Data Searching and Binary Search

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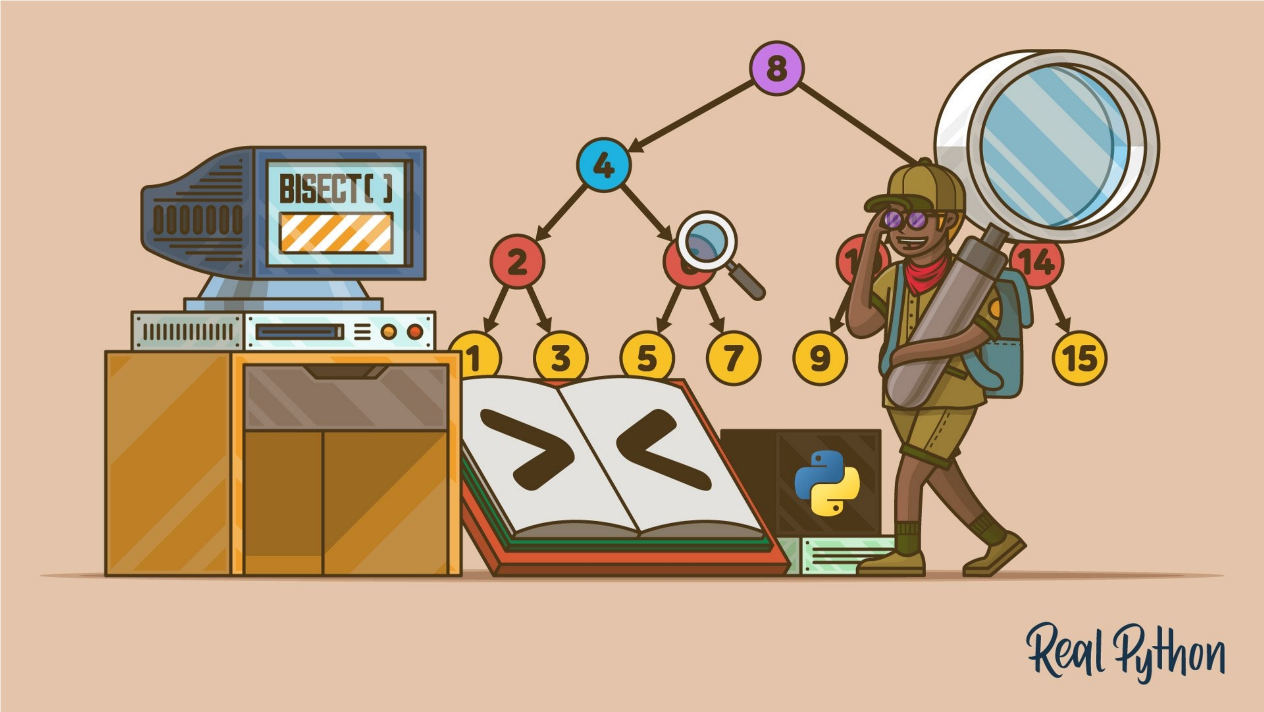
Slides adapted from Kaiqi Zhao

OUTLINE

- Definition of Search
- Types of Data Input
  - Unsorted Lists / Sorted Lists
- Types of Search
  - Sequential Search
  - Binary Search
- Time Complexity Analysis



Data Search in a Large Database

- Searching in a database D of records, such that each record has a key to use in the search.
- Example – search a student record by Album ID (as key).

Album ID	Album	Song Title	Singer Name	Release Year
1	Courage	"Flying On My Own"	Celine Dion	2019
2	Mind Games	"South Dakota"	Jordy	2019
3	Clarity	“Broken”	Kim Petras	2020
4	Nibiru	“Reggaeton en Paris”	Ozuna	2019
5	Basking in the Glow	"Morning Song"	Oso Oso	2019

Data Search in a Large Database

- Searching in a database D of records, such that each record has a key to use in the search.
- The search problem: Given a search key k, either
  - Return the record associated with k in D (a successful search: if k occurs several times, return any occurrence), or
  - Indicate that k is not found (an unsuccessful search).

- The purpose of the search
  - To access data in the record for processing, or
  - To update information in the record, or
  - To insert a new record or delete the record found.

Table ADT

- **Definition** (Table ADT): The table ADT is a set of ordered pairs, or table entries (k,v) where k is a unique key and v is a data value associated with the key k.

- Types of Operations
  - **RETRIEVE** the entry (k,v) based on the key v; if no entry exist, indicate the search is unsuccessful.
  - **REMOVE** the found entry from the table;
  - **UPDATE** its value v;
  - **INSERT** a new entry with key k if the table has no such entry.

Types of Search

- **Static search**: unalterable (fixed in advance) databases; no updates, deletions, or insertions.
- **Dynamic search**: alterable databases (allowable insertions, deletions, and updates).

Key		**Associated value** v		
Code	k	City	Country	State/Place
AKL	271	Auckland	New Zealand	North Island
DCA	2080	Washington	USA	District of Columbia (D.C.)
FRA	3822	Frankfurt	Germany	Hesse
SDF	12251	Louisville	USA	Kentucky

Implementation

- **Basic implementations** of the table ADT: lists and trees.

- An **elementary operation**
  - An update of a list element or tree node, or
  - Comparison of two of them.

Sequential Search in Unsorted Lists

- Starting at the head of a list and examining elements one by one until finding the desired key or reaching the end of the list.
- **Complexity**. Both successful and unsuccessful sequential search have worst-case and average-case time complexity $\Theta(n)$.
- **Proof**:
  - The unsuccessful search explores each of n keys, so the worst- and average-case time is $\Theta(n)$
  - The successful search examines n keys in the worst case and $\frac{n+1}{2}$ keys on the average, which is still $\Theta(n)$
- The sequential search is the **ONLY option** for unsorted lists of records.
- A sorted list implementation allows for much better search based on the divide-and-conquer paradigm.

Binary Search in a Sorted List of Records

Given a sorted list $L = \{(k_i, v_i) : i = 1, \dots, n;\ k_1 < k_2 < \cdots < k_n\}$

Recursive binary search for the key k:

1. If the list is empty, return "not found", otherwise
2. Choose the key k_m of the middle element of the list and
   - if $k_m = k$, return its record, otherwise
   - if $k_m > k$, make a recursive call on the head sublist, otherwise
   - if $k_m < k$, make a recursive call on the tail sublist.

- We can do this without recursion!

Non-recursive (Iterative) Binary Search in Array

- The performance of binary search on an array is much better than on a linked list because of the constant time access to a given element.

Algorithm 1 BinarySearch

1: **function** BinarySearch(a sorted integer $\boldsymbol{k} = (k_0, k_1, \dots, k_{n-1})$ of keys associated with items, a search key k)

2: $\quad l \leftarrow 0; r \leftarrow n-1$

3: $\quad$ **while** $l \leq r$ **do** $m \leftarrow \left\lfloor \frac{l+r}{2} \right\rfloor$

4: $\quad\quad$ **if** $k_m < k$ **then** $l \leftarrow m+1$

5: $\quad\quad$ **else if** $k_m > k$ **then** $r \leftarrow m-1$

6: $\quad\quad$ **else return** m

7: $\quad$ **return** *ItemNotFound*

Faster Binary Search with Two-way Comparisons

Algorithm 2 BinarySearch

function BinarySearch2(a sorted integer $\boldsymbol{k} = (k_0, k_1, \ldots, k_{n-1})$ of keys associated with items, a search key k)

    $l \leftarrow 0; r \leftarrow n - 1$

    while $l < r$ **do** $m \leftarrow \left\lfloor \frac{l+r}{2} \right\rfloor$

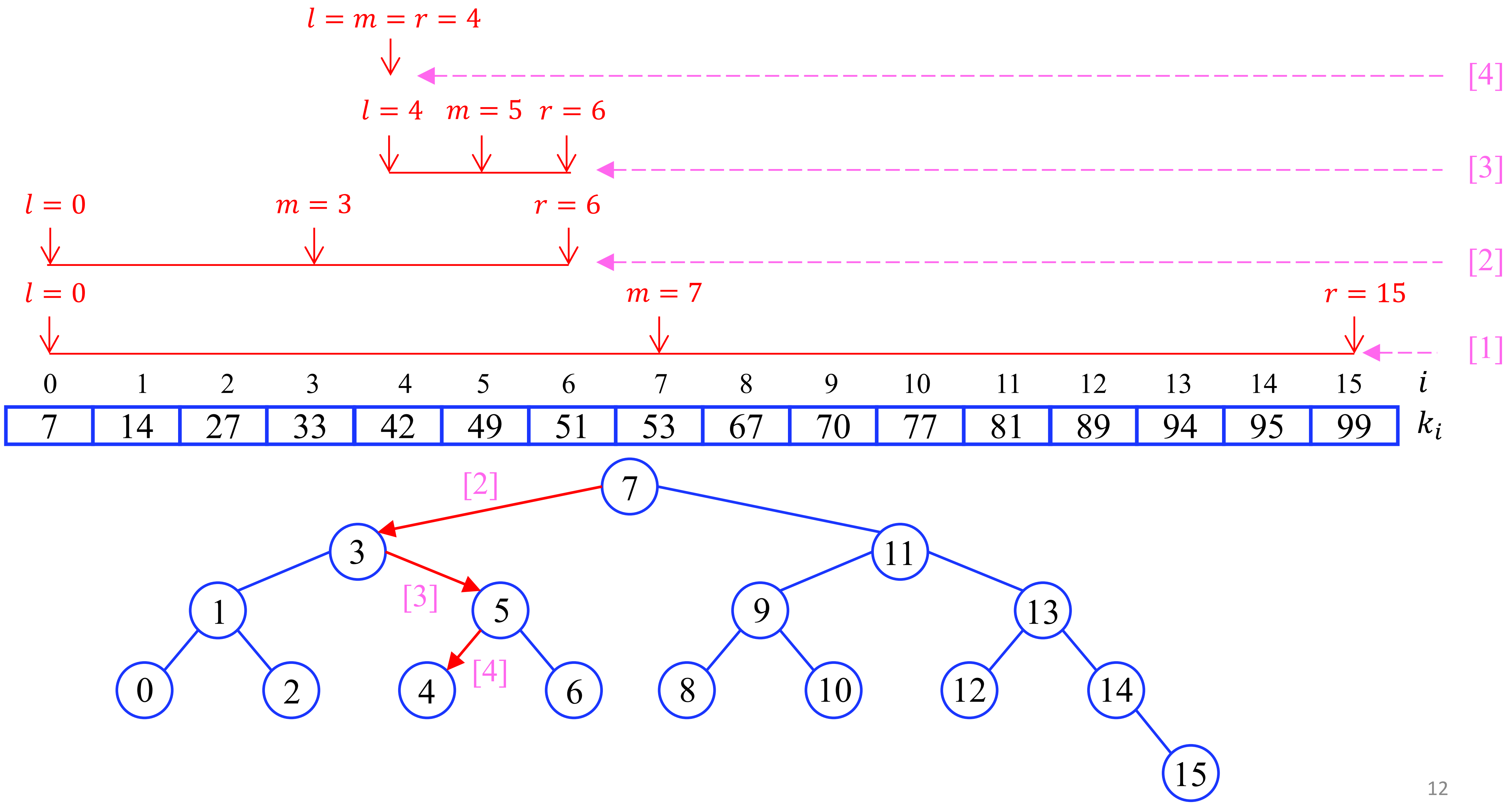
        if $k_m < k$ **then** $l \leftarrow m + 1$

        else $r \leftarrow m$

    if $k_l = k$ **then return** l

    else return *ItemNotFound*

Example: Binary Search in Array $\{k_0 = 7, \dots, k_{15} = 99\}$ for Key k=42



Time Complexity Analysis: Worst Case

- The worst-case time complexity of unsuccessful and successful binary search is $\Theta(\log n)$.

- The full binary tree of height $h-1$ has $n = 2^h - 1$ keys (each internal node has 2 children)
  - The comparison tree height is h with only the last level not full.
  - $l+1$ comparisons to find a key at level l.
  - The worst case: $h+1 = \lfloor \log_2 n \rfloor + 1$ comparisons.

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Time Complexity Analysis: Average Case

- **Lemma**: The average-case time complexity of successful and unsuccessful binary search in a balanced tree is $\Theta(\log n)$.

- **Proof**: The height of the tree is $h = \lfloor \log_2 n \rfloor$
  - At least half of the tree nodes have a depth at least $h - 1$.
  - The average depth over all nodes is at least $\frac{h-1}{2}$ and at most h, so that it is $\Theta(\log n)$
  - The average depth over all nodes of an arbitrary (not necessarily balanced) binary tree is $\Omega(\log n)$.

- The expected search time for an arbitrary balanced tree is equal to the average balanced tree depth $\Theta(\log n)$.

Interpolation Search

- Improvement of binary search if it is possible to guess where the desired key sits.
  - A simple practical example: the search for C or X in a phone directory.
  - Practical if the sorted keys are almost uniformly distributed over their range.

0	1	2	3	4	5	6
11	12	13	14	15	16	17

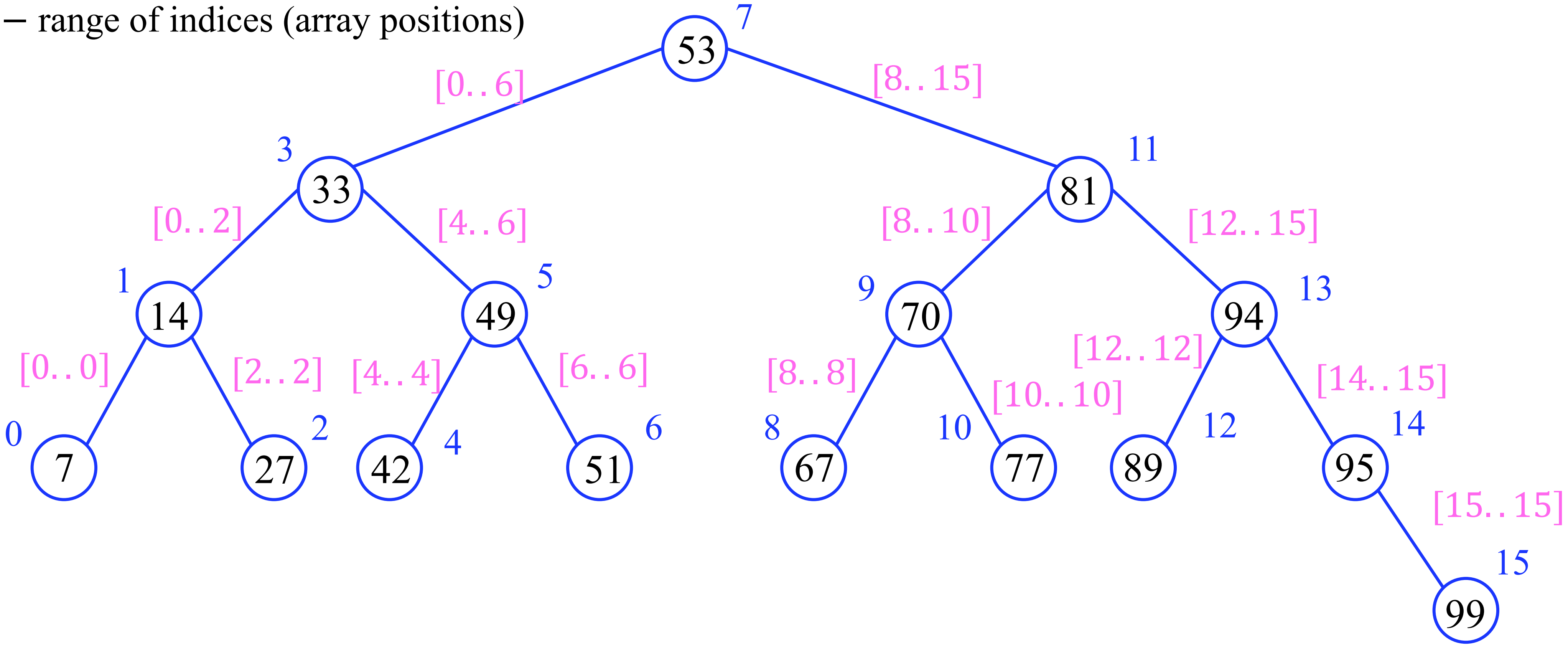
- **Binary search**: the middle position $m = \left\lfloor \frac{l+r}{2} \right\rfloor = l + \left\lfloor \frac{r-l}{2} \right\rfloor$.
- **Interpolation search**: the predicted position of key k if the keys are uniformly distributed between k_l and k_r :

$$m = l + \left\lfloor \frac{k - k_l}{k_r - k_l} (r - l) \right\rfloor$$

Tree Structure of Binary Search: Binary Search Tree

7	14	27	33	42	49	51	53	67	70	77	81	89	94	95	99

$[l..r]$ – range of indices (array positions)



SUMMARY

- Definition of Search
- Sequential Search on Unsorted Lists
- Binary Search on Sorted Lists
  - Iterative Binary Search
  - Faster Binary Search
- Time Complexity Analysis