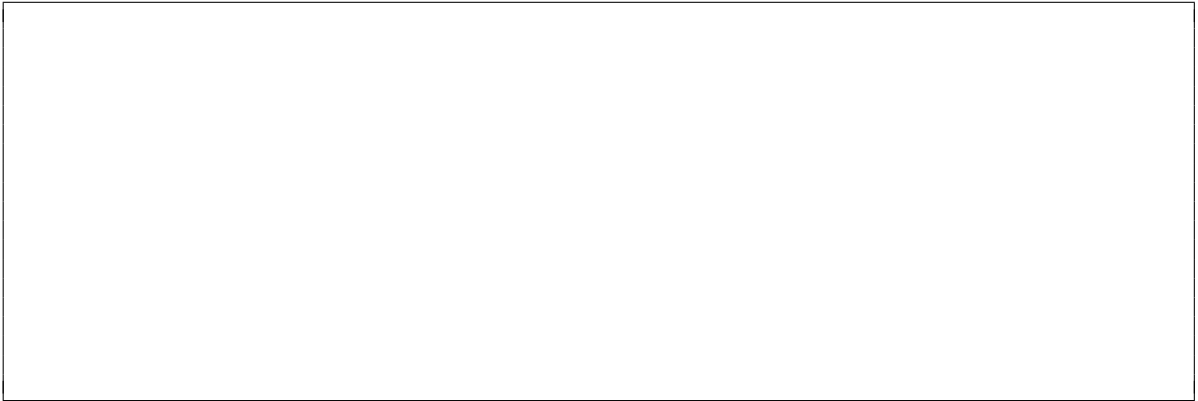1. Consider the adjacency list of a digraph G below:

   $a:$ b
   $b:$ c, e
   $c:$ b, e, f
   $d:$ e
   $e:$ b, c, d, f
   $f:$ c, e, g, h, i, j
   $g:$ f, h
   $h:$ f, j
   $i:$ k

   (A) Draw the digraph G.

   (B) What is the source node and sink node of G?

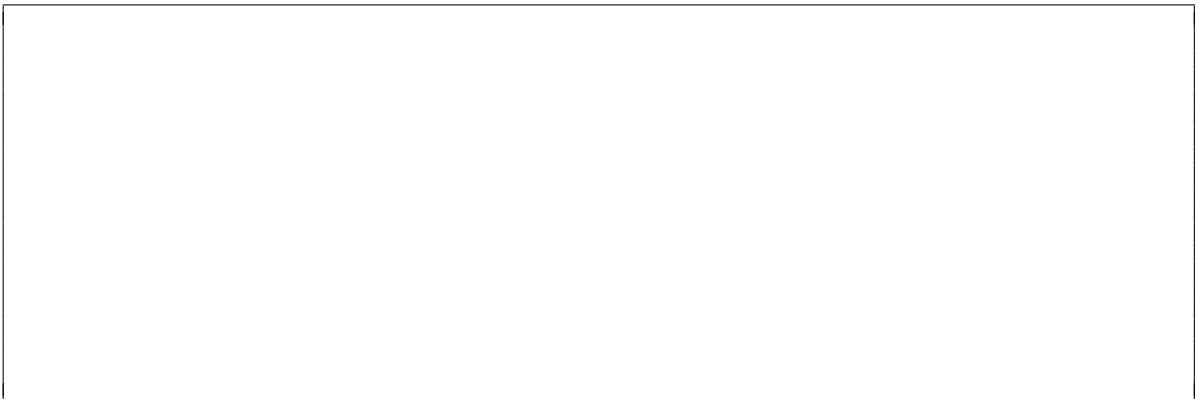   (C) What is the adjacency matrix of this digraph G?

2. Consider the same digraph G with the adjacency list above.

   (A) Find the depth first tree rooted at b, provided the vertices are ordered alphabetically. Please use the convention that nodes are chosen in ascending alphabetic order when there is a choice of nodes. Please show your working to justify your answer.

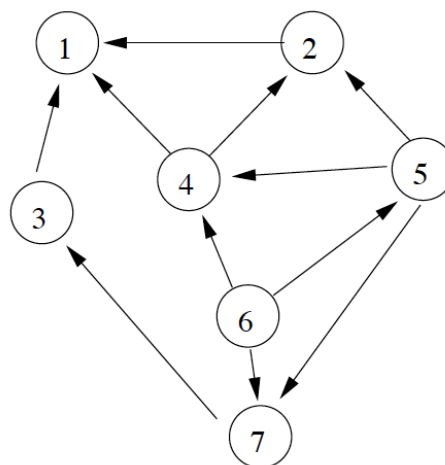   (B) Show the state of the stack after each change in its state.

   (C) Identify the type of each arc, the type being tree, forward, back, or cross.

3. Consider the same digraph G with the adjacency list above.

    (A) Find the breath-first tree of G, rooted at b, provided the vertices are ordered alphabetically. Please use the convention that nodes are chosen in ascending alphabetic order when there is a choice of nodes. Please show your working to justify your answer.

    (B) Show the state of the queue after each change in its state.

    (C) Identify the type of each arc, the type being tree, forward, back, or cross.

4. Consider the following directed graph G. Does the graph G have a topological sorting? If so give one with your working, if not why not?

5. Let v be a node of G. Which of the following lists the main steps in finding the length of a shortest directed cycle in a digraph G that contains v?

   (A) Run BFS starting from v and stop when a back arc (x, v) is found.

   (B) Run DFS starting from v and stop when a cross arc (x, v) is found.

   (C) Run BFS starting from v and stop when a cross arc (x, v) is found.

   (D) Run DFS starting from v and stop when a back arc (x, v) is found.

   (E) Run DFS and count the number of sink nodes in G.

6. Consider the problem of finding girth. Can we use Dijkstra's algorithm to calculate the girth? If yes, how can we achieve that?

7. Which of the following statements is FALSE?

   (A) Zero-indegree sorting can be used to find a topological order of a graph.

   (B) Suppose DFS is run on a digraph G. Then G is acyclic if and only if there are no cross arcs.

   (C) BFS can be used to find the connected components of a graph.

   (D) The girth of a directed graph can be strictly larger than its directed girth.

   (E) A digraph has a topological ordering if and only if it is acyclic.

8. We have mentioned an algorithm for finding the directed girth:

    1. For all nodes $v \in V(G)$ do:

        (a) Run BFSVISIT from node v.

        (b) As soon as the algorithm finds a back arc of the form (x, v), terminate, recording the length of such a cycle c, which will be $h + 1$, where h is the depth of node x in the given search tree.

    2. Return smallest c.

    Answer the following two questions:

    (A) Why is there no need to continue to the end of the level before halting the traversal?

    (B) What is the running time for doing so?