DPP 2.2

Limits of Algebraic and Trigonometric Functions

Single Correct Answer Type

1. The value of $\lim_{x\to 0}\frac{\sqrt{1-\cos x^2}}{1-\cos x}$ is

(a) $\frac{1}{2}$ (b) 2
(c) $\sqrt{2}$ (d) None of these

2. $\lim_{x\to\frac{\pi}{2}}(1-\sin x)\tan x =$

(a) $\frac{\pi}{2}$ (b) 1 (c) 0 (d) ∞

3. The value of $\lim_{x\to\infty} x^2\left(1-\cos\frac{1}{x}\right)$ is

(a) 0 (b) 1/4 (c) 1/2 (d) 1

4. $\lim_{x\to\infty}\sqrt[3]{x}\left(\sqrt[3]{(x+1)^2}-\sqrt[3]{(x-1)^2}\right)=$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

5. $\lim_{n\to\infty}\frac{3\cdot 2^{n+1}-4\cdot 5^{n+1}}{5\cdot 2^n+7\cdot 5^n}=$

(a) 0 (b) 3/5 (c) –4/7 (d) –20/7

6. $\lim_{x\to 2^+}\{x\}\frac{\sin(x-2)}{(x-2)^2}=$ (where {.} denotes the fractional part function)

(a) 0 (b) 2
(c) 1 (d) does not exist

7. $\lim_{x\to\infty}\frac{\cot^{-1}\left(\sqrt{x+1}-\sqrt{x}\right)}{\sec^{-1}\left\{\left(\frac{2x+1}{x-1}\right)^x\right\}}=$

(a) 1 (b) 0 (c) $\pi/2$ (d) non existent

8. $\lim_{x\to 0}\frac{3\tan 3x-4\tan 2x-\tan x}{4x^2\tan x}=$

(a) 0 (b) 1 (c) 3 (d) 4

9. $\lim_{x\to 0}\left[\frac{\sin^{-1}x}{\tan^{-1}x}\right]=$ (where [.] denotes the greatest integer function)

(a) 0 (b) 1
(c) –1 (d) none of these

10. The value of $\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{1-\sqrt{\sin 2x}}}{\pi-4x}$ is

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$
(c) 1 (d) does not exist

11. The value of $\lim_{x\to\infty}\left(e^{\sqrt{x^4+1}}-e^{(x^2+1)}\right)$ is

(a) 0 (b) e (c) $1/e$ (d) $-\infty$

12. The value of $\lim_{x\to\pi/4}\frac{\tan^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ is

(a) 8 (b) 4 (c) –8 (d) –2

13. $\lim_{x\to\frac{\pi}{2}}\frac{(1-\sin x)(8x^3-\pi^3)\cos x}{(\pi-2x)^4}$

(a) $-\frac{\pi^2}{16}$ (b) $\frac{3\pi^2}{16}$ (c) $\frac{\pi^2}{16}$ (d) $-\frac{3\pi^2}{16}$

14. $\lim_{x\to\infty}\frac{\sum_{r=1}^{10}(x+r)^{2010}}{(x^{1006}+1)(2x^{1004}+1)}=$

(a) 5 (b) 2010 (c) $\frac{502}{1005}$ (d) 0

15. If $\lim_{x\to 0}\frac{f(x)}{x^2}=a$ and $\lim_{x\to 0}\frac{f(1-\cos x)}{g(x)\sin^2 x}=b$ (where $b\neq 0$), then $\lim_{x\to 0}\frac{g(1-\cos 2x)}{x^4}$ is

DPP 2.3

Limits of Exponential and Logarithmic Functions, Limits of Functions $f(x)^{g(x)}$ Type

Single Correct Answer Type

1. The value of $\lim_{x\to 0^+}\frac{\tan x^{\frac{1}{5}}}{(\tan^{-1}\sqrt{x})^2}\frac{\log(1+5x)}{(e^{3\sqrt[5]{x}}-1)}$ is

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$
(c) 1 (d) none of these

2. The value of $\lim_{x\to 3}\frac{(x^3+27)\log_e(x-2)}{x^2-9}$ is

(a) 9 (b) 18 (c) 27 (d) 1/3

3. The value of $\lim_{x\to 0^+}\left(\frac{1-\cos(\sin^2 x)}{x^2}\right)^{\frac{\log_e(1-2x^2)}{\sin^2 x}}$ is

(a) 0 (b) e (c) –1 (d) ∞

4. $\lim_{x\to 0}\frac{1}{x^2}\begin{vmatrix}1-\cos 3x & \log_e(1+4x)\\ \sin^{-1}(e^x-1) & \tan^{-1}(2x)\end{vmatrix}$ is equal to

(a) 2 (b) –4 (c) 6 (d) 4

5. If graph of the function $y=f(x)$ is continuous and passes through point (3, 1) then $\lim_{x\to 3}\frac{\log_e(3f(x)-2)}{2(1-f(x))}$ is equal

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{3}{2}$ (d) $-\frac{1}{2}$

6. Let $f(x)$ be defined for all $x\in R$ such that $\lim_{x\to 0}\left[f(x)+\log\left(1-\frac{1}{e^{f(x)}}\right)-\log(f(x))\right]=0$. Then $f(0)$ is

(a) 0 (b) 1 (c) 2 (d) 3

7. $\lim_{x\to\infty}x^2\sin\left(\log_e\sqrt{\cos\frac{\pi}{x}}\right)$

(a) 0 (b) $-\frac{\pi^2}{2}$ (c) $-\frac{\pi^2}{4}$ (d) $-\frac{\pi^2}{8}$

8. If $\lim_{x\to\infty}\left(\frac{x+c}{x-c}\right)^x=4$ then the value of e^c is

(a) 1/4 (b) 1/2 (c) 1 (d) 2

9. If $\lim_{x\to 0}\left[1+x+\frac{f(x)}{x}\right]^{1/x}=e^3$, then $\lim_{x\to 0}\left[1+\frac{f(x)}{x}\right]^{1/x}=$

(a) e (b) e^2 (c) e^3 (d) none of these

10. $\lim_{x\to\frac{\pi}{2}^-}[1+(\cos x)^{\cos x}]^2=$

(a) Does not exist (b) 1
(c) e (d) 4

11. If $a>0$, $b>0$ then $\lim_{n\to\infty}\left(\frac{a-1+b^{\frac{1}{n}}}{a}\right)^n=$

(a) $b^{\frac{1}{a}}$ (b) $a^{\frac{1}{b}}$ (c) a^b (d) b^a

12. If $f(x)=\lim_{n\to\infty}\left(\cos\frac{x}{\sqrt{n}}\right)^n$, then the value of $\lim_{x\to 0}\frac{f(x)-1}{x}$ is

(a) 0 (b) 1 (c) 2 (d) 3/2

13. $\lim_{x\to 0}\frac{\log\left(e^{x^2}+2\sqrt{x}\right)}{\tan\sqrt{x}}$ is equal to

(a) 0 (b) 1 (c) e^2 (d) 2

14. Let $f: R\to R$ be such that $f(a)=1$, $f'(a)=2$. Then $\lim_{x\to 0}\left(\frac{f^2(a+x)}{f(a)}\right)^{1/x}$ is

(a) e^2 (b) e^4 (c) e^{-4} (d) $1/e$

15. The value of $\lim_{n\to\infty}\left(\frac{\sqrt{n^2+n}-1}{n}\right)^{2\sqrt{n^2+n-1}}$ is

(a) e (b) $1/e$ (c) e^2 (d) e^{-2}

16. If $f(n)=\lim_{x\to 0}\left(\left(1+\sin\frac{x}{2}\right)\left(1+\sin\frac{x}{2^2}\right)\dots\left(1+\sin\frac{x}{2^n}\right)\right)^{\frac{1}{x}}$ then $\lim_{n\to\infty}f(n)=$

(a) 1 (b) e (c) 0 (d) ∞

17. $\lim_{n\to\infty}(1-x+x\cdot\sqrt[n]{e})^n$ is equal to

(a) e^x (b) e^{-x}
(c) e^{2x} (d) none of these

Limits Using L'Hospital's Rule and Expansion Formula

Single Correct Answer Type

1. The value of $\lim_{x\to1}\frac{\sqrt[13]{x}-\sqrt[7]{x}}{\sqrt[5]{x}-\sqrt[3]{x}}$ is

(a) $\frac{44}{91}$ (b) $\frac{45}{91}$ (c) $\frac{45}{89}$ (d) $\frac{40}{93}$

2. $\lim_{x\to1}\frac{1+\log x-x}{1-2x+x^2}=$

(a) 1 (b) −1 (c) 0 (d) −1/2

3. The value of $\lim_{x\to0}\frac{1-\cos 2x}{e^{x^2}-e^x+x}$ is

(a) 1 (b) 2 (c) 4 (d) 8

4. If $f'(a)=\frac{1}{4}$, then $\lim_{h\to0}\frac{f(a+2h^2)-f(a-2h^2)}{f(a+h^3-h^2)-f(a-h^3+h^2)}=$

(a) 0 (b) 1
(c) −2 (d) none of these

5. $\lim_{x\to0^+}\frac{1}{x\sqrt{x}}\left(a\tan^{-1}\frac{\sqrt{x}}{a}-b\tan^{-1}\frac{\sqrt{x}}{b}\right)$ has the value equal to

(a) $\frac{a-b}{3}$ (b) 0
(c) $\frac{(a^2-b^2)}{6a^2b^2}$ (d) $\frac{a^2-b^2}{3a^2b^2}$

6. The value of $\lim_{x\to0}\left(\frac{1+2x}{1+3x}\right)^{\frac{1}{x^2}}\cdot e^{\frac{1}{x}}$ is

(a) $e^{\frac{5}{2}}$ (b) e^2 (c) e^{-2} (d) 1

7. If $f:R\to R$ be a differentiable function at $x=0$ satisfying $f(0)=0$ and $f'(0)=1$, then the value of

$$\lim_{x\to0}\frac{1}{x}\sum_{n=1}^{\infty}(-1)^n f\left(\frac{x}{n}\right)=$$

(a) 0 (b) −log 2 (c) 1 (d) e

8. The value of $\lim_{x\to\frac{3\pi}{4}}\frac{1+\sqrt[3]{\tan x}}{1-2\cos^2 x}$ is

(a) −1/2 (b) −2/3 (c) −3/2 (d) −1/3

9. Let $g(x)=\frac{(x-1)^n}{\log\cos^m(x-1)}$; $0<x<2$ m and n are integers, $m\neq0$, $n>0$ and. If $\lim_{x\to1^+}g(x)=-1$, then

(a) $n=1, m=1$ (b) $n=1, m=-1$
(c) $n=2, m=2$ (d) $n>2, m=n$

Comprehension Type

For Questions 10 and 11

Let $f(x)$ be the fourth degree polynomial such that $f'(0)=-6$ $f(0)=2$ and $\lim_{x\to1}\frac{f(x)}{(x-1)^2}=1$

10. The value of $f(2)$ is
(a) 1 (b) 0 (c) 2 (d) 3

11. The value of $f'(2)$ is
(a) 4 (b) 5 (c) 6 (d) 7

Answers Key

Single Correct Answer Type

1. (b) 2. (d) 3. (c) 4. (c) 5. (d)
6. (a) 7. (b) 8. (d) 9. (c)

Comprehension Type

10. (c) 11. (c)

DPP 2.5

Finding the Unknown

Single Correct Answer Type

1. Number of integral values of λ for which $\lim_{x\to1}\sec^{-1}\left(\frac{\lambda^2}{\log_e x}-\frac{\lambda^2}{x-1}\right)$ does not exist is

(a) 1 (b) 2 (c) 3 (d) 4

2. If $\lim_{x\to0}\frac{e^{ax}-e^x-x}{x^2}=b$ (finite), then

(a) $a=2, b=0$ (b) $a=0, b=\frac{3}{2}$
(c) $a=2, b=\frac{3}{2}$ (d) $a=0, b=2$

3. If $\lim_{x\to0}\frac{x^3}{\sqrt{a+x}\,(bx-\sin x)}=1$, $a>0$, then $a+b$ is equal to

(a) 36 (b) 37 (c) 38 (d) 40

4. If $\lim_{x\to\infty}x\log_e\left(\begin{vmatrix}\alpha/x & 1 & \gamma\\ 0 & 1/x & \beta\\ 1 & 0 & 1/x\end{vmatrix}\right)=-5$, where α, β, γ are finite real numbers, then

(a) $\alpha=2, \beta=1, \gamma\in R$
(b) $\alpha=2, \beta=2, \gamma=5$
(c) $\alpha\in R, \beta=1, \gamma\in R$
(d) $\alpha\in R, \beta=1, \gamma=5$

Multiple Correct Answers Type

5. If $\lim_{x\to0}\frac{ae^x+b\cos x+ce^{-x}}{e^{2x}-2e^x+1}=4$, then

(a) $a=2$ (b) $b=-4$
(c) $c=2$ (d) $a+b+c=-8$

6. If $a\in I$, then value of a for which $\lim_{x\to a}\frac{\tan([x^3]-[x]^3)}{(x-a)^3}$ exists finitely, is/are

(a) 0 (b) 1 (c) −1 (d) 2

Comprehension Type

For Questions 7 and 8

$$L=\lim_{x\to0}\frac{\sin(\sin x)-\sin x}{ax^5+bx^3+c}=-\frac{1}{12}$$

7. The value/values of a is
(a) $\in R$ (b) 2 (c) 0 (d) 1

8. The value/values of b is
(a) $\in R$ (b) 2 (c) 0 (d) 1

For Questions 9 and 10

If $f(x)=\lim_{n\to\infty}\frac{(x^2+ax+1)+x^{2n}(2x^2+x+b)}{1+x^{2n}}$ and $\lim_{x\to\pm1}f(x)$ exists, then

9. The value of a is
(a) −1 (b) 1 (c) 0 (d) 2

10. The value of b is
(a) −1 (b) 1 (c) 0 (d) 2

Answers Key

Single Correct Answer Type

1. (c) 2. (c) 3. (b) 4. (d)

Multiple Correct Answers Type

5. (a, b, c) 6. (a, b)

Comprehension Type

7. (a) 8. (b) 9. (b) 10. (c)