Shortest Path I: Dijkstra

Instructor: Meng-Fen Chiang

COMPSCI: WEEK 11.3

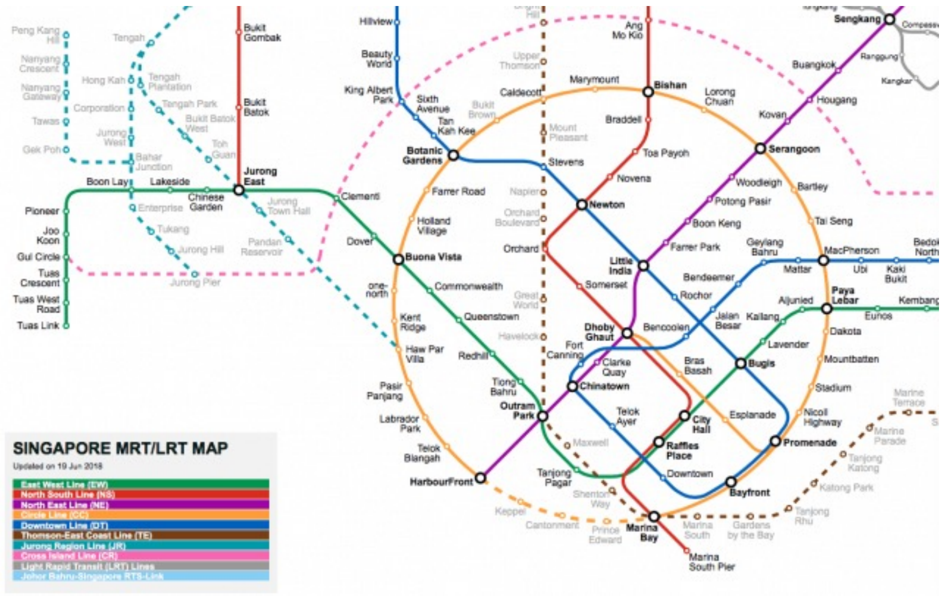
THE UNIVERSITY OF AUCKLAND
Te Whare Wananga o Tāmaki Makaurau
NEW ZEALAND

Slides adapted from Mark Wilson, Georgy Gimel'farb, Simone Linz, Tanya Gvozdeva, and Kaiqi Zhao

THE UNIVERSITY OF AUCKLAND
Te Whare Wananga o Tamaki Makaurau
NEW ZEALAND

OUTLINE

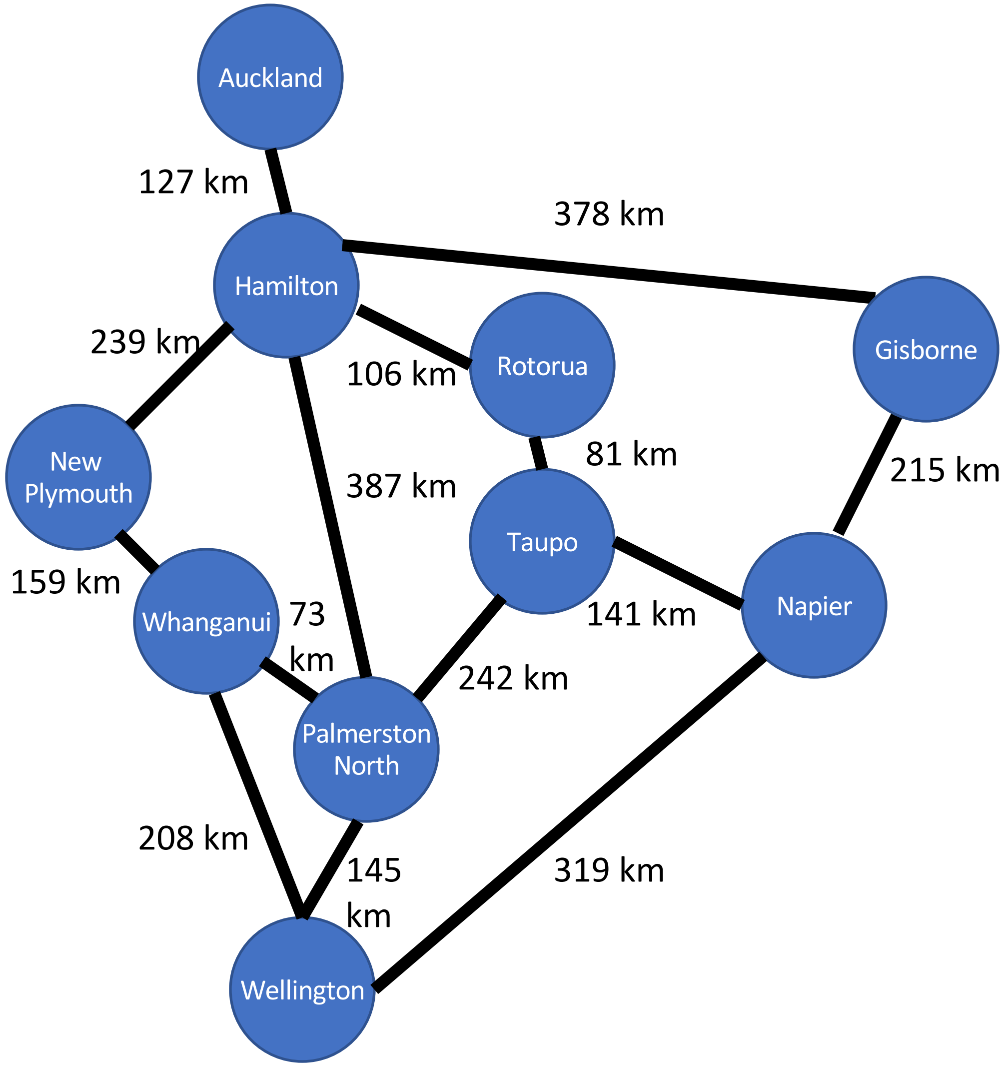
- Weighted Graphs
  - Representation
  - Weight as cost functions
- Algorithms on Weighted Graphs
  - Dijkstra
  - Bellman-Ford
  - Floyd-Warshall



Weighted (Di)graphs

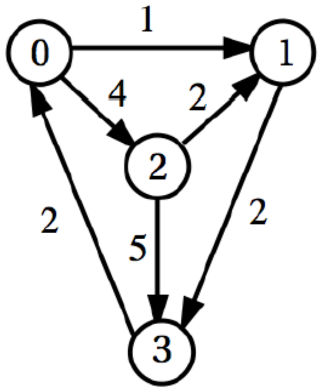
- Very common in applications, also called “networks". Optimization problems on networks are important in operations research.
- Each arc carries a real number “weight", usually positive, can be $+\infty$. Weight typically represents cost, distance, time. We may use the words “weight” and “cost” interchangeably in the slides.
- Representation: weighted adjacency matrix or double adjacency list.
- Standard problems concern finding a minimum or maximum weight path between given nodes (covered here), spanning tree, cycle or tour (e.g TSP), matching, flow, etc.

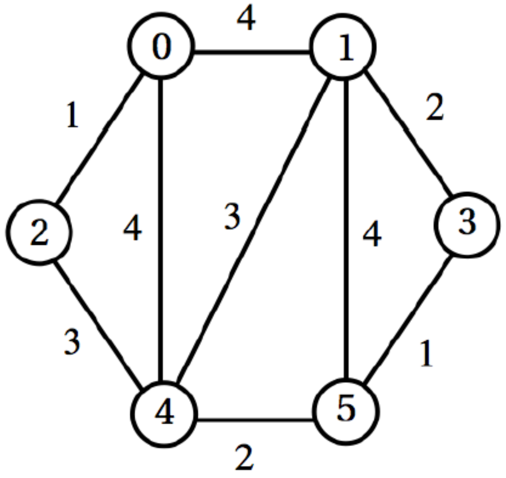
Example: Weighted Graph



Source of distances: Google Maps

Computer Representations of Weighted Digraphs

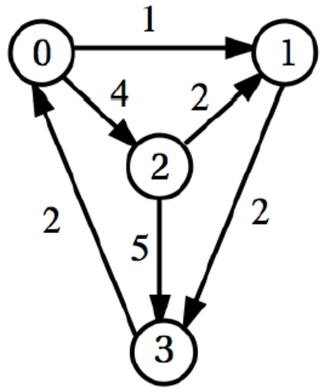


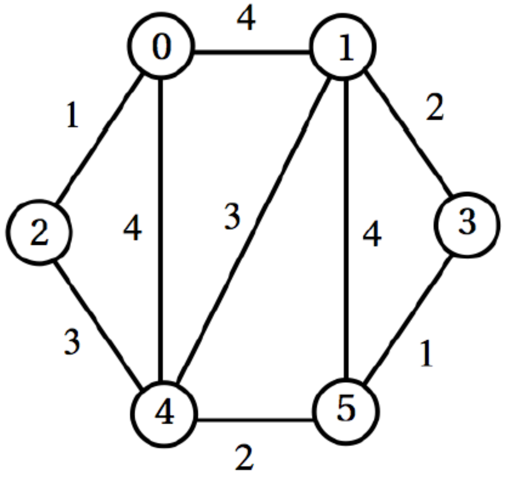


Weighted Adjacency Matrices (Cost Matrices):

$$\begin{bmatrix} 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 5 \\ 2 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 4 & 1 & 0 & 4 & 0 \\ 4 & 0 & 0 & 2 & 3 & 4 \\ 1 & 0 & 0 & 0 & 3 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 \\ 4 & 3 & 3 & 0 & 0 & 2 \\ 0 & 4 & 0 & 1 & 2 & 0 \end{bmatrix}$$

Computer Representations of Weighted Digraphs





Weighted (Double) Adjacency Lists:

1	1	2	4
3	2		
1	2	3	5
0	2		

1	4	2	1	4	4		
0	4	3	2	4	3	5	4
0	1	4	3				
1	2	5	1				
0	4	1	3	2	3	5	2
1	4	3	1	4	2		

Example: Distance Matrix Representation

$$
\begin{array}{l}
 & \text{Auckland} & \text{Gisborne} & \text{Hamilton} & \text{Napier} & \text{New Plymouth} & \text{Palmerston North} & \text{Rotorua} & \text{Taupo} & \text{Wellington} & \text{Whanganui} \\
\end{array}
$$

	Auckland	Gisborne	Hamilton	Napier	New Plymouth	Palmerston North	Rotorua	Taupo	Wellington	Whanganui
Auckland	0	∞	127	∞	∞	∞	∞	∞	∞	∞
Gisborne	∞	0	378	215	∞	∞	∞	∞	∞	∞
Hamilton	127	378	0	∞	239	387	106	∞	∞	∞
Napier	∞	215	∞	0	∞	∞	∞	141	319	∞
New Plymouth	∞	∞	239	∞	0	∞	∞	∞	∞	159
Palmerston North	∞	∞	387	∞	∞	0	∞	242	145	73
Rotorua	∞	∞	106	∞	∞	∞	0	81	∞	∞
Taupo	∞	∞	∞	141	∞	242	81	0	∞	∞
Wellington	∞	∞	∞	319	∞	145	∞	∞	0	208
Whanganui	∞	∞	∞	∞	159	73	∞	∞	208	0

Example: Adjacency List Representation

Auckland: Hamilton, 127

Gisborne: Hamilton, 378, Napier, 215

Hamilton: Auckland, 127, Gisborne 378, New Plymouth, 239, Palmerston North, 387, Rotorua, 106

Napier: Gisborne, 215, Taupo, 141, Wellington, 319

New Plymouth: Hamilton, 239, Whanganui, 159

Palmerston North: Hamilton, 387, Taupo, 242, Whanganui, 73, Wellington, 145

Rotorua: Hamilton, 106, Taupo, 81

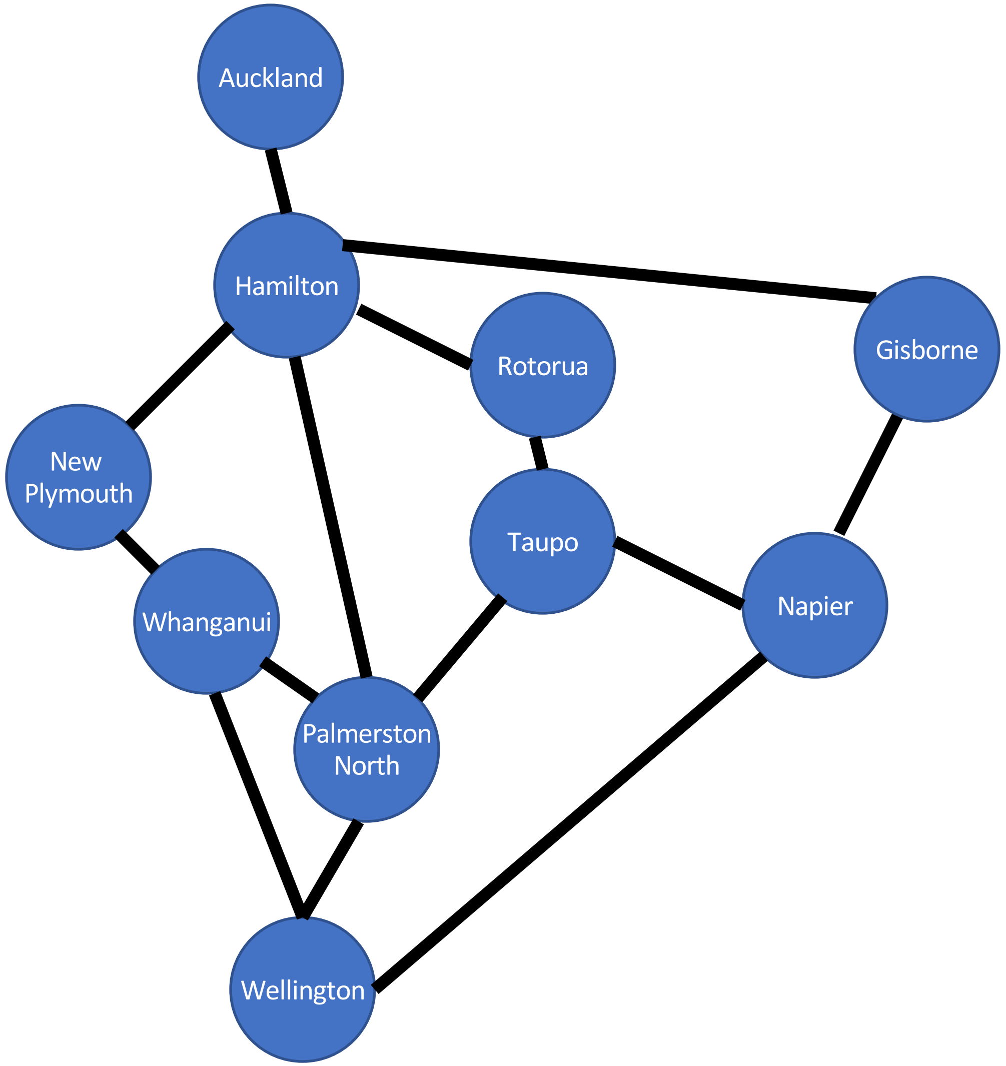
Taupo: Napier, 141, Palmerston North, 242, Rotorua, 81

Wellington: Napier, 319, Palmerston North, 145, Whanganui, 208

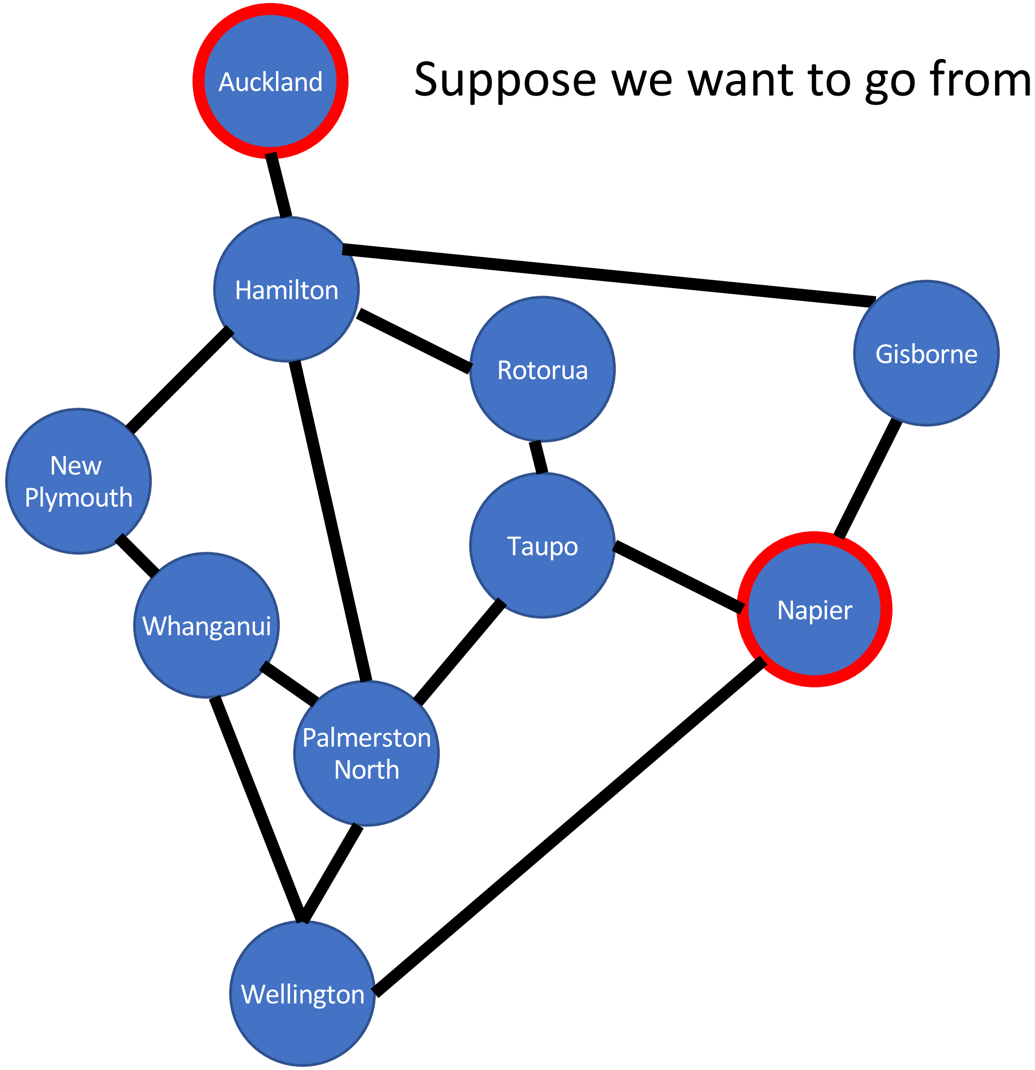
Whanganui: New Plymouth, 159, Palmerston North, 73, Wellington, 208

Essentially, we need the same number of extra storage spaces as there are objects, so the fundamental complexity does not change!

Graph: North Island Road Network



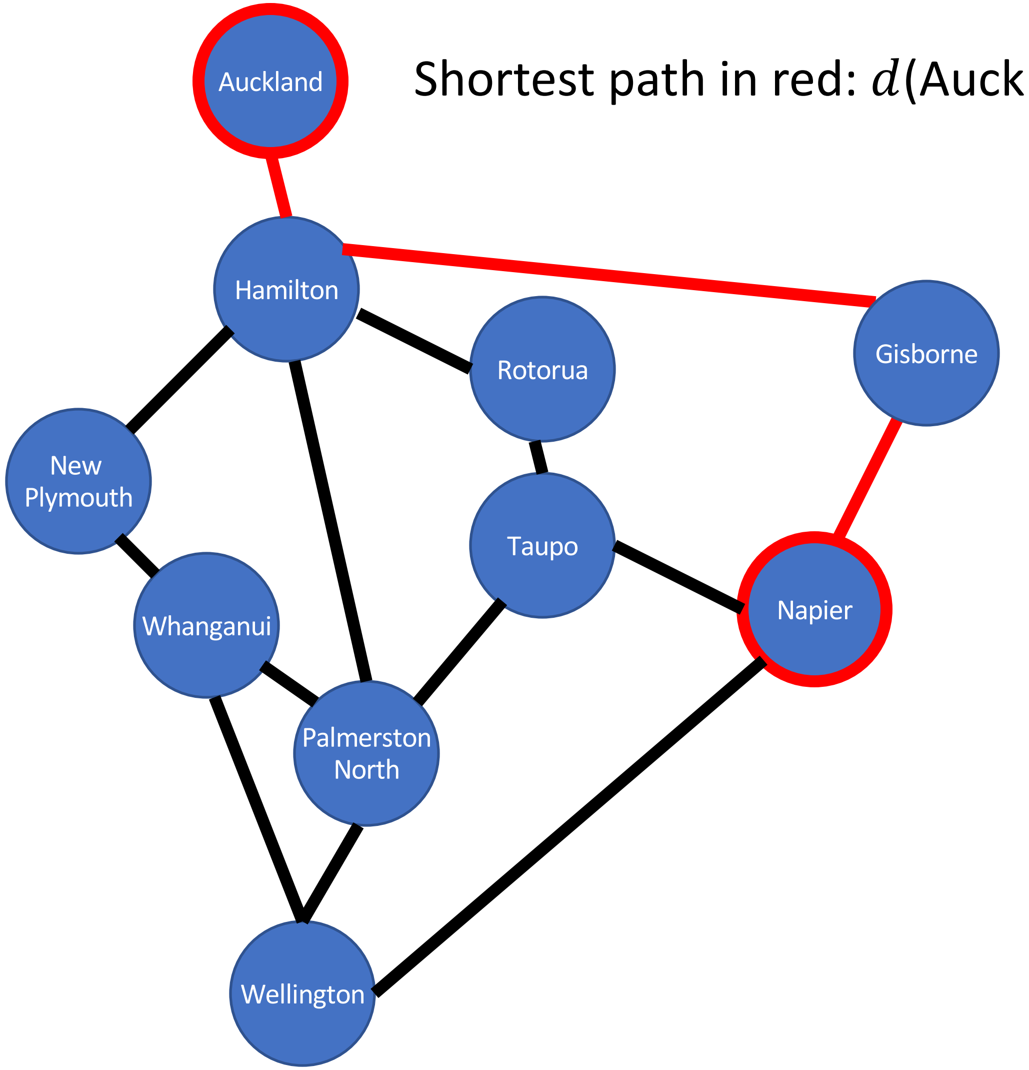
Graph: North Island Road Network



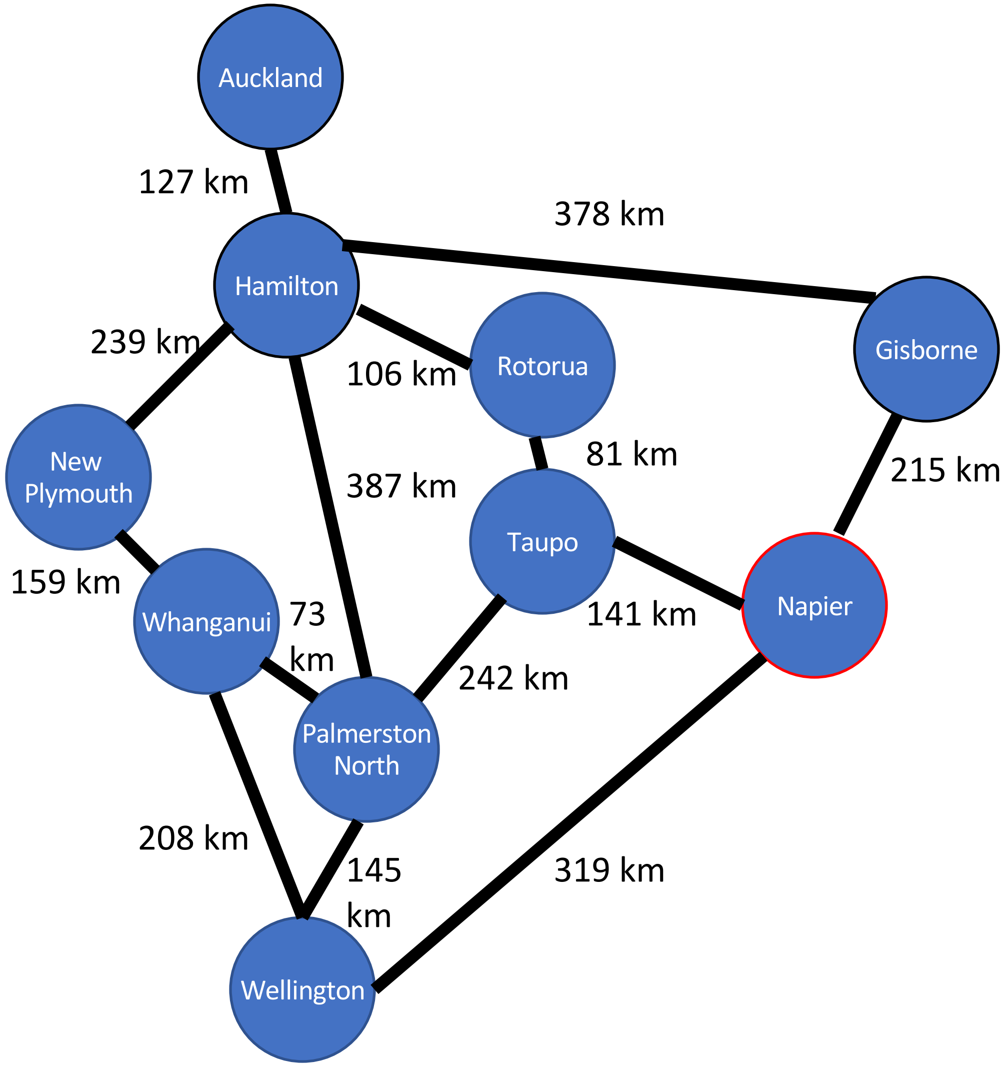
Suppose we want to go from Auckland to Napier

Graph: North Island Road Network

Shortest path in red: d(Auckland, Napier) = 3

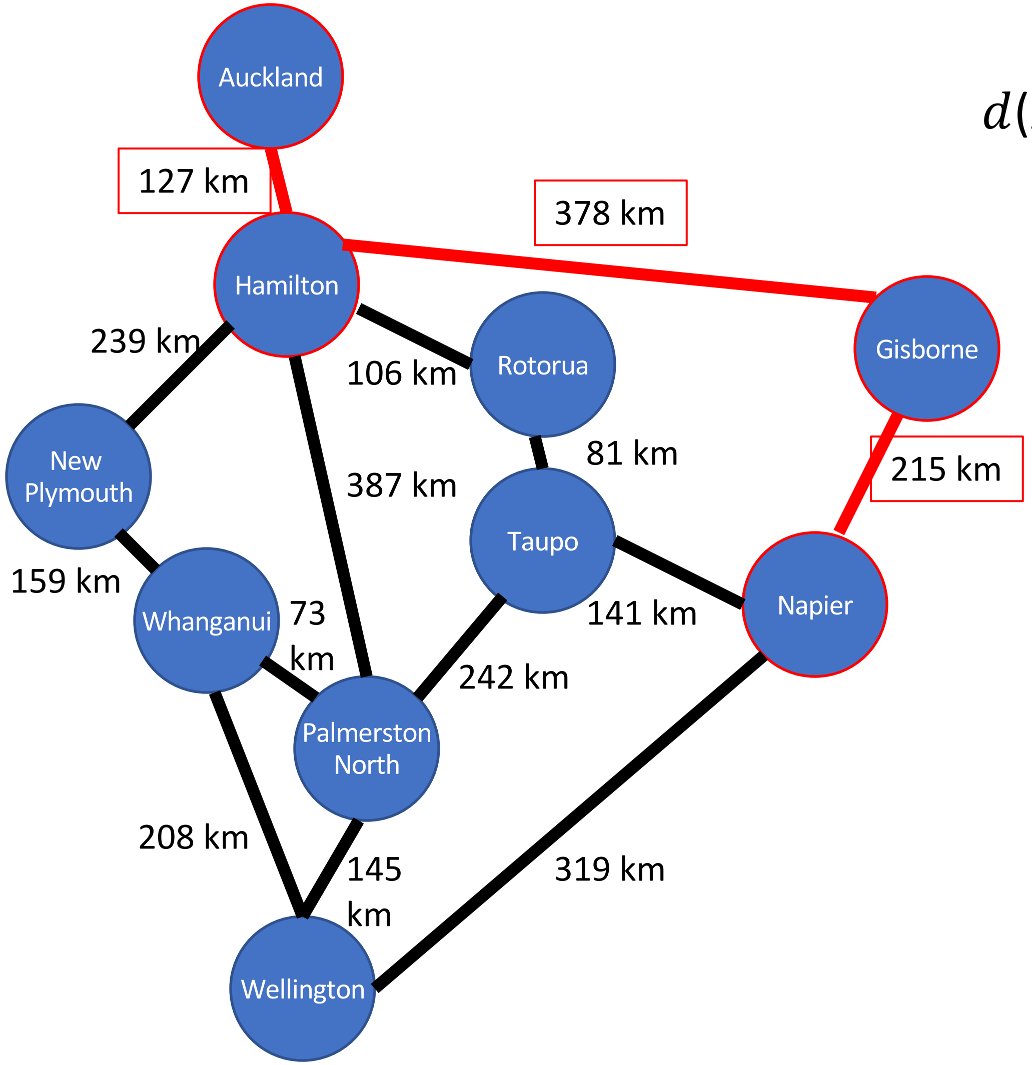


Weighted Graph: North Island Road Network



Source of distances: Google Maps

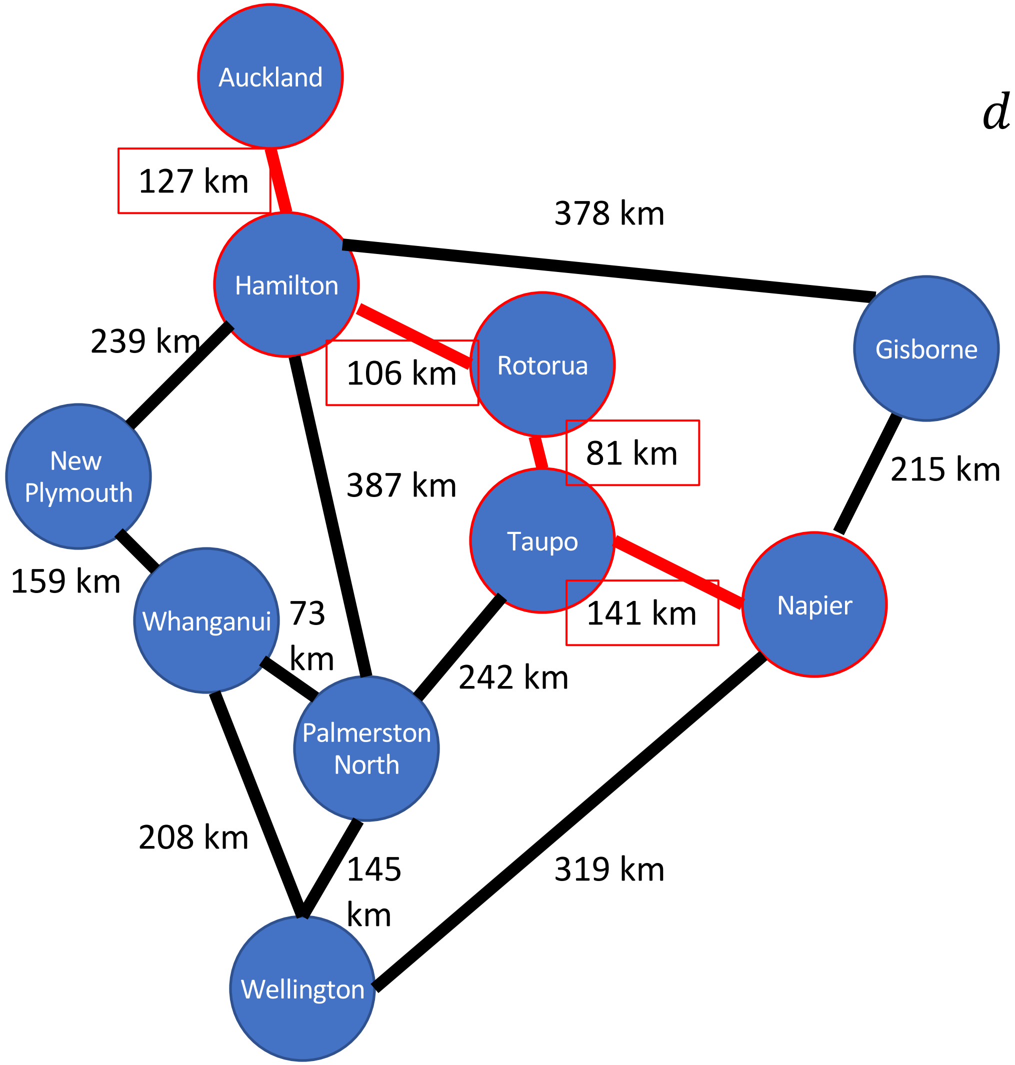
Weighted Graph: North Island Road Network



d(Auckland, Napier)= 720 km

Source of distances: Google Maps

Weighted Graph: North Island Road Network



d(Auckland, Napier)= 455 km

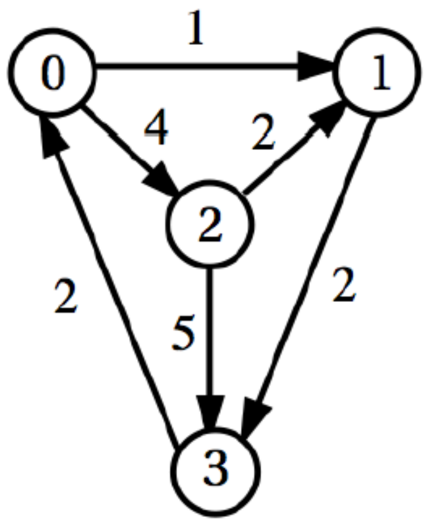
Source of distances: Google Maps

Paths/Distances (revisited)

- **Definition**. For a digraph (V, E) with arc weights $\{c(u,v)|(u,v) \in E\}$ we say that the distance $d(u,v)$ between two vertices u and v of V is the minimum cost of a path between u and v. The cost of a walk/path $v_0, v_1, \dots, v_k$ is $\sum_{i=0}^{k-1} c(v_i, v_{i+1})$

- **Definition**. The diameter of a (di-)graph $G = (V, E)$ is the maximum of $d(u,v)$ over all pairs $u, v \in V$. If the (di-)graph is not (strongly) connected, the diameter of G is not defined.

Example: Diameter



weighted adjacency matrix:

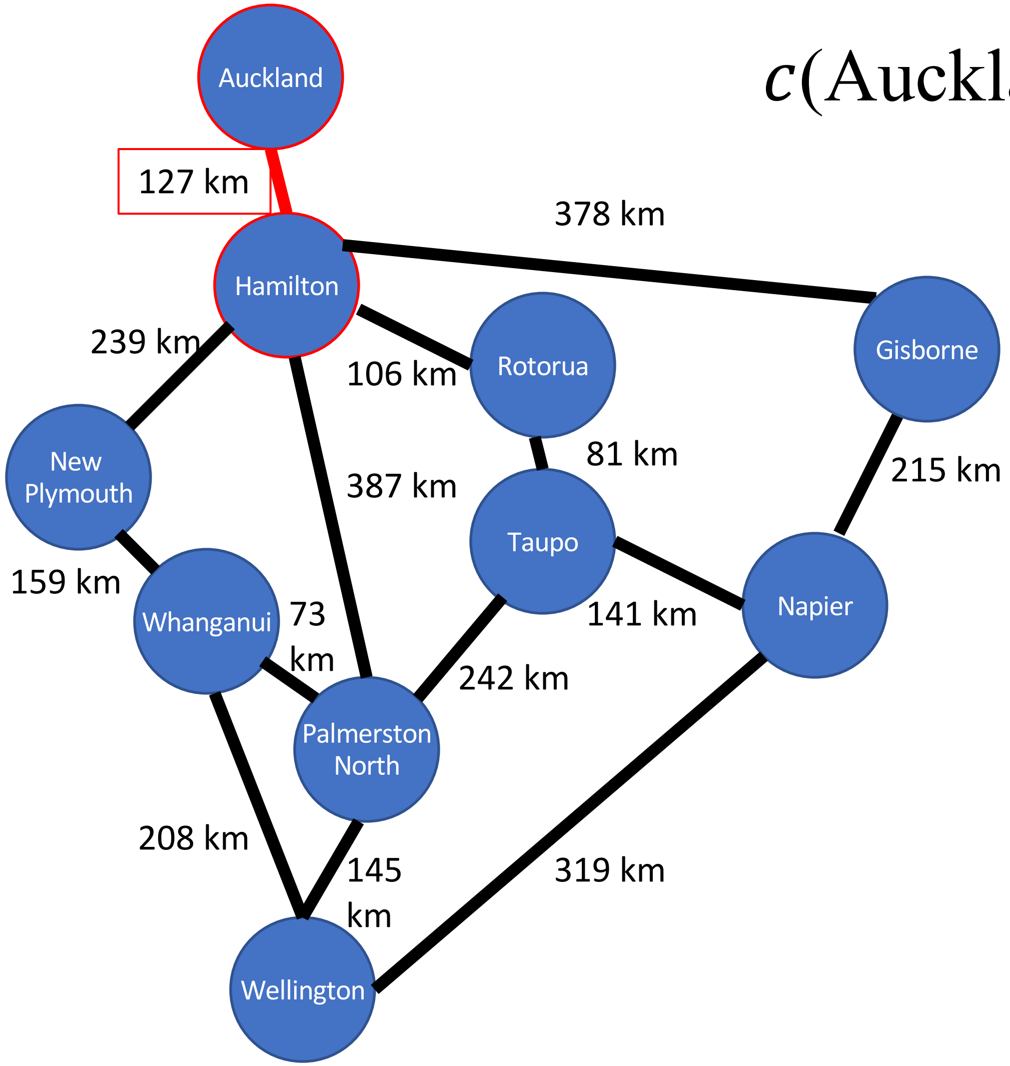
$$\begin{bmatrix} 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 5 \\ 2 & 0 & 0 & 0 \end{bmatrix}$$

distance matrix:

$$\begin{bmatrix} 0 & 1 & 4 & 3 \\ 4 & 0 & 8 & 2 \\ \mathbf{6} & 2 & 0 & \mathbf{4} \\ 2 & 3 & 6 & 0 \end{bmatrix}$$

Hence, the diameter is 8.

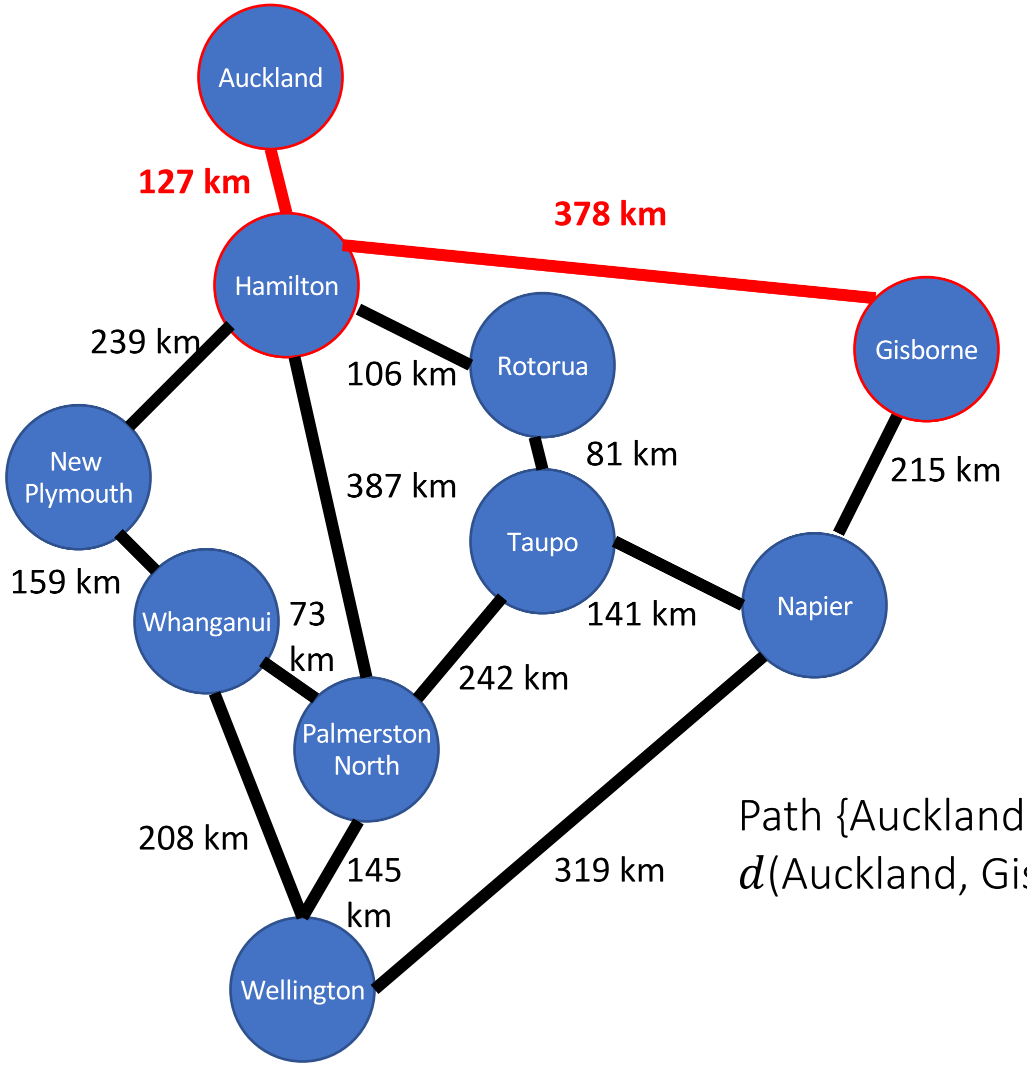
Example: Cost



$$c(\text{Auckland, Hamilton}) = 127 \text{ km}$$

Source of distances: Google Maps

Example: Distance v.s. Cost

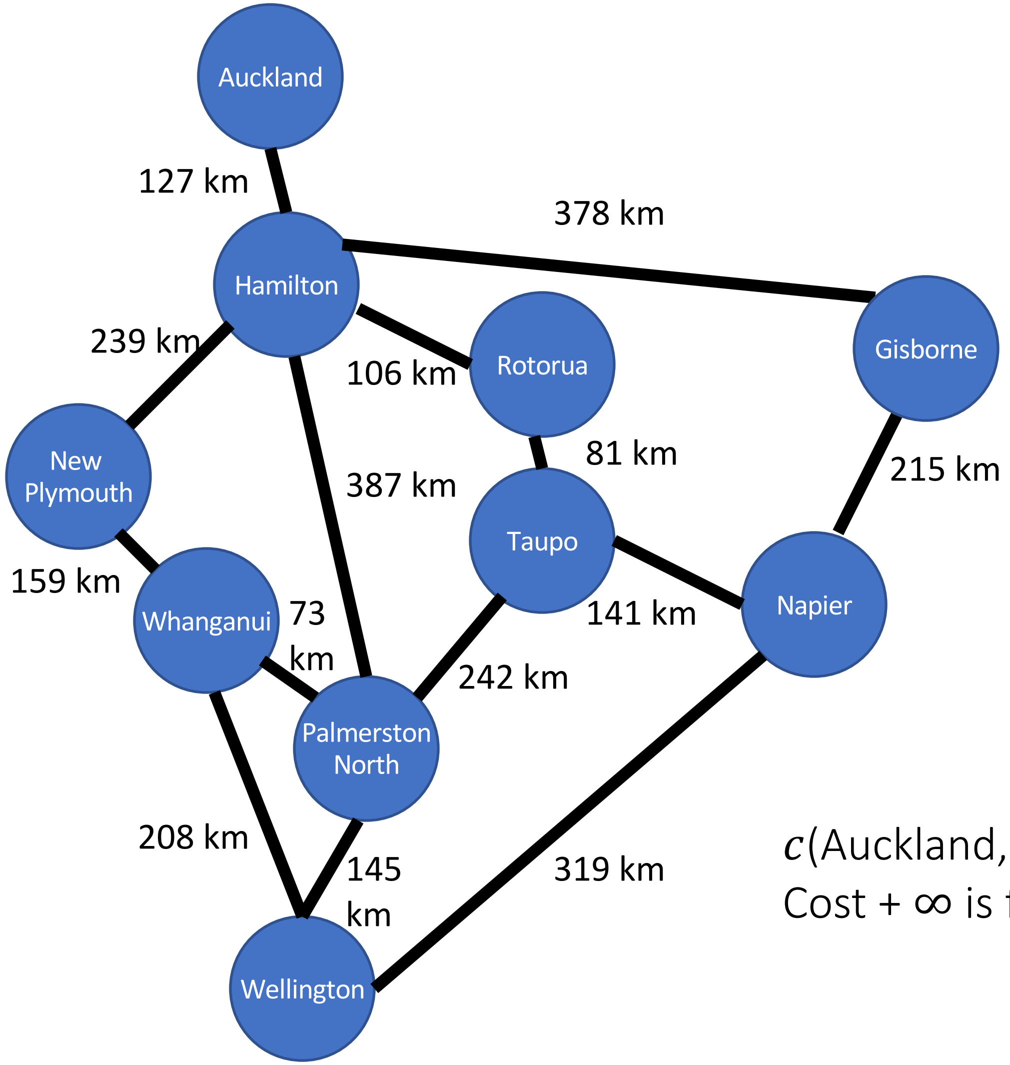


Path {Auckland, Hamilton, Gisborne} then

d(Auckland, Gisborne) = c(Auckland, Hamilton) + c(Hamilton, Gisborne)

Source of distances: Google Maps

Example: Distance v.s. Cost



c(Auckland, Hamilton) = 127 and c(Auckland, Gisborne) = $+\infty$

Cost $+ \infty$ is for cities that aren't directly connected.

Source of distances: Google Maps

Issue: Negative Weights

- The vast majority of weighted graph problems have positive weights
- However, exceptions exist where the weight of an edge/arc can be negative.

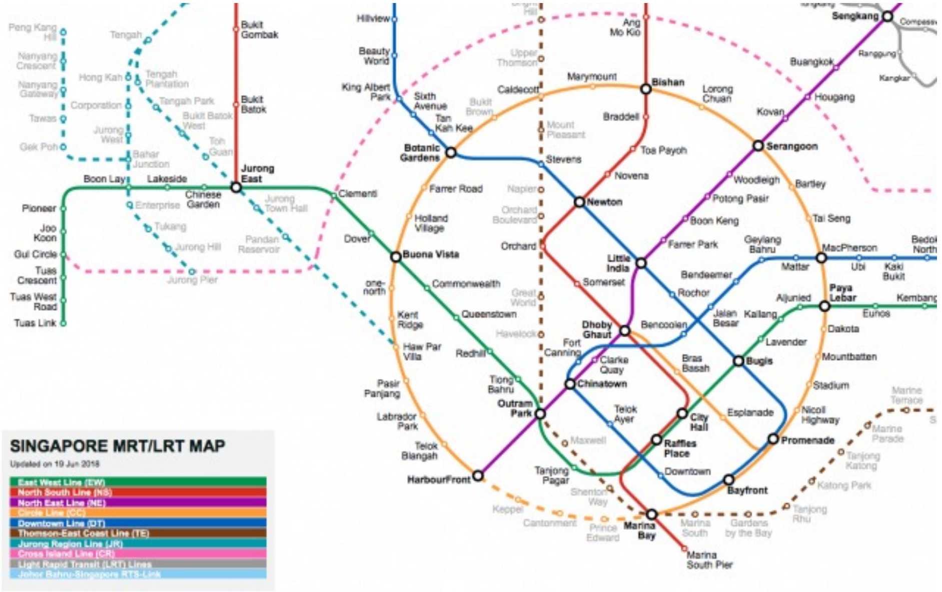
Example: Negative Weights

- Consider going on a holiday tripping around the world.
- Between some cities, we may need to buy a train/ferry ticket or an airfare (=cost, positive weight),
- … but between others we might have the opportunity to earn money by working as crew on a ship, train, or plane (=gain, negative weight).

OUTLINE

- Weighted Graphs
  - Representation
  - Weight as cost functions

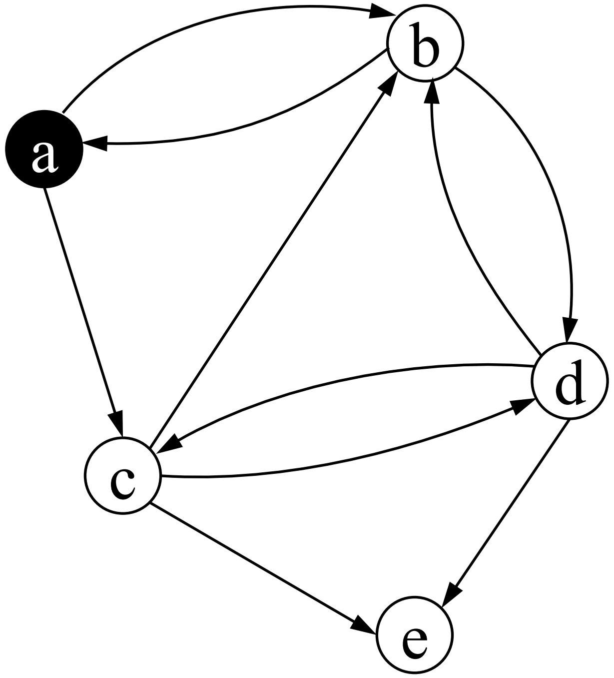
- Algorithms on Weighted Graphs
  - Dijkstra
  - Bellman-Ford
  - Floyd-Warshall



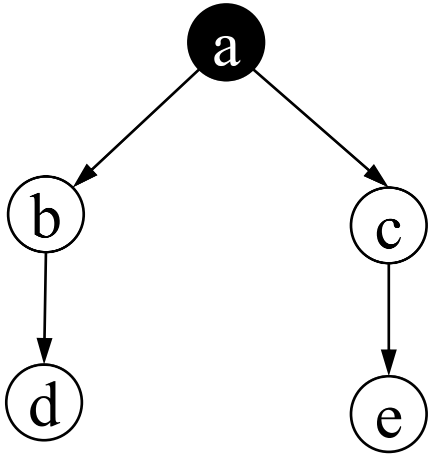
Single-source Shortest Path Problem (SSSP)

- Given an originating node v, find shortest (minimum weight) path to each other node. If all weights are equal then BFS works, otherwise not.

Unweighted digraph



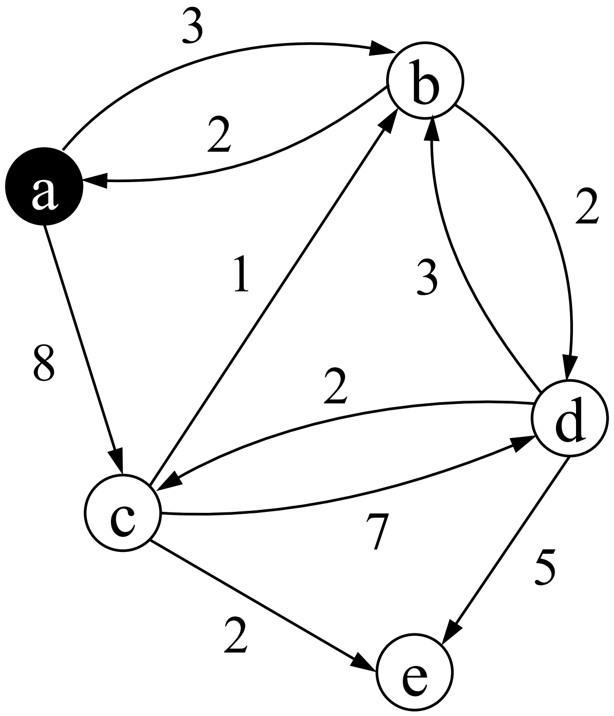
$d(a,b) = 1$, path: a, b
$d(a,c) = 1$, path: a, c
$d(a,d) = 2$, path: a, b, d or a, c, d
$d(a,e) = 2$, path: a, c, e



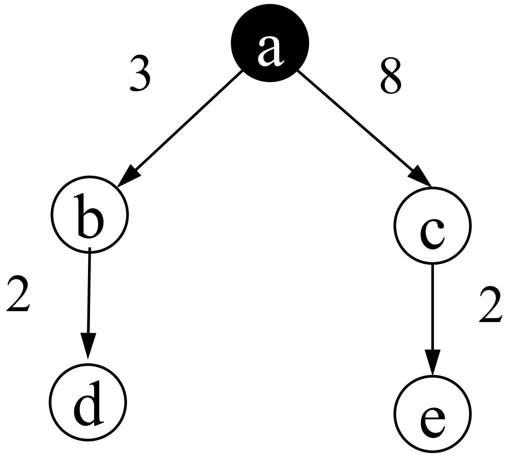
If BFS is used:
$d(a,b) = 1$, path: a, b
$d(a,c) = 1$, path: a, c
$d(a,d) = 2$, path: a, b, d
$d(a,e) = 2$, path: a, c, e

Single-source Shortest Path Problem (SSSP)

- Given an originating node v, find shortest (minimum weight) path to each other node. If all weights are equal then BFS works, otherwise not.



$d(a,b) = 3$, path: a, b
$d(a,c) = 7$, path: a, b, d, c
$d(a,d) = 5$, path: a, b, d
$d(a,e) = 9$, path: a, b, d, c, e



If BFS is used:
$d(a,b) = 3$, path: a, b
$d(a,c) = 8$, path: a, c
$d(a,d) = 5$, path: a, b, d
$d(a,e) = 10$, path: a, c, e

Algorithms on Weighted Graphs

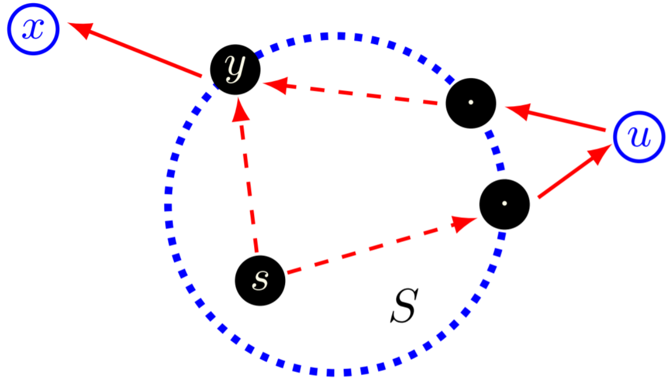
- **Dijkstra** (pronounced "Dyke-stra"): Used to find the cost to each destination vertex from a single source ("single source shortest path" - SSSP). Cannot handle negative weights.

- **Bellman-Ford**: solves SSSP as well, slower than Dijkstra but can handle negative weights

- **Floyd-Warshall**: solves all-pairs shortest path (APSP) problem – minimal cost between any given pair of vertices

Single-source Shortest Path Problem (SSSP)

- Several algorithms are known; we present one, Dijkstra's algorithm. An example of a greedy algorithm; locally best choice is globally best. Doesn't work if weights can be negative.
  1. Maintain list S of visited nodes.
  2. Choose an unvisited node u with shortest S-path and put it to S.
  3. Update distances (of remaining unvisited nodes) from starting node s in case adding u has established shorter paths.
  4. Repeat.



Let an S-path be a path starting at node s and ending at node u with all nodes in S except **possibly** u.

Dijkstra’s Algorithm

Algorithm 1 Dijkstra’s algorithm.

function DIJKSTRA(weighted digraph(G, c); node $s \in V(G)$)

  array $colour[0..n-1], dist[0..\mathrm{n}-1]$

  for $u \in V(G)$ **do**

    $dist[u] \leftarrow c(s, u); colour[u] \leftarrow$ WHITE

  $dist[s] \leftarrow 0; colour[s] \leftarrow$ BLACK

  while there is a white node **do**

    find a white node u so that $dist[u]$ is minimum

    $colour[u] \leftarrow$ BLACK

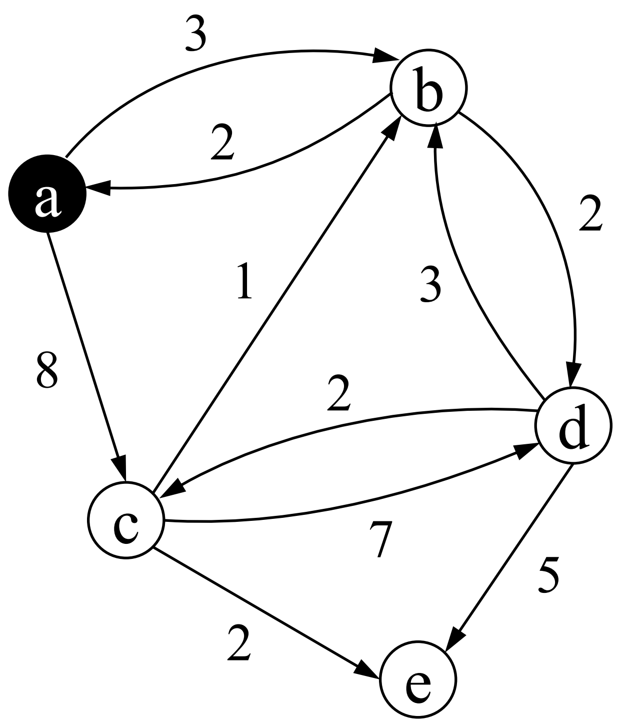
    for each x adjacent to u **do**

      if $colour[x] =$ WHITE **then**

        $dist[x] \leftarrow \min\{dist[x], dist[u] + c(u, x)\}$

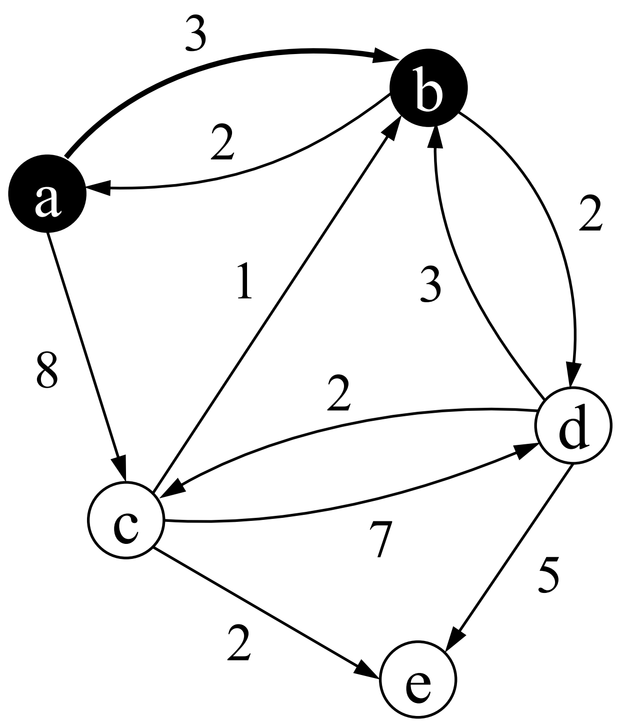
  return $dist$

Illustrating Dijkstra's algorithm



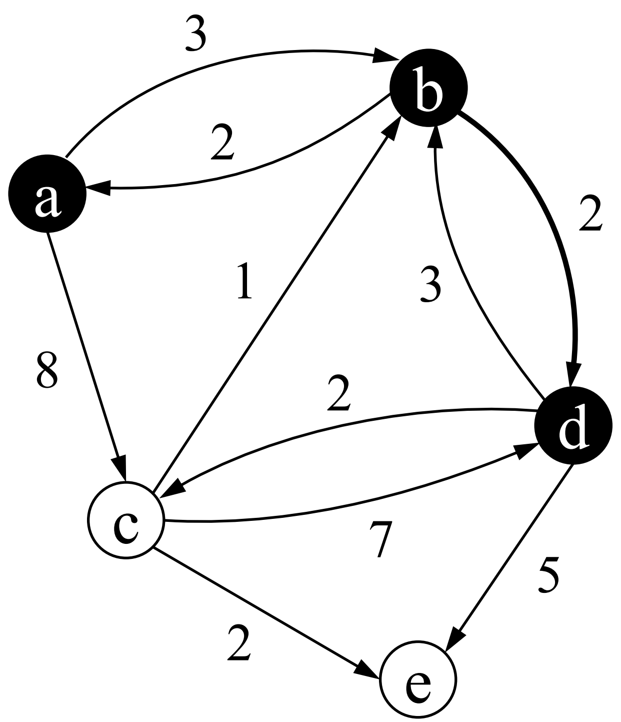
BLACK	$dist[x]$
	a, b, c, d, e
a	0, 3, 8, ∞, ∞

Illustrating Dijkstra's algorithm



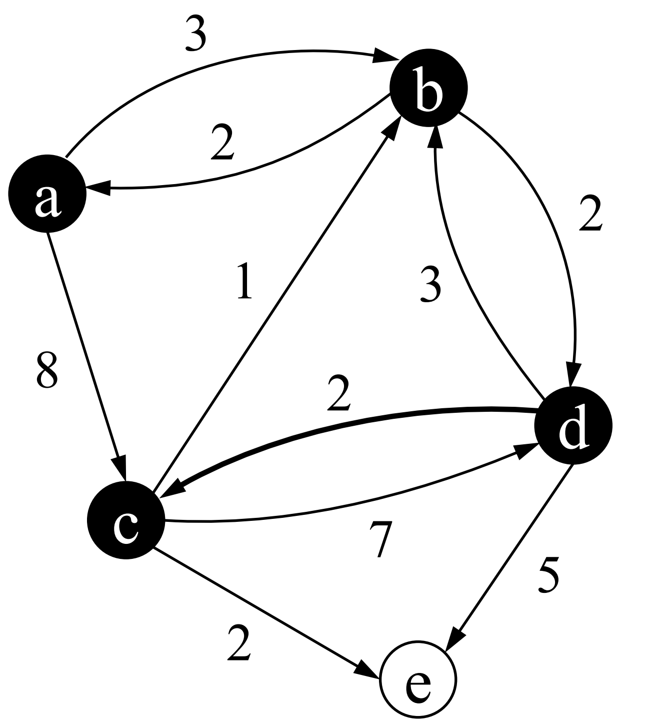
BLACK	$dist[x]$
	a, b, c, d, e
a	0, 3, 8, ∞, ∞
a, b	0, 3, 8, 3 + 2 = 5, ∞

Illustrating Dijkstra’s algorithm



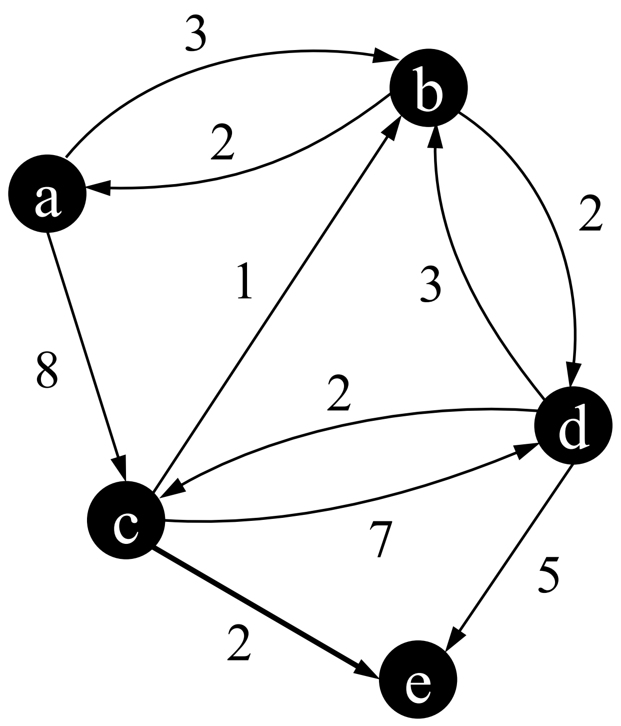
BLACK	$dist[x]$
	a, b, c, d, e
a	0, 3, 8, ∞, ∞
a, b	0, 3, 8, 3 + 2 = 5, ∞
a, b, d	0, 3, 3 + 2 + 2 = 7, 5, 10

Illustrating Dijkstra's algorithm



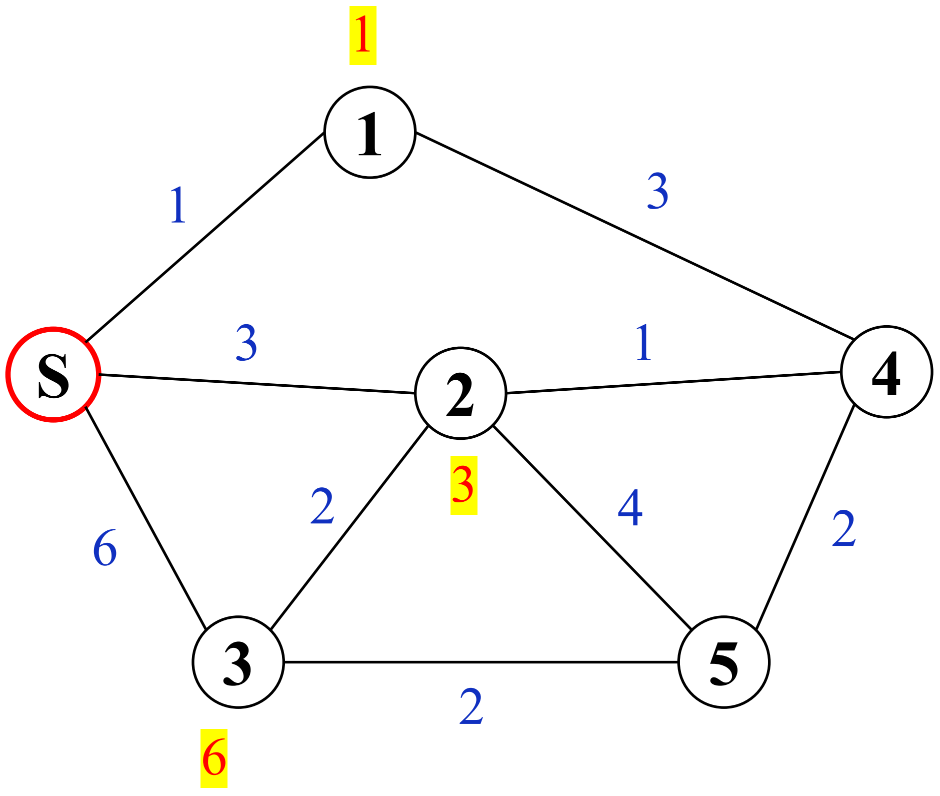
BLACK	$dist[x]$
	a, b, c, d, e
a	0, 3, 8, ∞, ∞
a, b	0, 3, 8, 3 + 2 = 5, ∞
a, b, d	0, 3, 3 + 2 + 2 = 7, 5, 10
a, b, c, d	0, 3, 7, 5, 7 + 2 = 9

Illustrating Dijkstra’s algorithm



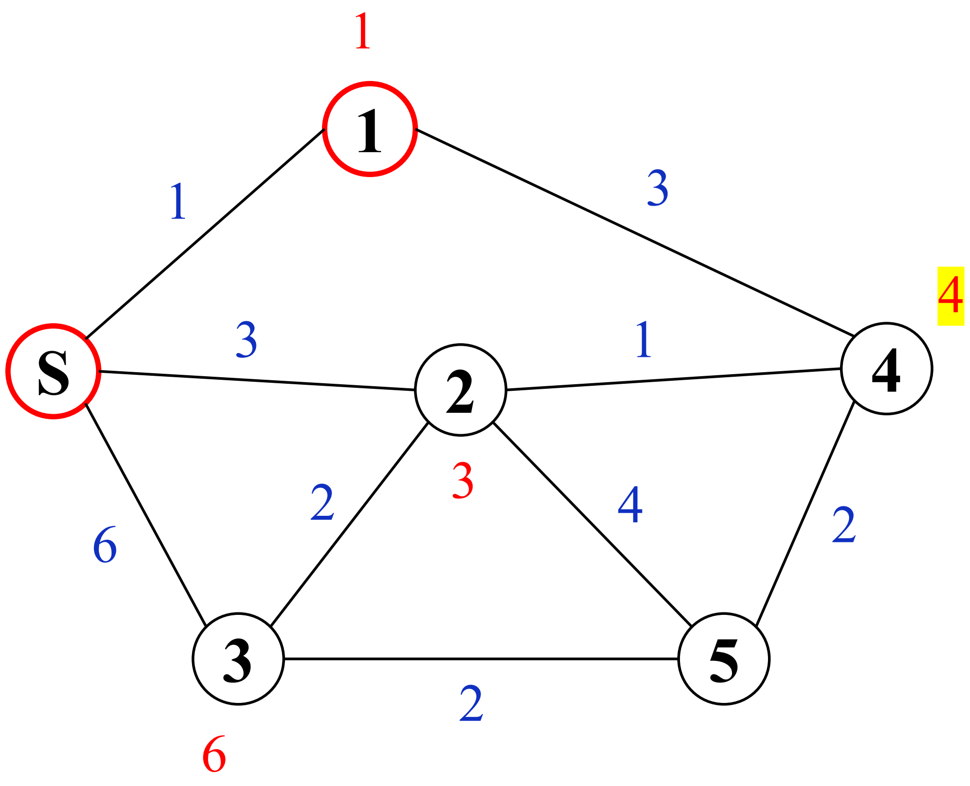
BLACK	$dist[x]$
	a, b, c, d, e
a	0, 3, 8, ∞, ∞
a, b	0, 3, 8, 3 + 2 = 5, ∞
a, b, d	0, 3, 3 + 2 + 2 = 7, 5, 10
a, b, c, d	0, 3, 7, 5, 7 + 2 = 9
$V(G)$	

Dijkstra’s algorithm - Pairwise



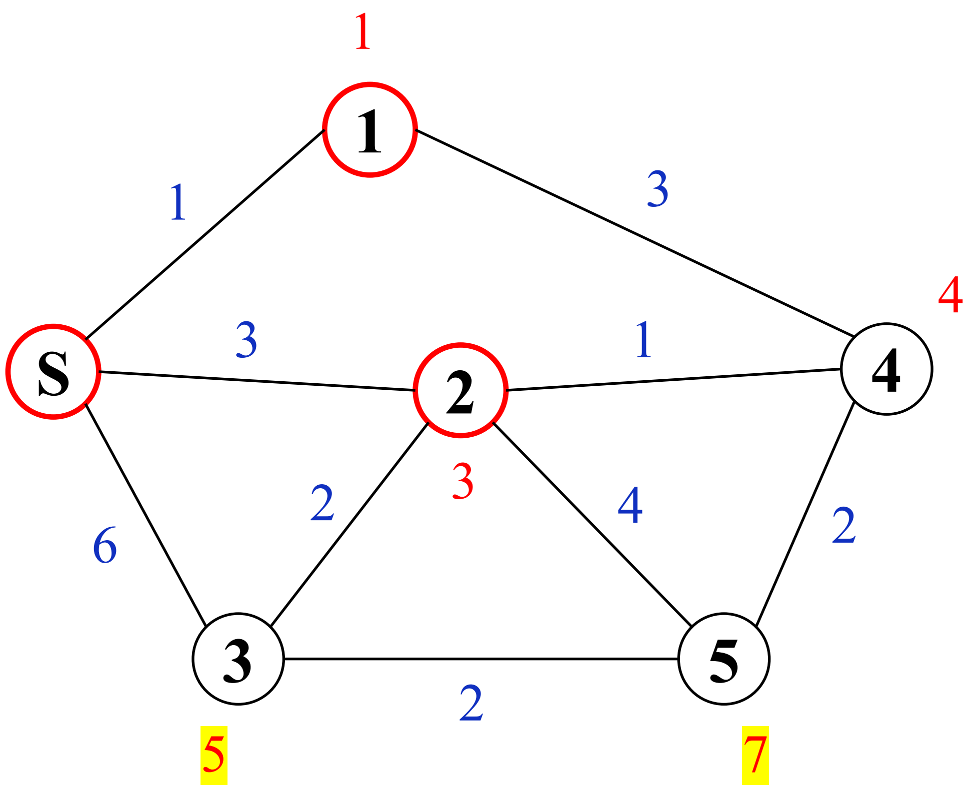
BLACK	$dist[x]$
	S, 1, 2, 3, 4, 5
S	0, 1, 3, 6, ∞, ∞

Dijkstra's algorithm - Pairwise



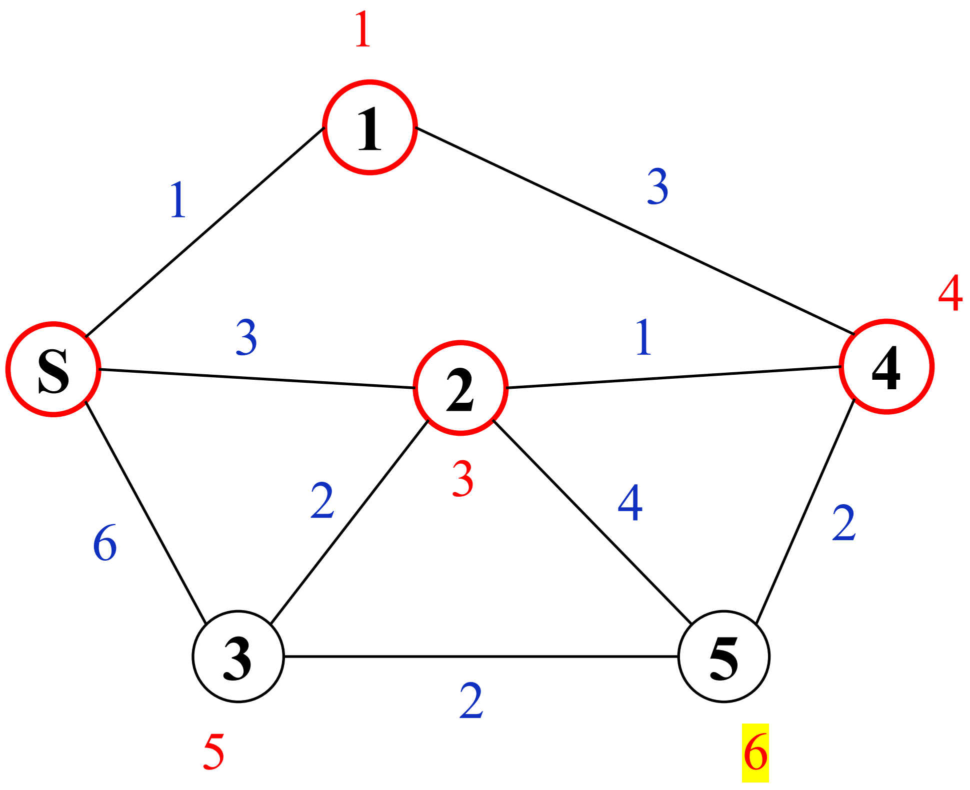
BLACK	$dist[x]$
	S, 1, 2, 3, 4, 5
S	0, 1, 3, 6, ∞, ∞
S, 1	0, 1, 3, 6, 4, ∞

Dijkstra's algorithm - Pairwise



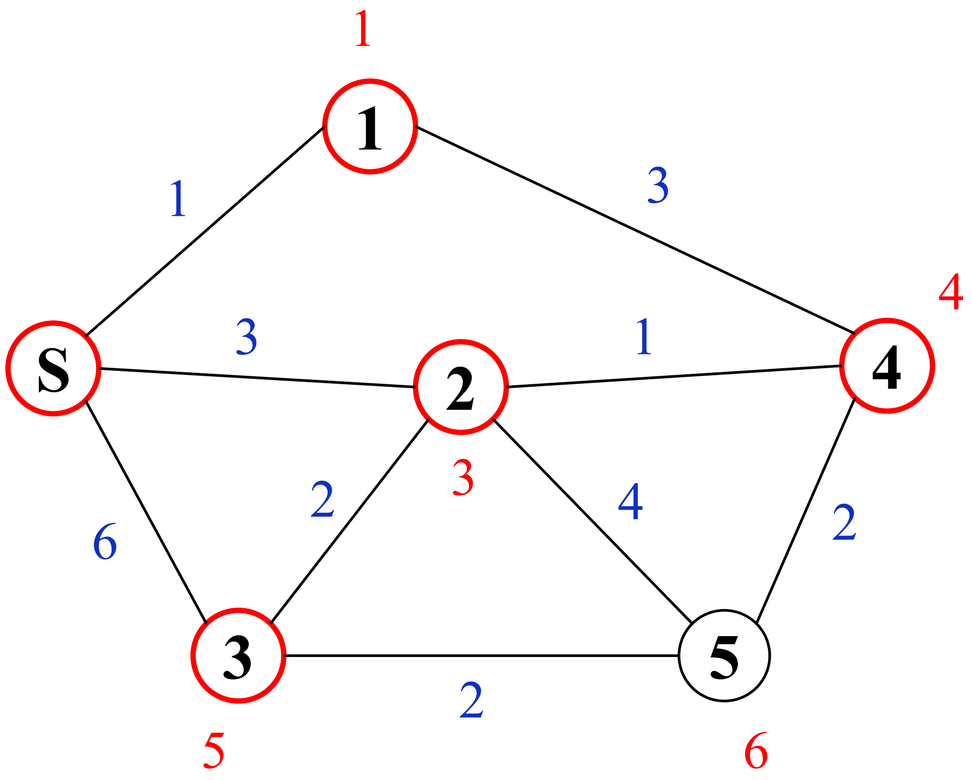
BLACK	$dist[x]$
	S, 1, 2, 3, 4, 5
S	0, 1, 3, 6, ∞, ∞
S, 1	0, 1, 3, 6, 4, ∞
S, 1, 2	0, 1, 3, 5, 4, 7

Dijkstra's algorithm - Pairwise



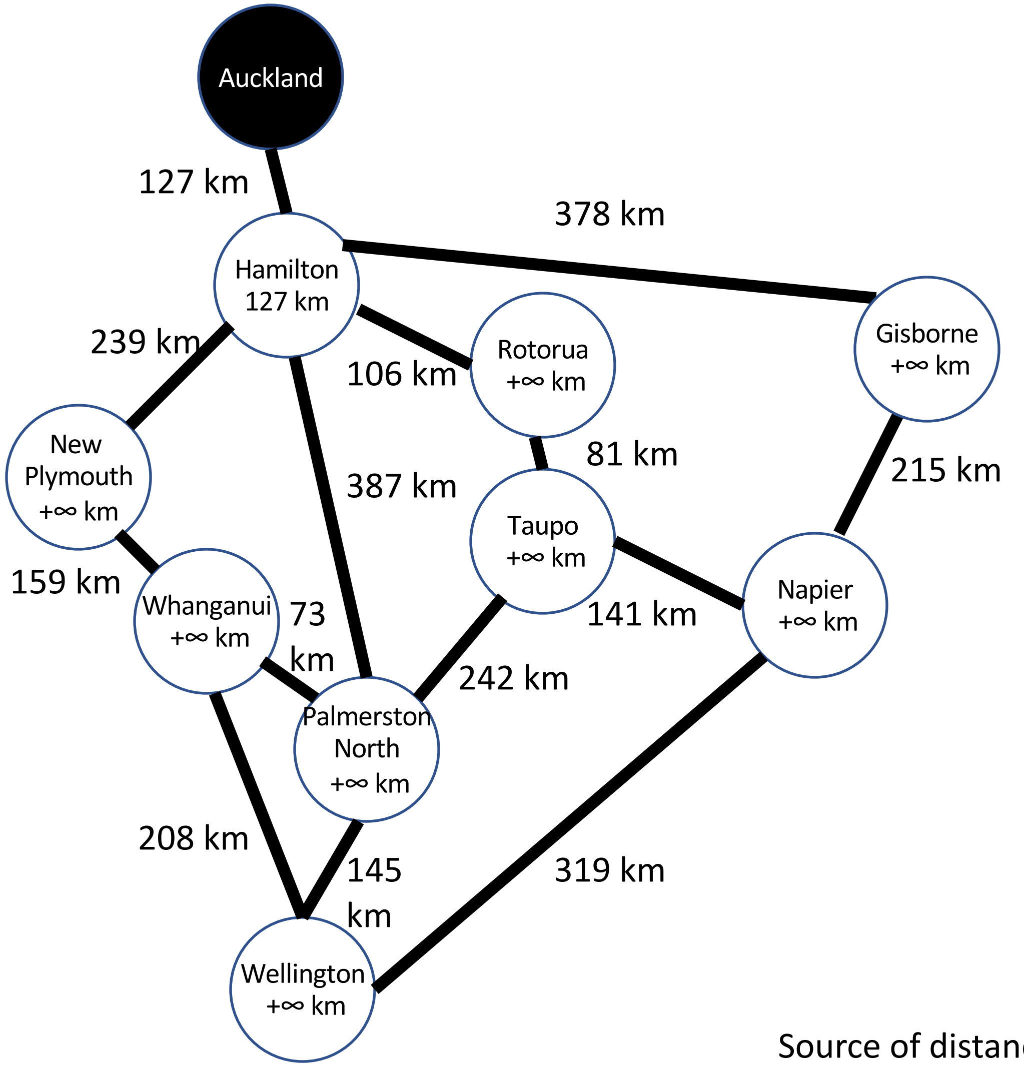
BLACK	$dist[x]$
	S, 1, 2, 3, 4, 5
S	0, 1, 3, 6, ∞, ∞
S, 1	0, 1, 3, 6, 4, ∞
S, 1, 2	0, 1, 3, 5, 4, 7
S, 1, 2, 4	0, 1, 3, 5, 4, 6

Dijkstra’s algorithm - Pairwise



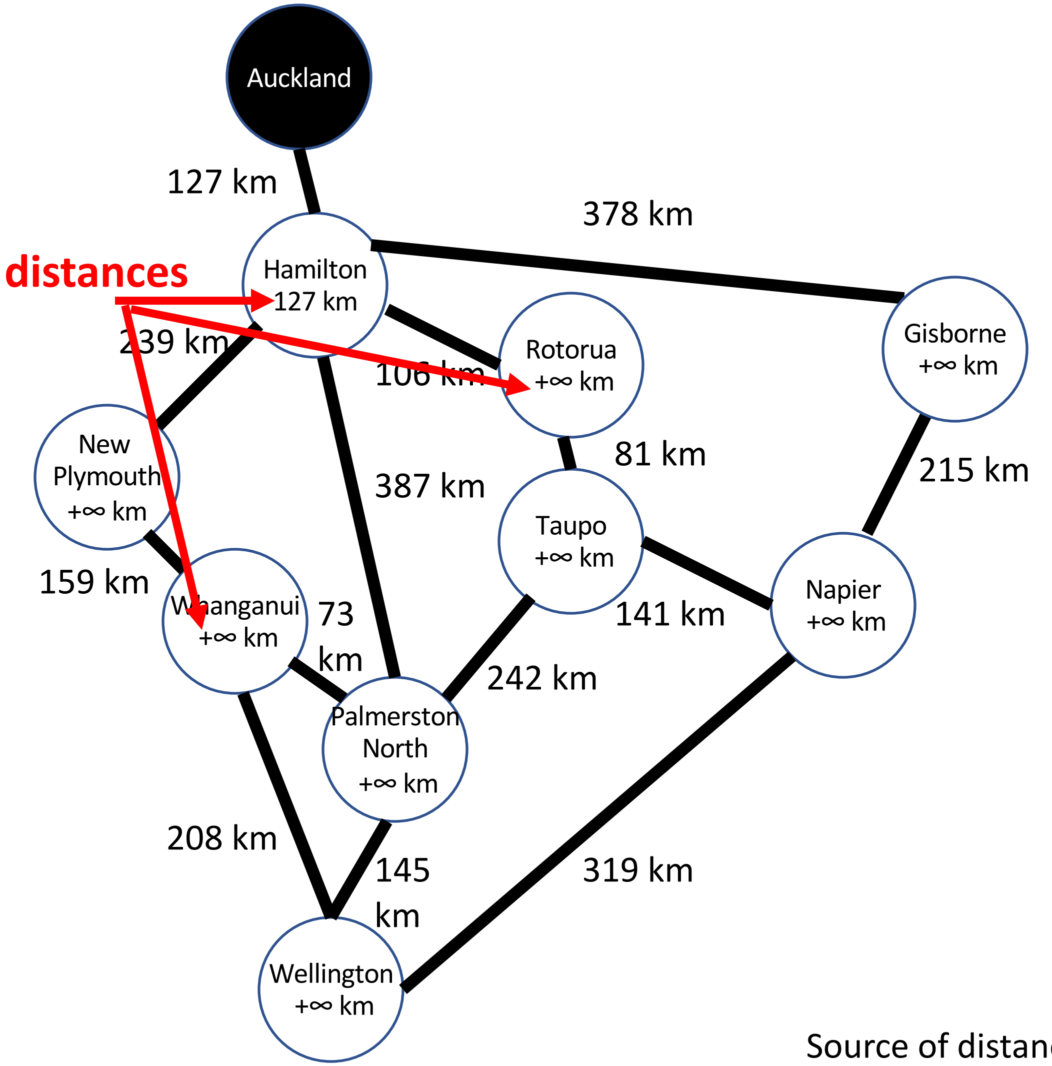
BLACK	$dist[x]$
	S, 1, 2, 3, 4, 5
S	0, 1, 3, 6, ∞, ∞
S, 1	0, 1, 3, 6, 4, ∞
S, 1, 2	0, 1, 3, 5, 4, 7
S, 1, 2, 4	0, 1, 3, 5, 4, 6
S, 1, 2, 4, 5	0, 1, 3, 5, 4, 6

Example: Dijkstra at Work



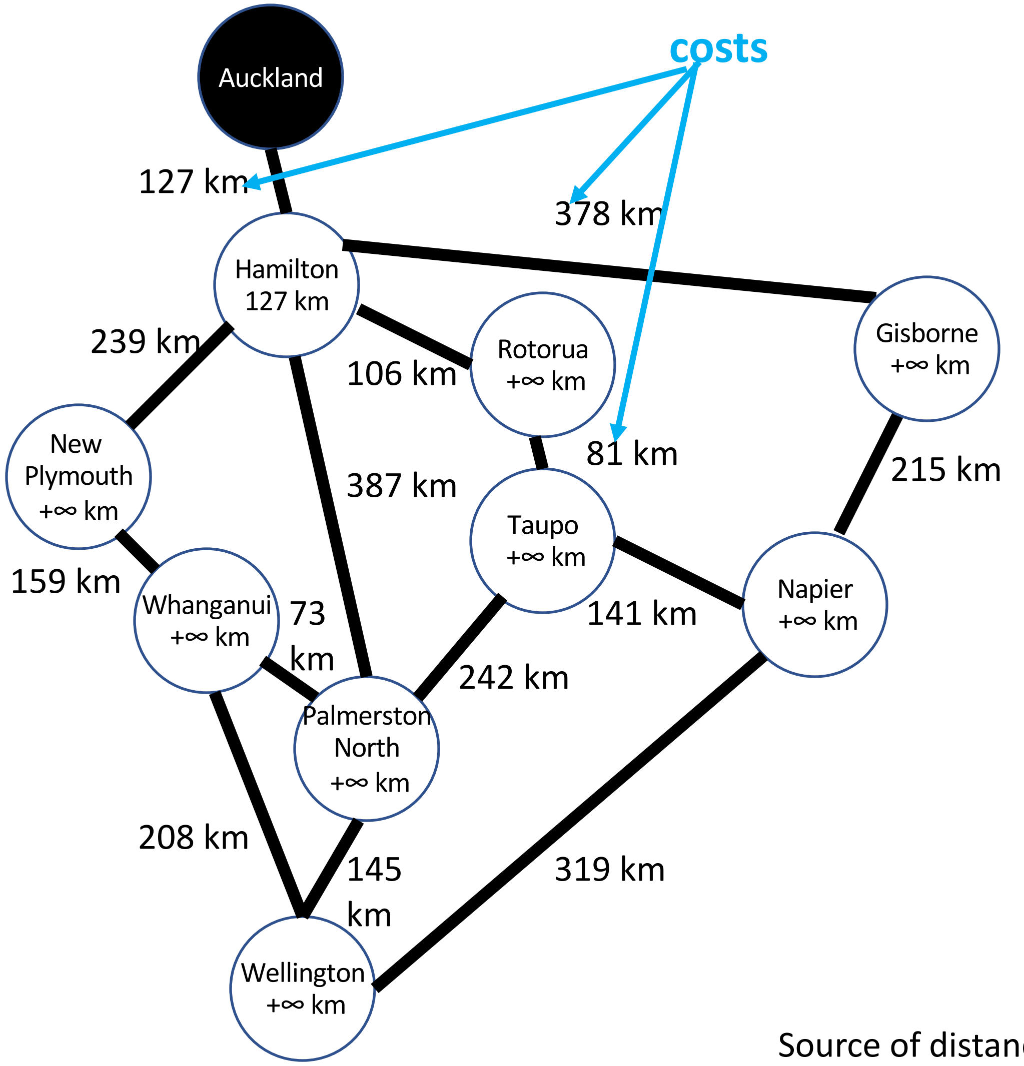
Source of distances: Google Maps

Example: Dijkstra at Work



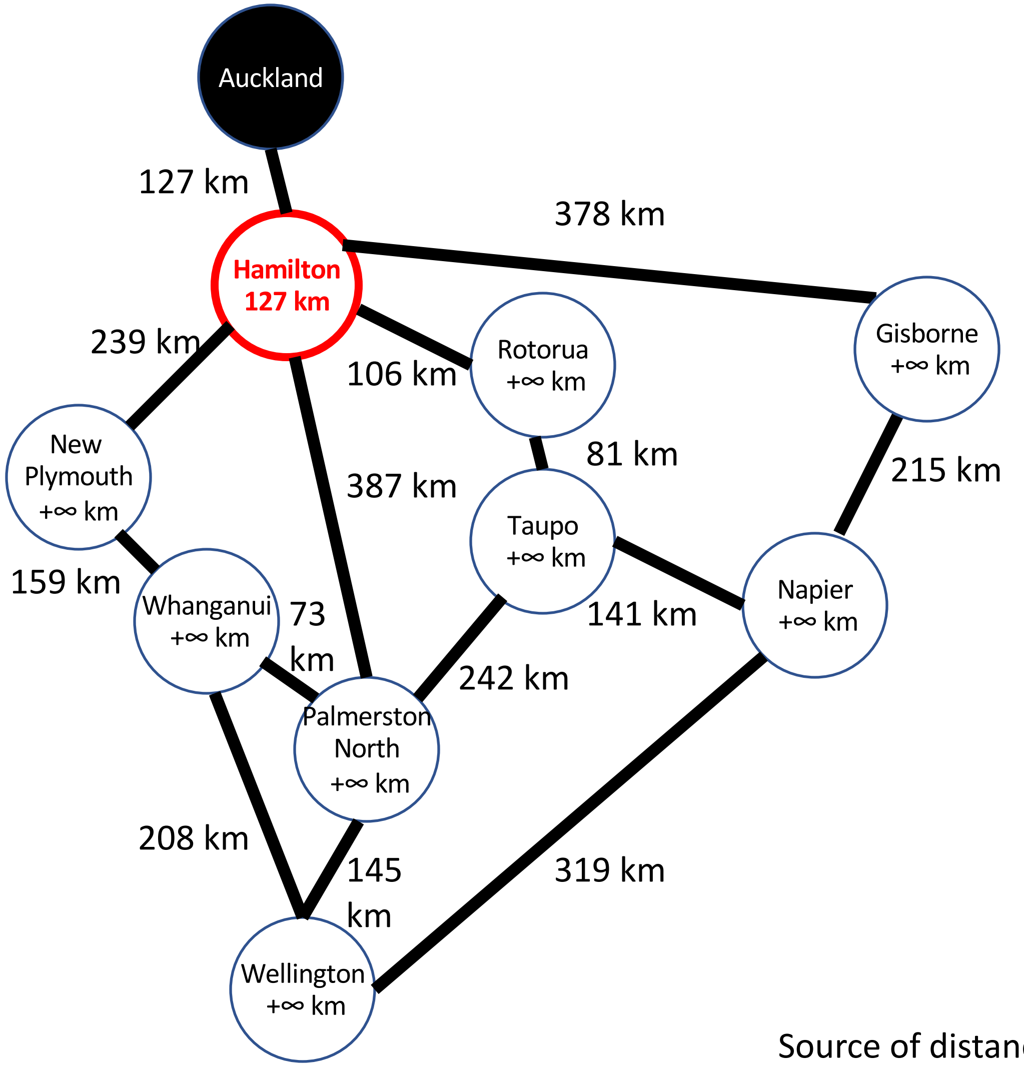
Source of distances: Google Maps

Example: Dijkstra at Work



Source of distances: Google Maps

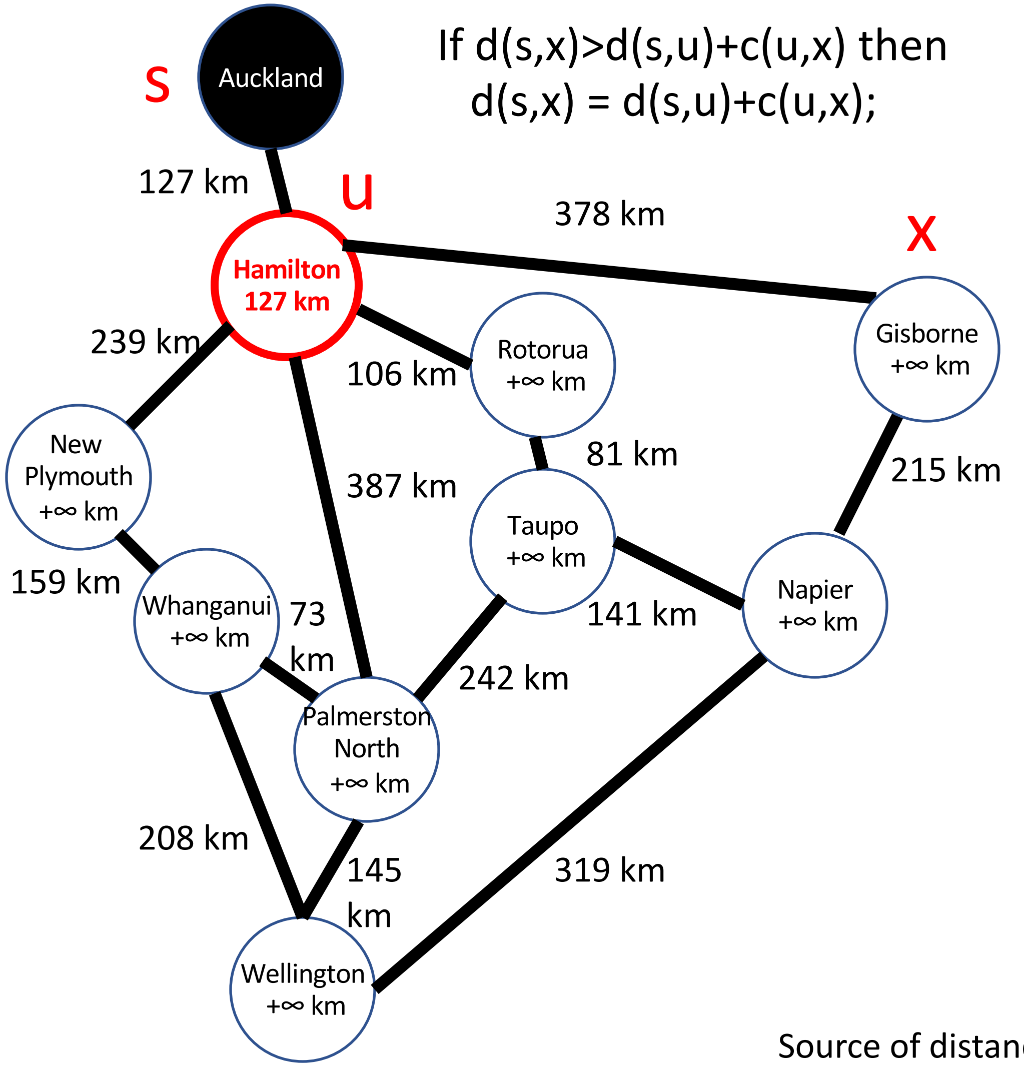
Example: Dijkstra at Work



Source of distances: Google Maps

Example: Dijkstra at Work

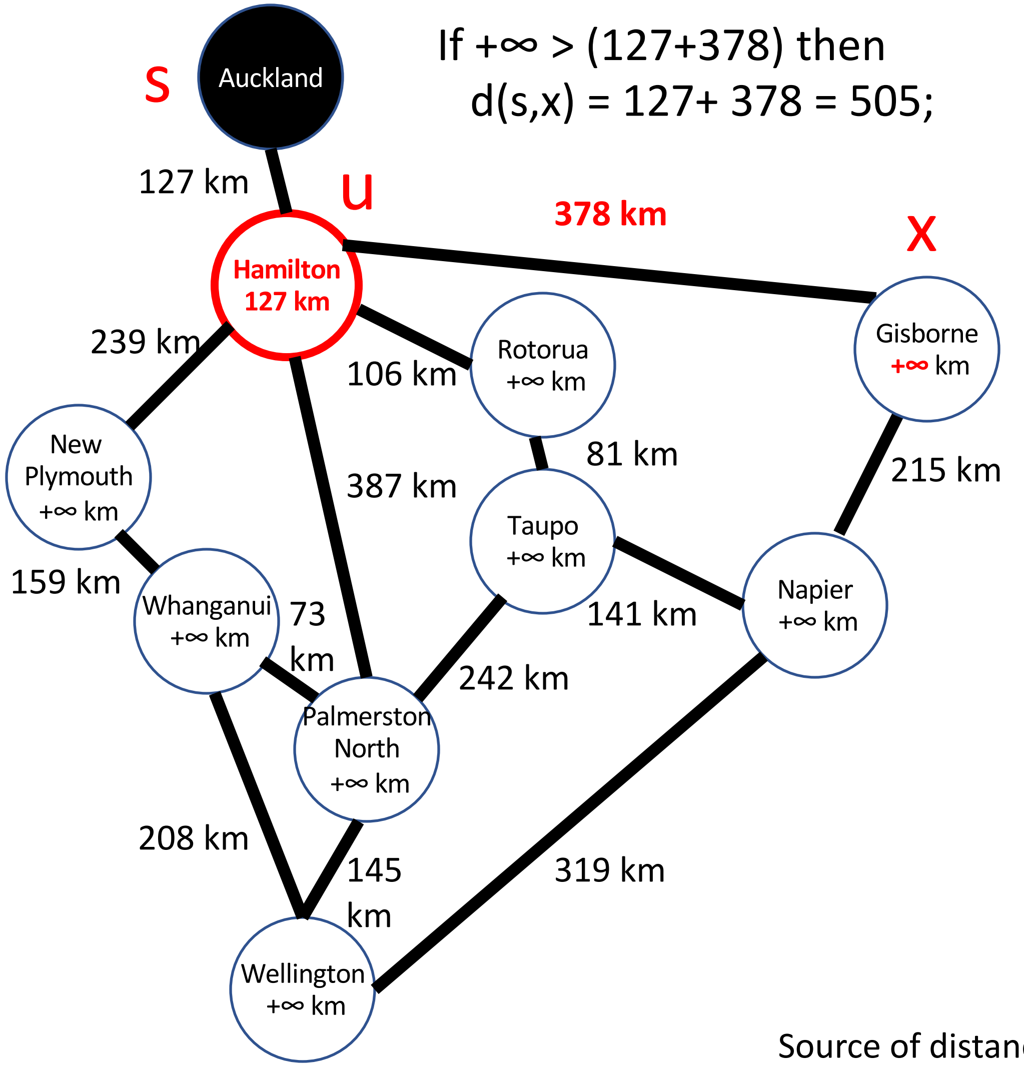
If d(s,x)>d(s,u)+c(u,x) then
d(s,x) = d(s,u)+c(u,x);



Source of distances: Google Maps

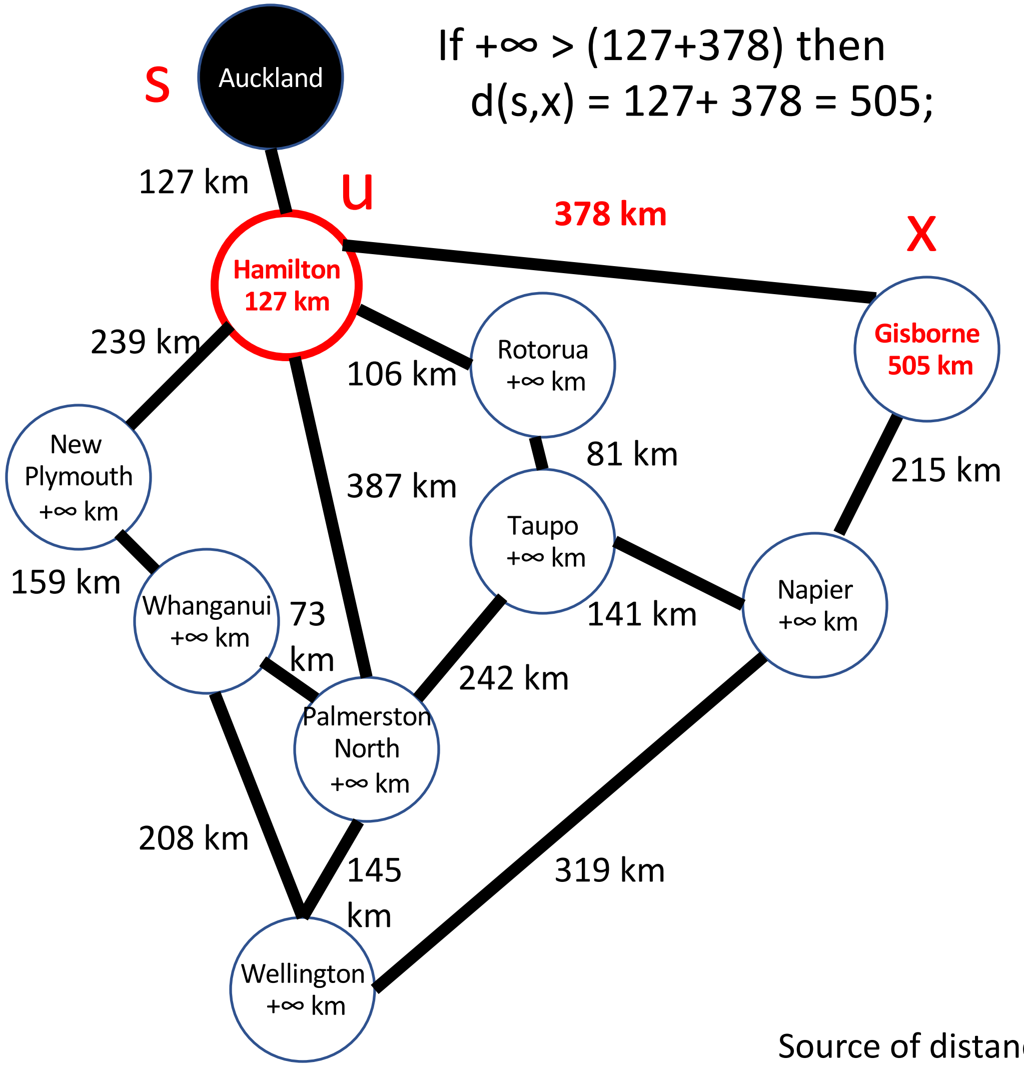
Example: Dijkstra at Work

If +∞ > (127+378) then
  d(s,x) = 127+ 378 = 505;



Source of distances: Google Maps

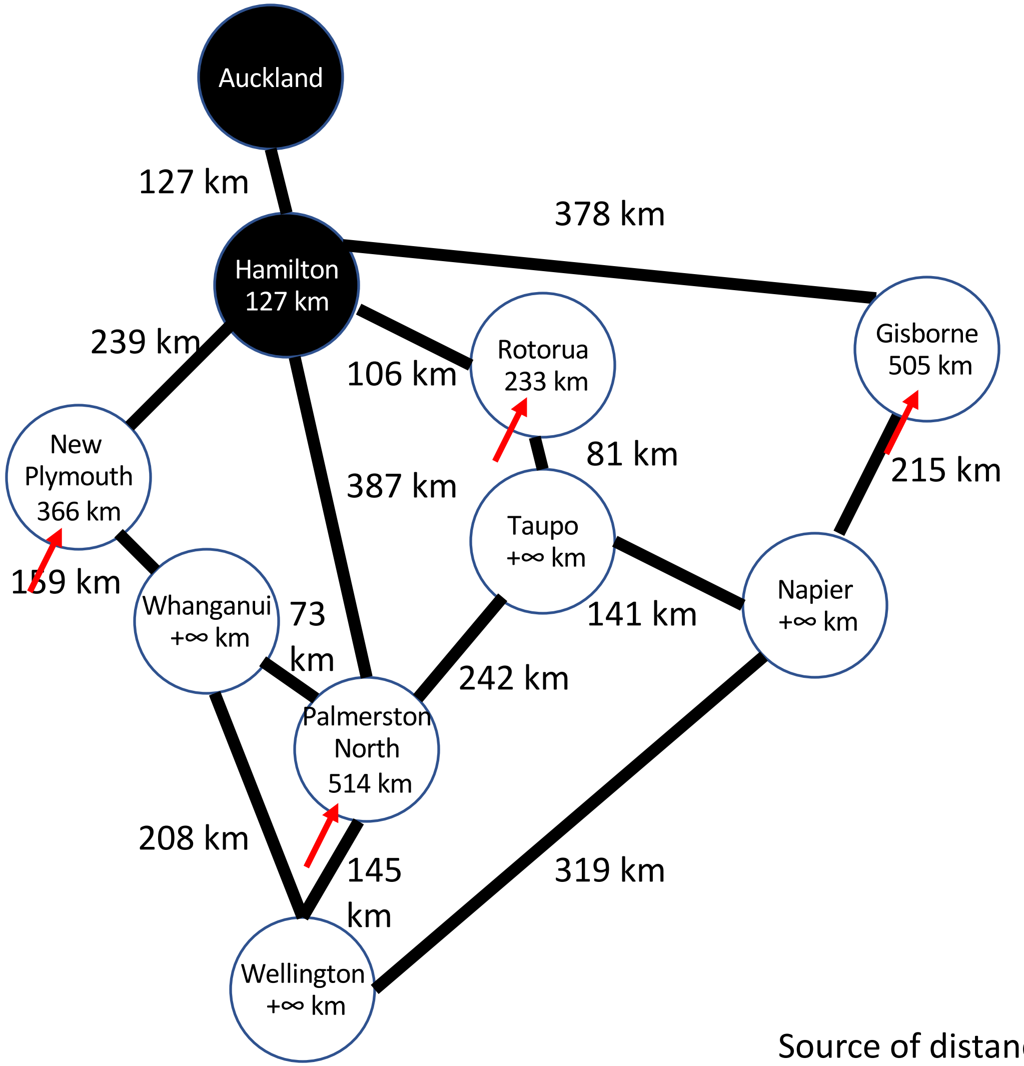
Example: Dijkstra at Work



If $+\infty > (127+378)$ then
$d(s,x) = 127 + 378 = 505$;

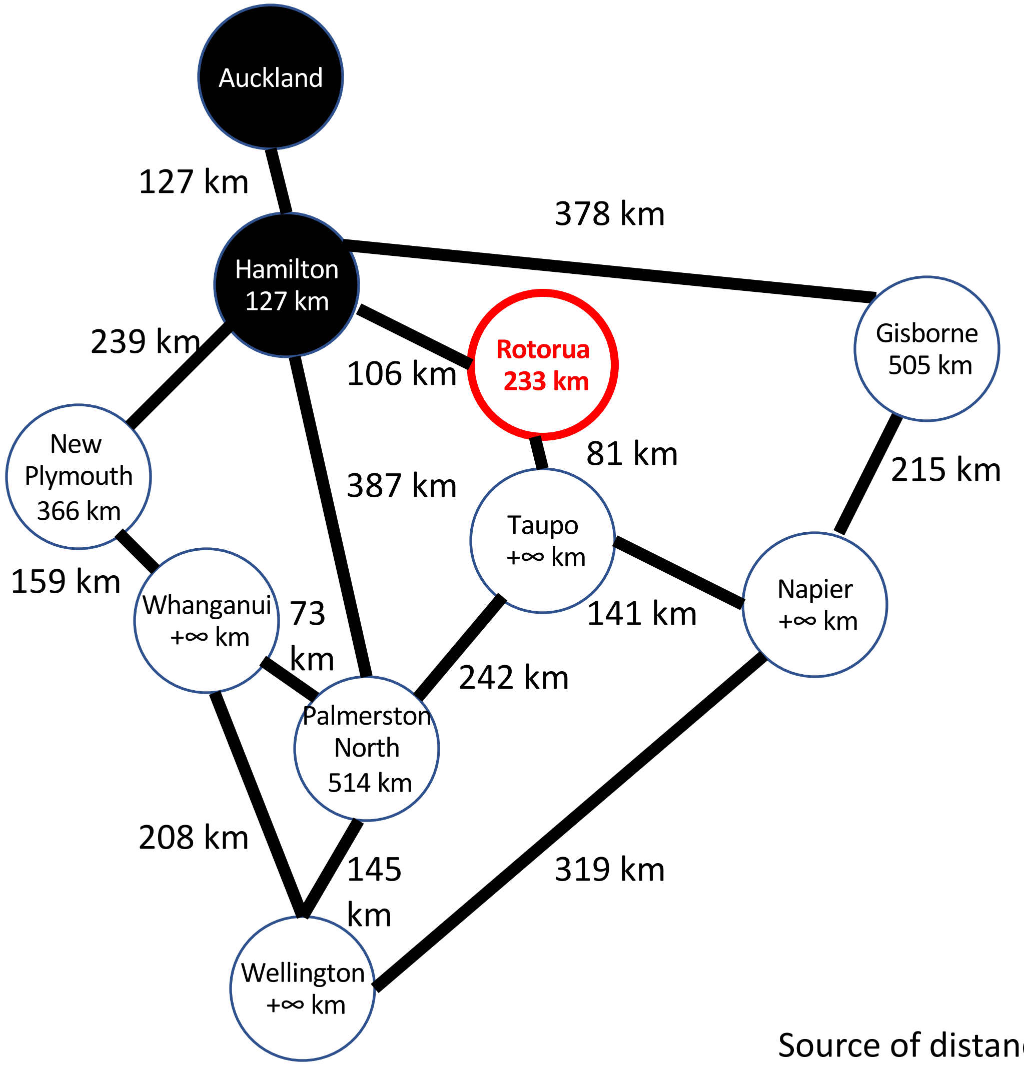
Source of distances: Google Maps

Example: Dijkstra at Work



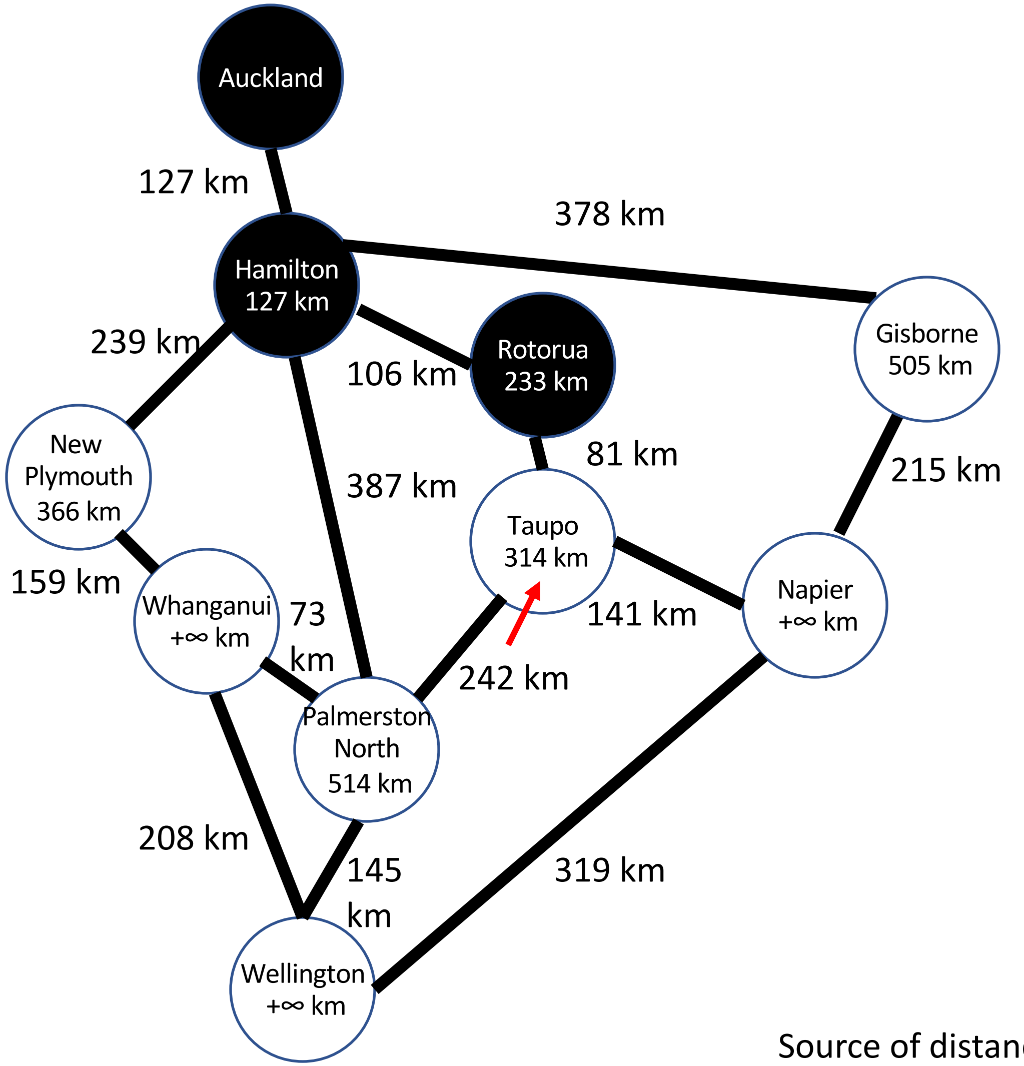
Source of distances: Google Maps

Example: Dijkstra at Work



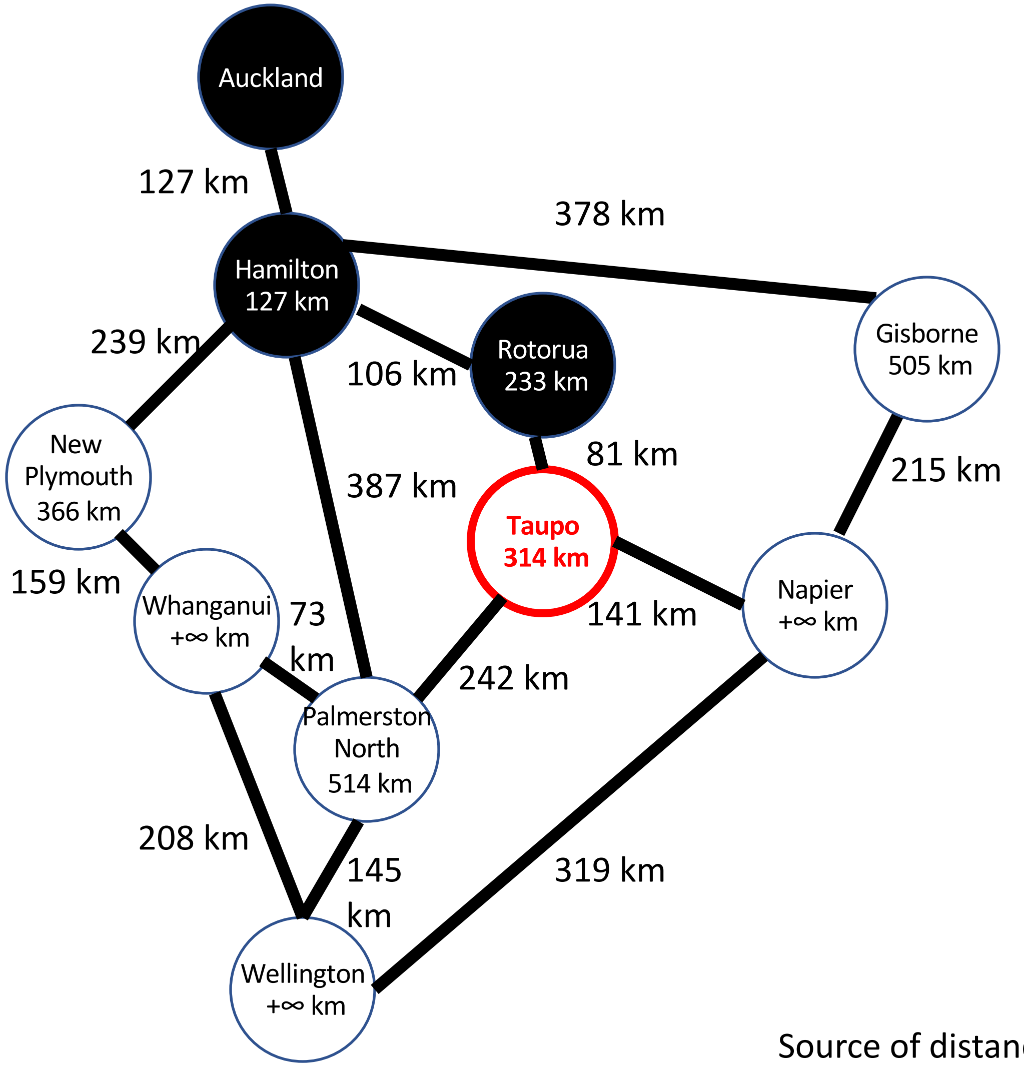
Source of distances: Google Maps

Example: Dijkstra at Work



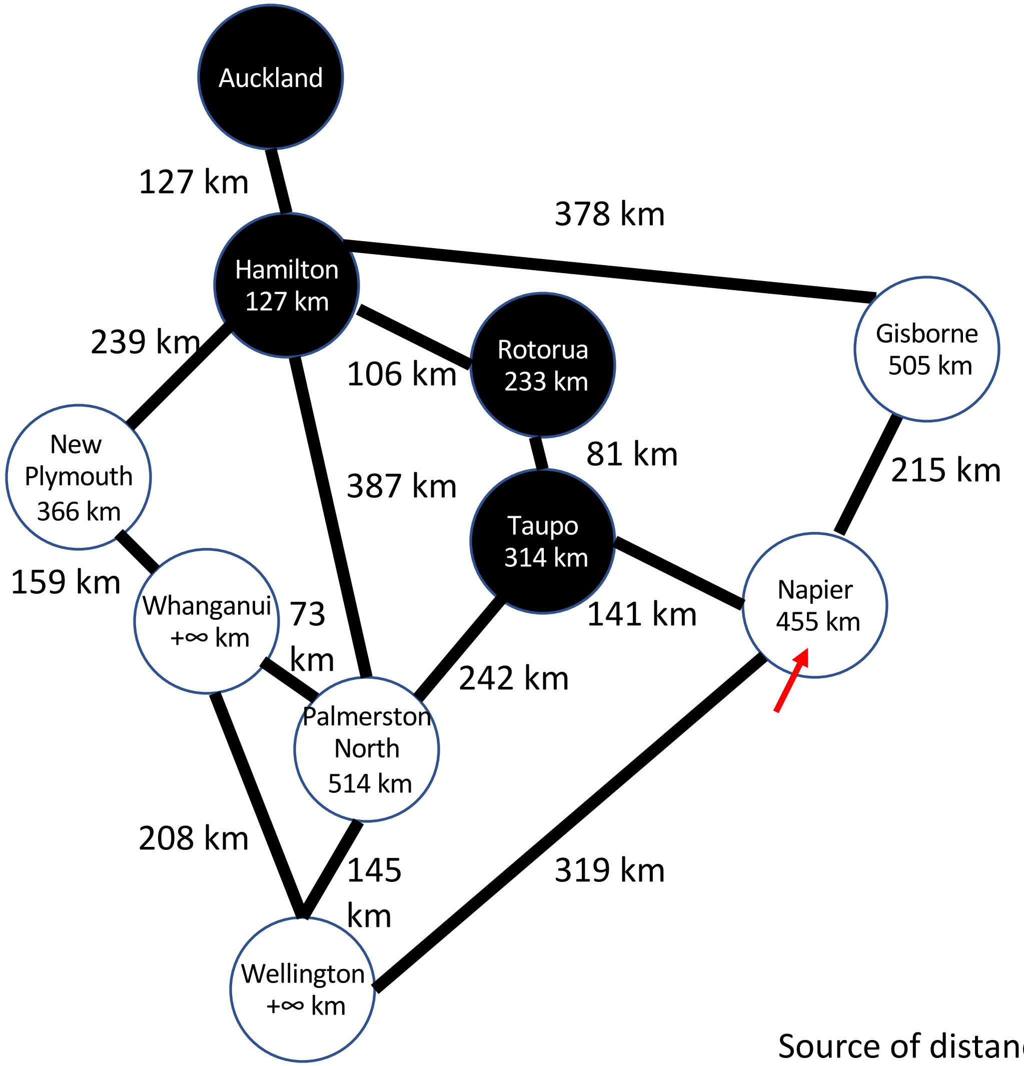
Source of distances: Google Maps

Example: Dijkstra at Work



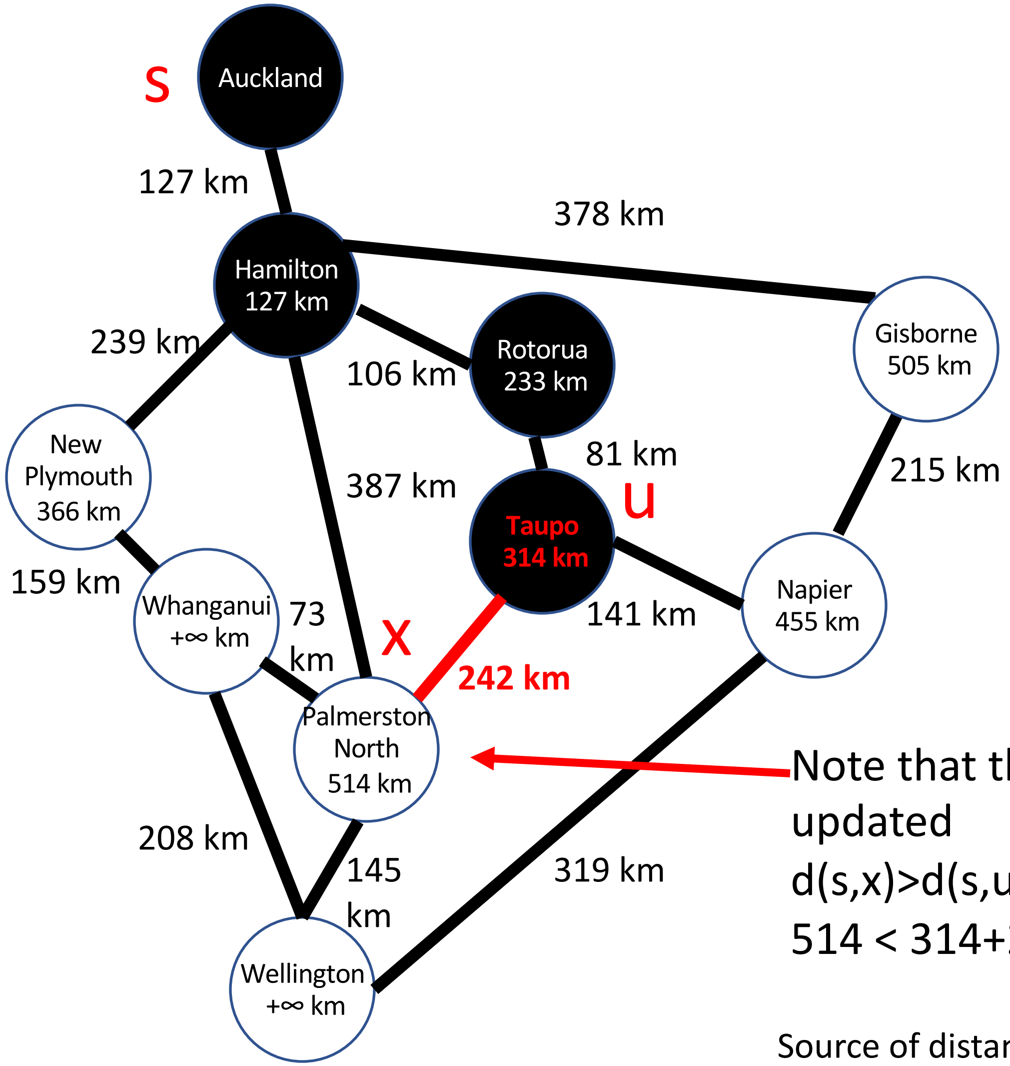
Source of distances: Google Maps

Example: Dijkstra at Work



Source of distances: Google Maps

Example: Dijkstra at Work



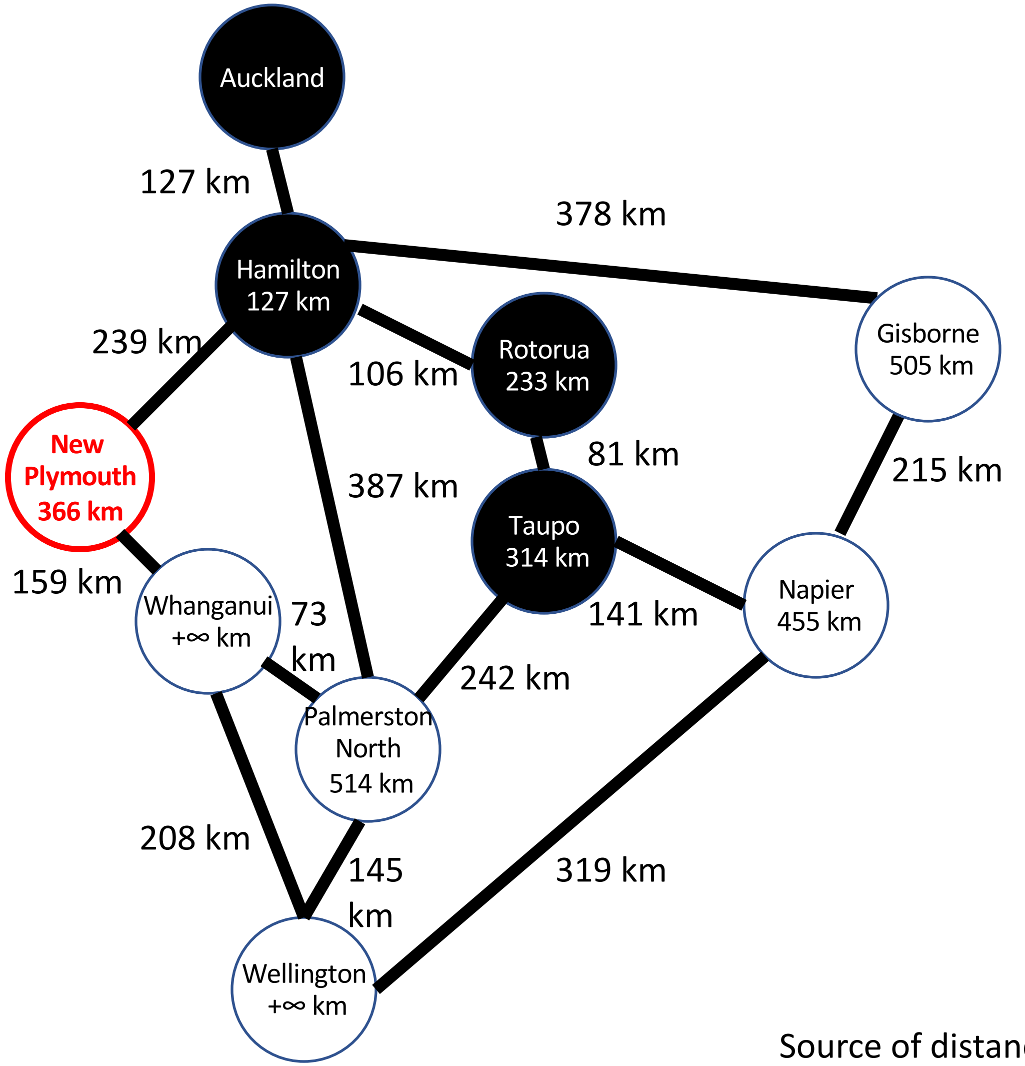
Note that this has not been updated

$d(s,x)>d(s,u)+c(u,x)$ is false

$514 < 314+242 = 556$

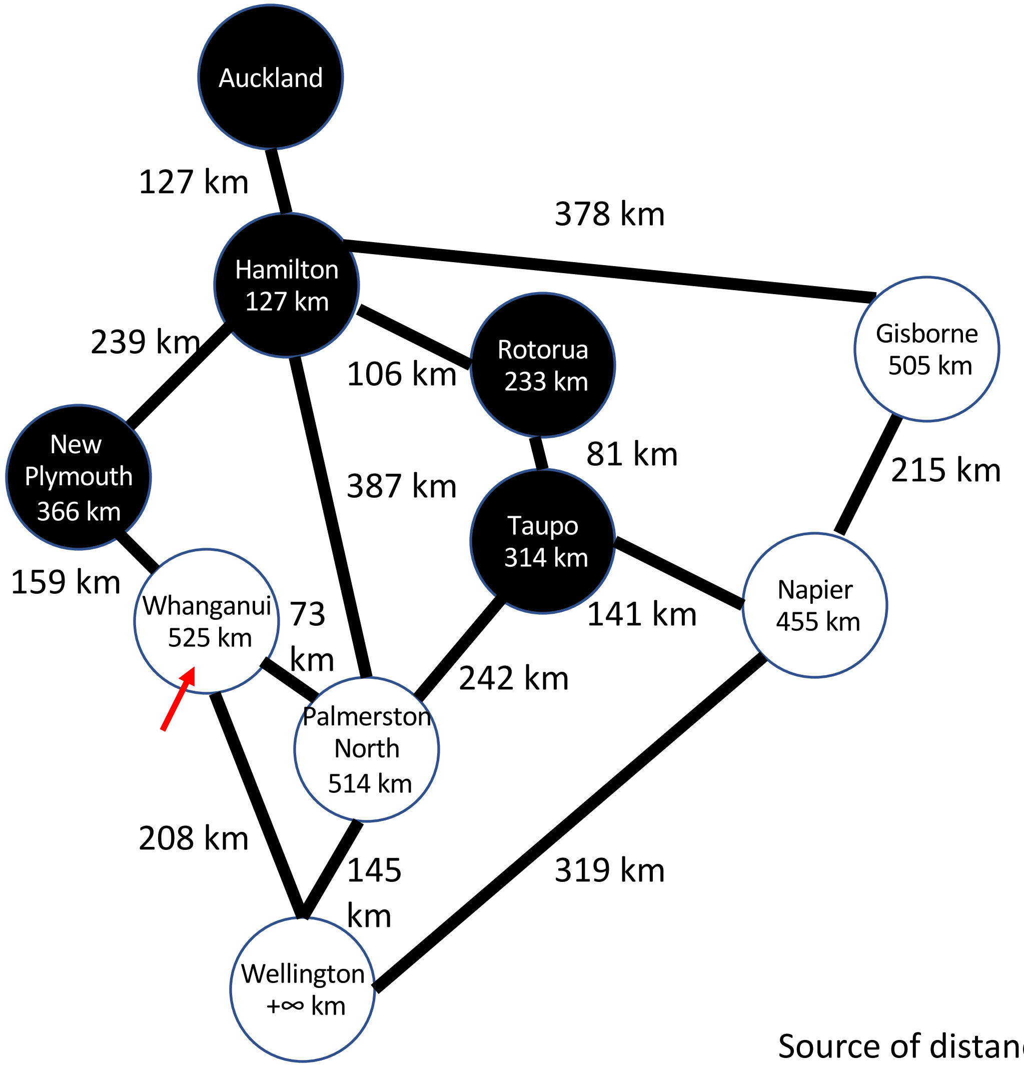
Source of distances: Google Maps

Example: Dijkstra at Work



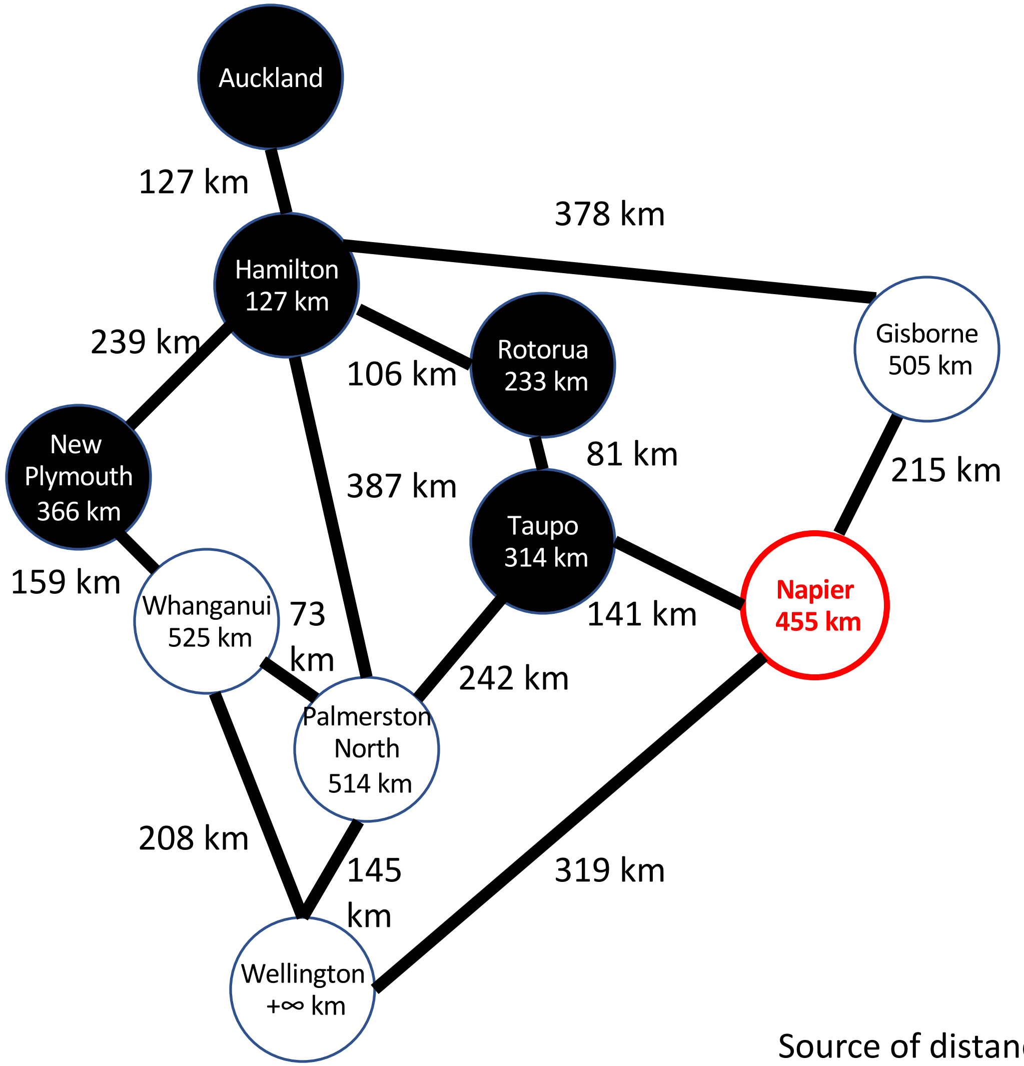
Source of distances: Google Maps

Example: Dijkstra at Work



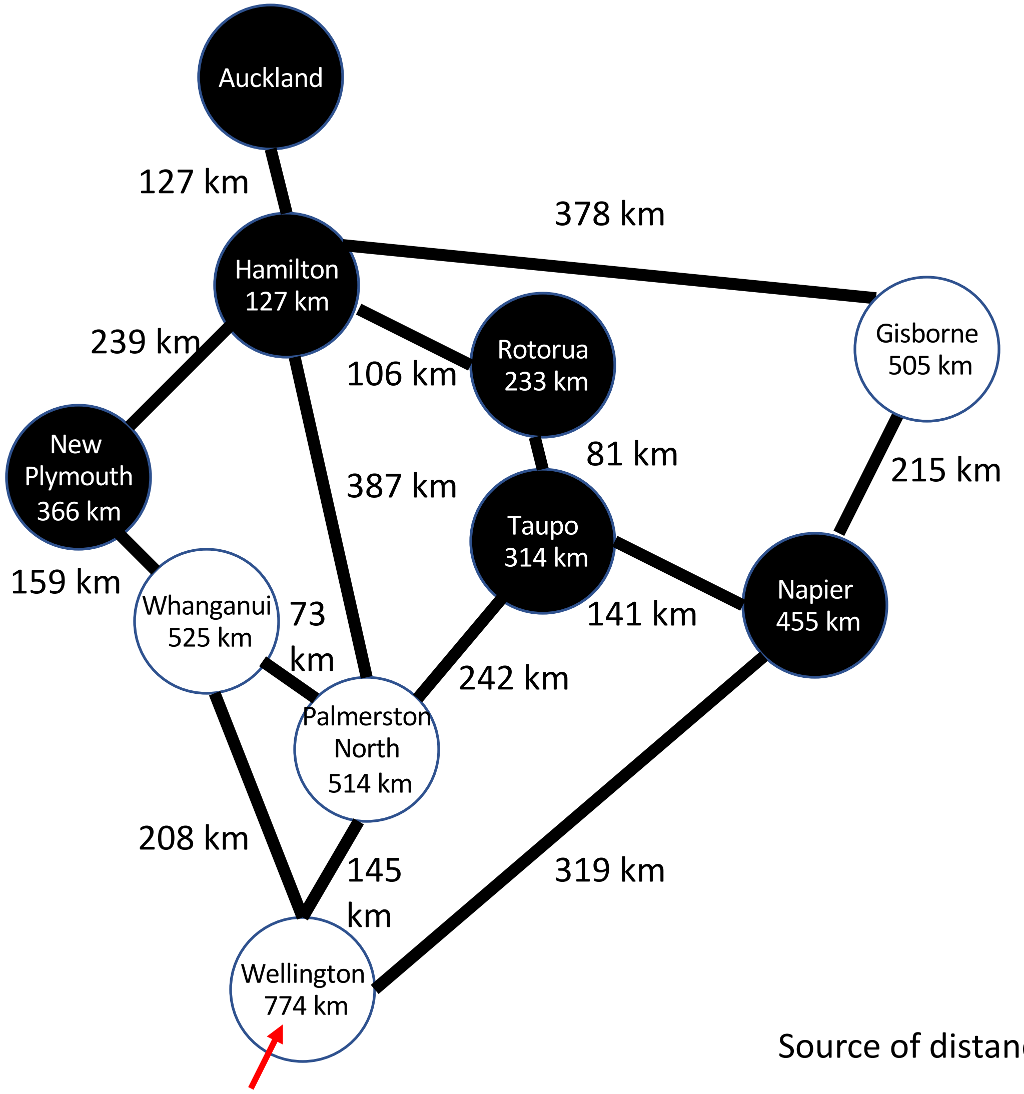
Source of distances: Google Maps

Example: Dijkstra at Work



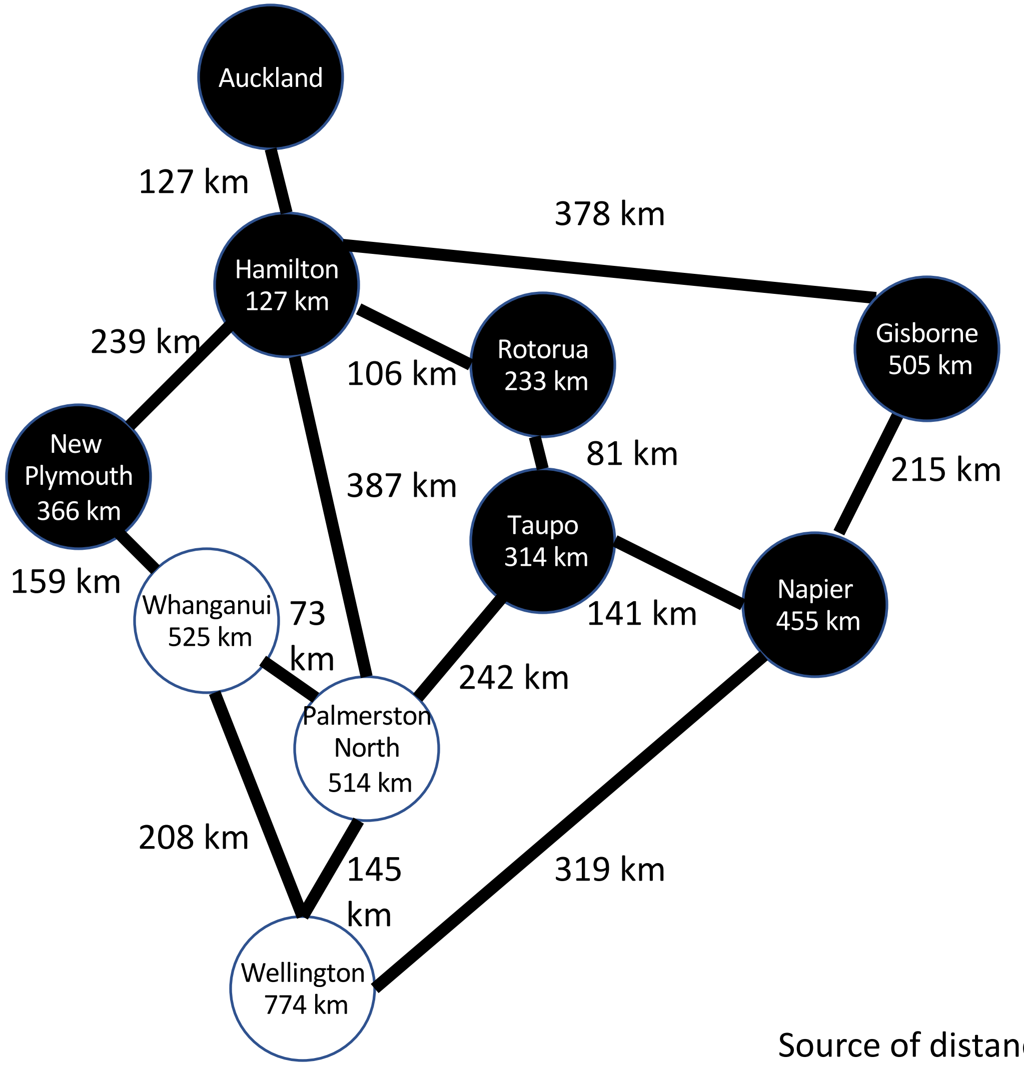
Source of distances: Google Maps

Example: Dijkstra at Work



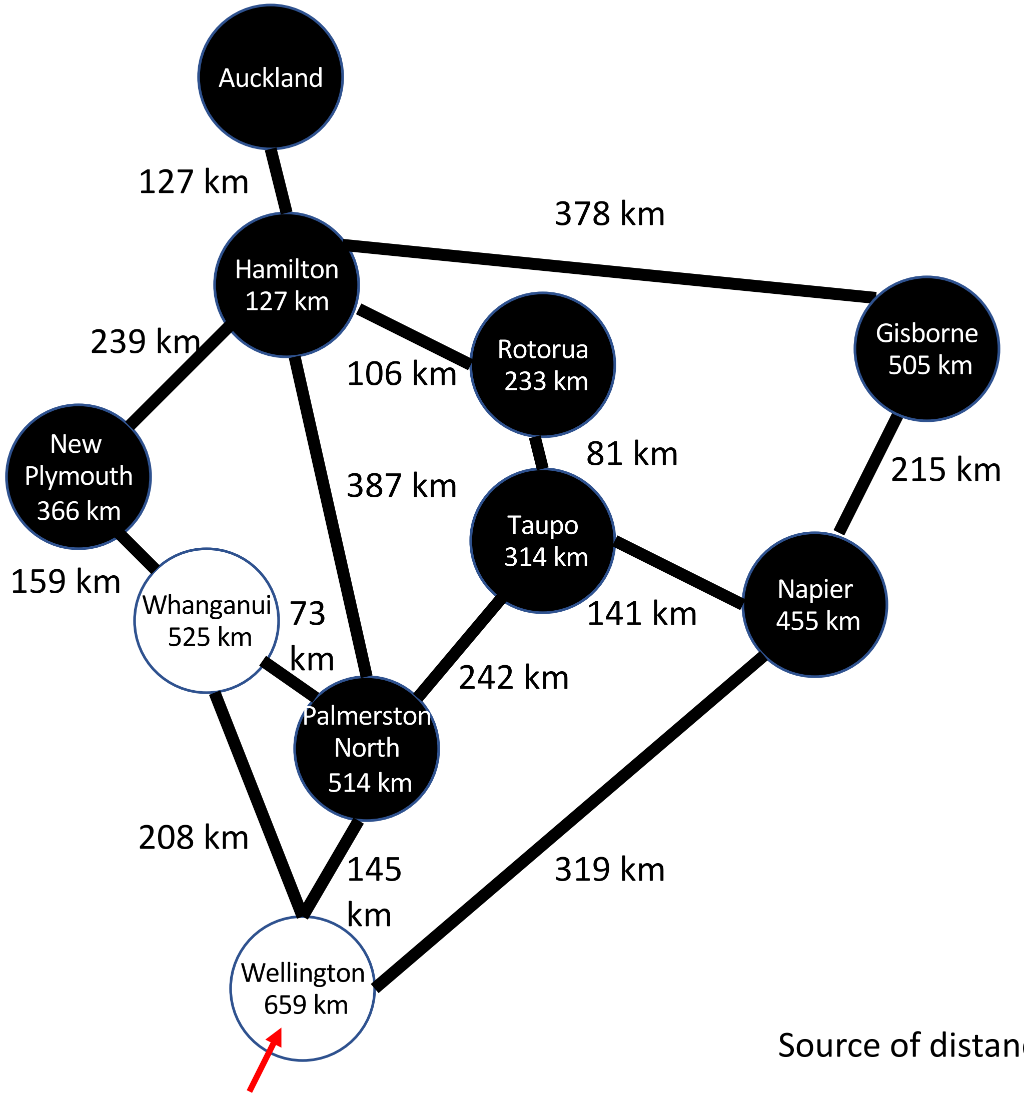
Source of distances: Google Maps

Example: Dijkstra at Work



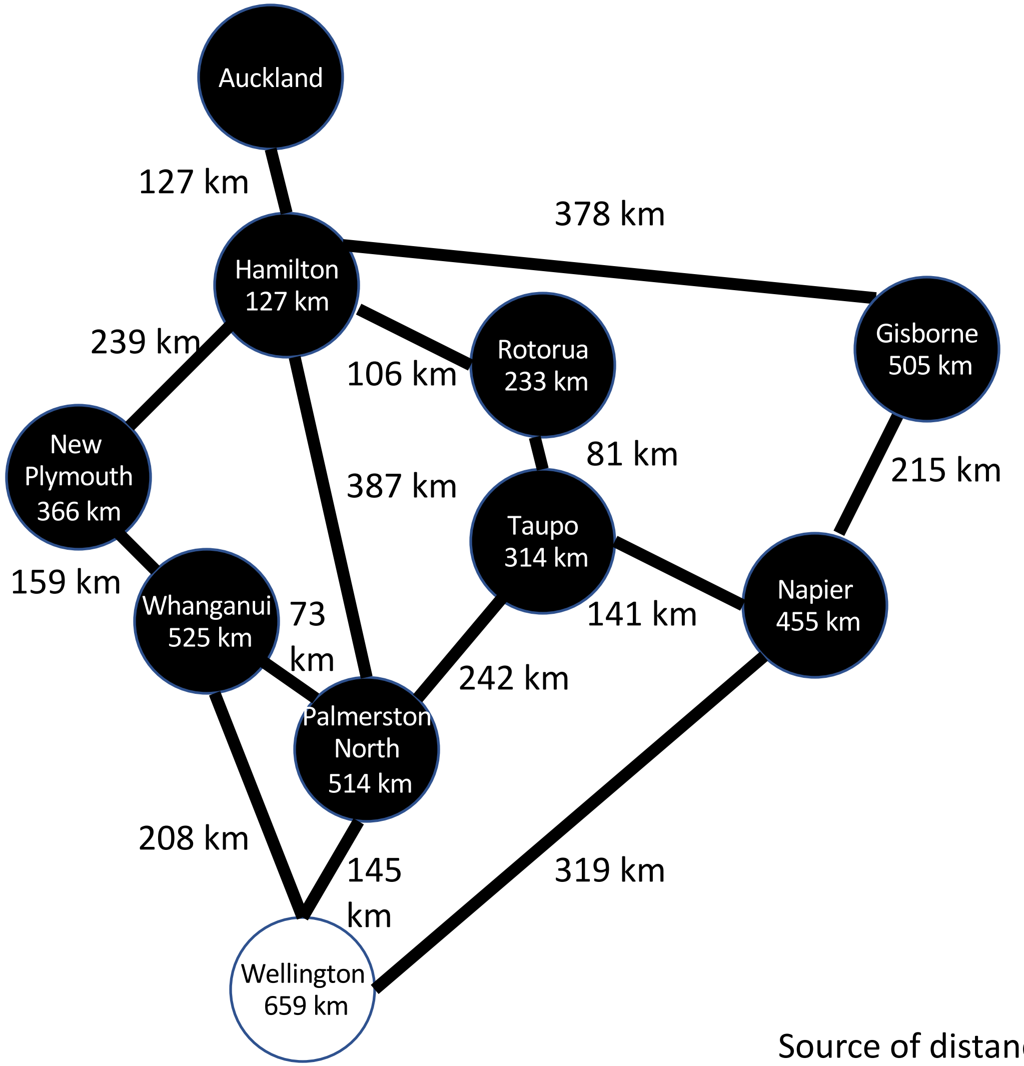
Source of distances: Google Maps

Example: Dijkstra at Work



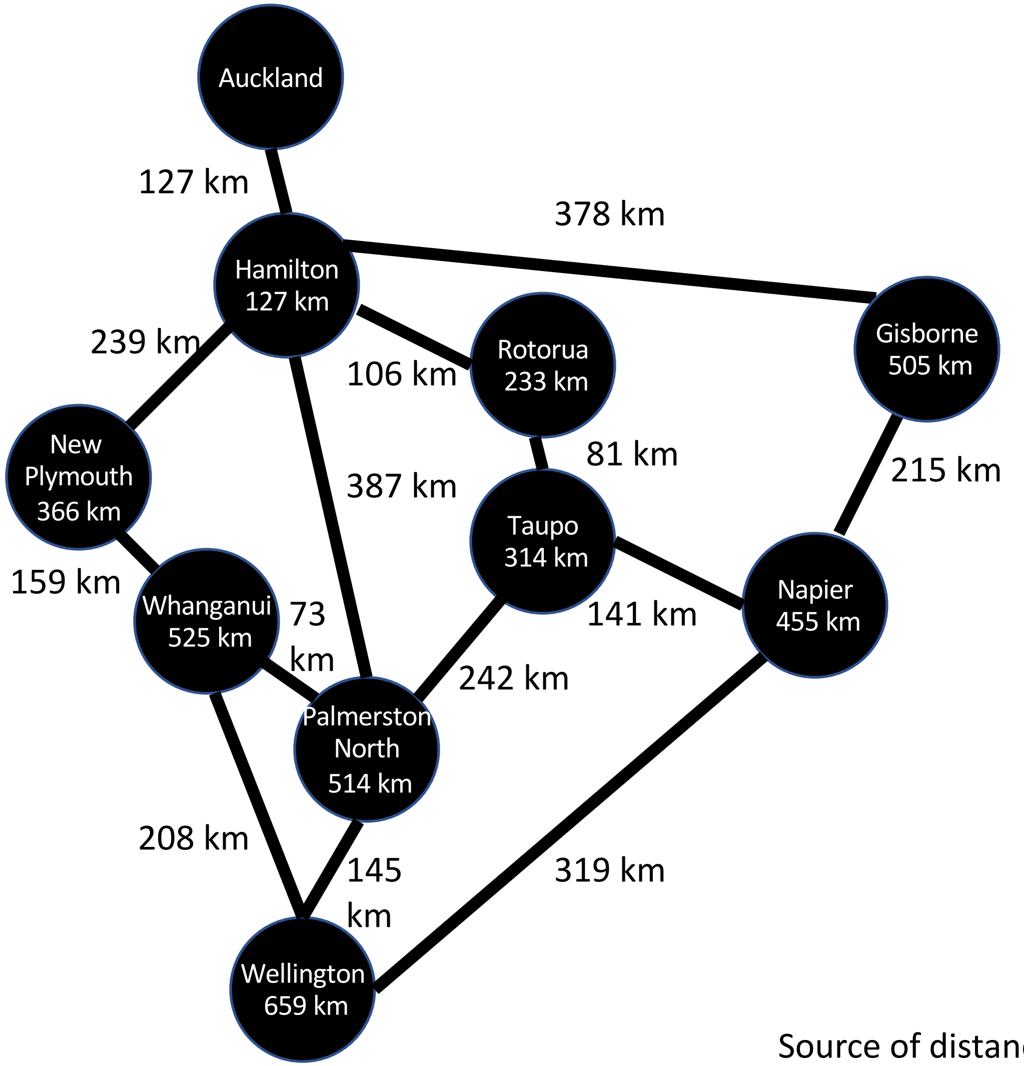
Source of distances: Google Maps

Example: Dijkstra at Work



Source of distances: Google Maps

Example: Dijkstra at Work



Source of distances: Google Maps

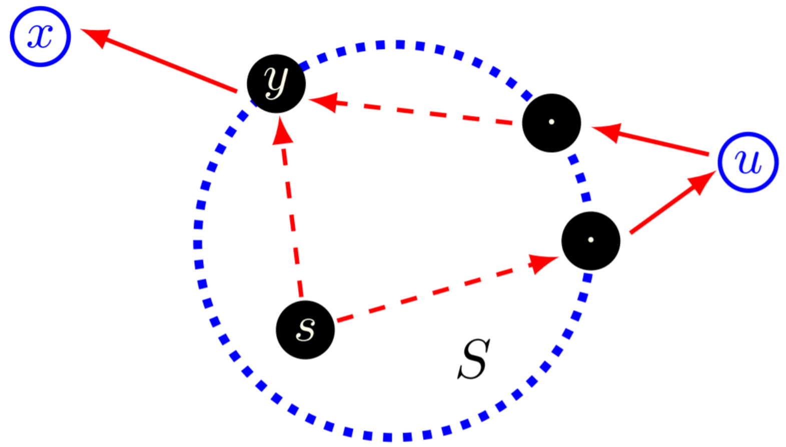
Time Complexity Analysis: Dijkstra’s Algorithm

- Complexity with array implementation to find the minimum value of dist: $\Theta(n^2)$
  - $n^2 + m$ Adjacency list representation
  - $n^2 + n^2$ Adjacency matrix representation
  - n: time taken to find the node with minimum value of dist

- Complexity depends on data structures used, especially for priority queue; $O((m + n)\log n)$ is possible.
  - n delete-min operations, and
  - m decrease-key operations at most
  - $\log n$: binary heap implementation of priority queue

Why Dijkstra's Algorithm Works

- **Fact**. Suppose that all weights are non-negative. At the top of $while$ loop, these properties hold for all $x \in V(G)$:
- P1: $dist[x]$ is the minimum weight of an S-path to x.
- P2: if $color[x]$=BLACK, $dist[x]$ is the minimum weight.



Let an *S*-path be a path starting at node s and ending at node u with all nodes in S except **possibly** u.

In Other Words …

- P1 guarantees that, in each iteration of the algorithm, we find a minimum weight path from s to a vertex $w \in V(G)$ that only uses vertices in S (black vertices) except possibly w.

- P2 guarantees that, for each black vertex w, we have already found a minimum weight path from s to w.

- Taken together, these two facts imply that Dijkstra's algorithm solves the single-source shortest path problem for weighted digraphs that do not have any negative arc weights.

Proof by Mathematical Induction

- Induction on the number of times k of going through the while-loop. We use S_k to denote the set of S at the k-th loop: $S_0 = \{s\}$; $dist[s] = 0$

$P1$: $dist[x]$ is the minimum weight of an S-path to x.
$P2$: if $color[x]$ = BLACK, $dist[x]$ is the minimum weight.

- **Base case**: both P1 and P2 hold when k=0
- If k=0, then only the starting vertex s is black and $dist[s]$=0. Hence, both properties hold.
- **Inductive hypothesis**: P1 and P2 hold for $k \geq 0$; $S_{k+1} = S_k \cup \{u\}$.

Inductive Step for P1

- Inductive hypothesis: P1 and P2 hold for $k \geq 0$; $S_{k+1} = S_k \cup \{u\}$.

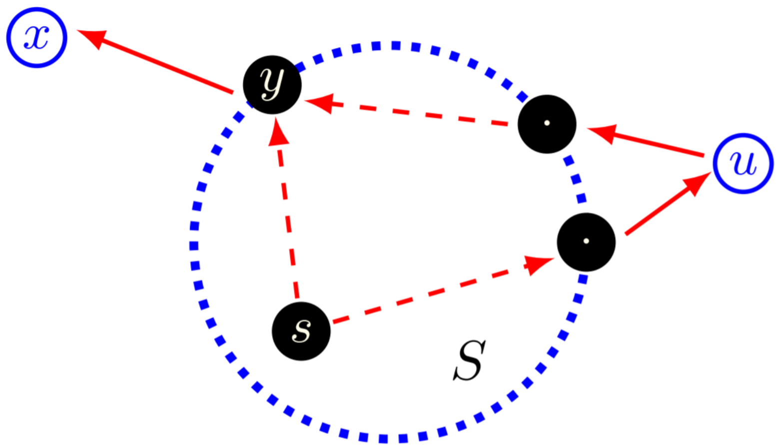
$P1$: $dist[x]$ is the minimum weight of an S-path to x.
$P2$: if $color[x] =$ BLACK, $dist[x]$ is the minimum weight.

- In iteration k+1, the algorithm colors u black.
- Let $x \in V(G)$ and γ be any S_{k+1} -path from starting vertex s to x.
- We want to show that $|\gamma| \geq dist[x]$, then $|\gamma| = dist[x]$ for min path

Case 1: u is not a vertex of γ.

- Then γ is an S_k -path and the result follows from the induction hypothesis: $dist[x]$ is the minimum weight of an S_{k+1} -path from the starting vertex to x.

Inductive Step for P1 (Contd.)



- Inductive hypothesis: P1 and P2 hold for $k \geq 0$; $S_{k+1} = S_k \cup \{u\}$

$P1$: $dist[x]$ is the minimum weight of an S-path to x. $P2$: if $color[x]$ = BLACK, $dist[x]$ is the minimum weight.

Case 2: u is a vertex of γ.

- Subcase 2a: γ=(s…u,x) Then let $\gamma_1 = (s \ldots u)$. So $|\gamma| = |\gamma_1| + c(u, x) \geq dist[x]$ // update formula
- Subcase 2B: γ=(s…u…y,x)

  Let β be the minimum S_k path from s to y (not including u): $|\beta|$=$dist[y]$

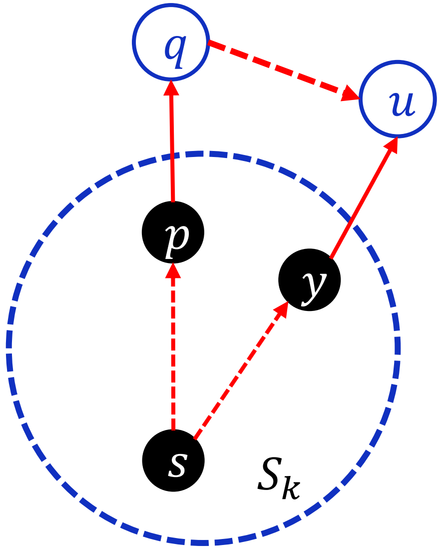
  By the induction hypothesis:

  $|\gamma| = |\gamma_1| + c(y, x) \geq |\beta| + c(y, x) = dist[y] + c(y, x) \geq dist[x]$ // induction hypothesis.

Inductive Step for P2

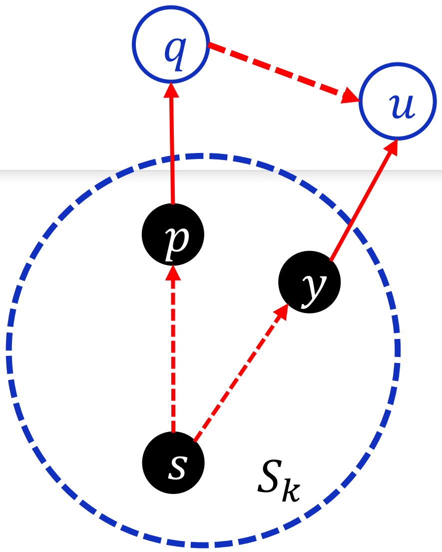
- Inductive hypothesis: P1 and P2 hold for $k \geq 0$; $S_{k+1} = S_k \cup \{u\}$.

$P1$: $dist[x]$ is the minimum weight of an S-path to x.
$P2$: if $color[x] = \text{BLACK}$, $dist[x]$ is the minimum weight.



Case 1: If $x \in S_{k+1}$ and $x \neq u$, $P2$ holds by hypothesis

Inductive Step for P2 (Contd.)



- Inductive hypothesis: P1 and P2 hold for $k \geq 0$; $S_{k+1} = S_k \cup \{u\}$.

$P1$: $dist[x]$ is the minimum weight of an S-path to x. $P2$: if $color[x]$ = BLACK, $dist[x]$ is the minimum weight.

Case 2: If x=u, for any S_k -path, we have $dist[u]$ be the shortest by hypothesis. We only need to show that there is no shorter path to u that contains any node that is not in S_{k+1}.

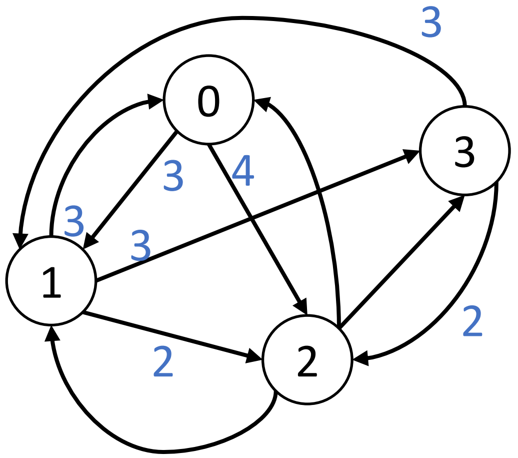
Let γ=(s,…,p,q,…,u) be any of the above path to u, and q is the first node in γ that leaves S_k. Let $\gamma_1 = (s, \ldots, p)$, $\gamma_2 = (q, \ldots, u)$

1. By hypothesis (P1): $|\gamma_1| + c(p,q) \geq dist[q]$ (γ_1 is an S_k path)
2. By non-negative weights: $|\gamma_2| \geq 0$

By 1 and 2, we have $|\gamma| = |\gamma_1| + c(p,q) + |\gamma_2| \geq dist[q]$. At time k+1, we choose u not q, this implies $dist[q] \geq dist[u]$, thus $|\gamma| \geq dist[u]$.

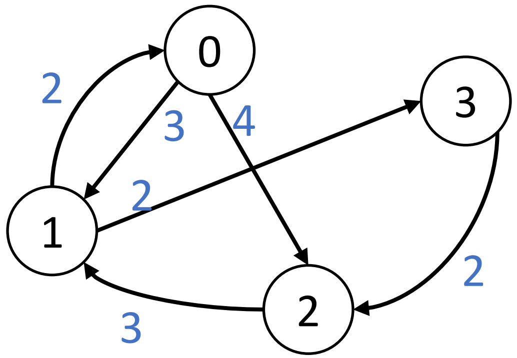
Example 28.3. Draw the weighted graph given by the weighted matrix below.

$$\begin{bmatrix} 0 & 3 & 4 & 0 \\ 3 & 0 & 1 & 3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 2 & 0 \end{bmatrix}$$

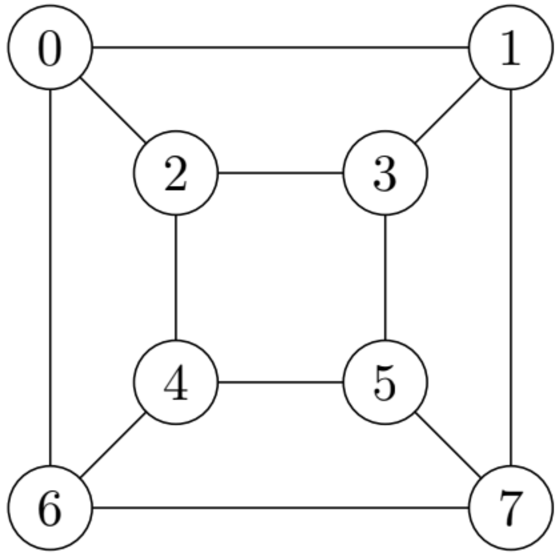


Draw the weighted digraph given by the weighted list representation below.

0	1	3	2	4
1	0	2	3	2
2	1	3		
3	2	1		



Example 28.7. What is the diameter of the 3-cube in Example 28.2?



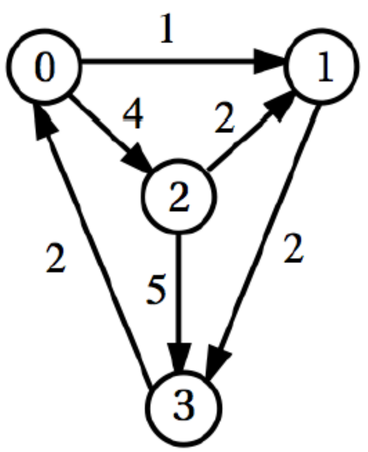
d(0,1)=1
d(0,2)=1
d(0,3)=2
d(0,4)=2
d(0,5)=3
d(0,6)=1
d(0,7)=2

d(1,0)=1
d(1,2)=2
d(1,3)=1
d(1,4)=3
d(1,5)=2
d(1,6)=2
d(1,7)=1

…

d(7,0)=2
d(7,1)=1
d(7,2)=3
d(7,3)=2
d(7,4)=2
d(7,5)=1
d(7,6)=1

Example 28.11. An application of Dijkstra's algorithm on the digraph below for each starting vertex s. Complete the table for the starting vertex 2.



The table illustrates that the distance array is updated at most $n - 1$ times (only before a new vertex is selected and added to S). Thus we could have omitted the lines with $S = \{0, 1, 2, 3\}$.

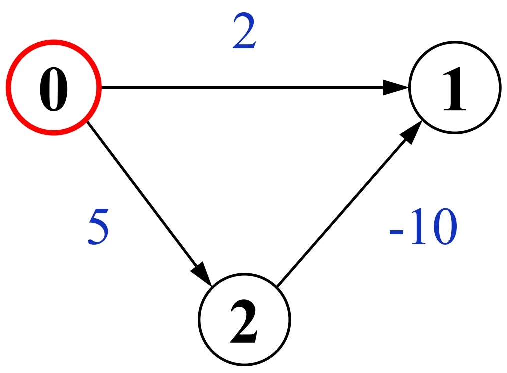
current $S \subseteq V$	**distance vector** `dist`
$\{0\}$	$0, 1, 4, \infty$
$\{0, 1\}$	$0, 1, 4, 3$
$\{0, 1, 3\}$	$0, 1, 4, 3$
$\{0, 1, 2, 3\}$	$0, 1, 4, 3$
$\{1\}$	$\infty, 0, \infty, 2$
$\{1, 3\}$	$4, 0, \infty, 2$
$\{0, 1, 3\}$	$4, 0, 8, 2$
$\{0, 1, 2, 3\}$	$4, 0, 8, 2$
$\{2\}$	$\infty, 2, 0, 5$
$\{ 1, 2 \}$	$\infty, 2, 0, 2 + 2$
$\{ 1, 2, 3 \}$	$2 + 2 + 2, 2, 0, 2 + 2$
$\{0, 1, 2, 3\}$	$6, 2, 0, 4$
$\{3\}$	$2, \infty, \infty, 0$
$\{0, 3\}$	$2, 3, 6, 0$
$\{0, 1, 3\}$	$2, 3, 6, 0$
$\{0, 1, 2, 3\}$	$2, 3, 6, 0$

Dijkstra Algorithm: Negative Weights

- One of the property of Dijkstra's algorithm is that once a node turns black its distance is not updated any longer

- This means that even if a new cheaper route is available, a black node's distance will not get updated – a case that would only be there when there are negative weights.

Fact: Dijkstra's algorithm does not work with negative weights!



starting vertex 0 :

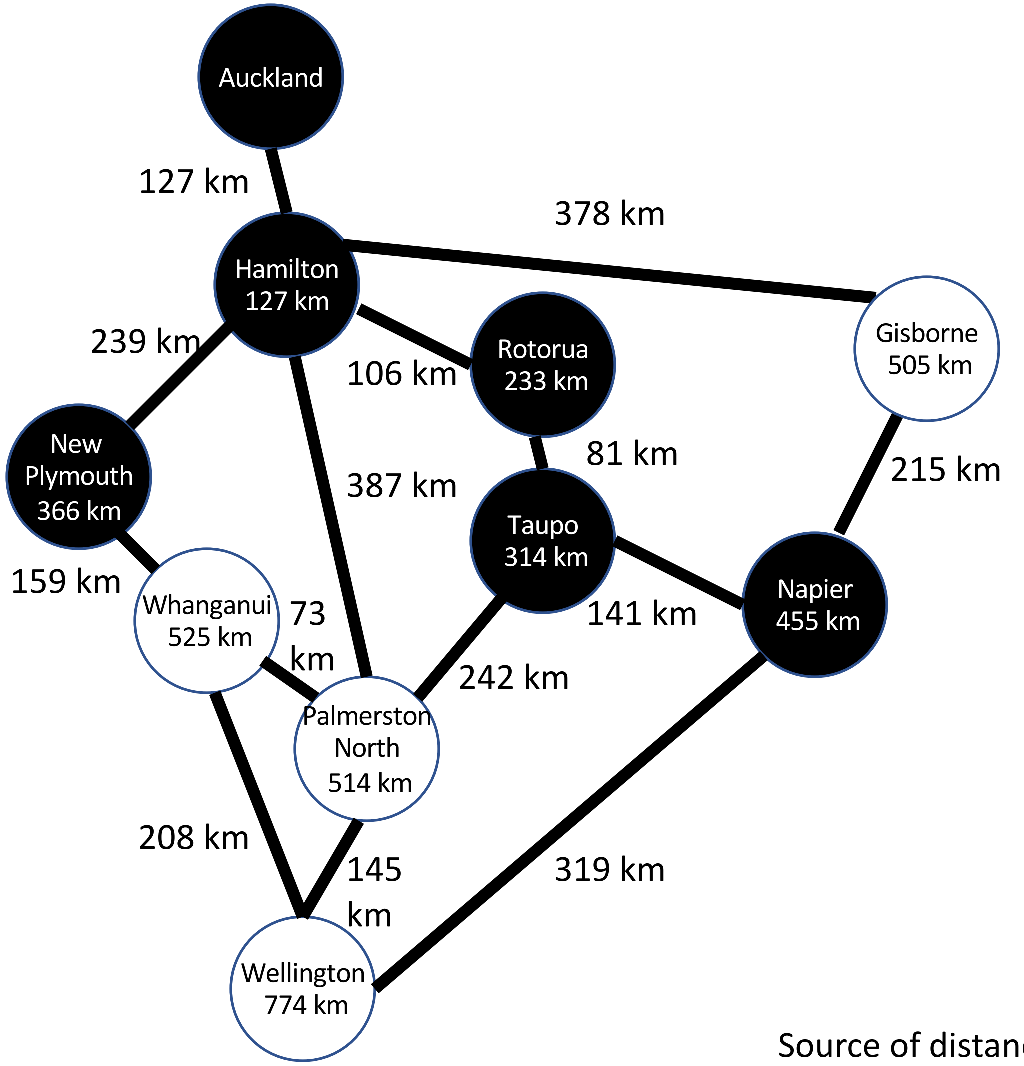
- shortest path from 0 to 2 is 5
- shortest path from 0 to 1 is 5 + (-10) = -5

1st iteration: 1 is colored black

2nd iteration: 2 is colored black

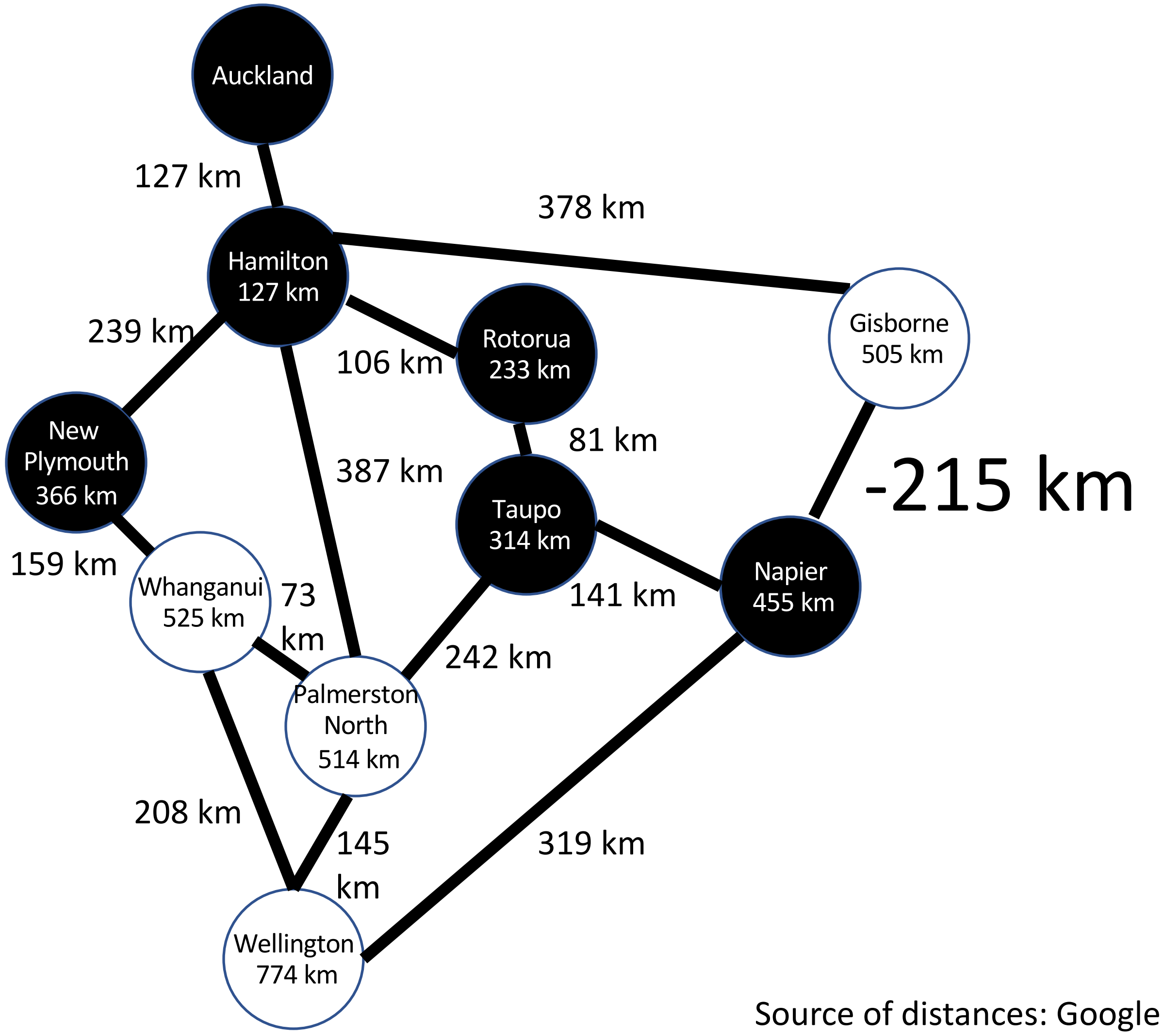
Shortest path from 0 to 1 is not updated since 1 is already black!

Example: Dijkstra at Work



Source of distances: Google Maps

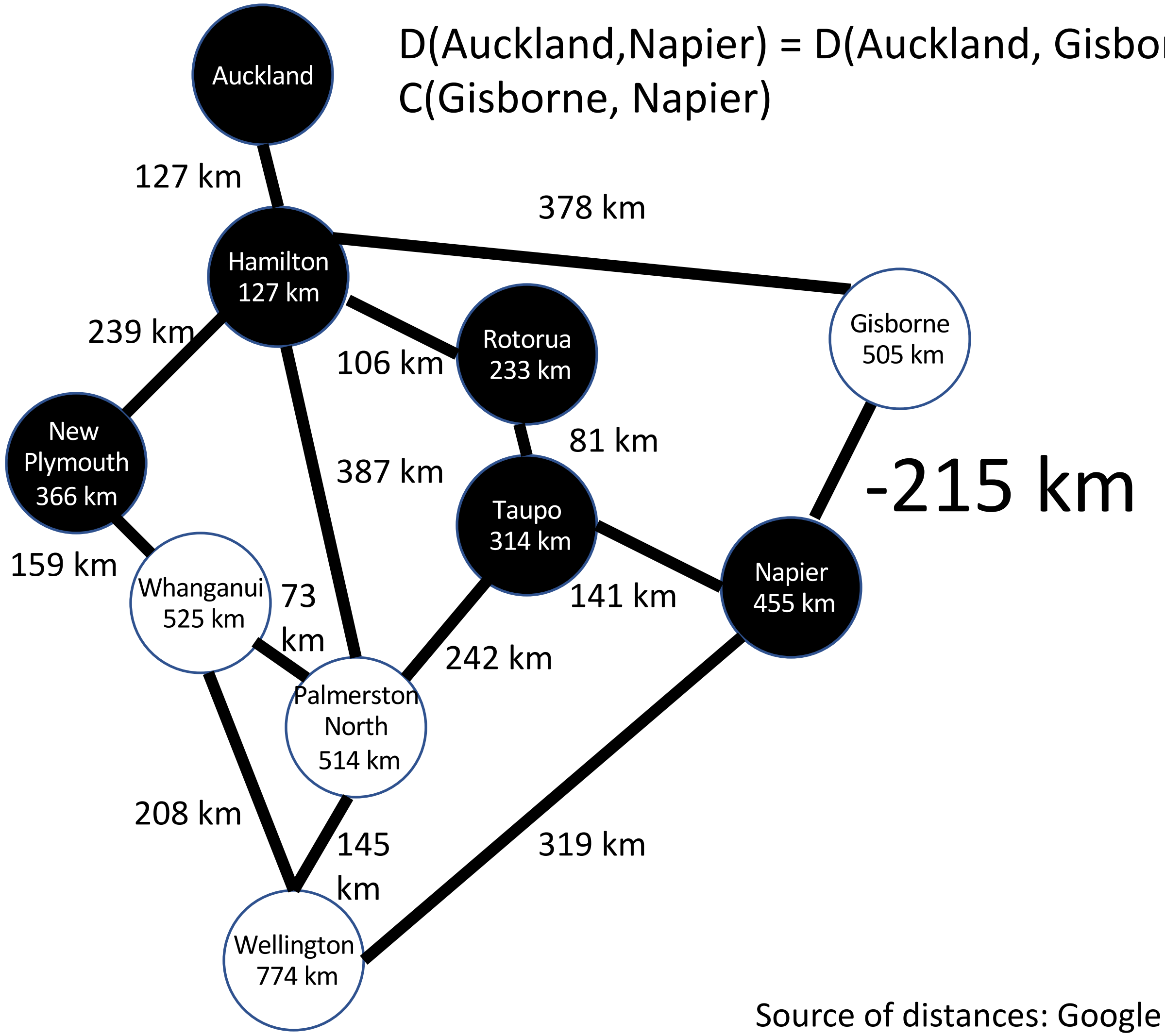
Example: Adding a Negative Weight



Source of distances: Google Maps

Example: Adding a Negative Weight

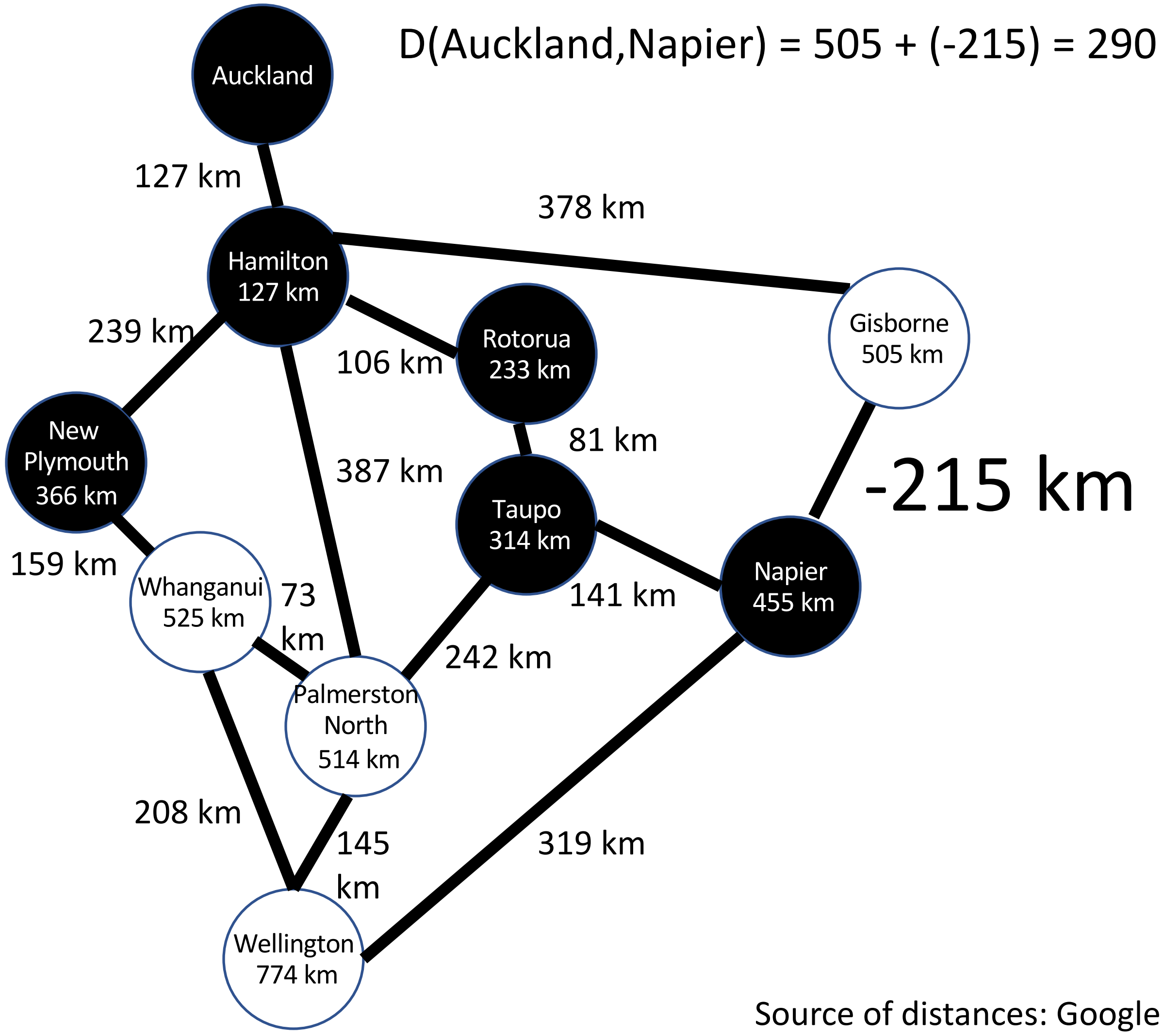
D(Auckland,Napier) = D(Auckland, Gisborne) + C(Gisborne, Napier)



Source of distances: Google Maps

Example: Adding a Negative Weight

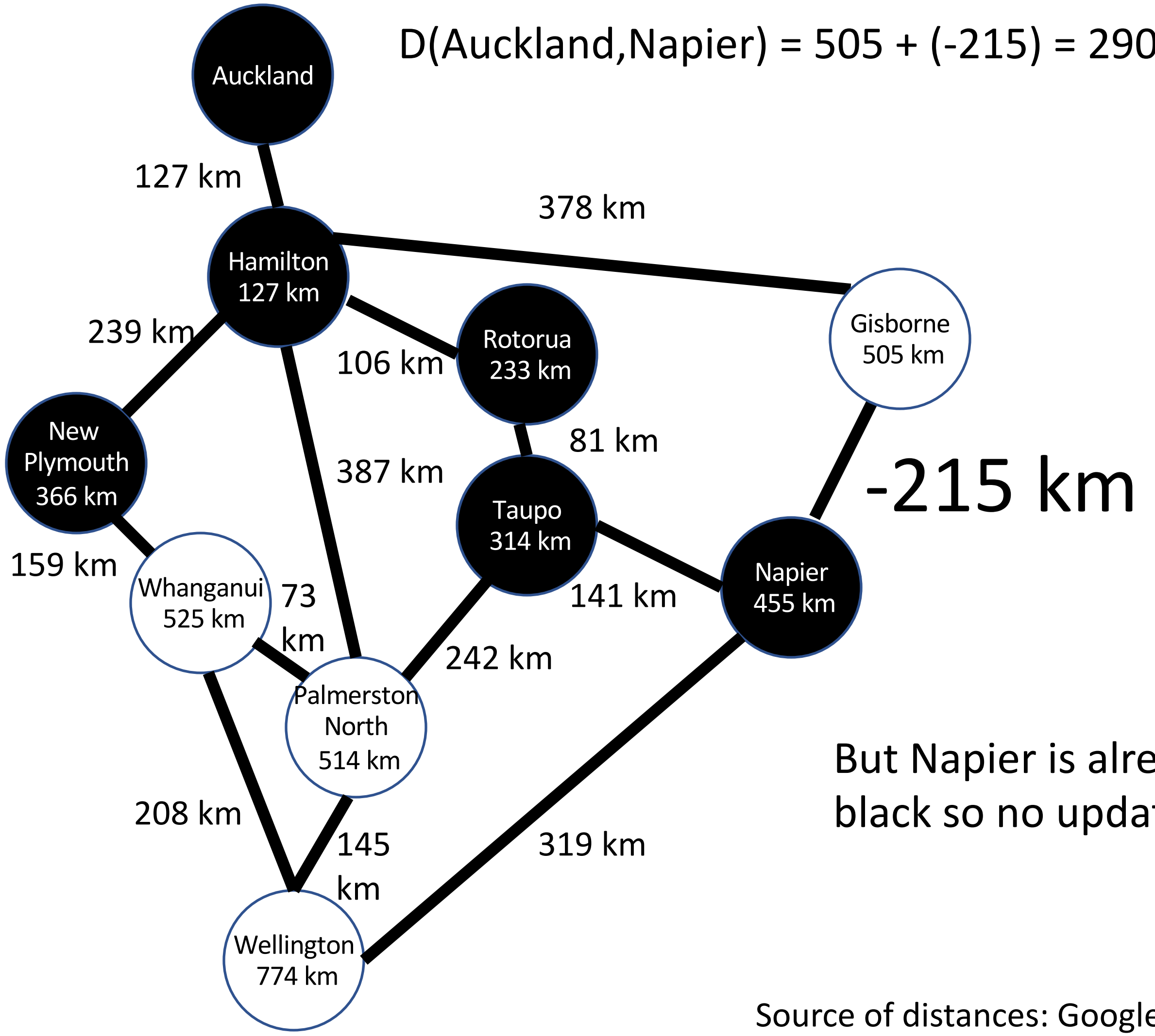
D(Auckland,Napier) = 505 + (-215) = 290



Source of distances: Google Maps

Example: Adding a Negative Weight

D(Auckland,Napier) = 505 + (-215) = 290

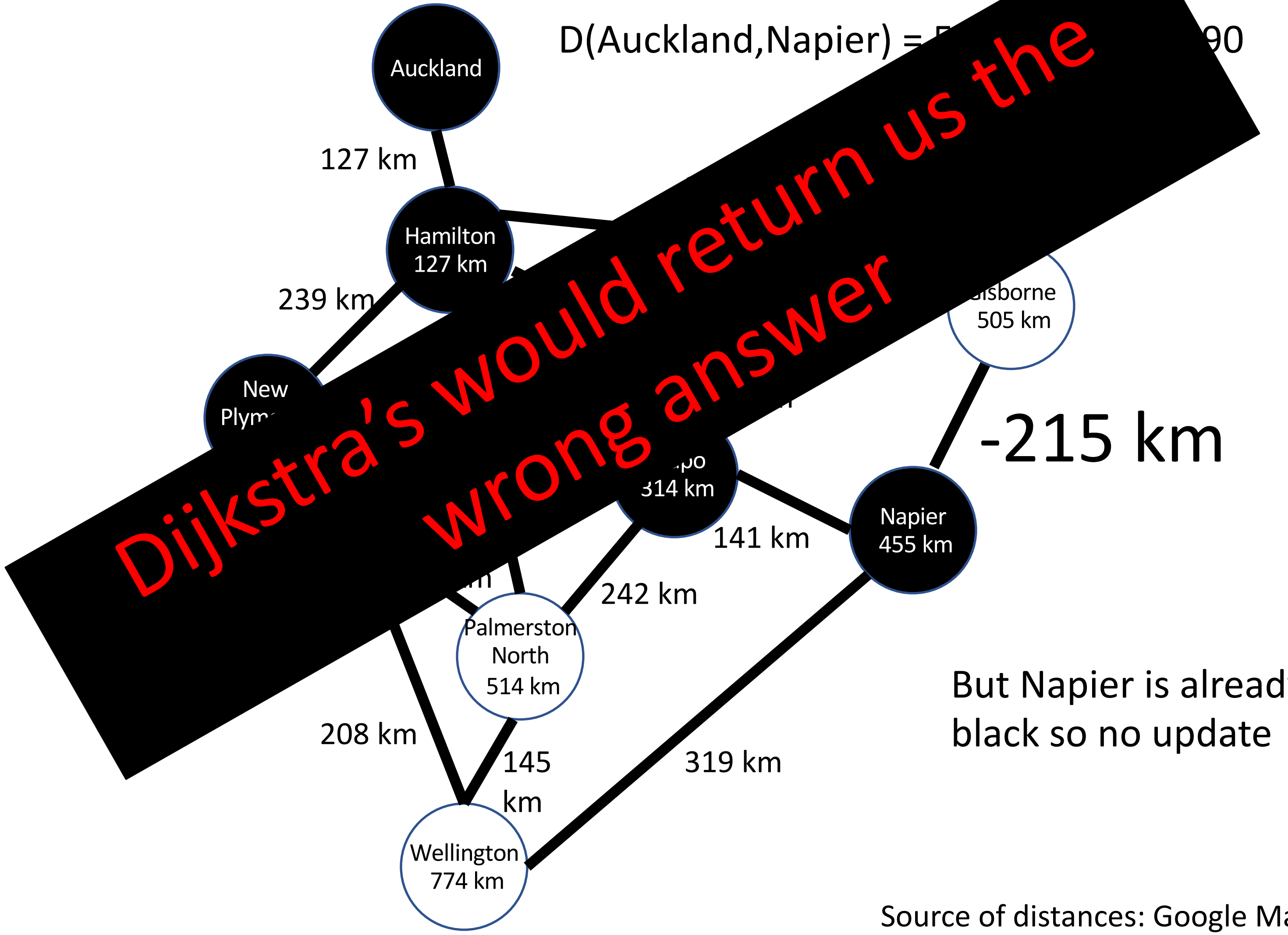


But Napier is already black so no update

Source of distances: Google Maps

Example: Adding a Negative Weigh

D(Auckland,Napier) =



But Napier is already black so no update

Source of distances: Google Maps

SUMMARY

- Weighted Graphs
  - Representation
  - Weight as cost functions
- Algorithms on Weighted Graphs
  - Dijkstra
  - Bellman-Ford
  - Floyd-Warshall