CONVERSE, CONTRA POSITIVE AND INVERSE OF AN IMPLICATION:

There are some related implication. They are as follows

- When $p \longrightarrow q$ an implication, then converse of $p \longrightarrow q$ is an implication $q \longrightarrow p$.
- When $p \longrightarrow q$ an implication, then contra positive of $p \longrightarrow q$ is an implication $\sim q \longrightarrow \sim$ p.
- When $p \longrightarrow q$ an implication, then inverse of $p \longrightarrow q$ is an implication $\sim p \longrightarrow \sim q$.

Remark: A conditional proposition and its converse or inverse are not logically equivalent. On other hand, a conditional proposition and its contra positive are logically equivalent (Can be check using the truth table).

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The importance of the contra positive derives from the fact that mathematical theorems in the form $p \longrightarrow q$ can sometimes be proved easily when restarted in the form $\sim q \longrightarrow \sim p$.

Example

Prove that if x^2 is divisible by 4, then x is even.

Solution:

Let p: x^2 is divisible by 4 and q: x is even.

The implication is of the form $p \longrightarrow q$. The contra positive is $\sim q \longrightarrow \sim p$, which state in words: If x is odd, then x^2 is not divisible by 4.

The proof of contra positive is easy.

Since x is odd , x=2k+1 , for some integer k.

Hence, $x^2=(2k+1)^2=4k^2+4k+1=4(k^2+k+1/4)$.

Since k^2+k is an integer, $k^2+k+1/4$ is not integer therefore x^2 is not divisible by 4.

<u>Example:</u> State Converse, Contra positive and Inverse of the following statement:

1) If it rain then the crop will grow.

Solution : p: It rains and q: The crop will grow.

<u>Converse</u> ($q \longrightarrow p$) If the crop grow, then there has been rain.

<u>Contra positive</u> ($\sim q \longrightarrow \sim p$)**:** If the crop do not grow, then there has been no rain.

<u>Inverse</u> ($\sim p \longrightarrow \sim q$)**:** If it does not rain, then crop will not grow.

2)If a triangle is not isosceles then it is not equilateral.

<u>Solution:</u>

p : A triangle is not isosceles

q : It is not equilateral.

<u>Converse</u> ($q \longrightarrow p$): If a triangle is not equilateral, then it is not isosceles.

<u>Contra positive</u> ($\sim q \longrightarrow \sim p$): If the triangle is equilateral, then it is isosceles.

<u>Inverse</u> ($\sim p \longrightarrow \sim q$) : If a triangle is isosceles then it is equilateral.

PRATICE EXAMPLE

Example: **State Converse, Contra positive and Inverse of the following statement:**

1) If a triangle is isosceles, then two of its sides are equal.

2) If there is no unemployment in India , then the Indian's won't go to the USA for employment.

Example: If P represents “This book is good“ and Q represent “This book is cheap”, then write the following sentences in symbolic form.

Solution: Here

Sentences	Symbol
This book is good and cheap.	$P \wedge Q$
This book is not good and cheap.	$\sim P \wedge Q$
This book is costly but good.	$\sim Q \wedge P$
This book is neither good nor cheap.	$\sim P \wedge \sim Q$
This book is either good or cheap.	$P \vee Q$

Example: If P: It is raining, Q: I have the time, R: I will go to a movie. Write the sentences in English corresponding to the following propositional form $(\sim P \wedge Q) \leftrightarrow R$, $(Q \to R) \wedge (R \to Q)$ $\sim Q \vee R$ and $R \to (\sim P \wedge Q)$.

$(\sim P \wedge Q) \leftrightarrow R$	I will go to a movie if and only if it is not raining and I have the time
$(Q \to R) \wedge (R \to Q)$	I will go to movie if and only if I have the time.
$\sim Q \vee R$	It is not the case that I have the time or I will go to a movie
$R \to (\sim P \wedge Q)$	I will go to a movie only if it is not raining and I have the time

Example: Construct a truth table for each compound proposition.

1) $p \wedge (\sim q \vee q)$

Solution :

$\boldsymbol{p}$	$\boldsymbol{q}$	$\boldsymbol{\sim q}$	$(\sim q \vee q)$	$p \wedge (\sim q \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

2) $(P \vee Q) \wedge (P \rightarrow Q) \wedge (Q \rightarrow P)$

Solution :

P	**Q**	$(P \vee Q)$	$P \rightarrow Q$	$(P \vee Q) \wedge (P \rightarrow Q)$	$Q \rightarrow P$	$(P \vee Q) \wedge (P \rightarrow Q) \wedge (Q \rightarrow P)$
T	T	T	T	T	T	T
T	F	T	F	F	T	F
F	T	T	T	T	F	F
F	F	F	T	F	T	F

PRATICE EXAMPLE

Obtain the truth table for the following:

$$1)\left(P \vee Q\right) \rightarrow (P \vee R) \rightarrow (Q \vee R)$$

$$2)(\sim P \leftrightarrow \sim Q) \leftrightarrow Q \leftrightarrow R$$

$$3)\left(P \wedge Q\right) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$$

$$4)\, p \wedge \left(q \vee r\right)$$

$$5) \sim (p \vee q) \vee (\sim p \wedge \sim q)$$

ALGEBRA OF PROPOSITION

Idempotent law	$p \vee p \equiv p$
	$p \wedge p \equiv p$
Associative Law	$p \vee (q \vee r) \equiv (p \vee q) \vee r$
	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$
Commutative Law	$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$
Distributive Law	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
De Morgan's Law	$\sim(p \vee q) \equiv \sim p \wedge \sim q$ $\sim(p \wedge q) \equiv \sim p \vee \sim q$

ALGEBRA OF PROPOSITION (Continued)

Involution law	$\sim(\sim p) \equiv p$
Complement Law	$\sim F \equiv T$
	$\sim T \equiv F$
	$p \vee \sim p \equiv T$
	$p \wedge \sim p \equiv F$
Identity Law	$p \vee F \equiv p$
	$p \vee T \equiv T$
	$p \wedge T \equiv p$
	$p \wedge F \equiv F$

SOME DERIVED CONNECTIVES

NAND: NAND is the negation of conjunction of two statements. Assume p and q are any two statement then NAND of p and q is a proposition which is false when both p and q are true otherwise true. It is denoted by $\boldsymbol{p \uparrow q}$.

$\boldsymbol{p}$	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

SOME DERIVED CONNECTIVES (Continued)

NOR: NOR is negation of disjunction of two statements. Assume p and q be two proposition .NOR of p and q is a proposition which is true when both p and q are false, otherwise false. It is denoted by It is denoted by $\boldsymbol{p \downarrow q}$.

$\boldsymbol{p}$	$\boldsymbol{q}$	$\mathbf{p \downarrow q}$
T	T	F
T	F	T
F	T	T
F	F	F

<u>SOME DERIVED CONNECTIVES (Continued)</u>

<u>**XOR (Exclusive or)**</u>**:** It is a proposition that is true when exactly one of p and q is true but not both and is false otherwise. It is denoted by $p \oplus q$.

$\boldsymbol{p}$	$\boldsymbol{q}$	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

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LOGICAL EQUIVALENCE

If two propositions P and Q have the same truth values in every possible case, the propositions are **logically equivalent**. It is denoted by $\boldsymbol{p \equiv q}$.

To test whether two proposition P and Q are logically equivalent

Method 1: Using Truth Table.

Method 2: Using Algebra of proposition (Properties of Proposition).

<u>Method 1: Using Truth Table</u>

Step 1: Construct the truth Table for compound statement P and Q.

Step 2: Check each combination of truth values of the propositional variables to see whether the value of P is same as the truth value of Q. If in each row the truth value of P is the same as the truth value of Q, then P and Q are logically equivalent.

Example: Use truth tables to prove

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

1	2	3	4	5	6	7	8
p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$(p \vee q)$	$(p \vee r)$	$(p \wedge q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T
F	F	F	F	F	F	F	F
F	F	T	F	F	F	T	F
F	T	F	F	F	T	F	F
T	F	F	F	T	T	T	T

Since the entries in 5 and 8 columns is same, therefore

$$p \vee (q \wedge r) \equiv (p \wedge q) \wedge (p \vee r)$$

Example: Use truth tables to prove

$$\sim(p \wedge q) \equiv (\sim p \vee \sim q)$$

1	2	3	4	5	6	7
p	q	$(p \wedge q)$	$\sim(p \wedge q)$	$\sim p$	$\sim p$	$(\sim p \vee \sim q)$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Since the entries in 4 and 7 columns is same, therefore $\sim(p \wedge q) \equiv (\sim p \vee \sim q)$.

Method 2: Using Algebra of proposition.

Example: Show that $(\boldsymbol{P} \wedge \boldsymbol{Q}) \vee (\boldsymbol{P} \wedge \sim\boldsymbol{Q}) \equiv \boldsymbol{P}$.

Solution: $(\boldsymbol{P} \wedge \boldsymbol{Q}) \vee (\boldsymbol{P} \wedge \sim\boldsymbol{Q}) \equiv \boldsymbol{P} \wedge (\boldsymbol{Q} \vee \sim\boldsymbol{Q})$

(Using Distributive Law)

$\equiv \boldsymbol{P} \wedge \boldsymbol{T}$

(By using Complement Law)

$\equiv P$

(By using Identity Law)

$= R.H.S$

$\therefore\ (\boldsymbol{P} \wedge \boldsymbol{Q}) \vee (\boldsymbol{P} \wedge \sim\boldsymbol{Q}) \equiv \boldsymbol{P}$

Example: Show that $(P \to Q) \wedge (R \to Q) \equiv (P \vee R) \to Q$.

Solution: $(P \to Q) \wedge (R \to Q) \equiv (\sim P \vee Q) \wedge (\sim R \vee Q)$

(Equivalent form of $P \to Q \equiv \sim P \vee Q$)

$\equiv (Q \vee \sim P) \wedge (Q \vee \sim R)$

(Commutative Law)

$\equiv Q \vee (\sim P \wedge \sim R)$

(Distributive Law)

$\equiv Q \vee (\sim(P \vee R))$

(De Morgan's Law)

$\equiv (\sim(P \vee R)) \vee Q$

(Commutative Law)

$\equiv (P \vee R) \to Q$

(Equivalent form of $P \to Q \equiv \sim P \vee Q$)

Hence Proved.

Example: Show that $\boldsymbol{\big(\sim P \wedge (\sim Q \wedge R)\big) \vee (Q \wedge R) \vee (P \wedge R) \equiv R}$**.**

Solution: L.H.S: $\boldsymbol{\big(\sim P \wedge (\sim Q \wedge R)\big) \vee (Q \wedge R) \vee (P \wedge R)}$

$$\equiv \big((\sim P \wedge \sim Q) \wedge R\big) \vee (Q \wedge R) \vee (P \wedge R)$$

(Associative Law)

$$\equiv (\sim(\boldsymbol{P} \vee \boldsymbol{Q}) \wedge R) \vee (Q \wedge R) \vee (P \wedge R)$$

(De Morgan Law)

$$\equiv (\sim(P \vee Q) \wedge R) \vee \big((Q \vee P) \wedge R\big)$$

(Distributive Law)

$$\equiv (\sim(P \vee Q) \wedge R) \vee \big((P \vee Q) \wedge R\big)$$

(Commutative Law)

$$\equiv \big(\sim(P \vee Q) \vee (P \vee Q)\big) \wedge R$$

$$\equiv T \wedge R$$ (Distributive Law)

$$\equiv R$$ (Identity Law)

Hence Proved.

PRATICE EXAMPLE

Example: Show that the following statements are logically equivalent using truth table.

1. $p \leftrightarrow q \equiv \big((p \rightarrow q) \wedge (q \rightarrow p)\big)$
2. $p \leftrightarrow q \equiv \big((p \vee q) \rightarrow (p \wedge q)\big)$
3. $(p \vee q) \rightarrow r \equiv \big((p \rightarrow r) \wedge (q \rightarrow r)\big)$

Show that the following statements are logically equivalent without using truth table.

4. $p \leftrightarrow q \equiv (p \vee q) \rightarrow (p \wedge q)$