# Math/Econ 317 HW5
# Section III 2023
# Duke Kunshan University

**Problem 1** (10 points). *Textbook (new) exercise 3.5.*

**Problem 2** (10 points). *Textbook (new) exercise 3.6.*

**Problem 3** (30 points). *Consider the simple linear regression*

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i, \tag{1}$$

*where $x_i$ are deterministic and $\epsilon_i$ are random variables with mean zero. Suppose we use the least square to estimate $\widehat{\beta}_0$ and $\widehat{\beta}_1$. We know $\mathbb{E}[\widehat{\beta}_0] = \beta_0$ and $\mathbb{E}[\widehat{\beta}_1] = \beta_1$.*

- *Calculate Var$(\widehat{\beta}_1)$.*
- *Calculate Var$(\widehat{\beta}_0)$*

**Problem 4** (15 points). *Consider the simple linear regression*

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i, \tag{2}$$

*where $x_i$ are deterministic and $\epsilon_i$ are random variables with mean zero. Suppose we use the least square to estimate $\widehat{\beta}_0$ and $\widehat{\beta}_1$, and we calculate the realized residual error as*

$$\widehat{\epsilon}_i = Y_i - \widehat{\beta}_0 - \widehat{\beta}_1 X_i. \tag{3}$$

*Calculate $\mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n}\widehat{\epsilon}_i^2\right]$.*

**Problem 5** (20 points). *Consider the simple regression framework*

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i, \tag{4}$$

*where $\epsilon_i$ follows i.i.d. normal distributions with mean zero. Estimate $\widehat{\beta}_0$ and $\widehat{\beta}_1$ using the least squares method. Numerically verify that the estimation of $\widehat{\beta}_1$ is consistent by showing that the error decreases as sample numbers increase. You can numerically verify the unbiasedness and accuracy by increasing the number of tries in the Monte Carlo simulation while maintaining the same number of $X_i$. Make a plot of all convergence results.*

**Problem 6** (15 points). *Consider the simple Capital Asset Pricing Model (CAPM) in the lecture. Suppose the daily return of the market factor follows normal distribution, $R_M \sim \mathcal{N}(\mu_M, \sigma_M^2)$. Suppose we consider $n$ stocks, each daily return has the relationship with $R_M$ as*

$$R_i = \alpha + \beta_i R_M + \epsilon_i, \tag{5}$$

*where $\epsilon_i \sim \mathcal{N}(0, \sigma_2^2)$ and are independent from each other. That is, for simplicity, we assume all stocks have the same $\alpha$ and $\sigma_2$, but they are generated independently. Let $n = 100$. The daily stepsize is $\Delta t = \frac{1}{252}$. Let $\mu_M = 0.08\Delta t$, $\sigma_M = 0.16\sqrt{\Delta t}$, and $\sigma_2 = 0.1\sqrt{\Delta t}$. Assume $\beta_i$ follows the uniform distribution such that $\beta_i \sim \mathcal{U}([0, 2])$. We collect the daily return for 1 year (252 days) and for all stocks. We use the regression to estimate $\widehat{\alpha}_i$ and $\widehat{\beta}_i$ and we find that $\max(\widehat{\alpha}_i) = 0.2\Delta t$. Can we conclude $\alpha > 0$? Answer the question with the help of simulations.*