

# Computer Network Homework # 1

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**Problem 1.** If the diameter of a network with 100 nodes is 1, what is the minimum number of links in this network?

**Solution:**

The diameter of the network is 1 indicating that, every node in the network is connected to every other nodes in the network with at least 1 link respectively, thus the minimum connection is  $\binom{100}{2}$  links in the network.

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**Problem 2.** If the diameter of a network with 100 nodes is 2, what is the minimum number of links in this network?

**Solution:**

Consider the network where there is a node which all other 99 nodes are only connected to it, thus there is 99 links and the network has diameter of 2.

Then remove one of a link from the network, this disconnects one node from the network, thus 99 links is the minimum links number.

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**Problem 3.** For a network of 100 nodes, if the degree of every node is at most 2, what is the minimum diameter of that network?

**Solution:**

A network is a connected graph. From theorem, the minimum edge number is  $(\text{vertex} - 1)$ .

Thus, for 100 nodes, there is at least 99 edges with 98 nodes with degree of 2 and 2 nodes with degree of 1. In this configuration the diameter is 99. Which is the distance between the two nodes with degree of 1

Then connect the two nodes that have degree of 1 in previous configuration, now all nodes have degree of 2, the network becomes a cycle. Label the nodes respectively from 1 to 100. Starting from node 1, the distance between node  $x$  with it is  $\min(x, 100 - x)$ . The maximum distance from a node to another is 50, thus the diameter in this configuration is 50.

From above, the minimum diameter of this network is 50.

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**Problem 4.** For a network of 100 nodes, if the degree of every node is at most 3, is it possible that the diameter of this network is not greater than 5?

**Solution:**

Build the network starting with one node, and then connect it with 3 other nodes, naming them the *1st generation child*. Since we want to maximize the connecting nodes with the degree limitation we will have to create binary branches from the *1st generation child*, and we will call the inherited child the *2<sup>nd</sup> generation child* and the inherited child from the previous generation the *n<sup>th</sup> generation child*

So for a tree with *n<sup>th</sup> generation child*, there are  $1 + \sum_{i=0}^{n-1} 3 \times 2^i$  nodes in the graph.

Since the maximum diameter is 5, the maximum node number is  $1 + 3 +$

$6 + 12 + 24 + 48 = 94$  nodes.

Thus, it is impossible to build a network of 100 nodes which has degree less than 3, and maximum diameter of 5.