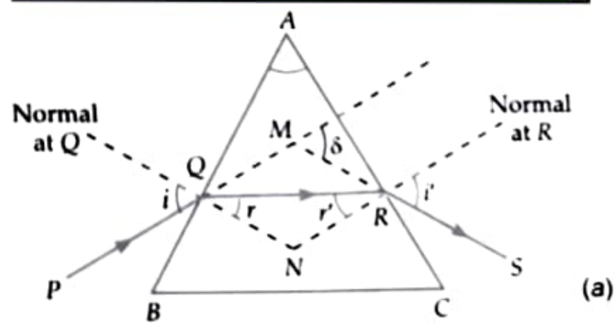
δ = Angle of Deviation

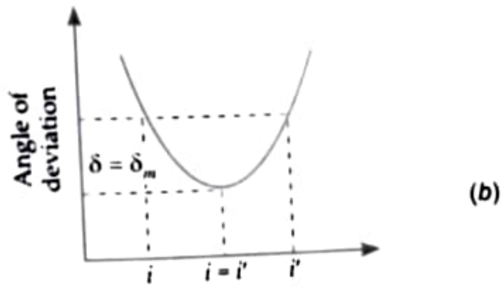
A = Angle of Prism

i = incidence angle

i' = emergent angle

Condition for minimum Deviation





deviation.

The minimum value of the angle of deviation suffered by a ray on passing through a prism is called the **angle of minimum deviation** and is denoted by δ_m.

Relation between refractive index and angle of minimum deviation. When a prism is in the position of minimum deviation, a ray of light passes symmetrically (parallel to the base) through the prism so that

$$i = i', \quad r = r', \quad \delta = \delta_m$$

As $\quad A + \delta = i + i'$

$\therefore \quad A + \delta_m = i + i \quad$ or $\quad i = \dfrac{A + \delta_m}{2}$

Also $\quad A = r + r' = r + r = 2r$

$\therefore \quad r = \dfrac{A}{2}$

From Snell's law, the refractive index of the material of the prism will be

$$\mu = \frac{\sin i}{\sin r} \quad \text{or} \quad \mu = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}}$$

By measuring the values of A and δ_m, with the help of a spectrometer, the refractive index μ of the prism glass can be determined accurately.

Simple microscope

A Simple microscope is just a convex lens of short focal length, held close to eye

Case 1) when the final image is formed at the least distance of distinct vision.

when an object AB is placed between focus F and optical centre O of a convex lens, a virtual, erect and magnified image is formed at a distance D (25cm) from the lens.

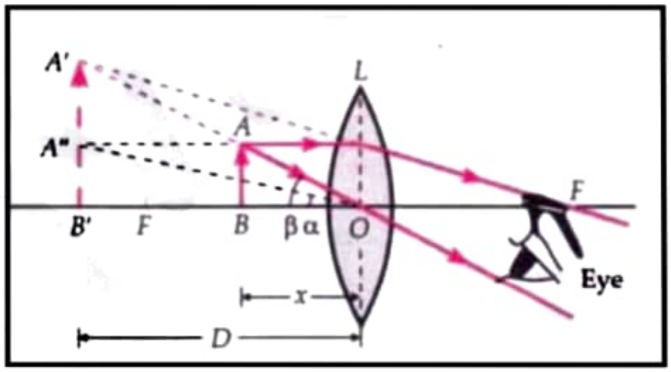


Diagram for image formation

$$\text{magnification} = \frac{\text{Angle formed by Image}}{\text{Angle formed by object}}$$

$$m = \frac{\beta}{\alpha} \approx \frac{\tan\beta}{\tan\alpha}$$

From ΔABO, $\tan\beta = \frac{AB}{BO}$

From ΔABC, $\tan\alpha = \frac{AB}{BC}$

Now

$$m = \frac{\tan\beta}{\tan\alpha} = \frac{AB/BO}{AB/BC} = \frac{AB}{BO} \times \frac{BC}{AB}$$

$$m = \frac{AC}{BO} = \frac{-D}{-x} = \frac{D}{x}$$

so $\boxed{m = \frac{D}{x}}$ — ①

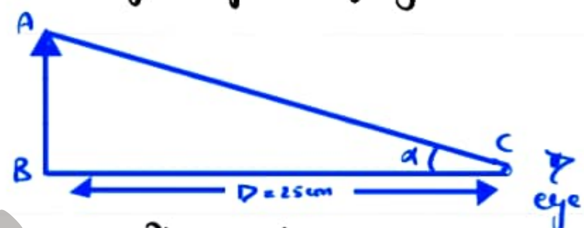


Diagram for Object

Now using thin lens formula:-

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-D} - \frac{1}{-x} = \frac{1}{f} \longrightarrow -\frac{1}{D} + \frac{1}{x} = \frac{1}{f}$$

motiply both sides with D, then

$$-\frac{D}{D} + \frac{D}{x} = \frac{D}{f} \longrightarrow -1 + \frac{D}{x} = \frac{D}{f}$$

Then $\boxed{\frac{D}{x} = 1 + \frac{D}{f}}$ — ②

On Comparing ① & ②

$\boxed{m = 1 + \frac{D}{f}}$ or $\boxed{\text{magnification} \propto \frac{1}{\text{focal length}}}$

Case 2:- when the final image is formed at infinity

when an object AB is placed at focus F of a convex lens, a virtual, erect and magnified image is formed at infinity.

$$\text{magnification} = \frac{\text{Angle formed by Image}}{\text{Angle formed by object}}$$

$$m = \frac{\beta}{\alpha} \approx \frac{\tan\beta}{\tan\alpha}$$

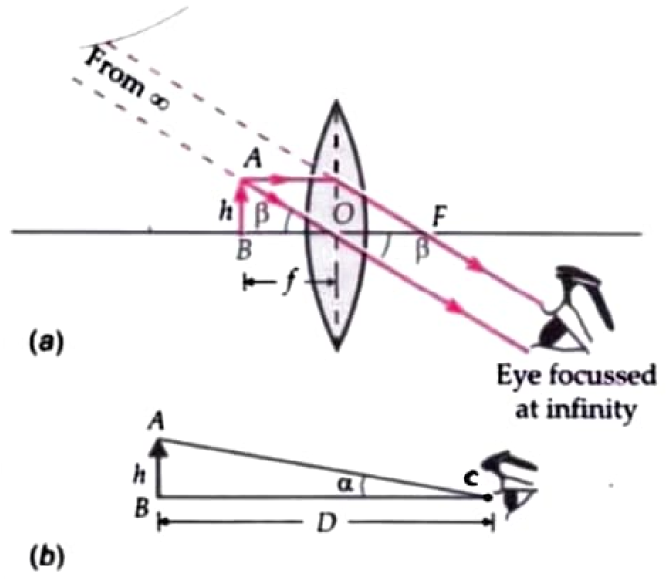
from ΔABO, $\tan\beta = \frac{AB}{BO}$

from ΔABC, $\tan\alpha = \frac{AB}{BC}$

$$m = \frac{\tan\beta}{\tan\alpha} = \frac{AB/BO}{AB/BC} = \frac{AB}{BO} \times \frac{BC}{AB}$$

$$m = \frac{BC}{BO} = \frac{-D}{-f} = \frac{D}{f}$$

So $\boxed{m = \frac{D}{f}}$



9.144 (a) With object at F, image is at infinity.
(b) Object at the near point.

Compound microscope A Compound microscope is an optical device used to see magnified image of tiny Objects.

Construction :-

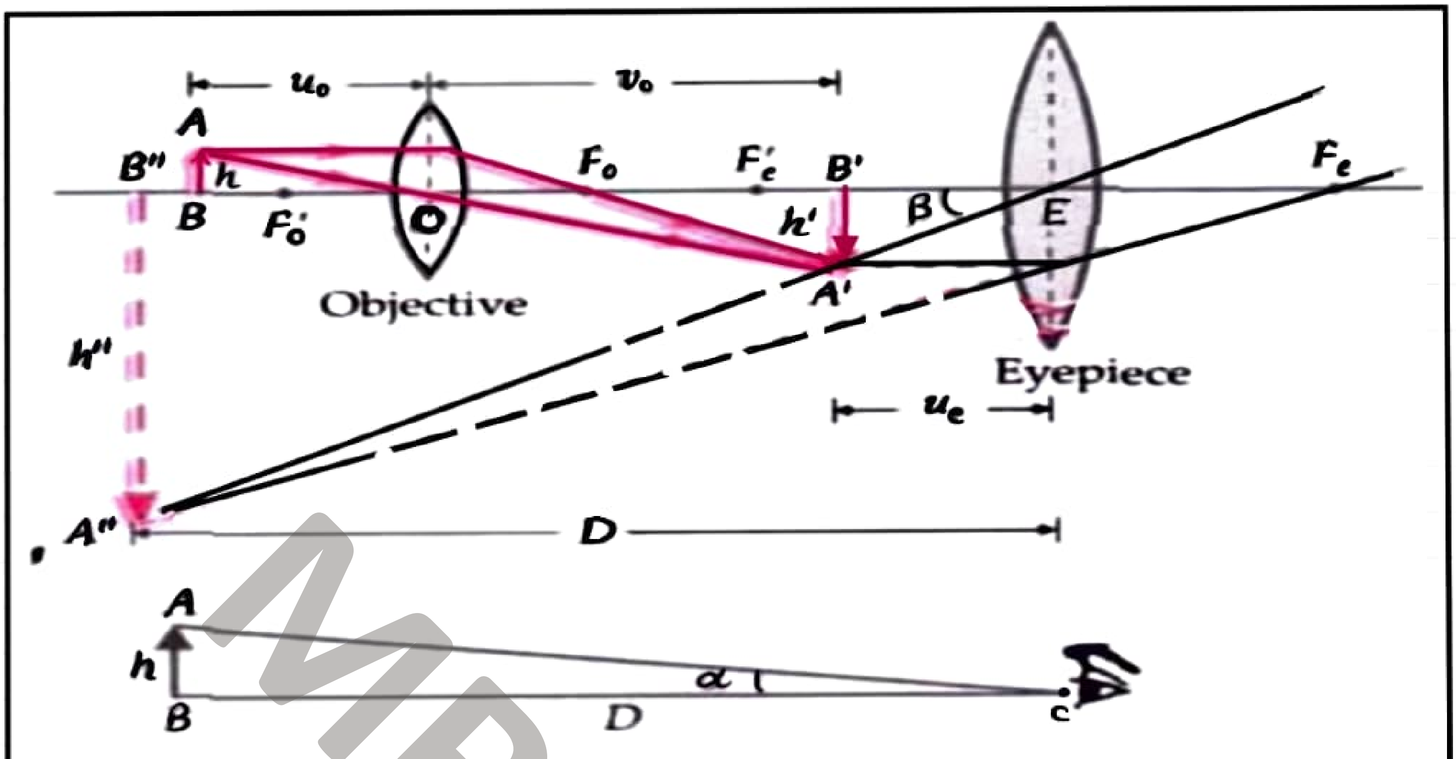
objective lens :- It is convex lens of very short focal length f_o and small aperture. It is placed near the object

Eyepiece lens :- It is convex lens of larger focal length f_e. and larger aperture. It is placed near the eye.

Case 1 :- when the final image is formed at the least distance of distinct vision.

An object AB is placed before a objective lens. The objective lens makes image A'B' which acts as a virtual object for eye piece lens. The eye piece lens makes final image A"B".

here u_o = object AB distance from objective lens.
v_o = Image A'B' distance from objective lens
u_e = Virtual object A'B' distance from eye piece lens
v_e = Image A"B" distance from eye piece lens
h = height of object AB
h' = height of image A'B' formed by objective lens.
h'' = height of image A"B" formed by eye piece lens.
f_o = focal length of objective lens
f_e = focal length of eye piece lens.
L = length of microscope = Distance between two lenses
D = least distance of Distinct vision.



Compound Microscope, final image at D.

$$\text{magnification} = \frac{\text{Angle formed by image}}{\text{Angle formed by object}}$$

$$m = \frac{\beta}{\alpha} \approx \frac{\tan\beta}{\tan\alpha}$$

from $\Delta A'B'E$, $\tan\beta = \frac{A'B'}{B'E}$ and from ΔABC, $\tan\alpha = \frac{AB}{BC}$

Now $m = \frac{\tan\beta}{\tan\alpha} = \frac{A'B'/B'E}{AB/BC} = \frac{A'B'}{B'E} \times \frac{BC}{AB} = \frac{h'}{u_e} \times \frac{D}{h} = \frac{h'}{h} \times \frac{D}{u_e}$

$m = \frac{h'}{h} \times \frac{D}{u_e}$ here $\frac{h'}{h}$ = magnification by objective lens $= m_o$

and $\frac{D}{u_e}$ = magnification by eye lens $= m_e$

$$\boxed{m = m_o \times m_e}$$

Now for magnification by eye piece lens, $\boxed{m_e = \frac{D}{u_e}}$ — (1)
using lens formula for eye piece

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$ Now multiply by D. Then,

$$\frac{D}{-D} + \frac{D}{u_e} = \frac{D}{f_e} \longrightarrow -1 + \frac{D}{u_e} = \frac{D}{f_e} \longrightarrow \boxed{\frac{D}{u_e} = 1 + \frac{D}{f_e}} \text{ — (2)}$$

from eqn (1) & (2) $\boxed{m_e = 1 + \frac{D}{f_e}}$

Now, for magnification by objective lens, $m_o = \frac{h'}{h}$

Now we know, $m_o = \frac{\text{Image distance}}{\text{object distance}} = \frac{v_o}{u_o}$

So $m_o = \frac{v_o}{u_o}$. Now let us take $v_o \simeq L$ (length of microscope)
$u_o \simeq -f_o$ (focal length of objective lens)

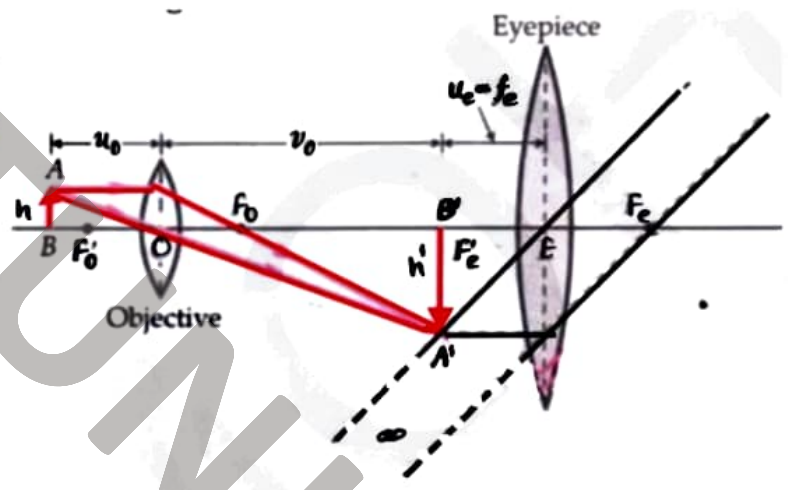
Then $m_o = \frac{v_o}{u_o} = \frac{L}{-f_o} = \frac{-L}{f_o}$

Then $\boxed{m_o = -\frac{L}{f_o}}$

Now total magnification $m = m_o \times m_e$

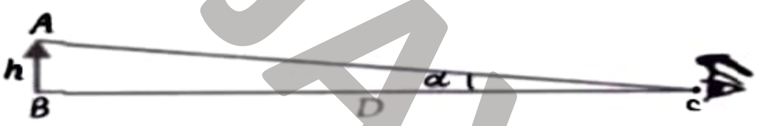
$$\boxed{m = -\frac{L}{f_o}\left[1 + \frac{D}{f_e}\right]}$$

Case 2:- When the final Image is formed at infinity



An object AB is placed before a objective lens, the objective lens makes image A'B' which acts as a virtual object for eye piece lens. The eye piece form an image at infinity.

u_o = object AB distance from objective lens.
v_o = Image A'B' distance from objective lens
u_e = Virtual object A'B' distance from eye piece lens
f_o = focal length of objective lens
f_e = focal length of eye piece lens.
L = length of microscope
D = least distance of Distinct vision.



$$\text{magnification} = \frac{\text{Angle formed by image}}{\text{Angle formed by object}}$$

$$m = \frac{\beta}{\alpha} \simeq \frac{\tan\beta}{\tan\alpha}$$

from $\Delta A'B'E$, $\tan\beta = \frac{A'B'}{B'E}$ and from ΔABC, $\tan\alpha = \frac{AB}{BC}$

Now $m = \frac{\tan\beta}{\tan\alpha} = \frac{A'B'/B'E}{AB/BC} = \frac{A'B'}{B'E} \times \frac{BC}{AB} = \frac{h'}{u_e} \times \frac{D}{h}$

here $u_e = f_e$ so, $m = \frac{h'}{f_e} \times \frac{D}{h} = \frac{D}{f_e} \times \frac{h'}{h}$.

$$m = \frac{h'}{h} \times \frac{D}{f_e} = m_o \times m_e$$

here $\frac{h'}{h}$ = magnification by objective lens = m_o

$$\boxed{m = m_o \times m_e}$$

and $\frac{D}{f_e}$ = magnification by eye lens = m_e

Now, for magnification by objective lens, $m_o = \frac{h'}{h}$

Now we know, $m_o = \frac{\text{Image distance}}{\text{object distance}} = \frac{v_o}{u_o}$

So $m_o = \frac{v_o}{u_o}$. Now let us take $v_o \approx L$ (length of microscope)

$u_o \approx -f_o$ (focal length of objective lens)

Then $m_o = \frac{v_o}{u_o} = \frac{L}{-f_o} = \frac{-L}{f_o}$

Then $\boxed{m_o = -\frac{L}{f_o}}$

total magnification $m = m_o \times m_e$

$$\boxed{m = -\frac{L}{f_o}\left[\frac{D}{f_e}\right]}$$

Reflecting Telescope

Newtonian reflecting telescope. The first reflecting telescope was set up by *Newton* in 1668. As shown in Fig. 15.150, it consists of a large concave mirror of large focal length as the objective, made of an alloy of copper and tin.

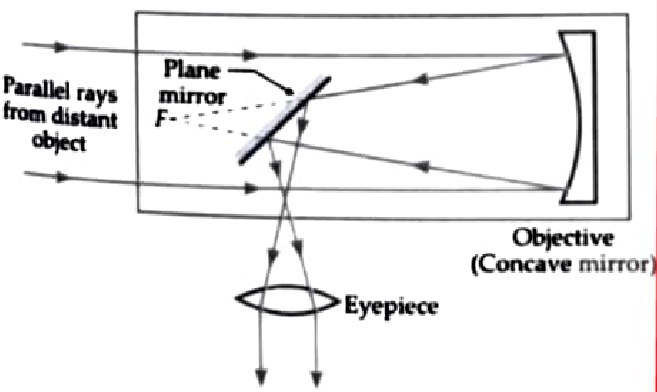


Fig. 15.150 **Newtonian reflecting telescope.**

A beam of light from the distant star is incident on the objective. Before the rays are focussed at *F*, a plane mirror inclined at 45° intercepts them and turns them towards an eyepiece adjusted perpendicular to the axis of the instrument. The eyepiece forms a highly magnified, virtual and erect image of the distant object.

Cassegrain reflecting telescope. Fig. 15.151 shows Cassegrainian type reflecting telescope. It consists of a large concave paraboloidal (primary) mirror having a hole at its centre. There is a small convex (secondary) mirror near the focus of the primary mirror. The eyepiece is placed on the axis of the telescope near the hole of the primary mirror.

The parallel rays from the distant object are reflected by the large concave mirror. Before these rays come to focus at *F*, they are reflected by the small convex mirror and are converged to a point *I* just

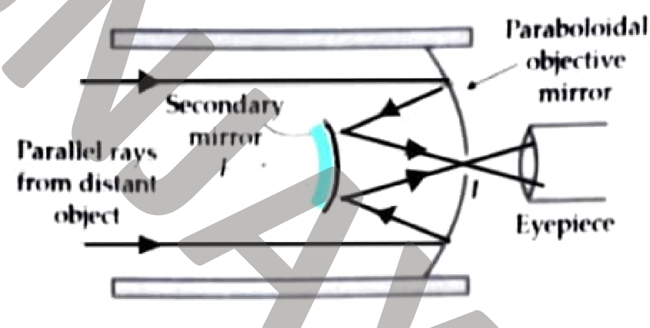


Fig. 15.151 **Cassegrain reflecting telescope.**

outside the hole. The final image formed at *I* is viewed through the eyepiece. As the first image at *F* is inverted with respect to the distant object and the second image *I* is erect with the respect to the first image *F*, hence the final image is inverted with respect to the object.

Let f_0 be the focal length of the objective and f_e that of the eyepiece.

For the final image formed at the least distance of distinct vision,

$$m = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

For the final image formed at infinity,

$$m = \frac{f_0}{f_e} = \frac{R/2}{f_e}.$$

Astronomical Telescope

Astronomical telescope. *It is a refracting type telescope used to see heavenly bodies like stars, planets, satellites, etc.*

Construction. It consists of two converging lenses mounted co-axially at the outer ends of two sliding tubes.

1. ***Objective.*** It is a convex lens of large focal length and a much larger aperture. It faces the distant object. In order to form bright image of the distant objects, the aperture of the objective is taken large so that it can gather sufficient light from the distant objects.

2. ***Eyepiece.*** It is a convex lens of small focal length and small aperture. It faces the eye. The aperture of the eyepiece is taken small so that whole light of the telescope may enter the eye for distinct vision.

Working. (*a*) When the final image is formed at the least distance of distinct vision. As shown in Fig. 15.147, the parallel beam of light coming from the distant object falls on the objective at some angle α. The objective focusses the beam in its focal plane and forms a real, inverted and diminished image $A'B'$. This image $A'B'$ acts as an object for the eyepiece. The distance of the eyepiece is so adjusted that the image $A'B'$ lies within its focal length. The eyepiece magnifies this image so that final image $A''B''$ is magnified and inverted with respect to the object. The final image is seen distinctly by the eye at the least distance of distinct vision.

Magnifying power. *The magnifying power of a telescope is defined as the ratio of the angle subtended at the eye by the final image formed at the least distance of distinct vision to the angle subtended at the eye by the object at infinity, when seen directly.*

As the object is very far off, the angle subtended by it at the eye is practically equal to the angle α subtended by it at the objective. Thus

$$\angle A'OB' = \alpha$$

Also, let $\angle A''EB'' = \beta$

∴ Magnifying power,

$$m = \frac{\beta}{\alpha} \simeq \frac{\tan\beta}{\tan\alpha} \qquad [\because \alpha, \beta \text{ are small}]$$

$$= \frac{A'B'/B'E}{A'B'/OB'} = \frac{OB'}{B'E}.$$

According to the new Cartesian sign convention,

$OB' = +f_0$ = focal length of the objective

$B'E = -u_e$ = distance of $A'B'$ from the eyepiece, acting as an object for it

$$\therefore \qquad m = -\frac{f_0}{u_e}$$

Again, for the eyepiece :

$$u = -u_e \quad \text{and} \quad v = -D$$

As $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \qquad \frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

or $$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Hence $$m = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Clearly for large magnifying power, $f_0 >> f_e$. The negative sign for the magnifying power indicates that the final image formed is *real* and *inverted*.

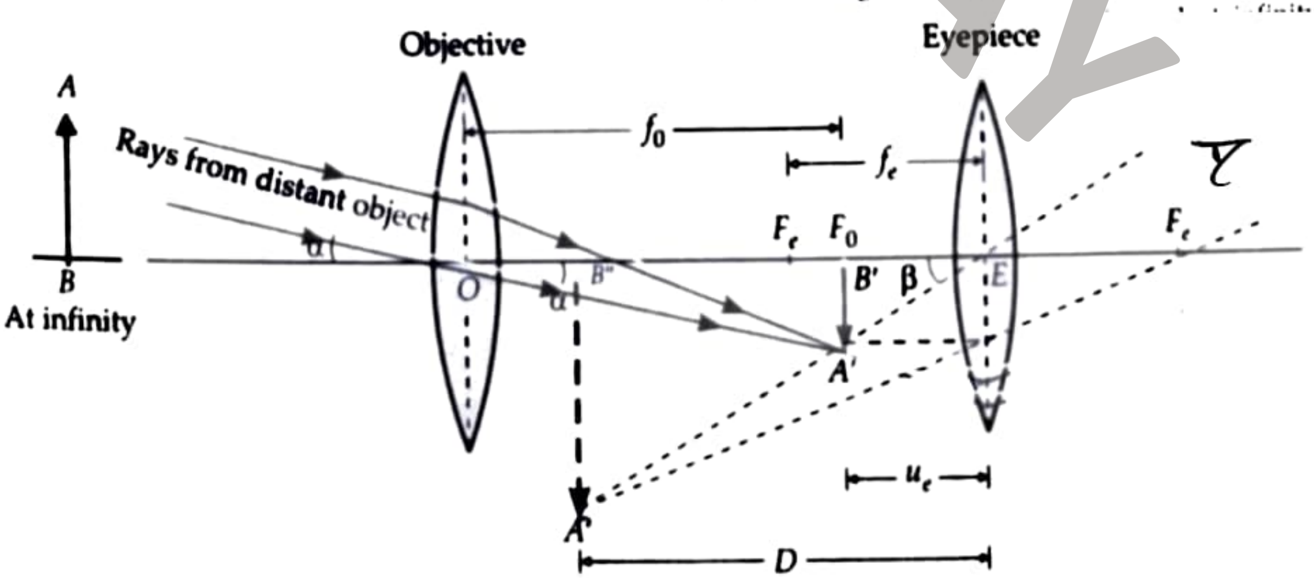


Fig. 15.147 **Astronomical telescope focussed for least distance of distinct vision.**

(b) When the final image is formed at infinity : Normal adjustment. As shown in Fig. 15.148, when a parallel beam of light is incident on the objective, it forms a real, inverted and diminished image $A'B'$ in its focal plane. The eyepiece is so adjusted that the image $A'B'$ exactly lies at its focus. Therefore, the final image is formed at infinity, and is highly magnified and inverted with respect to the object.

Magnifying power in normal adjustment. *It is defined as the ratio of the angle subtended at the eye by the final image as seen through the telescope to the angle subtended at the eye by the object seen directly, when both the image and the object lie at infinity.*

As the object is very far off, the angle subtended by it at the eye is practically equal to the angle α subtended by it at the objective.

Thus

$$\angle A'OB' = \alpha$$

and let $$\angle A'EB' = \beta$$

$\therefore$ Magnifying power,

$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} \quad [\because \alpha, \beta \text{ are small angles}]$$

$$= \frac{A'B'/B'E}{A'B'/OB'} = \frac{OB'}{B'E}$$

Applying new Cartesian sign convention,

$OB' = +f_0$ = Distance of $A'B'$ from the objective along the incident light

$B'E = -f_e$ = Distance of $A'B'$ from the eyepiece *against* the incident light

$$\therefore \qquad m = -\frac{f_0}{f_e}$$

Clearly for large magnifying power, $f_0 >> f_e$. The negative sign for m indicates that the image is *real* and *inverted*.

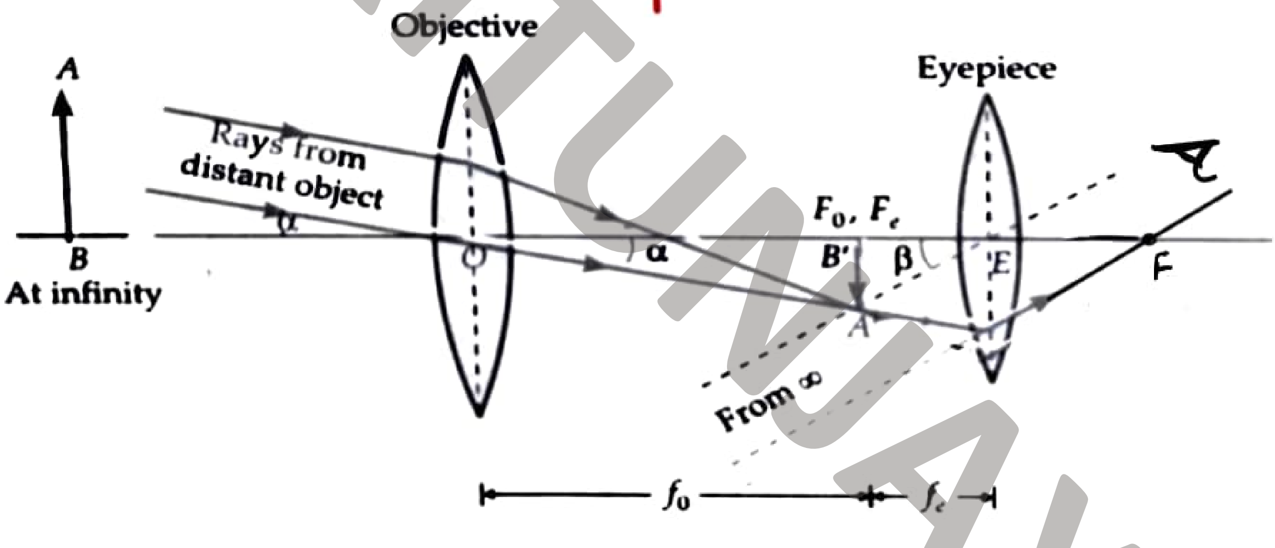


Fig. 15.148 **Astronomical telescope in normal adjustment.**

Advantages of a reflecting type telescope. A reflecting type telescope has the following advantages over a refracting type telescope :

1. A concave mirror of large aperture has high gathering power and absorbs very less amount of light than the lenses of large apertures. The final image formed in reflecting telescope is very bright. So even very distant or faint stars can be easily viewed.
2. Due to large aperture of the mirror used, the reflecting telescopes have ***high resolving power***.
3. As the objective is a mirror and not a lens, it is free from ***chromatic aberration*** (formation of coloured image of a white object).
4. The use of paraboloidal mirror reduces the ***spherical aberration*** (formation of non-point, blurred image of a point object).
5. A mirror requires grinding and polishing of one surface only. So it costs much less to construct a reflecting telescope than a refracting telescope of equivalent optical quality.
6. A lens of large aperture tends to be very heavy and, therefore, difficult to make and support by its edges. On the other hand, a mirror of equivalent optical quality weighs less and can be supported over its entire back surface.