Questions 1 and 2: SAT testing

The SAT is a standardized college admissions test used in the United States. The following two multi-part questions will ask you some questions about SAT testing.

Report 3 significant digits:

```
options(digits = 3)
```

This is a 6-part question asking you to determine some probabilities of what happens when a student guessed for all of their answers on the SAT. Use the information below to inform your answers for the following questions.

An old version of the SAT college entrance exam had a -0.25 point penalty for every incorrect answer and awarded 1 point for a correct answer. The quantitative test consisted of 44 multiple-choice questions each with 5 answer choices. Suppose a student chooses answers by guessing for all questions on the test.

Question 1a

1.0/1.0 point (graded)

What is the probability of guessing correctly for one question?

0.2 ✔ **Answer:** 0.2

0.2

Explanation

The following code can be used to calculate the probability:

```
p <- 1/5 # one correct choice of 5 options
p
```

Submit You have used 1 of 10 attempts

Answers are displayed within the problem

Question 1b

1.0/1.0 point (graded)
What is the expected value of points for guessing on one question?

0 ✔ **Answer:** 0

0

Explanation
The following code can be used to calculate the expected value:

```
a <- 1
b <- -0.25
mu <- a*p + b*(1-p)
mu
```

Submit You have used 1 of 10 attempts

Answers are displayed within the problem

Question 1c

1.0/1.0 point (graded)
What is the expected score of guessing on all 44 questions?

0 ✔ **Answer:** 0

0

Explanation
The following code can be used to calculate the expected score:

```
n <- 44
n*mu
```

Submit You have used 1 of 10 attempts

Answers are displayed within the problem

Question 1d

1.0/1.0 point (graded)
What is the standard error of guessing on all 44 questions?

3.32 ✔ **Answer:** 3.32

3.32

Explanation
The following code can be used to calculate the standard error:

```
sigma <- sqrt(n) * abs(b-a) * sqrt(p*(1-p))
sigma
```

Submit You have used 1 of 10 attempts

Answers are displayed within the problem

Question 1e

1.0/1.0 point (graded)
Use the Central Limit Theorem to determine the probability that a guessing student scores 8 points or higher on the test.

0.00798 ✔ **Answer:** 0.00793

0.00798

Explanation
The following code can be used to calculate the probability:

```
1-pnorm(8, mu, sigma)
```

Submit You have used 2 of 10 attempts

Answers are displayed within the problem

Question 1f

1.0/1.0 point (graded)

Set the seed to 21, then run a Monte Carlo simulation of 10,000 students guessing on the test.

(IMPORTANT! If you use R 3.6 or later, you will need to use the command `set.seed(x, sample.kind = "Rounding")` instead of `set.seed(x)`. Your R version will be printed at the top of the Console window when you start RStudio.)

What is the probability that a guessing student scores 8 points or higher?

0.0075 ✔ **Answer:** 0.008

0.0075

Explanation
The following code can be used to calculate the probability:

```
set.seed(21, sample.kind = "Rounding")
B <- 10000
n <- 44
p <- 0.2
tests <- replicate(B, {
  X <- sample(c(1, -0.25), n, replace = TRUE, prob = c(p, 1-p))
  sum(X)
})
mean(tests >= 8)
```

Submit You have used 2 of 10 attempts

Answers are displayed within the problem

The SAT was recently changed to reduce the number of multiple choice options from 5 to 4 and also to eliminate the penalty for guessing.

In this three-part question, you'll explore how that affected the expected values for the test.

Question 2a

1.0/1.0 point (graded)
Suppose that the number of multiple choice options is 4 and that there is no penalty for guessing - that is, an incorrect question gives a score of 0.

What is the expected value of the score when guessing on this new test?

11 ✔ **Answer:** 11

11

Explanation
The following code can be used to calculate the expected value:

```
p <- 1/4
a <- 1
b <- 0
n <- 44
mu <- n * a*p + b*(1-p)
mu
```

Submit You have used 3 of 10 attempts

Answers are displayed within the problem

Question 2b

0.0/1.0 point (graded)
What is the probability of scoring over 30 when guessing?
Report your answer using scientific notation with 3 significant digits in the format `x.xx*10^y`.

1.79*10^-11 ✖ **Answer:** 1.86*10^-11

$1.79 \cdot 10^{-11}$

Explanation
The following code can be used to calculate the probability:

```
sigma <- sqrt(n) * abs(b-a) * sqrt(p*(1-p))
1-pnorm(30, mu, sigma)
```

Submit You have used 10 of 10 attempts

Answers are displayed within the problem

Question 2c

1.0/1.0 point (graded)
Consider a range of correct answer probabilities `p <- seq(0.25, 0.95, 0.05)` representing a range of student skills.

What is the lowest `p` such that the probability of scoring over 35 exceeds 80%?

0.85

✔ **Answer:** 0.85

0.85

Explanation

The following code can be used to calculate the value for `p`:

```
p <- seq(0.25, 0.95, 0.05)
exp_val <- sapply(p, function(x){
  mu <- n * a*x + b*(1-x)
  sigma <- sqrt(n) * abs(b-a) * sqrt(x*(1-x))
  1-pnorm(35, mu, sigma)
})

min(p[which(exp_val > 0.8)])
```

Submit

You have used 1 of 10 attempts

Answers are displayed within the problem