Development of a Cost-Effective 1kN Liquid-Fueled Rocket Propulsion System

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1 Pipe Dimension Calculation

The following section describes the theoretical process used to dimensionalize the preliminiary plumbing framework in preparation for the first static cold flow test. By definition, these calculations are purely speculatory and are only used for the initial design process. The purpose of the cold flow test is to verify these parameters, and consequently adjust these parameters to better fit the system requirements.

1.1 Assumptions

To simplify the rigorous analysis and optimization processes often associated with viscous pipe flow, a couple of assumptions are applied in the following section to both shorten the development timeline and to avoid unecessarily complex or expensive methods outside of the scope of a high school amateur rocketry program. They are as follows:

1. Flow is driven by both pressure and gravity.
2. Pipe is circular and is of constant cross-sectional area.
3. No swirl, circumferential variation, or entrance effects.
4. No shaft-work or heat-transfer effects.
5. A simplified steady-flow energy equation due to Assumption 4.
6. Flow is fully developed (minimal boundary-layer effects).

1.2 Iterative Diameter Calculation

Given the simplifying assumptions mentioned in Section 1.1, we can begin by introducing the dimensionless Darcy friction factor f as a baseline relationship between roughness and pipe resistance:

$$\frac{8\tau_w}{\rho V^2} = f = F(Re_d, \frac{\epsilon}{d}) \tag{1.1}$$

where τ_w depicts the wall shear stress, ρ is density, V is volume, and F represents a later-defined function between the Reynold's Number Re_d and the average pipe roughness to diameter ratio $\frac{\epsilon}{d}$ (also known as relative roughness).

Using Equation 1.1 and the wall shear stress equation (see References), we obtain the desired expression for finding pipe-head loss, also known as the Darcy-Weisbach equation. It is valid for duct flows of any cross section and any Reynold's Number

$$h_f = f\frac{L}{d}\frac{V^2}{2g} \tag{1.2}$$

where h_f represents the head loss factor as a function of the dimensionless Darcy friction factor f, L is the length of the pipe, and g is equal to one standard Earth gravity.

1.3 Units

All units are implied with accordance to the Metric system (seconds, kilograms, Pascals, etc.), but are defined explicity below for ease-of-use: