Development of a Cost-Effective 1.5kN Liquid-Fueled Rocket Propulsion System

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1 Nomenclature

Symbols				**Acronyms**		
ϵ	=	Expansion ratio		*CEA*	=	Chemical equilibrium with applications
γ	=	Ratio of specific heats				
ρ	=	Density	g/cm^3	*COTS*	=	Commercial off-the-shelf
C^*	=	Characteristic velocity	m/s	*DAQ*	=	Data acquisition & control
C_d	=	Discharge coefficient		*FOD*	=	Foreign object debris
C_v	=	Valve flow coefficient		*GLOW*	=	Gross lift-off weight
f_d	=	Friction factor		*MECO*	=	Main engine cut-off
F_t	=	Thrust	N	*P&ID*	=	Plumbing and instrumentation diagram
g_0	=	Acceleration due to gravity	m/s^2			
I_{sp}	=	Specific impulse	s	*PT*	=	Pressure transducer
$\dot{m}$	=	Mass flow rate	kg/s	*TC*	=	Thermocouple
O/F	=	Oxidizer-to-fuel ratio		*SF*	=	Safety factor
Q	=	Volumetric flow rate	L/s	*VDC*	=	Direct current voltage

Note: Subscripts follow the convention outlined in [2]. Unless otherwise specified, subscript 0 indicates at stagnation or impact conditions, 1 indicates conditions at the nozzle inlet or combustion chamber, t indicates the nozzle throat, 2 is at the nozzle exit, and 3 is at ambient conditions.

2 Initial System Characterization

The following section outlines the characterization and design process for Aphlex 1B, our second-generation bi-propellant liquid rocket engine, set to fly on our 30 kg launch vehicle, Callisto 1. Project Caelus's unique position as a 501(c)(3) non-profit organization consisting entirely of high school students has laid the foundation for a design approach fully commited to cost-effectiveness, simplicity, and reliability, given certain mission constraints and objectives as discussed below.

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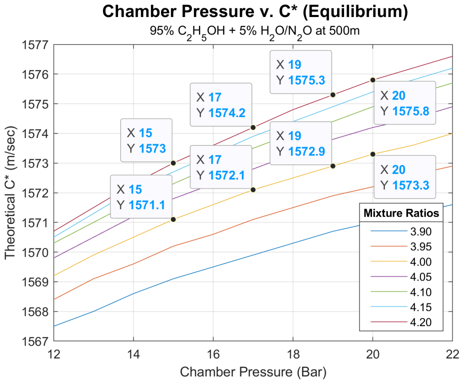

Figure 1: Theoretical C* efficiency vs chamber pressure and numerous mixture ratios.

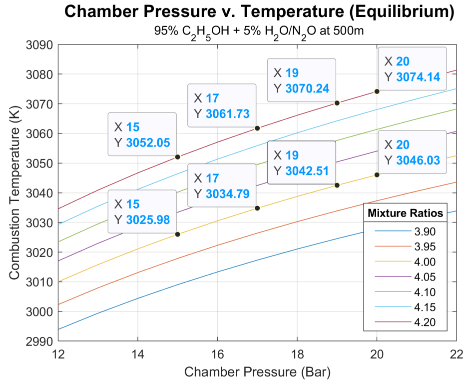

Figure 2: Combustion temperature vs chamber pressure and numerous mixture ratios.

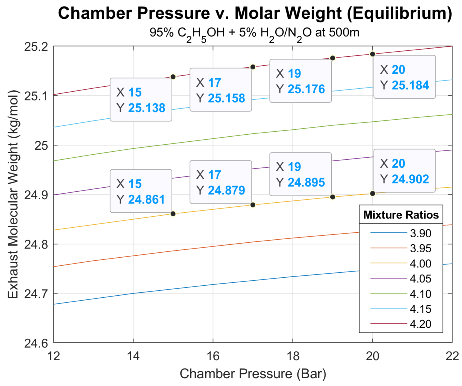

Figure 3: Gas molecular mass vs chamber pressure and numerous mixture ratios.

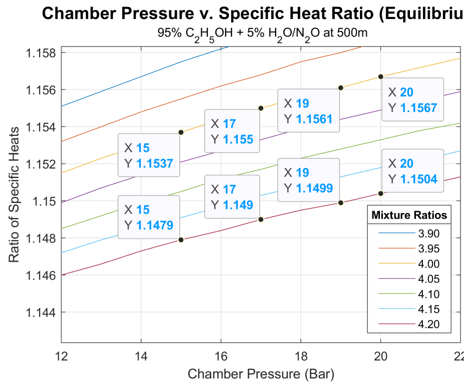

Figure 4: Ratio of specific heats vs chamber pressure and numerous mixture ratios.

2.1 Objectives

The Callisto 1 system is set to the following objectives:

1. Reach an altitude of 1500 m ($\approx$ 5000 ft).
2. A *GLOW* of no more than 30 kg ($\approx$ 70 lbsm).
3. A nominal main engine thrust of 1.5 kN ($\approx$ 350 lbsf).
4. A chamber pressure in the range of around 15 Bar to 20 Bar ($\approx$ 218 psi to 300 psi)
5. Consume a budget of no more than $10,000 USD.
6. Utililize 95% ethyl alcohol and nitrous oxide as the propellant combination.

2.2 Aphlex 1B

2.2.1 Chamber Pressure and Mixture Ratio Selection

The first step is to realize the theoretical maximum performance to be expected from our propellant combination. 95% ethyl alcohol (ethanol) was chosen for its availability, low pricing, and a modest specific impulse. A 95% dilution (by mass) with water was chosen as to lower the expected combustion chamber temperature. This tradeoff sacrifices some I_{sp} but reduces engineering complexity as regenerative and film cooling circuits may not be required. Industrial nitrous oxide was chosen as the main oxidizer for its self-pressurizing characteristics, non-cryogenic nature as opposed to liquid oxygen, relative ease to obtain, and a modest I_{sp} with ethanol. Using CEA, an open-source thermodynamics library provided by NASA's Glenn Research Center, critical data describing propellant combustion characteristics could be obtained. Numerical Python scripts were written to iterate through various combustion chamber pressures and mixture ratios and interface with CEA, and MATLAB scripts were used to parse and graph the CEA outputs as shown in Figures 1, 2, 3, and 4. Mixture ratios of 4.0 and 4.2 are labeled at certain chamber pressures. All dependent-variable properties (characteristic velocty C^*, combustion temperature T_c, exhaust molar mass M, and exhaust specific heat ratio γ) were permutated assuming shifting equilibrium flow and at an operational altitude of 500 m.

A mixture ratio of 4.0 was chosen for Aphlex 1B mainly in the interest of a conservative combustion temerature of around 3026 K and a middle-of-the-line theoretical C^* of around 1570 m/s, given a chamber pressure of 15 bar. This chamber pressure was chosen to minimize upstream fluid pressures and thus tank weight requirements, while also considering two additional design constraints that were not mentioned above but are nevertheless valid guiding parameters: a fluid flow velocity of less than 6 m/s to avoid water hammer effects [Zucrow Laboratories] and a pressure drop across the injector of around 25% of the designated chamber pressure [Michigan Aeronautical Science Association]. These guidelines further encourage a lower chamber pressure, and thus a conservative value of 15 Bar was chosen.

2.2.2 Nozzle Design

Design Parameters			
Name	Value	Unit	Uncertainty
Propellant (Fuel)	Ethanol (C_2H_5OH, 95%)	N/A	N/A
Propellant (Oxidizer)	Nitrous oxide (N_2O)	N/A	N/A
O/F, Oxidizer/fuel ratio	4.0	N/A	±1%
F_t, Nominal thrust	1.50	kN	±0.1%
P_c, Chamber static pressure	1.5×10^6	Pa	±0.1%
P_e, Ambient pressure	9.5540×10^4	Pa	±0.1%
T_c, Chamber static temperature	3025.98	K	±0.01%
M, Exhaust molecular mass	24.861	kg/mol	±0.01%
γ, Specific heat ratio	1.1537	N/A	±0.001%

Table 1: Summary of exhaust gas properties and fluid parameters.

$$T_3 = 15.04 - 0.00649h \tag{2.1}$$

$$P_3 = \left[101.29 \times \left(\frac{T + 273.1}{288.08}\right)^{5.256}\right] \times 1000 \tag{2.2}$$

The nozzle design process followed standard procedures outlined in Rocket Propulsion Elements [2] and open-source NASA documents. The thermodynamic properties of the exhaust gas and other important parameters are summarized and compiled in Table 1. The ambient pressure was calculated using NASA Glenn Research Center's Earth Atmosphere Model for an altitude within the troposphere (less than 11000 meters), as shown in Equations 2.1 and 2.2, where T_3 represents the ambient temperature in Kelvin, h is the altitude in meters, P_3 is the ambient pressure in Pascals.

Assuming fully isentropic flow (by definition both adiabatic and reversible) in the supersonic nozzle with choked flow conditions at the throat, an ideal converging-diverging (de Laval) nozzle can be characterized. The first parameter to calculate is the controlling area ratio

$$\frac{A_t}{A_2} = \left(\frac{\gamma+1}{2}\right)^{1/(\gamma-1)} \left(\frac{p_2}{p_1}\right)^{1/\gamma} \sqrt{\frac{\gamma+1}{\gamma-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}\right]} \tag{2.3}$$

where A_t is the cross-sectional area of the throat, A_2 is the cross-sectional area at nozzle exit, and p_1 and p_2 are the chamber pressure and exit pressure respectively. Equation 2.3 is also often referred to as the inverse of the expansion ratio ϵ, since $\epsilon = A_2/A_t$. Evaluating Equation 2.3 gives

$$\frac{A_t}{A_2} = \left(\frac{2.15}{2}\right)^{1/0.15} \left(\frac{9.6 \times 10^4}{1.5 \times 10^6}\right)^{1/1.15} \sqrt{\frac{2.15}{0.15}\left[1 - \left(\frac{9.6 \times 10^4}{1.5 \times 10^6}\right)^{0.15/1.15}\right]} = 0.307$$

Notice the substitution of p_2 for P_e as specified in Table 1, since in an ideal nozzle, the exhaust gas should expand to ambient pressure. The expansion ratio ϵ is simply

$$\epsilon = A_2/A_t = 1/AR = 1/0.307 = 3.26$$

Next, we can find the ideal exit velocity v_2, sometimes denoted as c:

$$v_2 = \sqrt{\frac{2\gamma}{\gamma-1}\left(\frac{R_u T_1}{M}\right)\left[1-\left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}\right]} \tag{2.4}$$

where R_u is the universal gas constant of 8314.3 J/kg mol-K and M is the molecular mass of the gas as shown in Table 1. Evaluating gives

$$v_2 = \sqrt{\frac{2*1.15}{0.15}\left(\frac{8314.3*3026}{24.86}\right)\left[1-\left(\frac{9.6\times10^4}{1.5\times10^6}\right)^{0.15/1.15}\right]} = 2162.3\ m/s$$

Next, the mass flow rate $\dot{m}$ can be calculated explicitly noting that $v_2 = c$, since earlier it was stated that $p_2 = p_3$:

$$\dot{m} = F_t/c = 1500/2162.3 = 0.694\ kg/s \tag{2.5}$$

Solving using our chosen O/F ratio of 4.0 for each independent propellant $\dot{m}$ gives

$$\dot{m}_f = \dot{m}*(4/5) = 0.694*(4/5) = 0.555\ kg/s$$
$$\dot{m}_o = \dot{m}*(1/5) = 0.694*(1/5) = 0.139\ kg/s$$

Next, we arrive at the throat area:

$$A_t = \frac{\dot{m}}{p_1}\sqrt{\frac{(R_u/M)T_1}{\gamma[2/(\gamma+1)]^{(\gamma+1)/(\gamma-1)}}} \tag{2.6}$$

Evaluating Equation 2.6 gives

$$A_t = \frac{0.694}{1.5\times10^6}\sqrt{\frac{(8314.3/24.86)*3026}{1.15[2/(2.15)]^{(2.15)/(0.15)}}} = 7.29\times10^{-4}\ m^2 = 7.29\ cm^2$$

Using this calculated throat area and the expansion ratio, the exit area is simply

$$A_2 = \epsilon * A_t = 3.26*7.29\times10^{-4} = 2.38\times10^{-3}\ m^2 = 23.8\ cm^2 \tag{2.7}$$

From the parameters calculated thus far, we can calculate some useful performance metrics such as I_{sp} and thrust coefficient C_F:

$$(I_{sp})_{opt} = F_t/(\dot{m}*g_0) = c/g_0 = 2162.3/9.81 = 220.42\ sec \tag{2.8}$$

where g_0 is the acceleration due to gravity at Earth's surface. C_F is

$$(C_F)_{opt} = \frac{F_t}{p_1 A_t} = \frac{1500}{1.5\times10^6*7.29\times10^4} = 1.372 \tag{2.9}$$

Note that under a more rigorous derivation, C_F can be seen to be a key parameter for analysis and varies depending on γ, the nozzle expansion ratio ϵ, and the pressure ratio p_1/p_2. Normally, C_F is experimentally determined by measuring chamber pressure, throat diameter, and thrust. The optimal C_F and therefore F_t occur when $p_2 = p_3$.

Finally, the physical dimensions of the nozzle can be determined using simple trigonometry and using a convergence half-angle α of 45° and a divergence half-angle β of 15°. First, the contraction ratio A_1/A_t, the ratio of chamber area to throat area, was set to a value of 8.0 for an acceptable engine form factor and since any value above 4.0 is satisfactory [2]. The characteristic chamber length L^*, a parameter used for characterizing the necessary chamber volume for adequate mixing and combustion of the propellants, must be more carefully considered. Ideally, L^* is purely a function of the chemistry of the propellant combination and is often based upon previous successful engine designs [2]. However,

Calculated Parameters Dimensions		
Name	Value	Unit
α, Convergence half-angle	45	deg
β, Divergence half-angle	15	deg
ϵ, Expansion ratio	3.26	N/A
$\dot{m}$, Total mass flow rate	0.694	kg/s
$\dot{m}_f$, Fuel mass flow rate	0.555	kg/s
$\dot{m}_o$, Oxidizer mass flow rate	0.139	kg/s
A_t, Throat area	7.29	cm^2
A_2, Exit area	23.8	cm^2
CR, Contraction ratio	8	N/A

Table 2: Summary of physical nozzle dimensions.

due to some nontrivial behaviors of nitrous oxide, such as exothermic decomposition after vaporization in the injector and a high density sensitivity to temperature, a more sophisticated model is needed to calculate L^*. Palacz proposes an explicit equation for finding the ideal L^* for a nitrous oxide system, and empirically determined an ideal range for an NO_x/ethanol system of L^* values from 125.6 cm to 167.8 cm [1]. A low-range L_* value of 1.25 m was chosen as a smaller form factor is desired over perfect combustion efficiency.

Table [something]

[Add section on finding I_{sp} and F_t at launch and MECO, along with a table of physical nozzle design parameters and their calculated dimensions.]

2.3 Callisto 1

2.4 Plumbing System

The following section describes the theoretical process used to dimensionalize the prelimiary plumbing framework in preparation for the first static cold flow test. By definition, these calculations are purely speculatory and are only used for the initial design process. The purpose of the cold flow test is to verify these parameters, and consequently adjust these parameters to better fit the system requirements.

2.4.1 Assumptions

To simplify the rigorous analysis and optimization processes often associated with viscous pipe flow, a couple of assumptions are applied in the following section to both shorten the development timeline and to avoid unecessarily complex or expensive methods outside of the scope of a high school amateur rocketry program. They are as follows:

1. Flow is driven by both pressure and gravity.
2. Pipe is circular and is of constant cross-sectional area.
3. No swirl, circumferential variation, or entrance effects.
4. No shaft-work or heat-transfer effects.
5. Flow is fully developed (minimal boundary-layer effects).

References

[1] Tomasz Palacz. "Nitrous Oxide Application for Low-Thrust and Low-Cost Liquid Rocket Engine". In: *7th European Conference for Aeronautics and Space Sciences* (July 2017).

[2] George P. Sutton and Oscar Biblarz. *Rocket Propulsion Elements, 9th Edition.* Wiley, Dec. 2016.

Design Parameters			
Name	Value	Unit	Uncertainty
μ, Dynamic viscosity	0.00196	$kg/m \cdot s$	$\pm 0.1\%$
ρ, Density	786	kg/m^3	$\pm 1\%$
L, Pipe length	8.0	m	$\pm 0.1\%$
d, Pipe diameter	12.7	mm	$\pm 0.1\%$
$\dot{m}$, Mass flow rate	0.9133	kg/s	$\pm 0.1\%$
Pipe material	Hard nylon	N/A	N/A
ϵ, Roughness	1.5 to 40.0	μm	$\pm 50\%$

Table 3: Summary of exhaust gas properties and fluid parameters.