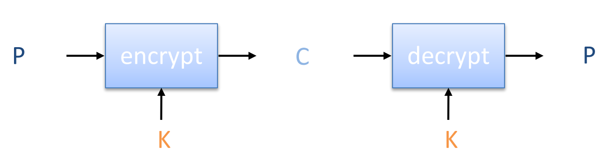
Cryptography, part I: Symmetric Key Encryption

Symmetric Cryptosystem

- Scenario
  - Alice wants to send a message (plaintext P) to Bob.
  - The communication channel is insecure and can be eavesdropped
  - If Alice and Bob have previously agreed on a symmetric encryption scheme and a secret key K, the message can be sent encrypted (ciphertext C)
- Issues
  - What is a secure symmetric encryption scheme?
  - What is the complexity of encrypting/decrypting?
  - What is the size of the ciphertext, relative to the plaintext?

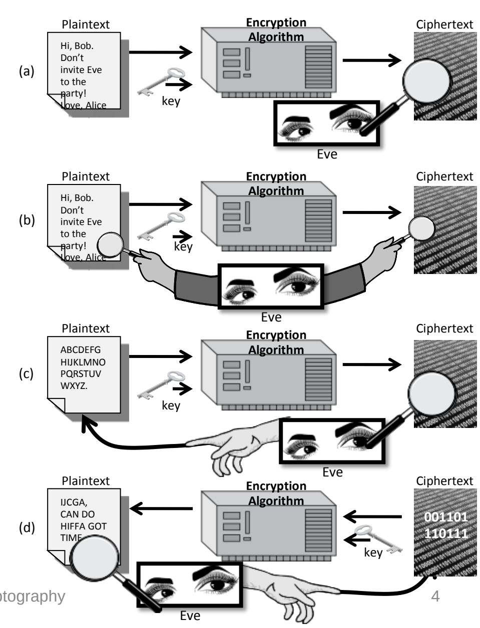


Basics

- Notation (simplified for fixed-length plaintexts)
  - Secret key K
  - Encryption function $E_K(P)$
  - Decryption function $D_K(C)$
  - Encryption and decryption are permutation functions (bijections) on the set of all n-bit arrays
- Efficiency
  - functions E_K and D_K should have efficient algorithms
- Consistency
  - Decrypting the ciphertext yields the plaintext
  - $D_K(E_K(P)) = P$

Attacks

- Attacker may have
  - a) collection of ciphertexts (ciphertext only attack)
  - b) collection of plaintext/ciphertext pairs (known plaintext attack)
  - c) collection of plaintext/ciphertext pairs for plaintexts selected by the attacker (chosen plaintext attack)
  - d) collection of plaintext/ciphertext pairs for ciphertexts selected by the attacker (chosen ciphertext attack)



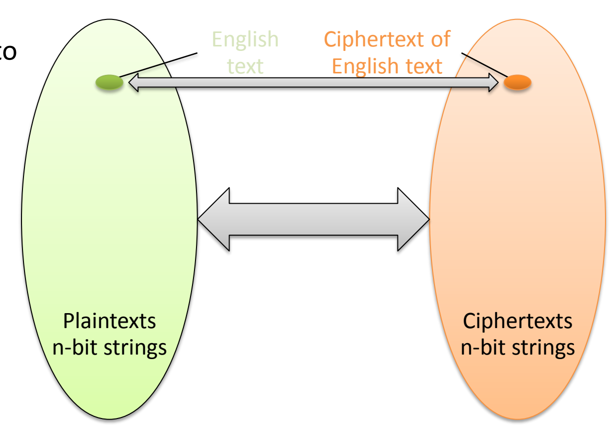
Brute-Force Attack

- Try all possible keys K and determine if $D_K(C)$ is a likely plaintext
  - Requires some knowledge of the structure of the plaintext (e.g., PDF file or email message)
- Key should be a sufficiently long random value to make exhaustive search attacks unfeasible

Image by Michael Cote from http://commons.wikimedia.org/wiki/File:Bingo_cards.jpg

Encrypting English Text

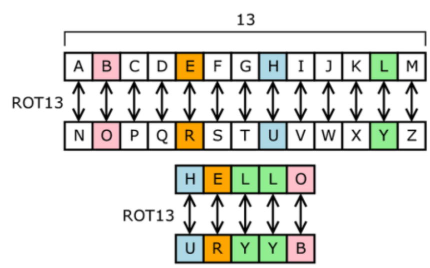
- English text typically represented with 8-bit ASCII encoding
- A message with t characters corresponds to an n-bit array, with n = 8t
- Redundancy due to repeated words and patterns
  - E.g., “th”, “ing”
- English plaintexts are a very small subset of all n-bit arrays



Substitution Ciphers

- Each letter is uniquely replaced by another.
- key = permutation between English letters
- There are 26! possible substitution cipher keys
- 26! > 4 x 10^{26}

- One popular substitution “cipher” for some Internet posts is ROT13.



Public domain image from http://en.wikipedia.org/wiki/File:ROT13.png

Frequency Analysis

- Letters in a natural language, like English, are not uniformly distributed.
- Knowledge of letter frequencies, including pairs and triples can be used in cryptologic attacks against substitution ciphers.

a:	8.05%	b:	1.67%	c:	2.23%	d:	5.10%
e:	12.22%	f:	2.14%	g:	2.30%	h:	6.62%
i:	6.28%	j:	0.19%	k:	0.95%	l:	4.08%
m:	2.33%	n:	6.95%	o:	7.63%	p:	1.66%
q:	0.06%	r:	5.29%	s:	6.02%	t:	9.67%
u:	2.92%	v:	0.82%	w:	2.60%	x:	0.11%
y:	2.04%	z:	0.06%				

Letter frequencies in the book *The Adventures of Tom Sawyer*, by Twain.

Substitution Boxes

- Permutations can also be done over binary alphabets, e.g. n-bit strings.
- Permutation on n-bit strings can be described by n/2 x n/2 "substitution box", or "S-box":

	00	01	10	11
00	0011	0100	1111	0001
01	1010	0110	0101	1011
10	1110	1101	0100	0010
11	0111	0000	1001	1100

(a)

	0	1	2	3
0	3	8	15	1
1	10	6	5	11
2	14	13	4	2
3	7	0	9	12

(b)

Figure 8.3: A 4-bit S-box (a) An S-box in binary. (b) The same S-box in decimal.

One-Time Pads

- There is one type of substitution cipher that is absolutely unbreakable.
  - The **one-time pad** was invented in 1917 by Joseph Mauborgne and Gilbert Vernam
  - Assume alphabet of p elements (e.g. p=26 for English)
  - We use a block of shift keys, $K=(k_1,k_2,\ldots,k_n)$, to encrypt a plaintext of length n, $M=(m_1,m_2,\ldots,m_n)$, i.e. $C=(c_1,c_2,\ldots,c_n)$ where $c_i = m_i + k_i$ (modulo p).
  - Each shift key is chosen uniformly at random in {1,…,p}
- Since each shift is random, every ciphertext is equally likely for any plaintext.

Weaknesses of the One-Time Pad

- In spite of their perfect security, one-time pads have some weaknesses
- The key has to be as long as the plaintext
- Keys can never be reused
  - Repeated use of one-time pads allowed the U.S. to break some of the communications of Soviet spies during the Cold War.

VENONA

TOP SECRET

[redacted]

Ref. No.: [redacted]

[redacted]

Issued: [redacted]

From: NEW YORK

To: MOSCOW

No: 812

29 May 1943

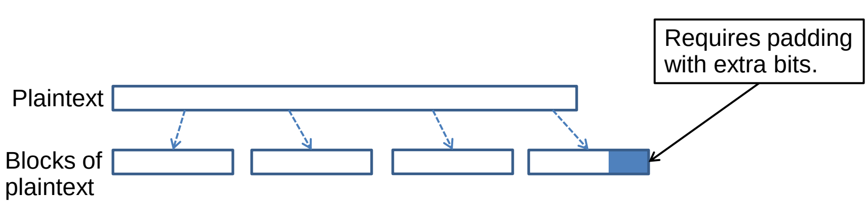
[Continued overleaf]

VENONA

TOP SECRET

Block Ciphers

- In a **block cipher:**
  - Plaintext and ciphertext have fixed block length b (e.g., 128 bits)
  - A plaintext of length n is partitioned into a sequence of m **blocks**, P[0], …, P[m−1], where $n \leq bm < n + b$
- Each message is encrypted or decrypted in terms of its blocks.



Padding

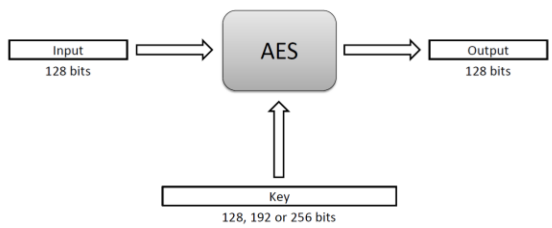
- Block ciphers require the length n of the plaintext to be a multiple of the block size b
- Padding the last block needs to be unambiguous (cannot just add zeroes)
- When the block size and plaintext length are a multiple of 8, a common padding method (PKCS5) is a sequence of identical bytes, each indicating the length (in bytes) of the padding
- Example for b = 128 (16 bytes)
  - Plaintext: "Roberto" (7 bytes)
  - Padded plaintext: "Roberto999999999" (16 bytes), where 9 denotes the number and not the character
- We need to always pad the last block, which may consist only of padding

Block Ciphers in Practice

- Data Encryption Standard (DES)
  - Developed by IBM and adopted by NIST in 1977
  - 64-bit blocks and 56-bit keys
  - Small key space makes exhaustive search attack feasible since late 90s
- Triple DES (3DES)
  - Nested application of DES with three different keys KA, KB, and KC
  - Effective key length is 168 bits, making exhaustive search attacks unfeasible
  - $C = E_{KC}(D_{KB}(E_{KA}(P)))$; $P = D_{KA}(E_{KB}(D_{KC}(C)))$
  - Equivalent to DES when KA=KB=KC (backward compatible)
  - Mode with KC=KA but different KB also seems to resist attacks so far.
- Advanced Encryption Standard (AES)
  - Selected by NIST in 2001 through open international competition and public discussion
  - 128-bit blocks and several possible key lengths: 128, 192 and 256 bits
  - Exhaustive search attack not currently possible
  - AES-256 is the symmetric encryption algorithm of choice

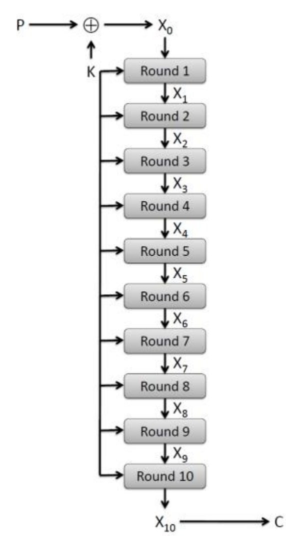
The Advanced Encryption Standard (AES)

- In 1997, the U.S. National Institute for Standards and Technology (NIST) put out a public call for a replacement to DES.
- It narrowed down the list of submissions to five finalists, and ultimately chose an algorithm that is now known as the **Advanced Encryption Standard** (**AES**).
- AES is a block cipher that operates on 128-bit blocks. It is designed to be used with keys that are 128, 192, or 256 bits long, yielding ciphers known as AES-128, AES-192, and AES-256.



AES Round Structure

- The 128-bit version of the AES encryption algorithm proceeds in ten rounds.
- Each round performs an invertible transformation on a 128-bit array, called **state**.
- The initial state X_0 is the XOR of the plaintext P with the key K:
- X_0 = P XOR K.
- Round i (i = 1, …, 10) receives state X_{i-1} as input and produces state X_i.
- The ciphertext C is the output of the final round: C = X_{10}.



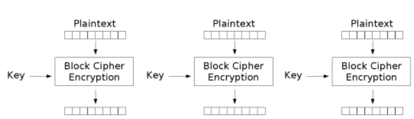
AES Rounds

- Each round is built from four basic steps:

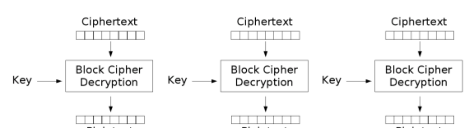
1. **SubBytes step**: an S-box substitution step
2. **ShiftRows step**: a permutation step
3. **MixColumns step**: a matrix multiplication step
4. **AddRoundKey step**: an XOR step with a **round key** derived from the 128-bit encryption key

Block Cipher Modes

- A block cipher mode describes the way a block cipher encrypts and decrypts a sequence of message blocks.
- Electronic Code Book (ECB) Mode (is the simplest):
  - Block P[i] encrypted into ciphertext block C[i] = E_K(P[i])
  - Block C[i] decrypted into plaintext block M[i] = D_K(C[i])



Electronic Codebook (ECB) mode encryption

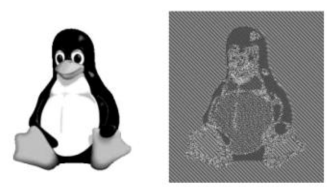


Electronic Codebook (ECB) mode decryption

Public domain images from http://en.wikipedia.org/wiki/File:Ecb_encryption.png and http://en.wikipedia.org/wiki/File:Ecb_decryption.png

Strengths and Weaknesses of ECB

- Strengths:
  - Is very simple
  - Allows for parallel encryptions of the blocks of a plaintext
  - Can tolerate the loss or damage of a block
- Weakness:
  - Not Chosen-Plaintext Secure
  - Intuitively: Related plaintexts always encrypt the same
  - Example: Documents and images are not suitable for ECB encryption since patters in the plaintext are repeated in the ciphertext:

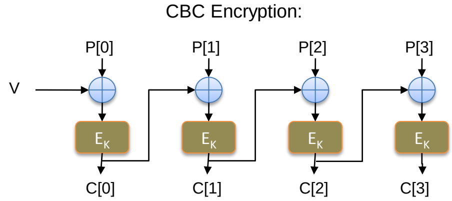


(a) (b)

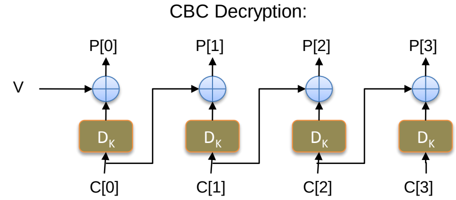
Cipher Block Chaining (CBC) Mode

- In Cipher Block Chaining (CBC) Mode
  - The previous ciphertext block is combined with the current plaintext block $C[i] = E_K(C[i-1] \oplus P[i])$
  - $C[-1] = V$, a random block separately transmitted encrypted (known as the initialization vector)
  - Decryption: $P[i] = C[i-1] \oplus D_K(C[i])$

CBC Encryption:



CBC Decryption:



Strengths and Weaknesses of CBC

- Strengths:
  - Doesn't show patterns in the plaintext
  - Is the most common mode
  - Is fast and relatively simple

- Weaknesses:
  - CBC requires the reliable transmission of all the blocks sequentially
  - CBC is not suitable for applications that allow packet losses (e.g., music and video streaming)