

Graph Definitions II

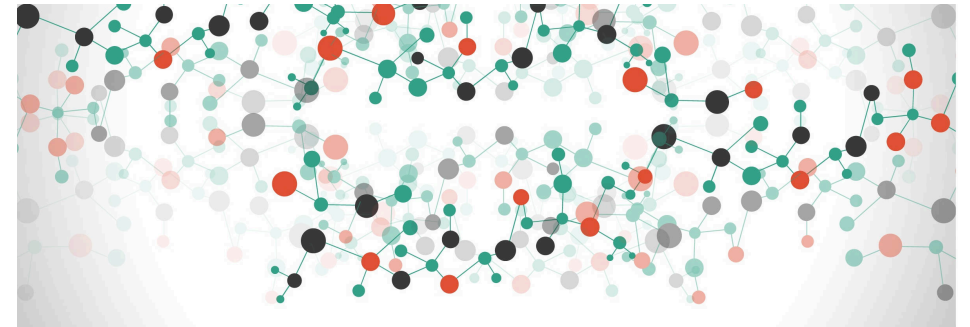
COMPSCI 220: WEEK 13.3

Instructor: Meng-Fen Chiang



OUTLINE

- Distance
- Eccentricity
- Diameter
- Radius
- Periphery and Center



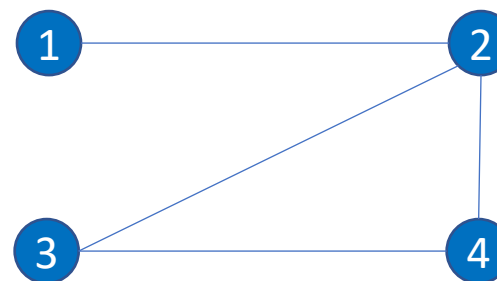
Distance

- **Definition.** Let u and v be two vertices in a graph G , then the distance between u and v , denoted as $d(u, v)$ is the length of shortest path between u and v in G .
- If G is disconnected and u and v are in different components then $d(u, v) = +\infty$

$$d(1,1) = 0$$

$$d(1,2) = 1$$

$$d(1,3) = 2$$



Eccentricity

- **Definition.** The eccentricity of a vertex v in $V(G)$, denoted as $e(v)$, is the maximum of the distances between v and any other vertex u in $V(G)$.

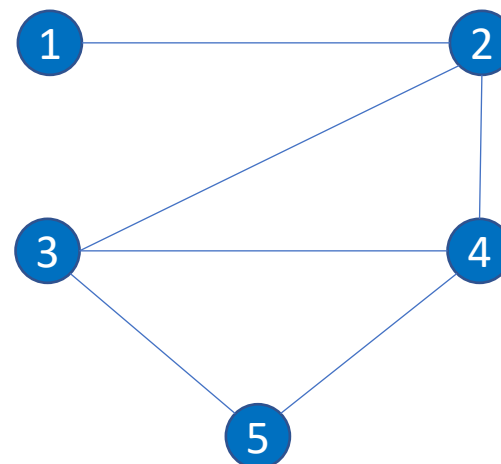
$$d(1,1) = 0$$

$$d(1,2) = 1$$

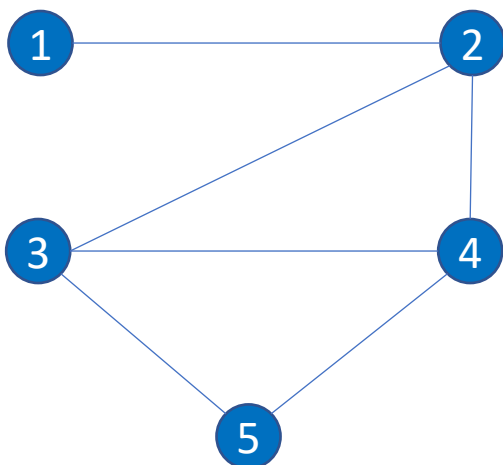
$$d(1,3) = 2$$

$$d(1,4) = 2$$

$$d(1,5) = 3$$



Eccentricity (Contd.)



$$d(1,1) = 0$$

$$d(1,2) = 1$$

$$d(1,3) = 2$$

$$d(1,4) = 2$$

$$d(1,5) = 3$$

$$e(1) = 3$$

$$e(2) = 2$$

$$e(3) = 2$$

$$e(4) = 2$$

$$e(5) = 3$$

Note: Distance is defined on two vertices while eccentricity is defined on a vertex

Diameter and Radius

- **Definition.** The **diameter** of a graph (or strongly connected digraph) G , denoted as $\text{diam}(G)$, is the **maximum eccentricity** of the vertices in $V(G)$.

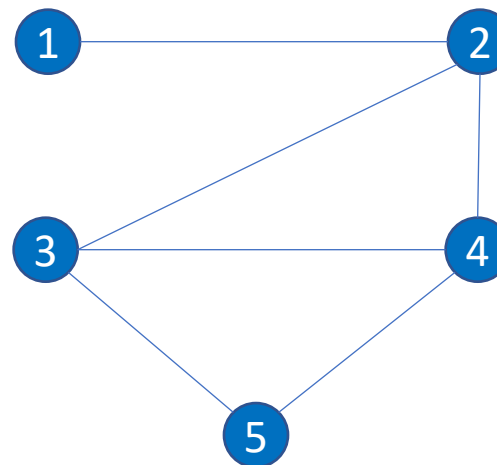
$$e(1) = 3$$

$$e(2) = 2$$

$$e(3) = 2$$

$$e(4) = 2$$

$$e(5) = 3$$

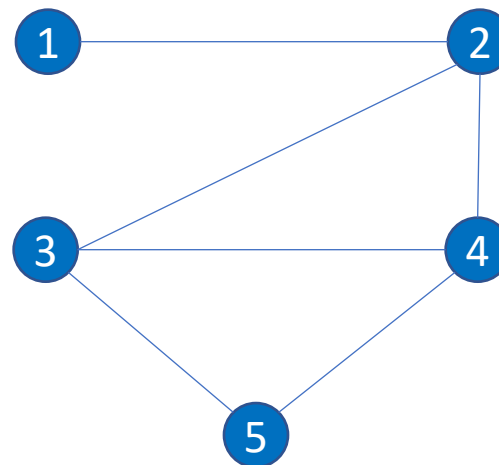


The diameter of this graph is 3

Diameter and Radius (Contd.)

- **Definition.** The **radius** of a graph (or strongly connected digraph) G , denoted as $\text{rad}(G)$, is the **minimum eccentricity** of the vertices in $V(G)$.

$$\begin{aligned}e(1) &= 3 \\e(2) &= 2 \\e(3) &= 2 \\e(4) &= 2 \\e(5) &= 3\end{aligned}$$

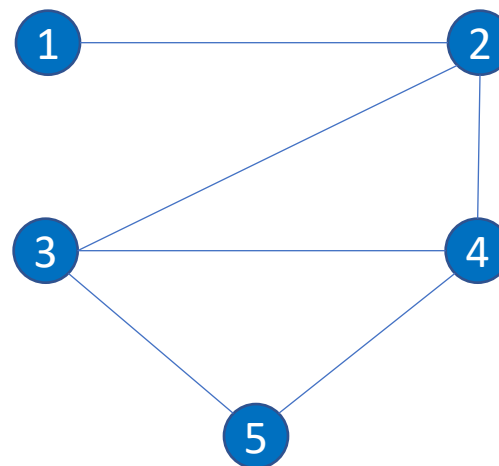


The radius of this graph is 2

Periphery and Centre in Graph

- **Definition.** A vertex v of a graph G with eccentricity equals to the diameter of G is said to be a **peripheral vertex**
- The set of all peripheral vertices in the graph is called the **periphery** of the graph

$$\begin{aligned}e(1) &= 3 \\e(2) &= 2 \\e(3) &= 2 \\e(4) &= 2 \\e(5) &= 3\end{aligned}$$



Node 1 and Node 5 form the periphery of the graph

Periphery and Centre in Graph

- **Definition.** A vertex v of a graph G with eccentricity equals to the radius of G is said to be a **central vertex**
- The set of all central vertices in the graph is called the **centre** of the graph

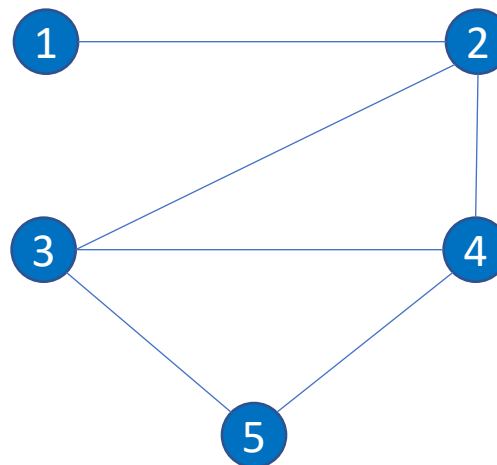
$$e(1) = 3$$

$$e(2) = 2$$

$$e(3) = 2$$

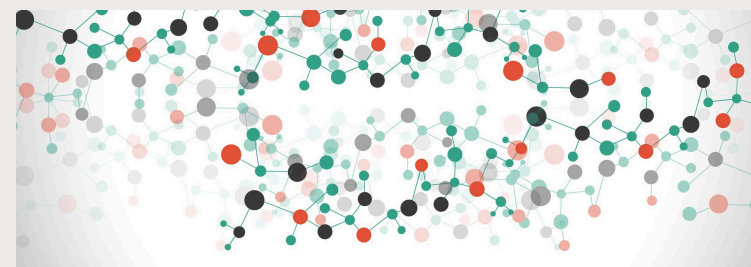
$$e(4) = 2$$

$$e(5) = 3$$



Nodes 2, 3 and 4 form the centre of the graph

SUMMARY



- The **eccentricity** of a vertex v in $V(G)$, denoted as $e(v)$, is the maximum of the distances between v and any other vertex u in $V(G)$.
- The **diameter** of a graph (or strongly connected digraph) G , denoted as $\text{diam}(G)$, is the **maximum eccentricity** of the vertices in $V(G)$.
- The **radius** of a graph (or strongly connected digraph) G , denoted as $\text{rad}(G)$, is the **minimum eccentricity** of the vertices in $V(G)$.
- A vertex v of a graph G with eccentricity equals to the diameter of G is said to be a **peripheral vertex**
- A vertex v of a graph G with eccentricity equals to the radius of G is said to be a **central vertex**