**EECS2040 Data Structure Hw #5 (Chapter 6 Graph)**

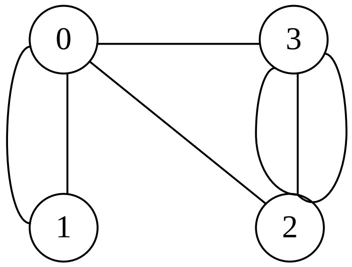
**due date 5/30/2022**

**by 108011235 陳昭維**

**(Part 1: 2% of final Grade)**

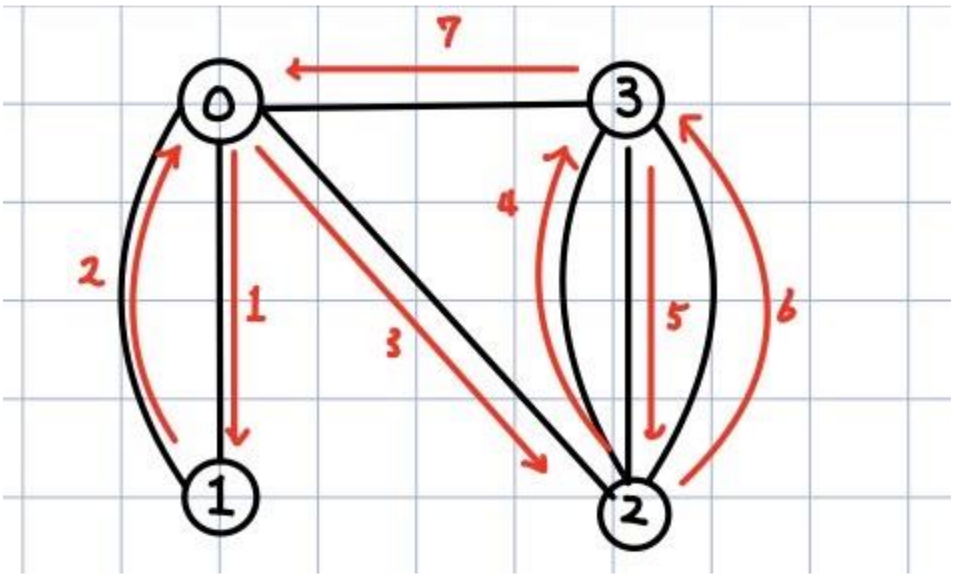
**Part 1**

1. (10%) Does the multigraph below have an Eulerian walk? If so, find one.

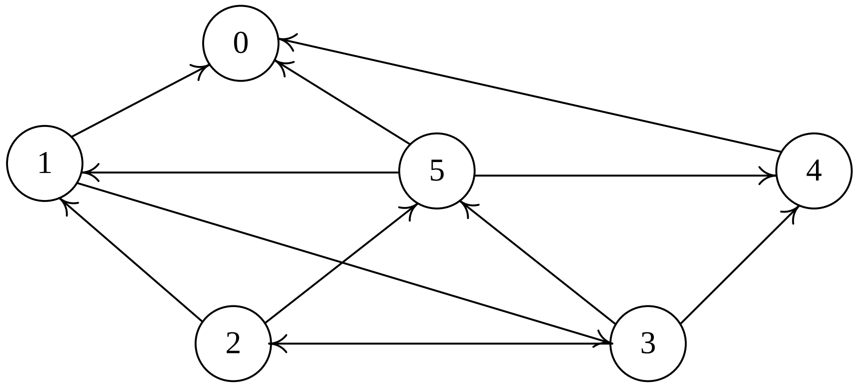



<answer>

Yes there is an Eulerian walk




2. (10%) For the digraph below obtain
   (a) The in-degree and out-degree of each vertex
   (b) Its adjacency-matrix
   (c) Its adjacency-list representation
   (d) Its strongly connected components



<answer>

(a)

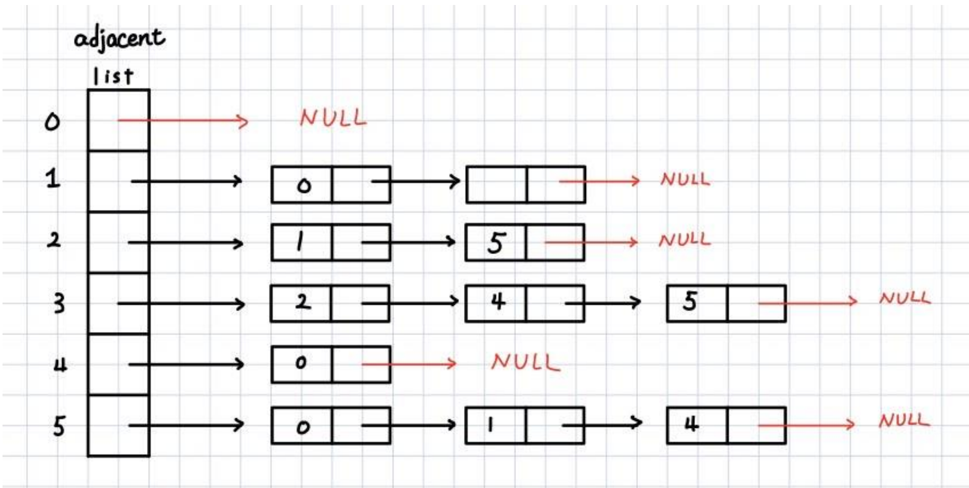
| Vertex | In-degree | Out-degree |
|---|---|---|
| 0 | 3 | 0 |
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 3 | 1 | 3 |
| 4 | 2 | 1 |
| 5 | 2 | 3 |

(b)

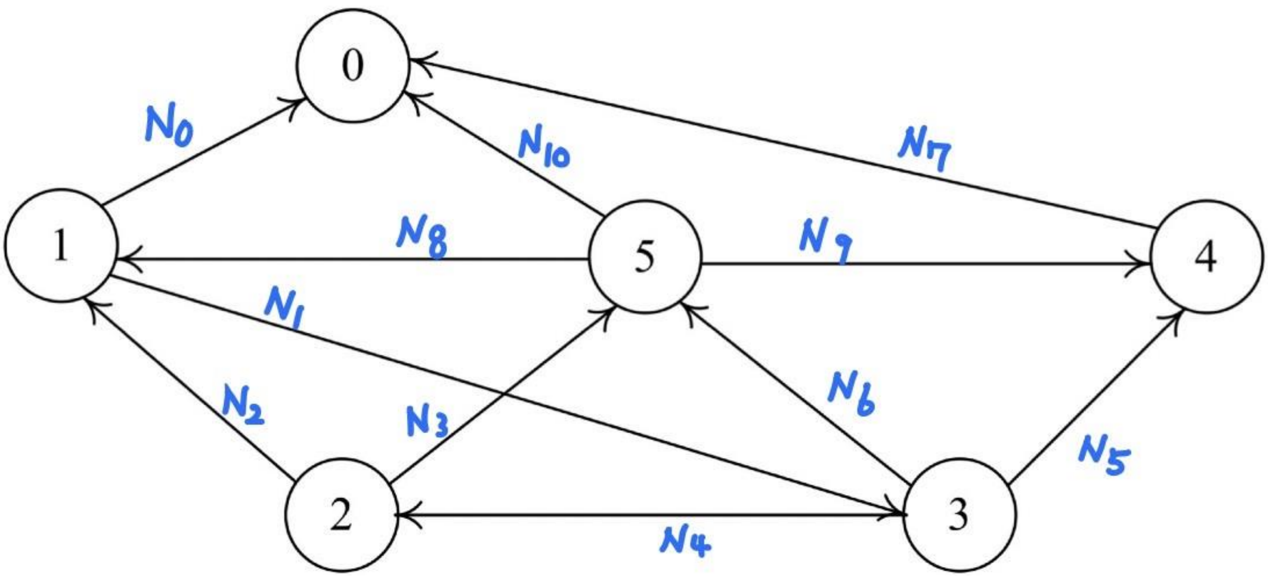
This column and row is indexed by (0~5) and (0~5), the graph is generated by LaTeX

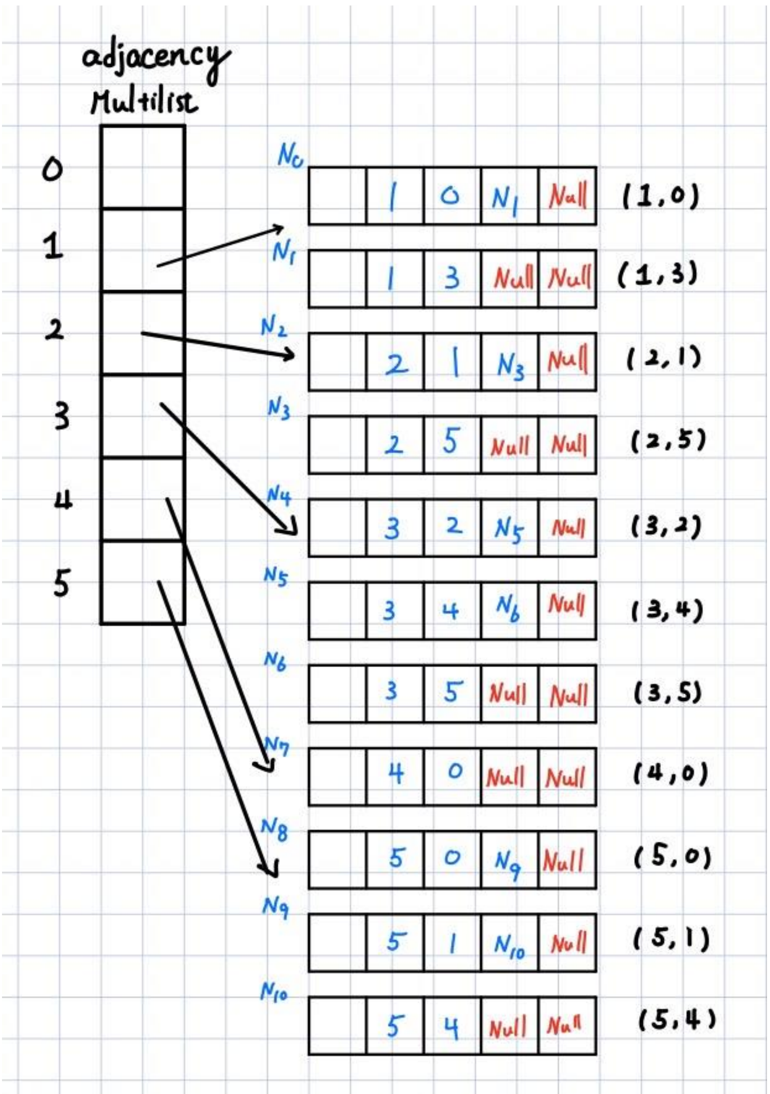
$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(c)

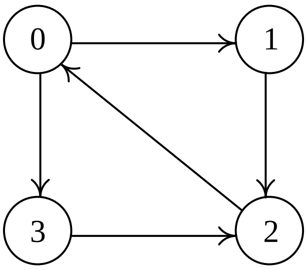



(d)




adjacency Multilist
0
1
2
3
4
5
N0
1 0 N1 Null
(1,0)
N1
1 3 Null Null
(1,3)
N2
2 1 N3 Null
(2,1)
N3
2 5 Null Null
(2,5)
N4
3 2 N5 Null
(3,2)
N5
3 4 N6 Null
(3,4)
N6
3 5 Null Null
(3,5)
N7
4 0 Null Null
(4,0)
N8
5 0 N9 Null
(5,0)
N9
5 1 N10 Null
(5,1)
N10
5 4 Null Null
(5,4)

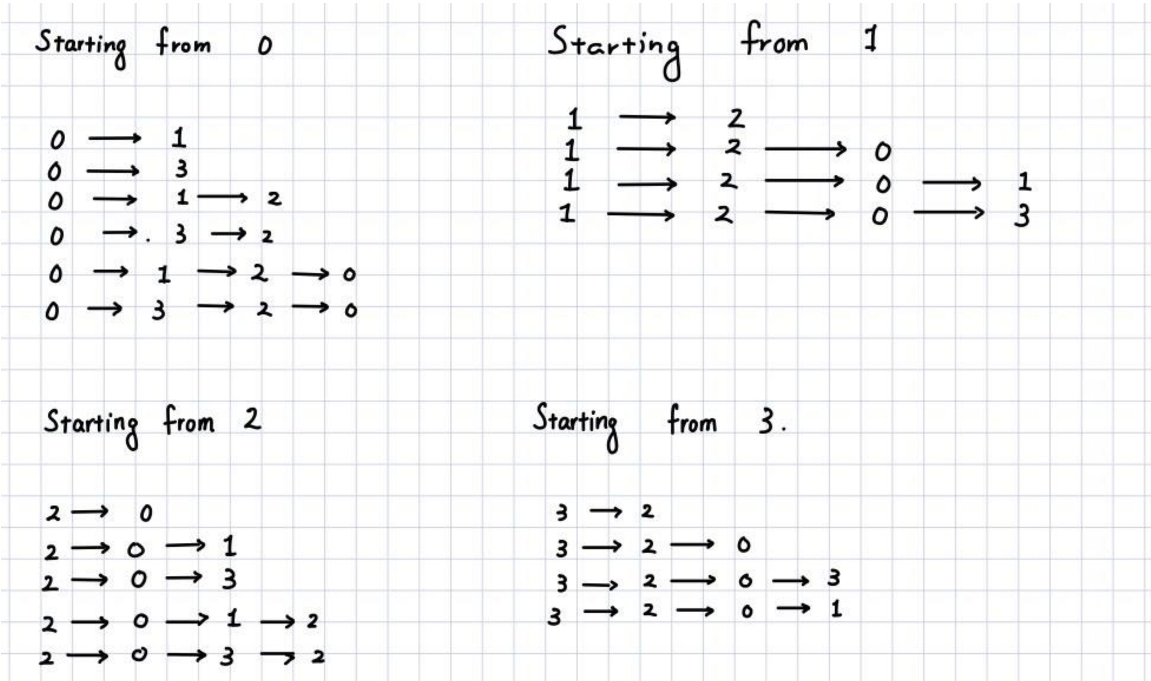
3. (10%) Is the digraph below strongly connected? List all the simple paths.




<answer>

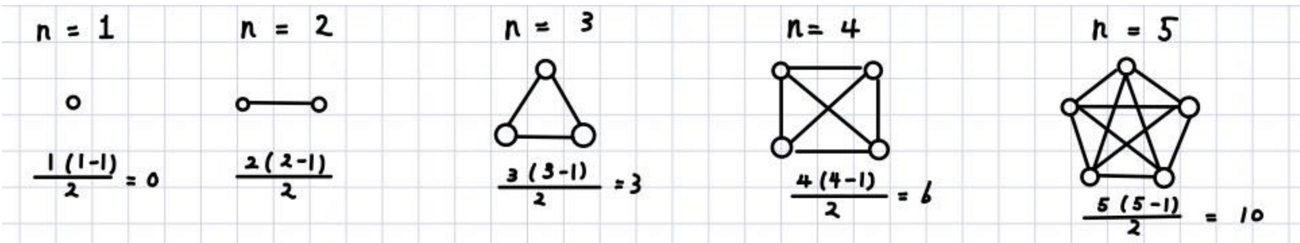
Yes, the graph is strongly connected.

Simple path of the graph:




4. (10%) Draw the complete undirected graphs on one, two, three, four, and five vertices. Prove that the number of edges in an n-vertex complete graph is n(n-1)/2.

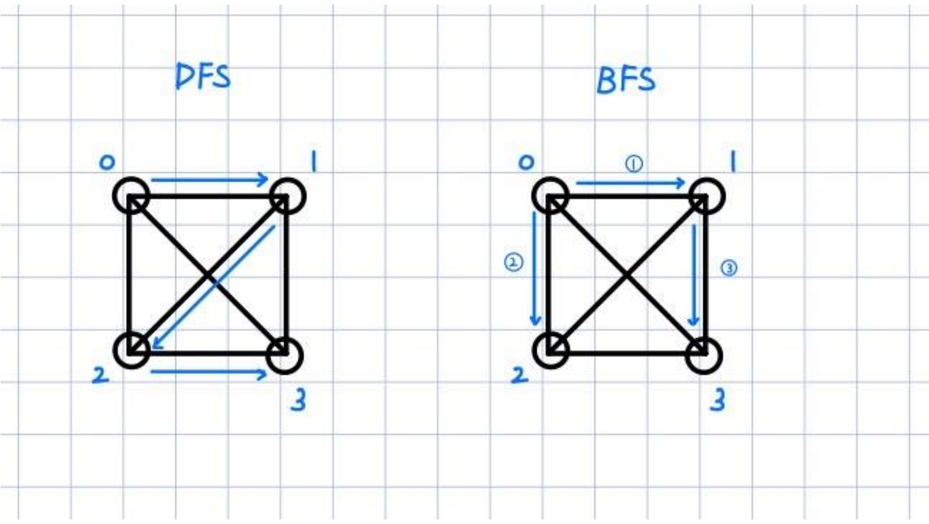
<answer>




For each n vertex to be connected to other vertex there are n – 1 edges, there are n vertices but so there are $\frac{n \times (n-1)}{2}$ edges, since 2 of each are replicates.

5. (4%) Apply depth-first and breadth-first searches to the complete graph on four vertices. Assume that vertices are numbered 0 to 3, are stored in increasing order in each list in the adjacency-list representation, and both traversals begin at vertex 0. List the vertices in the order they would be visited.

<answer>




Both the order of traversal for DFS and BFS are $0 \rightarrow 1 \rightarrow 2 \rightarrow 3$

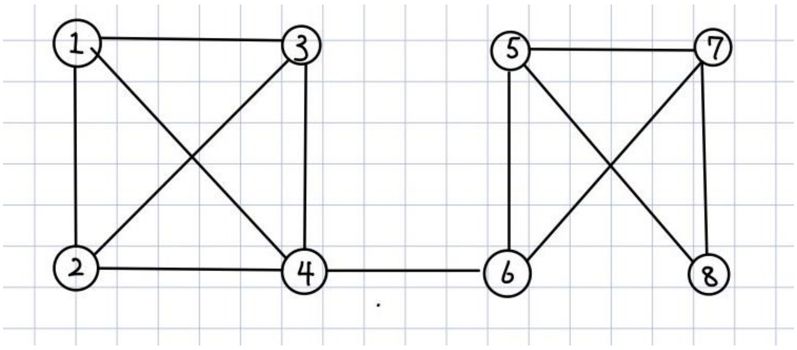
6. (6%) Let *G* be a graph whose vertices are the integers 1 through 8, and let the adjacent vertices of each vertex be given by the table below:

| ***Vertex*** | ***Adjacent Vertices*** |
|---|---|
| 1 | (2, 3, 4) |
| 2 | (1, 3, 4) |
| 3 | (1, 2, 4) |
| 4 | (1, 2, 3, 6) |
| 5 | (6, 7, 8) |
| 6 | (4, 5, 7) |
| 7 | (5, 6, 8) |
| 8 | (5, 7) |

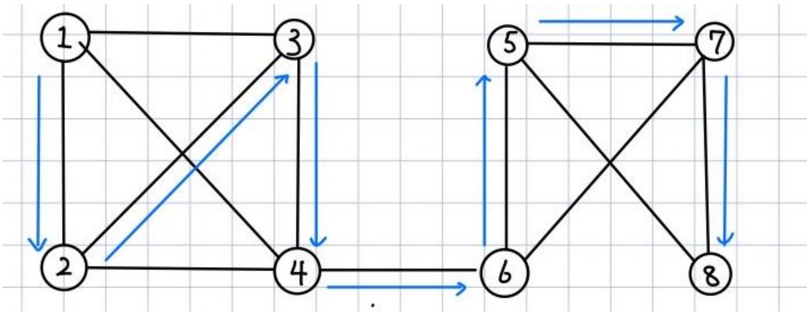
Assume that, in a traversal of *G*, the adjacent vertices of a given vertex are returned in the same order as they are listed in the table above.

<answer>

(a) Draw *G*.

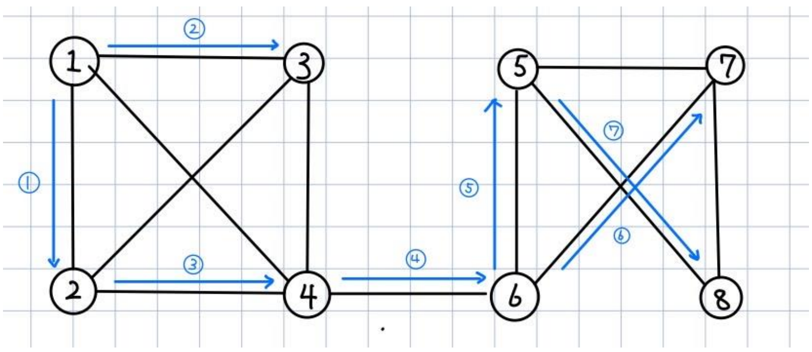



(b) Give the sequence of vertices of *G* visited using a DFS traversal starting at vertex 1.



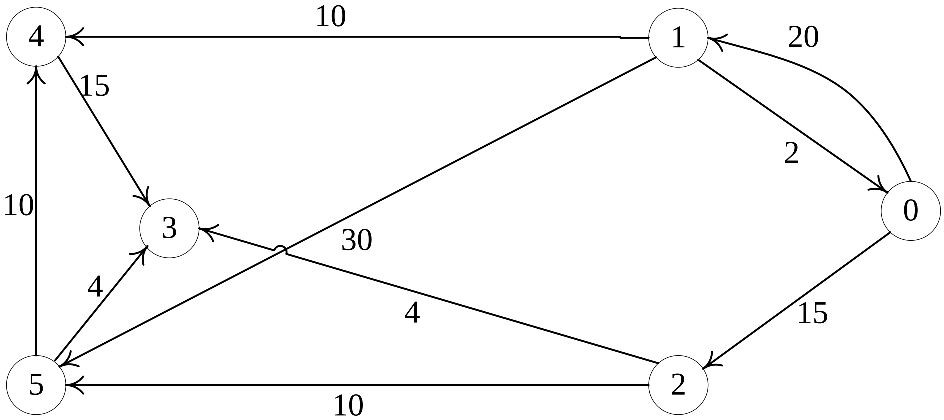

$1 \to 2 \to 3 \to 4 \to 6 \to 5 \to 7 \to 8$

(c) Give the sequence of vertices visited using a BFS traversal starting at vertex 1.




$1 \to 2 \to 3 \to 4 \to 6 \to 5 \to 7 \to 8$

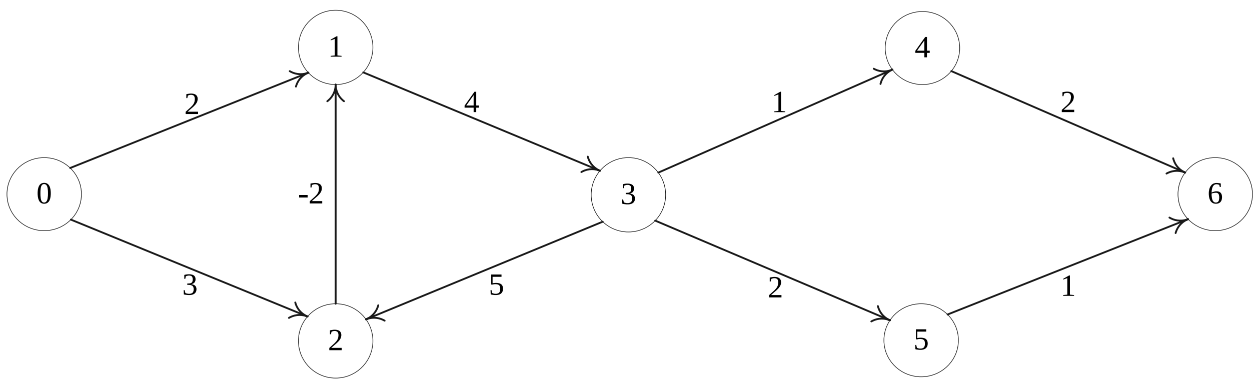
7. (10%) Use ShortestPath (Program 6.8) (Dijkstra's algorithm) to obtain, in nondecreasing order, the lengths and the paths of the shortest paths from vertex 0 to all remaining vertices in the graph below.




<answer>

| | path | lengths |
|---|---|---|
| 1 | 0 → 1 | 20 |
| 2 | 0 → 2 | 15 |
| 3 | 0 → 2 → 3 | 19 |
| 4 | 0 → 1 → 4 | 30 |
| 5 | 0 → 2 → 5 | 25 |

8. (10%) Using the directed graph below, explain why ShortestPath (Program 6.8) will not work properly. What is the shortest path between vertices 0 and 6?

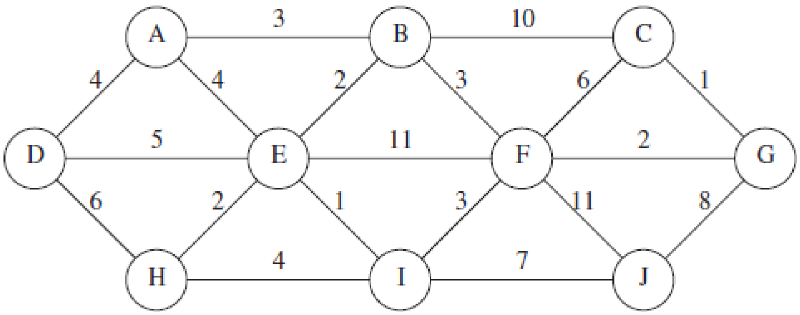



<answer>

The shortest path from 0 to 6 is 0 → 2 → 1 → 3 → 4 → 6 with length 8.

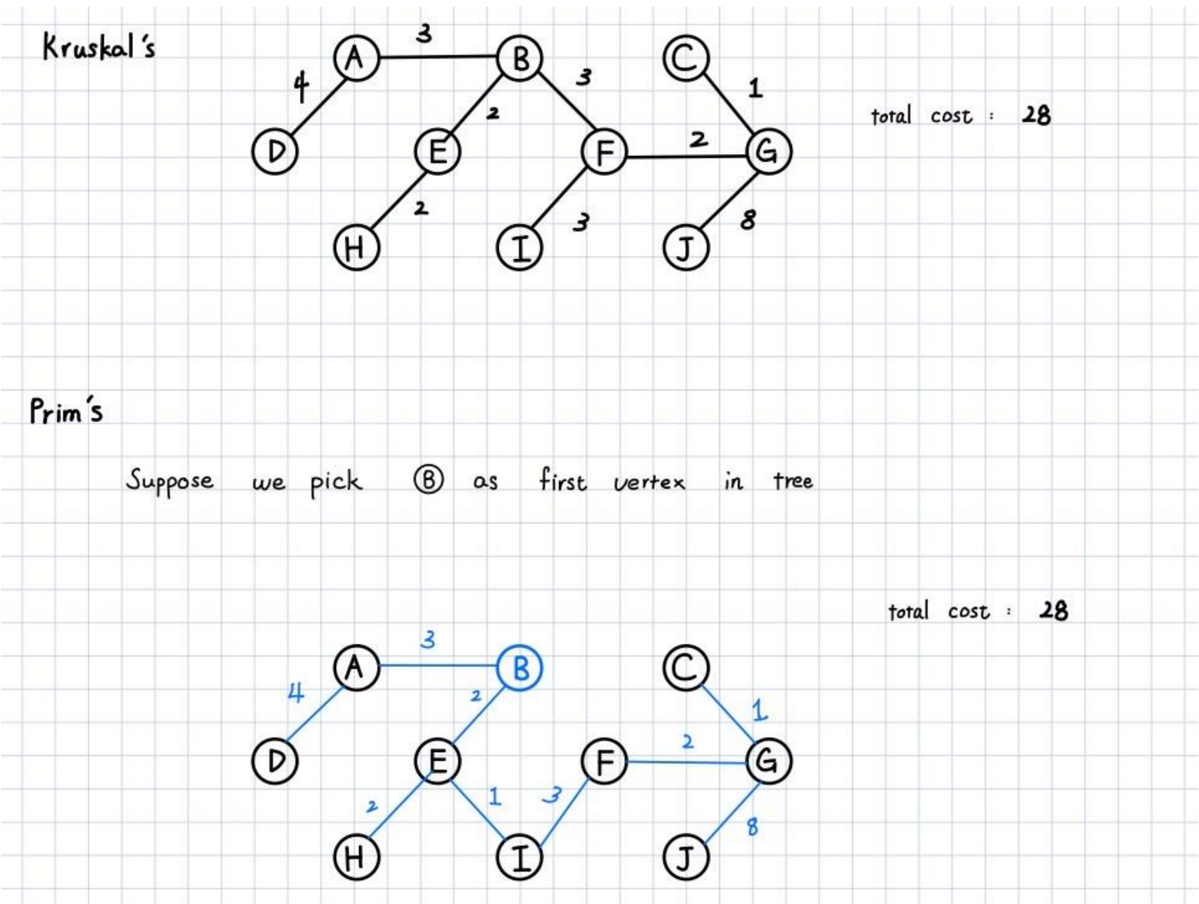
The Dijkstra's algorithm wouldn't work because, negative weight will not guaranteed a closed vertex is indeed minimal.

9. (10%) For the weighted graph G shown below,



(a) Find a minimum spanning tree for the graph using both Prim’s and Kruskal’s algorithms.

<answer>




(b) Is this minimum spanning tree unique? Why?

<answer>

No the minimum spanning tree is not unique, it varies with tie-break mechanism, but the total cost shouldn’t vary for distinct minimal spanning tree.

10. (10%) Does the following set of precedence relations (<) define a partial order on the elements 0 through 4? Why?

$0 < 1; 1 < 3; 1 < 2; 2 < 3; 2 < 4; 4 < 0$

<answer>

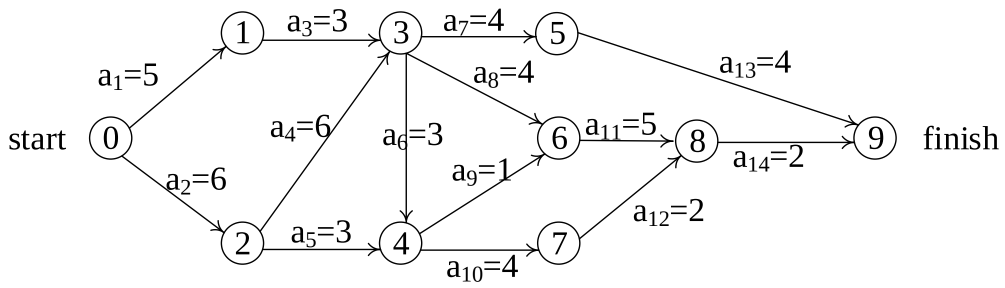
Assume a partial order is defined on the relations, since partial order guarantees transitive, thus from above relations, we get $0 < 0$ which is a mapping from 0 to 0, which violates the irreflexive properties of partial order. Thus, it doesn’t define a partial order.

11. (10%) For the AOE network shown below,

(a) Obtain the early, $e(a_i)$, and late, $l(a_i)$, start times for each activity. Use the forward-backward approach.

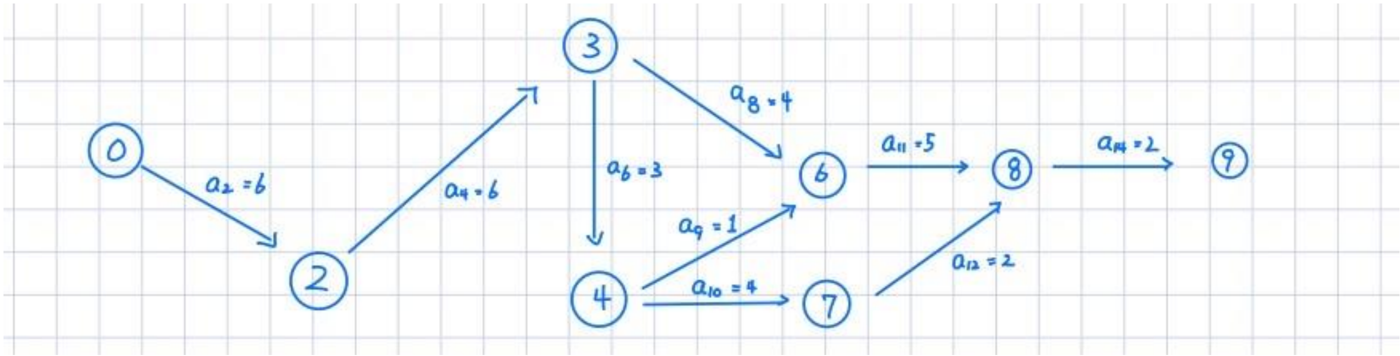
(b) What is the earliest time the project can finish?

(c) Which activities are critical? Fill the table below for answers to (a), (b), and (c).




| activity | Early time $e(a_i)$ | Late time $l(a_i)$ | slack $l(a_i) - e(a_i)$ | critical |
|---|---|---|---|---|
| $a_1$ | 0 | 4 | 4 | No |
| $a_2$ | 0 | 0 | 0 | Yes |
| $a_3$ | 5 | 9 | 4 | No |
| $a_4$ | 6 | 6 | 0 | Yes |
| $a_5$ | 6 | 12 | 6 | No |
| $a_6$ | 12 | 12 | 0 | Yes |
| $a_7$ | 12 | 15 | 3 | No |
| $a_8$ | 12 | 12 | 0 | Yes |
| $a_9$ | 15 | 15 | 0 | Yes |
| $a_{10}$ | 15 | 15 | 0 | Yes |
| $a_{11}$ | 16 | 16 | 0 | Yes |
| $a_{12}$ | 19 | 19 | 0 | Yes |
| $a_{13}$ | 16 | 19 | 3 | No |
| $a_{14}$ | 21 | 21 | 0 | Yes |

(d) Is there any single activity whose speed-up would result in a reduction of the project finish time?




The single activity that can speed up on the critical path is $a_2$, $a_4$, $a_{14}$