Tutorial

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COMPCSI220: WEEK 12

THE UNIVERSITY OF AUCKLAND
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NEW ZEALAND

Course Website: https://ankechiang.github.io/cs220_swu.html#week12

OUTLINE

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- Question 2: Bipartite
- Question 3: Shortest Path
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- Question 6: Graph Definition
- Question 7: Minimum Spanning Tree
- Question 8: Minimum Spanning Tree

Question 1

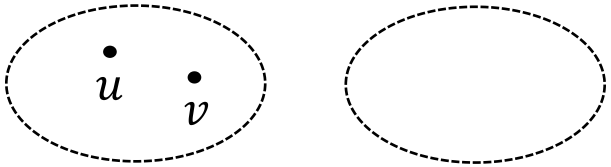
Determine the following statements are True/False for a digraph.

- If there exist $u, v \in G$ such that there is a path from u to v and from v to u, the digraph is strongly connected.

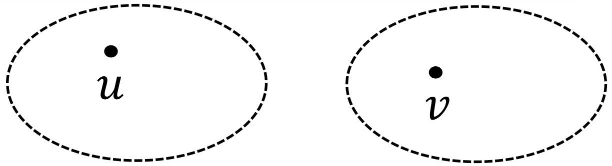
False. A digraph is strongly connected if and only if for every pair of nodes $u, v \in G$, there is a path from u to v and a path from v to u

- If there exist $u, v \in G$ such that there is a path from u to v but not from v to u, the digraph is weakly connected.

False. Consider if its underlying graph has two components, and u, v are from one of the two



Question 1 (Contd.)



Determine the following statements are True/False for a digraph.

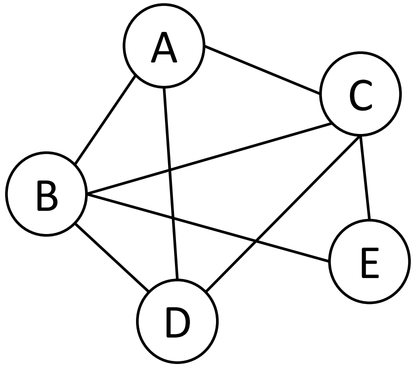
- If there exist $u, v \in G$ such that there is a path from u to v but not from v to u, the digraph contains no strong components.

False. Consider a digraph with two strong components, and only an arc (u, v) links a node u in one component to another node v in the other component.

- If there exist $u, v \in G$ such that there is no path from u to v or there is no from v to u, the digraph is not strongly connected

True. It is the negation of the definition of strongly connected digraph.

Question 2



- Consider the graph with vertices A, B, C, D, E and edges, which of the following statement is true?

{{A, B},{A, C},{A, D},{B, C},{B, D},{B, E},{C, D},{C, E}}

a. The graph has neither a 2-coloring, nor a 3- or a 4-coloring.

b. The graph has a 2- and a 4-coloring but no 3-coloring.

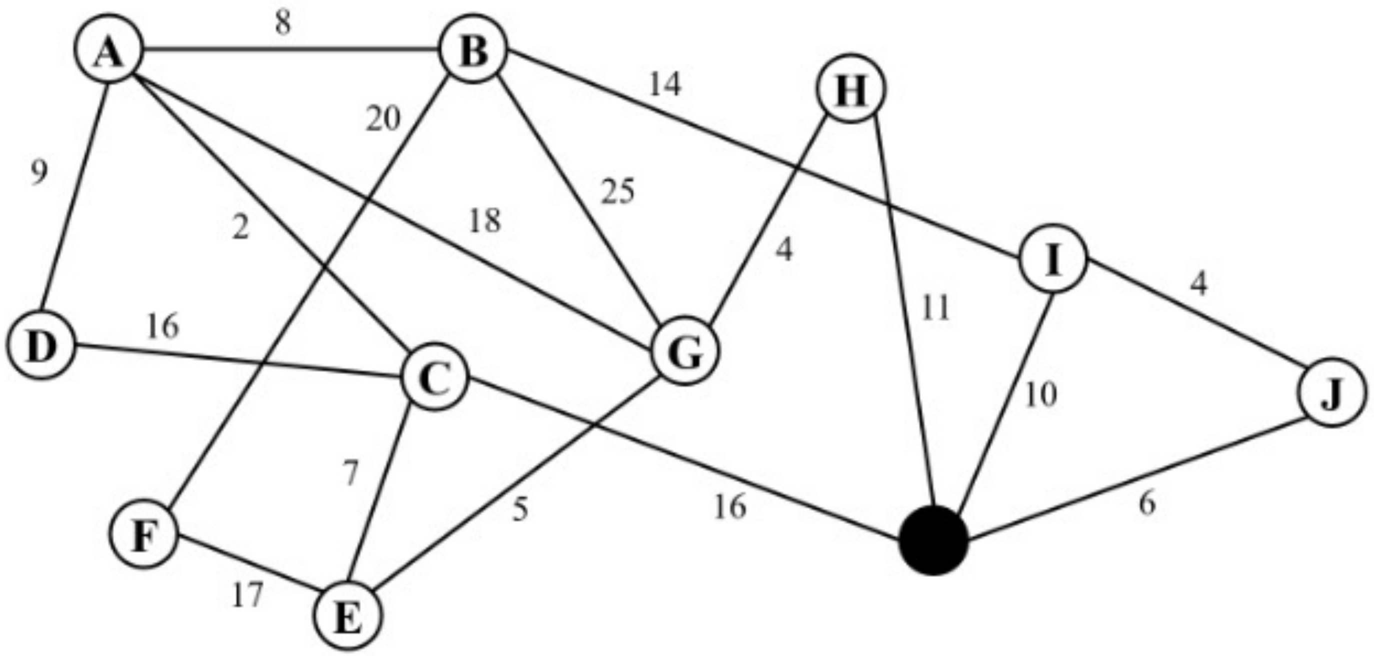
c. The graph has a 4-coloring but no 2- or 3-coloring.

d. The graph has a 2- and a 3-coloring, but no 4-coloringe The graph has a 3- and 4-coloring but no 2-coloring.

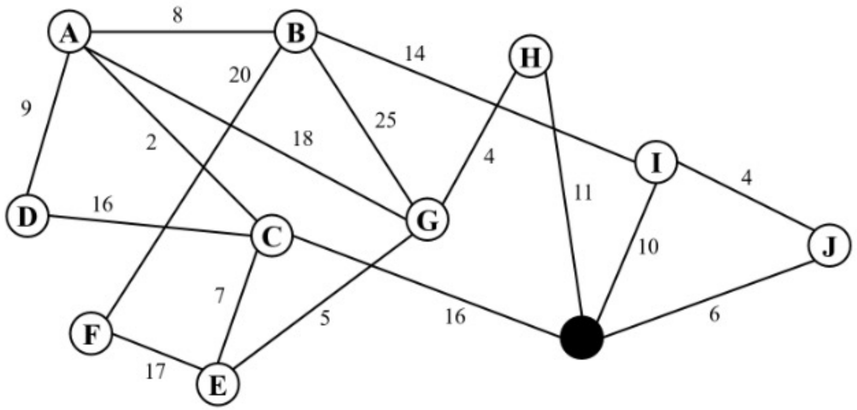
- If we look at vertices A, B, C and D, we see that they are all connected to one another. So just to color A, B, C and D, we need 4 colors.
- E connects to B and C only, so we can color it with the same color as A. Hence the minimum number of colors we need is 4. Therefore, option C is the correct answer.

Question 3

- Find the minimum weight paths from the black vertex to all the other vertices in the graph below using Dijkstra's algorithm. Show the value of the distance vector after each step



Question 3 (Contd.)



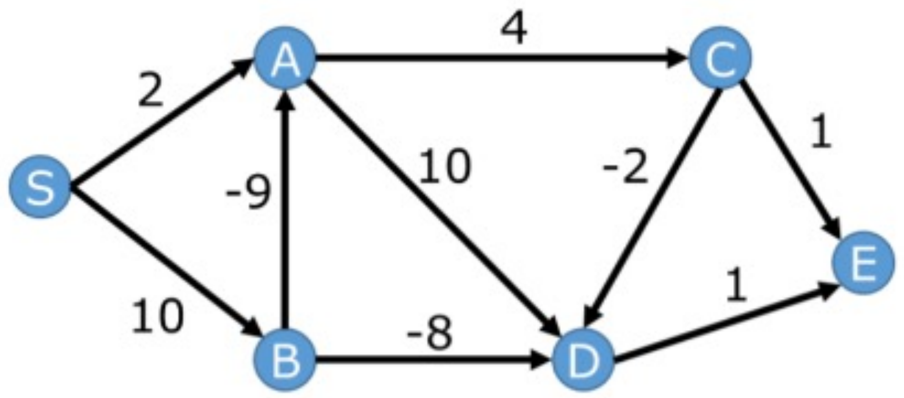
[*, A, B, C, D, E, F, G, H, I, J]

- minimum distance: dist[J] = 6 dist =0,∞,∞,16,∞,∞,∞,∞,11,10,6
- minimum distance: dist[I] = 10 dist =0,∞,24,16,∞,∞,∞,∞,11,10, 6
- minimum distance: dist[H] = 11 dist =0,∞,24,16,∞,∞,∞,15,11, 10, 6
- minimum distance: dist[G] = 15 dist =0,33,24,16,∞,20,∞,15, 11, 10, 6
- minimum distance: dist[C] = 16 dist =0,18,24,16,32,20,∞,15, 11, 10, 6
- minimum distance: dist[A] = 18 dist =0,18,24,16,27,20,∞,15, 11, 10, 6
- minimum distance: dist[E] = 20 dist =0,18,24,16,27,20,37,15, 11, 10, 6
- minimum distance: dist[B] = 24 . . . the following steps lead to no more updates, so the resulting
- dist vector is: dist = 0,18,24,16,27,20,37,15,11,10,6

[*, A, B, C, D, E, F, G, H, I, J]

Question 4

- Given the graph G shown, we find the shortest paths from node S using the Bellman-Ford algorithm. How many iterations does it take before the algorithm converges to the solution?



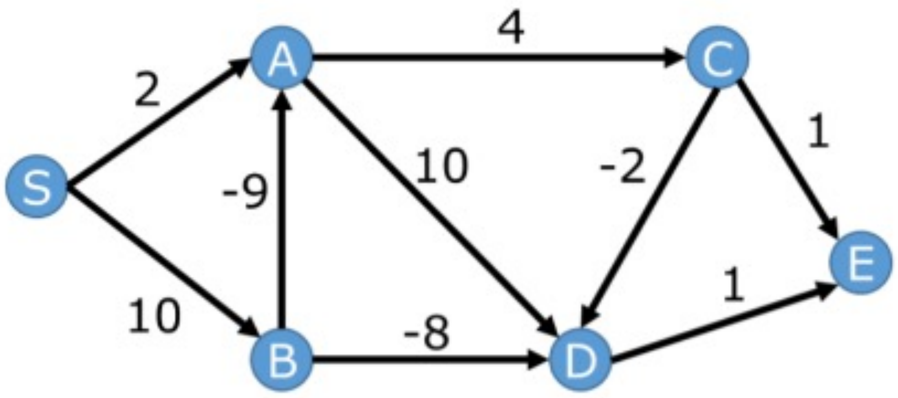
- We calculate the distance array dist[x] after each iteration i. We label S=0, A=1,B=2, C=3, D=4, E=5.

Question 4 (Contd.)

S, A, B, C, D, E

- Initially, we have dist= [0,∞,∞,∞,∞,∞]
- In iteration i = 0, we will do the following:
  - Take the arc (S, A) to update dist[A] = 2.
  - Take the arc (S, B) to update dist[B] = 10
- After iteration 0, dist= [0,2,10,∞,∞,∞]
  - Take the arc (A, C) to update dist[C] = 6.
  - 2. Take the arc (A, E) to update dist[E] = 12.
  - 3. Take the arc (B, A) to update dist[A] = 1.
  - 4. Take the arc (B, D) to update dist[D] = 2
- After iteration 1, dist= [0,1,10,6,2,∞]

S, A, B, C,D, E

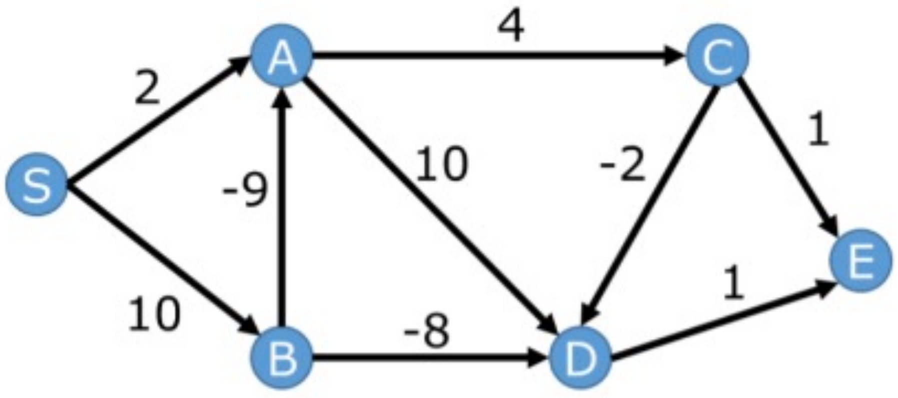


Question 4 (Contd.)

S, A, B, C, D, E

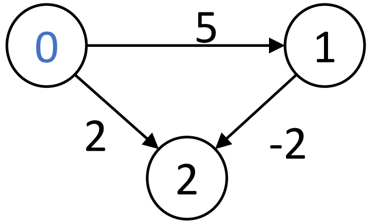
- After iteration 1,dist= [0,1,10,6,2,∞]
  - Take the arc (A, C) to update dist[C] = 5
  - Take the arc (C, E) to update dist[E] = 7
  - Take the arc (D, E) to update dist[E] = 3
- After iteration 2,dist= [0,1,10,5,2,3].

S, A, B, C, D, E



- All other iterations will give you the same distance array. So, three iterations will lead to converge.

Question 5



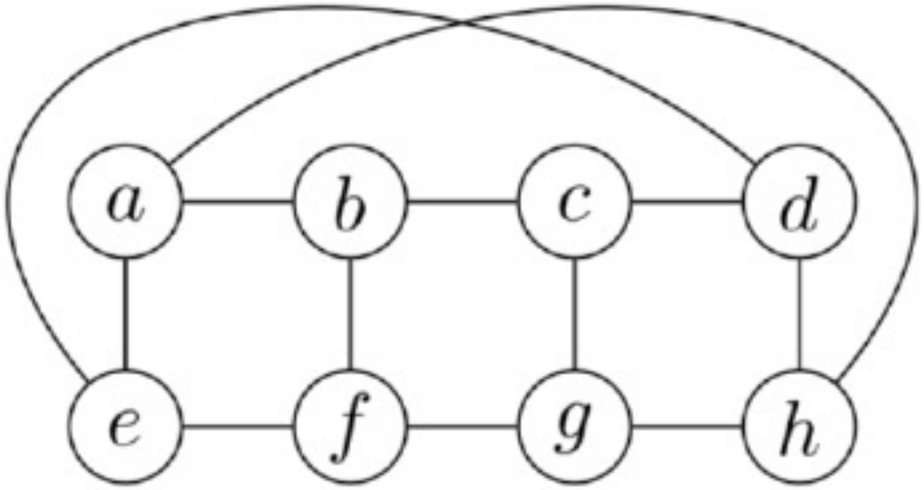
- Consider the digraph G with nodes 0, 1, 2 and arcs (0, 1) with weight 5, (0, 2) with weight 2, (1, 2) with weight -2. If we are solving the SSSP problem for node 0 then which of the following is true?
  A. Dijkstra gives wrong answer; Bellman-Ford gives right answer.
  B. Dijkstra and Bellman-Ford both give right answer.
  C. Dijkstra and Bellman-Ford both give wrong answer.
  D. There is no solution for this digraph.
  E. Dijkstra gives right answer; Bellman-Ford gives wrong answer

Dijkstra's algorithm will get the shortest path from 0 to 2 as 0,2. This gives the correct answer even though there is a negative weight arc.

Question 6

- What is the diameter of the following graph?
  - A. 5
  - B. 4
  - **C. 2**
  - D. 3
  - E. 1

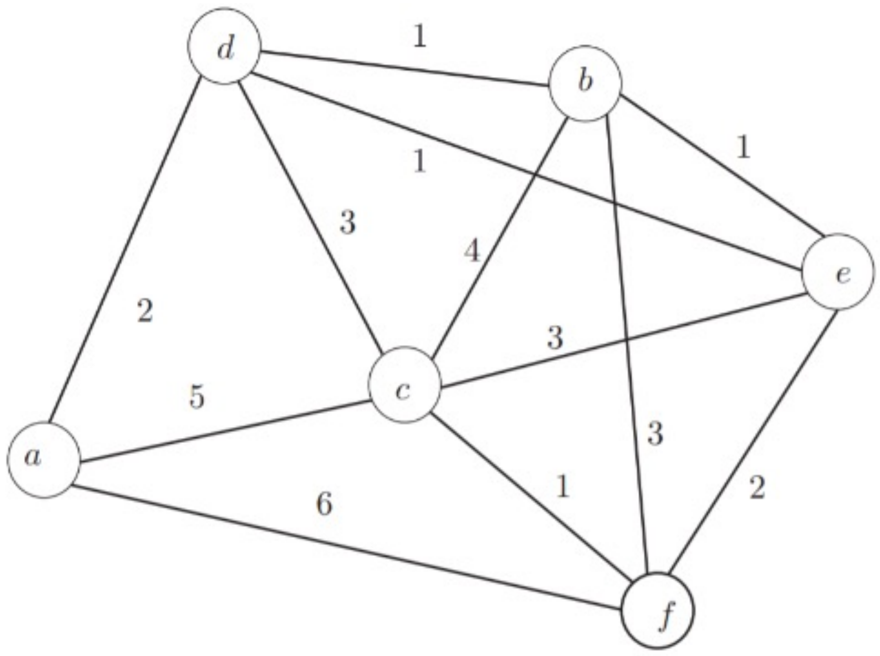
The diameter of a graph (or strongly connected digraph) G, denoted as diam(G), is the maximum eccentricity of the vertices in V(G).



d(a,b)=1	d(b,a)=1		d(h,a)=1
d(a,c)=2	d(b,c)=1		d(h,b)=2
d(a,d)=2	d(b,d)=2		d(h,c)=2
d(a,e)=1	d(b,e)=2	…	d(h,d)=1
d(a,f)=2	d(b,f)=1		d(h,e)=2
d(a,g)=2	d(b,g)=2		d(h,f)=2
d(a,h)=1	d(b,h)=2		d(h,g)=1
→ e(a)=2	→ e(b)=2		→ e(h)=2

Question 7

- For the graph shown below, what is the weight of a minimum spanning tree? Try Kruskal's algorithm for calculating the MST



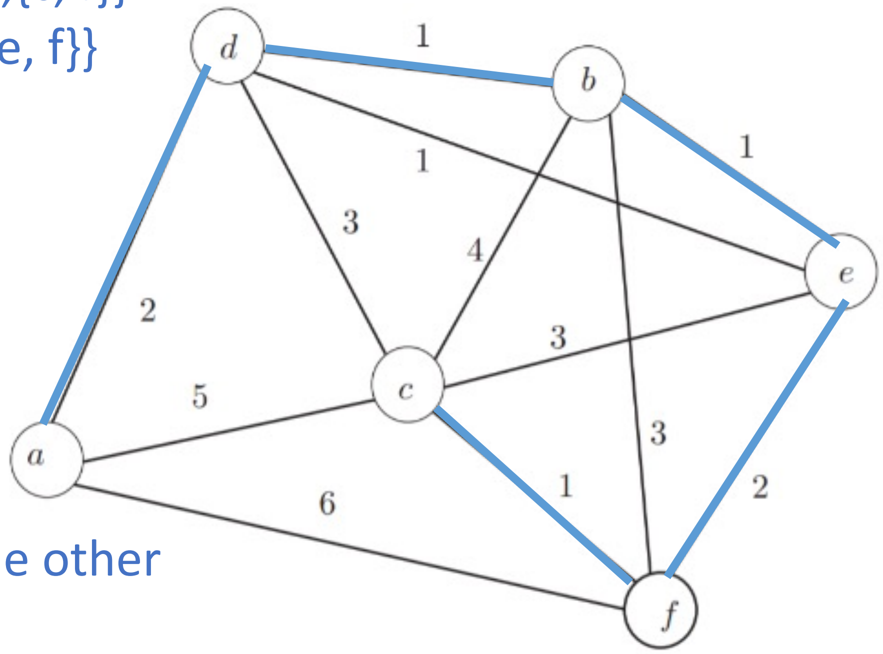
- We can maintain the disjoint sets:{{a},{b},{c},{d},{e},{f}}, and sort the edges by increasing order of the weights:
  (b, d):1, (b, e):1, (c, f):1, (d, e):1, (a, d):2, (e,f):2, (c, d):3, (c, e):3, (b, f):3, (b, c):4, (a, c):5, (a, f):6

- In iteration 1, we pick (b, d) to update the disjoint sets:{{a},{b, d},{c},{e},{f}}
- In iteration 2, we pick (b, e) to update the disjoint sets:{{a},{b, d, e},{c},{f}}

Question 7 (Contd.)

(b, d):1, (b, e):1, (c, f):1, (d, e):1, (a, d):2, (e,f):2, (c, d):3, (c, e):3, (b, f):3, (b, c):4, (a, c):5, (a, f):6

- In iteration 3, we pick (c, f) to update the disjoint sets:{{a},{b, d, e},{c, f}}
- In iteration 4, we pick (a, d) to update the disjoint sets:{{b, a, d, e},{c, f}}
- In iteration 5, we pick (e, f) to update the disjoint sets:{{b, a, c, d, e, f}}



The weight of the MST is 1+1+1+2+2=7.
However, this is not the unique MST of the graph.
By replacing the (b, d) or (b, e) with the edge (d, e), we can obtain the other two MSTs.

Question 8

- An undirected weighted graph G has n vertices. The cost matrix of G is given by an $n \times n$ square matrix whose diagonal entries are all equal to 0 and whose non-diagonal entries are all equal to 2. Which of the following statements is TRUE?

A. G has a minimum spanning tree of weight 0.

B. G has a unique minimum spanning tree of weight 2(n-1).

C. G has multiple minimum spanning trees, each of different weights

D. G has multiple minimum spanning trees, each of weight 2(n-1).

E. G has no minimum spanning tree.

$$\begin{bmatrix} 0 & 2 & \dots & 2 \\ 2 & 0 & \dots & 2 \\ \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & \dots & 0 \end{bmatrix}$$

Cost of Matrix

Resources

- Course Website
  - https://ankechiang.github.io/cs220_swu.html#week11
- Lecture Notes
  - https://github.com/ankechiang/ankechiang.github.io/tree/master/cs220_lecture
- Lecture Recordings
  - https://github.com/ankechiang/ankechiang.github.io/tree/master/cs220_recording