DPP 2.3

Limits of Exponential and Logarithmic Functions, Limits of Functions $f(x)^{g(x)}$ Type

Single Correct Answer Type

1. The value of $\lim_{x\to 0^+} \frac{\tan x^{\frac{1}{5}}}{(\tan^{-1}\sqrt{x})^2} \frac{\log(1+5x)}{(e^{3\sqrt[3]{x}}-1)}$ is

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) 1 (d) none of these

2. The value of $\lim_{x\to 3} \frac{(x^3+27)\log_e(x-2)}{x^2-9}$ is

(a) 9 (b) 18 (c) 27 (d) 1/3

3. The value of $\lim_{x\to 0^+} \left(\frac{1-\cos(\sin^2 x)}{x^2}\right)^{\frac{\log_e(1-2x^2)}{\sin^2 x}}$ is

(a) 0 (b) e (c) -1 (d) ∞

4. $\lim_{x\to 0} \frac{1}{x^2} \begin{vmatrix} 1-\cos 3x & \log_e(1+4x) \\ \sin^{-1}(e^x-1) & \tan^{-1}(2x) \end{vmatrix}$ is equal to

(a) 2 (b) -4 (c) 6 (d) 4

5. If graph of the function $y = f(x)$ is continuous and passes through point (3, 1) then $\lim_{x\to 3} \frac{\log_e(3f(x)-2)}{2(1-f(x))}$ is equal

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{3}{2}$ (d) $-\frac{1}{2}$

6. Let $f(x)$ be defined for all $x \in R$ such that $\lim_{x\to 0}\left[f(x)+\log\left(1-\frac{1}{e^{f(x)}}\right)-\log(f(x))\right]=0$. Then $f(0)$ is

(a) 0 (b) 1 (c) 2 (d) 3

7. $\lim_{x\to\infty} x^2 \sin\left(\log_e \sqrt{\cos\frac{\pi}{x}}\right)$

(a) 0 (b) $-\frac{\pi^2}{2}$ (c) $-\frac{\pi^2}{4}$ (d) $-\frac{\pi^2}{8}$

8. If $\lim_{x\to\infty}\left(\frac{x+c}{x-c}\right)^x = 4$ then the value of e^c is

(a) 1/4 (b) 1/2 (c) 1 (d) 2

9. If $\lim_{x\to 0}\left[1+x+\frac{f(x)}{x}\right]^{1/x} = e^3$, then $\lim_{x\to 0}\left[1+\frac{f(x)}{x}\right]^{1/x} =$

(a) e (b) e^2 (c) e^3 (d) none of these

10. $\lim_{x\to\frac{\pi}{2}^-} [1+(\cos x)^{\cos x}]^2 =$

(a) Does not exist (b) 1
(c) e (d) 4

11. If $a > 0$, $b > 0$ then $\lim_{n\to\infty}\left(\frac{a-1+b^{\frac{1}{n}}}{a}\right)^n =$

(a) $b^{\frac{1}{a}}$ (b) $a^{\frac{1}{b}}$ (c) a^b (d) b^a

12. If $f(x) = \lim_{n\to\infty}\left(\cos\frac{x}{\sqrt{n}}\right)^n$, then the value of $\lim_{x\to 0}\frac{f(x)-1}{x}$ is

(a) 0 (b) 1 (c) 2 (d) 3/2

13. $\lim_{x\to 0}\frac{\log\left(e^{x^2}+2\sqrt{x}\right)}{\tan\sqrt{x}}$ is equal to

(a) 0 (b) 1 (c) e^2 (d) 2

14. Let $f: R \to R$ be such that $f(a) = 1$, $f'(a) = 2$. Then $\lim_{x\to 0}\left(\frac{f^2(a+x)}{f(a)}\right)^{1/x}$ is

(a) e^2 (b) e^4 (c) e^{-4} (d) $1/e$

15. The value of $\lim_{n\to\infty}\left(\frac{\sqrt{n^2+n}-1}{n}\right)^{2\sqrt{n^2+n}-1}$ is

(a) e (b) $1/e$ (c) e^2 (d) e^{-2}

16. If $f(n) = \lim_{x\to 0}\left(\left(1+\sin\frac{x}{2}\right)\left(1+\sin\frac{x}{2^2}\right)\cdots\left(1+\sin\frac{x}{2^n}\right)\right)^{\frac{1}{x}}$ then $\lim_{n\to\infty} f(n) =$

(a) 1 (b) e (c) 0 (d) ∞

17. $\lim_{n\to\infty}(1-x+x\cdot\sqrt[n]{e})^n$ is equal to

(a) e^x (b) e^{-x}
(c) e^{2x} (d) none of these

Limits Using L'Hospital's Rule and Expansion Formula

Single Correct Answer Type

1. The value of $\lim_{x\to 1}\frac{\sqrt[3]{x}-\sqrt[7]{x}}{\sqrt[5]{x}-\sqrt[3]{x}}$ is

(a) $\frac{44}{91}$ (b) $\frac{45}{91}$ (c) $\frac{45}{89}$ (d) $\frac{40}{93}$

2. $\lim_{x\to 1}\frac{1+\log x - x}{1-2x+x^2} =$

(a) 1 (b) -1 (c) 0 (d) $-1/2$

3. The value of $\lim_{x\to 0}\frac{1-\cos 2x}{e^{x^2}-e^x+x}$ is

(a) 1 (b) 2 (c) 4 (d) 8

4. If $f(a) = \frac{1}{4}$, then $\lim_{h\to 0}\frac{f(a+2h^2)-f(a-2h^2)}{f(a+h^3-h^2)-f(a-h^3+h^2)} =$

(a) 0 (b) 1
(c) -2 (d) none of these

5. $\lim_{x\to 0^+}\frac{1}{x\sqrt{x}}\left(a\tan^{-1}\frac{\sqrt{x}}{a}-b\tan^{-1}\frac{\sqrt{x}}{b}\right)$ has the value equal to

(a) $\frac{a-b}{3}$ (b) 0

(c) $\frac{(a^2-b^2)}{6a^2b^2}$ (d) $\frac{a^2-b^2}{3a^2b^2}$

6. The value of $\lim_{x\to 0}\left(\frac{1+2x}{1+3x}\right)^{\frac{1}{x^2}}\cdot e^{\frac{1}{x}}$ is

(a) $e^{\frac{5}{2}}$ (b) e^2 (c) e^{-2} (d) 1

7. If $f: R \to R$ be a differentiable function at $x = 0$ satisfying $f(0) = 0$ and $f'(0) = 1$, then the value of $\lim_{x\to 0}\frac{1}{x}\sum_{n=1}^{\infty}(-1)^n f\left(\frac{x}{n}\right) =$

(a) 0 (b) $-\log 2$ (c) 1 (d) e

8. The value of $\lim_{x\to\frac{3\pi}{4}}\frac{1+\sqrt[3]{\tan x}}{1-2\cos^2 x}$ is

(a) $-1/2$ (b) $-2/3$ (c) $-3/2$ (d) $-1/3$

9. Let $g(x) = \frac{(x-1)^n}{\log\cos^m(x-1)}$; $0 < x < 2$ m and n are integers, $m \neq 0$, $n > 0$ and. If $\lim_{x\to 1+} g(x) = -1$, then

(a) $n = 1, m = 1$ (b) $n = 1, m = -1$
(c) $n = 2, m = 2$ (d) $n > 2, m = n$

Comprehension Type

For Questions 10 and 11

Let $f(x)$ be the fourth degree polynomial such that $f'(0) = -6$, $f(0) = 2$ and $\lim_{x\to 1}\frac{f(x)}{(x-1)^2} = 1$

10. The value of $f(2)$ is

(a) 1 (b) 0 (c) 2 (d) 3

11. The value of $f'(2)$ is

(a) 4 (b) 5 (c) 6 (d) 7

Answers Key

Single Correct Answer Type

1. (b) **2.** (d) **3.** (c) **4.** (c) **5.** (d)
6. (a) **7.** (b) **8.** (d) **9.** (c)

Comprehension Type

10. (c) **11.** (c)

DPP 2.5

Finding the Unknown

Single Correct Answer Type

1. Number of integral values of λ for which

$\lim_{x\to 1} \sec^{-1}\left(\frac{\lambda^2}{\log_e x} - \frac{\lambda^2}{x-1}\right)$ does not exist is

(a) 1 (b) 2 (c) 3 (d) 4

2. If $\lim_{x\to 0} \frac{e^{ax} - e^x - x}{x^2} = b$ (finite), then

(a) $a = 2, b = 0$ (b) $a = 0, b = \frac{3}{2}$

(c) $a = 2, b = \frac{3}{2}$ (d) $a = 0, b = 2$

3. If $\lim_{x\to 0} \frac{x^3}{\sqrt{a+x}\,(bx - \sin x)} = 1, a > 0$, then $a + b$ is equal to

(a) 36 (b) 37 (c) 38 (d) 40

4. If $\lim_{x\to\infty} x \log_e \left(\begin{vmatrix} \alpha/x & 1 & \gamma \\ 0 & 1/x & \beta \\ 1 & 0 & 1/x \end{vmatrix}\right) = -5$, where α, β, γ are finite real numbers, then

(a) $\alpha = 2, \beta = 1, \gamma \in R$

(b) $\alpha = 2, \beta = 2, \gamma = 5$

(c) $\alpha \in R, \beta = 1, \gamma \in R$

(d) $\alpha \in R, \beta = 1, \gamma = 5$

Multiple Correct Answers Type

5. If $\lim_{x\to 0} \frac{ae^x + b\cos x + ce^{-x}}{e^{2x} - 2e^x + 1} = 4$, then

(a) $a = 2$ (b) $b = -4$

(c) $c = 2$ (d) $a + b + c = -8$

6. If $a \in I$, then value of a for which $\lim_{x\to a} \frac{\tan([x^3] - [x]^3)}{(x-a)^3}$ exists finitely, is/are

(a) 0 (b) 1 (c) −1 (d) 2

Comprehension Type

For Questions 7 and 8

$$L = \lim_{x\to 0} \frac{\sin(\sin x) - \sin x}{ax^5 + bx^3 + c} = -\frac{1}{12}$$

7. The value/values of a is

(a) $\in R$ (b) 2 (c) 0 (d) 1

8. The value/values of b is

(a) $\in R$ (b) 2 (c) 0 (d) 1

For Questions 9 and 10

If $f(x) = \lim_{n\to\infty} \frac{(x^2 + ax + 1) + x^{2n}(2x^2 + x + b)}{1 + x^{2n}}$ and $\lim_{x\to\pm 1} f(x$ exists, then

9. The value of a is

(a) −1 (b) 1 (c) 0 (d) 2

10. The value of b is

(a) −1 (b) 1 (c) 0 (d) 2

Answers Key

Single Correct Answer Type

1. (c) **2.** (c) **3.** (b) **4.** (d)

Multiple Correct Answers Type

5. (a, b, c) **6.** (a, b)

Comprehension Type

7. (a) **8.** (b) **9.** (b) **10.** (c)