1. Determine the following statements are True/False for a digraph.

    1. If there exist $u, v \in G$ such that there is a path from u to v and from v to u, the digraph is strongly connected.
    2. If there exist $u, v \in G$ such that there is a path from u to v but not from v to u, the digraph is weakly connected.
    3. If there exist $u, v \in G$ such that there is a path from u to v but not from v to u, the digraph contains no strong components.
    4. If there exist $u, v \in G$ such that there is no path from u to v or there is no path from v to u, the digraph is not strongly connected.

2. Consider the graph with vertices A, B, C, D, E and edges

$$\{\{A, B\}, \{A, C\}, \{A, D\}, \{B, C\}, \{B, D\}, \{B, E\}, \{C, D\}, \{C, E\}\}$$

Which of the following statement is true?

a The graph has neither a 2-coloring, nor a 3- or a 4-coloring.

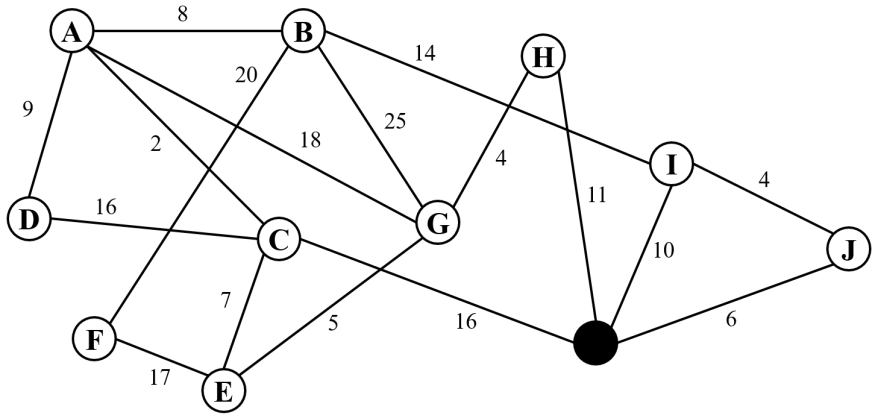
b The graph has a 2- and a 4-coloring but no 3-coloring.

c The graph has a 4-coloring but no 2- or 3-coloring.

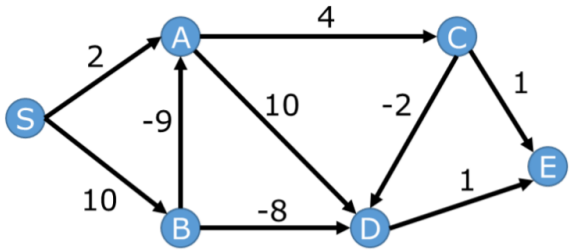
d The graph has a 2- and a 3-coloring, but no 4-coloring

e The graph has a 3- and 4-coloring but no 2-coloring.

3. Find the minimum weight paths from the black vertex to all the other vertices in the graph below using Dijkstra's algorithm. Show the value of the distance vector after each step.



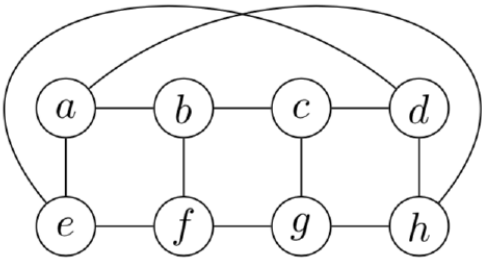
4. Given the graph G shown, we find the shortest paths from node S using the Bellman-Ford algorithm. How many iterations does it take before the algorithm converges to the solution?



5. Consider the digraph G with nodes 0, 1, 2 and arcs (0, 1) with weight 5, (0, 2) with weight 2, (1, 2) with weight -2. If we are solving the SSSP problem for node 0 then which of the following is true?

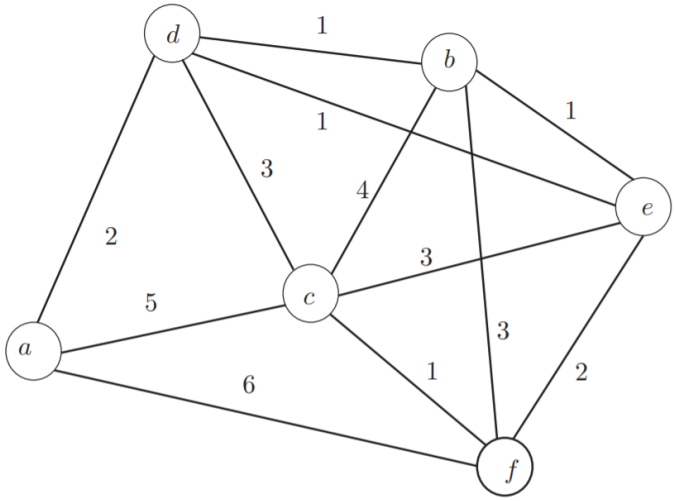
   A. Dijkstra gives wrong answer; Bellman-Ford gives right answer.
   B. Dijkstra and Bellman-Ford both give right answer.
   C. Dijkstra and Bellman-Ford both give wrong answer.
   D. There is no solution for this digraph.
   E. Dijkstra gives right answer; Bellman-Ford gives wrong answer.

6. What is the diameter of the following graph?



   A. 5
   B. 4
   C. 2
   D. 3
   E. 1

7. For the graph shown below, what is the weight of a minimum spanning tree? Try Kruskal's algorithm for calculating the MST.



8. An undirected weighted graph G has n vertices. The cost matrix of G is given by an $n \times n$ square matrix whose diagonal entries are all equal to 0 and whose non-diagonal entries are all equal to 2. Which of the following statements is TRUE?

A. G has a minimum spanning tree of weight 0.
B. G has a unique minimum spanning tree of weight 2(n-1).
C. G has multiple minimum spanning trees, each of different weights
D. G has multiple minimum spanning trees, each of weight 2(n-1).
E. G has no minimum spanning tree.