CHAROTAR UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF MATHEMATICAL SCIENCES

SEMESTER 3 B.Tech. CE, IT, CSE

DISCRETE MATHEMATICS AND ALGEBRA

MA253

UNIT 1

PREDICATE CALCULUS

OUTLINE OF UNIT 1 – PREDICATE CALCULUS

- **PROPOSITION, TYPES OF PROPOSITION, TAUTOLOGY, CONTRADICTIONS.**
- **CONNECTIVES, TYPES OF CONNECTIVES AND ITS PROPERTIES.**
- **LOGICAL EQUIVQLENCE, VERIFICATION USING TRUTH TABLE.**
- **CONVERSE, INVERSE AND CONTRAPOSITIVE.**
- **MINIMAL FUNCTIONALLY COMPLETE SET OF CONNECTIVES.**
- **NORMAL FORMS: DNF, CNF, PDNF, PCNF.**
- **LOGIC IN PROOF.**

SENTENCE:

A collection of words making a complete grammatical structure with meaning and sense is known as sentence.

PROPOSITION:

A declarative sentence which can be either true or false is known a Proposition. It is also known as statement.

Consider, for example, the following sentences in English

1. New Delhi is a capital of India.
2. The square of 4 is 16.
3. Bring me coffee.
4. Where are you going?
5. This statement is false.

The statement 1-2 is proposition as the sentence 1 and 2 are true. To sentence 3 we cannot decide whether it is true or false as it is not a declarative sentence (Command). Sentence 4 is a question so its not a proposition. Sentence 5 has both true as well as false value so 4 is not a proposition.

Propositions are denoted by *p*, *q*, *r*... or P, Q, R... and also known as propositional variables.

TRUTH VALUE:

The truth or falsity of a statement is known as its truth value. (True or False)

COMPOUND STATEMENT:

A proposition which is combination of two or more propositional variables is known as compound statement. It is also known as **molecular** or **composite statement**.

ATOMIC STATEMENT:

A proposition consisting of only a single propositional variable or a single propositional constant is known as atomic statement. It is also known as **primary** or **primitive proposition.**

TRUTH TABLE:

A truth table is a table that shows the truth value of a compound proposition for all possible value.

CONNECTIVES (Basic logical operations):

The words or phrases which are used to form a proposition are known as connectives. There are five basic connectives: Conjunction, Disjunction, Negation, Conditional and Bi-conditional.

<u>CONJUNCTION</u>: (AND)

If p and q are two statements then conjunction of p and q is the compound statement of the form " p and q" and it is denoted by $p \wedge q$, which is true when both p and q are true.

<u>Truth Table:</u>

$\boldsymbol{p}$	$\boldsymbol{q}$	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

<u>DISJUNCTION</u>: (OR)

If p and q are two statements then disjunction of p and q is the compound statement of the form " p or q" and it is denoted by $p \vee q$, which is false when both p and q are false.

<u>Truth Table:</u>

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

<u>NEGATION</u>: (NOT)

If p is any proposition, the negation of p is denoted by $\sim p$ or $\neg p$, is a proposition which is false when p is true and true when p is false. It is also known as **<u>unary operator.</u>**

<u>Truth Table:</u>

p	$\sim p$
T	F
F	T

CONDITIONAL:

If p and q are two statement then conditional statement of p and q is the compound statement of the form “ if p then q” and it is denoted by $\boldsymbol{p \to q}$,which is false when p is true and q is false.

Truth Table:

$\boldsymbol{p}$	$\boldsymbol{q}$	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

<u>Remark</u>:

In $p \rightarrow q$, the proposition p is called **antecedent or hypothesis** and proposition q is called **consequent or conclusion**.

The connective **if........then** can be also read as follows:

p implies q.

- **p is sufficient for q.**
- **p only if q.**
- **q is necessary for p.**
- **q if p.**
- **q follows from p.**
- **q is consequence of p.**
- $\boldsymbol{p \rightarrow q \equiv \sim p \vee q}$

<u>BI-CONDITIONAL</u>:

If p and q are two statement then bi-conditional statement of p and q is the compound statement of the form "if p only if q" and it is denoted by $p \leftrightarrow q$, which true when either p and q both are true or false simultaneously.

<u>Truth Table:</u>

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

CONVERSE, CONTRA POSITIVE AND INVERSE OF AN IMPLICATION:

There are some related implication. They are as follows

- When $p \longrightarrow q$ an implication, then converse of $p \longrightarrow q$ is an implication $q \longrightarrow p$.
- When $p \longrightarrow q$ an implication, then contra positive of $p \longrightarrow q$ is an implication $\sim q \longrightarrow \sim$ p.
- When $p \longrightarrow q$ an implication, then inverse of $p \longrightarrow q$ is an implication $\sim p \longrightarrow \sim q$.

Remark: A conditional proposition and its converse or inverse are not logically equivalent. On other hand, a conditional proposition and its contra positive are logically equivalent (Can be check using the truth table).

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The importance of the contra positive derives from the fact that mathematical theorems in the form $p \longrightarrow q$ can sometimes be proved easily when restarted in the form $\sim q \longrightarrow \sim p$.

Example

Prove that if x^2 is divisible by 4, then x is even.

Solution:

Let p: x^2 is divisible by 4 and q: x is even.

The implication is of the form $p \longrightarrow q$. The contra positive is $\sim q \longrightarrow \sim p$, which state in words: If x is odd, then x^2 is not divisible by 4.

The proof of contra positive is easy.

Since x is odd , x=2k+1 , for some integer k.

Hence, $x^2=(2k+1)^2=4k^2+4k+1=4(k^2+k+1/4)$.

Since k^2+k is an integer, $k^2+k+1/4$ is not integer therefore x^2 is not divisible by 4.

<u>Example:</u> State Converse, Contra positive and Inverse of the following statement:

1) If it rain then the crop will grow.

Solution : p: It rains and q: The crop will grow.

<u>Converse</u> ($q \longrightarrow p$) If the crop grow, then there has

been rain.

<u>Contra positive</u> ($\sim q \longrightarrow \sim p$)**:** If the crop do not grow, then there has been no rain.

<u>Inverse</u> ($\sim p \longrightarrow \sim q$)**:** If it does not rain, then crop will not grow.

2)If a triangle is not isosceles then it is not equilateral.

<u>Solution:</u>

p : A triangle is not isosceles

q : It is not equilateral.

<u>Converse</u> ($q \longrightarrow p$): If a triangle is not equilateral, then it is not isosceles.

<u>Contra positive</u> ($\sim q \longrightarrow \sim p$): If the triangle is equilateral, then it is isosceles.

<u>Inverse</u> ($\sim p \longrightarrow \sim q$) : If a triangle is isosceles then it is equilateral.

PRATICE EXAMPLE

Example: **State Converse, Contra positive and Inverse of the following statement:**

1) If a triangle is isosceles, then two of its sides are equal.

2) If there is no unemployment in India , then the Indian's won't go to the USA for employment.