1.
   - Prove that $T(n) = 3n^3 + 2n^2 + n + 90$ is both $O(n^3)$ and $O(n^4)$.
   - Determine the asymptotic relationship between $f(n) = n \ln(3n)$ and $g(n) = 3n^2$ using limit rule.

2. Let $T(n) = f(n)g(n) + h(n)$ be a function defined on three running time functions, where $f(n)$ is $\Theta(\sqrt{n})$; $g(n)$ is $\Omega(\log n^n)$ and $h(n)$ is $O(n^3)$. Show that $T(n)$ is $\Omega(n^{3/2} \log n)$.

3. What is the minimum and maximum number of comparisons needed when merging two nonempty sorted lists of size n into a single list?

4. Determine the order of the list after partitioning 23, 20, 6, 17, 13, 25, 14, assume you have a way to take the median of the list as the pivot.

5. Consider the following maximum heap: 39, 20, 37, 18, 6, 32, 13, 3, 14.

   1. insert 25 to the heap.
   2. delete 39 from the heap.

6. Given the binary search tree below:
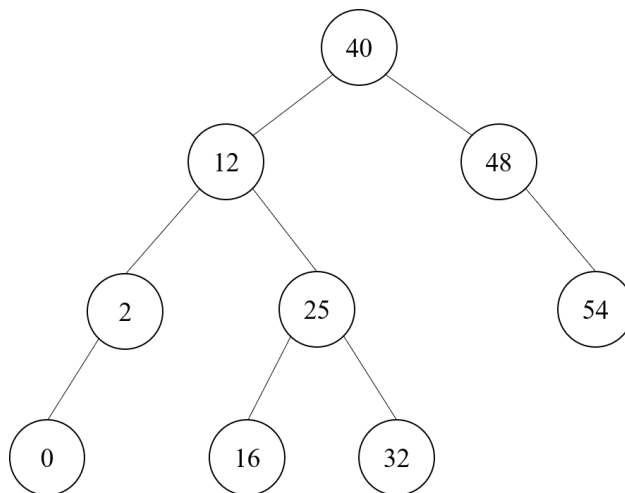
Describe the process and the outcome of the following deletion operations on the above BST.

1. Delete node 48 in the tree.
2. Delete node 16 in the tree.
3. Delete node 12 in the tree by using the minimum key in the right subtree.
4. Delete node 12 in the tree by using the maximum key in the left subtree.

7. Consider the following digraph G:
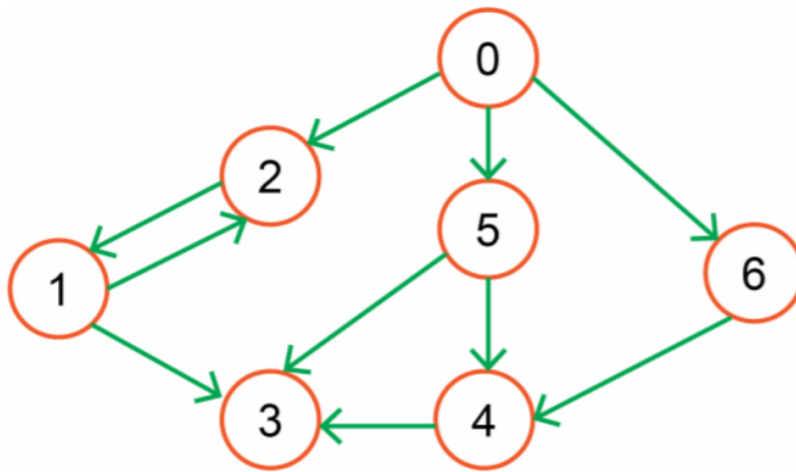
   1. What is the order and size of G?

   2. Identify all sources and sinks in G.

   3. Write down the adjacency matrix representation of G.

   4. Consider the vertex sequence 0, 2, 1, 3, 4, 5, is it a walk, path or cycle?

   5. Give the distance of the following pairs of nodes, $d(0,1)$, $d(0,3)$ and $d(5,6)$

8. Consider the adjacency list of a digraph G below:

   0: 1 2

   1: 2

   2: 1 3 4

   3: 4

   4:

      1. Draw the digraph G.

      2. Draw the sub-digraph induced by {1, 2, 3}.

      3. Draw the underlying graph of G.