hw_1

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1 AEROSP 536 Electric Propulsion: Homework 1

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1.1 Problem 3

1.1.1 Part (a)

We need to check for inconsistencies in the data provided. First, we can check the efficiency equation:

$$\eta = \frac{1}{2}\left(\frac{T^2}{\dot{m}}\right)\frac{1}{P_{in}}$$

against the matched exit condition thrust equation:

$$F_t = \dot{m}u_e = \dot{m}(I_{sp}g_0)$$

And subract the two to compute a residual. If the residual is significant, it means the table entries given are inconsistent.

```python
[7]: import numpy as np

data = {
    'P_in'     : np.array([4.46, 1.26, 1.25, 3.26, 3.24]) * 1e3,
    'F_t'      : np.array([357.8, 248.5, 264.0, 92.7, 95.0]) * 1e-3,
    'F_t_range': np.array([ 4.6,   3.3,   2.7,   2.7,   3.0]) * 1e-3,
    'eta'      : np.array([0.47, 0.39, 0.42, 0.44, 0.49]),
    'eta_range': np.array([0.01, 0.02, 0.03, 0.01, 0.02]),
    'isp'      : np.array([1196, 1086, 1132, 1163, 1235]),
    'isp_range': np.array([ 15,   32,   33,   16,   16])
}
num_entries = 5
for i in range(num_entries):
    mdot_1 = data['F_t'][i] / (data['isp'][i] * 9.81)
    mdot_2 = (1/2) * (data['F_t'][i]**2) / (data['eta'][i] * data['P_in'][i])
    print(f"Residual mass flow rate for Row {i+1}: {mdot_1 - mdot_2:.2e} kg/s")
```

```
Residual mass flow rate for 0: -4.060e-08 kg/s
Residual mass flow rate for 1: -3.951e-05 kg/s
Residual mass flow rate for 2: -4.260e-05 kg/s
```

```
Residual mass flow rate for 3: 5.130e-06 kg/s
Residual mass flow rate for 4: 4.999e-06 kg/s
```

We can see that all of these magnitudes are extremely small, so it is not the source of the inconsistency.

1.1.2 Part (b)

We're given a chart of F_t v.s. I_{sp} and lines of constant input power P_{in} and mass flow rate of Xenon $\dot{m}$. Since the company evaluated their performance at 100 W and 5.2 mN, we can read the chart:
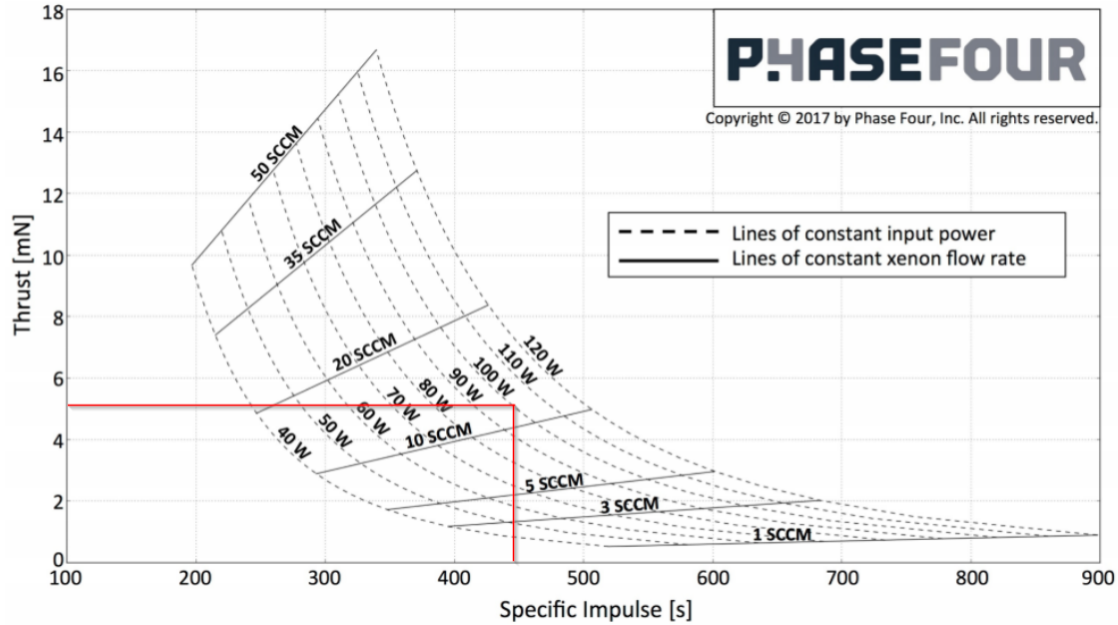
We see that the intersection approximately yields $\dot{V} = 12$ SCCM and $I_{sp} = 440$ s. Converting the volumetric flow rate to a mass flow rate requires knowledge that a standard cubic centimeter per minute is actually a molar flow rate, but expressed as a volumetric flow rate at standard conditions. Assuming the U.S. standard conditions of 293 K and 101.3 kPa, we get the density of Xenon at these conditions (via NIST REFPROP) as $\rho_{STP} = 5.49$ kg/m^3, and so $\dot{m}_{Xe} = \dot{V}_{SCCM}/\rho_{STP}$

Alternatively, if we don't want to guess the SCCM trace, we can also use Equation (2) to find $\dot{m}$ from the given thrust and I_{sp} values, for which we get $\dot{m} = 1.2 \times 10^{-6}$ kg/s. We can now plug into Equation (1) directly to get:

$$\eta = \frac{1}{2}\left(\frac{T^2}{\dot{m}}\right)\frac{1}{P_{in}} = 11.2\%$$

1.2 Problem 4

The scenario given is the extreme case where the propulsion system is a laser, where the reaction mass is photons.

1.2.1 Part (a)

Thrust to power ratio given 100% efficiency is:

$$\frac{T}{P_{in}} = \frac{2\eta}{u_e} = \frac{2}{c} \approx 6.67 \times 10^{-9}$$

1.2.2 Part (b)

Given that state-of-the-art EP systems can generate about 350 mN at 5 kW, how much power would it take for the laser propulsion system to generate the same thrust?

The inverse of what we found above can tell us (assuming a linear relationship between input power and thrust generation):

$$(350 \text{ mN}) \left(\frac{T}{P_{in}} \right)^{-1} = 52.5 \text{ MW}$$

1.2.3 Part (c)

I would not use laser propulsion. As seen above, the power requirement to operate laser propulsion at EP thrust levels (which is already very low, which contributes to a trade of transit time) is very high, which is over ~2 orders of magnitude more than the power produced by the solar arrays on the ISS. The thrust and power trade is simply too impractical, unless we are constructing an interstellar probe with a nuclear reactor.

1.3 Problem 5

To solve this problem, we just have to discretize the velocity vectors from the center to the radius $R = 0.5$ (let's say, create 1000 points), take the component that is useful thrust and therefore axial (by multiplying by the cosine of the divergence half angle $\theta = 25$), and take an average of those magnitudes to find the effective exhaust velocity. This is because the $\bar{u}$ at the exit plane of the nozzle is equal to u_{eff}. This is shown in the Python code below.

```
[11]: R = 0.5
      pts = np.linspace(0, 1, 1000) * R
      u_eff = np.cos(np.deg2rad(25)) * (3150) * np.exp((-pts**2)/(R**2))
      print(f"Effective exhaust velocity: {np.mean(u_eff):.2f} m/s")
```

Effective exhaust velocity: 2131.91 m/s

1.4 Problem 6

The Psyche mission makes use of four Maxar SPT-140 Hall thrusters that when operating at 4 kW have a thrust of 235 mN and a specific impulse of 1770 s each. The total impulse requirement for this mission is 13.6 MN-s.

1.4.1 Part (a)

To find the required operating time and propellant mass given the total impulse requirement, we use the definition of specific impulse as the total impulse over the weight of propellant:

$$I_{sp} = \frac{I_t}{m_p g_0}$$

so $m_p = 783.2$ kg. Then, we can use mass flow rate and the total propellant mass to find the needed burn time:

$$t_b = \frac{m_p}{\dot{m}} = \frac{m_p}{F/(I_{sp} g_0)} = 16{,}076 \text{ hours}$$

1.4.2 Part (b)

We cannot assume here that the efficiency stays constant, but we can solve for total impulse through another avenue. We know that the mass of the propellant required stays constant from Part (a) assuming I_{sp} is constant (which is a better assumption than power efficiency), meaning still, $m_p = 783.2$ kg. Now, we can solve for the mass flow rate that must be supplied given the propellant mass and operating time:

$$\dot{m} = \frac{m_p}{t_b} = \frac{783.2 \text{ kg}}{10362 \text{ hours}} = 2.1 \times 10^{-5} \text{ kg/s}$$

Now we can find the thrust that this corresponds with via $F = \dot{m}(I_{sp} g_0) = 365$ mN and multiply by the known burn time to get the total impulse:

$$I_t = F t_b = 13.599 \text{ MN-s}$$

This result is extremely close to the requirement of 13.6 MN-s and can be dismissed as rounding error, and the final answer is **yes, the requirement is satisfied.**

1.4.3 Part (c)

1.4.4 Part (d)

```python
import pandas as pd

csv_path = 'Psyche_Thrust_Variation.csv'
df = pd.read_csv(csv_path)
thrust3 = df['Simulation 3'].values * 1e-3   # mN to N
thrust4 = df['Simulation 4'].values * 1e-3   # mN to N
time_s = df['Time (Hours)'].values * 3600    # hours to seconds

# Trapezoidal integration for total impulse (N-s) using np.trapezoid
impulse3 = np.trapezoid(thrust3, time_s)
impulse4 = np.trapezoid(thrust4, time_s)

print(f"Total impulse for Simulation 3: {impulse3:.2f} N-s")
print(f"Total impulse for Simulation 4: {impulse4:.2f} N-s")
print(f"Difference: {abs(impulse3 - impulse4):.2f} N-s")
```

```
Total impulse for Simulation 3: 2482704.00 N-s
Total impulse for Simulation 4: 2435103.00 N-s
Difference: 47601.00 N-s
```