1.
   - Prove that $T(n) = 1000n^4 + 65n^3 + 3n^2 + 10n$ is both $O(n^4)$ and $O(n^5)$.
   - Determine the asymptotic relationship between $f(n) = n\ln(7n)$ and $g(n) = 10n^2$ using limit rule.

2. Let $T(n) = (n^{0.01} + \log_2 n)^2$ be processing time of an algorithm for input of size n. Which is the asymptotic time complexity of this algorithm, $\Theta(n^{0.02})$ or $\Theta((\log n)^2)$? Please show your working to justify your answer.

3. What is the minimum and maximum number of comparisons needed when merging two nonempty sorted lists of size n into a single list?

4. Determine the order of the list after partitioning [41, 30, -1, 20, 15, 77, 10]. Assume the pivot is 20. Please show your working to justify your answer.

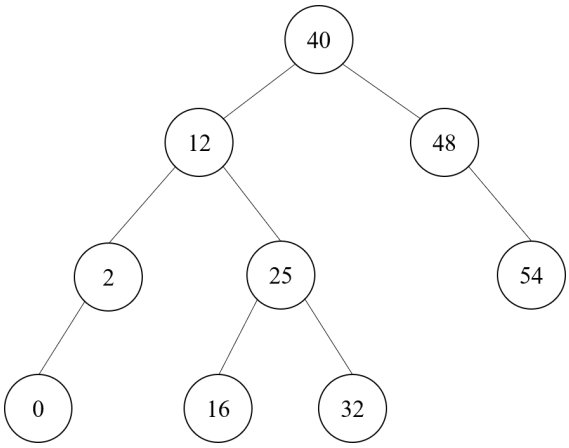
5. Use Quickselect to find the 3-rd largest element of the list [25, 65, 50, 21, 2, 67, 70, 31, 15, 8]. Show the state of the list after each step. Assume the first element is taken as the pivot.

6. Consider the following maximum heap: [41, 19, 37, 18, 8, 30, 10, 3, 17].

   (A) insert 25 to the heap.

   (B) delete 41 from the heap derived from previous step.

7. Given the binary search tree below:



Describe the process and the outcome of the following deletion operations on the above BST.

(A) Delete node 48 in the tree.

(B) Delete node 16 in the tree.

(C) Delete node 12 in the tree by using the minimum key in the right subtree.

(D) Delete node 12 in the tree by using the maximum key in the left subtree.

8. (A) Perform the linear time algorithm to construct a maximum heap on the following array. Please show the heap structure after each iteration.

$$A = [0, 54, 93, 2, 12, 32, 40, 16, 25, 51].$$

(B) Consider the same array A above. Is A a minimum heap? Justify your answer by briefly explaining the min-heap property. If A is not a min-heap, then restore the min-heap property and show the restored array A'.

(C) Given the array A' with min-heap property, delete the min value of A'. Please describe the heapification process and the outcome of the deletion operations on the min-heap constructed using A'.

(D) Prove that the height of a complete binary tree with n nodes is exactly $\lceil \lg(n+1) \rceil - 1$ using mathematical induction.