# DPP 2.3

## Limits of Exponential and Logarithmic Functions, Limits of Functions $f(x)^{g(x)}$ Type

### Single Correct Answer Type

1. The value of $\lim\limits_{x\to 0^+} \dfrac{\tan x^{\frac{1}{5}}}{(\tan^{-1}\sqrt{x})^2} \dfrac{\log(1+5x)}{(e^{3\sqrt[5]{x}}-1)}$ is

   (a) $\dfrac{3}{5}$ (b) $\dfrac{5}{3}$

   (c) 1 (d) none of these

2. The value of $\lim\limits_{x\to 3} \dfrac{(x^3+27)\log_e(x-2)}{x^2-9}$ is

   (a) 9 (b) 18 (c) 27 (d) 1/3

3. The value of $\lim\limits_{x\to 0^+} \left(\dfrac{1-\cos(\sin^2 x)}{x^2}\right)^{\frac{\log_e(1-2x^2)}{\sin^2 x}}$ is

   (a) 0 (b) $e$ (c) $-1$ (d) $\infty$

4. $\lim\limits_{x\to 0} \dfrac{1}{x^2}\begin{vmatrix} 1-\cos 3x & \log_e(1+4x) \\ \sin^{-1}(e^x-1) & \tan^{-1}(2x) \end{vmatrix}$ is equal to

   (a) 2 (b) $-4$ (c) 6 (d) 4

5. If graph of the function $y = f(x)$ is continuous and passes through point (3, 1) then $\lim\limits_{x\to 3} \dfrac{\log_e(3f(x)-2)}{2(1-f(x))}$ is equal

   (a) $\dfrac{3}{2}$ (b) $\dfrac{1}{2}$ (c) $-\dfrac{3}{2}$ (d) $-\dfrac{1}{2}$

6. Let $f(x)$ be defined for all $x \in R$ such that $\lim\limits_{x\to 0}\left[f(x)+\log\left(1-\dfrac{1}{e^{f(x)}}\right)-\log(f(x))\right]=0$. Then $f(0)$ is

   (a) 0 (b) 1 (c) 2 (d) 3

7. $\lim\limits_{x\to\infty} x^2 \sin\left(\log_e\sqrt{\cos\dfrac{\pi}{x}}\right)$

   (a) 0 (b) $-\dfrac{\pi^2}{2}$ (c) $-\dfrac{\pi^2}{4}$ (d) $-\dfrac{\pi^2}{8}$

8. If $\lim\limits_{x\to\infty}\left(\dfrac{x+c}{x-c}\right)^x = 4$ then the value of $e^c$ is

   (a) 1/4 (b) 1/2 (c) 1 (d) 2

9. If $\lim\limits_{x\to 0}\left[1+x+\dfrac{f(x)}{x}\right]^{1/x} = e^3$, then $\lim\limits_{x\to 0}\left[1+\dfrac{f(x)}{x}\right]^{1/x} =$

   (a) $e$ (b) $e^2$ (c) $e^3$ (d) none of these

10. $\lim\limits_{x\to\frac{\pi}{2}^-} [1+(\cos x)^{\cos x}]^2 =$

    (a) Does not exist (b) 1

    (c) $e$ (d) 4

11. If $a > 0$, $b > 0$ then $\lim\limits_{n\to\infty}\left(\dfrac{a-1+b^{\frac{1}{n}}}{a}\right)^n =$

    (a) $b^{\frac{1}{a}}$ (b) $a^{\frac{1}{b}}$ (c) $a^b$ (d) $b^a$

12. If $f(x) = \lim\limits_{n\to\infty}\left(\cos\dfrac{x}{\sqrt{n}}\right)^n$, then the value of $\lim\limits_{x\to 0}\dfrac{f(x)-1}{x}$ is

    (a) 0 (b) 1 (c) 2 (d) 3/2

13. $\lim\limits_{x\to 0}\dfrac{\log\left(e^{x^2}+2\sqrt{x}\right)}{\tan\sqrt{x}}$ is equal to

    (a) 0 (b) 1 (c) $e^2$ (d) 2

14. Let $f: R \to R$ be such that $f(a) = 1$, $f'(a) = 2$. Then $\lim\limits_{x\to 0}\left(\dfrac{f^2(a+x)}{f(a)}\right)^{1/x}$ is

    (a) $e^2$ (b) $e^4$ (c) $e^{-4}$ (d) $1/e$

15. The value of $\lim\limits_{n\to\infty}\left(\dfrac{\sqrt{n^2+n}-1}{n}\right)^{2\sqrt{n^2+n}-1}$ is

    (a) $e$ (b) $1/e$ (c) $e^2$ (d) $e^{-2}$

16. If $f(n) = \lim\limits_{x\to 0}\left(\left(1+\sin\dfrac{x}{2}\right)\left(1+\sin\dfrac{x}{2^2}\right)\ldots\left(1+\sin\dfrac{x}{2^n}\right)\right)^{\frac{1}{x}}$ then $\lim\limits_{n\to\infty} f(n) =$

    (a) 1 (b) $e$ (c) 0 (d) $\infty$

17. $\lim\limits_{n\to\infty}(1-x+x\cdot\sqrt[n]{e})^n$ is equal to

    (a) $e^x$ (b) $e^{-x}$

    (c) $e^{2x}$ (d) none of these

## Limits Using L'Hospital's Rule and Expansion Formula

### Single Correct Answer Type

1. The value of $\lim\limits_{x\to 1}\dfrac{\sqrt[13]{x}-\sqrt[7]{x}}{\sqrt[5]{x}-\sqrt[3]{x}}$ is

   (a) $\dfrac{44}{91}$ (b) $\dfrac{45}{91}$ (c) $\dfrac{45}{89}$ (d) $\dfrac{40}{93}$

2. $\lim\limits_{x\to 1}\dfrac{1+\log x-x}{1-2x+x^2} =$

   (a) 1 (b) $-1$ (c) 0 (d) $-1/2$

3. The value of $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{e^{x^2}-e^x+x}$ is

   (a) 1 (b) 2 (c) 4 (d) 8

4. If $f'(a) = \dfrac{1}{4}$, then $\lim\limits_{h\to 0}\dfrac{f(a+2h^2)-f(a-2h^2)}{f(a+h^3-h^2)-f(a-h^3+h^2)} =$

   (a) 0 (b) 1

   (c) $-2$ (d) none of these

5. $\lim\limits_{x\to 0^+}\dfrac{1}{x\sqrt{x}}\left(a\tan^{-1}\dfrac{\sqrt{x}}{a}-b\tan^{-1}\dfrac{\sqrt{x}}{b}\right)$ has the value equal to

   (a) $\dfrac{a-b}{3}$ (b) 0

   (c) $\dfrac{(a^2-b^2)}{6a^2b^2}$ (d) $\dfrac{a^2-b^2}{3a^2b^2}$

6. The value of $\lim\limits_{x\to 0}\left(\dfrac{1+2x}{1+3x}\right)^{\frac{1}{x^2}}\cdot e^{\frac{1}{x}}$ is

   (a) $e^{\frac{5}{2}}$ (b) $e^2$ (c) $e^{-2}$ (d) 1

7. If $f: R \to R$ be a differentiable function at $x = 0$ satisfying $f(0) = 0$ and $f'(0) = 1$, then the value of

   $$\lim_{x\to 0}\frac{1}{x}\sum_{n=1}^{\infty}(-1)^n f\left(\frac{x}{n}\right) =$$

   (a) 0 (b) $-\log 2$ (c) 1 (d) $e$

8. The value of $\lim\limits_{x\to\frac{3\pi}{4}}\dfrac{1+\sqrt[3]{\tan x}}{1-2\cos^2 x}$ is

   (a) $-1/2$ (b) $-2/3$ (c) $-3/2$ (d) $-1/3$

9. Let $g(x) = \dfrac{(x-1)^n}{\log\cos^m(x-1)}$; $0 < x < 2$ $m$ and $n$ are integers, $m \neq 0$, $n > 0$ and. If $\lim\limits_{x\to 1+} g(x) = -1$, then

   (a) $n = 1, m = 1$ (b) $n = 1, m = -1$

   (c) $n = 2, m = 2$ (d) $n > 2, m = n$

### Comprehension Type

**For Questions 10 and 11**

Let $f(x)$ be the fourth degree polynomial such that $f'(0) = -6$, $f(0) = 2$ and $\lim\limits_{x\to 1}\dfrac{f(x)}{(x-1)^2} = 1$

10. The value of $f(2)$ is

    (a) 1 (b) 0 (c) 2 (d) 3

11. The value of $f'(2)$ is

    (a) 4 (b) 5 (c) 6 (d) 7

### Answers Key

*Single Correct Answer Type*

1. (b) 2. (d) 3. (c) 4. (c) 5. (d)
6. (a) 7. (b) 8. (d) 9. (c)

*Comprehension Type*

10. (c) 11. (c)



# DPP 2.5

## Finding the Unknown

### Single Correct Answer Type

1. Number of integral values of $\lambda$ for which $\lim\limits_{x\to 1}\sec^{-1}\left(\dfrac{\lambda^2}{\log_e x}-\dfrac{\lambda^2}{x-1}\right)$ does not exist is

   (a) 1 (b) 2 (c) 3 (d) 4

2. If $\lim\limits_{x\to 0}\dfrac{e^{ax}-e^x-x}{x^2} = b$ (finite), then

   (a) $a = 2, b = 0$ (b) $a = 0, b = \dfrac{3}{2}$

   (c) $a = 2, b = \dfrac{3}{2}$ (d) $a = 0, b = 2$

3. If $\lim\limits_{x\to 0}\dfrac{x^3}{\sqrt{a+x}\,(bx-\sin x)} = 1$, $a > 0$, then $a + b$ is equal to

   (a) 36 (b) 37 (c) 38 (d) 40

4. If $\lim\limits_{x\to\infty} x\log_e\left(\begin{vmatrix}\alpha/x & 1 & \gamma \\ 0 & 1/x & \beta \\ 1 & 0 & 1/x\end{vmatrix}\right) = -5$, where $\alpha$, $\beta$, $\gamma$ are finite real numbers, then

   (a) $\alpha = 2, \beta = 1, \gamma \in R$

   (b) $\alpha = 2, \beta = 2, \gamma = 5$

   (c) $\alpha \in R, \beta = 1, \gamma \in R$

   (d) $\alpha \in R, \beta = 1, \gamma = 5$

### Multiple Correct Answers Type

5. If $\lim\limits_{x\to 0}\dfrac{ae^x+b\cos x+ce^{-x}}{e^{2x}-2e^x+1} = 4$, then

   (a) $a = 2$ (b) $b = -4$

   (c) $c = 2$ (d) $a + b + c = -8$

6. If $a \in I$, then value of $a$ for which $\lim\limits_{x\to a}\dfrac{\tan([x^3]-[x]^3)}{(x-a)^3}$ exists finitely, is/are

   (a) 0 (b) 1 (c) $-1$ (d) 2

### Comprehension Type

**For Questions 7 and 8**

$$L = \lim_{x\to 0}\frac{\sin(\sin x)-\sin x}{ax^5+bx^3+c} = -\frac{1}{12}$$

7. The value/values of $a$ is

   (a) $\in R$ (b) 2 (c) 0 (d) 1

8. The value/values of $b$ is

   (a) $\in R$ (b) 2 (c) 0 (d) 1

**For Questions 9 and 10**

If $f(x) = \lim\limits_{n\to\infty}\dfrac{(x^2+ax+1)+x^{2n}(2x^2+x+b)}{1+x^{2n}}$ and $\lim\limits_{x\to\pm 1} f(x)$ exists, then

9. The value of $a$ is

   (a) $-1$ (b) 1 (c) 0 (d) 2

10. The value of $b$ is

    (a) $-1$ (b) 1 (c) 0 (d) 2

### Answers Key

*Single Correct Answer Type*

1. (c) 2. (c) 3. (b) 4. (d)

*Multiple Correct Answers Type*

5. (a, b, c) 6. (a, b)

*Comprehension Type*

7. (a) 8. (b) 9. (b) 10. (c)