International Finance

Uncertainty and Current Account

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Uncertainty and Current Account

- We will consider how uncertainty affects current account dynamics.
- First of all, we need to know
  1. how uncertainty affects optimal choices?
  2. how people form expectation?

Section 1

Making Choices in Risky Situations

Criteria for Choice Over Risky Prospects

- Consider three assets with two states.
  - The good and bad states occur with equal probability.

		Price Next Year	
	Price Today	Good State	Bad State
Asset 1	100	120	105
Asset 2	100	160	50
Asset 3	100	160	105

- In practice, we prefer to examine stock return:

$$R = \left(\frac{P_{t+1} - P_t}{P_t} \right) \times 100$$

Criteria for Choice Over Risky Prospects

Criterion 1: State-by-State Dominance

	Returns (%)	
	Good State	Bad State
Asset 1	20	5
Asset 2	60	-50
Asset 3	60	5

- Asset 3 exhibits state-by-state dominance over assets 1 and 2.
- But the choice between assets 1 and 2 is not as clear cut.

Criteria for Choice Over Risky Prospects

Criterion 2: Mean-Variance Dominance

	Return (%)			
	Good State	Bad State	$E(R)$	$\sigma(R)$
Asset 1	20	5	12.5	7.5
Asset 2	60	-50	5	55
Asset 3	60	5	32.5	27.5

- Asset 1 exhibits mean-variance dominance over asset 2.
- By the mean-variance criterion, asset 3 dominates asset 2 but not asset 1, even though on a state-by-state basis, asset 3 is clearly to be preferred.

Criteria for Choice Over Risky Prospects

Criterion 3: Sharpe Ratios

- Consider two more assets:

	Return (%)			
	Good State	Bad State	$E(R)$	$\sigma(R)$
Asset 4	5	3	4	1
Asset 5	8	2	5	3

  - Neither exhibits state-by-state dominance.
  - Neither asset exhibits mean-variance dominance either.
  - William Sharpe (1944–, Nobel Laureate 1990) suggests that in these circumstances, it can help to compare the two assets' Sharpe ratios, defined as $E(R)/\sigma(R)$.

Criteria for Choice Over Risky Prospects

- Comparing Sharpe ratios, asset 4 is preferred to asset 5.

	Return (%)				
	Good State	Bad State	$E(R)$	$\sigma(R)$	$E(R)/\sigma(R)$
Asset 4	5	3	4	1	4
Asset 5	8	2	5	3	1.67

Criteria for Choice Over Risky Prospects

- Using the Sharpe ratio to choose between assets means assuming that investors weight the mean and standard deviation equally, in the sense that a doubling of $\sigma(R)$ is adequately compensated by a doubling of $E(R)$.
- Investors who are more or less averse to risk will disagree.

Criteria for Choice Over Risky Prospects

Summary:

- State-by-state dominance is the most robust criterion, but often cannot be concluded.
- Mean-variance dominance is more widely-applicable, but cannot always be concluded as the state-by-state dominance.
- The Sharpe ratio can always be applied, but requires a very specific assumption about consumer attitudes towards risk.

Criteria for Choice Over Risky Prospects

We need a more careful and comprehensive approach to comparing random payoffs.

$\Rightarrow$ We need first to understand people's preferences.

$\Rightarrow$ Economists use a utility function to represent preferences.

- Under uncertainty, we consider an expected utility function.

Expected Payoff

Let $u(\cdot)$ denote the utility function. Assume that there are two possible payoffs: X_G and X_B, with probabilities π and $1-\pi$.

- Would you prefer that risky (uncertain) situation, or $\pi X_G + (1-\pi)X_B$ with certainty?

$$\pi u(X_G) + (1-\pi)u(X_B) \text{ vs. } u\left(\pi X_G + (1-\pi)X_B\right)$$

- It is argued that most people prefer $\pi X_G + (1-\pi)X_B$.
  - Concavity of the utility function, proposed by Gabriel Cramer and Daniel Bernoulli.

Concave Utility Function

- Gabriel Cramer (1704–1752)
- Daniel Bernoulli (1700–1782)

Concave Utility Function

- Assume that the utility function $u(\cdot)$ over payoffs in any given state is concave: $u' > 0$, $u'' < 0$.
- This implies that investors prefer more to less, but have diminishing marginal utility as payoffs increase.
- Hence, the expected utility is given by

$$E[u(X)] = \pi u(X_G) + (1-\pi)u(X_B)$$

and by concavity (recall Jensen's Inequality),

$$E[u(X)] < u\left(E(X)\right)$$

- Does such an expected utility provide good description of people's preferences over risky situations?
    - Nobody knows.

Expected Utility Function

- About two centuries later, John von Neumann (Hungary, 1903-1957) and Oskar Morgenstern (Germany, 1902-1977) worked out the conditions under which investors' preferences over risky situations could be described by an expected utility function as

$$U = E[u(X)] = \pi u(X_G) + (1-\pi)u(X_B),$$

which is called the von Neumann-Morgenstern expected utility function

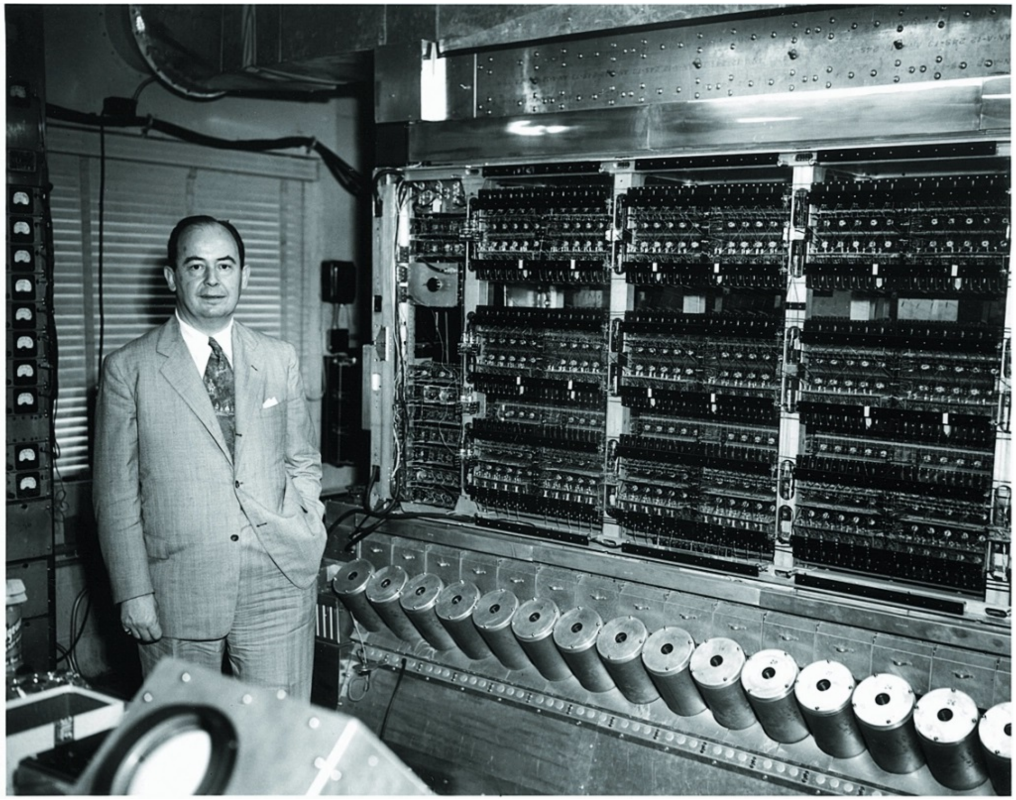
  - $u(\cdot)$ is the Bernoulli utility function, which is concave.
  - The von Neumann-Morgenstern expected utility function U is linear in probabilities.

Expected Utility Function

John von Neumann and Oskar Morgenstern.

- John von Neumann (1903-1957), Princeton University.
- Oskar Morgenstern (1902-1977), New York University.

John von Neumann and IAS computer



Section 2

Risk-Averse Behavior

Risk-Averse Behavior

- Recall that given a Bernoulli utility function,

$$u(E(X)) > E(u(X))$$

- The so-called risk-averse behavior.

Implications

- Use consumption as an example,

$$u\left(\frac{1}{2}C^H + \frac{1}{2}C^L\right) > \frac{1}{2}u(C^H) + \frac{1}{2}u(C^L)$$

- The risk-averse behavior implies that people prefers to have a steady stream of consumption rather than a volatile consumption path.
  - Consumption smoothing behavior.
- If the utility function is linear, then the individual is risk neutral and does not care about consumption fluctuation.

Section 3

Measure of Risk Aversion

The Measure of Absolute Risk Aversion

How to measure the degree of risk-aversion?

- Arrow-Pratt risk aversion
- If utility function u_A has higher degree of risk aversion than u_B, then

$$-\frac{u''_A(X)}{u'_A(X)} > -\frac{u''_B(X)}{u'_B(X)}$$

  - The measure of absolute risk aversion.

The Measure of Relative Risk Aversion

- In some circumstances, the absolute risk aversion fails to work.
- Given initial wealth X, the payoffs under different states are $k^H X$ and $k^L X$
- For example, $k^H = 1.1$ and $k^L = 0.9$:

$$110/90 \text{ vs. } 1,100,000/900,000$$

- Solution: The measure of relative risk aversion.

$$-\frac{Xu''(X)}{u'(X)}$$

Kenneth Arrow

- Kenneth Joseph Arrow (1921–2017), Stanford, Nobel Laureate (1972)
  - General equilibrium theory
  - Fundamental theorems of welfare economics
  - Arrow's impossibility theorem
  - Adverse selection and moral hazard

Example: Constant Relative Risk Aversion

- CRRA is one of the often-used utility function.

$$u(C) = \begin{cases} \frac{C^{1-\gamma}-1}{1-\gamma} & \text{for} \quad \gamma > 0 \text{ and } \gamma \neq 1, \\ \log C & \text{for} \quad \gamma = 1. \end{cases}$$

Section 4

Uncertainty and Rational Expectation

A Two-Period Model with Certainty

- Consider a simple two-period model:

$$\max_{\{C_1, C_2\}} u(C_1) + \beta u(C_2)$$

s.t.

$$\begin{cases} C_1 + B_2 & = Y_1 \\ C_2 & = Y_2 + (1+r)B_2 \end{cases}$$

- It is assumed in the previous lecture that when decisions are made in period 1, the values of exogenous variables in period 2 are known.
    - That is, Y_2 is known at time $t = 1$.

Expectations: Different Assumptions

Because of uncertainty, we need to form expectations.

- Perfect Foresight

$$Y^e_{t,t+1} = Y_{t+1}$$

- Adaptive Expectations (partial adjustment)

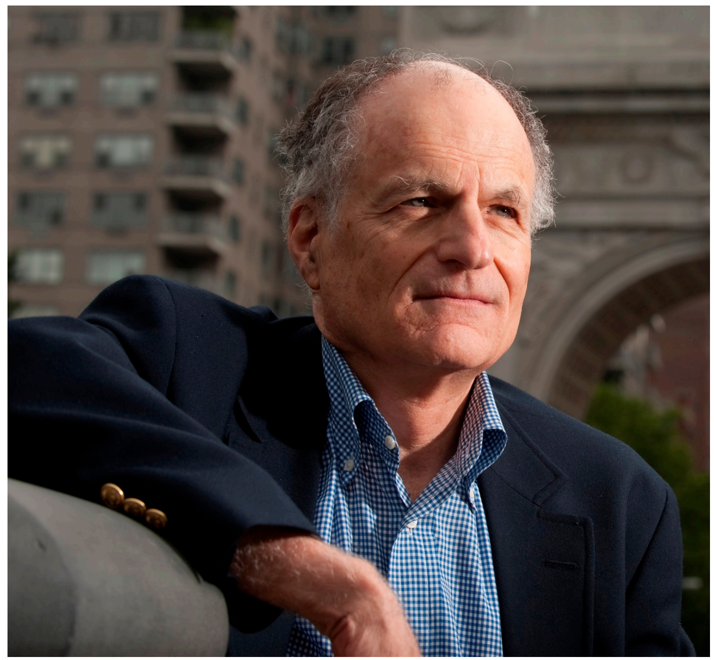
$$Y^e_{t,t+1} = Y^e_{t-1,t} + \underbrace{\lambda(Y_t - Y^e_{t-1,t})}_{\text{error-adjustment}} = (1-\lambda)Y^e_{t-1,t} + \lambda Y_t$$

- Rational Expectations

$$Y^e_{t,t+1} = E(Y_{t+1}|\Omega_t) = E_t Y_{t+1}$$

Modern Approach to Macroeconomics

- Pioneers and Representatives



- Robert Lucas, Jr. (1937–), Chicago. Nobel Laureate (1995)
- Thomas J. Sargent (1943–), NYU. Nobel Laureate (2011)
- Robert J. Barro (1944–), Harvard.

Rational Expectations

Rational expectations usually means two things:

- They understand the structure of the model economy and base their expectations of variables on this knowledge.
- They use publicly available information in an efficient manner. Thus, they do not make systematic mistakes when formulating expectations.

Two-Period Model with Uncertainty

- Setup
  - Two-period model.
  - Small open economy. Consumer can borrow and lend in period one at rate $1+r$, known with certainty. (A risk free international bond.)
  - First period endowment Y_1 is known with certainty.
  - Second period endowment is uncertain: $Y_2 = Y_2^i : i = \{H, L\}$ with conditional probability of $i = H$ given by

$$P(Y_2 = Y_2^H | Y_1) = \pi$$

and $i = L$ given by

$$P(Y_2 = Y_2^L | Y_1) = (1 - \pi)$$

Consumer's Problem

- Consumer chooses $\{C_1, C_2^H, C_2^L, B_2\}$ to maximize expected utility:

$$\begin{aligned} U &= u(C_1) + \beta E_1 u(C_2) \\ &= u(C_1) + \beta \Big[\pi u(C_2^H) + (1-\pi) u(C_2^L) \Big] \end{aligned}$$

subject to

$$\begin{aligned} C_1 + B_2 &= Y_1 \\ C_2^H &= Y_2^H + (1+r) B_2 \\ C_2^L &= Y_2^L + (1+r) B_2 \end{aligned}$$

Consumer's Problem

- The lifetime budget constraints are
  - For $i = H$
$$C_1 + \frac{C_2^H}{1+r} = Y_1 + \frac{Y_2^H}{1+r}$$
  - For $i = L$
$$C_1 + \frac{C_2^L}{1+r} = Y_1 + \frac{Y_2^L}{1+r}$$

Consumer's Problem

- The Lagrangian is

$$\mathcal{L} = u(C_1) + \beta\Big[\pi u(C_2^H) + (1-\pi)u(C_2^L)\Big]$$
$$+ \lambda^H\left(Y_1 + \frac{Y_2^H}{1+r} - C_1 - \frac{C_2^H}{1+r}\right) + \lambda^L\left(Y_1 + \frac{Y_2^L}{1+r} - C_1 - \frac{C_2^L}{1+r}\right)$$

- The FOCs are

$$u'(C_1) = \lambda^H + \lambda^L$$
$$\beta\pi u'(C_2^H) = \frac{\lambda^H}{1+r}$$
$$\beta(1-\pi)u'(C_2^L) = \frac{\lambda^L}{1+r}$$

Consumer's Problem

- Hence,

$$u'(C_1) = \beta(1+r)\Big[\pi u'(C_2^H) + (1-\pi)u'(C_2^L)\Big]$$

- That is, the stochastic Euler equation is

$$u'(C_1) = \beta(1+r)E_1(u'(C_2))$$

In General...

- Given Y_2 random, the optimization problem is:

$$\max_{\{C_1,C_2\}} u(C_1) + \beta E_1 u(C_2)$$

subject to

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

- Rewrite it as a unconstrained optimization

$$\max_{\{C_1\}} u(C_1) + \beta E_1 u\Big((1+r)Y_1 + Y_2 - (1+r)C_1\Big)$$

- Hence, by FOC we obtain:

$$u'(C_1) = \beta(1+r)E_1 u'(C_2)$$

Stochastic Euler Equation

- Take a 2nd order Taylor approximation on $u'(C_2)$ around $E_1(C_2)$:

$$u'(C_2) = u'(E_1(C_2)) + u''(E_1(C_2))(C_2 - E_1(C_2)) + \frac{u'''(E_1(C_2))}{2!}(C_2 - E_1(C_2))^2$$

- Take a conditional expectation:

$$E_1(u'(C_2)) = u'(E_1(C_2)) + \frac{u'''(E_1(C_2))}{2}Var_1(C_2)$$

- By $C_2 = Y_2 + (1+r)(Y_1 - C_1)$,

$$Var_1(C_2) = Var_1(Y_2)$$

Stochastic Euler Equation

- Hence, the stochastic Euler equation can be written as

$$u'(C_1) = \beta(1+r)\left[u'(E_1(C_2)) + \frac{u'''(E_1(C_2))}{2} Var_1(Y_2)\right]$$

Certainty Equivalent

Consider the following definition of certainty equivalent:

Definition (Certainty Equivalent)

$$Y_2^{CE} = E_1(Y_2)$$

- Under certainty, $Y_2^{CE} = E_1(Y_2)$, the Euler equation is

$$u'(C_1^{CE}) = \beta(1+r)u'(C_2^{CE})$$

and the budget constraint is

$$C_2^{CE} = Y_2^{CE} + (1+r)(Y_1 - C_1^{CE})$$

Certainty Equivalent

- We thus have

$$u'(C_1) = \beta(1+r)u'(E_1(C_2)) + \beta(1+r)\left[\frac{u'''(E_1(C_2))}{2}Var_1(Y_2)\right]$$
$$u'(C_1^{CE}) = \beta(1+r)u'(C_2^{CE})$$

- Hence,

$$u'(C_1) - u'(C_1^{CE}) - \beta(1+r)[u'(E_1(C_2)) - u'(C_2^{CE})]$$
$$= \beta(1+r)\left[\frac{u'''(E_1(C_2))}{2}Var_1(Y_2)\right]$$

Prudence

Definition (Prudence)

When marginal utility function $u'(\cdot)$ is convex, that is,

$$u'' < 0, \;\; u''' > 0$$

it is called prudence (Kimball 1990).

Theorem

Theorem

If $u'''(\cdot) > 0$, *then*

$$C_1 < C_1^{CE}$$

Theorem

Proof.

Suppose not, $C_1 \geq C_1^{CE}$,

$$\begin{aligned} E_1(C_2) &= E_1(Y_2) + (1+r)(Y_1 - C_1) \\ &\leq E_1(Y_2) + (1+r)(Y_1 - C_1^{CE}) \\ &= Y_2^{CE} + (1+r)(Y_1 - C_1^{CE}) = C_2^{CE} \end{aligned}$$

Since $C_1 \geq C_1^{CE}$ and $E_1(C_2) \leq C_2^{CE}$,

$$\begin{aligned} &\beta(1+r)\left[\frac{u'''(E_1(C_2))}{2} Var_1(Y_2)\right] \\ &= u'(C_1) - u'(C_1^{CE}) - \beta(1+r)[u'(E_1(C_2)) - u'(C_2^{CE})] \leq 0 \end{aligned}$$

a contradiction. $\square$

Precautionary Saving

- According to the previous theorem, under uncertainty about future income, and $u'''(\cdot) > 0$ (prudence), we have

$$C_1 < C_1^{CE}$$

where $C_1^{CE} - C_1$ is called precautionary saving.

- Moreover,

$$CA_1 = Y_1 - C_1 > Y_1 - C_1^{CE} = CA_1^{CE}$$

Section 8

Uncertainty and Investment

Production Economy

- Consider the following production function:

$$Y_t = A_t F(K_t)$$

where A_t is random.

- The capital accumulation equation is

$$K_{t+1} = I_t$$

where we assume $\delta = 1$.

- The resource constraint is

$$A_t F(K_t) + (1 + r) B_t = C_t + K_{t+1} + B_{t+1}$$

Production Economy

- The optimization problem is

$$\max_{\{C_1, C_2, K_2, B_2\}} u(C_1) + \beta E_1 u(C_2)$$

subject to

$$A_t F(K_t) + (1+r)B_t = C_t + K_{t+1} + B_{t+1}$$

- Given $B_1 = B_3 = K_3 = 0$, and $K_1 > 0$,

$$I_1 = K_2, \quad I_2 = K_3 = 0$$

$$A_1 F(K_1) = C_1 + K_2 + B_2$$

$$A_2 F(K_2) + (1+r)B_2 = C_2$$

Production Economy

- The constrained optimization problem can be rewritten as:

$$\max_{\{C_1, K_2\}} u(C_1) + \beta E_1[u(A_2 F(K_2) + (1+r)[A_1 F(K_1) - C_1 - K_2])]$$

- The FOCs are

$$u'(C_1) - \beta(1+r)E_1[u'(C_2)] = 0$$
$$\beta E_1[u'(C_2)(A_2 F'(K_2) - (1+r))] = 0$$

Production Economy

- Let's first focus on investment (note that K_2 is the chosen variable at $t = 1$):

$$F'(K_2)E_1[u'(C_2)A_2] = (1+r)E_1[u'(C_2)]$$

  - If A_2 and $u'(C_2)$ are independent,

$$F'(K_2)E_1[A_2] = 1+r$$

- Under certainty equivalent $A_2^{CE} = E_1[A_2]$, we can define K_2^{CE} as

$$F'(K_2^{CE})E_1[A_2] = 1+r$$

  - Hence, if A_2 and $u'(C_2)$ are independent,

$$K_2 = K_2^{CE}$$

Production Economy

- If A_2 and $u'(C_2)$ are NOT independent,

$$Cov_1(A_2, C_2) > 0 \;\Rightarrow\; Cov_1(A_2, u'(C_2)) < 0$$

then $E_1[u'(C_2)A_2] < E_1[u'(C_2)]E_1[A_2]$. and

$$\begin{aligned} F'(K_2)E_1[u'(C_2)]E_1[A_2] &> F'(K_2)E_1[u'(C_2)A_2] \\ &= (1+r)E_1[u'(C_2)] \end{aligned}$$

That is,

$$F'(K_2)E_1[A_2] > (1+r) = F'(K_2^{CE})E_1[A_2]$$

$$K_2 < K_2^{CE}$$

Uncertainty and Current Account

- By definition,

$$CA_1 = B_2 - B_1 = B_2 = A_1 F(K_1) - C_1 - K_2$$

- When there exists uncertainty,

$$C_1 < C_1^{CE}, \quad K_2 < K_2^{CE}$$

and

$$CA_1 > CA_1^{CE}$$