Tests For Convergence/Divergence

Test	Requirements	Possible Outcomes	Test
Test for Divergence (nth Term Test)	N/A	Divergent/Inconclusive	Series is Divergent if $\lim_{n\to\infty} a_n \neq 0$ (Otherwise Inconclusive)
Geometric Series	Series must be geometric $a + ar + ar^2 + ar^3 + \dots$	Divergent/Convergent	Convergent if $\lvert r\rvert < 1$ (series converges to $\frac{a}{1-r}$) Divergent if $\lvert r\rvert \geq 1$
P-Series	Series must be P-series $\sum \frac{1}{n^p}$	Divergent/Convergent	Convergent if $p > 1$ Divergent if $p \leq 1$
Integral Test	Positive/Continuous/Decreasing	Divergent/Convergent	(note: $a_n = f(n)$) Convergent if $\int_1^\infty f(x)\,dx$ converges Divergent if $\int_1^\infty f(x)\,dx$ diverges
Direct Comparison	Positive	Div./Conv./Inc.	• If $\sum b_n$ converges and $a_n \leq b_n$ for all positive n, then $\sum a_n$ converges • If $\sum b_n$ diverges and $a_n \geq b_n$ for all positive n, then $\sum a_n$ diverges (Other outcomes – Inconclusive)
Limit Comparison	Positive	Div./Conv./Inc.	• If $\lim_{n\to\infty} \frac{a_n}{b_n} = k > 0$, then either both series converge or both series diverge. • If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges. • If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.
Alternating Series	Series must be alternating $a_1 - a_2 + a_3 - a_4 + \dots + (-1)^{n-1} a_n + \dots$ or $-a_1 + a_2 - a_3 + \dots (-1)^n a_n + \dots$	Conv./Inconclusive	If $0 < a_{n+1} < a_n$ and $\lim_{n\to\infty} a_n = 0$, then convergent. Otherwise, Inconclusive
Ratio	all terms must be nonzero	Div./Conv./Inc.	• If $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L < 1$, then convergent • If $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L > 1$, then divergent • If $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$, then inconclusive