Assessment: Finding a missing airplane
Background

Bayesian search theory is the use of Bayes' Theorem to find lost objects. It has been used to find planes crashed at sea, sunken submarines, missing people and more. Prior assumptions about the probability of finding the object in various locations are continually revised as the search proceeds.

A search team is tasked with finding a crashed plane. Their initial data suggest the plane is found somewhere within one of four areas with the following probabilities:

Area	A	B	C	D
Pr(plane in area)	0.2	0.6	0.15	0.05

The team can search one area per day and will always search the area with the highest probability. When the team searches an area, there is a 90% chance that they find a plane if one is present and a 10% chance that they overlook the plane. (There is always a 100% chance of not finding a plane if one is not present in the area.)

Use Bayes' Theorem to determine where the team should search on the second day if necessary and the probability of finding the plane within 2 days.

Equation summary

$$\text{Pr(plane in A)} = 0.2$$

$$\text{Pr(plane in B)} = 0.6$$

$$\text{Pr(plane in C)} = 0.15$$

$$\text{Pr(plane in D)} = 0.05$$

$$\text{Pr(plane not found in area} \mid \text{plane in area)} = 0.1$$

$$\text{Pr(plane not found in area | plane in different area)} = 1$$

Pre-exercise code

Report your answers to 3 significant digits:

```
options(digits = 3)
```

Questions?

Ask any questions about this assessment in the discussion forum at the bottom of this page.

Question 1

3/3 points (graded)
On day 1, the team will search area B. The probability that the plane is in area B is $\text{Pr(plane in B)} = 0.6$.

What is the probability the plane is not in B?

0.4

✔ **Answer:** 0.4

0.4

Explanation
You can calculate the probability the plane is not in B with the following code:

```
pr_B <- 0.6
1 - pr_B
```

What is the probability the plane is in B but is not found?

0.06

✔ **Answer:** 0.06

0.06

Explanation
You can calculate the probability the plane is in B but not found with the following code:

```
pr_missed <- 0.1
pr_B * pr_missed
```

What is $\text{Pr(plane not found in B)}$, the probability the plane is not found in B on day 1?

0.46

✔ **Answer:** 0.46

0.46

Explanation

The probability the plane is not found in B on day 1 is the sum of the previous two probabilities:

```
1 - pr_B + pr_B*pr_missed
```

Submit You have used 2 of 5 attempts

ℹ Answers are displayed within the problem

Question 2

1/1 point (graded)

Suppose the plane will not be found on day 1. What is the equation for $\Pr(\text{plane in B} \mid \text{plane not found in B})$, the posterior probability that the plane is in area B given that it is not found in area B on day 1?

Select the correct equation for $\Pr(\text{plane in B} \mid \text{plane not found in B})$:

○ $\dfrac{\Pr(\text{plane not found in B}) \times \Pr(\text{plane in B})}{\Pr(\text{plane not found in B} \mid \text{plane in B})}$

● $\dfrac{\Pr(\text{plane not found in B} \mid \text{plane in B}) \times \Pr(\text{plane in B})}{\Pr(\text{plane not found in B})}$ ✔

○ $\dfrac{\Pr(\text{plane not found in B} \mid \text{plane in B}) \times \Pr(\text{plane not found in A})}{\Pr(\text{plane not in A})}$

○ $\dfrac{\Pr(\text{plane not found in B} \mid \text{plane in B}) \times \Pr(\text{plane not found in B})}{\Pr(\text{plane in B})}$

Submit You have used 1 of 2 attempts

ℹ Answers are displayed within the problem

Question 3

3/3 points (graded)

Use Bayes' Theorem to calculate the posterior probabilities of finding the plane in each of the 4 grid locations. Recall that area B will be searched on day 1.

What is the posterior probability that the plane is in area B given that it is not found on day 1?

| 0.13 | ✔ **Answer:** 0.130 |

0.13

Explanation

You can calculate the posterior probability the plane is in B with the following code:

```
pr_B <- 0.6
        pr_missed <- 0.1    # prob plane in B but not found
        pr_not_found <-  1 - pr_B + pr_B*pr_missed
        pr_missed * pr_B / pr_not_found
```

What is the posterior probability that the plane is in area C given that it is not found on day 1?

| 0.326 | ✔ **Answer:** 0.326 |

0.326

Explanation

You can calculate the posterior probability the plane is in C with the following code:

```
pr_C <- 0.15
        pr_missed_not_present <- 1
        pr_C * pr_missed_not_present / pr_not_found
```

Which area has the highest posterior probability and should be searched on day 2?

- ⦿ A ✔

- ○ B

- ○ C

- ○ D

Explanation

Calculate the posterior probabilities for A and D, then compare the 4 posterior probabilities from this problem and select the highest probability region:

```
            pr_A <- 0.2
            pr_D <- 0.05

            pr_A_post <- pr_A * pr_missed_not_present / pr_not_found
            pr_B_post <-  pr_missed * pr_B / pr_not_found
            pr_C_post <- pr_C * pr_missed_not_present / pr_not_found
            pr_D_post <- pr_D * pr_missed_not_present / pr_not_found

            results <- c(pr_A_post, pr_B_post, pr_C_post, pr_D_post)
            names(results) <- c("A", "B", "C", "D")
            names(which.max(results))
```

Submit | You have used 2 of 5 attempts

ℹ Answers are displayed within the problem

Question 4

3/3 points (graded)
Before the search begins, you have been asked to report the probability that you find the plane within two days.

What is the probability of finding the plane on the first day?

| 0.54 | ✔ **Answer:** 0.54 |

0.54

Explanation
This is the probability of the plane being in B times the probability of a successful search:

```
pr_B <- 0.6
        pr_missed <- 0.1    # prob of failed search
        pr_B * (1 - pr_missed)
```

What is the probability that the plane is not found on the first day but is found on the second day?
On day 2, you will search the region that had the highest posterior probability in question 3.

| 0.39 | ✔ **Answer:** 0.391 |

0.39

Explanation
This is the posterior probability that the plane is in A given that it was not found on day 1, times the probability of a successful search.

```
# use maximum posterior probability from previous question: region A
        pr_A <- 0.2
        pr_missed <- 0.1     # prob plane in B but not found
        pr_not_found <-  1 - pr_B + pr_B*pr_missed
        pr_missed_not_present <- 1
        pr_A_post <- pr_A * pr_missed_not_present / pr_not_found

        # multiply by chance plane is missed on day 2
        pr_A_post*(1-pr_missed)
```

What is the probability that the plane is found within 2 days?

| 0.931 |

✔ **Answer:** 0.931

| 0.931 |

Explanation

This is the sum of the probability of finding the plane on day 1 and the probability of not finding the plane on day 1 but finding it on day 2.

```
pr_B*(1-pr_missed) + pr_A_post*(1-pr_missed)
```

| Submit | You have used 1 of 5 attempts

ⓘ   Answers are displayed within the problem