US patent n°4,936,961 - Demonstration for efficiency

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Course

The mass per mol of hydrogen is equal to 1 gramm per mol.
Chemical formula for water electrolysis :
Cathode :

$$\boxed{2\,H^+ + 2\,e^- \longrightarrow H_{2(g)}}$$

Anode :

$$\boxed{2\,H_2O_{(l)} \longrightarrow O_{2(g)} + 4\,H_{(aq)}{}^+ + 4\,e^-}$$

Final reaction :

$$\boxed{2\,H_2O_{(l)} \longrightarrow O_{2(g)} + H_{2(g)}}$$

Calculations

We have :

$$\boxed{\begin{cases} M_{H_2} = 2 \\ n_{H_2}(t) = \frac{n_{e_-}(t)}{2} \\ m_{H_2}(t) = n_{H_2}(t).M_{H_2} \\ PCI_{H_2} = 34 \\ C = 1,10209271.10^{-9} \\ L = 2,03.10^{-5} \\ f = 10^4 \\ F = 96500 \\ T = \frac{1}{f} = 10^{-5} \\ U_2 = 78 \end{cases}}$$

$\boxed{F}$ is the Faraday constant in coulomb per mol ; $\boxed{U_2}$ is the voltage of the secondary coil of the transformer ; $\boxed{f}$ is the frequency of signal in Hz ; $\boxed{M_{H_2}}$ is the molar mass of the dihydrogen in g per mol ; $\boxed{PCI_{H_2}}$ is the energy of dihydrogen in kWh per kg ; $\boxed{n_{e_-}}$ is the quantity of matter of electrons in mol ; $\boxed{n_{H_2}}$ is the quantity of matter of dihydrogen in mol ; $\boxed{m_{H_2}}$ is the mass of dihydrogen produced in kg ; $\boxed{L}$ is the inductance of the coil in henry ;

We are here to resolve any ordinary differential equation with the reel world.

Between 0 and the half of the period, we have the following electronic circuit :

Thanks to the law of voltages, we have :

$$\boxed{\begin{aligned} U_2 = 2.U_L^{(1)}(t) + U_C^{(1)}(t) &\iff 78 = 2.L.\frac{\mathrm{d}i_C^{(1)}(t)}{\mathrm{d}t} + U_C^{(1)}(t) \\ &\iff 78 = 2.L.C.\frac{\mathrm{d}^2U_C^{(1)}(t)}{\mathrm{d}t^2} + U_C^{(1)}(t) \\ &\iff \frac{\mathrm{d}^2U_C^{(1)}(t)}{\mathrm{d}t^2} + \frac{U_C^{(1)}(t)}{2.L.C} - \frac{78}{2.L.C} = 0 \\ &\iff \frac{\mathrm{d}^2U_C^{(1)}(t)}{\mathrm{d}t^2} + 22,35.10^{12}.U_C^{(1)}(t) - 1,7.10^{15} = 0 \end{aligned}}$$

$$\boxed{\begin{cases} \alpha = \frac{1}{2.L.C} \\ \beta = \frac{78}{2.L.C} \\ \omega = \sqrt{\alpha} \\ (E) : \frac{\mathrm{d}^2U_C^{(1)}(t)}{\mathrm{d}t^2} + \alpha.U_C^{(1)}(t) = \beta \end{cases}}$$

The homogeneous equation :

$$\boxed{\frac{\mathrm{d}^2U_C^{(1)}(t)}{\mathrm{d}t^2} + \alpha.U_C^{(1)}(t) = 0}$$

has for solutions :

$$\boxed{(\gamma, \lambda) \in reel, t \in [0; \frac{T}{2}], U_C^{(1)}(t) = \gamma.\cos(\omega.t) + \lambda.\sin(\omega.t)}$$

$\boxed{\beta}$ is a polynomial function, so (E) owns a particular solution of the form P where P is a polynomial function as the same degree as $\boxed{\beta}$

Hence, $\boxed{t \in [0; \frac{T}{2}], P(t) = a}$ where a $\boxed{\in reel.}$

Find the value of a in injecting it into (E).
P is a particular solution of (E), so :

$$\boxed{\begin{aligned} \frac{\mathrm{d}^2 P(t)}{\mathrm{d}t^2} + \alpha.P(t) = \beta &\iff 0 + a.\alpha = \beta \\ &\iff a = \frac{\beta}{\alpha} = 78 \end{aligned}}$$

Hence,

$$\boxed{t \in [0; \frac{T}{2}], P(t) = 78}$$

All solution of (E) is sum of a particular solution of the homogeneous equation associated of (E) and a particular solution of (E).
Hence,

$$\boxed{(\gamma, \lambda) \in reel, t \in [0; \frac{T}{2}], U_C^{(1)}(t) = \gamma.\cos(\omega.t) + \lambda.\sin(\omega.t) + 78}$$

Hence,

$$\boxed{\begin{aligned} (\gamma, \lambda) \in reel, t \in [0; \frac{T}{2}], Q^{(1)}(t) &= C.U_C^{(1)}(t) \\ &= C.(\gamma.\cos(\omega.t) + \lambda.\sin(\omega.t) + 78) \end{aligned}}$$

Hence,

$$\boxed{\begin{aligned} (\gamma, \lambda) \in reel, t \in [0; \frac{T}{2}], i_C^{(1)}(t) &= C.\frac{\mathrm{d}U_C^{(1)}(t)}{\mathrm{d}t} \\ &= C.\frac{\mathrm{d}(\gamma.\cos(\omega.t) + \lambda.\sin(\omega.t) + 78)}{\mathrm{d}t} \\ &= C.(-\gamma.\omega.\sin(\omega.t) + \lambda.\omega.\cos(\omega.t)) \\ &= C.\omega.(\lambda.\cos(\omega.t)) - \gamma.\sin(\omega.t)) \end{aligned}}$$

Hence,

$$\boxed{\begin{aligned}(\gamma,\lambda)\in reel, t\in[0;\frac{T}{2}], n_{e^-}^{(1)}(t) &= \frac{Q^{(1)}(t)}{F}\\ &= \frac{C.(\gamma.\cos(\omega.t)+\lambda.\sin(\omega.t)+78)}{F}\end{aligned}}$$

Hence,

$$\boxed{\begin{aligned}(\gamma,\lambda)\in reel, t\in[0;\frac{T}{2}], n_{H_2}^{(1)}(t) &= \frac{n_{e^-}^{(1)}(t)}{2}\\ &= \frac{C.(\gamma.\cos(\omega.t)+\lambda.\sin(\omega.t)+78)}{2.F}\end{aligned}}$$

Hence,

$$\boxed{\begin{aligned}(\gamma,\lambda)\in reel, t\in[0;\frac{T}{2}], m_{H_2}^{(1)}(t) &= n_{H_2}^{(1)}(t).M_{H_2}\\ &= \frac{C.(\gamma.\cos(\omega.t)+\lambda.\sin(\omega.t)+78)}{F}\end{aligned}}$$

Hence,

$$\boxed{\begin{aligned}(\gamma,\lambda)\in reel, t\in[0;\frac{T}{2}], E_{H_2}^{(1)}(t) &= PCI_{H_2}.m_{H_2}^{(1)}(t)\\ &= \frac{34.C.(\gamma.\cos(\omega.t)+\lambda.\sin(\omega.t)+78)}{F}\end{aligned}}$$

Hence,

$$\boxed{\begin{aligned}(\gamma,\lambda)\in reel, t\in[0;\frac{T}{2}], E_{Transfo}^{(1)}(t) &= U_2(t).i_C^{(1)}(t).t\\ &= 78.C.\frac{\mathrm{d}U_C^{(1)}(t)}{\mathrm{d}t}.t\\ &= 78.t.C.\omega.(\lambda.\cos(\omega.t)) - \gamma.\sin(\omega.t))\end{aligned}}$$

Between the half of the period and the period, we have the following electronic circuit :

Thanks to the law of voltages, we have :

$$\boxed{(E) : \frac{\mathrm{d}^2 U_C^{(1)}(t)}{\mathrm{d}t^2} + U_C^{(1)}(t).\alpha = 0}$$

where $\boxed{\alpha = \frac{1}{(L_2 + 2.L).C}}$

The characteristic equation of the equation (E) is : $\boxed{X^2 + \alpha = 0}$

Let $\boxed{\varDelta}$ be the discriminant of the characteristic equation of the equation (E).

We have :

$$\boxed{\varDelta = 0^2 - 4.1.\alpha < 0}$$

Let r1 and r2 be the complex roots of the characteristic equation of the equation (E).

We have :

$$\boxed{\begin{cases} r1 = \frac{-0-i.\sqrt{|\varDelta|}}{2.1} = -i.\sqrt{\alpha} \\ r2 = i.\sqrt{\alpha} \end{cases}}$$

Hence,

$$\boxed{(\gamma, \lambda) \in reel, t \in [\frac{T}{2}; T], U_C^{(2)}(t) = \gamma.\cos(\omega.t) + \lambda.\sin(\omega.t)}$$

where

$$\boxed{\omega = \sqrt{\alpha}}$$

Hence,

$$\boxed{\begin{aligned} (\gamma, \lambda) \in reel, t \in [\frac{T}{2}; T], Q^{(2)}(t) &= C.U_C^{(2)}(t) \\ &= C.(\gamma.\cos(\omega.t) + \lambda.\sin(\omega.t)) \end{aligned}}$$

Hence,

$$\boxed{\begin{aligned}(\gamma,\lambda)\in reel, t\in[\frac{T}{2};T], i_C^{(2)}(t) &= C.\frac{\mathrm{d}U_C^{(2)}(t)}{\mathrm{d}t}\\ &= C.\frac{\mathrm{d}(\gamma.\cos(\omega.t)+\lambda.\sin(\omega.t))}{\mathrm{d}t}\\ &= C.(-\gamma.\omega.\sin(\omega.t)+\lambda.\omega.\cos(\omega.t))\\ &= C.\omega.(\lambda.\cos(\omega.t)-\gamma.\sin(\omega.t))\end{aligned}}$$

Hence,

$$\boxed{\begin{aligned}(\gamma,\lambda)\in reel, t\in[\frac{T}{2};T], n_{e^-}^{(2)}(t) &= \frac{Q^{(2)}(t)}{F}\\ &= \frac{C.(\gamma.\cos(\omega.t)+\lambda.\sin(\omega.t))}{F}\end{aligned}}$$

Hence,

$$\boxed{\begin{aligned}(\gamma,\lambda)\in reel, t\in[\frac{T}{2};T], n_{H_2}^{(2)}(t) &= \frac{n_{e^-}^{(2)}(t)}{2}\\ &= \frac{C.(\gamma.\cos(\omega.t)+\lambda.\sin(\omega.t))}{2.F}\end{aligned}}$$

Hence,

$$\boxed{\begin{aligned}(\gamma,\lambda)\in reel, t\in[\frac{T}{2};T], m_{H_2}^{(2)}(t) &= n_{H_2}^{(2)}(t).M_{H_2}\\ &= \frac{C.(\gamma.\cos(\omega.t)+\lambda.\sin(\omega.t)).2}{2.F}\\ &= \frac{C.(\gamma.\cos(\omega.t)+\lambda.\sin(\omega.t))}{F}\end{aligned}}$$

Hence,

$$\boxed{\begin{aligned}(\gamma,\lambda)\in reel, t\in[\frac{T}{2};T], E_{H_2}^{(2)}(t) &= PCI_{H_2}.m_{H_2}^{(2)}(t)\\ &= \frac{34.C.(\gamma.\cos(\omega.t)+\lambda.\sin(\omega.t))}{F}\end{aligned}}$$

Hence,

$$\boxed{\begin{aligned} t \in [\frac{T}{2};T], E^{(2)}_{Transfo}(t) &= U_2(t).i^{(2)}_C(t).t \\ &= 0.i^{(2)}_C(t).t \\ &= 0 \end{aligned}}$$

Hence,

$$\boxed{\begin{aligned} (\gamma,\lambda) \in reel, t \in [0;\frac{T}{2}], \eta^{(1)}(t) &= \frac{E^{(1)}_{H_2}(t)}{E^{(1)}_{Transfo}(t)} \\ &= \frac{\frac{34.C.(\gamma.\cos(\omega.t)+\lambda.\sin(\omega.t)+78)}{F}}{78.t.C.\omega.(\lambda.\cos(\omega.t)) - \gamma.\sin(\omega.t))} \\ &= \frac{34.C.(\gamma.\cos(\omega.t)+\lambda.\sin(\omega.t)+78)}{F.78.t.C.\omega.(\lambda.\cos(\omega.t)) - \gamma.\sin(\omega.t))} \\ &= \frac{17.(\gamma.\cos(\omega.t)+\lambda.\sin(\omega.t)+78)}{39.F.t.\omega.(\lambda.\cos(\omega.t)) - \gamma.\sin(\omega.t))} \end{aligned}}$$

Hence,

$$\boxed{\begin{aligned} (\gamma,\lambda) \in reel, t \in [\frac{T}{2};T], \eta^{(2)}(t) &= \frac{E^{(2)}_{H_2}(t)}{E^{(2)}_{Transfo}(t)} \\ &= \frac{\frac{34.C.(\gamma.\cos(\omega.t)+\lambda.\sin(\omega.t))}{F}}{0} \\ &= \infty \end{aligned}}$$

Hence,

$$\boxed{\begin{aligned}(\gamma,\lambda)\in reel, \eta^{(1)}(t=\frac{T}{2}) &= \frac{17.(\gamma.\cos(\omega.\frac{T}{2})+\lambda.\sin(\omega.\frac{T}{2})+78)}{39.F.\frac{T}{2}.\omega.(\lambda.\cos(\omega.\frac{T}{2})-\gamma.\sin(\omega.\frac{T}{2}))}\\ &= \frac{17.(\gamma.\cos(\frac{2.\pi.T}{T.2})+\lambda.\sin(\frac{2.\pi.T}{T.2})+78)}{39.F.\frac{T.2.\pi}{2.T}.(\lambda.\cos(\frac{2.\pi.T}{T.2})-\gamma.\sin(\frac{2.\pi.T}{T.2}))}\\ &= \frac{17.(-\gamma+78)}{39.F.\pi.(-\lambda)}\\ &= \frac{17.(78-\gamma)}{39.F.\pi.\lambda}\end{aligned}}$$

We are here to resolve any ordinary differential equation with the imaginary world or the complex number world.

$$\boxed{\begin{aligned}Equation(1); U_C^{(1)}(t=0) &= 0\\ &= \gamma.cos(\omega.0)+\lambda..sin(\omega.0)+78\\ &= \gamma+78\end{aligned}}$$

$$\boxed{Equation(1); \gamma=-78}$$

$$\boxed{\begin{aligned}Equation(1); i_C^{(1)}(t=0)=0 &= C.\omega.(\lambda.\cos(\omega.0))-\gamma.\sin(\omega.0))\\ &= C.\omega.\lambda.1\end{aligned}}$$

$$\boxed{Equation(1); \lambda=0; \gamma=-78}$$

$$\begin{cases} Equation(1); \lambda = 0; \gamma = -78 \\ t \in [0; \frac{T}{2}], U_C^{(1)}(t) = 78.(1 - cos(\omega.t)) \\ t \in [0; \frac{T}{2}], Q^{(1)}(t) = C.78.(1 - cos(\omega.t)) \\ t \in [0; \frac{T}{2}], i_C^{(1)}(t) = C.\omega.78.sin(\omega.t) \\ t \in [0; \frac{T}{2}], n_{e^-}^{(1)} = \frac{C.78.(1-cos(\omega.t))}{F} \\ t \in [0; \frac{T}{2}], n_{H_2}^{(1)}(t) = \frac{C.78.(1-cos(\omega.t))}{2.F} \\ t \in [0; \frac{T}{2}], m_{H_2}^{(1)}(t) = \frac{C.78.(1-cos(\omega.t))}{F} \\ t \in [0; \frac{T}{2}], E_{H_2}^{(1)}(t) = \frac{34.C.78.(1-cos(\omega.t))}{F} \\ t \in [0; \frac{T}{2}], E_{Transfo}^{(1)}(t) = (78)^2.t.C.\omega.sin(\omega.t) \\ t \in [0; \frac{T}{2}], \eta^{(1)}(t) = \frac{1326.(1-cos(\omega.t)}{3042.F.t.\omega.sin(\omega.t)} \end{cases}$$

$$\boxed{\begin{aligned} &\qquad\qquad\qquad Equation(2) \\ U_C^{(2)}(t = \frac{T}{2}) = U_C^{(1)}(t = \frac{T}{2}) = 156 &\iff \gamma.(-1) = 156 \\ &\iff \gamma = -156 \end{aligned}}$$

$$\boxed{\begin{aligned} &\qquad\qquad\qquad Equation(2) \\ i_C^{(2)}(t = \frac{T}{2}) = i_C^{(1)}(t = \frac{T}{2}) = 0 &\iff C.\omega.\lambda.(-1) = 0 \\ &\iff \lambda = 0 \end{aligned}}$$

$$\begin{cases} Equation(2); \lambda = 0; \gamma = -156 \\ t \in [\frac{T}{2}, T], U_C^{(2)}(t) = -156.cos(\omega.t) \\ t \in [\frac{T}{2}, T], Q^{(2)}(t) = -156.C.cos(\omega.t) \\ t \in [\frac{T}{2}, T], i_C^{(2)}(t) = 156.C.\omega.sin(\omega.t) \\ t \in [\frac{T}{2}, T], n_{e^-}^{(2)}(t) = \frac{-156.C.cos(\omega.t)}{F} \\ t \in [\frac{T}{2}, T], n_{H_2}^{(2)}(t) = \frac{-156.C.cos(\omega.t)}{2.F} \\ t \in [\frac{T}{2}, T], m_{H_2}^{(2)}(t) = \frac{-156.C.cos(\omega.t)}{F} \\ t \in [\frac{T}{2}, T], E_{H_2}^{(2)}(t) = \frac{-5304.C.cos(\omega.t)}{F} \\ t \in [\frac{T}{2}, T], E_{Transfo}^{(2)}(t) = 0 \\ t \in [\frac{T}{2}, T], \eta^{(2)}(t) = \infty \end{cases}$$

$$\boxed{\begin{aligned} m_{H_2}^{(1)}(t = \frac{T}{2}) &= \frac{156.C}{F} \\ &= 1,781.10^{-12} kg \end{aligned}}$$

I prefer to reduce the problem.

$$\boxed{\begin{cases} m_{H_2}^{(total)}(t \in [0,T]) = 1,781.10^{-12} kg \\ m_{H_2}^{(total)}(t \in [0,x.T]) = x.1,781.10^{-12} kg \end{cases}}$$