US patent n°4,936,961 - Demonstration for capacitor

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Course

A capacitor is a passive linear dipole made up of two armatures separated by a dielectric. Under the action of a voltage u(t) charges will accumulate opposite one another. The capacitor is characterized by the proportionality coefficient between the amount of charge stored and the voltage: $\boxed{i(t) = C * \frac{\mathrm{d}u}{\mathrm{d}t}}$

The capacitance C depends both on the geometry of the reinforcements and on the nature of the dielectric. Ex.: The capacity of a plane capacitor of section S whose dielectric has a thickness e: $\boxed{C = \epsilon * \frac{S}{e}}$

$\epsilon = \epsilon_0 * \epsilon_r$is the permittivity of the dielectric.

Energy stored: From an energy point of view, the behavior of the capacitor is very different from that of the resistance. As the latter dissipates electrical energy by transforming it into heat, the capacitor stores electrical energy when it charges and releases it when it discharges. There is very little loss of electrical energy. The charged capacitor therefore forms an energy reserve. This energy is expressed as a function of its capacity C and of the quantity of charge stored Q according to: $\boxed{E = \frac{1}{2} * u * Q = \frac{1}{2} * C * u^2}$

Electric power $\boxed{P \equiv \frac{\mathrm{d}E}{\mathrm{d}t}}$ received by the capacitor is then: $\boxed{P = u * C * \frac{\mathrm{d}u}{\mathrm{d}t} = u(t) * i(t)}$

Actual behavior: A capacitor never exhibits pure capacity. In particular, there are always losses in the dielectric. These losses are modeled as a first approximation by a resistor R placed either in series or in parallel with the capacitance C.

The capacitor is a cylindrical capacitor : $\boxed{C = \frac{2 * \pi * \epsilon_{water} * h}{\log_e \frac{R_2}{R_1}}}$

Calculations

We have :

$$\boxed{|Z_C| = \frac{1}{C.\omega}}$$

$$\boxed{|\underline{U_C}| = |Z_C|.|\underline{I_C}|}$$

$$\boxed{\omega = 2.\pi.f}$$

We have the features :

$$\boxed{\begin{cases} R_1 = 6,35 * 10^{-3} \\ R_2 = 9,525 * 10^{-3} \\ h = 0,1016 \\ \epsilon_r = 80,2 \\ \epsilon_{water} = 7 * 10^{-10} \\ \epsilon_{vacuum} = 8,8541878128 * 10^{-12} \\ C = 1,10209271 * 10^{-9} \\ f = 10^4 \\ \omega = 62831,85 \\ |Z_C| = 14441,16 \\ |\underline{i_C}| = 0,167 \\ |\underline{U_C}| = 2411,67 \end{cases}}$$

$\boxed{h}$ is the length of the cylinders in meter ; $\boxed{R_1}$ is the outside radius of the inner cylinder in meter ; $\boxed{R_2}$ is the outside radius of the outside cylinder in meter ; $\boxed{\epsilon_{vacuum}}$ is the vacuum permittivity in farad per meter ; $\boxed{\epsilon_r}$ is the relative permittivity of the water without unit ; $\boxed{\epsilon_{water}}$ is the water permittivity ; $\boxed{C}$ is the capacitance of the capacitor included in the water bath in farad ; $\boxed{\omega}$ is the frequency in radian per second ; $\boxed{Z_C}$ is the impedance of the capacitor in ohms ; $\boxed{I_C}$ is the current into the capacitor in amperes ; $\boxed{U_C}$ is the voltage of the capacitor in volts ; $\boxed{f}$ is the frequency of the circuit in Hz