Assignment 4

General Instructions

- Solutions due by 18^{th} March for Group A
- Solutions due by 20^{th} March for Group B

Hand calculations

1. (10 points) Consider a function $f(x)$ defined in $[0, 2\pi]$ as

$$f(x) = \begin{cases} -1 & x \in [-4, 0] \\ x - 1 & x \in [0, 2] \\ 1 & x \in [2, 4] \end{cases}$$

Sketch the function f vs x. What is the appropriate method to be used to solve for the root of $f(x) = 0$ and why? Perform five iterations using that method and report the solution at the end of each iteration.

2. (15 points) The hot combustion gases of a furnace are separated from the ambient air and its surroundings,

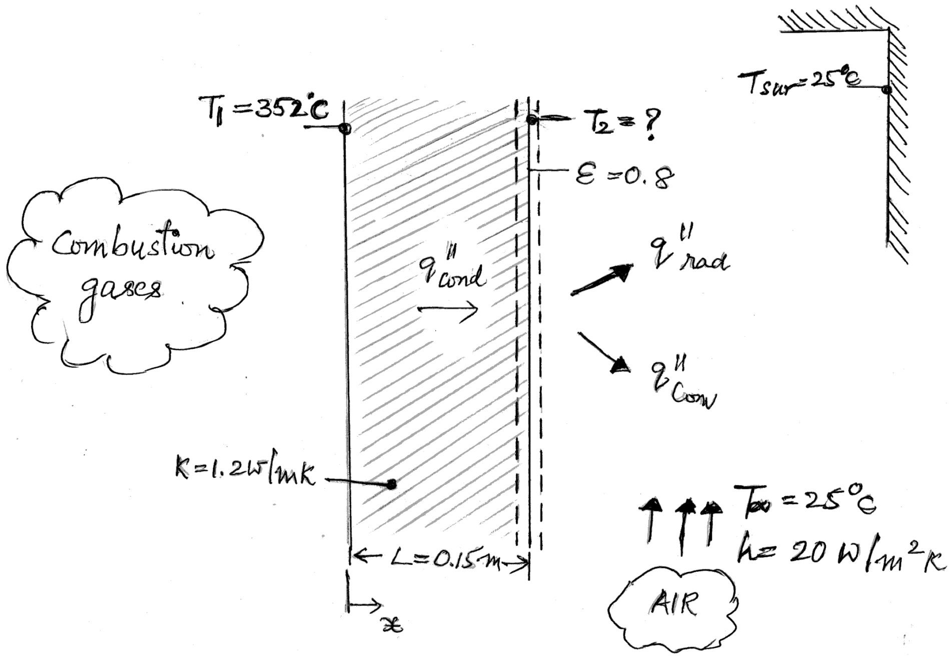

Figure 1: A schematic of the furnace indicating several heat transfer processes.

as shown in figure 1, which are at 25^oC, by a brick wall 0.15 m thick. The brick has a thermal conductivity of 1.2 W/m.K and a surface emissivity of 0.8. Under steady-state conditions an inner surface temperature of 352^oC is measured. Free convection heat transfer to the air adjoining the surface is characterised by a convection coefficient of $h = 20$ W/m^2. K. Calculate the brick outer surface temperature T_2 using Newoton's method by performing 5 iterations with an initial guess of 1000 K. An energy balance on the outer surface of the furnace yields the following equation:

$$k\frac{T_1 - T_2}{L} = h(T_2 - T_\infty) + \epsilon\sigma(T_2^4 - T_{sur}^4) \tag{1}$$

where $\sigma = 5.67 \times 10^{-8}$ W/m^2 K^4 is the Stefan-Boltzmann constant.

3. (10 points) A good approximation for $n!$ is given by the function

$$f(x) = (2\pi)^{1/2} x^{x+1/2} e^{-x} \left(1 + \frac{1}{12x} + \frac{1}{288x^2}\right)$$

Though $n!$ is defined for integer values of n, for large n, the approximation is very close. For this $f(x)$, if $f(x) = 1000$, calculate the value of x accurate up to 4 decimal places.

4. (10 points) Determine the order of convergence of the iterative method

$$x_{k+1} = \frac{x_0 f(x_k) - x_k f(x_0)}{f(x_k) - f(x_0)}$$

for finding a simple root of the equation $f(x) = 0$.

5. (15 points) Consider a thin, long metallic rod of length 0.1 m with its left end kept at $T_L = 100$ °C and its right end at $T_R = 30$ °C. This rod is subjected to natural convection in an ambient atmosphere at $T_\infty = 30$ °C. Under steady state, the differential equation governing this system is given by:

$$\frac{d^2T}{dx^2} = \lambda\,(T - T_\infty), \tag{2}$$

with the parameter λ signifying the thermal conductivity of the rod as well as the convective heat transfer coefficient.

(a) Discretize the above differential equation using a 3-point second order FD scheme with six grid points along the rod including its two ends, and write it in a general $\mathbf{A}\vec{T} = \vec{b}$ format. Here, $\mathbf{A} = \mathbf{A}(\lambda)$ is the coefficient matrix, $\vec{T} = [T_L\ T_2\ T_3\ T_4\ T_5\ T_R]'$ is the temperature (solution) vector, and $\vec{b} = \vec{b}(\lambda)$ is the RHS vector.

(b) Solve Eq. 2 analytically with $\lambda = 0$, i.e. in the absence of convection. Draw the resultant temperature profile on a graph sheet *to scale* with connected markers. Submit this sheet along with your answer.

(c) Solve the system again, but now with $\lambda = 500$ and by rounding all computations to *3 decimal places*. Draw the resultant temperature profile on the same graph sheet distinguishable from the previous one. Before plotting, round all temperature values to the nearest integer. Explain the physical reason behind the change, if any, in the two temperature profiles.

Programming

1. The Planck distribution for the emission of radiation from a balckbody is given by

$$E = \frac{c_1}{\lambda^5\,[exp(c_2/\lambda T) - 1]}$$

where, E is termed the monochromatic emissive power, λ is the wavelength of radiation, T is the temperature, and c_1 and c_2 are constants. We wish to find the value of λ at which E is maximum. Therefore, the first positive real root of the equation $dE/d\lambda =$ is to be determined.

(a) (10 points) Using the Newton-Rhapson method, write a program to find this value and compare with the analytical solution. Take the constants c_1 and c_2 as 3.741×10^8 and 1.439×10^4 in the appropriate units.

(b) (10 points) It is a good idea to combine bisection method with the Newton's method to get the best of both the worlds. In order to do the same limit the number of iterations to 10 in bisection method and use this value as initial guess (and continue couting iteration number as 11) for the Newton's. Plot λ_{max} as a function of iteration number. Determine the error after each iteration using the analytical solution. By plotting the error with iteration number, determine the rate of convergence for the bisection method, and the Newton-Rhapson method.

2. (20 points) Solve the system of non-linear equations given below with two starting guesses - $x = [0.1,\ 1.2,\ 2.5]^T$ and $x = [1,\ 0,\ 1]^T$. Do the two solutions converge to the same root? If not, why?

$$x + y + z = 3 \tag{3}$$
$$x^2 + y^2 + z^2 = 5 \tag{4}$$
$$e^x + xy - xz = 1 \tag{5}$$