Graph Definitions II

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COMPCSI220: WEEK 11

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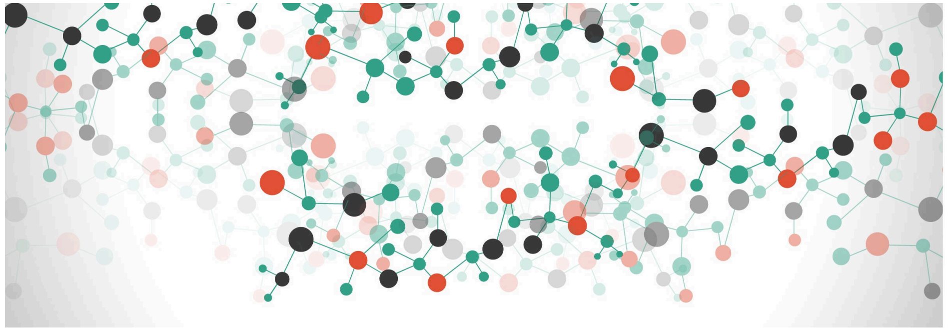
OUTLINE

- Distance
- Eccentricity
- Diameter
- Radius
- Periphery and Center

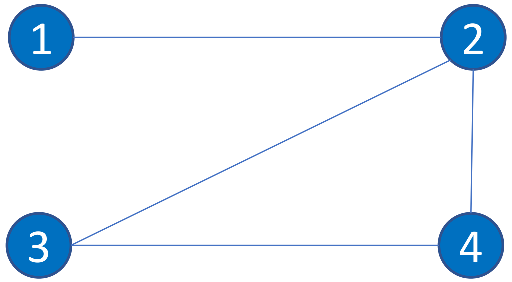


Distance

- **Definition.** Let u and v be two vertices in a graph G, then the distance between u and v, denoted as $d(u, v)$ is the length of shortest path between u and v in G.

- If G is disconnected and u and v are in different components then $d(u, v) = +\infty$

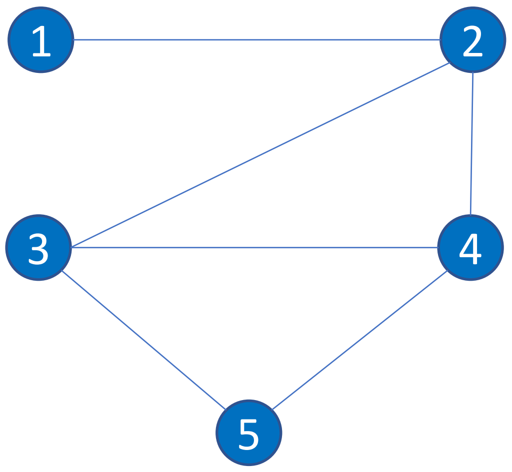
$$d(1,1) = 0$$
$$d(1,2) = 1$$
$$d(1,3) = 2$$



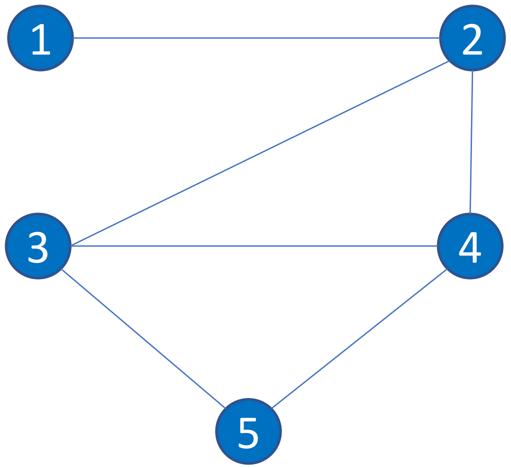
Eccentricity

- **Definition**. The eccentricity of a vertex v in $\mathrm{V(G)}$, denoted as $\mathrm{e}(v)$, is the maximum of the distances between v and any other vertex u in $\mathrm{V(G)}$.

$$\begin{aligned} d(1,1) &= 0 \\ d(1,2) &= 1 \\ d(1,3) &= 2 \\ d(1,4) &= 2 \\ d(1,5) &= 3 \end{aligned}$$



Eccentricity (Contd.)



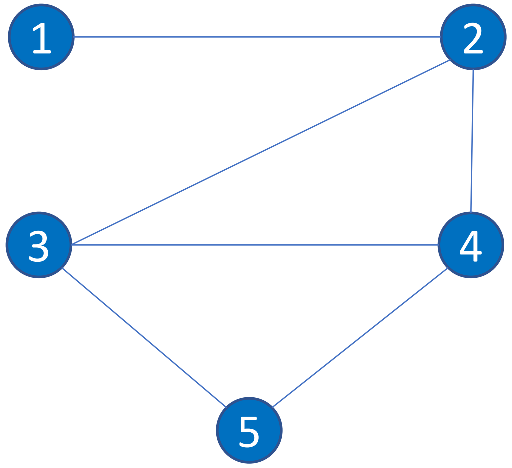
$$
\begin{aligned}
d(1,1) &= 0 \\
d(1,2) &= 1 \\
d(1,3) &= 2 \\
d(1,4) &= 2 \\
d(1,5) &= 3
\end{aligned}
\qquad\qquad
\begin{aligned}
e(1) &= 3 \\
e(2) &= 2 \\
e(3) &= 2 \\
e(4) &= 2 \\
e(5) &= 3
\end{aligned}
$$

Note: Distance is defined on two vertices while eccentricity is defined on a vertex

Diameter and Radius

- **Definition.** The diameter of a graph (or strongly connected digraph) G, denoted as $\mathrm{diam}(\mathrm{G})$, is the maximum eccentricity of the vertices in $\mathrm{V}(\mathrm{G})$.

$$
\begin{aligned}
e(1) &= 3\\
e(2) &= 2\\
e(3) &= 2\\
e(4) &= 2\\
e(5) &= 3
\end{aligned}
$$

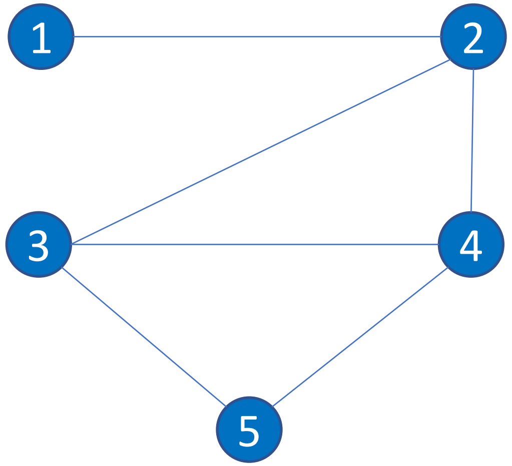


The diameter of this graph is 3

Diameter and Radius (Contd.)

- **Definition.** The radius of a graph (or strongly connected digraph) G, denoted as $\mathrm{rad}(\mathrm{G})$, is the minimum eccentricity of the vertices in $\mathrm{V}(\mathrm{G})$.

$$
\begin{aligned}
e(1) &= 3\\
e(2) &= 2\\
e(3) &= 2\\
e(4) &= 2\\
e(5) &= 3
\end{aligned}
$$

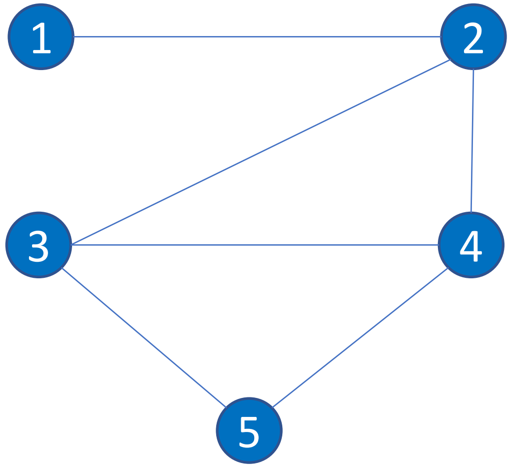


The radius of this graph is 2

Periphery and Centre in Graph

- **Definition.** A vertex v of a graph G with eccentricity equals to the diameter of G is said to be a peripheral vertex
- The set of all peripheral vertices in the graph is called the periphery of the graph

$$
\begin{aligned}
e(1) &= 3\\
e(2) &= 2\\
e(3) &= 2\\
e(4) &= 2\\
e(5) &= 3
\end{aligned}
$$

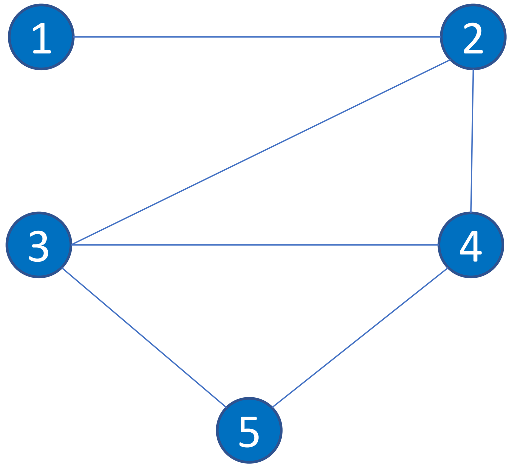


Node 1 and Node 5 form the periphery of the graph

Periphery and Centre in Graph

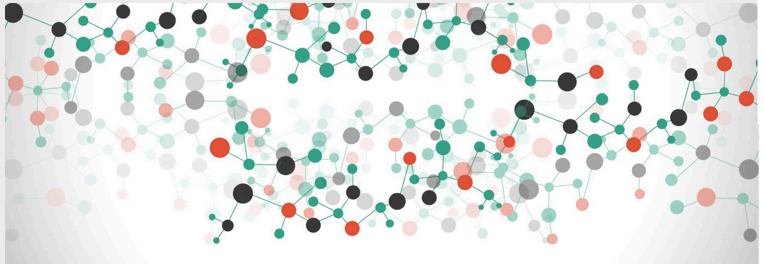
- **Definition.** A vertex v of a graph G with eccentricity equals to the radius of G is said to be a central vertex
- The set of all central vertices in the graph is called the centre of the graph

$$e(1) = 3$$
$$e(2) = 2$$
$$e(3) = 2$$
$$e(4) = 2$$
$$e(5) = 3$$



Nodes 2, 3 and 4 form the centre of the graph

SUMMARY



- The **eccentricity** of a vertex v in V(G), denoted as e(v), is the maximum of the distances between v and any other vertex u in V(G).

- The **diameter** of a graph (or strongly connected digraph) G, denoted as diam(G), is the **maximum eccentricity** of the vertices in V(G).

- The **radius** of a graph (or strongly connected digraph) G, denoted as rad(G), is the **minimum eccentricity** of the vertices in V(G).

- A vertex v of a graph G with eccentricity equals to the diameter of G is said to be a **peripheral vertex**

- A vertex v of a graph G with eccentricity equals to the radius of G is said to be a **central vertex**