# Data Searching and Binary Search

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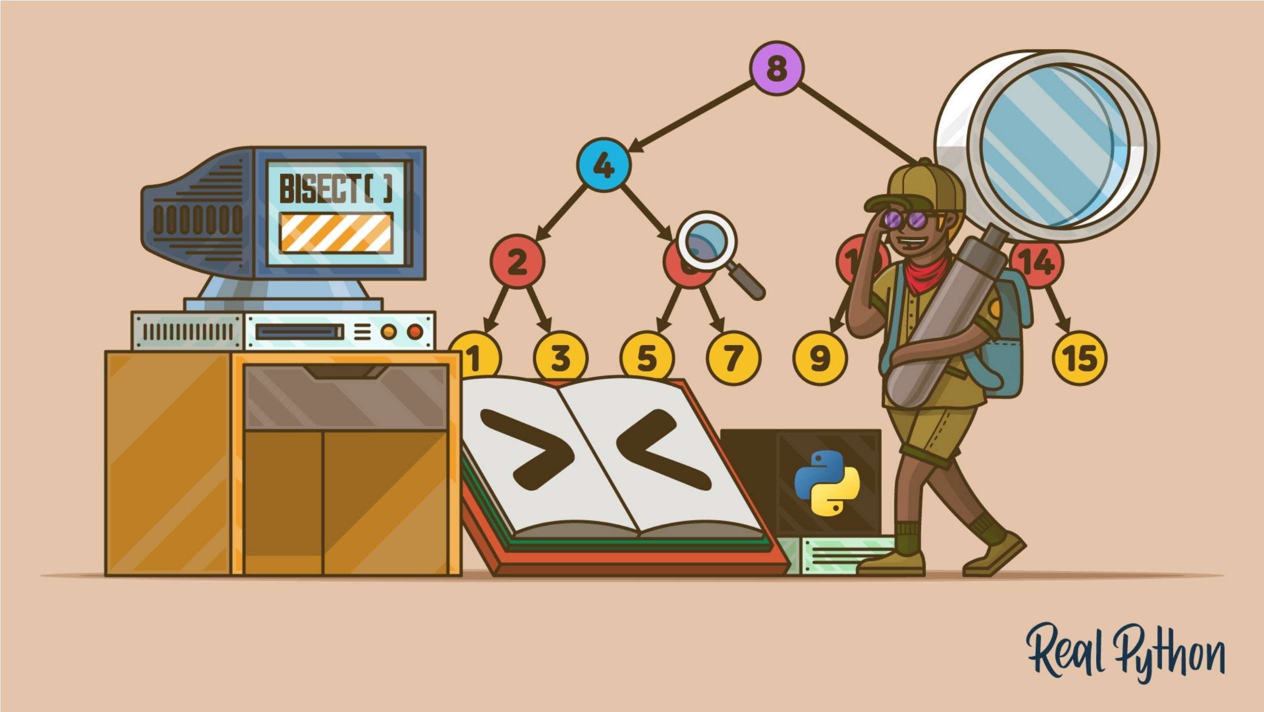
# OUTLINE

- Definition of Search
- Types of Data Input
    - Unsorted Lists / Sorted Lists
- Types of Search
    - Sequential Search
    - Binary Search
- Time Complexity Analysis



# Data Search in a Large Database

- Searching in a database D of records, such that each record has a key to use in the search.
- Example – search an album record by Album ID (as key).

| Album ID | Album | Song Title | Singer Name | Release Year |
|---|---|---|---|---|
| 1 | Courage | "Flying On My Own" | Celine Dion | 2019 |
| 2 | Mind Games | "South Dakota" | Jordy | 2019 |
| 3 | Clarity | “Broken” | Kim Petras | 2020 |
| 4 | Nibiru | “Reggaeton en Paris” | Ozuna | 2019 |
| 5 | Basking in the Glow | "Morning Song" | Oso Oso | 2019 |

# Data Search in a Large Database

- Searching in a database D of records, such that each record has a key to use in the search.
- The search problem: Given a search key $k$, either
  - Return the record associated with $k$ in D (a successful search: if $k$ occurs several times, return any occurrence), or
  - Indicate that $k$ is not found (an unsuccessful search).

- The purpose of the search
  - To access data in the record for processing, or
  - To update information in the record, or
  - To insert a new record or delete the record found.

# Table ADT

- **Definition** (Table ADT): The table ADT is a set of ordered pairs, or table entries $(k,v)$ where $k$ is a unique key and $v$ is a data value associated with the key $k$.

- Types of Operations
  - **RETRIEVE** the entry $(k,v)$ based on the key $v$; if no entry exist, indicate the search is unsuccessful.
  - **REMOVE** the found entry from the table;
  - **UPDATE** its value $v$;
  - **INSERT** a new entry with key $k$ if the table has no such entry.

# Types of Search

- **Static search**: unalterable (fixed in advance) databases; no updates, deletions, or insertions.
- **Dynamic search**: alterable databases (allowable insertions, deletions, and updates).

| **Key** | | **Associated value $v$** | | |
|---|---|---|---|---|
| Code | $k$ | City | Country | State/Place |
| AKL | 271 | Auckland | New Zealand | North Island |
| DCA | 2080 | Washington | USA | District of Columbia (D.C.) |
| FRA | 3822 | Frankfurt | Germany | Hesse |
| SDF | 12251 | Louisville | USA | Kentucky |

# Implementation

- **Basic implementations** of the table ADT: lists and trees.

- An **elementary operation**
  - An update of a list element or tree node, or
  - Comparison of two of them.

# Sequential Search in Unsorted Lists

- Starting at the head of a list and examining elements one by one until finding the desired key or reaching the end of the list.
- **Complexity**. Both successful and unsuccessful sequential search have worst-case and average-case time complexity $\Theta(n)$.
- **Proof**:
  - The unsuccessful search explores each of $n$ keys, so the worst- and average-case time is $\Theta(n)$
  - The successful search examines $n$ keys in the worst case and $\frac{n+1}{2}$ keys on the average, which is still $\Theta(n)$

- The sequential search is the **ONLY option** for unsorted lists of records.
- A sorted list implementation allows for much better search based on the divide-and-conquer paradigm.

# Binary Search in a Sorted List of Records

Given a sorted list $L = \{(k_i, v_i) : i = 1, \ldots, n;\ k_1 < k_2 < \cdots < k_n\}$

Recursive binary search for the key $k$:

1. If the list is empty, return "not found", otherwise
2. Choose the key $k_m$ of the middle element of the list and
   - if $k_m = k$, return its record, otherwise
   - if $k_m > k$, make a recursive call on the head sublist, otherwise
   - if $k_m < k$, make a recursive call on the tail sublist.

- We can do this without recursion!

# Non-recursive (Iterative) Binary Search in Array

- The performance of binary search on an array is much better than on a linked list because of the constant time access to a given element.

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**Algorithm 1** BinarySearch

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**function** BinarySearch(a sorted integer $\boldsymbol{k} = (k_0, k_1, \ldots, k_{n-1})$ of keys associated with items, a search key $k$)

  $l \leftarrow 0; r \leftarrow n - 1$

  **while** $l \leq r$ **do** $m \leftarrow \left\lfloor \frac{l+r}{2} \right\rfloor$

    **if** $k_m < k$ **then** $l \leftarrow m + 1$

    **else if** $k_m > k$ **then** $r \leftarrow m - 1$

    **else return** $m$

  **return** *ItemNotFound*

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# Faster Binary Search with Two-way Comparisons

**Algorithm 2** BinarySearch

**function** BinarySearch2(a sorted integer $\boldsymbol{k} = (k_0, k_1, \ldots, k_{n-1})$ of keys associated with items, a search key $k$)

    $l \leftarrow 0; r \leftarrow n - 1$

    **while** $l < r$ **do** $m \leftarrow \left\lfloor \frac{l+r}{2} \right\rfloor$

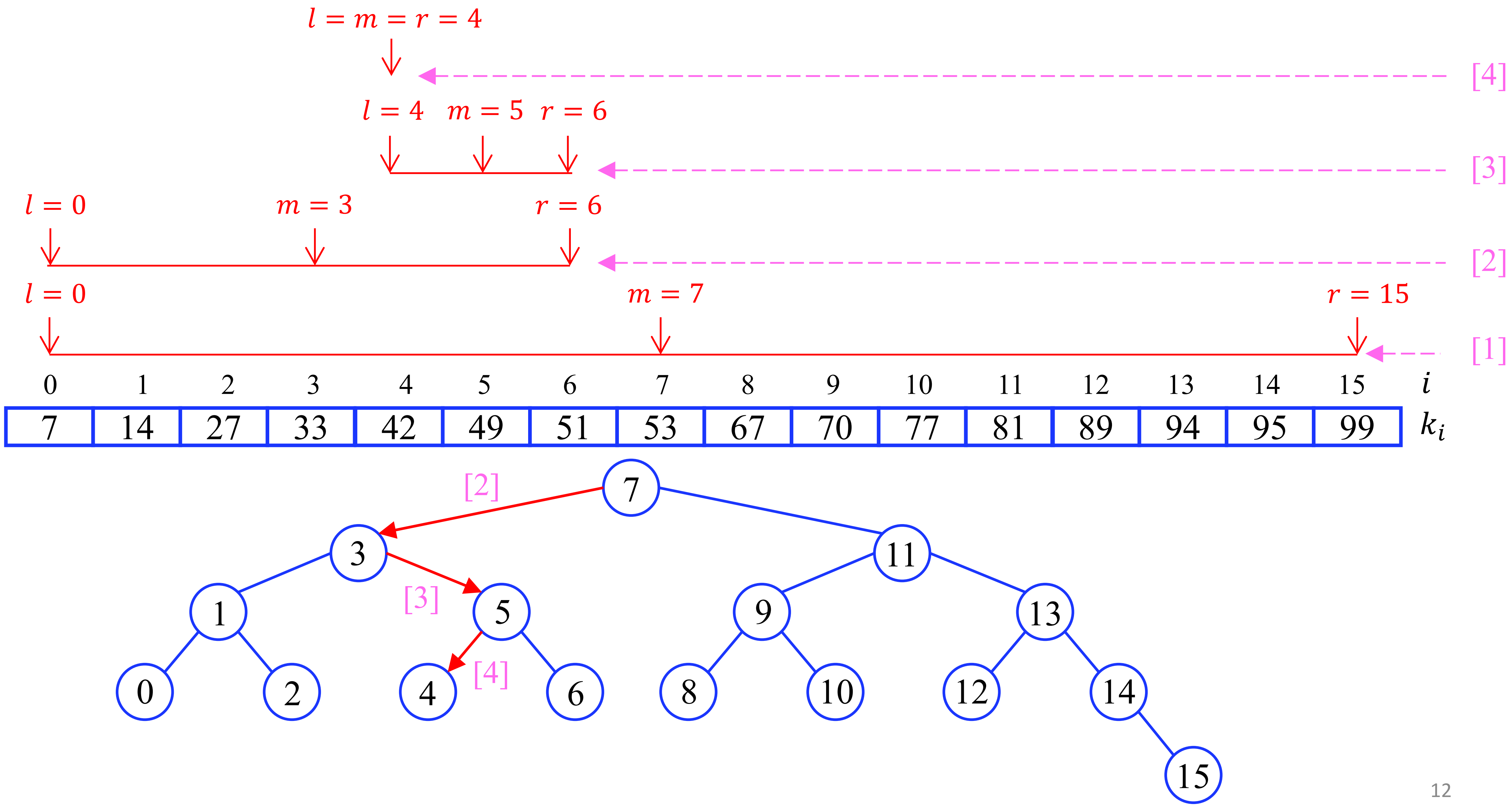
        **if** $k_m < k$ **then** $l \leftarrow m + 1$

        **else** $r \leftarrow m$

    **if** $k_l = k$ **then return** $l$

    **else return** *ItemNotFound*

# Example: Binary Search in Array $\{k_0 = 7, \ldots, k_{15} = 99\}$ for Key $k$=42



# Time Complexity Analysis: Worst Case

- The worst-case time complexity of unsuccessful and successful binary search is $\Theta(\log n)$.

- The full binary tree of height $h - 1$ has $n = 2^h - 1$ keys (each internal node has 2 children)
  - The comparison tree height is $h$ with only the last level not full.
  - $l + 1$ comparisons to find a key at level $l$.
  - The worst case: $h + 1 = \lfloor \log_2 n \rfloor + 1$ comparisons.

# Time Complexity Analysis: Average Case

- **Lemma**: The average-case time complexity of successful and unsuccessful binary search in a balanced tree is $\Theta(\log n)$.

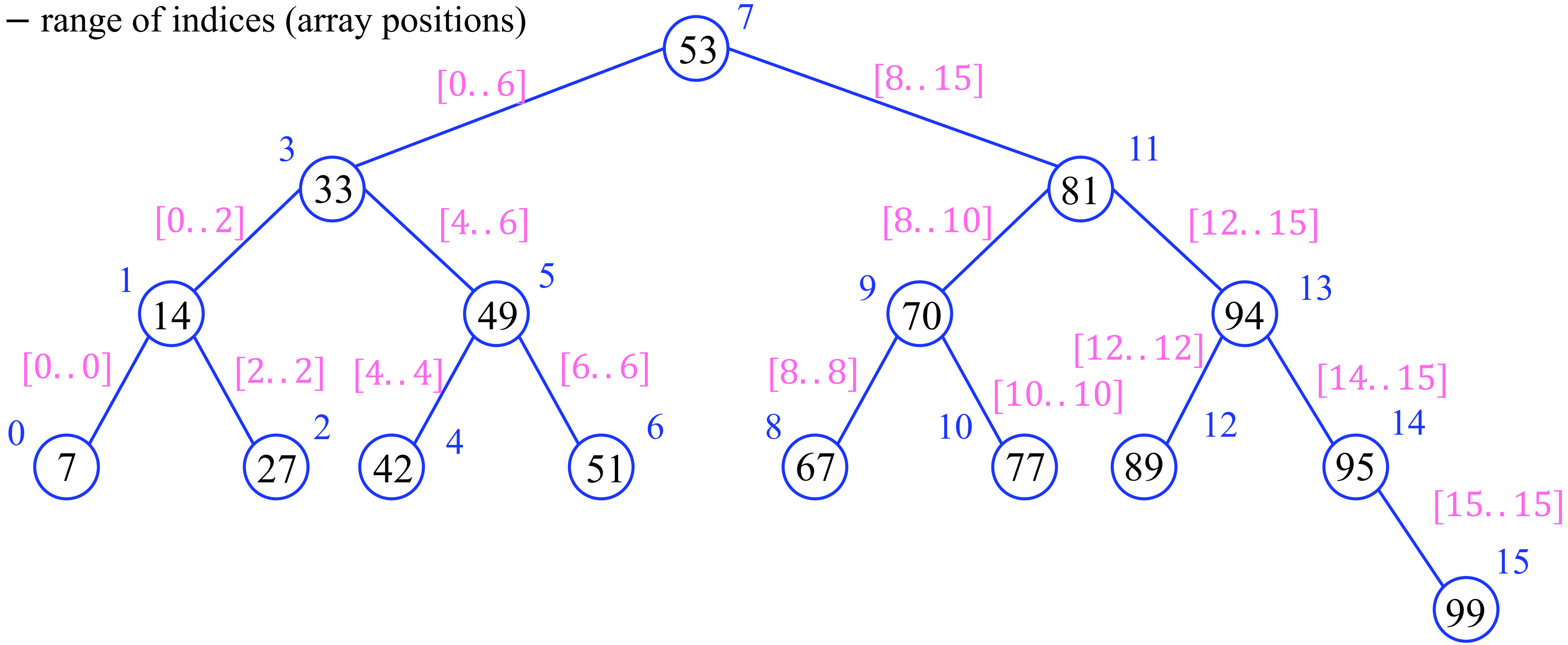
- **Proof**: The height of the tree is $h = \lfloor \log_2 n \rfloor$
  - At least half of the tree nodes have a depth at least $h - 1$.
  - The average depth over all nodes is at least $\frac{h-1}{2}$ and at most $h$, so that it is $\Theta(\log n)$
  - The average depth over all nodes of an arbitrary (not necessarily balanced) binary tree is $\Omega(\log n)$.

- The expected search time for an arbitrary balanced tree is equal to the average balanced tree depth $\Theta(\log n)$.

# Tree Structure of Binary Search: Binary Search Tree

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 14 | 27 | 33 | 42 | 49 | 51 | 53 | 67 | 70 | 77 | 81 | 89 | 94 | 95 | 99 |

$[l..r]$ − range of indices (array positions)



# SUMMARY

- Definition of Search
- Sequential Search on Unsorted Lists
- Binary Search on Sorted Lists
  - Iterative Binary Search
  - Faster Binary Search
- Time Complexity Analysis