International Finance

Intertemporal Models of Current Account Dynamics

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Outline

1. Two-period Models of the Current Account
   - The Endowment Model
   - Model with Production and Capital Accumulation
2. An Infinite Horizon Intertemporal Current Account Model

Section 1

Two-period Models of the Current Account

Subsection 1

The Endowment Model

Model: Small Country, One Good, Two Periods

- Assumptions:
  - Open: can borrow freely at the world real interest rate.
  - Small: actions of domestic agents do not affect the world capital market. So the world interest rate is exogenous. We assume here it is fixed at r.
  - One good used for consumption C.
  - Endowment economy: Y
  - Riskless bond: B
  - Representative agent lives two periods and chooses consumption for each period.
  - Discount rate: $0 < \beta < 1$
  - No uncertainty: perfect foresight

Preference

- Time-separable life-time utility

$$U(C_1) + \beta U(C_2)$$

  - Time separability (additive separability over time) implies that marginal utility in any period does not depend on consumption in other periods.
  - Counter-example:

$$U(C_1) + \beta U(C_2 - C_1)$$

Optimization Problem

- Hence, the optimization problem is

$$\max_{C_1, C_2, B_2} U(C_1) + \beta U(C_2) \qquad \text{s.t. } Y_t + (1+r)B_t = C_t + B_{t+1}$$

where B_t is the foreign bond holding at the end of period $t-1$.

- Assume $B_1 = 0$, and by transversality condition: $B_3 = 0$,

not assumption

$$Y_1 = C_1 + B_2$$

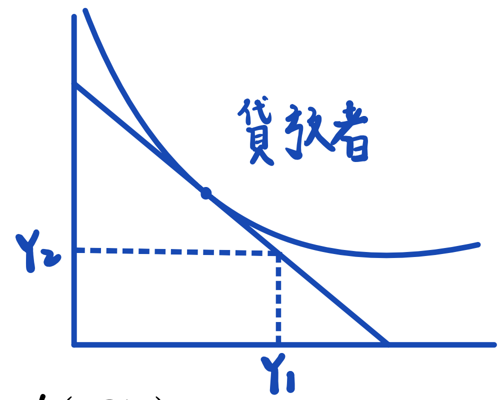
$$Y_2 + (1+r)B_2 = C_2$$

- The constrained optimization can be rewritten as unconstrained optimization

$Y_2 + (1+r)\underline{B_2}$ $Y_1 - C_1$

$$\max_{C_1} U(C_1) + \beta U(Y_2 + (1+r)(Y_1 - C_1))$$

Equilibrium



- F.O.C. (Euler equation)

$$U'(C_1) = (1+r)\beta U'(C_2)$$

- Equilibrium

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}, \qquad \frac{1}{1+\underline{r}} = \frac{\beta U'(C_2)}{U'(C_1)}$$

exogeneous, given

(if no international financing ⇒ endogenous r ⇒ $B_2 = 0$)

Current Account

- Recall that

$$CA_t \equiv \Delta NFA = B_{t+1} - B_t$$

- Economy starts with no net debt ($B_1 = 0$), so

$$CA_1 = B_2 - \underbrace{B_1}_{=0} = B_2 = Y_1 - C_1$$

- Moreover, economy ends up with no net debt ($B_3 = 0$),

$$CA_2 = \underbrace{B_3}_{=0} - B_2 = -B_2 = Y_2 + rB_2 - C_2$$

Clearly, $CA_1 = -CA_2$

- Graphical Analysis

Autarky

- r^A is the interest rate such that the economy does not want to borrow/lend in the global financial market:

$$C_1 = Y_1, \quad C_2 = Y_2, \qquad \frac{1}{1+r^A} = \frac{\beta U'(Y_2)}{U'(Y_1)}$$

⇒ one country does not want to lend / borrow

- It is called the autarky interest rate.
- Financial Openness and Welfare ↑

⇒

Special Case I: $\beta(1+r)=1$

$= \frac{1}{1+\rho}$ $=$

- Suppose that $\beta(1+r)=1$, we have

$$U'(C_2)=U'(C_1) \quad \Rightarrow \quad C_1=C_2$$

- Hence,

$\because C_1 = C_2$ consumption smoothing

$$C_1=C_2=\left(1+\frac{1}{1+r}\right)^{-1}\left(Y_1+\frac{Y_2}{1+r}\right)=\frac{1+r}{2+r}\left(Y_1+\frac{Y_2}{1+r}\right)$$

$$CA_1=Y_1-C_1=Y_1-\frac{1+r}{2+r}\left(Y_1+\frac{Y_2}{1+r}\right)=\frac{1}{2+r}(Y_1-Y_2)$$

Special Case I: $\beta(1+r)=1$

- Given

$$CA_1 = \frac{1}{2+r}(Y_1 - Y_2)$$

- Is it better to have a current-account deficit or surplus?
- From our simple model, the evolution of income in a country over time is an important factor in answering this question.
  - For a country that is growing fast ($Y_1 < Y_2$), the best strategy is to go into debt in the present. $CA_1 < 0$
  - For a country that has attended a high level of development and is growing at lesser rates ($Y_1 > Y_2$), the best strategy is to save so as to live from their return in the future. $CA_1 > 0$

Special Case II: Different Time Preference

- Now assume $Y_1 = Y_2 = Y$ but $\beta(1+r) \neq 1$.
- Suppose $\beta < \frac{1}{1+r}$ (less patient), then

$$\frac{U'(C_2)}{U'(C_1)} = \frac{1}{\beta(1+r)} > 1,$$

which suggests

$$U'(C_2) > U'(C_1) \quad \Rightarrow \quad C_1 > C_2$$

- It follows that

$$C_1 + \frac{C_1}{1+r} > C_1 + \frac{C_2}{1+r} = Y + \frac{Y}{1+r} \quad \Rightarrow \quad C_1 > Y$$

- Hence,

$$CA_1 = Y - C_1 < 0$$

Summary

Why current surplus?

- When $Y_1 > Y_2$: future output is expected to be less than current output.
- When more weight is put on future utility (more patient).

Evidence from the US

- "Expected output growth" hypothesis.
- Hoffmann, Krause, and Laubach (2019), The Expectations-driven US Current Account, *The Economic Journal*, 129:618, Pages 897–924.
    - In line with the intertemporal approach to the current account, a major part of the build-up and subsequent reversal of the US current account deficit appears to be consistent with an optimal response of households and firms to changing growth prospects.

International Evidence from the 70 Countries

- "Less patience" hypothesis.
- Nieminen (2022), Cross-country variation in patience, persistent current account imbalances and the external wealth of nations, *Journal of International Money and Finance*, Volume 121, 102517.
  - Countries inhabited by patient individuals run current account surpluses.
  - There is a positive relationship between patience and net foreign asset positions.

Government Spending

- Consumer's problem

$$\max_{C_1, C_2} U(C_1) + \beta U(C_2)$$

s.t.

$$C_1 + \frac{C_2}{1+r} = (Y_1 - T_1) + \frac{Y_2 - T_2}{1+r}$$

- Assume that taxes are funding government operations: $G_t = T_t$. The life-time budget constraint becomes

$$C_1 + \frac{C_2}{1+r} = (Y_1 - G_1) + \frac{Y_2 - G_2}{1+r}$$

T_t is absent in the optimization problem. The timing of taxes does not affect anything $\Rightarrow$ Ricardian Equivalence

Government Spending

- Hence, the optimal problem is now

$$\max_{C_1, C_2} U(C_1) + \beta U(C_2)$$

s.t.

$$C_1 + \frac{C_2}{1+r} = (Y_1 - G_1) + \frac{Y_2 - G_2}{1+r}$$

  - An increase in G_t acts like a decline in the endowment.
  - Can examine impact of changes in government spending.

$CA_t \downarrow$

Subsection 2

Model with Production and Capital Accumulation

Model with Production and Capital Accumulation

- Household produces output using capital:

$$Y_t = A_t F(K_t), \quad F' > 0, \quad F'' < 0$$

diminishing marginal product of k

- Capital accumulation

$$K_{t+1} - K_t = I_t - \delta K_t$$

$I_t = K_{t+1} - (1-\delta)K_t$

- Household budget constraint:

$A_t F(K_t)$

$$\underline{Y_t} + (1+r)B_t = C_t + K_{t+1} - (1-\delta)K_t + B_{t+1}$$

$$\begin{cases} \cancel{(1+r)B_1} + A_1 F(K_1) = C_1 + K_2 - (1-\delta)K_1 + B_2 \\ (1+r)B_2 + A_2 F(K_2) = C_2 + \cancel{K_3} - (1-\delta)K_1 + \cancel{B_3} \end{cases}$$

Two-period Model

- Assume
  - $B_1 = 0$
  - K_1 exogenous
- Transversality condition: $B_3 = K_3 = 0$
- Investment then is given by:

$$I_1 = K_2 - (1-\delta)K_1, \qquad I_2 = -(1-\delta)K_2$$

no K_3

- Household life-time budget constraint is:

I_1

$$\begin{aligned} C_2 &= A_2F(K_2) + (1-\delta)K_2 \\ &\quad + (1+r)\underbrace{\left(A_1F(K_1) - C_1 - [K_2 - (1-\delta)K_1]\right)}_{B_2} \end{aligned}$$

Household optimization

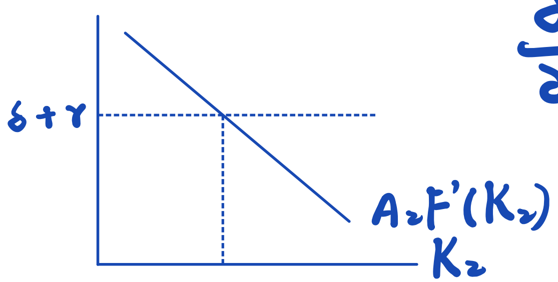
- The maximization problem is

C_1, K_2

$$\max_{C_1,K_2} U(C_1) + \beta U\Big(A_2F(K_2) + (1-\delta)K_2$$
$$+ (1+r)(A_1F(K_1) - C_1 - [K_2 - (1-\delta)K_1])\Big)$$

- F.O.C.

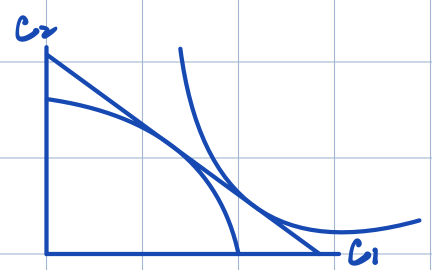
$$\frac{\partial U}{\partial C_1} = U'(C_1) = \beta U'(C_2)(1+r)$$



$$\frac{\partial U}{\partial K_2} = A_2F'(K_2) = \delta + r$$ independent to consumption

- Graphical Analysis: PPF, Autarky, Financial Openness, and Fisher's Separation Theorem.

$$(1-\delta)+A_2F'(K_2)=1+r$$



Autarky PPF. $\begin{cases} A_1F(K_1)=C_1+K_2-(1-\delta)K_1 \\ A_2F(K_2)=C_2-(1-\delta)K_2 \end{cases}$

$$A_2F[A_1F(K_1)-C_1+(1-\delta)K_1]$$
$$=C_2-(1-\delta)[A_1F(K_1)-C_1+(1-\delta)K_1]$$

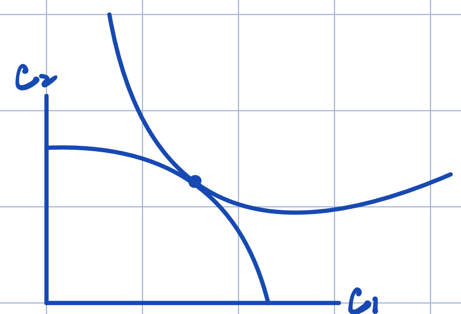
$$\Rightarrow C_2=A_2F[A_1F(K_1)-C_1+(1-\delta)K_1]+(1-\delta)[A_1F(K_1)-C_1+(1-\delta)K_1]$$

$$\frac{dC_2}{dC_1}=-A_2F'(K_2)-(1-\delta)<0,\quad \frac{d^2C_2}{dC_1}=A_2F''(K_2)<0$$

$$\max_{C_1}\ U(C_1)+\beta U[C_2(C_1)]$$

$$U'(C_1)=\beta U'(C_2)[A_2F'(K_2)+(1-\delta)]$$

$$\Rightarrow \frac{U'(C_1)}{\beta U'(C_2)}=A_2F'(K_2)+(1-\delta)$$



$$C_1=C_2=[A_2F(K_2)+(1-\delta)K_2]$$
$$+(1+r)(A_1F(K_1)-C_1-[K_2-(1-\delta)K_1])$$

$$\Rightarrow(2+r)C_1=[A_2F(K_2)+(1-\delta)K_2]$$
$$+(1+r)(A_1F(K_1)-[K_2-(1-\delta)K_1])$$

Equilibrium

- Assume $\beta(1+r)=1$,

$$C_1 = C_2 = \frac{1}{2+r}\Big(A_2F(K_2) + (1-\delta)K_2 + (1+r)(A_1F(K_1) - [K_2 - (1-\delta)K_1])\Big)$$

- In the form of permanent income,

$$C_1 = C_2 = \frac{1+r}{2+r}\Big(A_1F(K_1) - [K_2 - (1-\delta)K_1] + \frac{1}{1+r}[A_2F(K_2) + (1-\delta)K_2]\Big)$$

Equilibrium

- Current Account

$$CA_1 = B_2 - B_1 = B_2 = A_1 F(K_1) - C_1 - I_1$$

- Let's look at temporary, and anticipated productivity shocks first

Temporary Shock

current shock

- Consider $dA_1 > 0$ and $dA_2 = 0$

$$\frac{dCA_1}{dA_1} = F(K_1) - \frac{dC_1}{dA_1} - \underbrace{\frac{dI_1}{dA_1}}_{I_1 \text{ unrelated to } A_1} = F(K_1) - \frac{dC_1}{dA_1}$$

where

$$\frac{dC_1}{dA_1} = \frac{1+r}{2+r} F(K_1)$$

- Hence,

$$\frac{dCA_1}{dA_1} = F(K_1) - \frac{1+r}{2+r} F(K_1) = \frac{1}{2+r} F(K_1) > 0$$

- In this case, current consumption (C_1) will rise less than current output, which implies an increase in saving (consumption smoothing). $\uparrow F(K_1)$

Expected Future Shock

$I_1 = K_2 - (1-\delta)K_1$

affected by A_2

- Consider $dA_1 = 0$ and $dA_2 > 0$

$$\frac{dCA_1}{dA_2} = -\frac{dC_1}{dA_2} - \frac{dI_1}{dA_2} = -\underset{+}{\frac{dC_1}{dA_2}} - \underset{+}{\frac{dK_2}{dA_2}}$$

where

$A_2F'(K_2) = \delta + r$

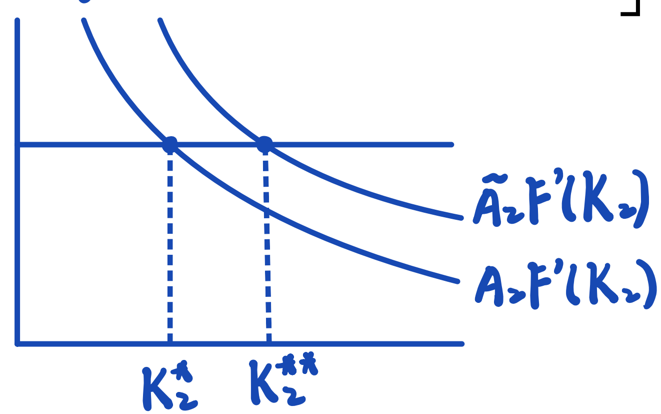
$\Rightarrow F'(K_2) + A_2F''(K_2)\frac{dK_2}{dA_2} = 0$

$$\frac{dK_2}{dA_2} = \frac{-F'(K_2)}{A_2F''(K_2)} > 0$$

product rule

$$\frac{dC_1}{dA_2} = \frac{1}{2+r}\left[F(K_2) + \underbrace{A_2F'(K_2)}_{\delta+r}\frac{dK_2}{dA_2} + (1-\delta)\frac{dK_2}{dA_2} - (1+r)\frac{dK_2}{dA_2}\right]$$

$$= \frac{1}{2+r}F(K_2) > 0$$



- Hence,

$$\frac{dCA_1}{dA_2} < 0$$

Expected Future Shock

- That is, conditional on expected future shocks,

$$\frac{dK_2}{dA_2} > 0, \quad \frac{dCA_1}{dA_2} < 0$$

- That is,

$$Cov(CA_1, K_2 | A_2) < 0$$

Interesting Articles

- 沈富雄, “本世紀以來, 順差不再是喜訊!”
  - See `http://0rz.tw/k7JWS`
- 沈富雄, “低油價的美麗與哀愁”
  - See `http://0rz.tw/cYiwj`
- When observing $Cov(CA_1, K_2) < 0$, is it

$$CA_1 \uparrow \Rightarrow K_2 \downarrow? \quad \text{or} \quad K_2 \uparrow \Rightarrow CA_1 \downarrow?$$

(1) Correlation does not necessary imply causality

(2) Macroeconomists think using conditional views

RBC Models

- The real business cycle literature extends the type of model we have been looking at. Its goal is to build comprehensive models that can account for both long-run and business-cycle frequency behavior.
- RBC models introduce stochastic shocks into a neoclassical growth model.
  - Brock and Mirman (1972)
  - Kydland and Prescott (1982)
- They extend our framework by, first, going from two periods to an infinite horizon. Households have an infinite lifetime. The models explicitly introduce random shocks, usually to productivity.

to match empirics

Pioneers of the RBC School

Section 2

An Infinite Horizon Intertemporal Current Account Model

An Infinite-horizon Model

- Infinite-horizon
- No Uncertainty (perfect foresight)
- Small Open Economy (exogenous world interest rates)
- What do we gain from looking at infinite-horizon models relative to two-period models?
  - We need to move beyond two periods if we want to take our models to data and test them.

An Infinite-horizon Model

- The infinite-horizon model we look at assumes forever-lived households have infinite horizons in making their plans.
- Huseholds at time t choose C_s for all s (that is, they choose a path of consumption, $\{C_t, C_{t+1}, \ldots\}$) to maximize:

$$\sum_{s=t}^{\infty} \beta^{s-t} U(C_s) = U(C_t) + \beta U(C_{t+1}) + \beta^2 U(C_{t+2}) + \cdots ,$$

discount factor / survival probability

where $\beta \in (0,1)$ and $U(\cdot)$ is strictly increasing and concave.

  - We could have models with no terminal date (i.e., infinite horizons for the economy), but in which all households have finite horizons. Overlapping Generations (OLG) models.

t t+1

two-period extended

t t+1

Comments on Intertemporal Utility

Given

$$\sum_{s=t}^{\infty} \beta^{s-t} U(C_s), 0 < \beta < 1.$$

- Time impatience (time discounting)
    - guarantees the infinite sum converges
    - β can also be viewed as a probability of survival
- Additive separability (against to habit formations)
    - Could be justified by the length of period in Macroeconomics (monthly, quarterly, yearly)

quarterly habit formation ? doubtable

⇒ justify additive separability

consumption data frequency

Budget Constraint

endowment model, no production side

- The period-by-period budget constraint:

$$Y_t + (1 + r_t)B_t = C_t + B_{t+1} \quad \forall t$$

$$C_t = Y_t + (1+r_t)B_t - B_{t+1}$$

where B_t is the foreign bond holding at the end of period $t - 1$.

- Note that

$$CA_t = \Delta NFA = B_{t+1} - B_t = Y_t + r_t B_t - C_t$$

$$\Rightarrow \max_{b_t \, \forall t} u(Y_t + (1+r)B_t - B_{t+1}) + \beta u(Y_{t+1} + (1+r)B_{t+1} - B_{t+2})$$
$$+ \beta^2 u(Y_{t+2} + (1+r)B_{t+2} - B_{t+3}) + \beta^3 u(Y_{t+3} + (1+r)B_{t+3} - B_{t+4}) + \cdots$$

$$\Rightarrow u'(C_1) = \beta u'(C_2)(1+r) \text{ subject to } C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

Budget Constraint

$$\left\{\begin{array}{l} Y_1 + (1+r)\underset{\text{non-zero}}{\underline{B_1}} = C_1 + B_2 \\ Y_2 + (1+r)B_2 = C_2 + B_3 \\ Y_3 + (1+r)B_3 = C_3 + \underbrace{\cancel{B_4}}_{=0} \end{array}\right.$$

$$\Rightarrow Y_1 + \frac{Y_2}{1+r} + (1+r)B_1 = C_1 + \frac{C_2}{1+r} + \frac{B_3}{1+r}$$

$$\Rightarrow Y_1 + \frac{Y_2}{1+r} + \frac{Y_3}{(1+r)^2} + (1+r)B_1 = C_1 + \frac{C_2}{1+r} + \frac{C_3}{(1+r)^2}$$

- Note we are taking the world interest rate as given and constant over time: $r_t = r$.
- If we iterate forward on this equation, we arrive at:

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s \underbrace{- \lim_{T\to\infty}\left(\frac{1}{1+r}\right)^{T} B_{t+T+1}}_{=0}$$

No-Ponzi Scheme Condition

- We impose the No-Ponzi scheme condition:

$$\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T B_{t+T+1} \geq 0$$

  - It is imposed exogenously on the model.
  - We have to impose this exogenously in a small-country model, because we are not modeling the behavior of the rest of the world, but we know they would not allow themselves to become victims of Ponzi schemes. required: $\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T B_{t+T+1} \geq 0$
  - However, it does NOT require B_{t+T+1} to be positive. We simply rule out the possibility of having debt ($B_t < 0$) that grows faster than the rate of interest.

No-Ponzi Scheme Condition

- To see why B_{t+T+1} can be negative while the debt does not grow faster than the interest rate, for instance,

$$B_{t+T+1} = B_t(1+g_B)^{T+1}$$

Hence

$$\lim_{T\to\infty} \frac{B_{t+T+1}}{(1+r)^T} = \lim_{T\to\infty} B_t(1+g_B)\left(\frac{1+g_B}{1+r}\right)^T = 0$$

- Even if $B_t < 0$ (and thus $B_{t+T+1} < 0$), this term goes to zero as $T \to \infty$ when $g_B < r$.
  - An alternative condition (which is more strict) would be

$$\lim_{T\to\infty} B_{t+T+1} \geq 0 \quad \text{institutional rule}$$

不可以債養債

Transversality Condition

- But the optimization rules out

$$\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^{T}B_{t+T+1} > 0$$

  Why? Rule out overaccumulation of wealth.

- So, we conclude

$$\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^{T}B_{t+T+1} = 0$$

  It is in general called a Transversality Condition (TVC).

Lifetime Budget Constraint

- Giving the TVC, the lifetime budget constraint is written as:

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s$$

  - In this model, we need to impose $r > g$, where g is the growth rate of output. If we did not impose that, then the discounted sum of current and future output would be infinite. Agents would have infinite wealth, and there would be no interesting economic problem.

Optimization

- The optimization problem is

$$\max_{\{C_s, B_{s+1}\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) = u(C_t) + \beta u(C_{t+1}) + \beta^2 u(C_{t+2}) + \cdots$$

subject to

$$Y_t + (1+r)B_t = C_t + B_{t+1}$$

- Rewrite as

$$\max_{\{B_{s+1}\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \beta^{s-t} u(Y_s + (1+r)B_s - B_{s+1})$$

Optimality Conditions

- Euler Equation

$$U'(C_s) = (1+r)\beta U'(C_{s+1}), \quad s = t, t+1, t+2, \ldots$$

- Budget Constraint

$$B_{t+1} - B_t = Y_t + r_t B_t - C_t$$

- TVC

$$\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T B_{t+T+1} = 0$$

Solution

- Assume that $\beta(1+r) = 1$, $C_t = C_{t+1} = C_{t+2} = \cdots$,

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s = \frac{1+r}{r} C_t$$

- Plug into lifetime budget constraint

$$\frac{1+r}{r} C_t = (1+r) B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s$$

- Hence,

$$C_t = r B_t + \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s$$

A Particular Solution

- Let's consider the special case that $Y_{s+1} = (1+g)Y_s$,

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s = \frac{1+r}{r-g} Y_t$$

- Hence,

$$C_t = rB_t + \frac{r}{r-g} Y_t$$

Current Account

- The current account for this economy is given by:

$$
\begin{aligned}
CA_t &= Y_t + rB_t - C_t \\
&= Y_t + rB_t - \left(rB_t + \frac{r}{r-g} Y_t \right) \\
&= \frac{-g}{r-g} Y_t
\end{aligned}
$$

Current Account: Some False Claims

- False Claim 1:
  - The current account must at some point be positive to offset current account deficits.
  - The current account must go to zero in the long run.
- Those claims are refuted in this model since the current account is always negative given $r > g$.

$$CA_t = Y_t + rB_t - C_t = \frac{-g}{r-g} Y_t$$

Current Account: Some False Claims

- False Claim 2:
  - Even if the current account is always in deficit, it must approach a constant in the long run.
- That is seen to be false here too. The current account deficit is proportional to output. Output is always growing, so the current account deficit will always grow.

$$CA_t = Y_t + rB_t - C_t = \frac{-g}{r-g} Y_t$$

- Why running CA deficit?
  - expect Y grows $(g > 0)$
  - but Y grows slower than its cost of debt: $r > g$.

Current Account: Some False Claims

- False Claim 3:
  - Debt position has to converge to a constant, and thus Debt/GDP ratio has to converge to zero (since GDP grows).
- To see that these are false, define $\gamma_t = \frac{B_t}{Y_t}$,

$$CA_t = B_{t+1} - B_t = \frac{-g}{r-g} Y_t$$

Hence, we can obtain a stable difference equation:

$$\gamma_{t+1} = \frac{1}{1+g}\gamma_t - \frac{g}{(1+g)(r-g)}$$

Current Account: Some False Claims

- Since the equation is stable, we can see that γ_t converges to $\bar{\gamma}$ defined by:

$$\bar{\gamma} = \frac{1}{1+g}\bar{\gamma} - \frac{g}{(1+g)(r-g)},$$

  and solve for $\bar{\gamma}$:

$$\bar{\gamma} = \frac{-1}{r-g}$$

- Since $r > g$, we must have $\bar{\gamma} < 0$
  - A non-zero steady-state debt/GDP ratio

Back to the General Solution

- Recall that

$$C_t = rB_t + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s$$

$$CA_t = Y_t + rB_t - C_t = Y_t - \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s$$

Model vs. Data

- Bouakez and Kano (2009) Tests of the present-value model of the current account: a note, Applied Economics Letters, 16:12, 1215-1219