

Assignment 5

General Instructions

- Solutions due by 25th March for Group A
- Solutions due by 27th March for Group B

Hand calculations

1. Consider a non-uniform grid with the left and right-hand side grid sizes of point x_i are h_l and h_r respectively and the grid size ratio is a constant ($h_r/h_l = r$). Denoting the function values at the grid points $x_i, (x_i - h_l), (x_i + h_r)$ as $u(x_i), u(x_i - h_l)$ and $u(x_i + h_r)$ respectively. Derive second-order accurate central-difference formulae for the first and second derivatives at the grid point x_i . Express your answer in terms of $u(x_i), u(x_i - h_l), u(x_i + h_r), r$ and h_l (20 points).
2. Derive a fourth-order accurate forward-difference formula for evaluating the second derivative on a uniform grid of size Δx (10 points).

Programming

1. Consider the following equation that arises in the solution of transient heat conduction in a plane wall

$$u(x) = Bi - x \tan(x) \quad (1)$$

where Bi is the Biot number in the present case taken to be 7 and $x = [-1, 1]$. Write a program to compute the first derivative using first, second and fourth-order accurate forward-difference formulae. Using grid sizes of $\Delta x = 0.1, 0.01, 0.001$ plot the ∞ -norm of the truncation-error versus the grid sizes for all the three schemes on a log-log plot. Provide the values of the slopes of these lines and compare them with the theoretical prediction (70 points).