

Development of a Cost-Effective 1.5kN Liquid-Fueled Rocket Propulsion System

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1 Nomenclature

Symbols			Acronyms	
ϵ	=	Expansion ratio	<i>CEA</i>	= Chemical equilibrium with applications
γ	=	Ratio of specific heats		
ρ	=	Density	g/cm^3	<i>COTS</i> = Commercial off-the-shelf
C^*	=	Characteristic velocity	m/s	<i>DAQ</i> = Data acquisition & control
C_d	=	Discharge coefficient		<i>FOD</i> = Foreign object debris
C_v	=	Valve flow coefficient		<i>GLOW</i> = Gross lift-off weight
f_d	=	Friction factor		<i>MECO</i> = Main engine cut-off
F_t	=	Thrust	N	<i>P&ID</i> = Plumbing and instrumentation diagram
g_0	=	Acceleration due to gravity	m/s^2	<i>PT</i> = Pressure transducer
I_{sp}	=	Specific impulse	s	<i>TC</i> = Thermocouple
\dot{m}	=	Mass flow rate	kg/s	<i>SF</i> = Safety factor
O/F	=	Oxidizer-to-fuel ratio		<i>VDC</i> = Direct current voltage
Q	=	Volumetric flow rate	L/s	

Note: Subscripts follow the convention outlined in [3]. Unless otherwise specified, subscript 0 indicates at stagnation or impact conditions, 1 indicates conditions at the nozzle inlet or combustion chamber, t indicates the nozzle throat, 2 is at the nozzle exit, and 3 is at ambient conditions.

2 Initial System Characterization

The following section outlines the characterization and design process for Aphlex 1B, our second-generation bi-propellant liquid rocket engine, set to fly on our 30 kg launch vehicle, Callisto 1. Project Caelus's unique position as a 501(c)(3) non-profit organization consisting entirely of high school students has laid the foundation for a design approach fully committed to cost-effectiveness, simplicity, and reliability, given certain mission constraints and objectives as discussed below.

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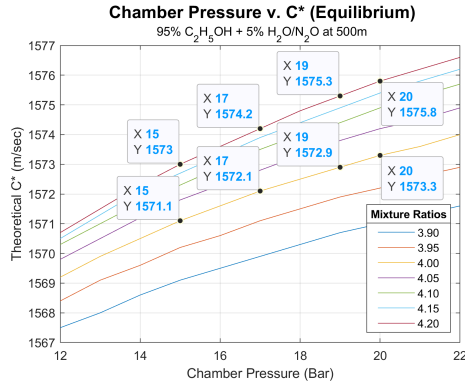


Figure 1: Theoretical C* efficiency vs chamber pressure and numerous mixture ratios.

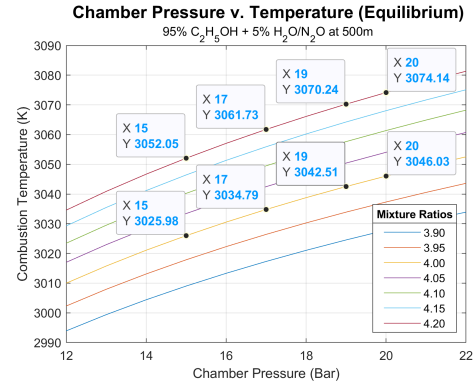


Figure 2: Combustion temperature vs chamber pressure and numerous mixture ratios.

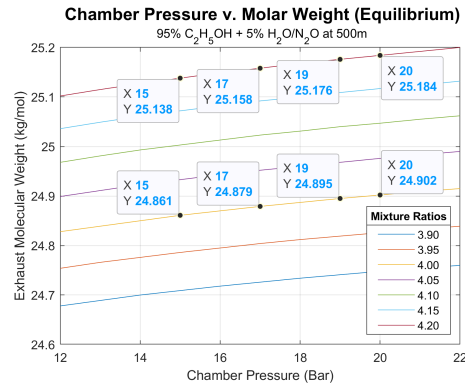


Figure 3: Gas molecular mass vs chamber pressure and numerous mixture ratios.

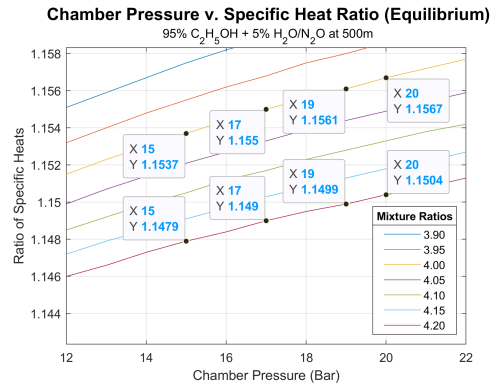


Figure 4: Ratio of specific heats vs chamber pressure and numerous mixture ratios.

2.1 Objectives

The Callisto 1 system is set to the following objectives:

1. Reach an altitude of 1500 m (\approx 5000 ft).
2. A *GLOW* of no more than 30 kg (\approx 70 lbsm).
3. A nominal main engine thrust of 1.5 kN (\approx 350 lbf).
4. A chamber pressure in the range of around 15 Bar to 20 Bar (\approx 218 psi to 300 psi)
5. Consume a budget of no more than \$10,000 USD.
6. Utilize 95% ethyl alcohol and nitrous oxide as the propellant combination.

2.2 Aphlex 1B

2.2.1 Chamber Pressure and Mixture Ratio Selection

The first step is to realize the theoretical maximum performance to be expected from our propellant combination. 95% ethyl alcohol (ethanol) was chosen for its availability, low pricing, and a modest specific impulse. A 95% dilution (by mass) with water was chosen as to lower the expected combustion chamber temperature. This tradeoff sacrifices some I_{sp} but reduces engineering complexity as regenerative and film cooling circuits may not be required. Industrial nitrous oxide was chosen as the main oxidizer for its self-pressurizing characteristics, non-cryogenic nature as opposed to liquid oxygen, relative ease to obtain, and a modest I_{sp} with ethanol. Using CEA, an open-source thermodynamics library provided by NASA's Glenn Research Center, critical data describing propellant combustion characteristics could be obtained. Numerical Python scripts were written to iterate through various combustion chamber

pressures and mixture ratios and interface with CEA, and MATLAB scripts were used to parse and graph the CEA outputs as shown in Figures 1, 2, 3, and 4. Mixture ratios of 4.0 and 4.2 are labeled at certain chamber pressures. All dependent-variable properties (characteristic velocity C^* , combustion temperature T_c , exhaust molar mass M , and exhaust specific heat ratio γ) were permuted assuming shifting equilibrium flow and at an operational altitude of 500 m.

A mixture ratio of 4.0 was chosen for Aphlex 1B mainly in the interest of a conservative combustion temperature of around 3026 K and a middle-of-the-line theoretical C^* of around 1570 m/s, given a chamber pressure of 15 bar. This chamber pressure was chosen to minimize upstream fluid pressures and thus tank weight requirements, while also considering two additional design constraints that were not mentioned above but are nevertheless valid guiding parameters: a fluid flow velocity of less than 6 m/s to avoid water hammer effects [Zucrow Laboratories] and a pressure drop across the injector of around 25% of the designated chamber pressure [Michigan Aeronautical Science Association]. The former of these additional guidelines is to avoid feed-coupled combustion instabilities that may be observed when the δp across the injector is not adequately high. These guidelines further encourage a lower chamber pressure, and thus a conservative value of 15 Bar was chosen.

2.2.2 Nozzle Design

Design Parameters			
Name	Value	Unit	Uncertainty
Propellant (Fuel)	Ethanol (C_2H_5OH , 95%)	N/A	N/A
Propellant (Oxidizer)	Nitrous oxide (N_2O)	N/A	N/A
O/F , Oxidizer/fuel ratio	4.0	N/A	$\pm 1\%$
F_t , Nominal thrust	1.50	kN	$\pm 0.1\%$
P_c , Chamber static pressure	1.5×10^6	Pa	$\pm 0.1\%$
P_e , Ambient pressure	9.5540×10^4	Pa	$\pm 0.1\%$
T_c , Chamber static temperature	3025.98	K	$\pm 0.01\%$
M , Exhaust molecular mass	24.861	kg/mol	$\pm 0.01\%$
γ , Specific heat ratio	1.1537	N/A	$\pm 0.001\%$

Table 1: Summary of exhaust gas properties and fluid parameters.

$$T_3 = 15.04 - 0.00649h \quad (2.1)$$

$$P_3 = \left[101.29 \times \left(\frac{T + 273.1}{288.08} \right)^{5.256} \right] \times 1000 \quad (2.2)$$

The nozzle design process followed standard procedures outlined in Rocket Propulsion Elements [3] and open-source NASA documents. The thermodynamic properties of the exhaust gas and other important parameters are summarized and compiled in Table 1. The ambient pressure was calculated using NASA Glenn Research Center's Earth Atmosphere Model for an altitude within the troposphere (less than 11000 meters), as shown in Equations 2.1 and 2.2, where T_3 represents the ambient temperature in Kelvin, h is the altitude in meters, P_3 is the ambient pressure in Pascals.

Assuming fully isentropic flow (by definition both adiabatic and reversible) in the supersonic nozzle with choked flow conditions at the throat, an ideal converging-diverging (de Laval) nozzle can be characterized. The first parameter to calculate is the controlling area ratio

$$\frac{A_t}{A_2} = \left(\frac{\gamma + 1}{2} \right)^{1/(\gamma-1)} \left(\frac{p_2}{p_1} \right)^{1/\gamma} \sqrt{\frac{\gamma + 1}{\gamma - 1} \left[1 - \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} \right]} \quad (2.3)$$

where A_t is the cross-sectional area of the throat, A_2 is the cross-sectional area at nozzle exit, and p_1 and p_2 are the chamber pressure and exit pressure respectively. Equation 2.3 is also often referred to as the inverse of the expansion ratio ϵ , since $\epsilon = A_2/A_t$. Evaluating Equation 2.3 gives

$$\frac{A_t}{A_2} = \left(\frac{2.15}{2} \right)^{1/0.15} \left(\frac{9.6 \times 10^4}{1.5 \times 10^6} \right)^{1/1.15} \sqrt{\frac{2.15}{0.15} \left[1 - \left(\frac{9.6 \times 10^4}{1.5 \times 10^6} \right)^{0.15/1.15} \right]} = 0.307$$

Notice the substitution of p_2 for P_e as specified in Table 1, since in an ideal nozzle, the exhaust gas should expand to ambient pressure. The expansion ratio ϵ is simply

$$\epsilon = A_2/A_t = 1/AR = 1/0.307 = 3.26$$

Next, we can find the ideal exit velocity v_2 , sometimes denoted as c :

$$v_2 = \sqrt{\frac{2\gamma}{\gamma-1} \left(\frac{R_u T_1}{M} \right) \left[1 - \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} \right]} \quad (2.4)$$

where R_u is the universal gas constant of 8314.3 J/kg mol-K and M is the molecular mass of the gas as shown in Table 1. Evaluating gives

$$v_2 = \sqrt{\frac{2 * 1.15}{0.15} \left(\frac{8314.3 * 3026}{24.86} \right) \left[1 - \left(\frac{9.6 \times 10^4}{1.5 \times 10^6} \right)^{0.15/1.15} \right]} = 2162.3 \text{ m/s}$$

Next, the mass flow rate \dot{m} can be calculated explicitly noting that $v_2 = c$, since earlier it was stated that $p_2 = p_3$:

$$\dot{m} = F_t/c = 1500/2162.3 = 0.694 \text{ kg/s} \quad (2.5)$$

Solving using our chosen O/F ratio of 4.0 for each independent propellant \dot{m} gives

$$\begin{aligned} \dot{m}_o &= \dot{m} * (4/5) = 0.694 * (4/5) = 0.555 \text{ kg/s} \\ \dot{m}_f &= \dot{m} * (1/5) = 0.694 * (1/5) = 0.139 \text{ kg/s} \end{aligned}$$

Next, we arrive at the throat area:

$$A_t = \frac{\dot{m}}{p_1} \sqrt{\frac{(R_u/M)T_1}{\gamma[2/(\gamma+1)]^{(\gamma+1)/(\gamma-1)}}} \quad (2.6)$$

Evaluating Equation 2.6 gives

$$A_t = \frac{0.694}{1.5 \times 10^6} \sqrt{\frac{(8314.3/24.86) * 3026}{1.15[2/(2.15)]^{(2.15)/(0.15)}}} = 7.29 \times 10^{-4} \text{ m}^2 = 7.29 \text{ cm}^2$$

Using this calculated throat area and the expansion ratio, the exit area is simply

$$A_2 = \epsilon * A_t = 3.26 * 7.29 \times 10^{-4} = 2.38 \times 10^{-3} \text{ m}^2 = 23.8 \text{ cm}^2 \quad (2.7)$$

From the parameters calculated thus far, we can calculate some useful performance metrics such as I_{sp} and thrust coefficient C_F :

$$(I_{sp})_{opt} = F_t/(\dot{m} * g_0) = c/g_0 = 2162.3/9.81 = 220.42 \text{ sec} \quad (2.8)$$

where g_0 is the acceleration due to gravity at Earth's surface. C_F is

$$(C_F)_{opt} = \frac{F_t}{p_1 A_t} = \frac{1500}{1.5 \times 10^6 * 7.29 \times 10^{-4}} = 1.372 \quad (2.9)$$

Note that under a more rigorous derivation, C_F can be seen to be a key parameter for analysis and varies depending on γ , the nozzle expansion ratio ϵ , and the pressure ratio p_1/p_2 . Normally, C_F is experimentally determined by measuring chamber pressure, throat diameter, and thrust. The optimal C_F and therefore F_t occur when $p_2 = p_3$.

Finally, the physical dimensions of the nozzle can be determined using simple trigonometry via a standard convergence half-angle α of 45° and a standard divergence half-angle β of 15° . The characteristic chamber length L^* , a parameter used for characterizing the necessary chamber volume for adequate mixing and combustion of the propellants, must be more carefully considered. Ideally, L^* is purely a function of the chemistry of the propellant combination and is often based upon previous successful engine designs [3]. However, due to the dynamical and complex nature of nitrous oxide, such as exothermic decomposition after vaporization in the injector and a high density sensitivity to temperature, a more

sophisticated model is needed to calculate L^* . Palacz proposes an explicit equation for finding the ideal L^* for a nitrous oxide system, and empirically determined an ideal range for an NO_x /ethanol system of L^* values from 125.6 cm to 167.8 cm [1]. This range is confirmed by Sutton and Biblarz, as L^* values of between 1.0 m and 1.5 m are expected with ethanol systems. A low-range L^* value of 1.25 m was chosen as a smaller form factor is desired over perfect combustion efficiency. L^* is mathematically defined as simply

$$L^* = \frac{V_c}{A_t} = \frac{\pi r_c^2 L_c}{A_t} \implies L_c = \frac{A_t L^*}{\pi r_c^2} \quad (2.10)$$

where L_c is the length of the chamber and r_c is the radius of the chamber. It is apparent that the a desired form factor can be obtained by adjusting either the radius or the length of the chamber. Through iteration, a chamber radius r_c of 4.0 cm was chosen. The only constraint on the chamber radius is the contraction ratio, defined as A_1/A_t (the ratio of chamber area to throat area), which must achieve a value of 4.0 or more [3]. A trivial area calculation can be performed to confirm that this constraint is satisfied: $CR = \pi r_c^2/A_t = 5.03 \times 10^{-4} \text{ m}^2/7.29 \times 10^{-4} \text{ m}^2 = 6.90 > 4.0$. The chamber length is therefore

$$L_c = \frac{A_t L^*}{\pi r_c^2} = \frac{(7.29 \times 10^{-4} \text{ m}^2)(1.25 \text{ m})}{5.03 \times 10^{-4} \text{ m}^2} = 0.1813 \text{ m} = 18.13 \text{ cm}$$

Calculated Performance Parameters		
Name	Value	Unit
\dot{m} , Total mass flow rate	0.694	kg/s
\dot{m}_f , Fuel mass flow rate	0.555	kg/s
\dot{m}_o , Oxidizer mass flow rate	0.139	kg/s
v_2 , Exhaust velocity	2162.3	m/s
$(I_{sp})_{opt}$, Specific impusle (optimal)	220.42	s
$(C_F)_{opt}$, Thrust coefficient (optimal)	1.372	N/A

Table 2: Summary of engine performace parameters.

Calculated Dimensional Parameters		
Name	Value	Unit
α , Convergence half-angle	45	deg
β , Divergence half-angle	15	deg
A_t , Throat area	7.29×10^{-4}	m^2
A_2 , Exit area	2.38×10^{-3}	m^2
A_c , Chamber cross-sectional area	5.03×10^{-4}	m^2
R_t , Throat radius	15.23	mm
R_2 , Exit radius	27.52	mm
R_c , Chamber radius	40.0	mm
L_c , Chamber length	181.3	mm
ϵ , Expansion ratio	3.26	N/A
CR , Contraction ratio	6.9	N/A

Table 3: Summary of physical nozzle dimensions.

Table 2 shows the calculated engine parameters and Table 3 shows the calculated physical nozzle dimensions. Using the data from Table 3, a basic conical nozzle can be realized.

Figure 5 shows the basic conical nozzle for Aphlex 1B created using the given dimensions. Although a conical nozzle may exhibit an advantage in its ease of manufacturing, it is suboptimal for performance since 1) the exhaust is not exiting the nozzle completely parallel to the nozzle axis, which results in lateral losses of energy, 2) the sharp convex corner at the throat generates shocks which are not adequately dissipated by a conical nozzle, and 3) the small divergence half-angle ensures a long diverging section, which increases engine mass. To address these issues, most modern nozzles are so-called bell nozzles, whose diverging section includes a straightening section to address the first issue and inherently addresses the third issue by having a more rapidly expanding cross-section as compared to conical nozzles. To neutralize the Prandtl-Meyer expansion fans generated at the sharp corner of the throat, a numerical technique called the Method of Characteristics (MOC) is used to generate the nozzle contour.

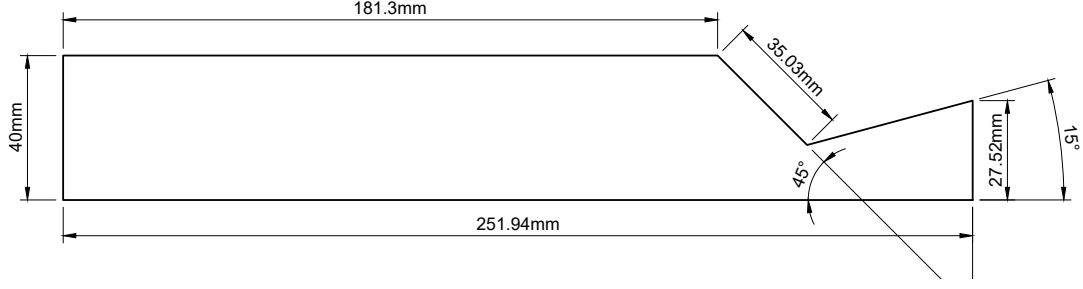


Figure 5: Aphlex 1B drawing with a conical nozzle and given dimensions.

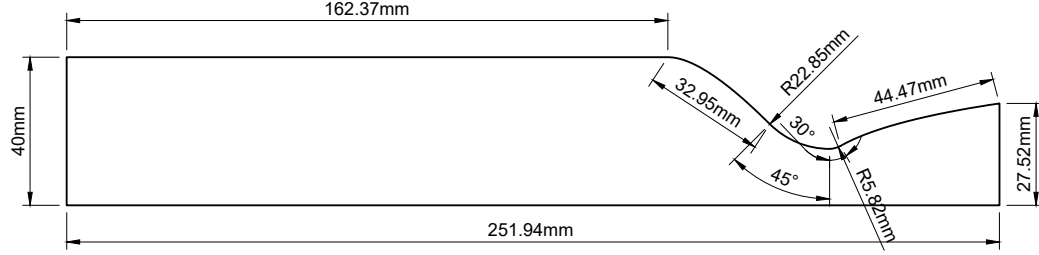


Figure 6: Aphlex 1B drawing with a MOC nozzle and given dimensions.

In brief, MOC propagates characteristic lines emanating from the throat, and by using equations to model characteristic interactions and reflections, it is able to generate a nozzle contour to precisely neutralize (reflect in a manner parallel to the nozzle axis) these expansion fans. A MATLAB script was written to execute and create the MOC nozzle, which can be found on our Project Caelus propulsion Github repository at <https://github.com/ProjectCaelus/propulsion>. The Cartesian points representing the generated wall contour were exported as a CSV file, which was then imported into our computer-aided design (CAD) program, Fusion 360, where a best-fit nth-order spline was used to finalize the diverging contour. Although the shape of the diverging section can be optimized significantly, the precise shape of the nozzle converging section has not been found to heavily impact engine performance and is therefore smoothened under the guidelines of Dr. GVR Rao's parabolic approximation method [3][2].

[Add section on finding I_{sp} and F_t at launch and MECO, along with a table of physical nozzle design parameters and their calculated dimensions.]

2.3 Callisto 1

2.4 Injector Design

The injector types considered were the triplet (fuel-centered) unlike impinging injector and the like-on-like doublet impinging injector, due to 1) our limited manufacturing capabilities 2) a fairly unbalanced O/F ratio 3) both types exhibiting good mixing and atomization properties and 4) the abundance of historical experience and data with both injector types. NASA's SP-8089 conference document on liquid engine injector design suggests that the best way to characterize both the individual orifice geometries and the overall injector geometry for unlike impinging injectors is through diameter ratios. Vigorous cold flow and other empirically testing methods at the time were used and have found correlations between the driving orifice diameter ratio with the optimum mixing efficiency. The correlation found was

$$\left(\frac{d_c}{d_{ou}}\right)^2 = M \left[\frac{\rho_{ou}}{\rho_c} \left(\frac{\dot{m}_c}{\dot{m}_{ou}} \right)^2 \right]^{0.7} \quad (2.11)$$

where d_c is the diameter of center orifice, d_{ou} is the diameter of an outside individual orifice, M is an experimentally-determined mixing factor coefficient, ρ represents liquid density, and \dot{c} and \dot{m}_{ou} are the center mass flow rate and outside mass flow rate respectively. It is cited that for a 2-on-1 element type,

M has a value of 1.6. Using Equation 2.11, we find that our controlling diameter ratio is

$$\frac{d_c}{d_{ou}} = \sqrt{M \left[\frac{\rho_{ou}}{\rho_c} \left(\frac{\dot{m}_c}{\dot{m}_{ou}} \right)^2 \right]^{0.7}} = \sqrt{1.6 \left[\frac{772.25 \text{ kg/m}^3}{789 \text{ kg/m}^3} \left(\frac{0.1389 \text{ kg/s}}{0.5556 \text{ kg/s}} \right)^2 \right]^{0.7}} = 0.4757$$

Since this diameter ratio is not reasonably near 1.22 (as suggested by NASA SP-8089), we can assume this correlation would not be accurate and that there will be potentially drastic losses in mixing efficiency. Thus, this suggests that a like-on-like system is required. Since like-on-like elements will have a 1 to 1 diameter and momentum ratio, we can begin determining the pressure drop and mass flow rate through each individual orifice. Rocket Propulsion Elements (RPE) provides an equation for the volumetric flow rate Q (and therefore \dot{m} since ρ is constant) as shown below:

$$\dot{m} = Q\rho = C_d A \sqrt{2\rho\Delta p} \implies \Delta p = \left[\left(\frac{\dot{m}}{C_d A} \right)^2 \right] / 2\rho \quad (2.12)$$

where C_d is a dimensionless discharge coefficient that is experimentally determined and a function of the orifice geometry, A is the area of the orifice, and Δp is the pressure drop across the orifice. Flow velocity is similar:

$$v = Q/A = C_d \sqrt{2\Delta p/\rho} \quad (2.13)$$

Since C_d is a measured parameter, initial design calculations must assume a value. RPE suggests a C_d value of around 0.88 for a 1 mm diameter orifice in a short tube with a rounded entrance ($L/D > 3.0$), and 0.9 for a similar configuration with a 1.57 mm diameter. Accordingly, a C_d value of 0.9 was chosen for the oxidizer and a C_d value of 0.88 was chosen for the fuel. For the orifice sizing, NASA SP-8089 found that smaller orifice sizes attributed to better mixing in all scenarios, although only to a certain extent (orifice diameters < 0.03 inches saw insignificant improvements in mixing). Due to limited manufacturing capabilities, a minimum hole size of 1 mm was chosen. Using this information, a design parameter of an overall injector pressure drop that is 25% of chamber pressure, and Equation 2.12, the mass flow rate across a single oxidizer and fuel orifice are

$$\dot{m}_o = 0.9 * (\pi * ((1.58 \text{ mm}/2) \times 10^{-3})^2) \sqrt{2(772.25 \text{ kg/m}^3)(1.5 \times 10^6 \text{ Pa} * 0.25)} = 0.0425 \text{ kg/s}$$

$$\dot{m}_f = 0.88 * (\pi * ((1.00 \text{ mm}/2) \times 10^{-3})^2) \sqrt{2(789 \text{ kg/m}^3)(1.5 \times 10^6 \text{ Pa} * 0.25)} = 0.0168 \text{ kg/s}$$

The same pressure drops can be used for each orifice due to Bernoulli's principle and the law of conservation of energy, similar to how voltage stays constant across a parallel circuit. The corresponding injection velocities (Equation 2.13) are

$$v_o = 0.9 \sqrt{(2(1.5 \times 10^6 \text{ Pa})(0.25)) / (772.25 \text{ kg/m}^3)} = 28.048 \text{ m/s}$$

$$v_f = 0.88 \sqrt{(2(1.5 \times 10^6 \text{ Pa})(0.25)) / (789 \text{ kg/m}^3)} = 27.132 \text{ m/s}$$

Dividing the mass flow rate by the individual orifice mass flow rates gives the total orifice count for each propellant:

$$\text{Number of oxidizer orifices} = \dot{m}_o / \dot{m}_{oi} = (0.4865 \text{ kg/s}) / (0.0425 \text{ kg/s}) = 11.447$$

$$\text{Number of fuel orifices} = \dot{m}_f / \dot{m}_{fi} = (0.139 \text{ kg/s}) / (0.0168 \text{ kg/s}) = 8.27$$

Since an integer and even amount of holes are required for a like-on-like impinging injector, Equation 2.12 is rearranged to compute the necessary diameter given the desired number of orifices, given a reasonable range of orifice count provided by the previous calculation:

Thus, to achieve [x] oxidizer orifices and [y] fuel orifices

an oxidizer orifice diameter of [x] mm and fuel orifice diameter of [y] mm are needed. Remaining impingement injector parameters were chosen due to a literature review, rather than explicit calculations. The angle of impingement, λ , is chosen to be 60 degrees (relative to the injector axis) and a point of impingement of around 3 cm downstream of the injector face is chosen for maximum ethanol/nitrous oxide combustion efficiency. The final injector parameters are summarized in the table below. [Still need to figure out angle of impingement, distance from injector face, though this is expected to be 60 degrees and around 3 cm respectively. Need to note that L/D is greater than 3 (or more), and that higher is better.]

2.5 Plumbing System

The following section describes the theoretical process used to dimensionalize the preliminary plumbing framework in preparation for the first static cold flow test. By definition, these calculations are purely speculative and are only used for the initial design process. The purpose of the cold flow test is to verify these parameters, and consequently adjust these parameters to better fit the system requirements.

2.5.1 Assumptions

To simplify the rigorous analysis and optimization processes often associated with viscous pipe flow, a couple of assumptions are applied in the following section to both shorten the development timeline and to avoid unnecessarily complex or expensive methods outside of the scope of a high school amateur rocketry program. They are as follows:

1. Flow is driven by both pressure and gravity.
2. Pipe is circular and is of constant cross-sectional area.
3. No swirl, circumferential variation, or entrance effects.
4. No shaft-work or heat-transfer effects.
5. Flow is fully developed (minimal boundary-layer effects).

Design Parameters			
Name	Value	Unit	Uncertainty
μ , Dynamic viscosity	0.00196	$kg/m \cdot s$	$\pm 0.1\%$
ρ , Density	786	kg/m^3	$\pm 1\%$
L , Pipe length	8.0	m	$\pm 0.1\%$
d , Pipe diameter	12.7	mm	$\pm 0.1\%$
\dot{m} , Mass flow rate	0.9133	kg/s	$\pm 0.1\%$
Pipe material	Hard nylon	N/A	N/A
ϵ , Roughness	1.5 to 40.0	μm	$\pm 50\%$

Table 4: Summary of exhaust gas properties and fluid parameters.

References

- [1] Tomasz Palacz. “Nitrous Oxide Application for Low-Thrust and Low-Cost Liquid Rocket Engine”. In: *7th European Conference for Aeronautics and Space Sciences* (July 2017).
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- [3] George P. Sutton and Oscar Biblarz. *Rocket Propulsion Elements, 9th Edition*. Wiley, Dec. 2016.